Capability, Technological Change and the Wage Distribution

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XXIX Annual Conference of the Central Bank of Chile
"Labor Markets and the Macroeconomy in a Changing World"
19-20 November 2026

THE UNIVERSITY OF CHICAGO

Introduction

- Two observations about skills, tasks and jobs
  1. Some tasks many people cannot do
  2. Some tasks can be performed by anyone at effectively the same level
- Capability:
  - Set of tasks ordered by their difficulty x
  - Worker has capability z if they can only perform tasks $x \leq z$
- Wage distribution: determined by relative supply of capability and demand for tasks
- Capability investment: incentives depend on how wages change with task difficulty
- Technological change: alters (i) set of tasks, (ii) who is capable, (iii) value they provide

This paper

1. Static task-assignment model, limiting case of Costinot-Vogel (2010)

   Output for worker z in task x

   $$k(x, z) = \mathbf{1}\{x \leq z\}$$

   Not strictly positive nor strictly log super-modular

   Equilibrium: pooling regions with flat wages and over-qualification

2. Steady-state experiments:
   - Difficult new tasks
   - Upskilling
   - Difficulty-biased technological change

3. Dynamic model of capability investment as a mean-field game

Related literature

- Assignment models: Sattinger (1975), Teulings (1995), Costinot and Vogel (2010), Garicano (2000), Garicano and Rossi-Hansberg (2006), and Boerma et al. (2025)

  ⇒ binary capability task production function

- Task-based models of technological change: Acemoglu and Autor (2011), Acemoglu and Restrepo (2018), Acemoglu et al. (2025), Acemoglu and Loebbing (2026), and Ide and Talamàs (2025)

  ⇒ pooling regions and overqualification

- Human capital investment: Ben-Porath (1967), Heckman et al. (1998), Guvenen and Kuruscu (2012), Blanchard and Willmann (2016), and Althoff and Reichardt (2026)

  ⇒ capability investment in an assignment model

Outline

1. Model

2. Equilibrium construction
   - Equilibrium with constant task weights
   - Equilibrium with increasing task weights

3. Technological change
   - Experiment 1: Difficult new tasks
   - Experiment 2: Upskilling
   - Experiment 3: Difficulty-biased technological change

4. Capability investment
   - Baseline dynamic model
   - Difficulty-biased technological change

5. Conclusion

Environment

- Set of tasks ordered by difficulty $x \in [0, \bar{X}]$, with task weights $g(x) > 0$
- Unit mass of workers indexed by capability $z \in [0, \bar{X}]$, $z \sim F(z)$
- Worker of capability z can perform any task of difficulty $x \leq z$
- Representative firm produces final good Y from tasks x

$$Y = \left[\int_0^{\bar{X}} g(x)^{\frac{1}{\sigma}} \left(\int_x^{\bar{X}} l(x,z)\, dz \right)^{\frac{\sigma-1}{\sigma}} dx \right]^{\frac{\sigma}{\sigma-1}}$$

  $l(x,z)$: mass of capability-z workers in task x

- Task output function $k(x,z) = \mathbf{1}\{x \leq z\}$ subsumed in lower limit of inner integral

Firm and worker optimality

- Firm: chooses $\{l(x,z)\}$ to maximize profits, taking wages $w(x,z)$ as given

$$\max_{\{l(x,z)\}} \; Y - \int_0^{\bar{X}} \int_x^{\bar{X}} l(x,z)\, w(x,z)\, dz\, dx \qquad \text{subject to} \quad Y = \left[\int_0^{\bar{X}} g(x)^{\frac{1}{\sigma}} m(x)^{\frac{\sigma-1}{\sigma}}\, dx \right]^{\frac{\sigma}{\sigma-1}}$$

  $m(x) \equiv \int_x^{\bar{X}} l(x,z)\, dz$: employment in task x

- Marginal product of labor in task x the same for every $z \geq x$

  $\Rightarrow$ wages $\omega(x)$ depend only on tasks, not of capability

$$\omega(x) = Y^{\frac{1}{\sigma}} \left(\frac{g(x)}{m(x)} \right)^{\frac{1}{\sigma}}$$

- Workers: choose the highest-paid task they can perform, taking $\omega(x)$ as given

$$x^*(z) \in \arg\max_{x \leq z} \; \omega(x)$$

Equilibrium

Given the capability distribution $F(z)$ and task weights $g(x)$, a competitive equilibrium is a wage schedule $\omega(x)$, an assignment rule from capabilities to tasks $\mu(\cdot \mid z)$, and a task-employment function $m(x)$ such that

1. Worker optimality: workers are assigned only to their highest-paid tasks

$$\mu(x \mid z) = 0 \quad \text{wherever } x \notin \arg\max_{x \leq z} \omega(x)$$

