Exchange Rate Puzzles and Policies

Oleg Itskhoki
University of California, Los Angeles

Dmitry Mukhin
London School of Economics

Pages 47 – 96 | Chapter printout

Credibility of Emerging Markets, Foreign Investors' Risk Perceptions, and Capital Flows

Álvaro Aguirre
Andrés Fernández
Şebnem Kalemli-Özcan
editors

29
Series on Central Banking, Analysis, and Economic Policies

Banco Central de Chile
Central Bank of Chile

The views and conclusions presented in the book are exclusively those of the authors and do not necessarily reflect the position of the Central Bank of Chile or its Board Members.

Published in Santiago, Chile
by the Central Bank of Chile
Manufactured in Chile

ISBN (print) 978-956-7421-71-8
ISBN (digital) 978-956-7421-72-5
Intellectual Property Registration 2020-A-2881

ISSN 0717-6686 (Series on Central Banking, Analysis, and Economic Policies)

Exchange Rate Puzzles and Policies

Oleg Itskhoki
University of California, Los Angeles

Dmitry Mukhin
London School of Economics

What is the optimal exchange rate policy? Should exchange rates be optimally pegged, managed, or allowed to freely float? What defines a freely floating exchange rate? Do open economies face a trilemma constraint in choosing between inflation and exchange rate stabilization, unlike divine coincidence in a closed economy? These are generally difficult questions, as the exchange rate is neither a policy instrument, nor a direct objective of the policy, but rather an endogenous general-equilibrium variable tied by equilibrium relationships in both goods and financial markets. At the same time, equilibrium exchange rate behavior features a variety of puzzles from the point of view of conventional business-cycle models typically used for policy analysis in open economy.

We address these questions by developing a general policy analysis framework with nominal rigidities and financial frictions that are both central for equilibrium exchange rate determination and result in an empirically realistic model of exchange rates. The model builds on Itskhoki and Mukhin (2021a,b) and is consistent with the exchange rate disconnect properties across floating and fixed regimes allowing for explicit policy analysis using both monetary policy and foreign exchange (FX) interventions in the financial market. The model features Balassa-Samuelson mechanism determining the value of the frictionless real exchange rate (departures from purchasing power

Prepared for the XXIV Annual Conference of the Central Bank of Chile "Credibility of Emerging Markets, Foreign Investors' Risk Perceptions, and Capital Flows." We thank Charles Engel for an insightful discussion and conference participants for useful comments.

Credibility of Emerging Markets, Foreign Investors' Risk Perceptions, and Capital Flows edited by Álvaro Aguirre, Andrés Fernández, and Şebnem Kalemli-Özcan, Santiago, Chile.

parity, PPP) and segmented financial markets resulting in endogenous equilibrium Uncovered Interest Rate Parity (UIP) deviations. The presence of both endogenous PPP and UIP deviations is essential for the optimal exchange rate policy analysis, as exchange rate variation is at the core of both deviations. We show that this framework is easily amenable to normative analysis and characterize the optimal exchange rate policies following Itskhoki and Mukhin (2022).

In section 1, we setup a simple small open economy model with a tradable and a nontradable sector. While highly stylized, this model allows us to illustrate the key mechanisms and derive the main policy insights that generalize in richer quantitative frameworks. In particular, in section 2, we show how this simple model captures the essential empirical properties of exchange rates, including the Meese-Rogoff disconnect and the Backus-Smith puzzles, in addition to PPP and UIP puzzles mentioned above. While macroeconomic aggregates are driven primarily by fundamental macroeconomic shocks such as productivity and monetary shocks, exchange rates are primarily driven by shocks emerging in international financial markets, for example, shifts in demand for different currencies that have little direct macroeconomic impact. This explains both vastly larger volatility of exchange rates relative to other macro variables—both nominal like inflation and real like consumption and GDP growth—and weak patterns of correlation between these variables and exchange rates.

More importantly, our simple model also reproduces Mussa facts on macroeconomic comovement with exchange rates associated with a switch between floating and fixed exchange rate regimes. As Mussa (1986) famously observed, the real exchange rate has changed dramatically its equilibrium behavior, along with the nominal exchange rate, immediately after the end of the Bretton Woods system of fixed exchange rates. This constitutes prime evidence in favor of non-neutrality of monetary policy regimes. At the same time, as first emphasized by Baxter and Stockman (1989), other macroeconomic aggregates, whether nominal or real, did not exhibit any comparable change in their statistical properties after the end of Bretton Woods. We argue that this set of Mussa facts requires that monetary non-neutrality emerges from the financial market, where international risk-sharing wedges endogenously respond to equilibrium exchange rate volatility. Indeed, a credible nominal exchange rate peg eliminates one of the main sources of risk in international financial transactions. As a result, financial arbitrageurs become more willing to intermediate international capital flows, resulting in smaller equilibrium UIP deviations. This, in turn, eliminates the primary source of exchange rate volatility under the float, allowing the government to achieve a credible peg without a major shift in equilibrium monetary policy. This explains why macroeconomic aggregates do not exhibit a dramatic change in their equilibrium behavior.

We describe the model of a segmented financial market with limits to arbitrage that is consistent with this Mussa mechanism. Endogeneity of international risk-sharing wedges and UIP deviations to the exchange rate regime is the key feature of the model to both explain the Mussa evidence and to provide new insights into the optimal exchange rate policy using a mix of monetary tools and FX interventions, which is the focus of section 3.

At the core of our analysis is the dual role played by the nominal exchange rate. First, it allows for adjustment of the real exchange rate when prices (or wages) are sticky. In the absence of such nominal exchange rate movements, the economy features an output gap resulting in welfare losses. Monetary policy can eliminate the output gap, but this generally requires a volatile nominal exchange rate. Second, the volatility of the nominal exchange rate limits the extent of international risk sharing in the financial market, as international financial transactions are intermediated by risk-averse market makers who need to hold the nominal exchange rate risk. This also leads to welfare losses. Financial-market interventions can redistribute the risk away from arbitrageurs, stabilizing resulting equilibrium UIP deviations and improving the extent of international risk sharing.

First, we prove a divine coincidence result in an open economy: if the frictionless real exchange rate is stable, then a fixed nominal exchange rate achieves both goals of output-gap and UIP stabilization, and thus is the optimal policy choice. Furthermore, direct nominal exchange rate targeting is favored over inflation stabilization, even though both policies have consistent goals. While the former policy ensures stable inflation as a result of exchange rate targeting, the latter policy may result in multiple equilibria in the international financial market, with and without nominal exchange rate volatility.

Second, we show that access to unconstrained monetary policy and FX interventions generally allows to implement the optimal allocation, independently of whether the frictionless real exchange rate is stable or not. The resulting equilibrium generally features volatile nominal exchange rate and inflation targeting, with financial interventions eliminating the intermediation friction and stabilizing UIP deviations. We also show that economies with segmented financial markets do not feature a conventional trilemma constraint, as market segmentation offers financial regulators an additional tool to stabilize the international financial market, even when monetary policy has an exclusive inward focus on domestic inflation and output-gap stabilization.[1]

Third, we explore various circumstances where either monetary policy is constrained (e.g., due to the zero lower bound) or financial interventions are constrained (e.g., due to non-negativity requirement on central-bank foreign reserves or value-at-risk constraints on the central bank's balance sheet). In this case, there are two independent policy goals—the output gap and the risk-sharing wedge—and only one unconstrained policy tool, thus making it generally impossible to replicate the optimal allocation. Fixing the exchange rate using monetary policy is generally feasible but is also generally suboptimal. Similarly, targeting the output gap alone is also suboptimal, and monetary policy trades off output-gap and exchange rate stabilization (a partial peg) in the absence of FX interventions. Using financial interventions to stabilize output gap is generally infeasible.

Lastly, we explore the ability of the government to extract rents in the international financial market by means of FX interventions. The government can generate expected rents for the country only in the presence of foreign noise traders by leaning against the wind of their liquidity currency demand. Arbitrageurs compete with the government for these rents, and greater equilibrium exchange rate volatility allows the government to capture a greater share of these rents by discouraging arbitrageurs from active intermediation. In general, the policymaker favors small departures from frictionless risk sharing and expected UIP deviations which result in expected incomes of the central bank against the losses of foreign noise traders. Capital controls are generally an imperfect substitute for FX interventions but could be used in combination to increase international rents of the country.

Related literature. We build on a vast literature studying the role of exchange rates in both goods and financial markets, as well as the optimal macroeconomic and financial policies in an open economy. Meese and Rogoff (1983), Mussa (1986), Backus and Smith (1993), Obstfeld and Rogoff (2001), Chari and others (2002), Engel and West

1. In other words, open market operations and sterilized interventions have a bite under financial market segmentation which is a source of departure from Wallace (1981)'s Modigliani-Miller (Ricardian) equivalence in an open economy.

(2005) are some of the most prominent papers studying exchange rate puzzles. The list of exchange rate models with frictional financial intermediation includes Kouri (1983), Jeanne and Rose (2002), Alvarez and others (2009), Gabaix and Maggiori (2015), Gourinchas and others (2019), Greenwood and others (2020), Jiang and others (2021), Bianchi and others (2021).

The normative implications of the expenditure switching channel of monetary policy is the focus of Friedman (1953), Clarida and others (2000), Corsetti and Pesenti (2001), Devereux and Engel (2003), Benigno and Benigno (2003), Gali and Monacelli (2005), Goldberg and Tille (2009), Corsetti and others (2010), Engel (2011), Farhi and others (2014), Egorov and Mukhin (2023), while the financial channel of monetary policy is studied in Farhi and Werning (2012), Rey (2013), Fanelli (2017), Basu and others (2020), Kekre and Lenel (2021), Fornaro (2021). Our analysis is also related to the recent studies of the costs and benefits of exchange rate interventions by Jeanne (2012), Amador and others (2019), Cavallino (2019), Fanelli and Straub (2021) and the optimal capital controls by Jeanne and Korinek (2010), Bianchi (2011), Costinot and others (2014), Farhi and Werning (2016, 2017), Schmitt-Grohé and Uribe (2016).

1. A Simple Model of Equilibrium Exchange Rates

We consider a simple small open economy model with a tradable and a nontradable sector. This stylized model allows us to illustrate the key mechanisms and derive the main policy insights that generalize in richer and more realistic frameworks analyzed in Itskhoki and Mukhin (2021a,b; 2022).

Households. We assume a separable log-linear utility of the households, which allows for a sharp analytical characterization of equilibrium exchange rates and optimal policies with stark policy motives:[2]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - (1-\gamma) L_t \right] \text{ with } C_t = \left(\frac{C_{Nt}}{1-\gamma} \right)^{1-\gamma} \left(\frac{C_{Tt}}{\gamma} \right)^{\gamma} \qquad (1)$$

2. This assumption combined with homogenous tradables in a small open economy eliminates all markup and terms of trade motives that typically complicate the optimal policy analysis. See Corsetti and Pesenti (2001), Benigno and Benigno (2003), Egorov and Mukhin (2023).

where C_t is the final consumption good, which has a $1 - \gamma$ cost share of nontradable inputs and a γ share of tradable inputs. Without loss of generality, we assume that the household sector assembles the final good from the two inputs minimizing expenditure $P_tC_t = P_{Nt}C_{Nt} + P_{Tt}C_{Tt}$, where P_{Nt} and P_{Tt} are the respective prices. This results in optimal demand $P_{Nt}C_{Nt} = (1 - \gamma)P_tC_t$ and $P_{Tt}C_{Tt} = \gamma P_tC_t$, where the price level $P_t = P_{Nt}^{1-\gamma}P_{Tt}^{\gamma}$.

