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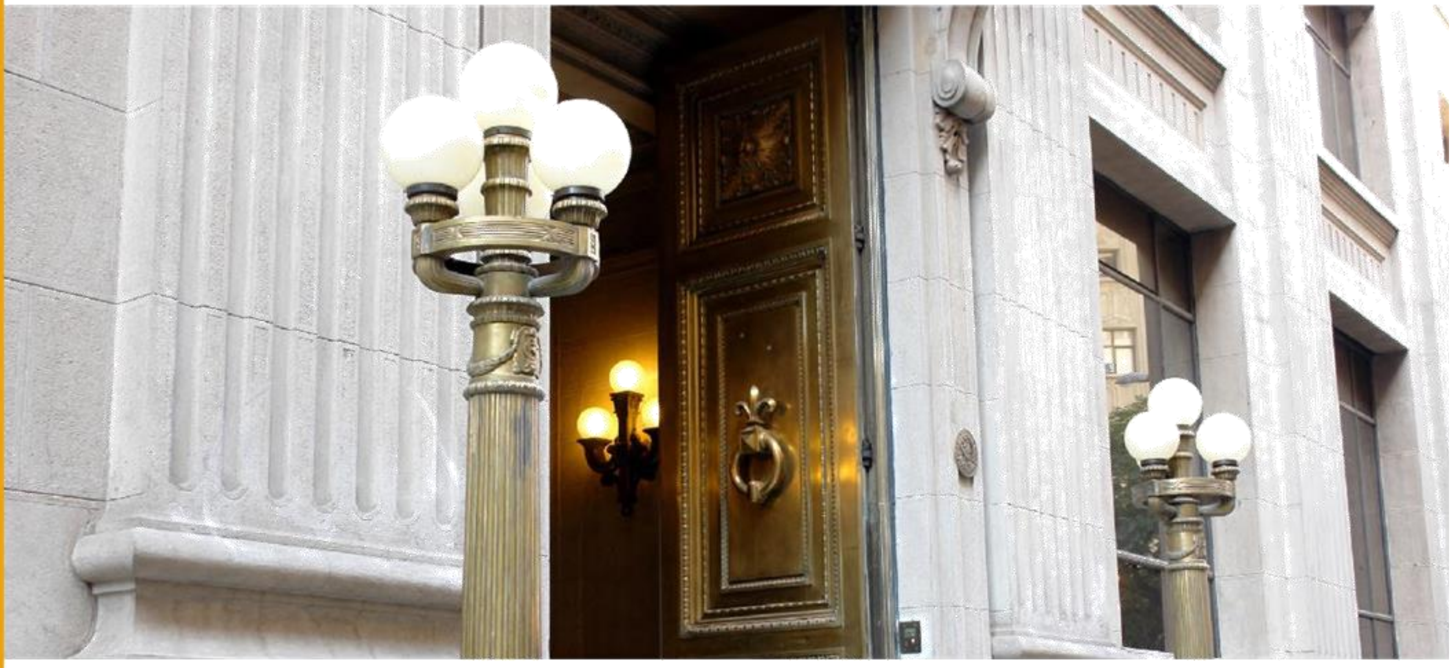

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The Incidence of Distortions*

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Resumen

Las distorsiones económicas—como el poder de mercado, los impuestos, las restricciones crediticias, etc.—son fundamentales para entender la diferencia en ingresos entre economías desarrolladas y en desarrollo. Trabajo reciente ha documentado la relevancia de estas distorsiones y como conllevan a pérdidas de productividad agregada. Se sabe mucho menos si estas distorsiones afectan a miembros de la sociedad de manera distinta. En este artículo se usan datos únicos de Chile, vinculando a individuos con empresas, a las empresas entre sí, empresas con consumidores y empresas y consumidores con el gobierno, para medir la incidencia de las distorsiones económicas por primera vez.

Abstract

Economic distortions—such as market power, taxes, credit constraints, etc.— are fundamental in understanding income differences across countries. Recent work has documented the pervasive extent of economic distortions and how they lead to sub-stantial aggregate productivity loss. Far less well understood is how these phenomena affect members of society differently. In this paper we combine unique datasets from Chile, linking workers and owners to firms, firms to each other, firms to consumers, and firms and consumers to the government, in order to quantify the incidence of distortions for the first time.

*This version: July 2025. Author contacts: atkin@mit.edu, bbernadac@ucla.edu, ddonald@mit.edu, tishara@mit.edu, and federico.huneeus@duke.edu. We are grateful to Joaquin Galeno, Pedro Pessoa, Melody Ying, Vincent Wang, Qirui Wang, and Jeremy Zhou for outstanding research assistance, to STEG for funding, and to numerous audience members for comments that have improved this work. The findings, interpretations, and conclusions are those of the authors and do not necessarily represent those of the Central Bank of Chile nor its board members.

1 Introduction

Economic distortions are pervasive and fundamental to understanding why countries sit at different levels of development. These distortions arise, for example, when firms enjoy market power, face borrowing constraints, or pay taxes or tariffs on outputs and inputs. Market failures such as these reduce the economy's overall efficiency, and hence its aggregate amount of production. A large literature has produced a deep understanding of the consequences that these distortions have for aggregate productivity (Restuccia and Rogerson (2008), Hsieh and Klenow (2009)). What is far less well understood, however, is the extent to which the burden of these distortions is shared equally across households—the question of "Who bears the incidence of distortions?" remains open. If the poor bear a greater incidence then distortions have clear equity costs as well as efficiency ones.

In this paper we build and analyze a new dataset from Chile that is designed to illuminate the incidence of economic distortions across households for the first time. In particular, we merge data on consumer-to-firm transactions and firm-to-firm transactions—both at the product level—with firm-to-employee wage payments, firm-to-individual ownership registries, and individual- and firm-level tax and transfer payments. The result is a micro-level system of national income and product accounts depicting the flow of goods and services, from each household to each firm in the supply of factors, from each firm to every other firm, and finally from each firm back to each household in the form of final consumption.

Central to this exercise is the fact that large stores and chains in Chile transmit to the tax authorities not only the quantities and prices of each product sold but also the tax ID of the consumer they are sold to. This granularity allows us to link individuals to both the products they purchase and the stores at which they shop, with the firm-to-firm transaction data further providing visibility on where that product was sourced from, all the way up the supply chain. In contrast to widely-used retail scanner data, the coverage goes beyond supermarket retail—with household consumption surveys at the store-brand level filling in the remaining gaps via statistical matching—and the consumer and retailer identifiers allow us to merge these data with firm-level datasets (on their employees, owners, suppliers, and tax payments) and household-level datasets (on their employment, firm ownership, pension claims, tax payments, and transfer receipts).

We then embed this new dataset inside a general equilibrium model of the Chilean economy. This model follows the framework laid out in Baqaee and Farhi (2020). Households supply heterogeneous amounts of factors of production (both capital and labor) to firms, firms use arbitrary technologies to make output with factors and intermediates, households consume firms' goods with heterogeneous preferences, and the government makes net tax/transfer payments from every household and firm in heterogeneous ways.

On top of these flows of goods and services, we then allow for an arbitrary set of distortions on every bilateral exchange. For example, when firms sell to any given firm or consumer, they may charge a markup as a result of their output market power. Analogously, when they buy from any factor or firm they may enjoy input market power and hence charge a markdown. Similarly, taxes, bribes, and credit constraints drive a wedge between the price that the seller effectively receives and the price that the buyer effectively pays, and this is the essence of any economic distortion.

While it is well known that such distortions lead to Pareto-inefficient outcomes, less focus has been paid to their heterogeneous consequences across the household distribution. Clearly, households may be differentially exposed to such distortions through their consumption (e.g. when they buy from suppliers with high product market power), their supply of factors of production (e.g. when they work for employers with high labor market power), their ownership of firms (and hence their capture of the rents that result from market power), and their participation in tax and transfer schemes. Further, in all of these cases, a household can be both directly and indirectly exposed due to their position in supply chains—for example, when a consumer buys a final good, they are exposed both to any markup charged by the final seller as well as to those charged by sellers further up the supply chain.

The data architecture that we pair with the Baqaee and Farhi (2020) model provides visibility on all of these interconnected phenomena for the first time. Our approach goes beyond the analysis of specific distortions in specific sectors that characterizes much of the limited existing work on this topic. Such comprehensiveness is important. When a second-best economy is simultaneously affected by multiple distortions the welfare effect of eliminating any specific distortion will be a function of other distortions in the economy. For example, what may appear as a harmful distortion when focusing within a sector may actually be beneficial if its presence happens to mitigate the effects of other distortions across sectors. Thus, to fully grasp both the equity and efficiency consequences of distortions one has to go beyond specific distortions in specific sectors. Our wide-reaching approach quantifies the relevance of this issue of overlapping distortions.

A central challenge when studying the impact of distortions arises in simply arriving at measures of the distortions themselves. We employ standard techniques from the misallocation literature to do so, noting both that these methods only uncover products of input and output distortions and that these same products are all that are required to estimate incidence.

Armed with such estimates of distortions on exchanges throughout the economy, as well as data on the network of such linkages between individuals, households, firms and the government, our final step is to conduct a series of counterfactual simulations that illuminate the incidence of distortions in our model economy. We adapt modern computational tools for solving linear systems of equations with dimensionality in the millions—as is necessary, given the need to do general equilibrium analysis with individual- and firm-level microdata on even a relatively small country such as Chile—in order to do so. Our main exercise reduces all distortions proportionately to their size. While this across-the-board reform is an extreme scenario, it relates to the typical goal of work in the misallocation literature, which is to assess the aggregate productivity gains that result from reducing distortions. We go beyond such aggregate impacts and answer the question of who is relatively harmed and helped by the presence of the distortions that are in place throughout the economy, uncovering differential incidence across groups based on income levels, age, and gender.

Our second exercise removes certain wedges or sets of wedges and asks which particular distortions are most responsible for the unequal incidence, both because some wedges may be larger than others and because their incidence differs conditional on size. Such an analysis also sheds light on the question posed above about whether certain distortions are countervailing or reinforcing. Finally, we study the trade-off between equity and efficiency that exists in Chile—as a result of the tax and non-tax distortions that appear to be in place throughout its economy—by comparing aggregate impacts to distributional ones.

Related Literature

This paper relates to several strands of the literature. First, we draw on theoretical and empirical tools in the literature that quantifies aggregate efficiency losses from distortions. This includes the seminal work of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), extended to a networked economy by Jones (2013) and Baqaee and Farhi (2019).

However, as discussed above, our primary interest lies not in the aggregate extent of misallocation—the effect on a hypothetical representative agent who buys all goods and owns all factors of production—but in its heterogeneous consequences—its incidence—across the many different types of agents throughout any modern economy. This interest is inspired by examples such as those stressed by Schmitz (2020), who has argued that the particularly odious consequences of monopoly power may derive more from its unequal incidence than from its effects on aggregate efficiency. This would be the case if, for example, poor households are more likely than rich households to consume goods with high markups, or to derive income from sources that are less likely to be connected (directly or indirectly) to firms that charge markups. We are unaware of attempts to quantify these broad inequality consequences of output market power. Nor are we aware of efforts to expand the scope of this point by seeing how it interacts with other distortions, beyond output market power, such as monopsony (vis-a-vis labor markets and intermediates markets), credit constraints, and taxes. This is surprising given the potential for countervailing incidence effects across different types of distortions.

By contrast, recent work has made large advances in documenting the incidence of distortions within particular sectors where previous data has allowed progress. For example, Faber and Fally (2022), Gupta (2022), and Sangani (2023) examine how retail markups differ across the income distribution. Similarly, Burstein, Cravino, and Rojas (2024), using a database from Chile that comprises one component of ours, document variation in markups on intermediates across types of buyer firms. Turning to other types of distortions, Faber (2014) and Acosta and Cox (2024) study the extent to which tariffs affect consumers across the income distribution heterogeneously, and Sharma (2023) evaluates the extent to which labor markdowns differ by gender of the employee. The study of the incidence of tax policies has, of course, a long tradition, ranging from partial equilibrium (seller vs. buyer incidence) to general equilibrium (often capital vs. labor). In this vein, an evolving modern approach, exemplified by Conlon, Rao, and Wang (2022), uses microdata to ask questions such as "Who pays sin taxes?" (on alcohol, cigarettes, and sugary drinks) across demographic categories. Our work aims to build on the lessons of studies like these in order to arrive at a bigger-picture understanding of how multiple types of distortions (markups, markdowns, credit constraints, corruption, taxes, etc.), with multiple types of exposure (firms, workers, and consumers, each in both their direct and indirect forms) overlap and interact to determine the overall incidence of distortions.

Finally, our work draws connections between recent contributions in general equilibrium measurement. For example, Adão et al. (2022) and Andersen et al. (2022) build micro-level versions of national accounts by connecting workers and owners to firms, firms to one another, and, in the latter case, firms to consumers. But these studies have considered neoclassical environments with no market distortions, and connecting consumers merely to the firms at which they shop may offer an incomplete picture if distortions vary across products as well as across firms. On the other hand, studies such as Atkin and Donaldson (2022) and Manelici et al. (2024) have built detailed pictures of market distortions in order to understand certain aggregate effects of trade and FDI shocks, respectively, but they have not emphasized the connections of individuals to the economy's distortions in order to quantify the incidence of misallocation.

2 Theory

This section describes the background theory of a general equilibrium economy with arbitrary distortions. We draw heavily on the presentation in Baqaee and Farhi (2020).

2.1 Set-up

The economy consists of C consumers, F primary factors, and N firms. We incorporate international trade and trade imbalances by modeling a "rest of the world" firm and consumer.

Consumers and factors. Each consumer $c \in \mathcal{C}$ has preferences

$$U_c(x_{c1}, \ldots, x_{cN}),$$

over the consumption amounts x_{ci} for each good $i \in \mathcal{N}$, and where each $U_c(\cdot)$ is homothetic (although each consumer can have arbitrarily different homothetic preferences). Each primary factor $f \in \mathcal{F}$ is in fixed supply, with each consumer c owning a share Φ_{cf} of the aggregate factor supply L_f. Similarly, consumer c owns a share Φ_{ci} of firm i. Denoting the prices of good i and factor f as p_i and w_f, respectively, each consumer faces the budget constraint

$$\sum_{i \in \mathcal{N}} v_i p_i x_{ci} = \sum_{f \in \mathcal{F}} \Phi_{cf} w_f L_f + \sum_{i \in \mathcal{N}} \Phi_{ci} \pi_i + T_c \equiv \chi_c,$$

where we let π_i denote any (after-tax) firm profits earned by firm i, v_i denote the tax rate on final consumption and T_c denote the net personal taxes paid by household c.[1] Finally, we take the total value of consumption in this economy as the numeraire (i.e. $\sum_i \sum_c v_i p_i x_{ci} = 1$) and let $b_{ci} \equiv p_i x_{ci}/\chi_c$ denote the share of consumer c's expenditure going to the seller of good i.

Firms. Each good i is produced by a unique and single-product firm that we also denote by i.[2] Each firm has access to its own arbitrary, but constant returns-to-scale, production

[1] T_c includes differences between observed income and expenditure. Letting T_c^g denote the net transfer from the government, $T_c^g - T_c$ captures the net savings of the household. We model a "rest of the world" household that absorbs the net national savings, ensuring that $\sum_c T_c^g - T_c = 0$. In counterfactuals, household savings, as a proportion of the GDP, is held constant.

[2] We map multi-product firms (including firms that sell a single product to multiple buyers) into single-product firms by assigning inputs to each product proportionally. We make an exception for retailers given their central role in mediating trade between consumers and firms and the nature of their production function. In particular, we break multi-product retailers into single-product retailers and assign any input that is later sold as output to the relevant single-product retailer. All remaining inputs are then shared proportionately across the single-product retailers.

function that uses potentially all factors and other goods as inputs. Denoting the firm's total output as y_i, its Domar weight (i.e. the share of its sales in total economy-wide consumption) is denoted by $\lambda_i \equiv p_i y_i$. We let p_j^I denote the price of any input (which could be either a good selling for p_j or a factor selling for w_f), such that when firm i purchases the quantity of inputs x_{ij}, then its after-tax profits are $\pi_i = p_i y_i - \sum_{j \in \mathcal{F}, \mathcal{N}} p_j^I x_{ij} - T_i$ when it rebates (as discussed further below) a total amount T_i to the government in net taxes. Finally, we let $\Omega_{ij} \equiv \frac{p_j^I x_{ij}}{p_i y_i}$ denote the share of input j in total revenues of firm i and let Ω denote the square matrix composed by the elements Ω_{ij}. Similarly, we define the Leontief inverse of this matrix by $\Psi \equiv (I - \Omega)^{-1}$.

Distortions. We allow for a general treatment of distortions. When a firm i buys an input (a good or a factor) at price p_j^I, it may nevertheless be the case that the as-if marginal cost to the buying firm, from buying one more unit of input j, is $\tau_{ij} p_j^I$ rather than the price p_j^I that the input supplier receives. When this is the case, the seller's and buyer's marginal incentives are not aligned via linear prices, which results in what we refer to as an input distortion (or wedge). The source of such distortions is potentially manifold. One example is monopsony power, in which the buyer recognizes that purchasing an extra unit costs not just p_j^I but also an additional amount (i.e. an extra $p_j^I(\tau_{ij} - 1)$) that comes from increasing the price of infra-marginal units.[3] Another example is simply an input tax collected by the firm, such as a payroll tax, or a value-added tax. Regardless of their underlying source, the combined effect of input distortions determines the value of τ_{ij} and this value is sufficient for what follows.[4] Given such distortions, we let $C_i \equiv \sum_{j \in \mathcal{F}, \mathcal{N}} x_{ij} \tau_{ij} p_j^I$ denote the firm's total costs, inclusive of the distortions, and $c_i \equiv C_i / y_i$ the corresponding unit (and marginal) cost.

In addition to these arbitrary distortions τ_{ij} on each input, we also allow for scenarios in which the price p_i that the customers of firm i pay is different from firm i's marginal cost c_i, where any such output wedge is denoted by $\mu_i \equiv p_i / c_i$. This allows for firm i to enjoy market power in its output market and hence charge a markup, for example). We then define $\widetilde{\Omega}_{ij} \equiv \tau_{ij} \mu_i \Omega_{ij}$ as the share of the distortion-inclusive cost of input j in firm i's total costs, with the matrices $\widetilde{\Omega}$ and $\widetilde{\Psi} \equiv (I - \widetilde{\Omega})^{-1}$ defined analogously.[5]

[3] Similarly, if firm i faces a binding credit constraint, the price it pays the lender of a marginal unit of capital (i.e. p_j^I) is lower than the shadow price of capital within firm i (i.e. $\tau_{ij} p_j^I$ for some $\tau_{ij} > 1$).

[4] Among multiple sources of τ_{ij}, one important distinction is between those due to taxes and those not. The revenues from the former accrue to the government and are included in T_i whereas those from the latter accrue to the firm. We therefore keep these distinct when evaluating counterfactuals.

[5] The expressions below hinge on the difference between $\widetilde{\Omega}_{ij}$ and Ω_{ij}, and hence only on the combined wedge $\tau_{ij} \mu_i$ rather than either of the components τ_{ij} or μ_i alone. We nevertheless keep the distinction between input and output wedges fully general for expositional purposes.