2. Firm optimality: wages equal marginal products

$$\omega(x) = Y^{\frac{1}{\sigma}} g(x)^{\frac{1}{\sigma}} m(x)^{-\frac{1}{\sigma}} \quad \text{wherever } m(x) > 0$$

3. Labor-market clearing: employment in each task equals the workers assigned to it

$$m(x) = \int_x^{\bar{x}} \mu(x \mid z)\, dF(z)$$

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Necessary and sufficient conditions on task employment

- Cumulative task employment $M(x) \equiv \int_0^x m(s)\, ds$
- $m(x)$ is part of an equilibrium $\Leftrightarrow$ satisfies three conditions

(i) Capability supply: tasks up to difficulty x employ at least all workers $z \in [0, x]$

$$M(x) \geq F(x) \quad \forall x \in [0, \bar{X}], \qquad M(\bar{X}) = F(\bar{X}) = 1$$

(ii) Wage monotonicity: wage $\omega(x)$ weakly increasing in difficulty

$$\frac{m(x)}{g(x)} \text{ weakly decreasing in } x$$

(iii) Over-qualification implies flat wages: $\omega(x)$ constant where workers are over-qualified

$$\frac{m(x)}{g(x)} \text{ constant on each interval with } M(x) > F(x)$$

regularity condition

Separating and pooling regions

- Separating regions $M(x) = F(x)$
  - Perfect sorting: worker z performs task $x = z$
  - Strictly increasing wage function $\omega'(x) > 0$
  - Wage given by labor supply and productivity: $\omega(x) = Y^{\frac{1}{\sigma}} \left(\frac{g(x)}{f(x)}\right)^{\frac{1}{\sigma}}$
- Pooling regions $M(x) > F(x)$
  - Flat wage function $\omega'(x) = 0$
  - Over-qualification: some workers z perform tasks $x < z$
  - Wage given by total labor supply and cumulative productivity in the pool

$$\omega_{[a,b]} = Y^{\frac{1}{\sigma}} \left(\frac{G(b) - G(a^-)}{F(b) - F(a^-)}\right)^{\frac{1}{\sigma}}$$

- Finding location of separating and pooling regions is a non-trivial global problem

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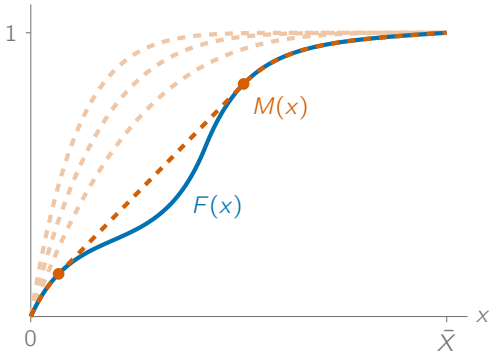
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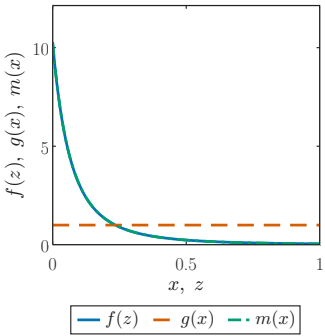
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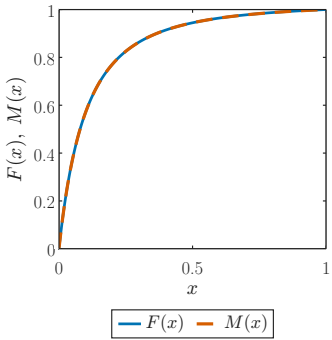
Least concave majorant with $g(x) = 1$

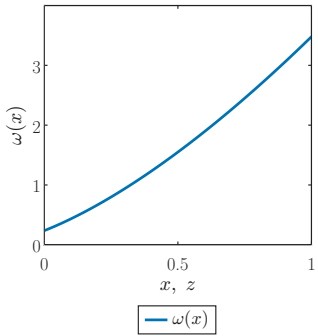
- With $g(x) = 1$, equilibrium conditions for $M(x)$ become
  - (i) Capability supply: $M(x) \geq F(x)$
  - (ii) Wage monotonicity: $M(x)$ weakly concave
  - (iii) Over-qualification flat wages: $M(x)$ linear where $M(x) > F(x)$
- Least concave majorant of $F(x)$: pointwise infimum of concave functions $\geq F(x)$