The households can borrow or lend using one-period risk-free home-currency and foreign-currency bonds (paying out one unit of respective currency next period):

$$P_{Nt}C_{Nt} + P_{Tt}C_{Tt} + \frac{B_t}{R_t} + e^{-\hat{\psi}_t}\frac{\mathcal{E}_tB_t^*}{R_t^*} = B_{t-1} + \mathcal{E}_tB_{t-1}^* + W_tL_t + \Pi_t + T_t, \tag{2}$$

where R_t and R_t^* are the gross nominal interest rates in the two currencies respectively, and $\hat{\psi}_t$ is the friction associated with holding foreign-currency bonds, which we microfound in section 3. The optimal bond holdings satisfy the Euler equations, which we write in the following way:

$$\beta R_t\mathbb{E}_t\left\{\frac{C_{Nt}}{C_{Nt+1}}\frac{P_{Nt}}{P_{Nt+1}}\right\} = 1, \tag{3}$$

$$e^{-\hat{\psi}_t}\beta R_t^*\mathbb{E}_t\left\{\frac{C_{Tt}}{C_{Tt+1}}\frac{P_{Tt}}{P_{Tt+1}}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right\} = 1, \tag{4}$$

where the nominal exchange rate $\mathcal{E}_t$ is the price of foreign currency in units of home currency (an increase in $\mathcal{E}_t$ is a home-currency depreciation). The household earns labor income W_tL_t, receives profits from home firms Π_t and transfers from the government T_t. Given the log-linear utility, we write the optimal labor supply condition as:

$$P_{Nt}C_{Nt} = W_t. \tag{5}$$

Firms and production. Competitive firms produce the nontradable good using labor, $Y_{Nt} = A_{Nt}L_t$, and are endowed with homogenous nontradable output $Y_{Tt} = A_{Tt}$, where productivity (A_{Nt}, A_{Tt}) follow exogenous and possibly correlated geometric random walk processes. Combined profits of all firms are given by $\Pi_t = P_{Tt}Y_{Tt} + P_{Nt}Y_{Nt} - W_tL_t$.

The law of one price holds for tradables, $P_{Tt} = \mathcal{E}_tR_{Tt}^*$, where P_{Tt}^* is the exogenous foreign-currency world price of tradables. Finally, the prices of nontradables are fully sticky in home currency, $P_{Nt} \equiv 1$. The firms hire necessary amount of labor L_t at flexible wage rate W_t to accommodate nontradable demand $C_{Nt} = Y_{Nt}$ given $P_{Nt} = 1$. We think of this as the limiting case of a Calvo economy where probability of price nonadjustment $\nu \to 1$ and the conventional New Keynesian Phillips curve for nontradable price inflation, $\pi_{Nt} = \beta \mathbb{E}_t \pi_{Nt} + \lambda \log\left(W_t / A_{Nt}\right)$, is degenerate with $\lambda \equiv \frac{(1-\nu)(1-\beta\nu)}{\nu} \to 0$ and $\pi_{Nt} = \Delta \log P_{Nt} = 0$ independently of the level of nominal marginal cost W_t / A_{Nt}.

This combination of stark assumptions—on the functional form of the utility, the endowment of homogenous tradables with the law of one price, and the permanent stickiness of nontradable prices—yields simple closed form solutions yet does not comprise the main qualitative properties of more general models, as we confirm in the other papers.

Government. The government sets domestic interest rate R_t by trading home-currency bond B_t with the households, and it returns the revenues from financial intermediation in the foreign-currency bond back to the households, $T_t = \left(e^{-\hat{\psi}_t} - 1\right)\frac{\mathcal{E}_t B_t^*}{R_t^*}$. Combining (5) with (3), we have $\beta R_t \mathbb{E}_t\left\{W_t / W_{t+1}\right\} = 1$, and thus the choice of R_t is equivalent to the choice of wage inflation, or the path of wages $\{W_t\}$.[3]

The first-best allocation in the nontradable sector requires $W_t / P_{Nt} = A_{Nt}$, and thus given sticky price $P_{Nt} \equiv 1$, the first-best nominal wage must track nontradable productivity, $\tilde{W}_t = A_{Nt}$. The realized wage can thus be written as $W_t = A_{Nt} X_t$, where X_t is the output gap induced by monetary policy ($X_t = 1$ corresponds to no output gap). We think of X_t as the monetary shock in the economy.

Substituting T_t and Π_t into the household budget constraint (2), and using the nontradable market clearing $C_{Nt} = Y_{Nt}$, the fact that home-currency bond is in zero net supply domestically, and the law of one price for tradables, we can write the home-country budget constraint in foreign-currency terms as follows:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = P_{Tt}^*\left(Y_{Tt} - C_{Tt}\right), \qquad (6)$$

where the right-hand side is home net exports in foreign-currency terms. $\{R_t^*, P_{Tt}^*\}$ correspond to foreign shocks in the financial and goods

3. Note from (5) that W_t corresponds to nominal nontradable expenditure $P_{Nt} C_{Nt}$, which is controlled by monetary policy.

markets. For simplicity, we shut them down and study the case with $P^*_{T_t} \equiv 1$ and $\beta R^*_t \equiv 1$, focusing on the productivity shocks (A_{Nt}, A_{Tt}) and monetary shocks X_t, as well as the risk-sharing wedge $\hat{\psi}_t$.

Equilibrium. The equilibrium in the nontradable sector is characterized by the labor supply condition (5) given sticky prices $P_{Nt} = 1$ and the market clearing $C_{Nt} = Y_{Nt} = A_{Nt} L_t$. We thus have:

$$Y_{Nt} = C_{Nt} = \frac{W_t}{P_{Nt}} = A_{Nt} X_t \text{ and } L_t = X_t. \tag{7}$$

The equilibrium in the tradable sector is an interplay of three equilibrium conditions—the expenditure switching between tradables and nontradables, the country budget constraint, and the foreign-currency Euler equation. The expenditure switching condition is the result of optimal expenditure on tradables and nontradables, and we rewrite it as:

$$\frac{\gamma}{1-\gamma} \frac{C_{Nt}}{C_{Tt}} = \frac{\mathcal{E}_t}{P_{Nt}}, \tag{8}$$

where we use the fact that $P_{Tt} = \mathcal{E}_t$ given the law of one price with the international price of tradables $P^*_{Tt} = 1$. Thus, shifts in nominal exchange rate, given sticky nontradable prices P_{Nt}, relocate expenditure between tradable and nontradable inputs of final consumption.

Finally, we rewrite the country budget constraint (6) and the Euler equation (4) as:

$$\beta B^*_t - B^*_{t-1} = Y_{Tt} - C_{Tt},$$

$$\mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = e^{\hat{\psi}_t},$$

where we used the facts that $\beta R^*_t = 1$ and $P^*_{T_t} = 1$. This system characterizes the solution for $\{C_{Tt}\}$, which we partition by analogy with nontradable consumption as

$$C_{Tt} = A_{Tt} Z_t, \tag{9}$$

where $\tilde{C}_{Tt} = A_{Nt}$ is approximately optimal path of tradable inputs in the absence of financial wedges $\hat{\psi}_t = 0$ (assuming $B^*_{-1} = 0$), while Z_t reflects the additional volatility in tradables due to wedges in the international financial market.

With this, we can write the equilibrium exchange rate as:[4]

$$\frac{\mathcal{E}_t}{P_{Nt}} = \frac{\gamma}{1-\gamma}\frac{A_{Nt}}{A_{Tt}}\frac{X_t}{Z_t}, \tag{10}$$

and the approximate expression for Z_t given by:

$$\Delta \log Z_t = -\frac{\beta}{1-\beta\rho}\left(\hat{\psi} - \frac{1}{\beta}\hat{\psi}_{t-1}\right), \tag{11}$$

assuming that $\hat{\psi}_t$ follows an AR(1) with persistence $\rho \in [0,1]$.[5] Equations (7)–(11) fully characterizes equilibrium in this economy where $\{A_{Nt}, A_{Tt}, X_t, \hat{\psi}_t\}$ are exogenous shocks.

Macroeconomic aggregates. We can now characterize macroeconomic aggregates in this economy—inflation (consumer price level), aggregate consumption, real GDP, employment, aggregate wage rate, and the real exchange rate. We express these macroeconomic aggregates as a function of exogenous shocks $\{A_{Nt}, A_{Tt}, X_t\}$ and the nominal exchange rate $\mathcal{E}_t$, which we characterized above.

In particular, consumer price level is given by $P_t = P_{Nt}^{1-\gamma}\mathcal{E}_t^{\gamma}$, where the two terms reflect the nontradable and tradable price inflation. Using the expenditure allocation condition and nontradable market clearing, we express aggregate consumption and real GDP as follows:

$$C_t = \frac{P_{Nt}C_{Nt}}{(1-\gamma)P_t} = \left(\frac{P_{Nt}}{\mathcal{E}_t}\right)^{\gamma}\frac{A_{Nt}X_t}{1-\gamma},$$

$$Y_t = \frac{P_{Nt}Y_{Nt} + P_{Tt}Y_{Tt}}{P_t} = \left(\frac{P_{Nt}}{\mathcal{E}_t}\right)^{\gamma} A_{Nt}X_t + \left(\frac{\mathcal{E}_t}{P_{Nt}}\right)^{1-\gamma} A_{Tt}.$$

4. See interpretation below following (16).

5. This solution relies on the fact that $Y_{Tt} = A_{Tt}$ follows a random walk and log-linearly approximates the equilibrium system around $B_t^* = 0$, which yields two dynamic equations (with $b_t^* \equiv B_t^*/Y_{T0}$):

$$\beta b_t^* - b_{t-1}^* = \text{dlog}Y_{Tt} - \text{dlog}C_{Tt} = -\,\text{dlog}Z_t,$$
$$\hat{\psi}_t = \mathbb{E}_t\Delta\log C_{Tt+1} = \mathbb{E}_t\Delta\log Z_{t+1},$$

where we use the facts that $\log Y_{Tt} = \log A_{Tt}$ is a random walk (i.e., $\mathbb{E}_t\Delta\log A_{t+1} = 0$) and $\log Z_t = \log C_{Tt} - \log A_{Tt}$. Solving this dynamic system with $\hat{\psi}_t \sim AR(1)$ yields $\text{dlog}\,Z_t = (1-\beta)b_{t-1}^* - \frac{\beta}{1-\beta\rho}\hat{\psi}_t$, which then results in (11).