Government. Profits of the firm are taxed at the rate t_i^p. This corporate profit tax is non-distortionary in our model (because firms are in fixed supply) but its presence has implications for inequality across firm owners and non-owners. In addition, the purchase of input j from firm i is taxed at the rate t_{ij}; together with the flexible consumption taxes v_i introduced above we can therefore fully capture cases such as value-added taxation and payroll taxes as special cases. Put together, these two sets of taxes levied on firms mean that total tax rebated to the government (excluding consumption tax, by convention) is

$$T_i = \sum_{j \in \mathcal{N}, \mathcal{F}} (t_{ij} - 1) p_j^I x_{ij} + (t_i^p - 1)\pi_i.$$

All tax rates are potentially firm-specific. This allows us to capture the distinction between formal and informal firms, discussed further below.[6] Finally, we set $\sum_c T_c = \sum_i T_i + \sum_i (v_i - 1) \sum_c p_i x_{ci}$ to close the government's budget constraint, where T_i is the total tax rebated by firm i to the government, excluding the tax on final consumption.

2.2 The Incidence of Distortions

Our strategy for quantifying the general equilibrium incidence of distortions proceeds by solving for the changes in individuals' real incomes that would occur if distortions were to change. For example, one such set of changes we consider in Section 5 below considers the complete removal of existing wedges.

To solve for the effects of such hypotheticals, begin by noting that the change in real income $\mathcal{Y}_c$ for consumer c is composed of two terms: the change in their nominal income χ_c and the change in the price index that is appropriate for their particular utility function. Using the envelope theorem to simplify the latter effect, the change in real income due to any vector of small price changes $d \ln p$ is

$$d \ln \mathcal{Y}_c = d \ln \chi_c - \sum_{i \in \mathcal{N}} v_i b_{ci} (d \ln v_i + d \ln p_i). \tag{1}$$

In turn, the price changes can themselves be written, using the envelope theorem applied to each firm's costs, as

$$d \ln p_i = \sum_{j \in \mathcal{N}} \widetilde{\Psi}_{ij} d \ln \mu_j + \sum_{k \in \mathcal{N}, \mathcal{F}} \widetilde{\Psi}_{ij} \widetilde{\Omega}_{jk} d \ln \tau_{jk} + \sum_{f \in \mathcal{F}} \widetilde{\Psi}_{if} d \ln w_f. \tag{2}$$

[6]In particular, formal firms must charge a value-added tax on sales to their customers, but get a tax rebate for all value-added tax paid on purchases; by contrast, informal firms do not charge a value-added tax but cannot claim any rebates either. Similarly, formal firms are subject to payroll taxes and profit taxes, whereas informal firms are not.

For example, if the markup on good j changes, this effect will propagate forward along the supply-chain to the price of good i in accordance with the Leontief inverse weight $\widetilde{\Psi}_{ij}$—this appropriately sums all senses in which the cost of good i depends, both directly and indirectly, on the price of input j.

In addition, the change in distortions under consideration will affect the prices w_f of each factor f, and these factor price changes will affect both good and factor prices (due to the last term of Equation (11), weighted by the Leontief-inverse exposure elements $\widetilde{\Psi}_{if}$) as well as the incomes $d\ln\chi_c$ in Equation (1). These changes in income themselves satisfy

$$d\chi_c = \sum_{f\in\mathcal{F}} \Phi_{cf} L_f dw_f + \sum_{i\in\mathcal{N}} \Phi_{ci} d\pi_i + dT_c, \tag{3}$$

where dT_c allows for change in the net government transfers that is desired to be simulated as part of the counterfactual. Similarly, the change in after-tax profits $d\pi_i$ satisfies

$$d\pi_i = \left(\frac{\pi_i + T_i}{\lambda_i}\right) d\lambda_i + \lambda_i \sum_{j\in,\mathcal{N},\mathcal{F}} \Omega_{ij}(d\ln\mu_i + d\ln\tau_{ij} - d\ln\widetilde{\Omega}_{ij}) - dT_i, \tag{4}$$

where dT_i is the change in net taxes rebated by the firm.[7]

The change in firm i's Domar weight is given by[8]

$$\begin{aligned} d\lambda_i =& - \sum_{l\in\mathcal{C},\mathcal{N};m\in\mathcal{N}} \lambda_l \Omega_{lm}(d\ln\mu_l + d\ln\tau_{lm})\Psi_{mi} \\ &+ \sum_{k\in\mathcal{C},\mathcal{N}} \mu_k^{-1}\lambda_k \text{Cov}_{\widetilde{\Omega}^{(k)}}(d\ln\widetilde{\Omega}^{(k)}, diag(\tau^{(k)})^{-1}\Psi_{(i)}) + \sum_{c\in\mathcal{C}} d\chi_c \sum_{k\in\mathcal{N}} b_{ck}\Psi_{ki}, \end{aligned} \tag{5}$$

where for any matrix A, the notation $A^{(k)}$ denotes the vector formed from row k (and analogously, $A_{(k)}$ that for column k), the weighted covariance operator $Cov_a(b,c)$ denotes the covariance of vectors b and c weighted by the vector a, and $diag(\tau^{(k)})$ is the diagonal matrix with the ith diagonal element equal to τ_{ki}. Because one can always think of a factor as a firm that uses no inputs, changes in factor prices satisfy an analogous expression to (14) but for dw_f instead of $d\lambda_i$.

The intuition behind Equation (14) is as follows. The first term captures the direct effect of wedges on demand for i: for example, an increase in the input wedge τ_{lm} will be a negative demand shock for supplier m, and this demand shock will propagate backward

[7]When solving for the effects of a change in wedges below, we keep track of the component that arises from any change in tax policy. Additionally, we impose that any increase in the tax collection either due to equilibrium changes in the tax base or the direct changes in taxes is distributed proportionally among the households. That is, we keep $\Phi_{cg} \equiv \frac{T_c^g}{\sum_c T_c}$ constant in the counterfactuals.

[8]Equation (14) extends the definition of wedges $\mu_k\tau_{ki}$, sales shares λ_k and cost shares $\tilde{\Omega}_{ki}$ to consumers, such that $\mu_c = 1$, $\tau_{ci} = v_i$, $\lambda_c = \chi_c$ and $\tilde{\Omega}_{ci} = v_i b_{ci}$ for $i \in \mathcal{N}$.

to firm i via Ψ_{mi}. The second term captures substitution within each buyer k: a relative change in input prices might cause this buyer to substitute input shares away from relatively costlier inputs and towards others. The consequences of these relative demand shocks for inputs for the sales of firm i depends on the direct plus indirect importance of these inputs in i's supply chain (i.e. the entries of $\Psi_{(i)}$) and the size of buyer k (i.e. λ_k). And the final term incorporates the fact that an income shock to consumer c can lead to more total spending on firm i even if the expenditure share spent on that good (the change in which is captured in the second term) is constant.

Finally, the change in input and profit tax collected from firm i is given by

$$dT_i = \sum_{j\in\mathcal{N},\mathcal{F}} \lambda_i\Omega_{ij}dt_{ij} + (t_{ij}-1)\Omega_{ij}\left(d\lambda_i - \lambda_i(d\ln\mu_i + d\ln\tau_{ij} - d\ln\tilde{\Omega}_{ij})\right) \\ + \pi_i dt_i^p + (t_i^p - 1)d\pi_i \tag{6}$$

where the first and third term capture direct changes due to the change in tax rate, while the rest captures the general equilibrium changes due to the change in tax base itself. Since the government budget is balanced, change in tax collected also impacts the household transfers T_c :

$$dT_c = \Phi_{cg}\sum_{i\in\mathcal{N}}\left(dT_i + dv_i\sum_{c'}\chi_{c'}b_{c'i} + (v_i-1)\sum_{c'}b_{c'i}\left(d\chi_{c'} + \chi_c(d\ln\tilde{\Omega}_{c'i} - d\ln v_i)\right)\right) \tag{7}$$

where Φ_{cg} is the proportion of the total taxes collected from firms that accrues to household c (held constant in the counterfactuals).

The final step is to solve for the change in input shares $d\ln\widetilde{\Omega}_{ij}$ that appear in Equations (14) - (7) above. Unlike all previous expressions that rely only on optimizing behavior (via the application of the envelope theorem), this final step requires an understanding of how buyers substitute across suppliers in response to changes in the prices that they pay. As in Baqaee and Farhi (2019), a unified treatment of such substitution is as follows. First, let any buyer i have a single CES utility/production function with elasticity θ_i among all inputs.[9] Then the change in input cost shares for such a buying entity i is given by

$$d\ln\widetilde{\Omega}_{ij} = (1-\theta_i)\left(d\ln p_j^I - \sum_{k\in\mathcal{F},\mathcal{N}}\widetilde{\Omega}_{ik}d\ln p_k^I\right). \tag{8}$$

Put together, the system of equations in (11)-(19) constitutes a linear system that can be solved for any given values of elasticities θ_i, wedges μ_i and τ_{ij}, values of the input share

[9]To allow for asymmetric substitution (as in the case of nested CES preferences or technologies) we create an additional fictitious good and a buyer that plays the role of a CES bundle in any lower nest.

$\widetilde{\Omega}$ and Leontief-inverse Ψ matrices, and values of the consumption shares b_{ci}. A solution to this system can then be used as a first-order approximation to the question of interest (valid to the extent that the exogenous changes in wedges fed into the system is small), or used in each step (updating the allocation each time) of a simple iterative algorithm that solves for arbitrary changes exactly.

Stepping back, these expressions make clear that the incidence of distortions in any economy will hinge on two considerations. The first is the analog of what tax analysts call "statutory" incidence: who actually pays the tax. Here, the broader question is: who pays the wedge and to whom do the rents from that wedge accrue? Combining Equations (1) and (11) demonstrates, for example, the simple sense in which consumers c with high values of $\sum_i b_{ci}\widetilde{\Psi}_{ij}$ are those who are, in the "statutory" sense, paying the wedge μ_j. For example, those consumers who buy from supermarkets with large markups or from supermarkets that themselves source from high-markup producers. The flip-side of these statements is clear from Equations (3) and (A.3): the owners (as embodied in the ownership matrix Φ_{ci}) of high-markup firms are earning income from large markups. Similarly, marked-down wages are being paid for by workers (in Φ_{cf}) and the owners of the firms marking down wages are receiving these payments.

However, as tax analysts recognize well, statutory incidence is not the end of the story for the "economic" incidence that ultimately matters. Indeed, in the simplest possible case of a competitive model of a single market, for example, the elasticities of supply and demand in that market uniquely determine the effect of a tax on consumer and producer prices, and hence the "economic" incidence among the two agents within that one market, and statutory incidence is irrelevant.

While this same force is at work above—in Equations (11) and (3) via the dependence on factor price responses $d\ln w_f$—the broader sense of "statutory" incidence of distortions does matter when we look across markets. The actors in any realistic economy participate in the bilateral exchange of each good and factor to vastly heterogeneous extents. That is, who buys and sells products that are directly or indirectly marked up, who works for and owns firms that buy inputs that are directly or indirectly marked down, etc., will vary enormously. The formulae above combine heterogeneous extents of statutory incidence—as measured in the spending patterns of consumers, the supply chain patterns of firms, and the ownership matrices of which consumers own which firms and factors—with general equilibrium elasticities of supply and demand in order to measure economic incidence correctly. The next two sections illustrate how we will measure these phenomena in order to put the theory of this section into practice.

3 Data

Our analysis draws on nine administrative datasets from Chile's Servicio de Impuestos Internos, their Internal Revenue Service equivalent (henceforth, IRS), and two survey datasets from the statistical agency (INE for its acronym in Spanish). These datasets cover the entire formal private sector in Chile. Below, we provide an overview of the main data sources and key variables, as well as how we deal with informality. We use data from 2022 in all cases, the most recent year where all sources are jointly available.[10]

First, we use a firm-to-firm electronic receipt dataset that is based on value-added tax (VAT) records. This dataset electronically records all transactions between formal firms in the economy. Thus, for each firm, we know the complete list of buyers and suppliers that the firm trades with (including those in the public sector). There are no reporting thresholds involved. The dataset includes the value of the transaction, the price and the product involved. Products are codified using around 2,500 standardized product categories via a string-matching procedure. Second, we use a firm-to-firm dataset similar to the first one but for international transactions that reports all imports and exports between domestic firms and foreign firms originating from customs records.

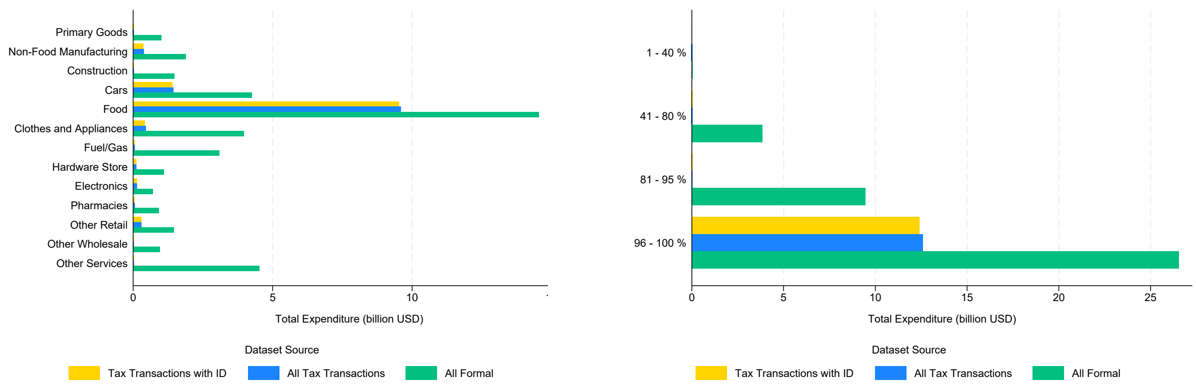
Third, we use a similar dataset to the two above but covering transactions of individual products between firms and households based on electronic receipts collected as part of the VAT system. This dataset electronically records all transactions between formal firms and individuals in the economy. Administrative data mapping firms to consumers are very uncommon and come about through the government's electronic filing system, which requests that every purchase be associated with a customer tax ID. Consumers routinely comply with this request, and do so almost always when shopping at larger retailers in part because these stores use this tax ID to link customers to their loyalty or rewards programs.[11] Thus, for each firm, we have a list of individuals the firm sells to and the products that they purchased. There are no reporting thresholds although coverage is incomplete, both for informal retailers and smaller stores that typically do not report tax IDs

[10] This paper was developed via an agreement within the scope of the research agenda conducted by the Central Bank of Chile (BCCh for its Spanish acronym) in economic and financial affairs in its purview. The BCCh has access to anonymized information from various public and private entities, by collaboration agreements signed with these institutions. To secure the privacy of workers and firms, the BCCh mandates that the development, extraction, and publication of the results should not allow the identification, directly or indirectly, of natural or legal persons. Officials of the BCCh processed the disaggregated data on our behalf. We implemented all analysis and neither involved nor compromised the IRS in doing so. The information contained in the databases of the IRS is of a tax nature originating in self-declarations of taxpayers presented to the Service; therefore, the veracity of the data is not the responsibility of the Service.

[11] This database provides customer-level itemized sales information for 2,124 of the country's largest firms in terms of their total final consumption sales.

in their submissions (an issue we address by apportioning this consumption using consumption surveys described below). Beyond having customer tax IDs, the variables are the same as those in the firm-to-firm dataset (value, price, detailed product classifications). For the remaining formal firms, only total sales to all customers are available.

Figure 1: Expenditures Across Retailer Type and Firm Size By Data Source



(a) Across Retailer Types

(b) Across Firm Size

Notes: Panel (a) displays total consumer expenditures across retailer types broken down by the source and level of detail in the consumption data: all formal sales to final consumers, formal sales recorded at the transaction level, and formal sales both recorded at the transaction level and attached to consumer tax IDs. Panel (b) reports the same for groups based on firm size.

Figure 1 displays the composition of expenditures captured by three different types of administrative consumption records described above. The green bars plot all formal sales data to final consumers, the blue bars plot all expenditures for which we have transaction-level data, and the yellow bars plot all expenditures for which we have both transaction-level data and tax IDs. Panel (a) shows the allocations of these three types of data across different types of retailer, for example food retailers, construction, or service firms, while Panel (b) shows the allocations across different firm size bins. The fact that the blue and yellow bars are almost identical shows that, conditional on shopping at a store for which we have transaction-level records, almost all customers provide their tax IDs. In terms of coverage by retailer type and firm size, stores reporting transaction-level data are under-represented in services and over represented in food retail and cars, and are much larger than the average store—consistent with the tax authorities sharing transaction-level data only for the largest retailers, primarily supermarkets, department stores and chains.[12]

Fourth, we use a matched employer-employee dataset (from IRS tax affidavits 1887

[12]Anecdotally, these larger stores are more likely to record tax IDs than smaller ones, so were the tax authorities to share transaction-level data for smaller stores there would likely be a smaller share of expenditures attached to tax IDs.

and 1879) that reports annual earnings from each job that a worker has. Earnings include wages, salaries, bonuses, tips, and other sources of labor income deemed taxable by the IRS. As earnings are reported net of social security payments, we adjust the earnings measure to include these payments. Fifth, we use the ownership linkages of firms from tax records (IRS tax affidavits 4415 and 4416). This dataset includes, for each firm, the complete list of owners of the firm (which can themselves be both firms and individuals) as well as the share of the firm that each owner possesses. Sixth, we use government pension records to expand these ownership linkages. Chile has a mandatory private pension system with individuals able to choose between seven pension providers with each provider offering five funds of varying risk levels. The investments of each fund are public, allowing us to obtain each individual's shareholdings of any companies their pension fund invests in.[13]

Seventh, we use government-to-household linkages that combine a dataset of transfers and another on income tax payments that allows us to build direct net transfers. The transfers data records the main direct transfers that the government makes to households every year. For this dataset, we know, for each type of transfer, the total amount of the transfer and the type of policy to which the transfer corresponds. The income tax dataset, on the other hand, records the income tax payments from households to the government (which apply only to the top income deciles of the country).

The seven aforementioned datasets record all the relevant transactions and relationships that firms and individuals have with different agents in the economy: other firms (both domestic and foreign), households, government, workers, banks, and capital owners.

Two other administrative datasets serve as complements to these bilateral administrative datasets. We use a civil registry database to provide year of birth, gender, marriages, place of birth and the father and mother of each individual. These data provide both demographic information useful for grouping individuals into groups when exploring differences in incidence and allow us to combine individuals into households, which is crucial since many consumption and income choices are made in part at the household level (for example, supermarket purchases that are recorded at the individual level). Eighth, and finally, we use an administrative dataset (IRS tax forms 29 and 22) that contains each firm's balance sheets to measure total sales, material costs, investments, and fixed assets for each firm.

[13] Due to server confidentiality rules, the administrative pension data can only be merged in once other parts of the analysis are complete. Thus, we currently substitute actual pension holdings with predicted pension holdings based on the consumption survey (described below) that records pension payments and receipts.

All individuals and all formal firms in Chile are assigned a unique tax ID that is consistently recorded across the datasets above, which enables all of the merges we require. In what follows, we define a firm as a tax ID.[14] Given the centrality of these bilateral administrative datasets to our analysis, Table 1 presents statistics about the scale and other attributes of these datasets.