Example: Full separation

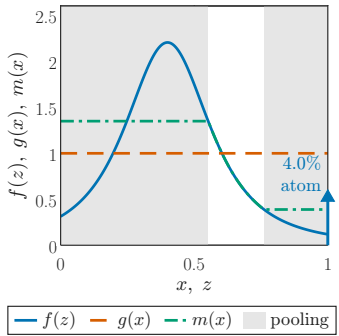


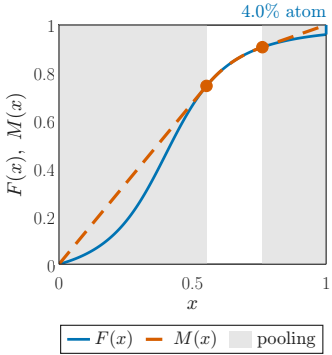


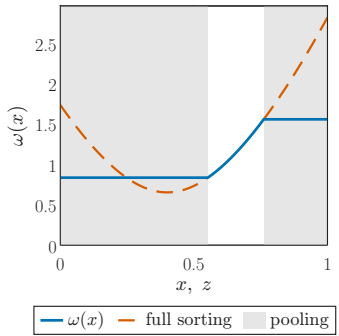


- Task weights: $g(x) = 1$ on $[0, 1]$,
- Worker capability: $z \sim$ truncated Pareto,
- CES: $\sigma = 2$
- Higher capability workers always scarcer $\Rightarrow$ wage strictly increasing in difficulty

Example: Pooling







- Task weights: $g(x) = 1$ on $[0, 1]$, • CES: $\sigma = 2$
- Worker capability: $z \sim$ winsorized Student t, with 4% mass point at $z = 1$
- Result: pooling at the bottom and at the top

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Transformation to task-capacity units

- Task-capacity units: $u \equiv G(x) = \int_0^x g(t)\,dt$

$$u \in [0, \bar{U}], \quad \bar{U} \equiv G(\bar{X}), \quad \widetilde{F}(u) \equiv F(G^{-1}(u)), \quad \widetilde{M}(u) \equiv M(G^{-1}(u))$$

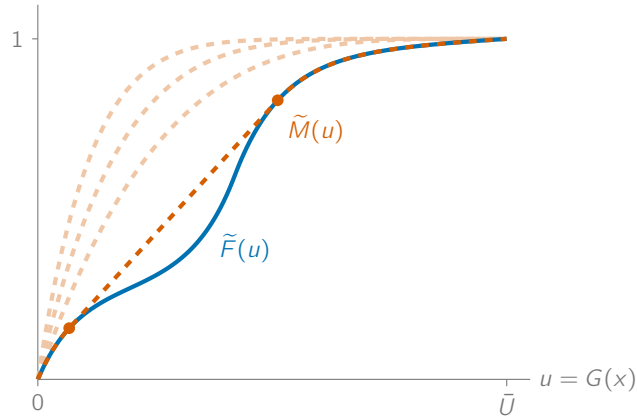
- Slope of $\widetilde{M}(u)$: $\widetilde{M}'(u) = \dfrac{m(x)}{g(x)}$

- Conditions (i)–(iii) become conditions on $\widetilde{M}(u)$:

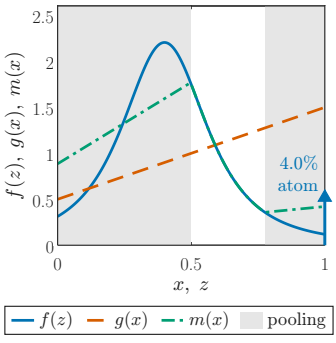
	In tasks x	In task-capacity units u
(i) Capability supply	$M(x) \geq F(x), \quad M(\bar{X}) = 1$	$\widetilde{M}(u) \geq \widetilde{F}(u), \quad \widetilde{M}(\bar{U}) = 1$
(ii) Wage monotonicity	$M(x)$ weakly concave	$\widetilde{M}(u)$ weakly concave
(iii) Flat wages	$M(x)$ linear where $M(x) > F(x)$	$\widetilde{M}(u)$ linear where $\widetilde{M}(u) > \widetilde{F}(u)$

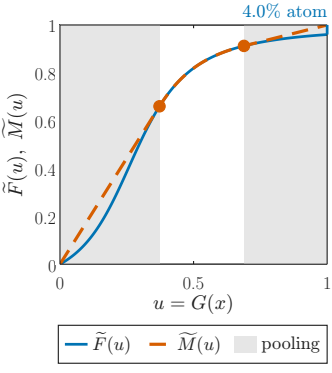
Least concave majorant in task-capacity units

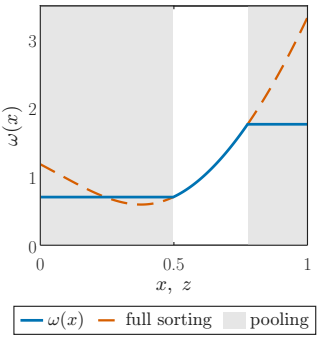
- $\widetilde{M}(u)$ satisfies (i)–(iii) $\Leftrightarrow$ $\widetilde{M}(u)$ is the least concave majorant of $\widetilde{F}(u)$