This allocation is supported with aggregate employment level $L_t = X_t$ given aggregate wage rate $W_t / P_{Nt} = A_{Nt} X_t$. Finally, the real exchange rate in this economy is given by:

$$\mathcal{Q}_t = \frac{P_t^* \mathcal{E}_t}{P_t} = \left(\frac{\mathcal{E}_t}{P_{Nt}} \right)^{1-\gamma},$$

where we assume $P_t^* = P_{Tt}^* = 1$. We kept P_{Nt} in the expressions above to illustrate how the results would generalize to a model where sticky prices P_{Nt} are allowed to adjust in response to output gap X_t.

We can now rewrite this macro quantities in log changes (growth rates), which by convention we denote with corresponding small letters (with the exception of inflation denoted with π_t):[6]

$$\pi_t = (1-\gamma)\pi_{Nt} + \gamma e_t, \tag{12}$$

$$c_t = a_{Nt} + x_t - \gamma(e_t - \pi_{Nt}), \tag{13}$$

$$y_t = (1-\gamma)(a_{Nt} + x_t) + \gamma a_{Tt}, \tag{14}$$

$$q_t = (1-\gamma)(e_t - \pi_{Nt}), \tag{15}$$

where $\pi_{Nt} = 0$ under fully sticky prices and more generally satisfies the dynamic Phillips curve $\pi_{Nt} = \beta \mathbb{E}_t \pi_{Nt+1} + \lambda \log X_t$ given the path of output gap X_t chosen by monetary policy. We assume the economy is subject to random-walk productivity and monetary shocks such that (a_{Tt}, a_{Nt}, x_t) are *idd* as growth rate shocks. Finally, the nominal exchange rate in (10) follows:

$$e_t = (a_{Nt} - a_{Tt}) + (\pi_{Nt} + x_t) - z_t \text{ where } z_t = -\frac{\beta}{1-\beta\rho}\left(\hat{\psi}_t - \frac{1}{\beta}\hat{\psi}_{t-1}\right). \tag{16}$$

Since $\hat{\psi}_t \sim AR(1)$, $z_t \sim ARMA(1,1)$ with autoregressive root ρ and moving average root $1/\beta$. When $\beta, \rho \approx 1$, this growth rate process is arbitrary close to white noise, so that the exchange rate is close to a random walk (recall that $e_t \equiv \Delta \log \mathcal{E}_t$), consistent with its empirical properties.

6. For real GDP, we approximate around balanced trade, so that $P_{Nt}Y_{Nt}$ and $P_{Tt}Y_{Tt}$ correspond to fraction $1-\gamma$ and γ of nominal GDP respectively (like consumption expenditure shares). Given this, the effects of the exchange rate on real GDP (via inflation and relative price of tradables) cancel out.

Before using these results to analyze a range of exchange rate puzzles, we offer a brief commentary. First, the nominal exchange rate in (16) has three components: (1) Balassa-Samuelson term $\tilde{q}_t \equiv a_{Nt} - a_{Tt}$ reflecting equilibrium pressures on the relative nontradable prices;[7] (2) nominal inflationary pressure $\pi_{Nt} + x_t$, which emerges from the output gap x_t under sticky prices, and then from price inflation π_{Nt} if they adjust; (3) financial shocks captured by z_t (i.e., relative demand shocks for foreign currency $\hat{\psi}_t$ causing home-currency depreciation). The relative nontradable prices evolve with $e_t - \pi_{Nt}$, which shapes the equilibrium dynamics of the real exchange rate q_t in (15).

What concerns macro aggregates (12)–(14), domestic consumer price inflation π_t, reflects nontradable and tradable inflation π_{Nt} and et with weights $(1 - \gamma)$ and γ respectively. Aggregate consumption evolves with productivity a_{Nt} and output gap x_t, as well as responds to the expenditure switching force due to the relative nontradables price with elasticity γ. In contrast, real GDP reflects relative productivities in the two sectors with weights $(1 - \gamma)$ and γ respectively, as well as responds to the output gap, which shapes aggregate employment in the economy. These are conventional macroeconomic forces typical in standard business-cycle models, and the only unconventional feature of the model is the presence of financial shocks z_t that affect the equilibrium exchange rate.

2. Exchange Rate Puzzles

2.1 Puzzles under Floating Exchange Rate

Backus-Smith. At the core of understanding the exchange rate under floating regime is the Backus-Smith puzzle.[8] While under complete asset markets and separable utility with risk aversion σ, the real exchange rate must satisfy $q_t = \sigma\,(c_t - c_t^*)$, in the data real exchange depreciations (increases in q_t) are associated with reductions in relative home consumption $(c_t - c_t^*)$, albeit with a weak correlation

7. See Obstfeld and Rogoff (1996), chapter 4.
8. See Backus and Smith (1993) and Kollmann (1995).

(see figure 2a below). The equilibrium conditions (13) and (15) provide an insight into this puzzle, as we can calculate:

$$\frac{\operatorname{cov}(c_t, q_t)}{\operatorname{var}(q_t)} = -\frac{\gamma}{1-\gamma} + \frac{1}{1-\gamma}\frac{\operatorname{cov}(a_{Nt} + x_t, e_t - \pi_{Nt})}{\operatorname{var}(e_t - \pi_{Nt})}$$

where $e_t - \pi_{Nt}$ is given by (16) and we assume $c_t^* = 0$ in line with our small open economy approach.

The first term reflects expenditure switching—a decline in consumption driven by a real depreciation (an increase in the relative price of foreign tradables)—and its effect is proportional to the openness of the economy to foreign tradables γ.[9] The second term reflects the comovement of the domestic component of consumption with the real exchange rate and equals the combined variance contribution of productivity shocks a_{Nt} and monetary shocks x_t (output gap) to the variance of the real exchange rate $q_t = (1-\gamma)(e_t - \pi_{Nt})$. This effect does not depend on the openness of the economy γ.

The decomposition above makes it clear what features of the model result in the Backus-Smith puzzle. Note that it is not about completeness of asset markets, as we assumed incomplete markets from the get-go. In fact, if monetary shocks x_t and/or productivity shocks a_{Nt} are the key drivers of the real exchange rate, then $\frac{\operatorname{cov}(a_{Nt} + x_t, e_t - \pi_{Nt})}{\operatorname{var}(e_t - \pi_{Nt})} \approx 1$ and thus $\frac{\operatorname{cov}(c_t, q_t)}{\operatorname{var}(q_t)} \approx 1$ irrespectively of asset market incompleteness and the openness of the economy γ. As a result, the persistence of the Backus-Smith puzzle is due to the fact that international Real Business Cycle (RBC) and New Keynesian models alike robustly reproduce it independently of the many features of such models as long as productivity and monetary shocks are the key driving forces in the economy.

What is the explanation for the Backus-Smith puzzle? It requires financial exchange rate shocks z_t to be the key driver of the nominal exchange rate in (16).[10] If this is the case, and z_t is

9. We write foreign tradables here since in a more general model with imperfectly substitutable home and foreign tradables, what matters for expenditure switching is the relative price of foreign tradables in the home market and their share in total consumption expenditure. See Itskhoki and Mukhin (2021a) and Itskhoki (2021).

10. In our simple model, it is also possible to explain the Backus-Smith puzzle if the key driver of the exchange rate is the homogenous tradable endowment shock a_{Tt}. This shock, however, is at odds with other exchange rate puzzles, in particular the exchange rate disconnect puzzle that we discuss next.

largely orthogonal with monetary and productivity shocks, then $\frac{\text{cov}(a_{Nt}+x_t, e_t-\pi_{Nt})}{\text{var}(e_t-\pi_{Nt})} \approx 0$ and thus $\frac{\text{cov}(c_t, q_t)}{\text{var}(q_t)} \approx -\frac{\gamma}{1-\gamma}$, consistent with the weak negative correlation in the data. Quantitatively, we show in Itskhoki and Mukhin (2021a) that financial shocks should account for around 80–90 percent of the nominal exchange rate volatility for the model to be quantitatively consistent with the Backus-Smith correlation in the data, given that most countries exhibit significant home bias and have a large nontradable share.

The purchasing power parity (PPP) puzzle. The PPP puzzle emphasizes the fact that the real exchange rate closely tracks the nominal exchange rate at most frequencies, inheriting both its volatility and persistence.[11] From the definition of the real exchange rate, this implies that inflation π_t is small and largely uncorrelated with exchange rate changes e_t. From (12) and (15), we see that the model is consistent with PPP puzzle if monetary inflation shocks π_{Nt} are small in the variation of the nominal exchange rate (16), and home bias is large (tradable share γ is small). In fact, the real and nominal exchange rates follow an equally persistent near-random walk process if financial shocks z_t are the main source of their volatility.

The simple model presented here is special as it assumes that the law of one price holds for a homogenous tradable good. In a more realistic model with home bias in imperfectly substitutable tradable goods and law-of-one-price violations due to sticky local-currency prices, the real exchange rate q_t perfectly traces the nominal exchange rate e_t even when $\gamma \gg 0$, as long as the volatility in the exchange rates is not due to monetary shocks.[12] The reason is that monetary policy can act to effectively stabilize consumer price inflation, while the nominal and real exchange rates are volatile and persistent in response to financial shocks z_t.[13]

Meese-Rogoff disconnect puzzle. Another crucial property of the nominal (and real) exchange rate is that it is largely uncorrelated with a whole range of macroeconomic fundamentals, both nominal and real, and tends to be an order of magnitude more volatile than

11. See Rogoff (1996) and Appendix figure A1a,b.

12. See Itskhoki and Mukhin (2021a), Eichenbaum and others (2021), Blanco and Cravino (2020).

13. Additionally, in the data, the wage-based real exchange rate tracks closely the nominal exchange rate. Given that $w_t = \pi_{Nt} + a_{Nt} + x_t$ and again assuming $w_t^* = 0$, we have $q_t^w = w_t^* + e_t - w_t = -(z_t - a_{Tt})$, which tracks e_t and q_t provided that z_t is the main source of variation.

various macroeconomic aggregates.[14] Figure 1 below and Appendix figure A1 illustrate the order-of-magnitude difference in the volatility of exchange rates and macroeconomic fundamentals under the floating exchange rate regime. Since we have already studied the exchange rate comovement with consumption and inflation above, we now focus on the real GDP given by (14):[15]

$$\frac{\operatorname{cov}(y_t, e_t)}{\operatorname{var}(e_t)} = \frac{\operatorname{cov}\big((1-\gamma)(a_{Nt} + x_t) + \gamma a_{Tt}, e_t\big)}{\operatorname{var}(e_t)}.$$

As long as productivity and monetary shocks (a_{Nt}, a_{Tt}, x_t) account for a small share of variation in the nominal exchange rate (16), which in turn is mostly driven by financial shocks z_t, the correlation between the nominal exchange rate and the real GDP is arbitrarily close to zero, while their relative volatility is arbitrarily large, in line with disconnect properties.