Table 1: Descriptive statistics on the scale of the bilateral administrative datasets

1. Firm-to-Firm Domestic Trade	# Buyers 1,354,408	# Suppliers 624,073	# Pairs 35,993,564	# Transactions 2.1 Billion
2. Firm-to-Firm International Trade	# Buyers 93,423	# Suppliers 155,283	# Pairs 273,110	# Transactions 5,298,769
3. Firm-to-Individual Consumption	# Consumers 13,453,311	# Suppliers 2,124	# Pairs 43,626,887	# Transactions 6 Billion
4. Firm-to-Workers Employment/Wages	# Firms 702,729	# Workers 8,242,191	# Pairs 13,138,247	# Jobs per Worker 1.6
5. Firm-to-Individual Ownership	# Owners 1,781,539	# Owned 1,445,504	# Pairs 3,172,853	Median Ownership Share 34%
6. Gov-to-Individuals Net Transfers	# Individuals 8,021,862	# Policies 10	# Policy Pairs 16,262,917	# Policy Transactions 16,495,680

Notes: This table presents statistics on the size of each transactional dataset described in Section 3. Each statistic is conditional on non-zero flows, which leads to differences in the number of firms and individuals across datasets. For example, not all firms engage in international trade.

While the administrative datasets cover close to the universe of formal economic transactions in the economy, they miss informal economic activity that is typically estimated to comprise about one quarter of employment (though considerably less of total output) in Chile's economy. Thus, in order to complement the administrative data, we use three large-scale government surveys that capture informal transactions. The first of these is a detailed consumption survey conducted by the Chilean government as a key input into inflation and poverty measurement (the IX Encuesta de Presupuestos Familiares fielded between October 2021 and September 2022 and surveying around 44,000 individuals). A lengthy questionnaire is administered to a large and representative sample of households. Households report the full list of consumption expenses they had in a year, including the associated prices, quantities, product description, and the store it was purchased from.[15] Surveyors both ask to see receipts of all purchases to increase accuracy and separately interview every member of the family to ensure no consumption is missed.

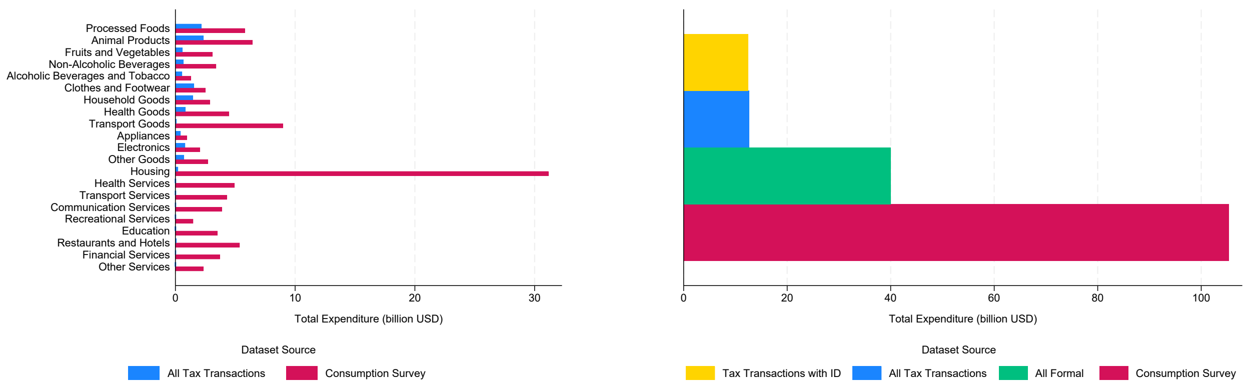
Panel (a) of Figure 2 compares product-group by product-group the coverage in the consumption survey (applying the survey weights) to the consumption captured at the

[14] As all tax forms are reported at the headquarters level, plant-level information is not available.

[15] Recall periods vary depending on the durability of products, with shorter recall periods for less durable products that we convert to annual expenditures.

transaction-level in the firm-to-individual administrative data that covers expenditure at the largest retailers. As would be expected, the consumption survey total expenditures are much larger but the gap also differs across sectors, with the largest shortfalls in the administrative data found in services, particularly housing where the consumption survey includes imputed rent paid by owner-occupiers. Panel (b) displays the aggregates across all four types of data we use to measure consumption; all formal sales to final consumers, formal sales recorded at the transaction level, formal sales both recorded at the transaction level and attached to consumer tax IDs, and all sales in the consumption survey (expanded via survey weights).

Figure 2: Consumption Data Coverage: Across Products and Aggregates



(a) Across Products

(b) Aggregates

Notes: Panel (a) displays total expenditure in consumption across product categories. The blue bars represent consumption documented in administrative tax data. The red bars represent consumption documented in the consumption survey (expanded via survey weights). Panel (b) presents aggregates across all four sets of sales data; all formal sales to final consumers, formal sales recorded at the transaction level, formal sales both recorded at the transaction level and attached to consumer tax IDs, and all sales in the consumption survey (expanded via survey weights).

The consumption survey also includes detailed demographics (including income data) that allow us to match surveyed individuals to similar groups of individuals in the administrative data for whom informal spending is missing, and for whom formal spending at small stores may not be attached to their tax IDs. Specifically, we follow Blanchet, Saez, and Zucman (2022) and draw on statistical matching methods that provide the matches that minimize the distances between common variables in two datasets via recent advances in optimal transport algorithms. This approach preserves the joint distribution of demographics and expenditures in the consumption survey data that is being brought into the administrative data and avoids extrapolation (and possibility of extreme outliers) that would occur using polynomial-based prediction models. We first form broad bins based on region, age and income. Within each group, survey respondents are matched (one-to-many) to individuals in the administrative data based on gender, income, labor and profit income shares, and the full vector of expenditures at supermarkets and department stores (expenditures that are well-measured in both the firm-to-individual administrative data and in the consumption surveys).

This match provides measures of expenditure for each individual-product-firm type triplet. However, we still must allocate all these expenditures to specific retailers. We start by forming region-product-firm type groups. Excess expenditures in the consumption data within a group are allocated proportionately to the corresponding stores of that type, e.g. a specialized shoe retailer, in the location that the individual resides. Consumers are randomly matched to specific formal stores for whom we do not have linked tax-id data but whose total sales we still see in the administrative data.[16] The remaining consumer expenditure predicted from the survey match is allocated to synthetic informal firms of the corresponding type (e.g. an informal specialized retailer).[17] These additional expenditures enable us to extend the detailed consumption patterns captured in administrative records to a more complete set of consumption transactions in the economy.

The income module of the household survey also contains multiple questions capturing income from government programs as well as from both informal and formal employment. We complement these data with a rich labor force survey that reports formal and informal labor market activity for a sample of representative workers in the economy (the 2021–2023 Encuesta Suplementaria de Ingresos surveying approximately 300,000 individuals). Again this data is matched to demographics that allow us to fill in missing data in the administrative data using a similar statistical matching process to how we deal with consumption above (in this case matching at the individual level on the vector of formal income by sector, again separately for each demographic group). Finally, we bring in a survey of small firms designed by the Chilean government to measure the informal sector (the Encuesta de Microemprendimiento surveys conducted in 2017, 2019 and 2022 surveying over 20,000 small business owners). Using the included survey weights, we use this survey to populate the informal firms in the economy. These informal firms are then statistically matched to the informal firm owners reported in the labor force surveys prior to performing the match between the labor force survey and the administrative data described above. This match allows us to supplement the administrative income data with

[16] Recall that we have total final goods sales for all formal firms but only firm-to-individual matched sales for the 2,124 largest sellers.

[17] To prevent very small or noisy predictions, we truncate extremely small consumers in these cases. We allow consumption to exceed the predicted consumption from the survey when expenditure in the firm-to-individual administrative data is greater than the matched total consumption from the survey.

informal income as well as ownership and profits of informal firms. Additionally, we augment the administrative firms' dataset with the informal firms created through this match.

Taken together, we believe these datasets may provide the most complete mapping of interactions between economic agents in an economy that have been assembled to date. As we have emphasized above, this level of detail is necessary for understanding incidence in the presence of overlapping distortions that may mitigate or exacerbate each other.

4 Measurement

Our implementation of the incidence analysis presented in Section 2 requires three empirical inputs: (i) matrices of household-level exposure to distortions; (ii) elasticities of substitution within firms' technologies and consumers' preferences; and (iii) the size of the various distortions themselves. We discuss each of these in turn.

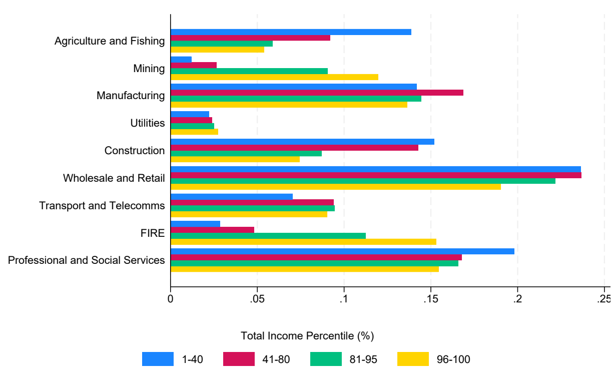
4.1 Exposure Matrices

To implement our incidence formulae from Section 2, we require knowledge of three fundamental matrices that together capture household exposure to distortions. Using the definitions from Section 2, these are the matrices of households' consumption shares b, input-output revenue shares Ω, and household-to-firm and -factor ownership shares Φ. Household consumption shares for each household i (b_i) are measured directly from the firm-to-household electronic VAT receipt data combined with the consumption survey for non-captured shopping trips and are at the store-product level. Input-output revenue shares between a buyer i and a supplier j (Ω_{ij}) come directly from the spending by buyers on particular suppliers, relative to total sales of buyers, in the firm-to-firm electronic transaction data and employee-employer data. Recall that we break retailers and wholesalers—multiproduct firms that sells inputs with minimal transformation—into multiple fictitious single-product firms, one for each final product in the original retailer/wholesaler, each of which uses as its input the corresponding product found in the original firm's input purchases. Any retailer/wholesaler inputs not sold as output are attributed to overhead costs that are shared across the single-product firms. For all other multi-product firms, we allocate all inputs proportionately to revenues. Firms' total sales—used also to measure the Domar weights λ_i—are recorded in firms' tax filings.

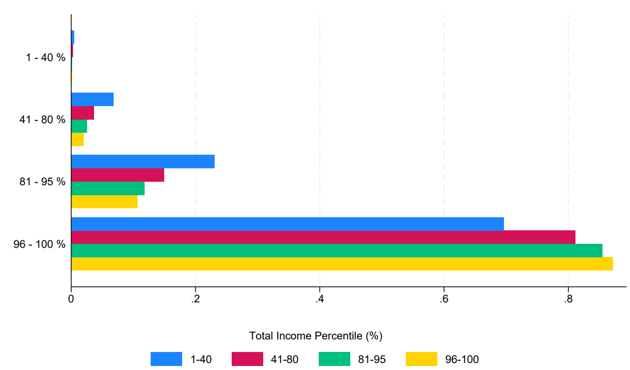
Finally, household-to-firm and -factor ownership shares between a household c and a firm i (Φ_{ci}) or factor f (Φ_{cf}) come directly from the firm ownership and pension records (in case of capital and profits) and employer-employee records (in the case of labor). Thus, except for the non-captured shopping trips that we impute using detailed store-product-level consumption surveys matched to household characteristics and informal labor matched from the same surveys, all the elements in the key exposure matrices are directly observed in Chile's expansive administrative datasets.

One example that illustrates the richness of the exposure matrices we observe is presented in Figure 3. In panel (a) we display elements of the matrix Φ_{cf}, where the set of individuals c is broken into four income categories and the set of factors f is taken to be sector-specific labor for each of nine sectors (the merge of individuals into households is still forthcoming). Large distinctions across income categories are apparent. For example, the lowest-income individuals are almost three times as dependent on the agricultural and fishing sector for their (labor) income as the richest individuals are, whereas the richest individuals are about ten times more dependent on the mining sector and six times more dependent on finance, real estate and insurance than the poorest are. Panel (b) presents an analogous depiction of Φ_{cf}, but now where the factor groups f are based on the size (four bins) of the firm size distribution at which individuals work. The poor are relatively more likely to work in smaller firms than the rich are. Both of these figures highlight how different the exposure to labor distortions in various sectors and firm sizes is depending on where in the income distribution an individual lies.

Figure 3: Exposure of Individuals to Labor Income



(a) By sector

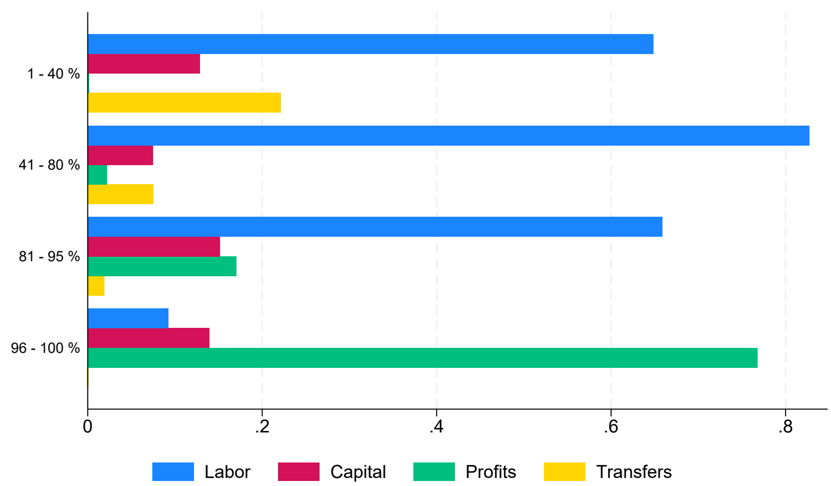


(b) By firm size

Notes: Panel (a) displays elements of the matrix Φ_{cf} for four types of individual c and nine types of sector-specific labor (i.e. f). By construction, $\sum_f \Phi_{cf} = 1$ for each c. Panel (b) does the same but for groups of f based on the size of the firm at which individuals are employed.

Other types of exposure matrices are also easy to visualize (in aggregated forms). For example, a natural next question is how total income divides into labor relative to capital and the role of government transfers. Figure 4 displays such a breakdown. In so doing, we disaggregate capital earnings into two components: (i) an amount that corresponds to the "fair" return on each firm's capital (based on Chile's interbank lending rate); and (ii) the additional profits that each firm earns, beyond labor and intermediate costs and the aforementioned fair return on their capital. Evidently, labor and transfer income comprise virtually all of the earnings for individuals in the bottom 80 percentiles of the income distribution. By contrast, for individuals in the top five percentiles of the income distribution we see that the vast majority of their income derives from firm ownership, and of that the bulk is profits rather than the fair return on capital.[18]

Figure 4: Income by Source



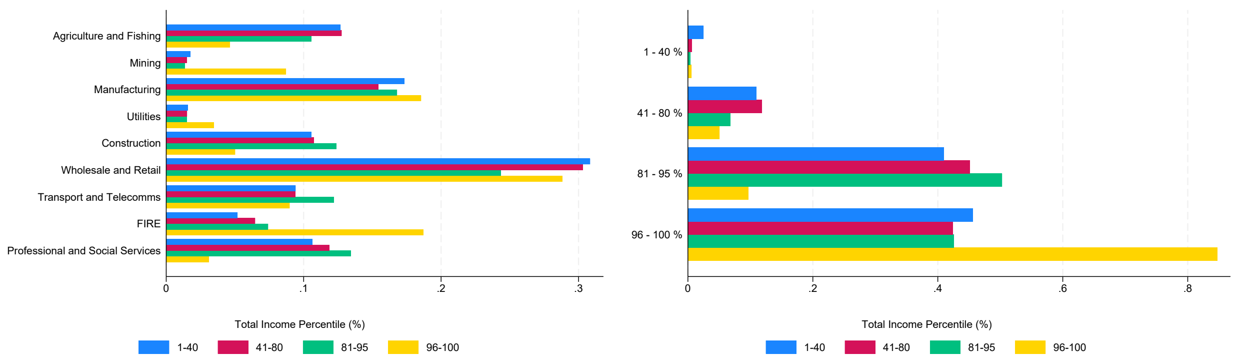
Notes: This figure displays the breakdown, for any type of individual (based on their position in the income distribution) c, of their earnings by source. Four types of source are indicated: that from all types of labor, that from all types of capital (based on a fair return), that from the profits that are earned in all types of firms, and that from government transfers.

As with labor income above, Figure 5 displays features of the ownership matrix Φ_{ci}, again for individuals c based on their position in the income distribution. In panel (a) the firms i are broken down by sector and in (b) they are broken down by firm size bins. As shown above, while rich individuals are of course more likely to own firms, the entries of Φ_{ci} sum to one within any c. Thus, these figures portray how a given type of income group's profit income is distributed across types of firms (sectors in (a), and sizes in (b)).

[18]Note that this result is in part driven by the fact that we are distributing all profits to income. In reality, a fraction of profits remain inside the firm with the value of the ownership stake rising (and so it would only be recorded as income to the IRS when the firm distributes it as dividends or is sold). An extreme case here are profits accruing to pension funds which we assign to the owners of those pension funds even though they only receive the returns to those claims upon retirement.

Again it is apparent that firm ownership type is heterogeneous across the income distribution; for example, poorer individuals are relatively more likely to own agriculture and fishing firms, and small- and medium-sized firms more generally.

Figure 5: Exposure of Individuals to Profit Income



(a) By sector

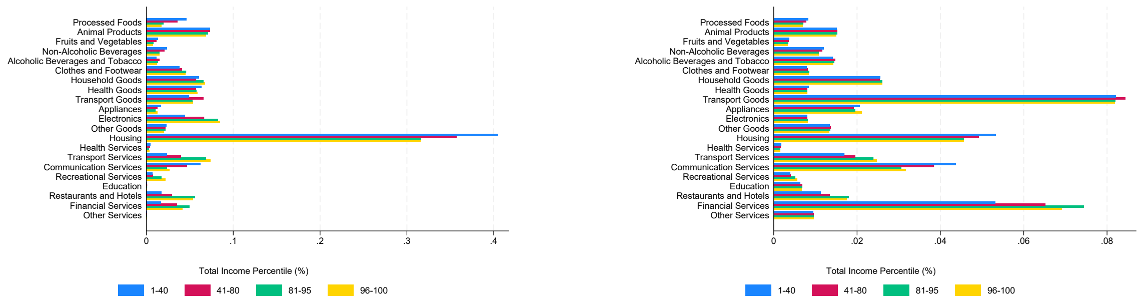
(b) By firm size

Notes: Panel (a) displays elements of the matrix Φ_{ci} for four types of individuals c and nine types of firms (i.e. i) owned, by sector. By construction, $\sum_i \Phi_{ci} = 1$ for each c. Panel (b) does the same but for groups of i based on the size of the firm owned.

Similarly, Figure 6 turns to the case of the matrix of consumer expenditure shares, beginning with a summary of direct shares (i.e. b_{ci}) in Panel (a). Using the information encoded in the linked firm-to-individuals transaction records as well as the statistically-matched expenditures from the consumption survey, we again summarize these expenditure shares by individuals' income groups. We break down each groups' expenditure shares by product category, revealing substantial cross-group heterogeneity in product consumption patterns across broad products. Of course, such summary measures hide the cross-group heterogeneity that our data uncovers within these broad categories. Panel (b) continues by examining the indirect consumption exposure embodied in $\sum_i b_{ci}\tilde{\Psi}_{ij}$. As expected, a number of products are bought little in a "direct" fashion by consumers (low b_{ci} in Panel a) but are "indirectly" purchased to a far greater extent (high $\sum_i b_{ci}\tilde{\Psi}_{ij}$ in Panel b). This is the case, for example, for any product that is a widespread production input but never features in final consumption.

Finally, Figure 7 displays elements of the firm-to-firm cost share matrix $\widetilde{\Omega}_{ij}$ (corresponding to a disaggregated version of standard input-output tables). Panel (a) reports aggregates based on narrow (6-digit) categories that are available in the data. Panel (b) shows how this matrix changes through supply chain linkages by displaying the indirect linkages constructed from elements of the Lenotief inverse of this matrix, $\widetilde{\Psi}_{ij} - \tilde{\Omega}_{ij} - I$.

