# Accounting for Nature in Economic Models and Decisionmaking

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The analysis and conclusions set forth are those of the authors and do not necessarily represent the views of the Sveriges Riksbank or its Board, nor those of the Central Bank of Chile or its Board.

## Outline

- Introduction (policy relevance, questions, our contributions)
- A two-bloc model with Nature
- The Social Planner Problem
- Calibration
- Model Dynamics
- Green Policies
- Conclusion

# Policy relevance & Questions

## Policy Relevance

- The decline or disappearance of biological diversity, also known as ¨biodiversity loss¨, has accelerated dramatically over the past half century
- Biodiversity loss is a risk for the environment and for humanity because biodiversity is key to producing what we need to live
- At the same time, the major drivers of biodiversity loss – are all "by-products" of economic growth and production (IPBES (2016))

## Policy Questions

- How should capital from Nature (*aka* ¨Natural Capital¨ or $K_N$) be accounted for in economic models and decisionmaking?
- What are the implications of accounting for Nature in economic models used to inform policy?
- Is there a role for green policies that can help conserve more Nature?

# Economics and Nature

- Traditional growth models, like the Solow-Swan 1956 neoclassical growth model (Solow (1956)), are silent about the natural foundation of production
- Following large oil shocks and rising pollution, in the 70s, attempts made to embed non-renewable natural resources; see Solow (1974), Stiglitz (1974); Dasgupta and Heal (1974), and Hartwick (1977)
- In the 1990s, limits to growth analysis begun (see Bovenberg and Smulders (1995), Howitt and Aghion (1998), Brock and Taylor (2010))
- Then came attempts to model $K_N$ as a renewable resource (Mourmouras (1991); Hinterberger et al. (1997); Bringezu et al. (2003); Copeland and Taylor (2009))
- A growing literature focuses on the political economy aspects of renewable resources management (Karp and Rezai (2014a), Karp and Rezai (2014b))
- Dasgupta (2021) proposes to add $K_N$ in production function as, both (1) a flow of ¨services¨ yield by natural assets (e.g. timber, catches of fishery products, produce, etc.), and as (2) a given stock of the biosphere necessary to ensure the continuation of key ecosystem services as well as life on Earth and human productive activities (see also Kornafel and Telega (2020))

# Our contribution

- Model $K_N$ as a *bounded factor input* in the supply side of a fully specified two-bloc growth model to proxy biophysical limits to economic production and allow for tipping points in its evolution
- Link value of $K_N$ to its 'exploited' value (e.g. a killed whale, fallen tree) *but also* to its economic value when conserved
- Derive optimal green policies to foster sustainable production
- **Show that:**
  (1) when natural assets are abundant, it is optimal to deplete some and conserve some, but less depletion should occur if there is a critical threshold beyond which Nature is irreversibly altered;
  (2) if production can push Nature to a level beyond which it can no longer regenerate itself, lower initial output and consumption levels make growth sustainable and stronger long run;
  (3) when Nature is a driver of TFP, subsidizing the conservation of Nature can improve economic welfare for all countries.

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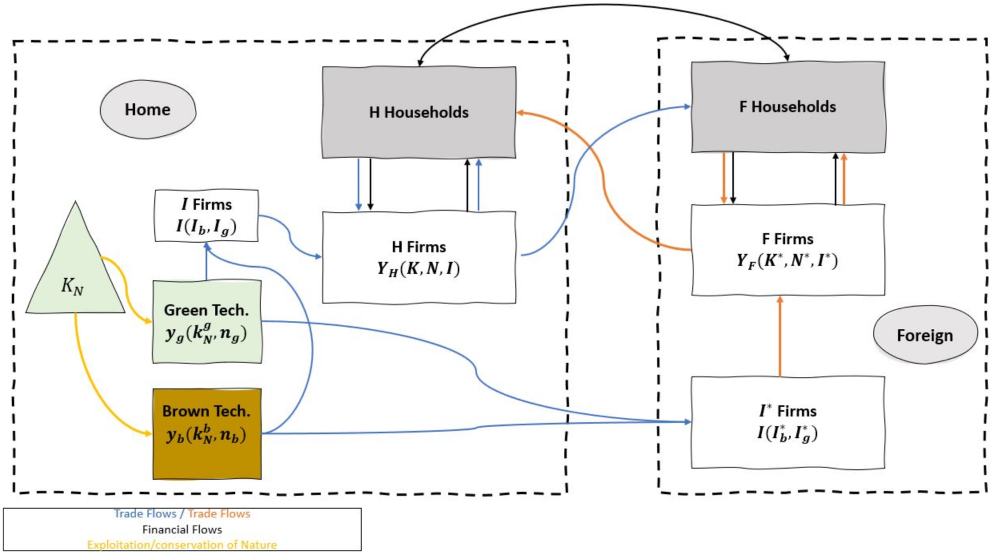
# A two-bloc model with Nature



# Macro ingredients of the two-bloc model

- Households ▸ Details
- Intermediate brown/green firms ▸ Details
- Intermediate input and final consumption good firms ▸ Details
- **Natural capital**
- Aggregation and market clearing ▸ Details

# Brown and Green Production

**Brown production:**

- It includes any type of technical process that uses the stock of Nature as an input in an unsustainable way (meaning that it has an impact on the accumulation of $K_N$ between two periods)
  - forest logging, industrial crop and animal farming, "$K_N$-consuming" manufacturing, etc.

**Green production** has two interpretations:

- It can include any type of technical process that relies on the stock of Nature as an input, in a sustainable way
  - poly-culture and regenerative land and ocean farming, conservation activities, sustainable forest management
  - other economic activities such as textiles, manufacturing, real estate, etc. where the amount of virgin materials used is reduced (for example through renewable energy production, recycling of material inputs etc.)
- It can be interpreted as environmental offsets

$\Rightarrow$ Importantly, green production can exploit the stock of Nature, the only restriction being that from one period to the next the exploitation must be such there is no perturbation in the natural accumulation of $K_N$

## Modeling Natural Capital

- The stock of natural capital ($K_{N,t}$) is assumed to evolve as a function of the amount that is exploited for production ($K^b_{N,t}$), the amount that is preserved ($K^g_{N,t}$), and parameters such as the intrinsic regeneration rate ($r_N$), the carrying capacity level ($CC$), and its critical threshold ($CT$)

$$K_{N,t+1} = K_{N,t+1}(K^b_{N,t}, K^g_{N,t}, r_N, CC, CT)$$
$$K_{N,t} = K^b_{N,t} + K^g_{N,t}$$

- The values of $r_N, CC, CT$ are going to be assumed exogenous,constant, known, and common across all firms
- $r_N$ contributes to the rate at which natural capital is regenerated when left untouched
- $CC$ indicates the upper ecological limit of natural capital (fauna and flora)
  $\Rightarrow$ The max amount of $K_N$ that can be sustained given the resources available in the ecosystem
- $CT$ refers to the critical threshold for natural capital, or its 'tipping point'
- To capture these dynamics we focus on two main alternative specifications for natural capital