Note that this does not mean that conventional macroeconomic shocks (a_{Nt}, a_{Tt}, x_t) are absent. In contrast, they are essential to ensure the conventional business-cycle dynamics of consumption, output and inflation. However, their relative contribution to the large exchange rate volatility is limited, as asset demand shocks $\hat{\psi}_t$ result in a considerably more volatile source of exchange rate fluctuations z_t, in particular when β and ρ are close to 1 in (16). These shocks feed back into macro dynamics via the expenditure switching effect on consumption in (13), which is proportionally small with the openness of the economy γ. More open economies exhibit both less volatile equilibrium exchange rates and less exchange rate disconnect—consistent with the model with imperfectly substitutable home and foreign tradable, as we show in Itskhoki and Mukhin (2021a).

UIP puzzle. Lastly, we turn to the forward premium puzzle, which emphasizes systematic UIP violations, namely that returns on a currency carry trade $R_t - R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$ co-move systematically with

14. See e.g., Meese and Rogoff (1983).

15. Similarly, we could focus on aggregate employment $(\ell_t = x_t)$ or nominal expenditure (e.g., $P_{Nt}C_{Nt} = P_{Nt}A_{Nt}X_t$).

the interest-rate differential $R_t - R_t^*$.[16] Combining together the two household Euler equations (3)-(4) and log-linearizing results in:[17]

$$i_t - i_t^* - \mathbb{E}_t e_{t+1} = \hat{\psi}_t, \tag{17}$$

where $i_t - i_t^* = \log R_t - \log R_t^*$ and $e_{t+1} = \Delta \log \mathcal{E}_{t+1}$. The foreign-currency demand shock $\hat{\psi}_t$ results in UIP deviations, which can be either long-run mean zero or nonzero, as is the case for developed versus developing countries.[18] The Fama regression, however, emphasizes that $i_t - i_t^* = \mathbb{E}_t e_{t+1}$ systematically increases with $i_t - i_t^*$, or in other words e_{t+1} tends to be negative (appreciate) when $i_t - i_t^*$ increases, albeit with a vanishingly small predictive power (i.e., $R^2 \approx 0.01$).

Our simple models predicts that the coefficient in the Fama regression of e_{t+1} on $i_t - i_t^*$ is indeterminant and the R^2=0, independently of the presence or absence of $\hat{\psi}_t$. This is because $i_t^* = 0$ and $i_t = \mathbb{E}_t \left\{c_{t+1} + \pi_{t+1}\right\} = \mathbb{E}_t \left\{a_{Nt+1} + \pi_{Nt+1} + x_{t+1}\right\} = 0$ under random walk shocks. This emphasizes the weak identifying power of the Fama regression. If we run a real version of the Fama regression of q_{t+1} on $r_t - r_t^*$, where $r_t = i_t - \mathbb{E}_t \pi_{t+1}$, we identify a negative coefficient as we regress $(1-\gamma)\, e_{t+1}$ on $-\gamma \mathbb{E}_t e_{t+1}$, provided that z_t shocks account for some variation in e_t. The R^2 of this regression would still be close to zero—capturing the robust empirical property of the Fama regression.

2.2 Mussa and Other Puzzles

The Mussa (1986) puzzle concerns the switch from a pegged to a floating nominal exchange rate regime, which empirically is associated with a dramatic increase in the volatility of both nominal and real exchange rates, yet little change in the properties of other macroeconomic variables. We illustrate this in figure 1 and Appendix figure A1, which show a dramatic increase in the volatility of both nominal and real exchange rates immediately after the end of Bretton Woods, while the behavior of consumption, real GDP, and inflation did not experience any discernible discontinuity around this breakpoint.

16. See Fama (1984) and figure 2b below.

17. The nonlinear condition is $\mathbb{E}_t \left\{ \frac{W_t}{W_{t+1}} \left[R_t e^{\hat{\psi}_t} - R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] \right\} = 0$, where we use the fact that optimal expenditure $P_{Tt} C_{Tt} \propto P_{Nt} C_{Nt} = W_t$. The higher order term, thus, depends on $\text{cov}_t\left(W_{t+1}^{-1}, \mathcal{E}_{t+1}\right)$, which is close to zero both in the data and in the model that satisfies the disconnect property, as discussed above.

18. See e.g., Hassan and Mano (2018) and Kalemli-Özcan and Varela (2021).

The simple model above allows us to investigate what features are necessary for a model to match these empirical patterns. The expression for the equilibrium nominal exchange rate in (16) shows how a change in monetary policy x_t can accommodate a fixed nominal exchange rate. Indeed, setting $x_t = z_t - \pi_{Nt} - (a_{Nt} - a_{Tt})$ ensures $e_t = 0$, while we think of a floating exchange rate regime as output-gap stabilization with $x_t = 0$ and $e_t = (a_{Nt} - a_{Tt}) + \pi_{Nt} - z_t$. Such switch in monetary policy has dramatic consequences for macroeconomic quantities.

On the one hand, under fully sticky prices with $\pi_{Nt} = 0$, we have $q_t = (1 - \gamma)e_t$, and indeed a change in the volatility of the nominal exchange rate induces a proportional change in the volatility of the real exchange rare q_t, in line with the empirical patterns. Note that this property of the model is independent of the nature of the shocks driving the exchange rate and is the consequence of stable inflation under both regimes, which empirically indeed remained stable even after a switch to volatile floating exchange rates.

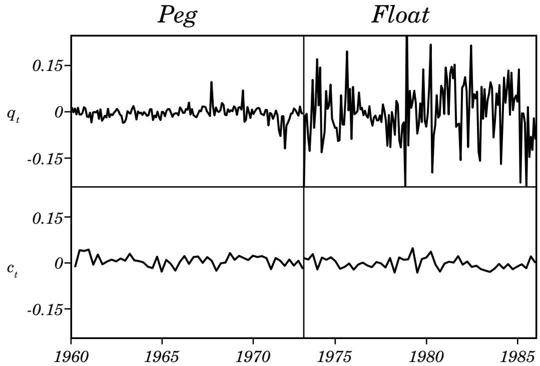
On the other hand, in contrast with the data, such change in monetary policy has equally large consequences for macroeconomic aggregates. We focus, for example, on real GDP and the results for aggregate consumption are similar:

$$y_t = \begin{cases} (1-\gamma)a_{Nt} + \gamma a_{Tt}, & \text{under float} \\ a_{Tt} + z_t, & \text{under peg.} \end{cases}$$

That is, under the float, the real GDP reflects average productivity of the economy given the stabilized output gap, while the financial shock z_t—the key drivers of the exchange rate (see above)—does not affect GDP, as it is absorbed by the exchange rate.[19] In contrast, under the peg, both real GDP and aggregate consumption reflect one-to-one financial shocks z_t, irrespectively of the openness of the economy. This is because monetary policy needs to absorb exchange rate shocks and thus pass on financial shocks into fluctuations of the output gap x_t, which affects employment, consumption, and output independently of the openness of the economy. This is in sharp contrast with the empirical Mussa patterns shown in figure 1.

19. The exact orthogonality of the real GDP with e_t (and thus with z_t) is a knife-edge implication of the Cobb-Douglas utility and other special assumption of our model; more generally, the real GDP is exposed to the exchange rate fluctuations, like aggregate consumption, with an elasticity proportional to the openness of the economy γ. Imperfectly substitutable tradable goods and local currency price stickiness of exports further mute this transmission along with low γ. See Itskhoki and Mukhin (2021a,b).

Figure 1. Real Exchange Rate and Aggregate Consumption during and after Bretton Woods



Source: Itskhoki and Mukhin (2021b).
Note: Monthly real exchange rate changes q_t (G7 countries plus Spain, without Canada against the U.S.) and quarterly aggregate consumption growth rates c_t (average for G7 countries); both series annualized and in log points (that is, 0.15 corresponds to 15 log points, approximately 15%). The breakup of Bretton Woods is dated 1973.1. See also Appendix figure A1.

Itskhoki and Mukhin (2021b) show that in a large class of conventional business-cycle models there exists a robust sufficient statistic $\sigma(c_t - c_t^*) - q_t$, where σ is risk aversion, that does not change its statistical properties with a switch in the monetary regime, even if consumption and the real exchange rate change their behavior. Indeed, this is the case in the model presented here with $\sigma = 1$:

$$\left(c_t - c_t^*\right) - q_t \quad = a_{Nt} + x_t - \gamma\left(e_t - \pi_{Nt}\right) - \left(1-\gamma\right)\left(e_t - \pi_{Nt}\right) = a_{Tt} + z_t,$$

where we used (13), (15), and (16). So long as the endowment shock a_{Tt} and the financial shock z_t do not change their properties across monetary regimes, changes in monetary policy x_t and the associated changes in the behavior of exchange rates do not affect this sufficient statistic. In the data, however, $(c_t - c_t^*) - q_t$ dramatically changes its behavior along with q_t following a switch between a peg and a float.

Resolution. The result above suggests a feature of the model that can lead to a resolution of the Mussa puzzle. Indeed, it requires that some shocks change their properties with a change in a monetary regime. In particular, in Itskhoki and Mukhin (2021b), we show that the volatility of financial shocks has to be endogenous to the exchange rate regime and, specifically, increasing in the equilibrium exchange rate volatility:[20]

$$\hat{\psi}_t = \chi\left(\sigma_e^2\right)\cdot\psi_t,\ \text{where}\ \chi\left(0\right)=0,\ \chi'\left(\cdot\right)>0\ \text{and}\ \sigma_e^2 = \text{var}_t\left(e_{t+1}\right). \tag{18}$$

In the following section, we describe a microfoundation for such an endogenous change in $\hat{\psi}_t$, which is also essential for the optimal policy analysis.

Under (18), an exchange rate peg with $\sigma_e^2 = 0$ results in $\hat{\psi}_t = 0$ and consequently $z_t = 0$, eliminating financial shocks as a driver of both nominal and real exchange rates. Recall that exchange rate disconnect under the float requires that financial shocks z_t are the key drivers of the floating exchange rates, explaining the dramatic shift in their volatility with the exchange rate regime, as observed in figure 1 and Appendix figure A1.

As the exchange rate changes from $e_t = (a_{Nt} - a_{Tt}) - z_t$ to $e_t = 0$, the output gap needs to change only from $x_t = 0$ to $x_t = a_{Nt} - a_{Tt}$ to accommodate a switch to a peg. To the extent z_t accounts for the bulk of the exchange rate variation under the float and $a_{Nt} - a_{Tt}$ are (relatively) stable, this requires only a minor change in monetary policy. Consequently, the real GDP and aggregate consumption also change only mildly, e.g., from $y_t = (1 - \gamma)\, a_{Nt} + \gamma a_{Tt}$ under the float to $y_t = a_{Tt}$ under the peg. This explains why we do not observe a major breakpoint in the behavior of these macroeconomic aggregates.

Home bias in consumption and nontradables (lowγ) shield macroeconomic aggregates from exchange rate volatility under the float, as we discussed above. More importantly, however, endogenous financial volatility in (18) shields monetary policy and consequently macroeconomic aggregates from financial volatility under the peg. Without this, monetary policy would need to absorb volatile financial shocks to stabilize the exchange rate, and consequently pass on this volatility into inflation, consumption, and output, irrespectively of the openness of the economy.