Figure 6: Consumer Expenditure Shares From Firm-to-Individual Transactions and Consumption Surveys



(a) Direct consumption: b_{ci} (b) Indirect consumption: $\sum_i b_{ci}\tilde{\Psi}_{ij}$

Notes: Panel (a) displays elements of the matrix b_{ci}, as revealed in the transaction-level individual-firm matched consumption data as well as the statistically-matched expenditures from the consumption survey, for four types of individuals c (based on income groups) by product category i (aggregated across firms in each category. By construction, $\sum_i b_{ci} = 1$ for each c. Panel (b) displays the analog but for indirect consumption, defined as $\sum_i b_{ci}\tilde{\Psi}_{ij}$.

Figure 7: Firm-to-Firm Expenditure Shares

(a) Direct Linkages: $\widetilde{\Omega}_{ij}$ (b) Indirect Linkages: $\widetilde{\Psi}_{ij} - \tilde{\Omega}_{ij} - I$

Notes: Panel (a) displays elements of the matrix of firm-to-firm input cost shares $\widetilde{\Omega}_{ij}$ for groups of buying and selling firms i and j using 6-digit sectors. Panel (b) does the same but for the indirect linkages through supply chains captured through the Lenotief inverse of this matrix, $\widetilde{\Psi}_{ij} - \tilde{\Omega}_{ij} - I$.

4.2 Elasticities of Substitution

The second requirement of our simulation procedure is the elasticities of substitution θ_i that appear in Equation (19), in both technology and preferences. Technological elasticities also play a role in our estimation of distortions, as discussed further below. In both cases we use functional form choices that have been common in prior work.

Beginning with preferences, we specify the nested demand system from Equation (19) as a two-tier system across products and sectors. In particular, at the lower level, the elasticity θ_s denotes substitution across products within sector s, and the upper-level elasticity across sectors is denoted by θ_U. We obtain estimates of the parameters θ_s from Gervais and Jensen (2019) and the parameter θ_U from Redding and Weinstein (2024), which consider products and sectors using similar levels of aggregation as we do. The former study is based on US data and the latter is based on Chilean data.

On the production side, we assume that firms' production functions take the Cobb-Douglas form across groups of inputs: capital, labor, and material inputs coming from each sector. This amounts to setting $\theta_i = 1$ in Equation (19) for every firm when it concerns substitution across such groups. Then, as with common specifications of Cobb-Douglas technologies, we assume that inputs within these groups (e.g. different versions of capital) are perfect substitutes.

4.3 Distortions

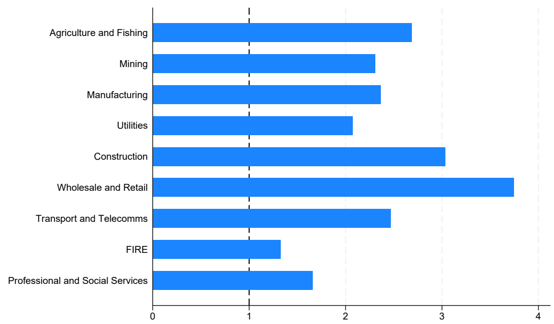
The final input into our analysis is the size of the wedges μ_i and τ_{ij} themselves. As emphasized by De Loecker and Warzynski (2012), for example, this is straightforward once estimates of firms' technologies (as discussed above) are known. A key identity in Hall (1986) expresses a firm's markup (or more generally, its output wedge) μ_i as the ratio of the firm's output elasticity, for any distortion-free input, to the firm's expenditure on that input as a share of the firm's total revenues. In our case, with potential bilateral distortions τ_{ij} on each input j, the analog of this result is

$$\frac{\eta_{ij}}{\Omega_{ij}} = \mu_i \tau_{ij}, \tag{9}$$

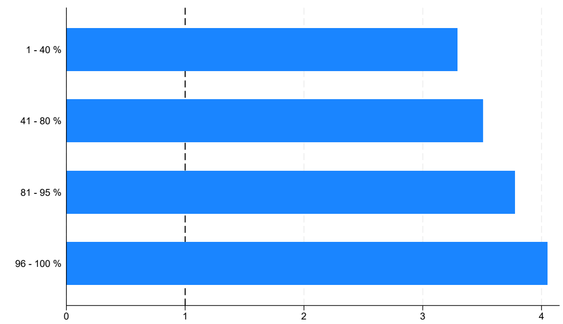
where η_{ij} is the elasticity of firm i's output with respect to input j. Equation (9) highlights how the combination $\mu_i \tau_{ij}$ is identified but the separate components μ_i and τ_{ij} are not. However, this is inconsequential for our study of incidence because, as discussed above, our formulae depend only on $\mu_i \tau_{ij}$.

In the case of Cobb-Douglas technologies, as assumed above, we follow the approach developed in Hsieh and Klenow (2009) for measuring the output elasticities η_{ij}. This method assumes that US firms use (on average within each sector) inputs in a non-misallocated fashion, and that their technologies are Cobb-Douglas over capital, labor, and each sector's type of materials. In this case the output elasticities η_{ij} in Equation (9) are obtained from the average shares (across firms, within each sector) of each type of input in the costs of US firms. The resulting wedge estimates from applying this procedure, $\mu_i \tau_{ij}$ for j equals labor, capital, and materials show considerable dispersion, as found by Hsieh and Klenow (2009) for the cases of China and India. There is also substantial dispersion of average wedges across sectors and across firm size bins, with Figure 8 displaying these averages by input type. Particularly notable are the large wedges on wholesale and retail labor, and the finding that wedges tend to be bigger in larger firms—patterns that will interact with the cross-group heterogeneity in exposure detailed above when calculating incidence.

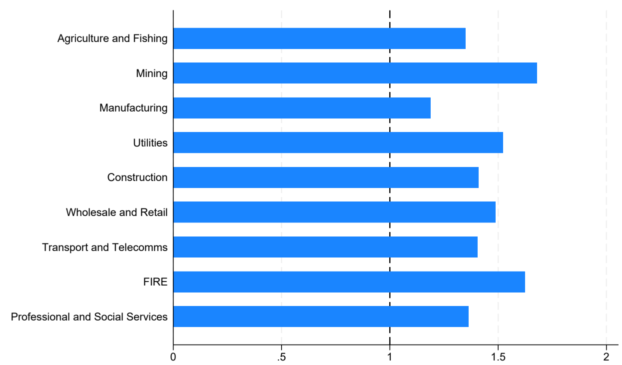
Figure 8: Estimates of Average Wedges By Sector and Firm Size



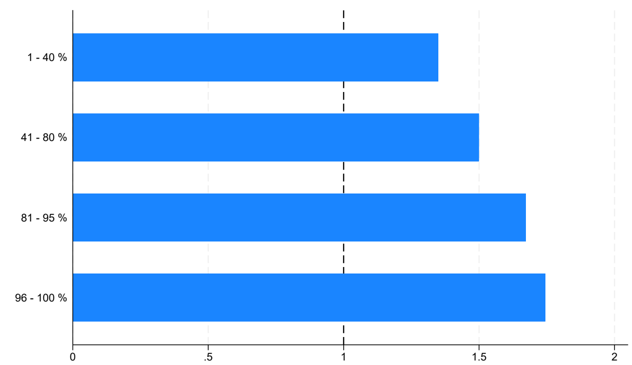
(a) Labor Wedges by Sector



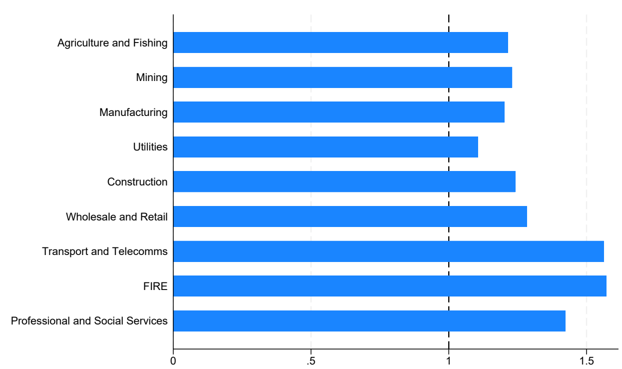
(b) Labor Wedges by Firm Size



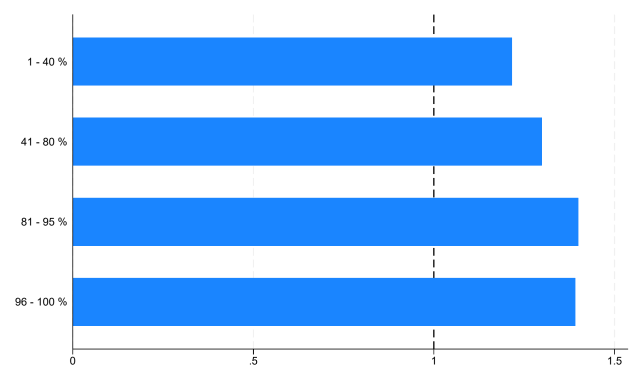
(c) Capital Wedges by Sector



(d) Capital Wedges by Firm Size



(e) Materials Wedges by Sector



(f) Materials Wedges by Firm Size

Notes: Panel (a) displays average values of the estimated wedge $\mu_i \tau_{ij}$, where j is labor, obtained by using the Hsieh and Klenow (2009) method, for groups of firms i arranged by sector. Panel (c) and (e) do the same for the wedge $\mu_i \tau_{ij}$, where j is capital and materials, respectively. Panel (b), (d) and (f) repeats these same wedges for labor, capital, and materials respectively, but for groups of firms i arranged by firm size rather than sector.

5 Measuring the Incidence of Distortions

Having populated the formulae in Section 2 with the data and estimates described in Sections 3 and 4, we now perform a number of counterfactual simulations designed to address the key question at the heart of this paper: what is the incidence of distortions?

The first set of counterfactuals asks how the burden of distortions is shared across various groups in society. To do so, we can use Equation (1) to calculate the welfare changes experienced by different groups if all distortions in the economy were removed. Specifically, we eliminate all capital, labor and materials-input wedges such that consumers and downstream firms pay only the marginal costs of production, and firms pay labor and capital their value marginal products. We focus on differences in incidence not only across the income distribution, but also by gender and age. This allows us to explore hypotheses such as do the poor bear more of the incidence of distortions, or do certain age groups, or do women? And are these differences quantitatively important? While executing any such counterfactuals, we hold constant features of the Chilean tax and transfers system such as the corporate profit tax rate and the value-added tax system.[19]

To better understand where any differences come from, we decompose the total welfare effects for each demographic group in a number of ways. First, it is straightforward to decompose the total effect into that coming from the consumption side, from factor services, and from changes in non-factor services such as rents and transfers. Furthermore, we can break down the consumption-side effects in two additional ways. First, we can decompose the impacts from reducing different combinations of distortion—labor, capital and material wedges, in all three cases multiplied by output wedges. Second, we further decompose each of these into those coming from changing goods and services prices (the first two terms in Equation (11)) and those coming from changing factor prices (the last terms in Equation (11)).

Our second set of counterfactuals asks which particular distortions are most responsible for the unequal burden of distortions. By removing certain wedges or sets of wedges we explore which distortions matter most for welfare and to which groups, both because some wedges are larger in magnitude and because their incidence differs conditional on size. In addition, this set of counterfactuals asks which wedges reinforce each other and which are countervailing, in the sense that interacting distortions may partially correct the harmful effects that each would cause in isolation. For example, distortions a firm faces on the output side may counteract those they face on the input side, or greater heterogene-

[19]In particular, we assign to each firm the profit tax rate that they were paying in 2022, and to each firm-product the relevant statutory value-added tax rate (which varies by product).

ity in wedges across firms within a sector may mitigate misallocation due to that sector having a relatively low average level of wedges.

Finally, our third set of counterfactuals explore a related question: what are the trade-offs between equity and efficiency of specific policy changes, and what is the equity-efficiency frontier that policymakers face? For example, how much would policies that reduce wedges in credit markets or the monopsony power of firms raise aggregate output and would this come at the expense of increases in certain forms of inequality? Are there other policy mixes that achieve the same increase in output but in a more equitable manner?

In this preliminary draft, the calculations that follow are based on a version that is simplified in a number of dimensions:

- When reducing wedges we solve only for the linearized effect of a 1% change in any wedge. This calculates the local elasticity of responses to such small shocks, which amounts to the first step in an iterative algorithm for calculating the effects of large shocks (such as the complete elimination of wedges).
- Individuals are not grouped into households via the civil registry.
- Indirect firm ownership via pensions is not included. (But all firms' remaining ownership shares are allocated proportionally across direct owners so that ownership shares always sum to one.)
- We set $\theta_U = 1$ and $\theta_s = 1.3$ for all s.
- Types of labor are broken down by region but not further by, for example, education group.

We also focus on impacts across the income distribution, leaving the study of wider demographic impacts to future versions.

5.1 The Distributional Impact of Eliminating Wedges

As a first counterfactual calculation, we begin with the exercise described above in which all wedges are removed. To be consistent with the first-order approximation underlying the results of Section 2, we implement a reduction of 1% of all wedges, thereby reducing each wedge in proportion to its size.

Figure 9 displays the results of this exercise. The hypothetical change in wedges in the economy induces considerable improvements in the real income of individuals who start in all of the four income bands that we have introduced above. These effects range from

0.4% to 0.55%.[20] Members of the upper middle-class group gain the most (in terms of percentage growth in their real income), though the difference across groups is relatively small. The fact that there are broad gains for all groups is not surprising as the removal of distortions raises efficiency and hence the total size of the pie.

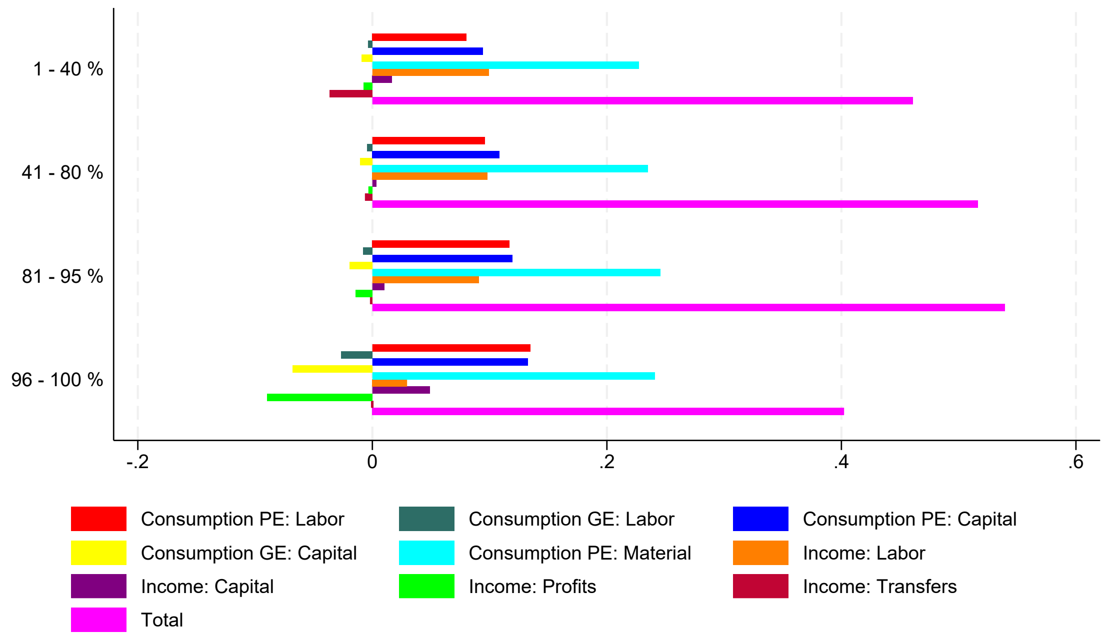


Figure 9: Welfare Impacts of Reducing All Distortions by 1%: Across Income

Notes: This figure displays the impacts on the real income $\mathcal{Y}_c$ (expressed as percent changes) of individuals c across income (grouped by their percentile in the initial income distribution) of a counterfactual exercise that reduces all distortions in the economy by 1%. Impacts are broken down by 4 channels. First, the partial-equilibrium consumption exposure to labor, capital and material wedges. Second, the general equilibrium changes in factor income coming from changes in factor prices. Third, the impact that factor price changes have on consumption prices (labelled GE). Finally, the impact on non-factor income, which involves profits and transfers.

Following Section 2, we break down these results into four sets of channels. First, we examine a partial-equilibrium consumption channel, which ignores factor price changes. We report this channel separately for labor, capital and materials wedges. Figure 9 shows that all income groups gain from this channel. The biggest gains come from shrinking material input wedges, then typically the gains from capital and labor wedges are similar but smaller than those from input wedges. These effects are again similar across income groups, though capital and labor wedge effects are slightly pro-rich and material wedge effects are slightly pro-middle class.

[20]Given the 1% reduction in the wedges, these numbers can be interpreted as elasticities.

Second, we consider the general equilibrium forces that arise due to changes in factor income of labor and capital.[21] As a response to the reduction in wedges, both factor prices increase. However, gains are unevenly distributed between income groups. The poorest groups gain relatively more from increases in labor income, whereas the richest group gains significantly less. On the other hand, gains from capital income changes tend to be pro-rich. This is to be expected given the evidence in Figure 4, namely that richer households rely relatively more on capital income than poorer households.

The third channel we investigate corresponds to the role that general equilibrium forces due to factor price changes have on final consumption prices. Figure 9 also show that individuals lose from this channel. This is natural given that we showed increases in factor income, which imply that the cost of production goes up. These losses are relatively large but fairly evenly distributed across the income distribution.[22]

Finally, we turn to the channel of non-factor income—that is, profits and (net) transfers. We find that these are unevenly distributed across the income distribution. Given that wedges go down, tax revenues also go down and thus transfers go down because the government keeps a balanced budget. We distribute these reductions in transfers proportional to the initial distribution of transfers across the income distribution. Thus, the poorest households lose the most out of this whereas the richest households are close to unaffected. On the other hand, the reduction in wedges reduces overall profits. These losses are largest for the richest group of individuals, since they rely the most on profit income, and are very sizable. Overall, the heterogeneity of this profit channel drives most of the heterogeneity we see in total welfare impacts across income groups—at least in these preliminary results, which omit a number of potentially important sources of heterogeneity.

Having examined incidence through the lens of heterogeneity across individuals grouped by income, we turn now to three additional differences. Figure 10 explores incidence by age, Figure 11 by gender, and Figure 12 by region. The gains from reducing wedges are positive for all age groups, though older individuals gain relatively less from reduction in wedges. This result comes primarily from the fact that older individuals are more exposed to profit losses than younger individuals are. Turning to gender differences, Figure 11 shows that women gain relatively more than men from reducing distortions. This difference comes mostly from consumption-based heterogeneity, with women more exposed (on the consumption side) to labor wedges, capital wedges, and material wedges. Finally,

[21] Given that factor supply is fixed and that the numeraire is GDP, these effects can be interpreted as changes in factor shares of GDP.

[22] We note that such conclusions may change once labor factors are defined by skill group and location rather than just location as in the current draft.