# Specification 1: Natural capital with no critical threshold

- The stock of natural capital can always recover to its original, carrying capacity level, no matter what amount of depletion occurs between periods
- The stock of natural capital depends non-linearly on its ¨background¨ or ¨natural¨ regeneration rate, $r_N$, and its $CC$ level

$$K_{N,t+1} = \underbrace{K_{N,t}\left(1 + r_N\left(1 - \frac{K_{N,t}}{CC}\right)\right)}_{\textit{Natural Accumulation}} - \underbrace{K^b_{N,t}}_{\textit{Economic Activity}} \tag{1}$$

- We define the accumulation rate of natural capital as:

$$A_{N,t} \equiv r_N K_{N,t}\left(1 - \frac{K_{N,t}}{CC}\right)$$

⇒ This is the traditional specification used in the literature (see Copeland and Taylor (2009))

## Specification 2: Natural capital with critical threshold

- The stock of natural capital is also dependent on $CT$
- The equation for natural capital under this specification becomes

$$K_{N,t+1} = K_{N,t}\left(1 + r_N\left(1 - \frac{K_{N,t}}{CC}\right)\left(\frac{K_{N,t}}{CT} - 1\right)\right) - K_{N,t}^b \tag{2}$$

- In this case, once $K_{N,t} < CT$, the existing stock of natural capital converges progressively to zero. The accumulation rate is then given by:

$$A_{N,t} \equiv r_N K_{N,t}\left(1 - \frac{K_{N,t}}{CC}\right)\left(\frac{K_{N,t}}{CT} - 1\right)$$

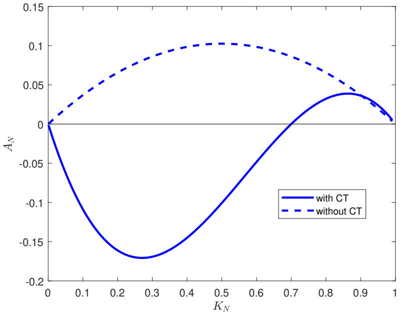
- In the presence of a critical threshold, the rate at which natural capital accumulates/decumulates depends not only on $CC$ but also on $CT$

$\Rightarrow$ Less common specification (emphasized in Dasgupta (2021))

# Natural capital accumulation rates: A graphical representation

- Figure 1 plots the rate at which natural capital evolves (that is, its accumulation rate $A_N$) with or without $CT$, normalizing the value of $CC$ to 1 and assuming $CT = 0.7$ and setting $r_N = \{0.41, 1.41\}$

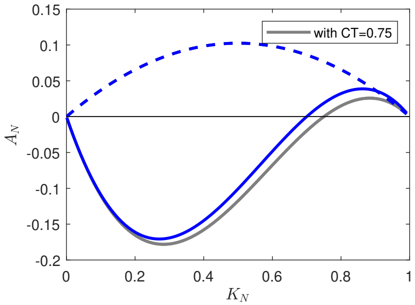
**Figure 1:** Nature Accumulation Rates ($A_N$)



# Natural capital accumulation rates: A graphical representation

- A higher $CT$ ($CT = 0.75$) compresses the region where there is positive accumulation of $K_N$ and impairs the regeneration rate when $K_N$ is close to the $CT$

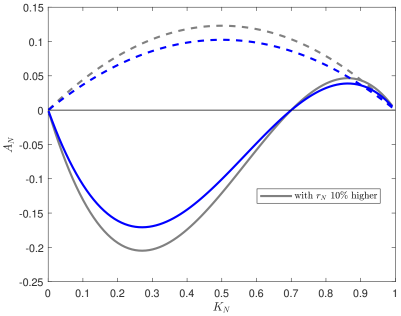
**Figure 2:** Nature Accumulation Rates ($A_N$)



# Natural capital accumulation rates: A graphical representation

- A higher $r_N$ (by 10%) increases the accumulation rate of $K_N$

**Figure 3:** Nature Accumulation Rates ($A_N$)



# Accounting for uncertainty in accumulation rates

- In practice, under both specifications, the accumulation rate of $K_N$ remains uncertain because shocks to each specification may affect the evolution of natural capital
- To capture this we go one step further in modeling $K_N$ and postulate that there are shocks that affect multiplicatively the accumulation rate

$$ln\left(z_{t+1}\right) = \rho^N ln\left(z_t\right) + \sigma\epsilon_{t+1}$$
$$\epsilon_t \sim \mathcal{N}(0,1)$$

- We thus re-write the law of motion of natural capital (in the absence of critical threshold) as:

$$K_{N,t+1} = K_{N,t}\left(1 + z_t r_N\left(1 - \frac{K_{N,t}}{CC}\right)\right) - K^b_{N,t}$$

- and similarly for the case when there is a $CT$
- The multiplicative assumption implies that the greater $A_N$, the larger the uncertainty that the agents or Social Planner face when making optimal decisions

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# The Social Planner Problem

- To study the role of policy in the optimal management of natural assets, we introduce in the model an unconstrained global Social Planner (SP)
- Her role is to maximize global welfare subject to world resources constraints and the dynamics of natural capital
- Global welfare depends on domestic and foreign consumption $(C_t, C_t^*)$ through the utility functions $U(\bullet)$ of the households living in the H and F blocs, and does not have a specific preference nor target for a certain level of the natural capital stock

$$U(C_t) \equiv \frac{C_t^{1-\sigma} - 1}{1-\sigma}$$

$$U(C_t^*) \equiv \frac{C_t^{*1-\sigma} - 1}{1-\sigma}$$

## The Social Planner Problem: $\max_{\Psi} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t(\chi U(C_t) + (1-\chi)U(C_t^*))$ s.t.

$$Y_{F,t} = C^*_{F,t} + C_{F,t} + K^*_{t+1} - (1-\delta^*)K^*_t \tag{3}$$

$$Y_{H,t} = C_{H,t} + C^*_{H,t} + K_{t+1} - (1-\delta)K_t \tag{4}$$

$$Y_{b,t}(1-\kappa(K_{N,t}, K^b_{N,t})) = I^*_{b,t} + I_{b,t} \tag{5}$$

$$Y_{g,t} = I^*_{g,t} + I_{g,t} \tag{6}$$

$$K_{N,t} = K^g_{N,t} + K^b_{N,t} \tag{7}$$

$$K_{N,t+1} = K_{N,t+1}(K^b_{N,t}, K^g_{N,t}, r_N, CT, CC) \tag{8}$$

together with a list of definitions and clearing conditions.