Other puzzles. Consider three exchange rate puzzles that change their properties with the exchange rate regime. First, consider Balassa-Samuelson that suggests that the real exchange rate should evolve with relative nontradable productivity $a_{Nt} - a_{Tt}$. Indeed, we have:

$$q_t = (1 - \gamma)[(a_{Nt} - a_{Tt}) + x_t - z_t].$$

20. See also Kollmann (2005).

Thus to the extent z_t dominates the volatility of exchange rates under the float, it is difficult to isolate the Balassa-Samuelson forces from the time series properties of the real exchange rate. In contrast, if z_t disappears under the peg without a change in monetary policy $x_t = 0$, then q_t is shaped entirely by the Balassa-Samuelson forces under the peg, for which there is indeed empirical evidence since the introduction of euro.[21]

Second, UIP holds considerably better under the peg than under the float, in line with (17), provided that $\hat{\psi}_t$ has an endogenously reduced volatility under the peg. Furthermore, the negative sign of the Fama regression coefficient persistent under the float, either turns zero or becomes positive under the peg, closer to the theoretical benchmark (see figure 2b). Similarly, the Backus-Smith correlation turns from negative to positive under the peg, which is again in line with the Mussa mechanism. We rewrite the Backus-Smith covariation in the model as follows:

$$\frac{\mathrm{Cov}\left(c_t, q_t\right)}{\mathrm{var}\left(q_t\right)} = -\frac{\gamma}{1-\gamma} + \frac{1}{1-\gamma}\frac{\mathrm{var}\left(a_{Nt} - a_{Tt}\right) + \mathrm{cov}\left(a_{Tt}, a_{Nt} - a_{Tt}\right)}{\mathrm{var}\left(a_{Nt} - a_{Tt}\right) + \mathrm{var}\left(z_t\right)}$$

where we assumed for simplicity that $x_t = 0$ under both the float and the peg and that the financial shock z_t is orthogonal with productivity $(a_{Nt} - a_{Tt})$.[22] If z_t is the dominant shock under the float, then the Backus-Smith covariation is mildly negative, as in the data. If the variance of z_t declines towards zero under the peg, the Backus-Smith covariance increases and turns positive, provided a_{Tt} and a_{Nt} are not strongly negatively correlated. This is again consistent with the data, as we show in figure 2a.[23]

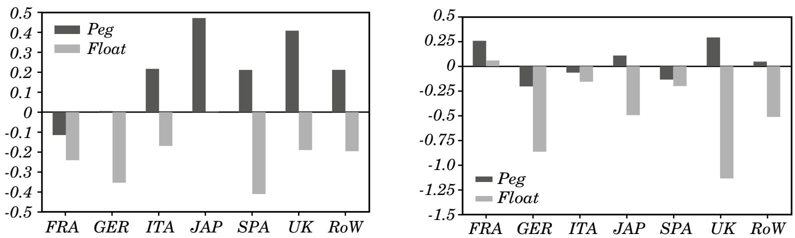
21. See Berka and others (2012, 2018).

22. In our simple model, a monetary policy x_t that fully stabilizes nominal exchange rate also fully stabilizes the real exchange rate, and thus the Backus-Smith moment we focus on is zero or indeterminant under the peg. More generally, the real exchange rate reflects relative inflation under the peg, which is nonzero (see Appendix figure A1), and the Backus-Smith correlation is well-defined in the data, consistent with the description we offer in the text.

23. See also Devereux and Hnatkovska (2020) and Colacito and Croce (2013).

Figure 2. Backus-Smith Correlation and Fama Coefficient before and after the End of Bretton Woods

(*a*) *Backus-Smith correlations* (*b*) *Fama coefficient,* β_F



Source: Itskhoki and Mukhin (2021b).
Note: The left panel displays the Backus-Smith correlation, corr $(c_t - c_t^*, q_t)$ in growth rates, using annual data for 1960–71 for Peg and 1973–1989 for Float. The right panel displays Fama regression coefficient β_F, obtained from an OLS regression of depreciation rate $e_{(t+1)}$ on $(i_t - i_t^*)$, using monthly data for 1960.1–1971.7 for Peg and 1973.1–1989.12 for Float. G7 countries (plus Spain, without Canada) against the United States.

3. Exchange Rate Policies

Two key features are essential for the model to be consistent with the combined empirical properties of exchange rates. First, financial shocks $\hat{\psi}_t$ must account for the bulk of exchange rate volatility under a floating regime. A range of models of the international financial market can give rise to such shocks.[24] Second, the evidence on the switch of the floating regime to an exchange rate peg further requires that the volatility of these financial shocks endogenously decreases with a reduction in equilibrium exchange rate volatility, that is $\hat{\psi}_t = \chi(\sigma_e^2)\,\hat{\psi}_t$, where $\chi(\cdot) = 0$ is an increasing function of exchange rate volatility $\sigma_e^2 = \text{var}_t\,(e_{t+1})$. We next describe a micro-founded model for this reduced form, which then allows us to proceed with the analysis of the optimal exchange rate policies.

24. Exogenous UIP shocks are commonly used in the international macro literature (see e.g., Devereux and Engel, 2002; Kollmann, 2005; Farhi and Werning, 2012), and can be viewed to emerge from exogenous asset demand, as in the literature following Kouri (1976, 1983). Models of UIP deviations include models with incomplete information, expectational errors and heterogeneous beliefs (Evans and Lyons, 2002; Gourinchas and Tornell, 2004; Bacchetta and van Wincoop, 2006), financial frictions (Gabaix and Maggiori, 2015; Adrian and others, 2015; Camanho and others ,2018), liquidity premia (Jiang and others, 2021; Bianchi and others, 2021), habits, long-run risk, and rare disasters (Verdelhan, 2010; Colacito and Croce, 2013; Farhi and Gabaix, 2016), and alternative formulations of segmented markets (Jeanne and Rose, 2002; Alvarez and others, 2009).

3.1 A Model of the Financial Market

The general modeling environment is the same as in section 1, with the only difference that household do not have direct access to the foreign-currency (dollar) bond, i.e., $B_t^* \equiv 0$ and the Euler equation (4) no longer applies. The households can only save and borrow using the home-currency bond B_t with interest rate R_t according to the optimality condition (3). In addition, we introduce an explicit model of the financial market which intermediates international capital flows.

Apart from the households, three types of agents trade home- and foreign-currency bonds in the international financial market. Namely, these are the government, noise traders and arbitrageurs. The government holds a portfolio of (F_t, F_t^*) units of home- and foreign-currency bonds, respectively, with the value of the portfolio (government net foreign assets) given by $F_t/R_t + \mathcal{E}_t F_t^*/R_t^*$. Changes in F_t and F_t^* correspond to open market operations of the government.

Noise traders hold a zero capital portfolio (N_t, N_t^*) of the two bonds, such that $N_t/R_t + \mathcal{E}_t N_t^*/R_t^* = 0$, and $N_t^*/R_t^* = \hat{\psi}_t$ is the liquidity demand for foreign currency by the noise traders, that is $\hat{\psi}_t$ is a random variable uncorrelated with macroeconomic fundamentals. A positive $\hat{\psi}_t$ means that noise traders short home-currency bonds to buy foreign-currency bonds, and vice versa.

Finally, the arbitrageurs also hold a zero capital portfolio (D_t, D_t^*) such that $D_t/R_t + \mathcal{E}_t D_t^*/R_t^* = 0$, with a return on one foreign-currency unit holding of such portfolio given by $\tilde{R}_{t+1}^* = R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$ in dollars. In other words, the income from this carry trade is given by $\pi_{t+1}^{D*} = D_t^* - \frac{D_t}{\mathcal{E}_{t+1}} = \tilde{R}_{t+1}^* \cdot \frac{D_t^*}{R_t^*}$ in foreign currency, where we used the zero-capital constraint linking D_t and D_t^*. Arbitrageurs choose their portfolio (D_t, D_t^*) to maximize min-variance preferences over profits, $V_t\left(\pi_{t+1}^{D*}\right) = \mathbb{E}_t\left\{\Theta_{t+1}\pi_{t+1}^{D*}\right\} - \frac{\omega}{2}\operatorname{var}_t\left(\pi_{t+1}^{D*}\right)$, where $\Theta_{t+1} = \beta\frac{C_{Tt}}{C_{Tt+1}}$ is the stochastic discount factor of home households, and the second term in $V_t(\cdot)$ reflects the additional risk penalty of the arbitrageurs with ω being the risk aversion parameter. The optimal portfolio choice satisfies:

$$\frac{D_t^*}{R_t^*} = \frac{\mathbb{E}_t\left\{\Theta_{t+1}\tilde{R}_{t+1}^*\right\}}{\omega\sigma_t^2},$$

where $\sigma_t^2 \equiv \operatorname{var}_t\left(\tilde{R}_{t+1}^*\right) = R_t^2 \cdot \operatorname{var}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)$ measures the carry-trade risk which is associated with the nominal exchange rate volatility.

The market clearing in the financial market requires that the home-currency bond positions of all four types of agents balance out:

$$B_t + N_t + D_t + F_t = 0.$$

The foreign-currency bond is in perfect elastic international supply at an exogenous interest rate R_t^*.

The government budget constraint from operations in the financial market is given by:

$$\frac{F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*} = F_{t-1} + \mathcal{E}_t F_{t-1}^* + \tau \mathcal{E}_t \pi_t^* - T_t \quad \text{with } \pi_t^* = \tilde{R}_t^* \cdot \frac{N_{t-1}^* + D_{t-1}^*}{R_{t-1}^*},$$

where T_t is the lump-sum transfer to the home households and π_t^* is the combined income from the financial transactions of noise traders and arbitrageurs (in dollars). Note that parameter $\tau \in [0,1]$ can be viewed as either the home country's ownership share of the financial sector or a tax on financial transactions imposed by the home government.[25]

Equilibrium. Define the net foreign asset (NFA) position of the home country, B_t^* in foreign currency, which has the home-currency value:

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} = \frac{B_t + F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*},$$

that is the value of the combined position of the home households and the government. Using B_t^*, we prove in Appendix B the following lemma that characterize the open economy equilibrium conditions.

Lemma 1. *The NFA of the home country equals the combined foreign-currency bond position in the financial market, $B_t^* = F_t^* + N_t^* + D_t^*$, and the combined home-country budget constraint in foreign-currency terms is given by:*

$$\frac{B_t^*}{R_t^*} - B_t^* = \left(Y_{Tt} - C_{Tt}\right) - \left(1 - \tau\right) \tilde{R}_t^* \frac{B_{t-1}^* - F_{t-1}^*}{R_{t-1}^*}. \tag{19}$$

25. Note that the arbitrageur's problem omits τ without loss of generality, as a change in income share τ is isomorphic to a re-parameterization of the risk aversion ω, and we take both ω and τ as fixed parameters in our analysis.