Example: Pooling with Increasing task weights







- Task weights: $g(x) = 0.5 + x$ on $[0, 1]$, • Worker capability: $z \sim$ Student t with a 4% mass point at $z = 1$ • CES: $\sigma = 2$
- Relative to $g(x) = 1$: smaller bottom pool with lower wage, steeper wages in separating region

Existence, uniqueness, and efficiency

- Existence: a competitive equilibrium (ω, μ, m) is given by

$$m(x) = \widetilde{M}'(G(x))\, g(x), \qquad \omega(x) = Y^{\frac{1}{\sigma}} \left(\frac{g(x)}{m(x)} \right)^{\frac{1}{\sigma}}$$

  $\mu(\cdot \mid z)$: assigns the worker at rank $r \in [0,1]$ of the capability distribution to task $M^{-1}(r)$

- Uniqueness: every equilibrium has the same $m(x)$ and $\omega(x)$ at almost every task, and the same Y

- Efficiency: $n(x) = m(x)$ at almost every task, where $n(x)$ solves the planner's problem

$$\max_{n \geq 0} \left[\int_0^{\bar{X}} g(x)^{\frac{1}{\sigma}} n(x)^{\frac{\sigma-1}{\sigma}}\, dx \right]^{\frac{\sigma}{\sigma-1}} \quad \text{subject to } N(x) \geq F(x), \quad N(\bar{X}) = 1$$

  $N(x) \equiv \int_0^x n(s)\, ds$: cumulative task employment under $n(x)$

- $m(x)$ and location of pools independent of σ

proof sketches

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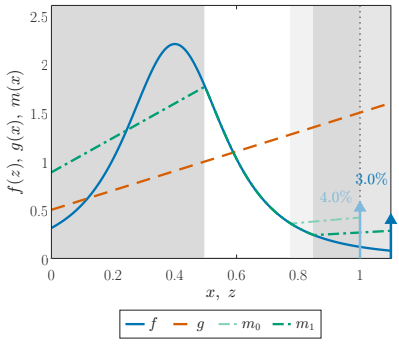
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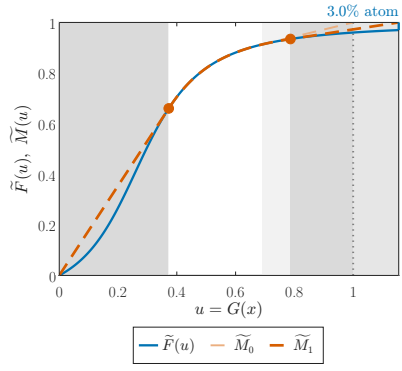
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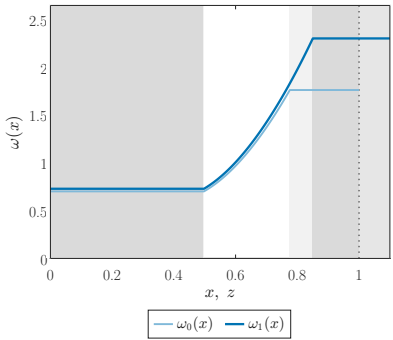
Experiment 1: Difficult new tasks: assignment





- New tasks at the top: $\bar{X} = 1 \to 1.1$, keep $g(x) = 0.5 + x$ and $\sigma = 2$
- Mass of fully capable workers: 4% $\to$ 3%
- Top pool grows in the set of tasks, but holds fewer workers, bottom pool unchanged

Experiment 1: Difficult new tasks: wages



- Workers in new top pool gain most, including those not capable of the new tasks

Wage decomposition

- Output: potential output $\times$ match quality, with relative task weight $\hat{g}(x) \equiv g(x)/\bar{U}$

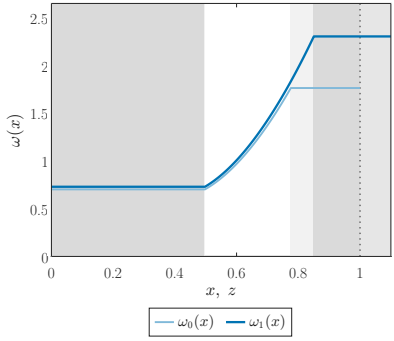
$$Y = \underbrace{\bar{U}^{\frac{1}{\sigma-1}}}_{\text{potential output}} \underbrace{\left[\int_0^{\bar{X}} \hat{g}(x)^{\frac{1}{\sigma}} m(x)^{\frac{\sigma-1}{\sigma}} \, dx\right]^{\frac{\sigma}{\sigma-1}}}_{\Phi}$$