- $\kappa(K_{N,t}, K^b_{N,t})$ is a cost function that is increasing in the amount of exploited natural capital for a given stock
- $Y_H$ ($Y_F$) is domestic (foreign) final consumption goods output, $C_F$ ($C^*_F$) domestic (foreign) consumption of foreign goods, $C_H$ ($C^*_H$) domestic (foreign) consumption of home goods, $K$ ($K^*$) domestic (foreign) physical capital, $Y_b$ ($Y_g$) brown (green) output and $I_b$ ($I_g$) brown (green) intermediate inputs
- FOCS Here

## The Social Planner Problem: Shadow Price of Nature

- A special FOC pertains to the shadow price of natural capital, $\mu_t$. Based on the SP problem's FOC with respect to $K^g_{N,t}$, this is given by:

$$\mu_t = \underbrace{-\lambda_{3,t}\left(\frac{\partial Y_{b,t}}{\partial K^g_{N,t}}\left(1-\kappa(K_{N,t},K^g_{N,t})\right) - Y_{b,t}\frac{\partial \kappa(K_{N,t},K^g_{N,t})}{\partial K^g_{N,t}}\right)}_{\text{Marginal Costs from conservation}} - \underbrace{\lambda_{4,t}\frac{\partial Y_{g,t}}{\partial K^g_{N,t}}}_{\text{Marginal Benefits from conservation}}$$

where

$$\frac{\partial Y_{b,t}}{\partial K^g_{N,t}} < 0; \quad \frac{\partial \kappa(K_{N,t},K^g_{N,t})}{\partial K^g_{N,t}} < 0$$

- The shadow price of natural capital can be interpreted as the difference between two magnitudes: the marginal cost from foregoing an additional unit of $K_N$ in the production of brown intermediate goods; and the marginal benefit associated with preserving one additional unit of $K_N$ for the production of green intermediate goods
- This means that in our set up, the shadow price of $K_N$ is positive when there exists a trade-off between exploitation and conservation of the stock of natural capital

# The Social Planner Problem: Hotelling Condition

- $K_N$'s shadow price must satisfy an intertemporal condition. This relates the current shadow price with expected future benefits from changes in the amount of available $K_N$
- Following from the FOC with respect to $K_{N,t+1}$, this Hotelling condition is given by:

$$\mu_t = \beta \mathbb{E}_t \left[ \mu_{t+1} \frac{\partial A_{N,t+1}}{\partial K_{N,t+1}} + \lambda_{3,t+1} \left( \frac{\partial Y_{b,t+1}}{\partial K_{N,t+1}} (1 - \kappa(K_{N,t+1}, K^g_{N,t+1})) - Y_{b,t+1} \frac{\partial \kappa(K_{N,t+1}, K^g_{N,t+1})}{\partial K_{N,t+1}} \right) \right]$$

- Assuming $\mu_t > 0$, the pace of natural capital accumulation/decumulation rate thus depends on two forces: first, the change in $K_N$ accumulation rate, which in turn depends on the level of the natural capital stock relative to $CC$ (and $CT$ if this is assumed); second, the marginal value of changes to next period's natural capital stock on brown production

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# Calibration

- We assume that one period corresponds to 5 years, in line with the literature that focuses on modeling long-run climate economics phenomena
- We align common macroeconomic parameters with prior existing empirical literature
- We assume no Home bias in consumption
- We experiment with parameters governing natural capital basing our priors on environmental research in connection to each parameter, since the economic literature on this is scarce or non-existent
- Our analysis should be taken as a numerical exploration rather than a quantitative one

# Calibration: Macro Parameters

- For the intratemporal rate of substitution between intermediate inputs, $g$, $b$, previous literature has found a vast range of estimates highlighting the difficulty in pinning down a value for our parsimonious model $\Rightarrow$ we use a value equal to 1.5, so as to emphasize imperfect substitution
  - this value is below 2.4, which is the value reported for domestic and imported intermediates by Blaum et al. (2018) but above the value of 0.9 proposed by Peter and Ruane (2023) to characterize the elasticity between energy, material and services inputs

- For the share of green inputs in the intermediate goods production function we chose 10%. This allows to incorporate well the trade-off that a SP is likely to face, conditional on the current economic structure

$$I_t = \Big( \underbrace{\omega_g}_{10\%} I_{g,t}^{\frac{\rho-1}{\rho}} + \underbrace{\omega_b}_{90\%} I_{b,t}^{\frac{\rho-1}{\rho}} \Big)^{\frac{\rho}{\rho-1}}$$

and similarly for $I_t^*$

- Other details Here; full Table with values Here

# Calibration: Environmental Parameters

- To set the $CT$ level, we rely on current estimates of what is considered a safe decline in biodiversity levels, and ecosystems more generally
  - We use the Atlantic rainforest as a reference; the forest itself when in a self-sustainable state can recycle much of the rain that falls on it, generating a self preserving cycle. Removing as little as 30% of the forest cover *could* impede this self-perpetuating stabilizing cycle (see Nepstad et al. (2007),Salati (1987), Farley (2008)). A similar threshold applied for fish populations ((Bousquet et al. (2008) andThorpe et al. (2015)))

⇒ Normalizing the level of CC to 1 we set $CT = 0.7$

- To calibrate the regeneration rate $r_N$, we follow Poorter et al. (2021), who analyzed 2,200 patches of forest in West Africa and Central and South America, including areas of the Atlantic and Amazon rainforests
  - The main results of the study suggest after deforestation the forests' structure and function had fully returned after 60 years

⇒ We set $r_N = 1.4$ for the model without $CT$ and 0.4 for the model with $CT$

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# Model Dynamics

- We study how decisions by the Social Planner in the model are affected by different initial levels of natural capital and by natural capital's own dynamics
- We start by assuming that there is no TFP growth. We look at four natural capital scenarios:
  - (1) initially abundant $K_n$, no $CT$;
  - (2) initially abundant $K_n$, $CT$;
  - (3) critically depleted $K_n$, no $CT$;
  - (4) critically depleted $K_n$, $CT$.
- In all cases we trace the behavior of both natural capital and key macro-variables as the economy evolves starting from the given levels of the state variables toward the model long-run stationary equilibrium
- We then move to a world with positive (exogenous) TFP growth and perform a similar analysis

# Model Dynamics: Cases (1) (2) -Abundant Initial $K_N$

- To be able to compare the model dynamics we start by computing initial (time $t0$) values of the key macroeconomic variables between the case with CT and the case with no CT
- This allows subsequently to put relative growth rates into context across the two scenarios
- The Table reports such values showing that in the presence of a CT there is lower economic activity than in a world in which there is no CT ...
- with the exception of green production and green labor

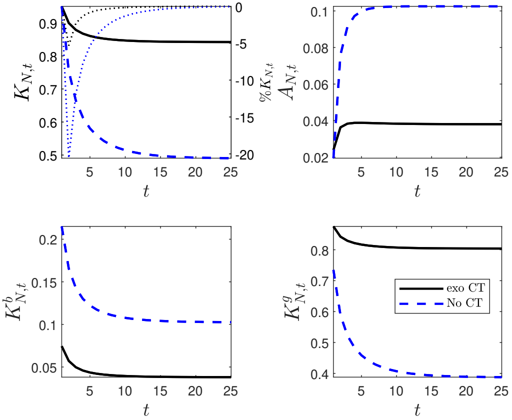
**Table 1:** Percentage (%) in levels at $t0$ ($CT$ versus no-$CT$)

| Var. | $\Delta$(%) |
|---|---|
| $C$ | -21.8 |
| $Y_b$ | -40.5 |
| $Y_g$ | 20.9 |
| $N_b$ | -2.0 |
| $N_g$ | 22.6 |
| $Inv$ | -29.2 |
| $Inv^*$ | -28.9 |
| $Y_H$ | -22.2 |
| $Y_F$ | -23.6 |