The international risk-sharing condition is given by:

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} = 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \text{ where } \sigma_t^2 = R_t^2 \cdot \text{var}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right). \quad (20)$$

The international risk-sharing wedge is $\hat{\Psi}_t \equiv \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}$.

Conditions (19) and (20) are the segmented markets counterparts to the equilibrium conditions (6) and (4) in the baseline model in section 1. The last term in the budget constraint (19) reflects the international transfer of financial-sector income from the home country to the rest of the world. When $\tau = 1$, that is either all income is taxed away or the financial sector is owned by the domestic residents, there is no international transfer and the budget constraint is simply $B_t^*/R_t^* - B_t^* = Y_{Tt} - C_{Tt}$, exactly as before in (6).

The international risk-sharing condition (20) specializes (4) to the case of a segmented market equilibrium, which provides a particular structural interpretation $\hat{\Psi}_t$ to the reduced-form risk-sharing wedge $\hat{\psi}_t$ in (4). When $\hat{\psi}_t = 0$, the international risk-sharing condition reduces to the conventional Euler equation for the foreign-currency bond, $\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} = 1$, a property of the constrained optimal risk sharing in this economy. Combining international risk sharing (20) with the home household Euler equation (3), we obtain the modified UIP condition that holds in this economy:

$$\mathbb{E}_t\left\{\frac{\beta C_{Tt}}{C_{T,t+1}}\left[R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]\right\} = \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*} = \hat{\Psi}_t. \quad (21)$$

Note that $\hat{\Psi}_t$ is the UIP wedge. When $\hat{\Psi}_t = 0$, whether due to $\omega\sigma_t^2 = 0$ or to $D_t^* = B_t^* - N_t^* - F_t^* = 0$, the UIP holds from the perspective of the home households. Thus, in the limit of risk neutral arbitrageurs $\omega \to 0$, the international financial market converges to a frictionless two-bond market where UIP holds.

To summarize, condition (7) still characterizes the equilibrium allocation $\{C_{Nt}, L_t, Y_{Nt}\}$ in the nontradable sector given sticky prices $P_{Nt} \equiv 1$ and where we think of W_t as directly controlled by monetary policy R_t.[26] Given (7) and the expenditure switching condition (8), the

26. Recall that the choice of domestic policy rate R_t allows to choose the path of nominal wages W_t, as they are linked by the household Euler equation (3), which in light of (5) can be written as $\beta R_t \mathbb{E}_t\{W_t / W_{t+1}\} = 1$; as usual, one needs to ensure the uniqueness of the implemented equilibrium path $\{W_t\}$.

dynamic equilibrium system (19)–(20) characterizes the equilibrium path of $\{C_{Tt}, B_t^*, \mathcal{E}_t\}$ and the implied $\{\sigma_t^2\}$ in the tradable sector. The equilibrium path is shaped by the endowment process $Y_{Tt} = A_{Tt}$, the initial condition B_{-1}^* ,the path of policies $\{R_t, F_t, F_t^*\}$ and exogenous shocks $\{A_{Nt}, A_{Tt}, R_t^*, N_t^*\}$, where recall that $N_t^* = \hat{\psi}_t$ is the noise trader liquidity shock for foreign versus home currency.[27]

3.2 Optimal Policy

We start with the analysis of optimal policies in the case with $\tau = 1$, namely when all income in the financial sector remains in the home country and there is no international transfer associated with noise traders and/or arbitrageurs. The planner's problem in this case delivers the constrained optimum as there is no incentive to manipulate risk sharing or monetary policy to achieve a monetary transfer from the rest of the world. We consider the case with $\tau < 1$ in section 3.2.4.

We use the equilibrium characterization to simplify the policy problem. In particular, we substitute the solution for equilibrium allocation in the nontradable sector (7), namely $C_{Nt} = W_t$ and $L_t = W_t / A_{Nt}$ given fully sticky prices $P_{Nt} = 1$, directly into the household utility function (1). This results in the following welfare objective:

$$\mathbb{W}_0 = max\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\gamma\log C_{Tt} + (1-\gamma)\left(\log W_t - \frac{W_t}{A_{Nt}}\right)\right]. \tag{22}$$

We treat the nominal wage W_t as the instrument of monetary policy, since any path of W_t can be implemented with a suitable interest-rate rule R_t, as we discussed above.

Given W_t and FX interventions F_t^*, tradable consumption must satisfy the country budget constraint (19), the international risk-sharing condition (20), and the expenditure switching condition (8), which we reproduce here as:

27. From $\{B_t^*, F_t^*, N_t^*\}$ we can recover the equilibrium position of intermediaries $D_t^* = B_t^* - F_t^* - N_t^*$ (by market clearing in Lemma 1), and the household home-currency bond position is $B_t / R_t = \mathcal{E}_t (B_t^* - F_t^*)/R_t^* - F_t/R_t$. Note that the home-currency position of the government F_t simply crowds out B_t one-for-one without changing the equilibrium path, a form of Ricardian equivalence in this economy.

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt} \tag{23}$$

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*} \text{ with } \sigma_t^2 = R_t^2 \cdot \text{var}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right), \tag{24}$$

$$\mathcal{E}_t = \frac{\gamma}{1-\gamma}\frac{W_t}{C_{Tt}} \tag{25}$$

where we used $\tau = 1$ in (19) and $C_{Nt} = W_t$ in (8).[28] The unconventional nature of this policy problem is that the equilibrium volatility of the nominal exchange rate σ_t^2 endogenously magnifies the intermediation friction in international risk sharing.

3.2.1 Full Optimal Policies

The planner chooses the path of monetary policy and FX interventions $\{W_t, F_t^*\}$, and the implied equilibrium allocation $\{C_{Tt}, B_t^*, \mathcal{E}_t, \sigma_t^2\}$, to maximize (22) subject to (23)–(25) and given the path of shocks $\{A_{Nt}, A_{Tt}, R_t^*, N_t^*\}$ with $Y_{Tt} = A_{Tt}$.

We note that the policy instrument F_t^* enters only in the international risk-sharing constraint (24), and thus it would be chosen to relax this constraint (that is, ensure a zero Lagrange multiplier). The optimal choice of B_t^* when (24) is not binding requires:

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 \tag{26}$$

that is international risk sharing without a wedge (i.e., $\hat{\Psi}_t = 0$ in Lemma 1). Combining this undistorted risk-sharing condition with the budget constraint (23) determines the unique optimal path of $\{C_{Tt}, B_t^*\}$.

By consequence, this requires setting $F_t^* = B_t^* = N_t^*$ to ensure zero wedge $\hat{\Psi}_t = 0$ independently of the equilibrium volatility of the nominal exchange rate σ_t^2. This characterizes the optimal FX interventions, which lean against the wind—in fact, fully eliminate the wind—by fully accommodating the NFA demand of the households B_t^* and the

28. Another side equation which defines R_t in (24) is the home-currency Euler equation (3), which we write as $\beta R_t \mathbb{E}_t\{W_t / W_{t+1}\} = 1$.

liquidity demand of the noise traders N_t^*. As a result, the arbitrageurs have no job left, and $D_t^* = 0$, the equilibrium risk premium is eliminated, and international intermediation is frictionless. Since imperfect intermediation under segmented markets is the only source of UIP deviations in this economy, the UIP holds under the optimal policy.[29]

Next, consider the optimal monetary policy, namely the choice of $\{W_t\}$. Note that with the undistorted risk sharing, the nominal exchange rate $\mathcal{E}_t$ no longer constrains the optimization over W_t, and the expenditure switching condition (25) acts merely as a side equation. The choice of W_t then becomes static:

$$\tilde{W}_t = \arg\max_{W_t} \{\log W_t - W_t/A_{Nt}\} = A_{Nt}. \tag{27}$$

Setting $W_t = A_{Nt}$ eliminates the state-by-state output gap, that is $X_t = W_t/A_{Nt} = 1$. The equilibrium nominal exchange rate obtains from (25) and equals $\varepsilon_t = \frac{\gamma}{1-\gamma}\frac{A_{Nt}}{C_{Tt}}$.

We summarize this discussion in:

Proposition 1. *The constrained optimum allocation denoted with* $\{\tilde{C}_{Tt}, \tilde{W}_t, \tilde{B}_t^*, \tilde{F}_t^*, \tilde{\varepsilon}_t^*\}$ *maximizes welfare (22) subject to the budget constraint (23) alone, and it is implemented with monetary policy* $\tilde{W}_t = A_{Nt}$, *which closes the state-by-state output gap, and FX interventions* $\tilde{F}_t^* = B_t^* - N_t^*$, *which eliminates the risk-sharing (UIP) wedge in (24). The optimum consumption path* $\{\tilde{C}_{Tt}\}$ *is the unique path that satisfies the dynamic system (23) and (26). The nominal exchange rate is given by* $\tilde{\mathcal{E}}_t = \frac{\gamma}{1-\gamma}\frac{A_{Nt}}{\tilde{C}_{Tt}}$. *The optimal policy is time consistent.*

Intuitively, there are two distortions—output gap due to sticky prices and imperfect risk sharing due to the intermediation friction (under limits to arbitrage)—and two policy instruments (monetary policy and FX interventions), which allow the planner to address both

29. By UIP condition we mean here the household indifference condition between the home- and foreign-currency bonds, that is $\mathbb{E}_t\left\{\frac{\beta C_{Tt}}{C_{T,t+1}}\left[R_t^* - R_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]\right\} = 0$, which features a representative household's UIP risk premium. More generally, the planner wants to illuminate the intermediation wedge, leaving intact the fundamental sources of the risk premium.

distortions and deliver the constrained optimum.[30] The property of the constrained optimum is zero wedges in production (output gap) and in international risk sharing, $X_t = 1$ and $\hat{\Psi}_t = 0$. The maximum utility is given by $\tilde{\mathbb{W}}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log \tilde{C}_{Tt} + (1-\gamma)(\log A_t - 1) \right]$, and we use it as the benchmark for the remaining analysis:

$$\mathbb{W}_0 - \tilde{\mathbb{W}}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log \frac{C_{Tt}}{\tilde{C}_{Tt}} + (1-\gamma)\left(\log \frac{W_t}{A_t} - \frac{W_t - A_t}{A_t} \right) \right] \leq 0.$$

where the first term is the loss from risk-sharing distortions and the second term is the loss from the output gap.

Importantly, the optimal policy is time consistent, as both instruments remove the respective distortions contemporaneously and require no intertemporal promises. As a result, the implementation of the constrained optimum allocation does not require commitment on the part of the monetary authority.