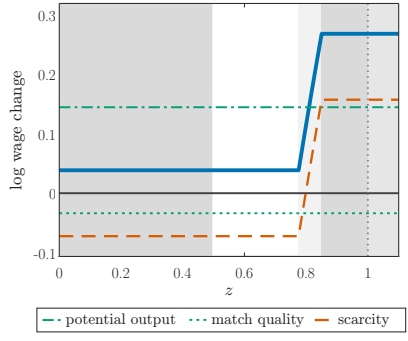
  - $\bar{U}$: total task weight
  - $\Phi \leq 1$, equality only if $m(x) = \hat{g}(x)$

- Log wage: potential output and match quality common to all tasks, relative scarcity task-specific

$$\log \omega(x) = \underbrace{\frac{1}{\sigma-1} \log \bar{U}}_{\text{potential output}} + \underbrace{\frac{1}{\sigma} \log \Phi}_{\text{match quality}} + \underbrace{\frac{1}{\sigma} \log \frac{\hat{g}(x)}{m(x)}}_{\text{relative scarcity}}$$

Experiment 1: Difficult new tasks: wages





- Workers in new top pool gain most, including those not capable of the new tasks
- Scarcity effect ripples down through the old top pool, stops where separation begins
- Wage level depends on σ: potential output term changes by $\dfrac{1}{\sigma - 1}\Delta \log \bar{U}$

$\sigma \leq 1$

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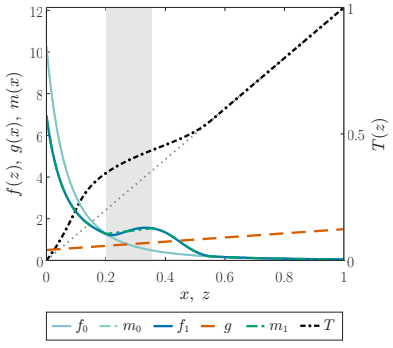
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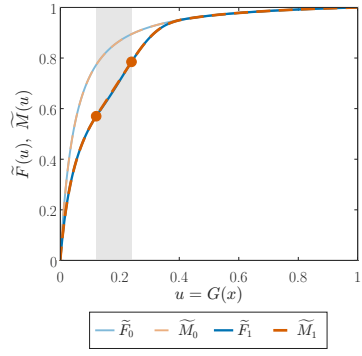
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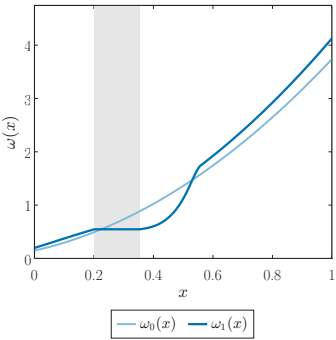
Experiment 2: Upskilling: assignment

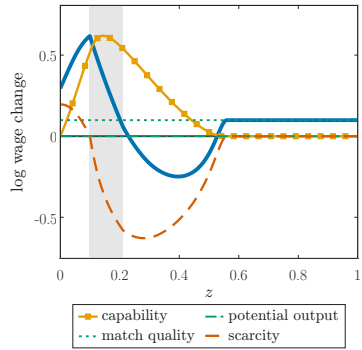




- Capability distribution shifts right: $z \to T(z) \geq z$ for all z, with $T(z) > z$ for $z < 0.56$
- New mass in the middle $\Rightarrow$ new pool
- Workers in new pool assigned to tasks easier than their new capability $T(z)$

Experiment 2: Upskilling: wages





- Direct capability effect of upskilling on wages: $\log \omega_0(T(z)) - \log \omega_0(z)$
- Race between direct capability effect and scarcity effect
- Lowest capabilities: wages rise
- Middle capabilities: wages fall

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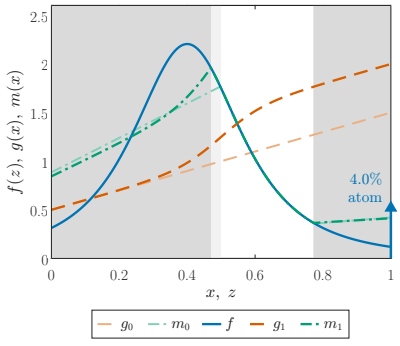
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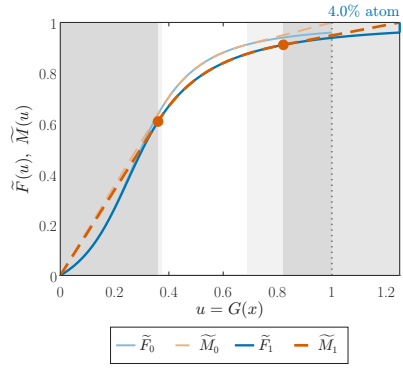
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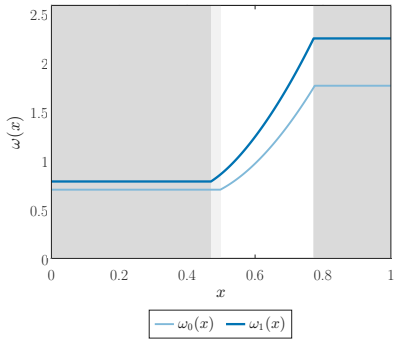
Experiment 3: Difficulty-biased technological change