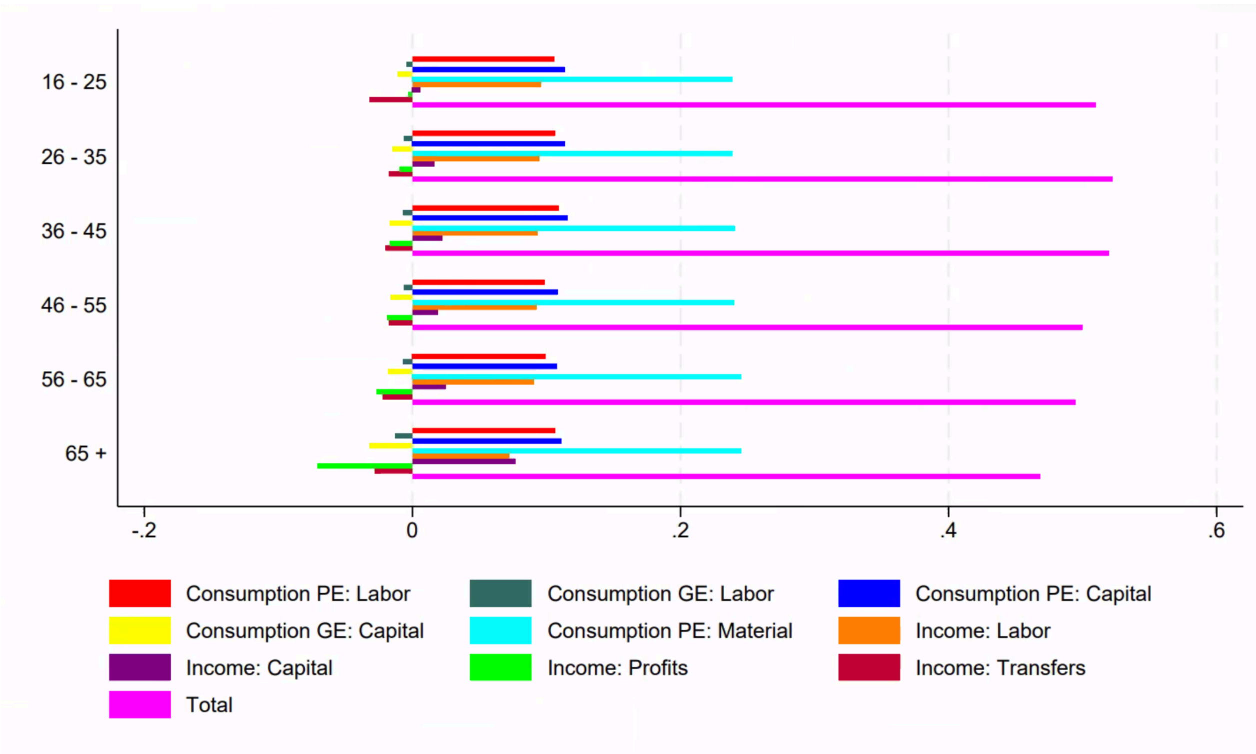


Figure 10: Welfare Impacts of Reducing All Distortions by 1%: Across Age

Notes: This figure displays the impacts on the real income $\mathcal{Y}_c$ of individuals c across age groups of a counterfactual exercise that reduces all distortions in the economy by 1%. Impacts are broken down by four channels. First, the partial-equilibrium consumption exposure to labor, capital and material wedges. Second, the general equilibrium changes in factor income coming from changes in factor prices. Third, the impact that factor price changes have on consumption prices (labeled GE). Finally, the impact on non-factor income, which involves profits and transfers.

with respect to geographic differences, in Figure 12 we split up Chile into three regions: Santiago (the country's political and commercial capital), other urban, and all rural. Evidently, residents of Santiago benefit least from reducing distortions, and this effect derives primarily from the exposure there of consumers to wedges (on all of labor, capital, and materials).

Figure 11: Welfare Impacts of Reducing All Distortions by 1%: Across Gender

Notes: This figure displays the impacts on the real income $\mathcal{Y}_c$ of individuals c across gender of a counterfactual exercise that reduces all distortions in the economy by 1%. Impacts are broken down by 4 channels. First, the partial-equilibrium consumption exposure to labor, capital and material wedges. Second, the general equilibrium changes in factor income coming from changes in factor prices. Third, the impact that factor price changes have on consumption prices (labeled GE). Finally, the impact on non-factor income, which involves profits and transfers.

5.2 Which Distortions Matter More?

The previous counterfactual exercise examined the impact of reducing all distortions throughout the economy. But it is natural to ask whether the incidence of certain types of distortions are borne more or less unequally than others. Towards this end, Figure 13 reports results from six separate simulations: the first is the baseline exercise of Section 5.1 (for purposes of comparison); the next three reduce (by 1%) only the labor, capital, and materials wedges in the economy, respectively; the fifth reduces all wedges, but only those in large firms; and the sixth reduces all wedges, but only those in manufacturing firms. Our

Figure 12: Welfare Impacts of Reducing All Distortions by 1%: Across Geography

Notes: This figure displays the impacts on the real income $\mathcal{Y}_c$ of individuals c across the location of individuals of a counterfactual exercise that reduces all distortions in the economy by 1%. Impacts are broken down by 4 channels. First, the partial-equilibrium consumption exposure to labor, capital and material wedges. Second, the general equilibrium changes in factor income coming from changes in factor prices. Third, the impact that factor price changes have on consumption prices (labeled GE). Finally, the impact on non-factor income, which involves profits and transfers.

baseline findings about the incidence of distortions remain strikingly similar across these different types of distortions.

5.3 Equity-Efficiency Tradeoffs

As we have seen in the counterfactual simulations above, reductions in distortions have implications for inequality, especially at the very top. But they also have consequences for the economy's overall efficiency level. Our final analysis compares these two impacts for every simulation discussed thus far. Figure 14 reports the results. Each dot represents a separate counterfactual simulation, with the y-axis value referring to the impact of the counterfactual change on equality (as measured by the negative of the Gini coefficient across individual-level nominal income changes) and the x-axis value referring to the impact on aggregate efficiency (as measured by the Baqaee and Burstein 2022 value). Broadly, there is no evidence for an equity-efficiency tradeoff across these exercises—those that offer larger efficiency gains (such as that which reduces wedges on large firms only)

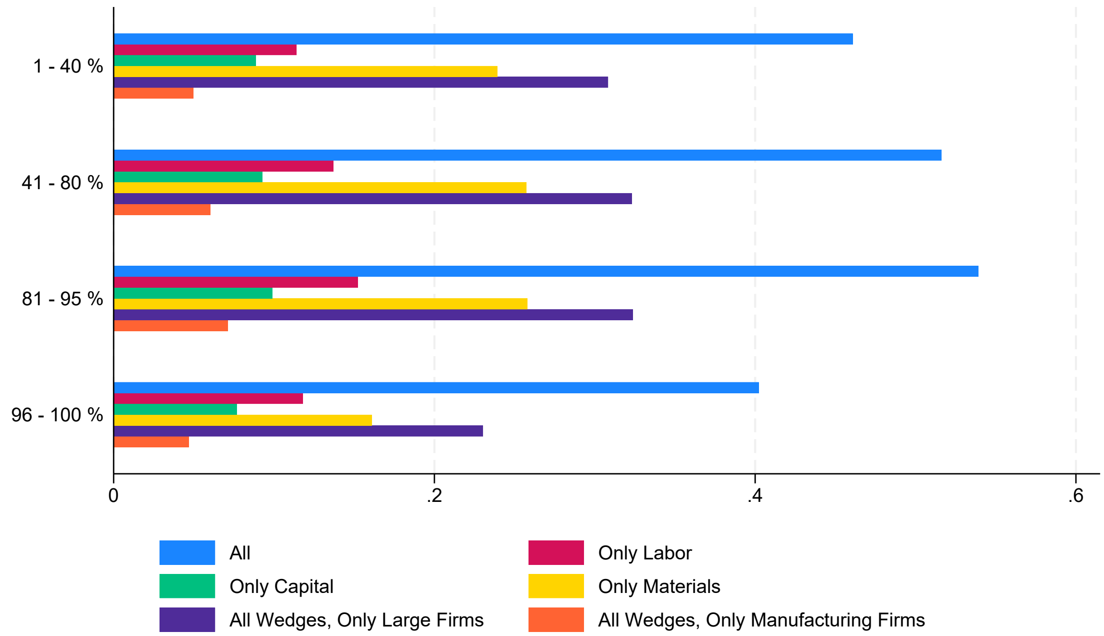


Figure 13: Welfare Impacts of Reducing Different Types of Distortions by 1%

Notes: This figure displays the impacts on the real income $\mathcal{Y}_c$ of individuals c across income groups of counterfactual exercise that reduce distortions in the economy by 1%. In the first, all distortions in the economy are reduced. Subsequent exercises reduce distortions only on labor, capital, materials, for large firms, and for manufacturing firms, respectively.

tend to also give rise to the greatest increases in (Gini-based) equality.

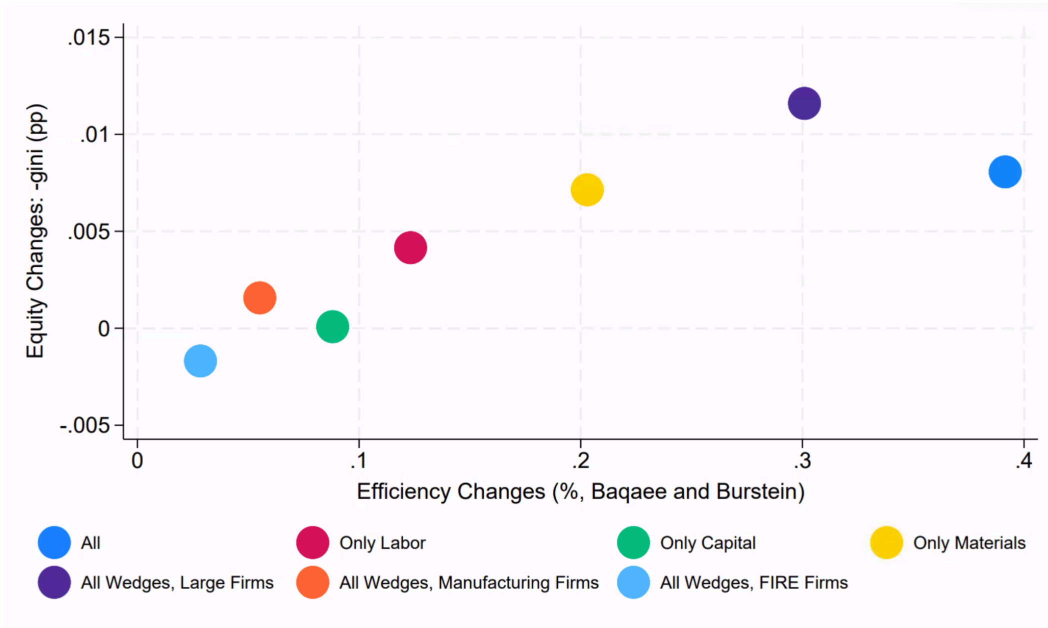


Figure 14: Equity-Efficiency Comparisons from Reducing Different Types of Distortions by 1%

Notes: Each dot in this figure displays the impacts of a separate counterfactual exercise. The y-axis reports the impact on the level of the negative of the Gini coefficient for nominal income (taken across all individuals). And the x-axis reports the impact on the level of the Baqaee and Burstein 2022 measure of aggregate efficiency.

6 Conclusion

Recent work has documented the pervasive extent of economic distortions—such as market power, taxes, tariffs, credit constraints, etc.—and how they lead to substantial misallocation, or aggregate productivity loss. Far less well understood is how these phenomena affect members of society differently. In this paper we combine unique datasets from Chile, linking workers and owners to firms, firms to each other, firms to consumers, and firms and consumers to the government, in order to quantify the full incidence of distortions for the first time.

References

Acosta, M. and L. Cox (2024). *The Regressive Nature of the US tariff code: Origins and Implications*. Tech. rep. Working Paper.

Adão, R., P. Carrillo, A. Costinot, D. Donaldson, and D. Pomeranz (2022). "Imports, Exports, and Earnings Inequality: Measures of Exposure and Estimates of Incidence". *The Quarterly Journal of Economics* 137, pp. 1553–1614.

Andersen, A. L., K. Huber, N. Johannesen, L. Straub, and E. T. Vestergaard (Nov. 2022). *Disaggregated Economic Accounts*. Working Paper 30630. National Bureau of Economic Research.

Atkin, D. and D. Donaldson (2022). "The Role of Trade in Economic Development". *Handbook of International Economics: International Trade, Volume 5*. Ed. by G. Gopinath, E. Helpman, and K. Rogoff. Elsevier, pp. 1–59.

Baqaee, D. and A. Burstein (2022). *Aggregate Welfare and Output with Heterogeneous Agents*. Tech. rep.

Baqaee, D. and E. Farhi (2019). "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem". *Econometrica* 87, pp. 1155–1203.

Baqaee, D. and E. Farhi (2020). "Productivity and Misallocation in General Equilibrium". *The Quarterly Journal of Economics* 135, pp. 105–163.

Blanchet, T., E. Saez, and G. Zucman (July 2022). *Real-Time Inequality*. Working Paper 30229. National Bureau of Economic Research.

Burstein, A., J. Cravino, and M. Rojas (2024). *Input Price Dispersion Across Buyers and Misallocation*. Tech. rep. Central Bank of Chile.

Conlon, C., N. Rao, and Y. Wang (2022). "Who Pays Sin Taxes? Understanding the Overlapping Burdens of Corrective Taxes". *Review of Economics and Statistics*, pp. 1–27.

De Loecker, J. and F. Warzynski (2012). "Markups and Firm-Level Export Status". *American Economic Review* 102, pp. 2437–71.

Faber, B. (2014). "Trade Liberalization, the Price of Quality, and Inequality: Evidence from Mexican Store Prices". UC Berkeley Department of Economics.

Faber, B. and T. Fally (May 2022). "Firm Heterogeneity in Consumption Baskets: Evidence from Home and Store Scanner Data". *The Review of Economic Studies* 89.3, pp. 1420–1459.

Gervais, A. and J. B. Jensen (2019). "The tradability of services: Geographic concentration and trade costs". *Journal of International Economics* 118, pp. 331–350.

Gupta, A. (2022). "Demand for Quality, Variable Markups and Misallocation: Evidence from India". Dartmouth working paper.

Hall, R. E. (1986). “Market Structure and Macroeconomic Fluctuations”. *Brookings Papers on Economic Activity* 2, pp. 285–338.

Hsieh, C.-T. and P. J. Klenow (2009). “Misallocation and Manufacturing TFP in China and India”. *Quarterly Journal of Economics* 124.4, pp. 1403–1448.

Jones, C. I. (2013). “Misallocation, Economic Growth, and Input–Output Economics”. *Advances in Economics and Econometrics*, pp. 419–456.

Manelici, I., J. P. Vasquez, M. Ulate, and R. D. Zarate (2024). *The Gains from Foreign Multinationals in an Economy with Distortions*. Tech. rep.

Redding, S. J. and D. E. Weinstein (2024). “Accounting for trade patterns”. *Journal of International Economics* 150, p. 103910.

Restuccia, D. and R. Rogerson (2008). “Policy distortions and aggregate productivity with heterogeneous establishments”. *Review of Economic Dynamics* 11.4, pp. 707–720.

Sangani, K. (2023). *Markups Across the Income Distribution: Measurement and Implications*. Tech. rep.

Schmitz, J. A. (2020). *Monopolies Inflict Great Harm on Low-and Middle-Income Americans*. Tech. rep. Federal Reserve Bank of Minneapolis.

Sharma, G. (2023). *Monopsony and Gender*. Tech. rep.

A Theory Appendix

In this appendix, we first provide detailed derivations of the key equations from Section 2. Then, we express the system of equations in matrix form, which we use for computational implementation.

A.1 Change in Welfare

Proposition 1.

$$d\ln\mathcal{Y}_c = d\ln\chi_c - \sum_{i\in\mathcal{N}} v_i b_{ci}(d\ln v_i + d\ln p_i). \tag{10}$$

Proof. Under homothetic preferences, $\frac{de_c(p,u)}{du} = e(p,1) \equiv P$. Moreover, by Shephard's lemma we have that $\frac{de_c(p,u)}{dp_i} = x_{ci}$, where x_{ci} is the consumption of good i by consumer c. For each unit of good i, the consumer pays an ad-valorem tax over the price, thus each unit costs $v_i p_i$. Then, with a slight abuse of notation, by totally differentiating the expenditure function we have

$$\begin{aligned}
& e_c(vp,\mathcal{Y}_c) = \chi_c \\
\implies\ & \frac{de_c(vp,u)}{dvp}dvp + \frac{de_c(vp,u)}{du}d\mathcal{Y}_c = d\chi_c \\
\implies\ & x_{ci}dvp + Pd\mathcal{Y}_c = d\chi_c \\
\implies\ & x_{ci}vpd\ln vp + P\mathcal{Y}_c d\ln\mathcal{Y}_c = \chi_c d\ln\chi_c \\
\implies\ & \sum_i v_i x_{ci} p_i(d\ln v_i + d\ln p_i) + \chi_c d\ln\mathcal{Y}_c = \chi_c d\ln\chi_c \\
\implies\ & \sum_i v_i b_{ci}(d\ln v_i + d\ln p_i) + d\ln\mathcal{Y}_c = d\ln\chi_c,
\end{aligned}$$

where the last line uses the definition $b_{ci} \equiv \frac{p_i x_{ci}}{\chi_c}$. The result follows.

□

A.2 Change in Prices

Proposition 2.

$$d\ln p_i = \sum_{j\in\mathcal{N}} \widetilde{\Psi}_{ij} d\ln\mu_j + \sum_{j\in\mathcal{N};\ k\in\mathcal{N},\mathcal{F}} \widetilde{\Psi}_{ij}\widetilde{\Omega}_{jk} d\ln\tau_{jk} + \sum_{f\in\mathcal{F}} \widetilde{\Psi}_{if} d\ln w_f. \tag{11}$$

Proof. By totally differentiating the CRS unit cost function and using Shephard's lemma, we can write

$$
\begin{aligned}
dc_i &= \sum_j \frac{dc_i}{d(\tau_{ij}p_j)} d(\tau_{ij}p_j) + \sum_f \frac{dc_i}{d(\tau_{if}w_f)} d(\tau_{if}w_f) \\
\implies dc_i &= \sum_j x_{ij} d(\tau_{ij}p_j) + \sum_f x_{if} d(\tau_{if}w_f) \\
\implies c_i d\ln c_i &= \sum_j x_{ij}\tau_{ij}p_j d(\ln \tau_{ij}p_j) + \sum_f x_{if}\tau_{if}w_f d(\ln \tau_{if}w_f) \\
\implies d\ln c_i &= \sum_j \tilde{\Omega}^p_{ij}(d\ln \tau_{ij} + d\ln p_j) + \sum_f \tilde{\Omega}^w_{if}(d\ln \tau_{if} + d\ln w_f),
\end{aligned}
$$

where $\tilde{\Omega}^p \equiv (\tilde{\Omega}_{ij})_{i\in\mathcal{N},j\in\mathcal{N}}$ and $\tilde{\Omega}^w \equiv (\tilde{\Omega}_{ij})_{i\in\mathcal{N},j\in\mathcal{F}}$. We also have $d\ln p_i = d\ln \mu_i + d\ln c_i$, and hence

$$
d\ln p_i = d\ln \mu_i + \sum_{j\in\mathcal{N}} \tilde{\Omega}^p_{ij}\left(d\ln \tau_{ij} + d\ln p_j\right) + \sum_{f\in F} \tilde{\Omega}^w_{if}(d\ln \tau_{if} + d\ln w_f),
$$

where $\tilde{\Omega}^p = (\tilde{\Omega}_{ij})_{i\in\mathcal{N},j\in\mathcal{N}}$ and $\tilde{\Omega}^w = (\tilde{\Omega}_{ij})_{i\in\mathcal{N},j\in\mathcal{F}}$

The first term captures the direct change in firm i's price due to the change in its markup, the second term captures the (direct and indirect) change induced by a change in goods prices faced by firm i, and the third term captures the (direct and indirect) change due to changes in factor prices faced by firm i.