# Model Dynamics: Cases (1) (2) -Abundant Initial $K_N$

- In our set up it is not optimal to maintain $K_N$ close to its $CC$ level because this requires setting aside a large portion of natural capital for conservation, which in turn implies reducing substantially the production of brown goods
- The output trade-off when $K_N$ is close to $CC$ is particularly stark because–both and without a $CT$–the rate at which natural capital accumulates is low, as in the proximity of $CC$ this rate declines as natural capital increases (Figure Accumulation Rates)
- It follows that when the economy finds itself with a high level of $K_N$ close to $CC$, the SP, efficiently chooses to lower the natural capital stock gradually over time

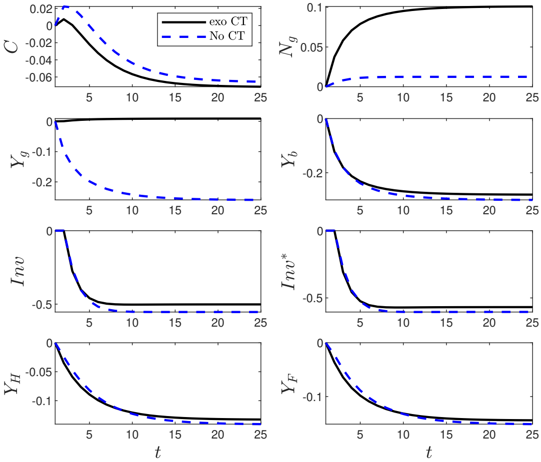
**Figure 4:** Natural Capital Dynamics



# Model Dynamics: Cases (1) (2) -Abundant Initial $K_N$ (Continued)

- There is a smooth decline of brown production
- Physical investment also declines
- Since the speed of physical disinvestment is higher than the speed at which output declines, in equilibrium consumption can temporarily increase, before falling
- When there is no *CT* green output falls; this outcome is efficient due to the relatively small share that green production plays in generating the final consumption goods
- When there is a *CT*, green output increases; this is possible because of a sufficiently strong re-allocation of labor from the brown sector towards the green sector

**Figure 5:** Growth Rates of Macroeconomic Variables



# Model Dynamics: Cases (3) (4) -Critically Depleted Initial $K_N$

- To be able to compare the model dynamics in this case relative to the previous case we compute initial (time $t0$) values of the key macroeconomic variables in this case versus the abundant $K_N$ case
- This allows to put relative growth rates into context across the two scenarios.
- The Table reports such values for both the no-$CT$ and the $CT$ assumptions, showing that in both cases, there is lower economic activity than in a world in which natural capital is initially abundant
- The discrepancy in activity is visibly larger in the presence of a $CT$ than in the absence of it, with the exception of green production and green labor

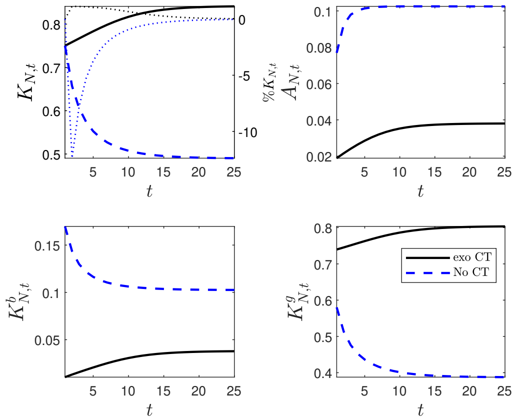
**Table 2:** Percentage (%) in levels at $t0$ (Critical vs. Abundant)

| Var. | Exo CT | No CT |
|---|---|---|
| $C$ | -32.3 | -4.9 |
| $Y_b$ | -61.9 | -10.7 |
| $Y_g$ | 8.0 | -10.7 |
| $N_b$ | -4.1 | 0.0 |
| $N_g$ | 35.5 | 0.0 |
| $Inv$ | -90.5 | -17.2 |
| $Inv^*$ | -85.6 | -16.6 |
| $Y_H$ | -39.0 | -6.5 |
| $Y_F$ | -41.6 | -7.0 |

# Model Dynamics: Cases (3) (4) -Critically Depleted Initial $K_N$ (Continued)

- Like in the abundant case, natural capital adjusts to its economically efficient long-run levels
- The *rate* at which natural capital accumulates in this scenario is much lower, however, when a *CT* is present
- When the economy starts with a low stock of natural assets, and there exists a CT, it is obviously optimal to conserve more natural assets than when the economy starts in an abundant $K_N$ state
- Moving away from *CT* also raises the rate at which Nature can regenerate itself, which in turn allows the social planner to count on more natural capital in the future while still allowing for more $K_N^b$ accumulation in the near term–a virtuous cycle

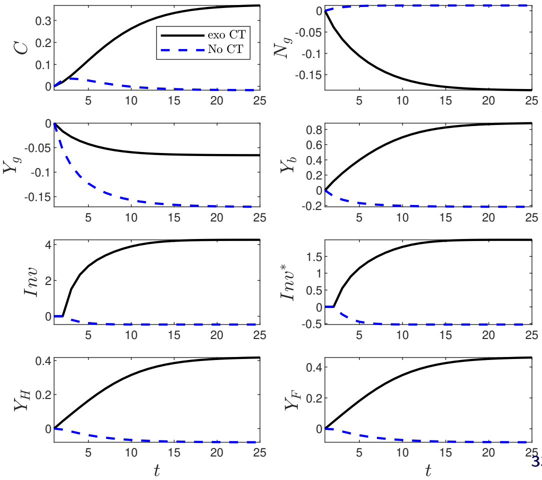
**Figure 6:** Natural Capital Dynamics



# Model Dynamics: Cases (3) (4) -Critically Depleted Initial $K_N$ (Continued)

- Brown output can expand *while* green output declines due to labor reallocation; overall consumption increase through time
- This happens because while, with $K_N$ initially close to $CT$, it is optimal for the social planner at first to reduce brown output and divert resources to green production–in order to raise the level of $K_N$ from its critically-depleted state...
- as the economy moves away from its tipping point it becomes increasingly inefficient to sacrifice brown production to favor green production
- When there is no critical threshold, there are no large qualitative differences from an abundant natural capital state