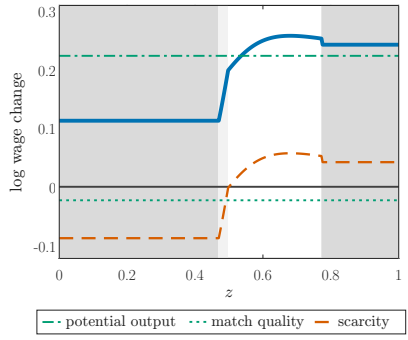




- Difficult tasks more productive: $g_0(x) = 0.5 + x \to$ S-shaped $g_1(x)$, $\bar{U} = 1 \to 1.25$
- Bottom pool shrinks, $[0, 0.50] \to [0, 0.47]$, from 66% to 61% of workers; top pool nearly unchanged

Experiment 3: Difficulty-biased technological change: wages





- All wages rise, more at high x where $g_1(x) - g_0(x)$ is large
- Bottom: negative scarcity effect, so wage gains require higher potential output $\Rightarrow$ sensitive to σ

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Dynamic model

- Demographics: enter at age 0, exit at age $\bar{a}$, replaced by an entrant $\Rightarrow$ ages uniform on $[0, \bar{a}]$
- Preferences: linear utility, discount rate ρ
- Entry capability: $z \sim \pi_0(z)$
- Capability investment: choose rate q at convex cost $c(q) = \dfrac{q^{1+\beta}}{\alpha(1+\beta)}$, depreciation δ, idiosyncratic shocks ν, reflecting barriers at 0 and $\bar{X}$

$$dz = (q - \delta z)\, dt + \nu\, dB$$

- Production: static assignment equilibrium at each instant, wage $W(z) = \max_{x \le z} \omega(x) = \omega(z)$

Stationary recursive competitive equilibrium

A stationary equilibrium is a value function $V(z,a)$, an investment policy $q(z,a)$, a density of workers over capability and age $\psi(z,a)$, and a wage schedule $\omega(x)$ such that

1. Worker optimality: given $\omega(x)$, $V(z,a)$ and $q(z,a)$ satisfy the HJB

$$\rho V(z,a) = \max_{q \geq 0} \left\{ W(z) - c(q) + (q - \delta z) V_z(z,a) + \frac{\nu^2}{2} V_{zz}(z,a) + V_a(z,a) \right\}$$

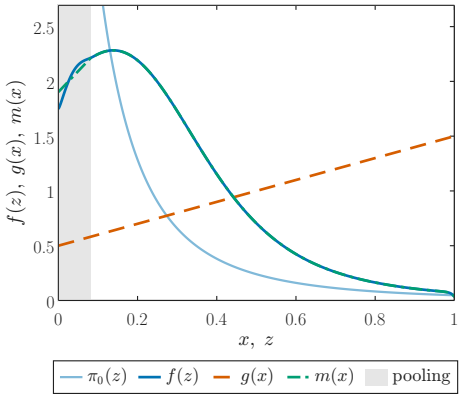
   with boundary conditions: $V(z,\bar{a}) = 0$ and $V_z(0,a) = V_z(\bar{X},a) = 0$

2. Aggregation: given $q(z,a)$, $\psi(z,a)$ satisfies the KFE

$$\frac{\partial \psi}{\partial a}(z,a) = -\frac{\partial}{\partial z}\Big[\big(q(z,a) - \delta z\big)\psi(z,a) \Big] + \frac{\nu^2}{2} \frac{\partial^2 \psi}{\partial z^2}(z,a), \qquad \psi(z,0) = \frac{\pi_0(z)}{\bar{a}}$$

3. Assignment: given the capability density $f(z) = \int_0^{\bar{a}} \psi(z,a)\, da$, $\omega(x)$ is the wage schedule of the static competitive equilibrium