Define $d\ln \tilde{U}_{ij} \equiv \tilde{\Omega}_{ij} d\ln \mu_i \tau_{ij}$ and $d\ln \tilde{U}_i \equiv \sum d\ln \tilde{U}_{ij}$. Using $\sum_{j\in\mathcal{N},\mathcal{F}} \tilde{\Omega}_{ij} = 1$ for all $i \in \mathcal{N}$, we can rewrite the above equation in the following matrix notation as

$$
d\ln p = d\ln \tilde{U} + \tilde{\Omega}^p d\ln p + \tilde{\Omega}^w d\ln w.
$$

Simplifying we have the desired result:

$$
d\ln p = \tilde{\Psi}^p d\ln \tilde{U} + \tilde{\Psi}^w d\ln w. \tag{12}
$$

□

A.3 Changes in Profits

Proposition 3.

$$
d\pi_i = \left(\frac{\pi_i + T_i}{\lambda_i}\right) d\lambda_i + \lambda_i \sum_{j\in\mathcal{F},\mathcal{N}} \Omega_{ij}(d\ln \mu_i + d\ln \tau_{ij} - d\ln \widetilde{\Omega}_{ij}) - dT_i, \tag{13}
$$

Proof. Firm profits are given by

$$\pi_i = p_i y_i - \sum_{j \in \mathcal{N},\mathcal{F}} p_{ij} x_{ij} - T_i = p_i y_i \left(1 - \sum_{j \in \mathcal{N},\mathcal{F}} \frac{p_{ij} x_{ij}}{p_i y_i}\right) - T_i.$$

Since total expenditure is the numeraire, we can express the above equation in terms of sales shares λ_i. Using $\Omega_{ij} = \frac{p_{ij} x_{ij}}{p_i y_i}$, we have

$$\pi_i = \lambda_i \sum_{j \in \mathcal{N},\mathcal{F}} (1 - \Omega_{ij}) - T_i.$$

Totally differentiating this expression we obtain

$$d\pi_i = d\left(\lambda_i \sum_{j \in \mathcal{N},\mathcal{F}} (1 - \Omega_{ij})\right) - dT_i = \lambda_i d \sum_{j \in \mathcal{N},\mathcal{F}} (1 - \Omega_{ij}) + d\lambda_i \sum_{j \in \mathcal{N},\mathcal{F}} (1 - \Omega_{ij}) - dT_i.$$

Finally, we arrive at the desired equation by substituting $d\sum_j (1 - \Omega_{ij}) = -\sum_j d\Omega_{ij} = -\sum_j \Omega_{ij} d\ln\Omega_{ij} = \sum_j \Omega_{ij}(d\ln\tilde{\Omega}_{ij} - d\ln(\mu_j \tau_{ij}))$ and $\pi_i = \lambda_i \sum_{j \in \mathcal{N},\mathcal{F}} (1 - \Omega_{ij}) - T_i$, or hence $\sum_{j \in \mathcal{N},\mathcal{F}} (1 - \Omega_{ij}) = \frac{\pi_i + T_i}{\lambda_i}$ in the expression above.

□

A.4 Changes in Sales and Factor Income Shares

Proposition 4. *For $i \in \mathcal{N}, \mathcal{F}$*

$$\begin{aligned} d\lambda_i = &- \sum_{l \in \mathcal{C},\mathcal{N}; m \in \mathcal{N}} \lambda_l \Omega_{lm} (d\ln\mu_l + d\ln\tau_{lm}) \Psi_{mi} \\ &+ \sum_{k \in \mathcal{C},\mathcal{N}} \mu_k^{-1} \lambda_k Cov_{\widetilde{\Omega}^{(k)}}(d\ln\widetilde{\Omega}^{(k)}, diag(\tau^{(k)})^{-1}\Psi_{(i)}) + \sum_{c \in \mathcal{C}} d\chi_c \sum_{k \in \mathcal{N}} b_{ck} \Psi_{ki}. \end{aligned} \tag{14}$$

Proof. First note that, for $i \in \mathcal{N}$, we have

$$\lambda_i = \sum_{c \in \mathcal{C}} \chi_c b_{ci} + \sum_{j \in \mathcal{N}} \lambda_j \Omega_{ji}.$$

Using this equation, we have

$$\lambda_i = \sum_{c \in \mathcal{C}, j \in \mathcal{N}} \chi_c b_{cj} \Psi^p_{ji}$$

where $\Psi^p \equiv (I - \Omega^p)^{-1}$. For $f \in \mathcal{F}$, we have

$$\lambda_f = \sum_{i \in \mathcal{N}} \lambda_i \Omega_{if} = \sum_{i \in \mathcal{N}} \sum_{c \in \mathcal{C}, k \in \mathcal{N}} \chi_c b_{ck} \Psi^p_{ki} \Omega_{if} = \sum_{i \in \mathcal{N}} \sum_{c \in \mathcal{C}} \chi_c b_{ck} \Psi_{kf}$$

where the last equality uses

$$\Psi = \begin{bmatrix} \Psi^p_{N\times N} & \Psi^p_{N\times N}\Omega^w_{N\times F} \\ \mathbf{0} & I \end{bmatrix}$$

where, recall, $\Omega^w = (\Omega_{ij})_{i\in\mathcal{N};\, j\in\mathcal{F}}$. Therefore, for $i \in \mathcal{N}, \mathcal{F}$, we have

$$\lambda_i = \sum_{c\in\mathcal{C}, j\in\mathcal{N}} \chi_c b_{cj} \Psi_{ji}, \tag{15}$$

which implies

$$d\lambda_i = \sum_{c\in\mathcal{C}, j\in\mathcal{N}} \chi_c b_{cj} (d\Psi_{ji}) + \chi_c (db_{cj}) \Psi_{ji} + (d\chi_c) b_{cj} \Psi_{ji}. \tag{16}$$

The third term of this equation coincides with the third term of (14).

To further characterize the first term of (16), note that

$$\begin{aligned} & \Psi = (I-\Omega)^{-1} \\ \Longrightarrow\ & \Psi(I-\Omega) = I \\ \Longrightarrow\ & \Psi = I + \Psi\Omega \\ \Longrightarrow\ & d\Psi = d\Psi\Omega + \Psi d\Omega \\ \Longrightarrow\ & d\Psi(I-\Omega) = \Psi d\Omega \\ \Longrightarrow\ & d\Psi = \Psi d\Omega\Psi. \end{aligned}$$

This implies that we can substitute the expression for $d\Psi_{ji}$ in (16) to obtain

$$\begin{aligned} \sum_{c\in\mathcal{C}, j\in\mathcal{N}} \chi_c b_{cj}(d\Psi_{ji}) &= \sum_{c\in\mathcal{C}, j\in\mathcal{N}} \sum_{l\in\mathcal{N}, m\in\mathcal{N}} \chi_c b_{cj} \Psi_{jl} d\Omega_{lm} \Psi_{mi} \\ &= \sum_{l\in\mathcal{N}, m\in\mathcal{N}} \lambda_l d\Omega_{lm} \Psi_{mi} \\ &= \sum_{l\in\mathcal{N}, m\in\mathcal{N}} \lambda_l \Omega_{lm} d\ln\Omega_{lm} \Psi_{mi} \\ &= \sum_{l\in\mathcal{N}, m\in\mathcal{N}} \lambda_l \Omega_{lm} \left(d\ln\tilde{\Omega}_{lm} - d\ln\mu_l \tau_{lm} \right) \Psi_{mi} \\ &= -\sum_{l\in\mathcal{N}, m\in\mathcal{N}} \lambda_l \Omega_{lm} \left(d\ln\mu_l + d\ln\tau_{lm} \right) \Psi_{mi} + \sum_{l\in\mathcal{N}, m\in\mathcal{N}} \lambda_l \Omega_{lm} d\ln\tilde{\Omega}_{lm} \Psi_{mi}, \end{aligned} \tag{17}$$

where the second equality uses (15).

Also note that

$$\sum_{c\in\mathcal{C},j\in\mathcal{N}} \chi_c(db_{cj})\Psi_{ji} = \sum_{c\in\mathcal{C},j\in\mathcal{N}} \chi_c b_{cj}(d\ln\tilde{b}_{cj} - d\ln v_j)\Psi_{ji} \tag{18}$$
$$= -\sum_{c\in\mathcal{C},j\in\mathcal{N}} \chi_c b_{cj}(d\ln v_j)\Psi_{ji} + \sum_{c\in\mathcal{C},j\in\mathcal{N}} \chi_c b_{cj} d\ln\tilde{b}_{cj}\Psi_{ji},$$

where $\tilde{b}_{ci} \equiv \frac{v_i p_i x_{ci}}{\chi_c}$. Using the definitions

$$b_{ci} = \Omega_{ci},\ \chi_c = \lambda_c,\ \mu_c = 1,\ \tau_{ci} = v_i,$$

we can verify that the first terms of Equations (17) and (18) when added coincide with the first term of (14). It reamins to demonstrate that

$$\sum_{l\in\mathcal{N},m\in\mathcal{N}} \lambda_l\Omega_{lm} d\ln\tilde{\Omega}_{lm}\Psi_{mi} + \sum_{c\in\mathcal{C},j\in\mathcal{N}} \chi_c b_{ci} d\ln\tilde{b}_{cj}\Psi_{ji} = \sum_{l\in\mathcal{N},\mathcal{C};\ m\in\mathcal{N}} \lambda_l\Omega_{lm} d\ln\tilde{\Omega}_{lm}\Psi_{mi}$$

coincides with the second term of (14). To see this, note that

$$\begin{aligned}
&\sum_{l\in\mathcal{N},\mathcal{C};\ m\in\mathcal{N}} \lambda_l\Omega_{lm} d\ln\tilde{\Omega}_{lm}\Psi_{mi} \\
&\quad= \sum_{k\in\mathcal{N},\mathcal{C};\ m\in\mathcal{N}} \lambda_k(\mu_k)^{-1}\tilde{\Omega}_{km} d\ln\tilde{\Omega}_{km}(\tau_{km})^{-1}\Psi_{mi} \\
&\quad= \sum_{k\in\mathcal{N},\mathcal{C}} \lambda_k(\mu_k)^{-1} \sum_{m\in\mathcal{N}} \tilde{\Omega}_{km} d\ln\tilde{\Omega}_{km}(\tau_{km})^{-1}\Psi_{mi} \\
&\quad= \sum_{k\in\mathcal{N},\mathcal{C}} \lambda_k(\mu_k)^{-1} \left(\sum_{m\in\mathcal{N}} \tilde{\Omega}_{km} d\ln\tilde{\Omega}_{km}(\tau_{km})^{-1}\Psi_{mi} - \sum_{m\in\mathcal{N}} \tilde{\Omega}_{km} d\ln\tilde{\Omega}_{km} \sum_{m\in\mathcal{N}} \tilde{\Omega}_{km}(\tau_{km})^{-1}\Psi_{mi} \right) \\
&\quad= \sum_{k\in\mathcal{N},\mathcal{C}} \lambda_k(\mu_k)^{-1} Cov_{\tilde{\Omega}^{(k)}}(d\ln\tilde{\Omega}^{(k)}, diag(\tau^{(k)})^{-1}\Psi_{(i)})
\end{aligned}$$

where the third equality uses the fact that $\sum_m \tilde{\Omega}_{km} = 1$ implies $\sum_m d\tilde{\Omega}_{km} = \sum_m \tilde{\Omega}_{km} d\ln\tilde{\Omega}_{km} = 0$. □

A.5 Change in Cost Shares

Proposition 5.

$$d\ln\widetilde{\Omega}_{ij} = (1-\theta_i)\left(d\ln v_j + d\ln p^I_j - \sum_{k\in\mathcal{F},\mathcal{N}} \widetilde{\Omega}_{ik}(d\ln v_k + d\ln p^I_k) \right). \tag{19}$$

Proof. Under CES demand[23], the consumption share for buyer $i \in \mathcal{N},\mathcal{C}$ of good $j \in \mathcal{N},\mathcal{F}$

[23] We will derive the equation using the consumer problem. The same logic applies to the firm's problem,

is given by

$$\tilde{\Omega}_{ij} = \frac{\beta_{ij}^{\theta_i}(v_j p_j^I)^{1-\theta_i}}{\sum_{l\in\mathcal{N},\mathcal{F}} \beta_{il}^{\theta_i}(v_l p_l^I)^{1-\theta_i}},$$

where β_{ij} is an arbitrary preference parameter, and v_i is a tax. Taking logs and differentiating both sides we have

$$d\ln\tilde{\Omega}_{ij} = (1-\theta_i)(d\ln v_j + d\ln p_j^I) - \frac{\sum_{k\in\mathcal{N},\mathcal{F}} \beta_{ik}^{\theta_i}(1-\theta_i)(v_k p_k^I)^{1-\theta_i} d\ln(v_k p_k^I)}{\sum_{l\in\mathcal{N},\mathcal{F}} \beta_{il}^{\theta_i}(v_l p_l^I)^{1-\theta_i}} =$$
$$= (1-\theta_i)(d\ln v_j + d\ln p_j^I - \sum_{k\in\mathcal{N},\mathcal{F}} \tilde{\Omega}_{ik}(d\ln v_k + d\ln p_k^I),$$

where in the second equality we have used the definition of $\tilde{\Omega}_{ik}$. □

A.6 System of Equations in Matrix Form

We first reduce the system of equations further in three sets of unknowns: $d\ln\lambda_i$, $d\ln\lambda_f$, and $d\ln\chi_c$. Substituting the expression for $d\ln\tilde{\Omega}$ into the expressions for $d\lambda_i$ we obtain

$$d\ln\lambda_i = -\sum_{l\in\mathcal{C},\mathcal{N},\; m\in\mathcal{N}} \frac{\lambda_l\Omega_{lm}}{\lambda_i}(d\ln\mu_l + d\ln\tau_{lm})\Psi_{mi}$$
$$+ \sum_{k\in\mathcal{C},\mathcal{N}} \mu_k^{-1}\frac{\lambda_k}{\lambda_i}(1-\theta_k)\mathrm{Cov}_{\widetilde{\Omega}^{(k)}}(d\ln\mathbf{v} + d\ln\mathbf{p}, diag(\tau^{(k)})^{-1}\Psi_{(i)}) + \sum_{c\in\mathcal{C}} \frac{\sum_{k\in\mathcal{N}}\chi_c b_{ck}\Psi_{ki}}{\lambda_i} d\ln\chi_c,$$

where $\mathbf{v}$ denotes the vector of consumption taxes if $k\in\mathcal{C}$ and are markups if $k\in\mathcal{N}$. Substituting dT_i in the expression for $d\pi_i$, we have

$$d\pi_i = \frac{1}{t_i^p}\left((1-\sum_{j\in\mathcal{N},\mathcal{F}} t_{ij}\Omega_{ij})d\lambda_i + \lambda_i\sum_{j\in\mathcal{F},\mathcal{N}} \Omega_{ij}t_{ij}(d\ln\mu_i + d\ln\tau_{ij} - d\ln\widetilde{\Omega}_{ij}) - \sum_{j\in\mathcal{N},\mathcal{F}} \lambda_i\Omega_{ij}dt_{ij} - \pi_i dt_i^p\right),$$

or in logs

$$d\ln\pi_i = d\ln\lambda_i + \frac{\lambda_i}{t_i^p\pi_i}\sum_{j\in\mathcal{F},\mathcal{N}} \Omega_{ij}t_{ij}(d\ln\mu_i + d\ln\tau_{ij} - d\ln t_{ij} - d\ln\widetilde{\Omega}_{ij}) - d\ln t_i^p.$$

---

but instead of consumer taxes v_i we have markups τ_{ij}.

Substituting dT_i in the expression for dT_c, we arrive at

$$dT_c = \Phi_{cg} \sum_{i\in\mathcal{N},\mathcal{C};\ j\in\mathcal{N},\mathcal{F}} \lambda_i \Omega_{ij} dt_{ij} + (t_{ij}-1)\Omega_{ij}\left(d\lambda_i - \lambda_i(d\ln\mu_i + d\ln\tau_{ij} - d\ln\tilde{\Omega}_{ij})\right)$$
$$+ \sum_{i\in\mathcal{N}} \left(\pi_i dt_i^p + (t_i^p - 1)d\pi_i\right).$$

In logs this is

$$\begin{aligned} d\ln T_c =& \Phi_{cg} \sum_{i\in\mathcal{N},\mathcal{C};\ j\in\mathcal{N},\mathcal{F}} \frac{(t_{ij}-1)\lambda_i\Omega_{ij}}{T_c}\left(d\ln\lambda_i + d\ln(t_{ij}-1) - (d\ln\mu_i + d\ln\tau_{ij} - d\ln\tilde{\Omega}_{ij})\right) \\ &+ \Phi_{cg}\sum_{i\in\mathcal{N}} \frac{\pi_i(t_i^p-1)}{T_c}\left(d\ln\pi_i + d\ln(t_i^p - 1)\right). \end{aligned}$$

The fact that firms have Cobb-Douglas technologies implies that $d\ln\tilde{\Omega}_{ij} = 0$ for all $i\in\mathcal{N},\ j\in\mathcal{N},\mathcal{F}$. This implies that we can write the equation for $d\ln\pi_i$ as

$$d\ln\pi_i = d\ln\lambda_i + \frac{\lambda_i}{t_i^p\pi_i}\sum_{j\in\mathcal{F},\mathcal{N}} \Omega_{ij}t_{ij}(d\ln\mu_i + d\ln\tau_{ij} - d\ln t_{ij}) - d\ln t_i^p.$$

Substituting the expression for $d\ln\pi_i$ into the expression for $d\ln T_c$, and denoting $\sum_c T_c = T_g$ we have

$$\begin{aligned} d\ln T_c =& \Phi_{cg}\sum_{i\in\mathcal{N};\ j\in\mathcal{N},\mathcal{F}} \frac{(t_{ij}-1)\lambda_i\Omega_{ij}}{T_c}\left(d\ln\frac{t_{ij}-1}{t_{ij}}\right) \\ &+ \Phi_{cg}\sum_{c\in\mathcal{C};\ j\in\mathcal{N}} \frac{(v_j-1)\chi_c b_{cj}}{T_c}\left(d\ln\chi_c + d\ln\frac{v_j-1}{v_j} + d\ln\tilde{\Omega}_{cj})\right) \\ &+ \Phi_{cg}\sum_{i\in\mathcal{N}} \frac{\lambda_i\sum_{j\in\mathcal{N},\mathcal{F}}(t_{ij}-1)\Omega_{ij} + \pi_i(t_i^p-1)}{T_c}\left(d\ln\lambda_i + d\ln\frac{t_i^p-1}{t_i^p}\right) \\ &+ \Phi_{cg}\sum_{i\in\mathcal{N};\ j\in\mathcal{N},\mathcal{F}} \frac{(\frac{t_i^p-1}{t_i^p} - \frac{t_{ij}-1}{t_{ij}})\lambda_i\Omega_{ij}t_{ij}}{T_c}(d\ln\mu_i + d\ln\tau_{ij} - d\ln t_{ij}). \end{aligned}$$