**Figure 7:** Growth Rates of Macroeconomic Variables



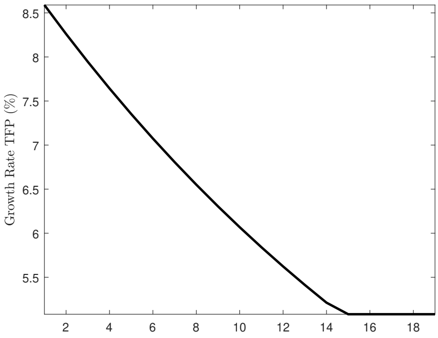
# Adding Technological Growth

- In this case the policy functions are time-inhomogenous
- The growth processes are formalized following Barrage and Nordhaus (2023):

$$A_{j,t} = A_{j,t-1}/(1 - g_{0,j} exp(-g_{1,j} N(t-1)))$$

for $j \in \{H, F, c, h\}$
- We calibrate the initial exogenous productivity levels of $H, F$ by matching the relative average TFP in 2019 between the two groups
- To calibrate $A_{H,t}, A_{F,t}$ we follow Christensen et al. (2018)
- We use the same growth rates between the two groups
- We make the simplifying assumption that $A_{h,t} = A_{c,t} = A_{H,t}$
- we assume that after 15 periods the economy reaches a steady growth rate
- From period 20 (i.e. after 100 years), we assume a stationary solution, in which we revert to the naive-approach where the agents policy functions do not incorporate the future path of productivity growth

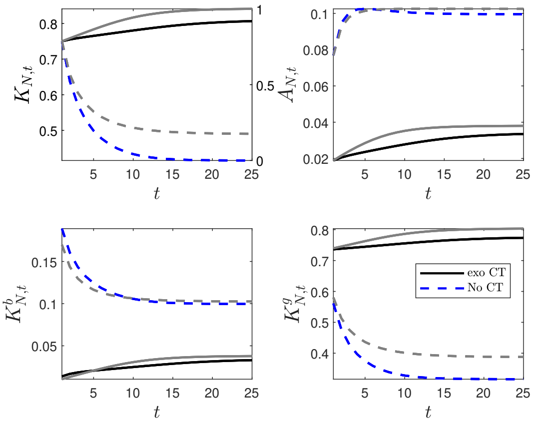
**Figure 8:** Growth Rate of Technology



# Model Dynamics with Technological Growth

- The stock of natural capital declines faster and more (accumulates slower and less) than in the absence of technological growth, when we assume the absence (presence) of a *CT*
- This is because SP internalizes that, for a given level of natural capital conserved today, the marginal benefit that arise from producing brown goods tomorrow–i.e. goods which entail depleting natural capital–are relatively higher compared to a scenario without technological growth
- This in turn generates an equilibrium where it is optimal to deplete relatively more natural capital (or equivalently, in the *CT* scenario, to conserve relatively less natural capital)

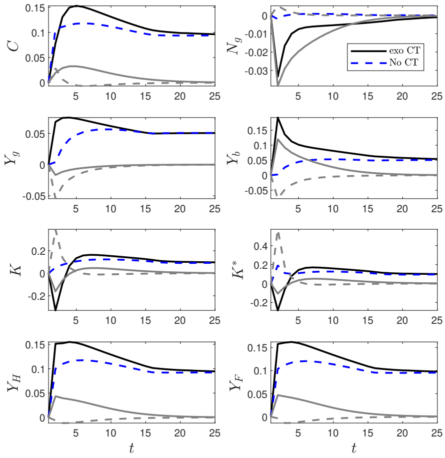
**Figure 9:** Natural Capital Dynamics



# Model Dynamics with Technological Growth (Continued)

- The dynamics remain coherent with the scenario without technological growth
- The simulation highlights that in the presence of a CT it is optimal to contain the accumulation of physical capital (that is, to invest less), compared to a scenario without a CT
- Given the current calibration and assumptions, the exercise suggests a slower accumulation of domestic (foreign) capital in the range of -4.0% (-4.5%) per year on average over the course of the next 10 years

**Figure 10:** Growth Rates in Macro Variables



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# Introducing the TFP externality of Nature

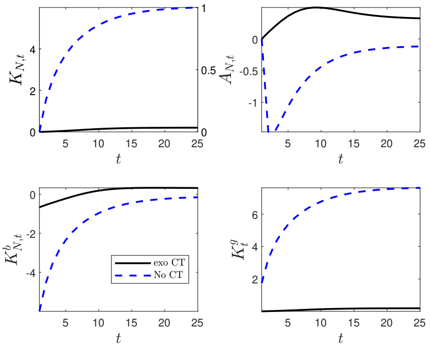
- We assume the following externalities:

$$y_{g,t} \equiv \underbrace{A^0_{g,t}(K^g_{N,t})^{\phi_g}}_{\equiv A_{g,t}}(k^g_{N,t})^{\alpha_g} n^{1-\alpha_g}_{g,t}$$

$$y_{b,t} \equiv \underbrace{A^0_{b,t}(K^g_{N,t})^{\phi_b}}_{\equiv A_{b,t}}(k^b_{N,t})^{\alpha_b} n^{1-\alpha_b}_{b,t}$$

- As expected, natural capital accumulation is stronger when there is a CT and the SP internalizes the TFP externality
- The same virtuous cycle allows eventually for more exploitation and more conservation
- In both cases there is *front-loading* and the speed of accumulation (decumulation) of natural capital is stronger (milder) during the first periods, as demonstrated by the non-monotonic evolution of the percentage differences in accumulation rates

**Figure 11:** Natural Capital (% Differences SP and CE)




Note: $\phi_g = 0.05$, $\phi_b = 0.05$, $K_{N,t_0} = 0.75$.

# Green Policies: The case of a Domestic Subsidy Scheme on Intermediate Goods Firms

- A tax authority in the Home country could establish a (lump-sum financed) subsidy offered to the intermediate goods firms on the stock of conserved nature ($k^g_{N,t}$)
- This subsidy can be expressed in nominal units (that is in the country "currency"), as similarly done in the case of carbon taxes, which are usually expressed in terms of $ per ton of $CO_2$
- In our case, the relevant measure is the "unit of nature"; this unit could be made to correspond to the stock of forests, the stock of fishing population, or the stock of freshwater available net of withdraws

**Proposition**

*A nominal, lump-sum financed, subsidy to the intermediate goods firms equal to $P_t \tau_t k^g_{N,t}$ where $\tau_t$*

$$\tau_t = p_{b,t}\phi_b \frac{y_{b,t}}{K^g_{N,t}}(1 - \kappa(k_{N,t}, k^g_{N,t})) + p_{g,t}\phi_g \frac{y_{g,t}}{K^g_{N,t}}$$

*offsets the TFP-related externality associated with the stock of nature.*

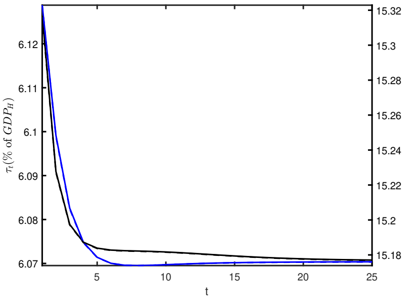
# Green Policies: The case of a Domestic Subsidy Scheme on Intermediate Goods Firms (Continued)

- The subsidy (as a % of H bloc output)

$$\frac{\tau_t K^g_{N,t}}{p_{H,t} Y_{H,t} - p_{I,t} I_t}$$

- The numerical illustration shows that the subsidy rate (as a percentage of home GDP) is decreasing through time as the economy adjusts (LHS: $\phi_g = \phi_b = 0.02$; RHS $\phi_g = \phi_b = 0.05$)
- The dynamics of the subsidy are similar, independently of the assumed elasticity

**Figure 12:** Optimal Green Subsidy (%) Home GDP




Note: Left $y$ axis corresponds to the case $\phi_g = \phi_b = 0.02$; right $y$ axis corresponds to the case $\phi_g = \phi_b = 0.05$.