There is no closed form characterization of $\tilde{C}_{Tt}$ in the presence of uninsured country risk in Y_{Tt}, but when $Y_{Tt} = A_{Tt}$ follows a random walk, $\tilde{C}_{Tt}$ follows a near-random walk with changes in $\tilde{C}_{Tt}$ approximately equal to changes in A_{Tt}. What are the implications of this for the nominal and real exchange rate? The nominal exchange rate $\tilde{\mathcal{E}}_t = \frac{\gamma}{1-\gamma} \frac{A_{Nt}}{\tilde{C}_{Tt}}$, as well as the real exchange rate $\tilde{Q}_t = \tilde{\mathcal{E}}_t^{1-\gamma}$, appreciates with the relative productivity in the tradable sector, that is, when tradable endowment A_{Tt} increases sharper than nontradable productivity A_{Nt}. Indeed, this is the Balassa-Samuelson force, which shapes the path of the real exchange rate in proportion with the relative tradable-nontradable productivity. Under sticky prices, implementing this path for the real exchange rate requires the nominal exchange rate to follow the same relative productivities.

Implementing the constrained optimum in an economy with sticky prices and frictional financial market requires an active use of both monetary policy and FX interventions but does not require the use

30. Note that the constrained optimum is not first best as international financial market is incomplete and only allows to share risk in expectation given the foreign interest rate R_t^*. This is equivalent to a single foreign-currency bond economy. Interestingly, the presence of the home-currency bond is irrelevant for the optimal allocation, as R_t is merely a side variable and does not affect the equilibrium allocation in this case, and the planner has no incentive to use any additional instrument (e.g., capital controls; see below).

of capital controls. The goal of FX interventions is not to eliminate exchange rate volatility, but rather to eliminate the risk-sharing wedge—the UIP deviation $\hat{\Psi}_t$ due to the intermediation friction. No UIP deviations are, in fact, consistent with a volatile nominal exchange rate, which itself is generally a consequence of the optimal monetary policy stabilizing output gap.[31] In segmented financial markets, FX interventions provide the government with an important additional tool, which allows to fix distortions associated with frictional intermediation. The use of FX interventions does not interfere with monetary policy, which is focused on domestic output-gap stabilization, as in the closed economy, and does not generally require the use of capital controls. In this sense, such economy does not feature the trilemma trade-off present in conventional monetary models with a frictionless financial market.[32]

3.2.2 Divine Coincidence: Fixed Exchange Rate

In the constrained optimum allocation, FX interventions $\tilde{F}_{Tt} = B_t^* - N_t^*$ eliminate the risk-sharing wedge ($\hat{\Psi}_t = 0$), but do not result in a stable exchange rate ($\mathcal{E}_t \neq const$ in general). Indeed, the nominal exchange rate traces the frictionless real exchange rate, which in turn reflects the relative movements in nontradable productivity (relative to tradable endowment). We now explore the special case when a fixed exchange rate implements the constrained optimum.

Note also that the constrained optimum implementation requires the use of both instruments—monetary policy W_t and FX interventions F_t^*—and, in general, it cannot be implemented with monetary policy alone. There exists, however, an important special, yet robust, case when monetary policy alone can simultaneously implement both goals—output-gap stabilization and elimination of the international risk-sharing wedge—without any need to use FX interventions. This case relies on the full stabilization of the nominal exchange rate—the fixed exchange rate—which can be achieved by means of monetary

31. As shown above, the nominal exchange rate implementing the first best follows the relative nontradable productivity. Arguably, the volatility of relative productivities is not as large as the observed volatility of floating exchange rates, e.g., dollar/euro (10% annualized standard deviation). Thus, it is likely that optimal FX interventions partially stabilize the exchange rate relative to laissez-faire, as we further discuss below.

32. Note that this does not mean however that any path of the exchange rate can be implemented without compromising the ability of monetary policy to stabilize inflation and output gap, and in this sense the trilemma is still present.

policy and thus eliminates the need to use FX interventions. We refer to this special case as the *divine coincidence* in an open economy.

Indeed, examining the general policy problem (22), the limiting case with a commitment to fixed exchange rate $\mathcal{E}_t = const$ implies $\sigma_t^2 = 0$, and thus eliminates the risk-sharing wedge (ensures $\hat{\Psi}_t = 0$), irrespective of the use of the other instrument F_t^*. Furthermore, since $\mathcal{E}_t = \frac{\gamma}{1-\gamma}\frac{W_t}{C_{Tt}}$, monetary policy can always ensure a fixed exchange rate by setting $W_t / C_{Tt} = const$.

The only remaining question is when such monetary policy can also be optimal from the point of view of the output-gap stabilization, that is, ensure that $X_t = W_t / A_{Nt} = 1$. While being a knife-edge case, it is an important one and can be formulated as follows: if the first-best real exchange rate—i.e., the real exchange rate corresponding to the first-best allocation with zero output gap—is constant, then fixed nominal exchange rate is the optimal policy stabilizing simultaneously output gap and international risk sharing. Indeed, recall that the real and nominal exchange rates perfectly comove under sticky prices, $\mathcal{Q}_t = \mathcal{E}_t^{1-\gamma}$, so that if the first-best real exchange rate $\tilde{Q}_t = (\frac{\gamma}{1-\gamma}\frac{A_{Nt}}{\tilde{C}_{Tt}})^{1-\gamma} = const$, then it can always be implemented with $\mathcal{E}_t = const$ independently of the degree of price stickiness. Furthermore, this is an "if and only if" statement, and the fixed exchange rate is necessarily suboptimal whenever $\tilde{Q}_t \neq const$ and prices are (at least partially) sticky.

Proposition 2. *The fixed nominal exchange rate implements the constrained optimum allocation if and only if the first-best real exchange rate is stable,* $\tilde{Q}_t = const$. *In this case, monetary policy alone can achieve both goals of output-gap stabilization,* $X_t = 1$, *and elimination of the international risk-sharing wedge,* $\hat{\Psi}_t = 0$, *without the use of FX interventions or capital controls.*

When can we expect the first-best real exchange rate to be stable? In our setup, this is the case when Balassa-Samuelson forces exactly offset each other and, in particular, the nontradable productivity and tradable endowment comove in lockstep. Formally, this would require a near-random walk perfectly correlated processes in both $Y_{Tt} = A_{Tt}$ and A_{Nt}, so that C_{Tt} tracks Y_{Tt} and thus $A_{Nt} / C_{Tt} = const$.[33] More generally, the real exchange rate may also vary because of the differential

33. In a linearized environment, this is exactly the case, as $c_{Tt} = y_{Tt}$ under a random walk endowment, but in a full nonlinear problem, the path of C_{Tt} differs from that of Y_{Tt} due to precautionary savings from uninsured idiosyncratic risk.

evolution of home and foreign tradable productivity under home bias in tradable consumption. The divine coincidence principle generalizes to those environments and still suggests that if one can argue that the first-best real exchange rate is stable, then a fixed nominal exchange rate regime implements the constrained optimum and achieves both policy objectives without the need to use other instruments such as exchange rate interventions or capital controls. In other words, divine coincidence is exactly the case where inflation (output-gap) stabilization does not come into conflict with a fixed exchange rate and thus the trilemma, if present, is not binding.

Implementation. We focused above on the direct implementation of the peg using W_t. Two remarks are in order. First, the same allocation can be implemented using an interest rate R_t rule, as pointed out above. Second, and more importantly, either W_t or R_t implementation can either target output gap or nominal exchange rate directly. Indeed, divine coincidence implies that fixed exchange rate equilibrium corresponds to the zero output-gap equilibrium. However, the implementation of the policy does matter, as targeting output gap may be consistent with multiple exchange rate equilibria, one with $\sigma_t^2 = 0$ and another with $\sigma_t^2 > 0$, and only the former one ensures undistorted international risk sharing.[34] Therefore, in terms of implementation, a monetary policy that explicitly targets the nominal exchange rate can be superior to that stabilizing the output gap, even under divine coincidence. In this sense, the model captures the idea of using a nominal peg to anchor expectations, although the focus is on the financial-market expectations rather than inflation expectations of households and firms.[35]

3.2.3 Single Instrument without Divine Coincidence

Proposition 1 characterized the optimal joint use of monetary policy and FX interventions, which allows to implement the optimal

34. Formally, compare the case with $W_t = A_{Nt}$ and $W_t = \kappa C_{Tt}$ for some appropriately chosen $\kappa > 0$, which under divine coincidence are both consistent with the optimal allocation. While the latter implementation ensures $\mathcal{E}_t = const$ from (25) and thus $\sigma_t^2 = 0$, the former may be consistent with multiple equilibria that solve $\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^*}{R_t^*}$ where $\sigma_t^2 = R_t^2 \cdot \mathrm{var}_t\left(\frac{C_{Tt+1}/C_{Tt}}{A_{Nt+1}/A_{Nt}}\right)$, in addition to the budget constraint (23). The multiplicity of solutions for (C_{Tt}, σ_t^2) translates into the multiplicity of solutions for $\mathcal{E}_t$, with $\sigma_t^2 = 0$ solution welfare dominating other possible solutions.

35. *Cf*. Marcet and Nicolini (2003).

allocation by eliminating both the output gap and the international risk-sharing wedge state by state. Proposition 2 shows how monetary policy can fully stabilize the nominal exchange rate, which immediately eliminates the risk-sharing wedge without the use of FX interventions, and further characterizes circumstances when it is also optimal from the point of output-gap stabilization. As a corollary, when prices are flexible and thus the output gap is absent irrespective of monetary policy, the optimal risk sharing can be always achieved by monetary policy that stabilizes the nominal exchange rate, without the use of FX interventions. In other words, equilibrium nominal exchange rate volatility can be desirable only under sticky prices, when it needs to accommodate the real exchange rate variation that cannot be achieved via adjustment of prices.

We now consider the reverse case of whether the output gap can be stabilized by FX interventions alone, when monetary policy is constrained, e.g., by the zero lower bound $R_t \geq \underline{R}$ or fixed exchange rate $\mathcal{E}_t = \bar{\mathcal{E}}$.[36] In contrast to the previous case, it is *not* possible to implement the first-best allocation with FX interventions. In particular, fixed exchange rate implies $\sigma_t^2 = 0$ in (20) and, while it immediately eliminates the risk-sharing wedge, it also makes FX interventions F_t^* irrelevant for the equilibrium allocation. F_t^* can still affect allocation $\{C_{Tt}, C_{Nt}\}$ under the zero-lower-bound constraint if $\sigma_t^2 > 0$. However, under separable utility, F_t^* is optimally used to only eliminate the risk-sharing wedge in tradables without targeting the allocation of nontradables and the output gap.[37]

This analysis in particular suggests that FX interventions cannot substitute for monetary policy. We next explore the optimal use of monetary policy in the presence of both frictions when FX

36. Recall that, under sticky prices, $P_{Nt} = 1$, we have $C_{Nt} = W_t$, and the loss from the output gap can be written as $\log\frac{C_{Nt}}{A_t} - \frac{C_{Nt} - A_t}{A_t} \leq 0$. Furthermore, C_{Nt} must satisfy $\beta R_t \mathbb{E}_t\left\{C_{Nt} / C_{N,t+1}\right\} = 1$ and $\mathcal{E}_t = \frac{\gamma}{1-\gamma}\frac{C_{Nt}}{C_{Tt}}$, with the former possibly constrained by the zero lower bound and the latter by the fixed exchange rate.