Substituting the expression for $d\ln\tilde{\Omega}_{cj}$ above, we get

$$
\begin{aligned}
d\ln T_c =&\Phi_{cg}\sum_{i\in\mathcal{N};\ j\in\mathcal{N},\mathcal{F}}\frac{(t_{ij}-1)\lambda_i\Omega_{ij}}{T_c}\left(d\ln\frac{t_{ij}-1}{t_{ij}}\right)\\
&+\Phi_{cg}\sum_{c\in\mathcal{C};\ j\in\mathcal{N}}\frac{(v_j-1)\chi_c b_{cj}}{T_c}\left(d\ln\chi_c+d\ln\frac{v_j-1}{v_j}\right)+\sum_{c\in C}\frac{\chi_c}{T_c}(1-\theta_c)Cov_{\tilde{\Omega}^{(c)}}(d\ln\mathbf{v}+d\ln\mathbf{p},\frac{\mathbf{v}-\mathbf{1}}{\mathbf{v}})\\
&+\Phi_{cg}\sum_{i\in\mathcal{N}}\frac{\lambda_i\sum_{j\in\mathcal{N},\mathcal{F}}(t_{ij}-1)\Omega_{ij}+\pi_i(t_i^p-1)}{T_c}\left(d\ln\lambda_i+d\ln\frac{t_i^p-1}{t_i^p}\right)\\
&+\Phi_{cg}\sum_{i\in\mathcal{N};\ j\in\mathcal{N},\mathcal{F}}\frac{(\frac{t_i^p-1}{t_i^p}-\frac{t_{ij}-1}{t_{ij}})\lambda_i\Omega_{ij}t_{ij}}{T_c}(d\ln\mu_i+d\ln\tau_{ij}-d\ln t_{ij})
\end{aligned}
$$

Finally, we can write the final system of equations. For $i\in\mathcal{N},\mathcal{F}$

$$
\begin{aligned}
d\ln\lambda_i =&-\sum_{l\in\mathcal{C},\mathcal{N},\ m\in\mathcal{N}}\frac{\lambda_l\Omega_{lm}}{\lambda_i}(d\ln\mu_l+d\ln\tau_{lm})\Psi_{mi}\\
&+\sum_{c\in\mathcal{C}}\frac{\chi_c}{\lambda_i}(1-\theta_c)\mathrm{Cov}_{\tilde{\Omega}^{(c)}}(d\ln\mathbf{v}+\tilde{\Psi}^p d\ln\tilde{U}+\tilde{\Psi}^w d\ln\lambda^w, diag(\mathbf{v})^{-1}\Psi_{(i)})\\
&+\sum_{c\in\mathcal{C}}\frac{\sum_{k\in\mathcal{N}}\chi_c b_{ck}\Psi_{ki}}{\lambda_i}d\ln\chi_c, \qquad (20)
\end{aligned}
$$

where we have used the fact that $d\ln\mathbf{p}=\tilde{\Psi}^p d\ln\tilde{U}+\tilde{\Psi}^w d\ln\lambda^w$. Now, denote the total tax revenue collected from firms (or total transfers made to the households) as $T^g=\sum_c T_c^g=\sum_c T_c$. For $c\in\mathcal{C}$, we have

$$
\begin{aligned}
d\ln\chi_c =&\sum_{f\in\mathcal{F}}\frac{\Phi_{cf}\lambda_f}{\chi_c}d\ln\lambda_f\\
&+\sum_{i\in\mathcal{N}}\frac{\Phi_{ci}\pi_i}{\chi_c}\left(d\ln\lambda_i+\frac{\lambda_i}{t_i^p\pi_i}\sum_{j\in\mathcal{F},\mathcal{N}}\Omega_{ij}t_{ij}(d\ln\mu_i+d\ln\tau_{ij}-d\ln t_{ij})-d\ln t_i^p\right)\\
&+\frac{\phi_{cg}T^g}{\chi_c}d\ln T^g \qquad (21)
\end{aligned}
$$

where $d\ln T^g$ is given by

$$\begin{aligned}
d\ln T^g =& \sum_{i\in\mathcal{N};\ j\in\mathcal{N},\mathcal{F}} \frac{(t_{ij}-1)\lambda_i\Omega_{ij}}{T^g}\left(d\ln\frac{t_{ij}-1}{t_{ij}}\right) \\
&+ \sum_{c\in\mathcal{C};\ j\in\mathcal{N}} \frac{(v_j-1)\chi_c b_{cj}}{T^g}\left(d\ln\chi_c + d\ln\frac{v_j-1}{v_j}\right) \\
&+ \sum_{c\in C}\frac{\chi_c}{T^g}(1-\theta_c)Cov_{\tilde{\Omega}^{(c)}}(d\ln\mathbf{v} + \tilde{\Psi}^p d\ln\tilde{U} + \tilde{\Psi}^w d\ln\lambda^w, \frac{\mathbf{v}-\mathbf{1}}{\mathbf{v}}) \\
&+ \sum_{i\in\mathcal{N}} \frac{\lambda_i\sum_{j\in\mathcal{N},\mathcal{F}}(t_{ij}-1)\Omega_{ij} + \pi_i(t_i^p-1)}{T^g}\left(d\ln\lambda_i + d\ln\frac{t_i^p-1}{t_i^p}\right) \\
&+ \sum_{i\in\mathcal{N};\ j\in\mathcal{N},\mathcal{F}} \frac{(\frac{t_i^p-1}{t_i^p} - \frac{t_{ij}-1}{t_{ij}})\lambda_i\Omega_{ij}t_{ij}}{T^g}(d\ln\mu_i + d\ln\tau_{ij} - d\ln t_{ij}). \qquad (22)
\end{aligned}$$

We the change in sales shares of firms, income shares of factors, and income shares of individuals with a $(N+F+C)\times 1$ dimensional vector $d\ln\boldsymbol{\lambda}$. We solve for the changes by finding a solution to the system of equations

$$d\ln\boldsymbol{\lambda} = Ad\ln\boldsymbol{\lambda} + B$$

or

$$d\ln\boldsymbol{\lambda} = (I-A)^{-1}B,$$

where A is a square matrix of dimension $(N+F+C)^2$ and B is a vector of dimension $(N+F+C)\times 1$. Finally, we partition these matrices such that

$$\begin{bmatrix} d\ln\lambda_{N+F\times 1} \\ d\ln\chi_{C\times 1}\end{bmatrix} = \begin{bmatrix} \mathbf{0}_{N+F\times N} & A^{12}_{N+F\times N} & A^{13}_{N+F\times C} \\ A^{21}_{C\times N} & A^{22}_{C\times F} & A^{23}_{C\times C}\end{bmatrix}\begin{bmatrix} d\ln\lambda^p_{N\times 1} \\ d\ln\lambda^w_{F\times 1} \\ d\ln\chi_{C\times 1}\end{bmatrix} + \begin{bmatrix} B^1_{N+F\times 1} \\ B^2_{C\times 1}\end{bmatrix}$$

where

$$
\begin{aligned}
A^{12}_{if} &= \sum_{c\in\mathcal{C}} \frac{\chi_c}{\lambda_i}(1-\theta_c)\text{Cov}_{\tilde{\Omega}^{(c)}}(\tilde{\Psi}_{(f)}, diag(\mathbf{v})^{-1}\Psi_{(i)}) \\
A^{13}_{ic} &= \frac{\sum_{k\in\mathcal{N}} \chi_c b_{ck}\Psi_{ki}}{\lambda_i} \\
A^{21}_{ci} &= \frac{\Phi_{ci}\pi_i + \Phi_{cg}\left(\lambda_i \sum_{j\in\mathcal{N},\mathcal{F}}(t_{ij}-1)\Omega_{ij} + \pi_i(t^p_i - 1)\right)}{\chi_c} \\
A^{22}_{cf} &= \frac{\phi_{cf}\lambda_f + \Phi_{cg}\left(\sum_{c'\in C}\chi_{c'}(1-\theta_{c'})Cov_{\tilde{\Omega}^{(c')}}(\tilde{\Psi}_{(f)}, \frac{\mathbf{v}-\mathbf{1}}{\mathbf{v}})\right)}{\chi_c} \\
A^{23}_{cc'} &= \frac{\phi_{cg}\sum_{j\in\mathcal{N}}(v_j - 1)\chi_{c'}b_{c'j}}{\chi_c}
\end{aligned}
$$

and

$$
\begin{aligned}
B^1_i = &- \sum_{l\in\mathcal{C},\mathcal{N},\ m\in\mathcal{N}} \frac{\lambda_l\Omega_{lm}}{\lambda_i}(d\ln\mu_l + d\ln\tau_{lm})\Psi_{mi} \\
&+ \sum_{c\in\mathcal{C}} \frac{\chi_c}{\lambda_i}(1-\theta_c)\text{Cov}_{\tilde{\Omega}^{(c)}}(d\ln\mathbf{v} + \tilde{\Psi}^p d\ln\tilde{U}, diag(\mathbf{v})^{-1}\Psi_{(i)}) \\
B^2_c = &\sum_{i\in\mathcal{N}} \frac{\Phi_{ci}}{\chi_c}\frac{\lambda_i}{t^p_i}\sum_{j\in\mathcal{F},\mathcal{N}}\Omega_{ij}t_{ij}(d\ln\mu_i + d\ln\tau_{ij} - d\ln t_{ij}) - d\ln t^p_i \\
&+ \frac{\phi_{cg}T^g}{\chi_c}d\ln T^{g,direct}
\end{aligned}
$$

where

$$
\begin{aligned}
d\ln T^{g,direct} = &\sum_{i\in\mathcal{N};\ j\in\mathcal{N},\mathcal{F}} \frac{(t_{ij}-1)\lambda_i\Omega_{ij}}{T^g}\left(d\ln\frac{t_{ij}-1}{t_{ij}}\right) \\
&+ \sum_{c\in\mathcal{C};\ j\in\mathcal{N}} \frac{(v_j-1)\chi_c b_{cj}}{T^g}d\ln\frac{v_j-1}{v_j} \\
&+ \sum_{c\in C}\frac{\chi_c}{T^g}(1-\theta_c)Cov_{\tilde{\Omega}^{(c)}}(d\ln\mathbf{v} + \tilde{\Psi}^p d\ln\tilde{U}, \frac{\mathbf{v}-\mathbf{1}}{\mathbf{v}}) \\
&+ \sum_{i\in\mathcal{N}}\frac{\lambda_i\sum_{j\in\mathcal{N},\mathcal{F}}(t_{ij}-1)\Omega_{ij} + \pi_i(t^p_i-1)}{T^g}d\ln\frac{t^p_i-1}{t^p_i} \\
&+ \sum_{i\in\mathcal{N};\ j\in\mathcal{N},\mathcal{F}}\frac{(\frac{t^p_i-1}{t^p_i} - \frac{t_{ij}-1}{t_{ij}})\lambda_i\Omega_{ij}t_{ij}}{T^g}(d\ln\mu_i + d\ln\tau_{ij} - d\ln t_{ij}).
\end{aligned}
$$

B Data Appendix

In this appendix we present all the data sources, the crucial variables, and a description of the cleaning process before we take the data to the theory. In all instances, we utilize data from the year 2022, representing the latest year in which all sources are concurrently accessible.

B.1 Raw datasets

The raw datasets we used can be divided into three types: administrative, survey and other auxiliary datasets. We describe each in turn.

B.1.1 Administrative datasets

We use administrative datasets from Chile's Servicio de Impuestos Internos (SII), the equivalent of their Internal Revenue Service (henceforth, IRS) and the Central Bank of Chile. These datasets cover the entire formal private sector in Chile. All individuals and all formal firms in Chile are assigned a unique tax ID that is consistently recorded across the datasets. We use this ID to merge all datasets.

- Domestic Firm-to-Firm Electronic Transaction Records: This dataset includes all the domestic transactions made by firms to firms and is at the buyer-origin-seller-destination-product-transaction level. For each firm-location-product pair (firms may have several production sites), the dataset reports the value of the transaction from origin to destination, month and year of transaction, the price and the non discounted price. Data is available from 2014 onwards. Since 2014, only the largest firms report their transactions, smaller firms did not have the equipment to report. Every firm starts reporting in 2017. There is no reporting threshold neither on the firm nor the transaction side. Three product classifications are available: COICOP (Classification of Individual Consumption by Purpose), CPC (Central Product Classification) and CUP (*Clasificación Uniforme de Productos*). The most disaggregated one is CPC with 2,563 products. The other classification, COICOP, has 1,187 products. COICOP is less disaggregated but allows for matching with the consumption survey that is described below. CUP is a Chilean version of CPC, which is used for national accounts.

- Customs Firm-to-Firm Transaction Records: This dataset reports international trade transactions at the domestic firm-country-foreign firm-product-transaction level. For each firm, it reports the total value of imports and exports as well as quantities, year of transaction, HS product classification, origin/destination country and foreign partner. Products are identified by 6-digit Harmonized Standard (HS) system, which describes 5,613 product categories.

- Firm-to-Individual Electronic Transaction Records: This dataset covers transactions between firms and individuals from 2022 to the present. This dataset comes about through customers providing firms with their tax IDs when making purchases, something which has become the norm for most large retailers who use the tax ID to link customers to their loyalty or rewards programs. The data is at the firm-consumer-product-transaction level. For each individual, product and firm, we observe the total value of transactions, number of transactions and the average price. There are no reporting thresholds although coverage is incomplete for both informal retailers and for smaller stores that typically do not store the accompanying tax IDs[24]. Both CPC and COICOP product classifications are available. Given the size of the data, currently, only data for the 2,124 largest sellers of final goods has been transferred to the Central Bank of Chile. Aggregate sales at the firm level are available for all formal firms.

- Employer-Employee Records: This data is gathered by the Chilean tax authority, Servicio de Impuestos Internos (SII), through affidavits 1887 and 1879. This dataset captures details about each job held by a worker within a specific year and it is at the individual-firm-year level. It has information on the amount earned by the employee in that year and dummies showing in which months the individual worked. Affidavit 1887 reports formal contracts, while affidavit 1879 reports invoices of labor services.

- Firm Ownership Records: We use ownership linkages of firms from tax records (IRS tax affidavits 4415 and 4416). This dataset includes, for each firm, the complete list of owners, with both firms and individuals being able to own a firm. It is at the owner-firm-year level. It has a dummy showing if the owner is an individual or a firm, initial investment, the ownership share and the share of the firm's capital and a dummy indicating if the invested amount has changed. Shares are displayed as a value between zero and one hundred. If total shares of a given firm do not sum up to one, the missing shares are assigned to the firm itself. That way, all firms' ownership shares sum up to one.

- Individual Pension Holdings: This dataset reports at the individual-pension type-

[24]We address this issue by using consumption surveys as described bellow.

year level the pension holdings. It includes the individuals ID of the holder, the firm providing the pension account (there are currently seven private pension fund administrators, known as AFPs), type of pension investment fund (there are 5 funds offered by each AFP with each varying in its level of risk) and the total pension amount, which corresponds to the accumulated pension holdings.

- Individual Income Declaration Forms and Government Transfers: We make use of government-to-household data by merging the universe of money transfer records with another dataset on income tax payments, enabling the creation of direct net transfers. The income tax dataset is at the individual-year level. For each individual, it reports his or her annual income declaration: labor, capital and total income. However, not all income is reported since there is a threshold on minimal earned income to be declared. Below this threshold, people do not appear in this form. The government transfer form is at the individual-year-type of transfer level. It reports the amount of all the different money transfers the government gives to individuals and describes the purpose of the transfer.

- Civil Registry Form: It reports for each individual, the date of birth, the gender, the tax ID of the mother and the tax ID of the father. This allows for building demographics of individuals such as age and gender as well as the household.

- Firms Form 22 and 29: We measure additional firms' characteristics with tax forms 22 and 29. Form 22 is at the firm-year level. This dataset reports the value of declared capital, assets, and liabilities by a firm for a year. Form 29 is at the firm-month level. This dataset contains firms' sector (a 5 digit number with 674 sectors), total sales, total exports, total material, total imports and investment.

- Unemployment Insurance Dataset: Contains information on the educational level of all formal workers in the economy hired by private firms.

B.2 Survey data

While administrative datasets cover close to the universe of formal economic transactions in the economy, they miss informal economic activity that is an important feature of middle-income countries such as Chile. Thus, we use three large-scale government surveys that capture informal transactions to complement administrative data (as well as to shed light on the products purchased at smalle rstores for which we do not have Firm-to-Individual Electronic Transaction Records).

- Consumption Survey: We use the *Encuesta de Presupuestos Familiares* or EPF for its acronym in spanish. This is a consumer survey run by the statistical agency (*Instituto Nacional de Estadistica* or INE for its acronym in Spanish) of the Chilean government and is a key input for measuring inflation and poverty. Each observation is at the individual-transaction level. For each individual, the survey reports the full list of consumption expenses they had in a year, including the associated prices, quantities, product description, and the store it was purchased from. The survey is conducted every 5 years. We use the waves VIII from 2017 and IX from 2022. In 2022, there are 15,239 households (who are either reported to live in the great Santiago or in other regions) and 1,187 products, identified using the COICOP product classification. The survey also has a module with income and demographics information of each individual.

- Employment Survey: We use a labor force survey that reports labor market activity for a sample of representative workers in the economy, called *Encuesta Suplementaria de Ingresos* or ESI for its acronym in Spanish. This is a survey run once a year and is conducted by the INE. This survey complements the standard employment survey that INE runs every month. It provides rich information about the sources of income of each individual, both formal and informal, and demographics of each individual.

- Micro-enterprise Survey: We use the survey *Encuesta de Microemprendimiento* or EME for its acronym in Spanish. This surveys small firms and is designed by the Chilean government to measure the informal sector. We use the waves from 2017, 2019 and 2022, which in total survey over 20,000 small business owners. Each business owner reports detail characteristics of their business including labor expenses (in particular, characteristics of the workers employed), materials expenses, capital expenses. This allows us to measure wedges of these firms, profits of the firm, as well as labor relationships.

B.3 Other data sources

We use auxiliary and more aggregated from different sources. These datasets do not have information on specific firms nor individuals.

- Price Deflators: We include in the analysis several price deflators provided by the Central Bank of Chile. First, we include the Capital Gross Price Index dataset which has the capital gross index price for each year since 2013 to deflate the firms' capital expenditure and stock. Second, we include the CPI table that reports the product categories and CPI time series since 1980 in Chile..

- Sectors Description: Dataset provided by the Central Bank of Chile that contains all the possible sector classification of firms. Several levels are available: sector (674 possibilities), subclase (485 possibilities), division (88 possibilities), seccion (21 possibilities) and sector9 (9 possibilities).

- List of Municipalities: Dataset provided by the Central Bank of Chile reporting for each municipalities (345 geographical area) GPS coordinates, population, density, region (16 possibilities) and province (55 possibilities).

- WIOD Socio Economic Account: University of Groningen provides Nominal and real sectoral value added, gross output, intermediate and final expenditure for the period 1965-2000 for 25 countries. We use this dataset to measure cost shares from the US and to populate the rest of the world.

- US National IO Table: University of Groningen released national Input Output Table (NIOT) for 43 countries. It describes input and output linkages and transaction value from 2000 until 2014. The values are denoted in million of dollars. We use the 2016 version. We use this dataset to measure cost shares from the US and to populate the rest of the world

- World KLEMS Dataset: Measures output, inputs and productivity at a detailed industry level across countries, including Chile. We use information on capital disaggregation into equipment versus structures across 1-digit sectors in Chile and also measures of capital depreciation across sectors in Chile. We use these dimensions for measuring capital.

B.4 Data Cleaning

Before describing how we build the sample of firms and individuals, we describe the data cleaning steps.