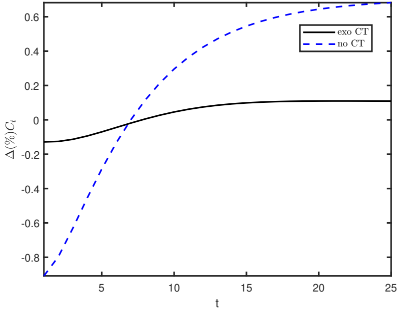
# Green Policies: Welfare Gains

- A subsidy initially determines a drop in consumption compared to the CE; however, higher Nature eventually allow for higher levels of consumption
- We quantify the proportional increase in consumption that would make the households indifferent between the economy with and without green policies; formally, we define the welfare gains of the households as the value $\Omega$ that would make the following equality hold:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\left((1+\Omega)C_t\right)^{1-\sigma}-1}{1-\sigma}\right]=\mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\left(C_t^I\right)^{1-\sigma}-1}{1-\sigma}\right]$$

where $C_t$ denotes consumption in the *CE* and $C_t^I$ denotes the consumption when the intervention is introduced

**Figure 13:** Consumption (% Diff. CE+subsidy vs. CE)

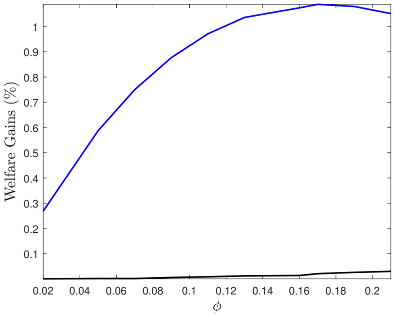



Note: $\phi_g = \phi_b = 0.05$

# Green Policies: Welfare Gains ($\phi_g = \phi_b$)

- Assuming, as done before, that all realizations of shocks are set to 0, we compute welfare gains as a function of $\phi_g, \phi_b$ in both the $CT$ and no-$CT$ case. Welfare gains are increasing in the size of the externality
- Welfare gains when having a $CT$ are close to zero due to the fact that the policymaker has a very limited margin of action when trying to boost conservation, since the accumulation rate is low
- Moreover, rational agents in the CE are already allocating a relatively large amount of resources to the green sector, so as to converge away from the CT
- Forgoing additional amounts of the exploited natural resources has very large costs in terms of produced output, since it requires a significant shift of labor from the brown to the green sector to compensate for the decline in the intermediate input
- Green Policies: Impact on ▸ Relative Prices

**Figure 14:** Welfare Gains (%)
Low $K_{N,t0}$



## Green Policies: Global Tax on Intermediate Inputs

- Even though direct subsidy on Nature represents a first best policy intervention it might be less practical than policies that affect economic quantities for which a market already exists
- We analyzed the case of global taxes applied on purchases of intermediate brown goods
- In the *CE* there is (at least initially) an excess of brown production – the tax is set such that in equilibrium the relative demand of brown inputs (to green inputs) reflects the allocation that arises under a *SP* (▸ Details)
- We found that only a small share of the original externality is resolved and that welfare gains are contained in both the *CT* and $no - CT$ scenarios (▸ Details)
- Intuitively, the brown tax affects the amount of conservation indirectly through the Hotelling equation: i) via a decline in the future marginal value of brown goods, ii) via labor markets readjustments following the decline in the demand of brown inputs

# Outline

- Introduction (policy relevance, questions, our contributions)
- A two-bloc model with Nature
- The Social Planner Problem
- Calibration
- Model Dynamics
- Green Policies
- Conclusion

# Conclusion

- We enriched a standard stochastic two bloc growth model with natural capital alongside man-made capital. Natural capital is an input to production in both blocs, but only one bloc is endowed with it
- Depletion of natural capital for production reduces its ability to regenerate itself, like in the real natural world
- We traced the efficient dynamics for natural capital and economic variables by comparing a world still rich in natural assets and a world in which these assets have been critically depleted.
- We then incorporated an externality in the model (linking the level of natural capital and TFP ) and derived optimal policies
- Our analysis opens the doors to further policy relevant applications, such as the study of the economic impact of greening the production structure of an economy and the welfare costs from delaying the green transition.

Thank You!

# Annex

# Calibration: Macro Parameters Summary Table

| Parameters | *Value* | *Explanation* |
|---|---|---|
| $\sigma$ | 2 | HHs Intratemporal Elast. of Subst. |
| $\beta$ | 0.985 | HHs Discount Factor |
| $\zeta_N$ | 0.30 | Labor Share in H Production |
| $\zeta_{N*}$ | 0.25 | Labor Share in F production |
| $\zeta_K$ | 0.11 | Capital Share in H production |
| $\zeta_{K*}$ | 0.11 | Capital Share in F production |
| $\omega_g$ | 0.10 | share of $c$ intermediate inputs in H production |
| $\omega_g^*$ | 0.10 | Share of $c$ intermediate inputs in F production |
| $\rho$ | 1.50 | Intratemporal Elast. of H intermediate inputs |
| $\rho^*$ | 1.50 | Intratemporal Elast. of F intermediate inputs |
| $\alpha_b$ | 0.48 | Share of Natural Capital in $h$ production |
| $\alpha_g$ | 0.48 | Share of Natural Capital in $c$ production |
| $\phi_g, \phi_b$ | ** | Elasticity of intermed. output to stock of $K_N$ |
| $b_1$ | 10 | Parameters in Cost Function of $h$ firms |
| $\gamma$ | 0.50 | Share F consumption goods |

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# Calibration of Macro Parameters: Details

- We include in the $H$ bloc countries where the share of total natural resources rents in the data is equal to, or above, the $70^{th}$ percentile of the full sample item To calibrate the $F$ bloc we include all the remaining countries, with the exclusion of "low income" and "lower middle income" countries
- We use World Bank data on total resources rents as a share of GDP to distinguish between the two blocs
- We calibrate the size of each bloc (population) and depreciation rates pf physical capital from the Penn World Tables (PWT) as of 2019
- We calibrate the size of each bloc using population data from the Penn World Tables (PWT) as of 2019
- The households intratemporal elasticity of substitution of intermediate goods and the discount factor are set within standard ranges (see for example Golosov et al. (2014)).
- For the shares of intermediate inputs, labor and capital in the production of final consumption goods, we follow results from regressions in Baptist and Hepburn (2013) which, while fit to U.S. data, can be used as a rough starting point sufficient for our numerical illustrations.
- We calculate the depreciation rates of physical capital using the PWT

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# The Global Social Planner Problem: Consumption FOCs