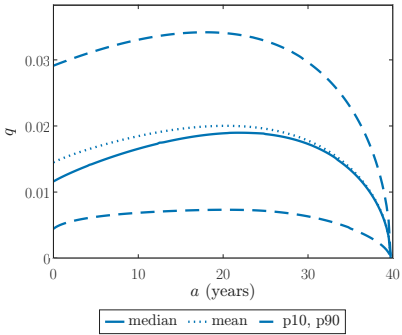
Baseline stationary equilibrium



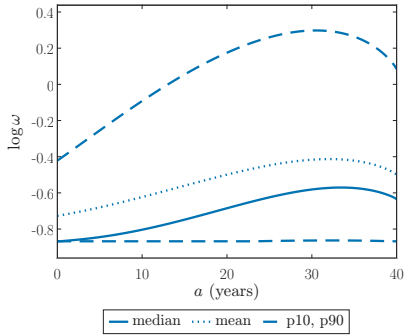
- Parameters (time in years)
  - Career length $\bar{a} = 40$, discount rate $\rho = 6.25\%$
  - Entry density $\pi_0(z)$: the static model's Pareto
  - Task weights $g(x) = 0.5 + x$, elasticity $\sigma = 2$
  - Depreciation $\delta = 5\%$, shock volatility $\nu = 2\%$
  - Investment cost: $\beta = 2$, $\alpha = 4 \times 10^{-5}$
- Stationary equilibrium
  - Hump-shaped stationary capability density $f(z)$
  - Mean capability 0.27, against 0.14 at entry
  - Pool with 17% of workers
  - Static model with the same Pareto results in full separation

Life-cycle investment and wages

Investment by age



Log wage by age



- Investment: peaks mid-career, falls to zero at retirement
- Wages: mean log wage rises until about age 32, then falls

Outline

1. Model

2. Equilibrium construction
   - Equilibrium with constant task weights
   - Equilibrium with increasing task weights

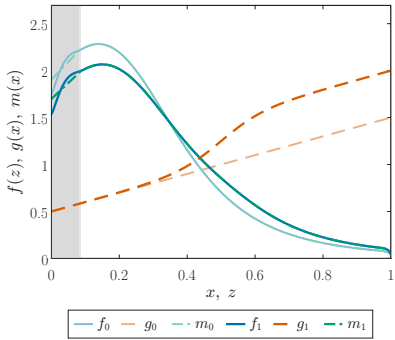
3. Technological change
   - Experiment 1: Difficult new tasks
   - Experiment 2: Upskilling
   - Experiment 3: Difficulty-biased technological change

4. Capability investment
   - Baseline dynamic model
   - Difficulty-biased technological change

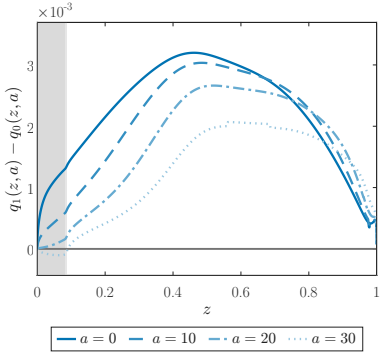
5. Conclusion

Difficulty-biased technological change in dynamic model

Capability density, task weights, and task employment

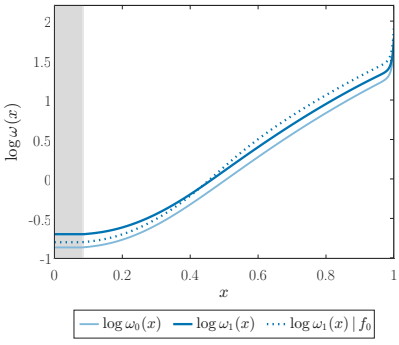


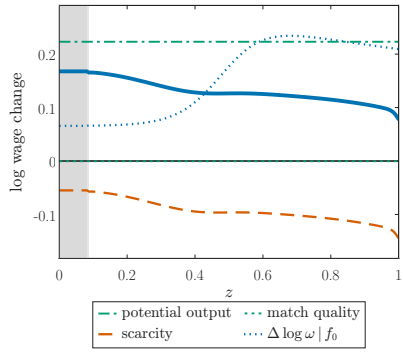
Change in the investment policy by age



- Before: hump-shaped capability density, 17% of workers in the pool
- New task weights ⇒ investment rises, most for young workers just below the gaining tasks
- Capability density shifts right

Difficulty-biased technological change in dynamic model



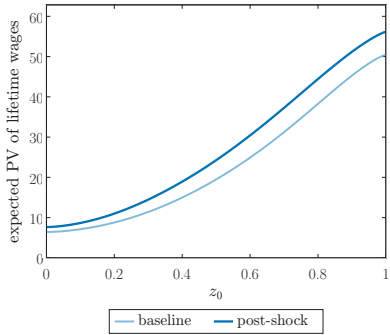


- No investment response (dotted): gains go to workers capable of the difficult tasks
- With investment: scarcity higher at the bottom, lower at the top than with no investment response