B.4.1 Censoring

To reduce the prevalence of outliers, we censor the labor payments of the top 0.01% of employer-employee payments. Firm-to-firm and firm-to-individuals transactions are censored both for the upper and lower 0.01% of the distribution of flows.

B.4.2 Identify Capital Transactions

To separate consumption from investment, we need to distinguish between durable and non-durable transactions. For CUP product description, our dataset comes with a ”durable” dummy that indicates whether the product is durable or not. We get 290 durable products. For COICOP products, our descriptive dataset comes with a variable called ”tipo”, which indicates durable goods. Out of 303 products, 34 are durable.

B.4.3 The Rest of the World

Since we allow for international trade, we need to keep track of transactions, both imports and exports, between Chileans and the rest of the world. We create a representative individual living in the rest of the world and a representative firm operating in the rest of the world. We have two sources of data for these agents. The first one is international transactions with Chilean firms. Whenever we observe a Chilean firm exporting in the customs data, we interpret this as sales from the Chilean firm to the rest of the world individual. Similarly, any Chilean firm import, conditional on the product being non-durable from the customs data is interpreted as a material purchase of the Chilean firm from the rest of the world firm. This international output is assumed to be produced with a technology that directly transforms labor into output, hence the international firm does not generate any profit from this sale. The second source of international data is the WIOD. From this dataset, we get the rest of the world consumption from the rest of the world (hence excluding Chilean exports), labor expenses, capital stock and sales to all countries, other than Chile.

B.4.4 Factor Markets

We define as many labor markets as regions in Chile, which are 16. We also define a residual labor market in Chile, to which we assign labor usage by firms when we do not know their sector. Finally, we define one more labor market, corresponding to the one used by the rest of the world firm.

We define two capital markets. The first one is for every firm that is located in Chile, regardless of the region. We define another capital market, the rest of the world market, that corresponds to the one used by the rest of the world firm.

B.4.5 Multiproduct Retailers and Wholesalers

All firms are assumed to be monoproduct. For firms outside the retail and wholesale sectors, we ignore any available product-level information and assign them a missing product value. For wholesalers and retailers, we proceed in two steps. First, we compile a list of products that each firm sells to both other firms and households. Second, we identify which of these products the firm also purchases from other firms. For each product that meets this criteria, we create a new entity—referred to as a ”subfirm”, which represents the combination of the original firm and that specific product. If a retailer or wholesaler does not purchase any product that it also sells, we treat it as a standard monoproduct firm, consistent with firms in other sectors.

Each subfirm is assigned a weight that reflects its relative importance within the parent firm. This weight is used to allocate portions of the parent firm's inputs and outputs. Specifically, labor and capital usage, material purchases (for products not also sold by the firm), and sales of outputs that are not also inputs. The subfirm weight is defined as the share of the product's purchases in the total value of all products that the firm both buys and sells.

The remaining sales and input and factor expenses of these parent firms that are allocated to these subfirms, are then allocated to a standard monoproduct firm.

B.5 Sample of firms

We create a sample of all Chilean firms operating in a year. We create a dataset that lists all firm-product pairs that satisfy the following conditions:

- Employ at least two workers.
- Report some capital.
- Purchase material from other firms.
- Report some sales.
- Report their operating sector.

One key challenge in our data construction is to always identify the two sides of any given any economic transaction. This subsection describes how we build all the relevant firm-specific information from bilateral administrative datasets.

B.5.1 Geography, sector, geography and product information

- Geography: A firms' main location comes from form 29. It is characterized by 345 municipalities, 55 provinces and 16 regions. Using information from the municipalities, we create a categorical urban variable (either urban, rural or Santiago) and a macroregion categorical variable (either North, South or Santiago).
- Sectors: Using form 29 sector information, we attribute to each firm a sector and a division, corresponding respectively to 674 and 88 possible values.

- Product: This depends on the operating sector of the firm. If the firm is not a retailer or a wholesaler, then we just consider it a monoproduct firm and define its product to be a ”missing” value. For retailers and wholesalers, only the subfirms[25] enter the final firm dataset. They already have an assigned product value.

B.5.2 Firms labor, capital, material, output

- Labor: Information on labor payments come from the employer-employee data. We collapse the amount earned by each individual at the firm-year level and count the number of employees by firm for each year. Thus, we construct a dataset of total number of employees and total annual earnings paid by each firm.

- Capital: Information on firms' capital comes from forms 22, 29 and capital's gross price index dataset. Firms' capital stock is built using the perpetual inventory method. Using a depreciation rates from KLEMS for Chile, capital has the standard law of motion: $K_{i,t} = (1-\delta)K_{i,t-1} + I_{i,t}$, where $K_{i,t}$ is firm i capital at year t, $I_{i,t}$ is i's investment at year t and δ is the depreciation rate. We measure the initial condition of capital, $K_{i,0}$, using the first report of fixed assets from form 22 that a firm reports over the 2005–2022 period. Investment is measured from form 29. We deflate capital and investment using the gross capital index from the Central Bank of Chile. Since the measure of investment is better at capturing capital expenses in machines and equipment, instead of structures and buildings, we adjust firms' capital by the ratio of structures to equipment by sector reported in KLEMS. After measuring the capital stock, we measure the user cost of capital as the risk-free nominal interest rate of 10% plus the sectoral depreciation rate measured from KLEMS.

- Materials: Material purchases are obtained from combining non-durable purchases from Chilean firms through the firm-to-firm electronic transactions dataset and from the rest of the world through the customs data.

- Output sales: There are four sources of output sales:annual sales from form 29, annual exports from customs data, sales to other firms from the electronic firm-to-firm data, and sales to individuals from firm-to-individuals electronic records. We are thus able to observe output sold at a disaggregated level. Total sales of each firm is measured from form 29. For that reason, output which is not sold to firms, individuals or foreign market, i.e. the difference between form 29 total sales and total exports as well as intermediate sales to individuals and other firms, is allocated as output to a residual individual. We obtain for each firm the total sales value and a breakdown

[25] Remember that in our vocabulary, a subfirm is a firm-product combination

of these sales by buyer: other domestic firms, consumers, rest of the world and the residual individual. In order to avoid singularity of the input-output matrix, we do not consider output sales of firms for which we only observe transactions to themselves. This could happen if a firm has several sites and the only sales of one plant are to other plants in the same firm. Foreign firm output corresponds to final sales to the foreign individual and intermediate sales to firms in Chile and to the foreign firm.

B.5.3 Residual firm

For measuring wedges, we have to focus on firms for which we can measure their labor, capital, material, and output sales, and an assigned 5 digit sector. We also focus on firms with at least two employees because some tax IDs are not used for production purposes. The main issue of dropping firms that do not meet these criterion is to lose track of an income or an expense which breaks the circularity of the economy. Consider a firm that sells to a firm who does not possess any capital. The benefit from this transaction will be counted as the first firm's output sale. However, no one will receive the good if the receiving firm is dropped from the sample. To deal with this possibility, we create an additional firm, that we label the ”residual firm”. The role of this entity is to combine all of the firms that are dropped from the analysis when cleaning the data. That way, we never lose economic transactions.

B.6 Sample of individuals

Similarly to what we do with firms, we create a sample of Chilean individuals, covering their income and consumption over a year. Income is here defined in a very broad sense as it combines labor, capital and profits from the ownership of firms and government transfers. To be considered in the sample of individuals, we need to be able to measure:

- Income.
- Consumption.
- Age of at least 15.
- Gender.

We describe in this subsection how to measure each of these variables. We focus on formal income and consumption measured from tax records. In Section B.7 we describe how we measure informal income and consumption using surveys and statistical matching to combine it with the administrative data.

For measuring formal income, we need first to measure the ownership matrix, which allows us to distribute capital payments and profits from firms to individuals.

B.6.1 Compute the ownership matrix

To allocate individuals their capital and profit income, we use the ownership records. It comes in the form of shares of firms owned by entities that can either be individuals or firms. We transform it into shares of firms that are owned only by individuals. To do so, we create two matrices.

- Direct ownership: Rows are individuals, while columns are owned firms.
- Indirect ownership: Rows are firms that own other firms, columns are the owned firms.

The ownership matrix we use is the product of the direct ownership matrix and the Leontief inverse of the indirect ownership matrix. This way, we take into account firms owning other firms similarly to an input-output table and translate all ownership shares to individuals, both directly and indirectly.

Because of the firms we created above such as the residual firm, and some data correction issues, we need to make a few assumptions.

- Residual firm: Its ownership shares are computed using individuals' ownership shares of the firms that make up this residual firm. We measure each firm's "contribution" to the aggregate one by computing their profit size relative to the aggregate profit share. Because these contribution weights sum up to one, we can multiply initial ownership shares by the weights to recover residual firm's ownership shares.
- Rest of the world firm: We define a rest of the world individual that owns the entire rest of the world firm.
- Firms not appearing in ownership records: We allocate the ownership of these firms to the residual individual. Thus, all firms are owned entirely by individuals.

B.6.2 Individual's consumption, labor income, capital income, profit income

- Consumption: We define two types of consumption. The first one is "direct" consumption. This comes from firm-to-individuals electronic records. The second one is "indirect" consumption. This corresponds to the capital expenses from firm-to-firm electronic records. Since we do not allows for savings and investment, we treat capital expenses as consumption from individuals perspective. To implement this, we rebate firms' expense on durable goods as consumption of the individuals that own firms', using the ownership measure described above. Thus, total consumption of an individual is the sum of her direct purchases from firms and the amount of capital the firms they own purchase from other firms, weighted by the individual's ownership share of the firm.

As before, we implement some adjustments to consider special cases.

  - Rest of the world: Foreign individual consumption is equal to total domestic exports plus the rest of the world consumption.
  - Residual individual: When dealing with firms output, we assumed that the difference between the firm's output sold over a year and the sales we can accrue to entities (individual, firm or rest of the world) was given to a fourth category that we labeled the residual individual. This is the additional individual that consumes the amount of output required to match firms' total sales.

- Labor income: From the employer-employee data, we compute annual labor earnings for each individual. We also define the rest of the world labor income to be the labor expenses from the rest of the world reported in the WIOD.

- Capital income: We rebate firms' capital expenses described above to individuals as capital income according to our constructed ownership shares.

- Profit income: We compute firms profit using labor, capital, material and output firms' values we measured when we established our sample of firms. We distribute firms' profits to individuals using our computed ownership matrix.

B.6.3 Additional individuals

As with firms, we create additional individuals to ensure the circularity of the economy so that standard aggregate identities hold. Here, we recap the additional individuals we create while measuring income and consumption:

- A "foreign" individual who consumes all of the domestic exports. They have labor income, which is all the revenue the foreign firm makes (i.e. total domestic imports). They also consume the rest of the world production that is not sold to Chilean firms.

- Residual individual: Consumes all the firms' output that remains after taking into account sales to other firms (domestic and foreign) and to individuals (domestic and foreign, with and without tax ID). It also absorbs characteristics of individuals for which we do not observe income, consumption, age or gender.

B.6.4 Other individuals' characteristics

- Age and gender: We obtain age and gender from the civil registry dataset. We drop individuals with age below 15. Each individual is either classified as female or male and in an age category: 15-25, 25-35, 35-45, 45-55, 55-65 or 65-100.
- Location: We measure the location of an individual using the most common location from which they purchase final goods as reported in the firm-to-individual electronic transaction records.

B.6.5 Group individuals into households

Using civil registry data, we group individuals into households. We keep a version at the individual level which is still useful to study differences at the individual characteristic level such as gender or age.

B.7 Statistical Matching Between Administrative and Survey

Despite the richness of the administrative tax data, there are two limitations. First, informality plays a role in a country like Chile where around 27% workers are employed informally. Second, the firm-to-individual transactions are not exhaustive because we cannot observe these transactions for smaller formal firms. Thus, we use several surveys from both individuals and firms to enrich the measurement of income and consumption. Since survey data does not have the tax IDs used in the administrative data, we use statistical matching techniques to combine both types of sources.

B.7.1 Matching Many Surveys

We use the consumption (EPF), employment (ESI) and micro-enterprise (EME) surveys to incorporate individual's labor income from working in informal firms, profit and capital income from the ownership of informal firms and transfers from the government.

For each of the surveys, we first construct a distinct ID for each individual. Then, we group individuals into several bins and apply statistical matching within bins. We use variables for matching that are common in both the administrative sources and the surveys. We minimize the distance between these variables and get a one-to-one matching between each survey ID and the tax ID in the administrative data. For this, we use the optimal transport algorithms that have been used for building distributional national accounts (see Blanchet, Saez and Zucman, 2023). Statistical matching preserves the joint distribution of demographics and incomes or expenditures in the survey data that is being brought into the administrative data and avoids extrapolation (and possibility of extreme outliers) that would occur using polynomial-based prediction models.

After implementing the statistical matching approach, we perform checks on how well each match performs by comparing the original distribution of the target variable with the matched one. Finally, we add the information from the survey to the administrative data we described in the previous sections and update individual's income accordingly.

In total, we build five bridges: ESI to EME employers, ESI to EME employees, administrative data to ESI, administrative data to EPF and EME to administrative data. The variables used for creating bins, for matching and the target variables of each statistical matching procedure is presented in Table 2.

B.7.2 Informal Firms

In order to capture the informal side of the Chilean economy, we create informal firms. Just like our formal firms, we assign them labor, capital, material and output values. We recover all of these values from the EME dataset. Contrary to formal firms, informal ones do not pay taxes. The main challenge for us is to connect these new firms with the rest of the economy.

We obtain region, labor values, capital values, material values, output values and 1-digit sector for informal firms directly from the EME survey. We use statistical matching to fill in information on detailed sector and product for informal firms. We match them to similar small formal firms in the same region and sector, minimizing the distance between their total sales, material expenses, labor expenses, capital expenses and employment with optimal transport algorithms.

For labor transactions, we use the information that EME has on workers of informal firms. With it, we are able to define a list of workers for the informal firm. We can match these workers to their ESI counterpart. We then use the match between ESI and admin data to ultimately allocate informal firm labor payroll to individuals.

For firm-to-firm transactions, given that we do not have information about suppliers of informal firms, we match informal firms with formal firms based on common characteristics we have on both types of firms, and we sample the same number of suppliers as formal firms within the same industry, region and size bins to allocate them to informal suppliers.

For firm-to-individual transactions, we assume that informal firms are downstream, and all their output is sold to final consumers. As with suppliers, we match these informal firms with similar formal firms that only sell to final consumers and sample final consumers conditional on keeping the total number of consumers the same. We do this allocation considering the product and region allocation described above.

With complete information about informal firms, we append these firms to the formal firms' dataset and create unique firm IDs for these newly added firms. The ownership dataset that measures the ownership of firms is also updated accordingly directly from the EME survey.

B.7.3 Individual's Income

Having increased our set of firms to include informal firms, we adjust individuals income in two ways. First, we redistribute factor payment of these informal firms to their owners. For informal labor income, we use information on employer and employee from statistical matching. For capital and profit income, we define ownership of the informal firms. To do so, we first do statistical matching between EME and ESI, and then use the statistical matching between ESI and administrative data.

Second, we recover government transfers. We perform a statistical matching between the EPF and the administrative data. This allows us to identify the share of individuals receiving government transfers. The transfers we recover are smaller that those recorded in government expenditures that we identify from administrative datasets. To make identities hold, we proportionally increase transfers to individuals such that government's budget is balanced.

B.7.4 Individual's Consumption

Our objective is to build a complete firm-to-individual consumption matrix, including both administrative and survey sources. The consumption from firm-to-individual electronic transaction records in administrative data does not include informal consumption or consumption at small formal stores while EPF covers all kinds of consumption. Similar to individual income, first we use statistical matching to bring the complete consumption data from the EPF survey into our administrative datasets using statistical matching techniques. Individuals are classified into demographic bins based on a set of variables shared across datasets, including macroregion, age bins and income bins. Age bins are defined using 25, 45 and 65 as cutoffs. We keep individuals aged 15 or above in our sample. We divide individuals into five income bins based on quantiles computed within each gender-by-age-group cell. Individuals with zero income or zero consumption are grouped separately. Individuals within each macroregion $\times$ age bin $\times$ income bin are matched to the closest individual that minimizes the distance between matching variables using optimal transport algorithms.

Matching variables include gender, total income, labor income share, profit income share and the full vector of expenditures at supermarkets and department stores (expenditures that are well-measured in both the firm-to-individual administrative data and in the consumption surveys). In the EPF, we classify store types using two complementary approaches: keyword-based classification that uses store names, and vector-based classification that compares the distribution of a store's product sales to the typical product mix observed within each sector. Then, sectors are matched to 5 store types (supermarkets, department stores, traditional stores, specialized stores and others).

After the statistical matching, we get a complete individual consumption data by product and store type. Next, we build the firm-to-individual consumption matrix, i.e. we identify which firms each individual purchases from. For each region-product-firm type cell, we first compute residuals between matched consumption and observed admin consumption. Then, we allocate this residual to firms in the corresponding region-product-firm type cell that have no-tax ID transactions. We impose that total expenditure at each formal firm equals their final consumption sales in the administrative records. For region-product-firm type cells that only appear in the administrative data, we apportion them evenly to a random sample of consumers in the same region. For region-product-firm type cells that only appear in the consumption survey, they are classified as informal consumption and are allocated to informal firms created in Section B.7.2.

B.8 Wedges

B.8.1 Industry-Specific Cost Shares for the US

We need industry-specific cost shares for the United States to build wedges for the Chilean economy. We compute them by combining WIOD Socio Economic Account database with the US National IO table. For 56 industries, we compute cost share of labor, capital and material. From the Socio Economic table, we get the total labor, capital and material expenses for the 56 industries in 2014. From the US National IO table, we get material from other sectors for the same year and industries. Finally, using these two sources of information we build cost shares of labor, capital and material for 56 industries in the US.

B.8.2 Assign each Chilean firm its American counterpart input cost shares

Using firms' sector information described above, we create a concordance to map those disaggregated sectors into the 56 industries in the US. Thus, we can measure the cost shares for each firm.

B.8.3 Input Wedges

We compute three types of input wedges for each firm: labor (τ_i^l), capital (τ_i^k) and material (τ_i^m). Note that we do not separately estimate input wedges from output wedges. Hence, our measure of "input wedge" corresponds to the product of the factor-specific input wedge and the firm's output wedge.

Table 2: Summary of Statistical Matching Approaches

Datasets	**EME-ESI**	**ESI-Admin**	**EPF-Admin**	**EPF-Admin**
Target Variables	Informal Firms' profit and related demographics, informal firms employers' information	Informal income	Transfers	Expenditures
Common Variables	**Bin Variables:** Sector9, Firm size, informality **Matching Variables:** Age, Total income, Occupation, Female, Macroregion, Education, Household head	**Bin Variables:** Region, Age bin, Gender, Household size, Income bin **Matching Variables:** Age, Formal income	**Bin Variables:** Macroregion, Age bin, Gender, Household size, Income bin **Matching Variables:** Age, Number of people in the household within age range	**Bin Variables:** Macroregion, Age bin, Income bin **Matching Variables:** Gender, Total income, Labor/Profit income share, Expenditure at product-store level

Note: Bin variables are used to create strata prior to matching; matching variables are used for within-stratum matching. Only people in the same bin will be matched. The second EPF-Admin match refers to individuals consumption section described below.

Input wedge is equal to the product of US equivalent's sector input cost share (α_{sf}^{US}) and value of output produced ($p_i y_i$) divided by value of input consumed ($f_i p_{if}$). The formula is:

$$\tau_i^f \mu_i = \alpha_{sf}^{US} \frac{p_i y_i}{f_i p_{if}}$$

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