The first order conditions associated with consumption $(C_F, C_F^*, C_H, C_H^*)$ are given by:

$$FOC(C_{F,t}): \ \chi \frac{\partial U'(C_t)}{\partial C_{F,t}} = \lambda_{1,t}$$

$$FOC(C^*_{F,t}): \ (1-\chi) \frac{\partial U'(C^*_t)}{\partial C^*_{F,t}} = \lambda_{1,t}$$

$$FOC(C_{H,t}): \ \chi \frac{\partial U'(C_t)}{\partial C_{H,t}} = \lambda_{2,t}$$

$$FOC(C^*_{H,t}): \ (1-\chi) \frac{\partial U'(C^*_t)}{\partial C^*_{H,t}} = \lambda_{2,t}$$

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# The Global Social Planner Problem: Intermediate Input FOCs

The first order conditions associated with $I_{b^*}, I_b$ give:

$$FOC(I^*_{b,t}): \ \lambda_{1,t}\frac{\partial Y_{F,t}}{\partial I^*_{b,t}} = \lambda_{3,t}$$

$$FOC(I_{b,t}): \ \lambda_{2,t}\frac{\partial Y_{H,t}}{\partial I_{b,t}} = \lambda_{3,t}$$

$$FOC(I^*_{j,g,t}): \ \lambda_{1,t}\frac{\partial Y_{F,t}}{\partial I^*_{g,t}} = \lambda_{4,t}$$

$$FOC(I_{g,t}): \ \lambda_{2,t}\frac{\partial Y_{H,t}}{\partial I_{g,t}} = \lambda_{4,t}$$

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# The Global Social Planner Problem: Home Labor FOCs

The first order conditions with respect to $N_t^b$ and $N_t^g$ are given by:

$$FOC(N_t^b): \quad -\lambda_{2,t}\frac{\partial Y_{H,t}}{\partial N_t^b} = \lambda_{3,t}\frac{\partial Y_{b,t}}{\partial N_t^b}(1-\kappa(K_{N,t}, K_{N,t}^g))$$

$$FOC(N_t^g): \quad -\lambda_{2,t}\frac{\partial Y_{H,t}}{\partial N_t^g} = \lambda_{4,t}\frac{\partial Y_{g,t}}{\partial N_t^g}$$

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# The Global Social Planner Problem: Euler FOCs

The first order conditions with respect to $K_{t+1}, K^*_{t+1}$ produce standard Euler conditions:

$$FOC(K^*_{t+1}): \ \beta \mathbb{E}_t \left[ \lambda_{1,t+1} \left( \frac{\partial Y_{F,t+1}}{\partial K^*_{t+1}} + (1-\delta^*) \right) \right] = \lambda_{1,t}$$

$$FOC(K_{t+1}): \ \beta \mathbb{E}_t \left[ \lambda_{2,t+1} \left( \frac{\partial Y_{H,t+1}}{\partial K_{t+1}} + (1-\delta) \right) \right] = \lambda_{2,t}$$

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# The Global Social Planner Problem: Specifications and Identities

**Consumption Good:**

$$C_t \equiv C_{H,t}^{1-\gamma} C_{F,t}^{\gamma} \ , \ C_t^* \equiv C_{H,t}^{*1-\gamma} C_{F,t}^{*\gamma} \tag{9}$$

**Foreign Firm:** The intermediate input $F$ country firm uses the following CES technology:

$$I_t^* \equiv \left( \omega_g^* I_{g,t}^{*\frac{\rho^*-1}{\rho^*}} + \omega_b^* I_{b,t}^{*\frac{\rho^*-1}{\rho^*}} \right)^{\frac{\rho^*}{\rho^*-1}} \tag{10}$$

The second type of firms' technology uses a Cobb-Douglas CRS Aggregator:

$$Y_{F,t} \equiv A_{F,t} K_t^{*\zeta_K^*} N_t^{*\zeta_N^*} I_t^{*1-\zeta_N^*-\zeta_K^*} \tag{11}$$

**Home Firm:** The intermediate good firm is analogously specified as:

$$I_t \equiv \left( \omega_g I_{g,t}^{\frac{\rho-1}{\rho}} + \omega_b I_{b,t}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{12}$$

and the final good firm as:

$$Y_{H,t} \equiv A_{H,t} K_t^{\zeta_K} N_t^{\zeta_N} I_t^{1-\zeta_N-\zeta_K} \tag{13}$$

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# The Global Social Planner Problem: Specifications and Identities (Continued)

The Home intermediate production technologies are given by:

$$Y_{g,t} \equiv \underbrace{A^0_{g,t}(K^g_{N,t})^{\phi_g}}_{\equiv A_{g,t}}(K^g_{N,t})^{\alpha_g} N^{1-\alpha_g}_{g,t} \tag{14}$$

$$Y_{b,t} \equiv \underbrace{A^0_{b,t}(K^g_{N,t})^{\phi_b}}_{\equiv A_{b,t}}(K^b_{N,t})^{\alpha_b} N^{1-\alpha_b}_{b,t} \tag{15}$$

The cost function is given:

$$\kappa(K_{N,t}, K^b_{N,t}) = e^{-\frac{b_1}{2}\left(1-\frac{K^b_{N,t}}{K_{N,t}}\right)^2}$$

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# Other Ingredients of the two-bloc model: Households

- We assume a mass $N_t$ of $H$ households and a mass $N_t^*$ of $F$ households
- Households in both bloc maximize their utility ($U(c_t), U(c_t^*)$) subject to their respective budget constraints

The budget constraints read as:

$$c_t + p_{H,t}k_{t+1} + \tilde{a}_{t+1} \leq W_t + (R_t^K - \delta)p_{H,t}k_t + R_t\tilde{a}_t + +t_t + o_t \tag{16}$$

$$c_t^* + p_{F,t}^*k_{t+1}^* + \tilde{a}_{t+1}^* \leq W_t^* + (R_t^{K*} - \delta^*)p_{F,t}^*k_t^* + R_t^*\tilde{a}_t^* + t_t^* + o_t^* \tag{17}$$

- In both countries households have access to a risk free internationally traded financial asset, can save through physical capital and supply one unit of labor. We assume complete markets.