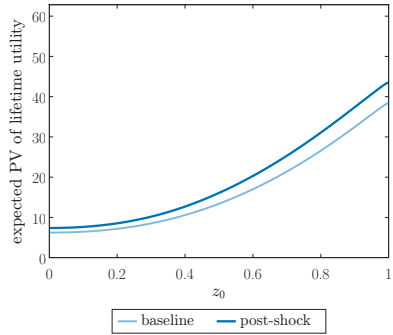
  ⇒ wage gains much more equal across capabilities

Lifetime wages and utility

Expected PV of lifetime wages at birth

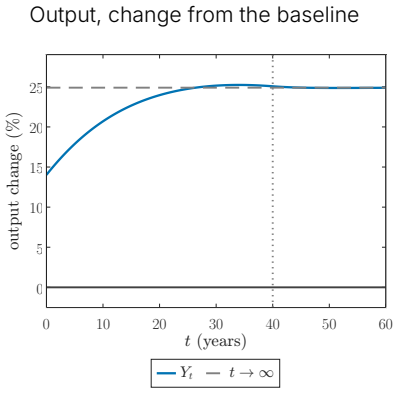


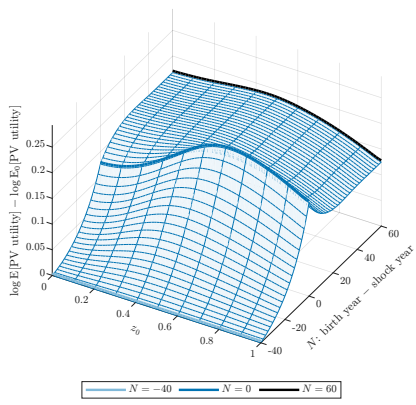
Expected PV of lifetime utility at birth



- Stationary equilibria: expected PV of a full career, by capability at birth z_0

Transition dynamics





- Cohorts: N = birth year − shock year; $N = -40 \equiv a = 40$ when the shock hits
- Lifetime value: much lower for older cohorts, and more equal for younger cohorts

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   - Difficulty-biased technological change

5. Conclusion

A Capability Theory of the Wage Distribution

- Framework for studying effects of technological change on the wage distribution that embodies a notion of capability
- Wage distribution and task allocation determined by worker capability distribution and task difficulty weight function
- Equilibrium gives rise to pooling regions: workers of different capabilities earn same wages and some workers perform tasks for which they are overqualified
- Algorithm to find pooling regions through least concave majorant of capability supply in task-capacity units.
- Capability investment can mitigate distributional effects of difficulty-biased technological change

Appendix

Regularity condition

- Assumption 1: for some constants $K > 0$ and $c > 0$,

$$\frac{F(x)}{G(x)} \leq K \quad \forall x \in (0, \bar{X}] \qquad \text{and} \qquad \frac{1 - F(x)}{\bar{U} - G(x)} \geq c \quad \forall x \in [0, \bar{X})$$

  - Labor per unit of task weight bounded at the bottom, bounded away from zero at the top
  - Holds for the truncated Pareto and the winsorized Student t

→ back to conditions → back to results

Elasticity of substitution

Experiment 1, new tasks at the top: Student t capability density, $g(x) = 0.5 + x$

	$\sigma = 2$	$\sigma = 1$	$\sigma = 0.5$
Change in output (%)	8.0	−14.1	−43.3
Log wage change			
Potential output	0.144	0	−0.288
Match quality	−0.034	−0.152	−0.558
Scarcity			
Bottom pool	−0.072	−0.144	−0.288
Top pool	0.157	0.313	0.627
Total			
Bottom pool	0.038	−0.296	−1.134
Top pool	0.267	0.162	−0.219
Workers with higher wages (%)	100.0	7.2	0.0

→ back $\sigma = 1$

Cobb–Douglas limit

$$Y = \bar{U}^{\frac{1}{\sigma-1}} \underbrace{\left[\int_0^{\bar{X}} \hat{g}(x)^{\frac{1}{\sigma}} m(x)^{\frac{\sigma-1}{\sigma}} \, dx \right]^{\frac{\sigma}{\sigma-1}}}_{\Phi}$$

- As $\sigma \to 1$, $\bar{U}^{\frac{1}{\sigma-1}} \to 0$ or ∞ for $\bar{U} \neq 1$

  $\Rightarrow$ specify production in the Cobb–Douglas limit as

$$Y_{\sigma=1} \equiv \lim_{\sigma \to 1} \Phi = \exp\left(\int_0^{\bar{X}} \hat{g}(x) \log \frac{m(x)}{\hat{g}(x)} \, dx \right)$$

- Log wage difference given by:

$$\Delta \log \omega(x) = \underbrace{\Delta \log \Phi}_{\text{match quality}} + \underbrace{\Delta \log \frac{\hat{g}(x)}{m(x)}}_{\text{relative scarcity}}$$

→ back