The final consumption good is an aggregate of $H$ and $F$ consumption goods

$$C_t = C_t(C_{H,t}, C_{F,t}) \tag{18}$$

$$C_t^* = C_t^*(C_{H,t}^*, C_{F,t}^*) \tag{19}$$

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# Other Ingredients of the two-bloc model: Intermediate Goods Green/Brown Firms

- We assume a continuum of production units
- The mass of these continua is fixed and normalized to 1.
- Atomistic firms own a stock of natural capital ($k_{N,t}$) and have available two technologies
- Each firm can chose how much labor to hire for the production of both green and brown intermediate goods
- in the case of the brown good, labor is combined with an exploited amount of natural capital ($k^b_{N,t}$)
- in the case of the green good, labor is combined with the stock of existing natural capital that is not being exploited for the production of the brown good ($k^g_{N,t}$)

The production technologies are given by:

$$y_{g,t} = y_{g,t}(A_{g,t}, k^g_{N,t}, n_{g,t}) \tag{20}$$

$$y_{b,t} = y_{b,t}(A_{b,t}, k^b_{N,t}, n_{b,t}) \tag{21}$$

We assume Cobb-Douglas (C-D) specification (with CRS)

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# Other Ingredients of the two-bloc model: Green/Brown Intermediate Goods Firms (Continued)

- We assume that $A_{i,t} = A^0_{i,t}(K^g_{N,t})^{\phi_i}$, for $i \in \{b, g\}$, to capture the fact that productivity also depends on the aggregate stock of natural capital, where $\phi_i > 0$ is a parameter. $A^0_{i,t}$ is an exogenous productivity process, which we assume common across firms of the same sector (e.g., brown or green)
- We assume variable costs associated with producing the brown good
- The cost function is assumed to be quadratic and dependent on the remaining stock of natural capital:

$$\kappa(k_{N,t}, k^b_{N,t}) = e^{-\frac{b_1}{2}\left(1 - \frac{k^b_{N,t}}{k_{N,t}}\right)^2}$$

costs are a function of $b$ output and are given by:

$$\kappa(k_{N,t}, k^b_{N,t}) y_{b,t}$$

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# Other Ingredients of the two-bloc model: Intermediate Inputs and Final Consumption Goods Firms

In both blocs:

- There is a representative firm that aggregates the intermediate imported goods into the intermediate composite good
- There is a representative firm that uses labor, capital, and an intermediate composite good to produce final output

$$I_t^* = I_t^*\left(I_{b,t}^*, I_{g,t}^*\right) \tag{22}$$

$$I_t = I_t\left(I_{b,t}, I_{g,t}\right) \tag{23}$$

$$Y_{F,t} = Y_{F,t}(A_{F,t}, K_t^*, N_t, I_t^*) \tag{24}$$

$$Y_{H,t} = Y_{H,t}(A_{H,t}, K_t, N_t, I_t) \tag{25}$$

$K$ denotes physical capital, $N$ denotes labor, $I$ denotes the intermediate input.
We assume CES specification for the intermediate input firms and CD specification for the final good firm, both with CRS assumption.
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# Other Ingredients of the two-bloc model: Aggregation and Market Clearing

$$Y_{g,t} = \int_0^1 y_{g,t}(i)di, \quad Y_{b,t} = \int_0^1 y_{b,t}(i)di, \quad K_{N,t} = \int_0^1 k_{N,t}(i)di,$$
$$K^g_{N,t} = \int_0^1 k^g_{N,t}(i)di, \quad K^b_{N,t} = \int_0^1 k^b_{N,t}(i)di, \quad N_{b,t} = \int_0^1 n_{b,t}(i)di,$$
$$N_{g,t} = \int_0^1 n_{g,t}(i)di, \quad C_t = \int_0^{N_t} c_t(i)di, \quad C^*_t = \int_0^{N^*_t} c^*_t(i)di,$$
$$\tilde{A}^*_t = \int_0^{N^*_t} \tilde{a}^*_t(i)di, \tilde{A}_t = \int_0^{N_t} \tilde{a}_t(i)di, K_t = \int_0^{N_t} k_t(i)di, K^*_t = \int_0^{N^*_t} k^*_t(i)di$$

$$\tilde{A}_t + \tilde{A}^*_t = 0 \tag{26}$$

$$N_t = N_{H,t} + N_{b,t} + N_{g,t} \tag{27}$$

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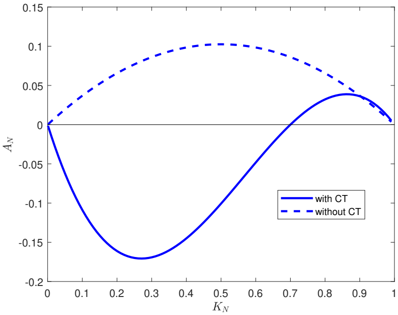
**Figure 15:** Nature Accumulation Rates ($A_N$)

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**Theorem**

*An "inputs-efficient", lump sum financed, global tax scheme can be expressed as a function of the relative marginal value of labor across sectors in the H bloc:*

$$\tau_{b,t} = \frac{\frac{\partial Y_{g,t}^{SP}}{\partial N_{g,t}^{SP}}}{\frac{\partial Y_{b,t}^{SP}}{\partial N_{b,t}^{SP}}(1-\kappa(\bullet)^{SP})} \Big/ \frac{\frac{\partial Y_{g,t}^{CE}}{\partial N_{g,t}^{CE}}}{\frac{\partial Y_{b,t}^{CE}}{\partial N_{b,t}^{CE}}(1-\kappa(\bullet)^{CE})} - 1 \tag{28}$$

*where the superscript $SP, CE$ refer to the allocations arising in the social planning and competitive equilibrium, respectively. This tax allows to align the relative purchase of brown intermediate inputs with the allocation that arise in a SP outcome.*

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# Green Policies: Impact on Relative Prices

- Brown goods prices increases in the medium term but eventually settle at a relatively lower level
- Green goods prices decrease
- Terms of trade worsens in the short-medium-run but eventually improves

**Figure 16:** Rel. Prices - Low $K_{N,t0}$
(% Diff. CE+subs. vs. CE)

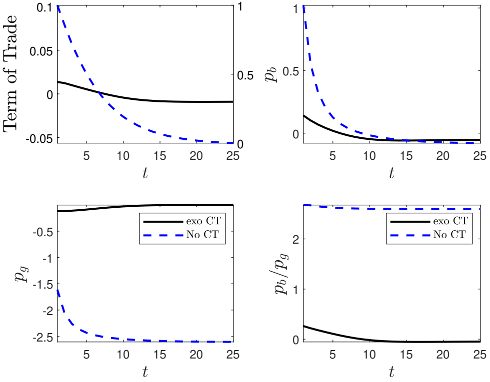


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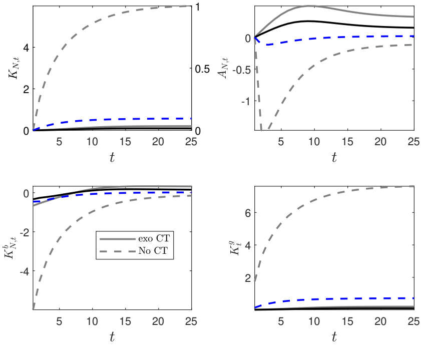
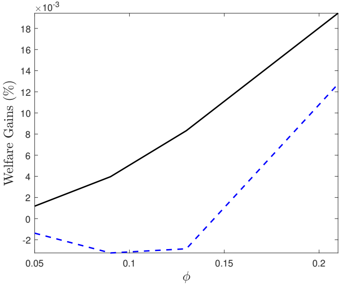
**Figure 17:** Natural Capital
(% Diff. CE+tax and CE)




$\phi_g = \phi_b = 0.05$

**Figure 18:** Welfare Gains (%)
Low $K_{N,t0}$

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