37. With nonseparable utility in (C_{Tt}, C_{Nt}), FX interventions can depart from the optimal risk sharing $\beta R_t^* \mathbb{E}_t\left\{u_{Tt}/u_{Tt+1}\right\} = 1$ in order to relax the constraint imposed by $\beta R_t \mathbb{E}_t\left\{u_{Nt} / u_{Nt+1}\right\} = 1$ when R_t cannot adjust (where u_{Tt} and u_{Nt} correspond to marginal utility of tradable and nontradable consumption, respectively). As in the general theory of second best, the constrained optimal policy introduces a wedge into international risk sharing if it allows to reduce the domestic output gap. Unlike capital controls or other taxes, however, which can directly distort $\beta R_t \mathbb{E}_t\left\{\frac{u_{Tt}}{u_{Tt+1}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right\} = 1$, FX interventions are less capable and operate exclusively via their indirect effect on C_{Tt} in (20). Cf. Farhi and Werning (2012), Correia and others (2013), Farhi and others (2014).

interventions F_t^* are not available. In this case, the optimal monetary policy closes the output gap on average and trades off the state-by-state variation in output gap ex post with a reduction in the risk-sharing wedge ex ante by partially stabilizing the future nominal exchange rate. Formally, the optimal monetary policy ensures $\mathbb{E}_t X_{t+1} = 1$, where $X_{t+1} = W_{t+1}/A_{Nt+1}$ is the output gap, but varies $X_{t+1} \neq 1$ state by state to reduce σ_t^2, in particular in periods following large risk-sharing wedges $\hat{\Psi}_t = \omega\sigma_t^2 \frac{N_t^* - B_t^*}{R_t^*} \neq 0$.[38] The policy reduces $C_{Nt+1} = W_{t+1}$ below A_{Nt+1} when C_{Tt+1} is low, and vice versa, which reduces the volatility of $\mathcal{E}_{t+1} = \frac{\gamma}{1-\gamma}\frac{W_{t+1}}{C_{Tt+1}}$ by making tradable and nontradable consumption more correlated. This is the optimal trade-off between the two frictions, namely giving up on fully stabilizing the output gap at $t + 1$ to reduce the international risk-sharing wedge at t to smooth tradable consumption.

We summarize these results in the following proposition and provide a formal proof in Appendix B:

Proposition 3. *(i) Monetary policy can eliminate the risk-sharing wedge, while FX interventions cannot close the output gap when monetary policy is constrained and can only ensure constrained optimal international risk sharing. (ii) Optimal monetary policy in the absence of FX interventions eliminates the output gap on average and uses the state-by-state variation in output gap to partially reduce the volatility of the nominal exchange rate and the ex-ante risk-sharing (UIP) wedge.*

This proposition emphasizes that FX interventions are a direct instrument to offset international risk-sharing wedges emerging as a result of imperfect intermediation. This result generalizes beyond segmented market models and applies in noncompetitive environments with rents and markups and in models with financial constraints.[39] As the same time, FX interventions are ineffective to address other frictions such as output gap or, in richer models, inefficiencies arising from overborrowing due to pecuniary externalities.[40]

The proposition also suggests that pure floats are generally suboptimal when monetary policy focuses exclusively on output-gap and inflation stabilization and FX interventions are not used.

38. In contrast, $\mathbb{E}_t X_{t+1} = 1$ state by state in periods following $\hat{\Psi}_t = 0$, i.e., when risk-sharing UIP deviations are small due to a combination of small risk aversion ω, small exchange rate volatility σ_t^2, and/or small equilibrium financial flows $N_t^* - B_t^*$.

39. For example, Gabaix and Maggiori (2015), Adrian and others (2015), Jiang and others (2021), Bianchi and others (2021).

40. For example, Basu and others (2020).

Instead, partial and crawling pegs whereby either FX interventions or monetary policy are used to partially stabilize or eliminate short-run exchange rate volatility are generally superior to pure floats, as well as to outright pegs. Full pegs are optimal under divine coincidence and pure floats are optimal when wedges arising from intermediation frictions are negligible. The latter happens when either risk-bearing capacity is large (small ω) or financial flows $N_t^* - B_t^*$ are small relative to the absorption capacity of the financial market (a deep financial market). A sign of a deep financial market are small UIP deviations despite large ex-post exchange rate volatility. In contrast, when UIP deviations are large, this may indicate frictional intermediation and call for policy intervention to smooth out UIP deviations. In other words, large ex-ante UIP deviations is a necessary condition for a welfare improving exchange rate intervention.[41]

Discretionary policy. An important property of the optimal policies in Proposition 1 was time consistency and no need for commitment to implement them. As described above, the optimal monetary policy in the absence of FX interventions trades off output-gap stabilization at $t + 1$ for a reduction in the risk-sharing wedge at t. This requires commitment on the part of the monetary authority, as the only time-consistent discretionary outcome is the state-by-state output-gap stabilization, $X_{t+1} = 1$, which leaves a laissez-faire international risk-sharing wedge $\hat{\Psi}_t$. This is suboptimal, as shown in Proposition 3.

3.2.4 International Transfers. Capital Controls

We now consider the case with international transfers when $\tau < 1$ in the country budget constraint (19), which we rewrite as:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = \left(Y_{Tt} - C_{Tt}\right) - \tilde{\tau}\tilde{R}_t \frac{B_{t-1}^* - F_{t-1}^*}{R_{t-1}^*}, \tag{28}$$

where we denoted $\tilde{\tau} \equiv 1 - \tau > 0$ and carry trade return $\tilde{R}_t \equiv R_{t-1}^* - R_{t-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}$. Thus, the planner maximizes the objective (22) subject to (28), (24)–(25) and the Euler equation (3), which determines R_t. For convenience, we

41. If UIP deviations reflect default or counterparty risk rather than intermediation friction or rents, then FX interventions are not justified as a policy response. See Amador and others (2019).

combine (3) with (24) to write the constraint as the UIP condition (21) on R^*_{t+1}, which we reproduce here as follows:

$$\mathbb{E}_t\left\{\Theta_{t+1}\tilde{R}^*_t\right\} = \omega\sigma_t^2\,\frac{B^*_t - N^*_t - F^*_t}{R^*_t} = \hat{\Psi}_t, \tag{29}$$

where recall that $\Theta_{t+1} = \beta\frac{C_{Tt}}{C_{Tt+1}}$ is the home household's stochastic discount factor (SDF) for returns in foreign currency and $\hat{\Psi}_t$ is the UIP wedge.

The last term in the budget constraint (28) corresponds to the international wealth transfer, which obtains when the noise traders and arbitrageurs jointly make losses on their financial positions, as their losses are the gains of the combined home households and government sector. Under these circumstances, while it is still feasible, it is no longer optimal for the government to fully eliminate the risk-sharing wedge $\hat{\Psi}_t$ in (29). First, consider the optimal policies from Proposition 1, namely $W_t = A_t$ and $F^*_t = B^*_t - N^*_t$, which still eliminate both the output gap and the risk-sharing wedge. In this case, the country budget constraint becomes:

$$\frac{B^*_t}{R^*_t} - B^*_t = \left(Y_{Tt} - C_{Tt}\right) - \tilde{\tau}\tilde{R}^*_t\psi_{t-1}.$$

Where $\psi_{t-1} = \frac{N^*_{t-1}}{R^*_{t-1}}$ is the exogenous noise trader liquidity demand for dollar relative to home currency. As a result, this allocation is associated with mean-zero idiosyncratic international transfers (evaluated using the home household SDF):

$$\mathbb{E}_{t-1}\left\{\Theta_t\tilde{\tau}\tilde{R}^*_t\psi_{t-1}\right\} = \tilde{\tau}\psi_{t-1}\mathbb{E}_{t-1}\left\{\Theta_t\tilde{R}^*_t\right\} = 0$$

and they contribute to the national income volatility of the home country thus reducing welfare. Can the government improve upon this allocation? In particular, is it feasible to eliminate income risk or even create systematic transfers from the rest of the world.

One can show that departures from $W_t = A_t$, if UIP still holds in expectation, generate at most third-order benefits, while creating second-order losses from departures from output gap. Thus, we focus here for concreteness on monetary policy that stabilizes output gap, $W_t = A_t$, and explore the use of FX interventions F^*_t in the presence of international transfers. We rewrite the budget constraint (28) as:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = \left(Y_{Tt} - C_{Tt}\right) - \tilde{\tau}\tilde{R}_t^* \left[\psi_{t-1} + \frac{\mathbb{E}_{t-1}\Theta_t \tilde{R}_t^*}{\omega\sigma_{t-1}^2} \right],$$

and the government has a direct control over the size of the UIP deviation, $\mathbb{E}_t\Theta_{t+1}\tilde{R}_{t+1}^* = \hat{\Psi}_t$ by means of FX interventions F_t^* in (29). Therefore, the tradeoff faced by the policymaker is whether to engineer ex-ante UIP deviations, which distort risk sharing, yet can generate additional national income under certain circumstances.

The expected discounted income (using home SDF) from FX interventions that allow for UIP deviations ($\hat{\Psi}_t \neq 0$) is given by:

$$-\tilde{\tau}\mathbb{E}_t\Theta_{t+1}\tilde{R}_{t+1}^* \left[\psi_t + \frac{\mathbb{E}_t\Theta_{t+1}\tilde{R}_{t+1}^*}{\omega\sigma_t^2} \right] = -\tilde{\tau} \left[\psi_t \hat{\Psi}_t + \frac{\hat{\Psi}_t^2}{\omega\sigma_t^2} \right].$$

Therefore, the expected income is (weakly) negative in the absence of noise trader demand (when $\psi = 0$), and thus $\hat{\Psi}_t = 0$ is optimal in this case as it guarantees both efficient risk sharing and no expected income losses. A corollary of this result is that, if noise traders are domestic and arbitrageurs are foreign, the government can generate no expected income and should ensure $\hat{\Psi}_t = 0$ by setting $F_t^* = B_t^* - N_t^*$ as in Proposition 1.[42]

In the presence of international noise trader demand, the policymaker can generate expected incomes by partially "leaning against the wind" of their currency demand and choosing F_t^* such that:

$$\psi_t \hat{\Psi}_t \propto N_t^* \cdot \left(B_t^* - N_t^* - F_t^*\right) < 0.$$

The income gains of the government are limited, however, by the arbitrageurs, who take positions in the same direction as the government and inversely proportionally to $\omega\sigma_t^2$. As a result, in the limit of $\omega\sigma_t^2 \to 0$, the government cannot sustain any expected income gains, even in the presence of noise traders, and should not attempt to choose $\hat{\Psi}_t \neq 0$, which would be futile anyways. Finally, for any $\omega\sigma_t^2 > 0$, UIP deviations $\hat{\Psi}_t$ in response to $\psi_t \neq 0$ generate income gains that are first order in $\hat{\Psi}_t$ and welfare losses from the resulting risk-sharing wedge that are second order in $\hat{\Psi}_t$, around $\hat{\Psi}_t = 0$. Therefore, nonzero

42. *Cf.* Amador and others (2019), Fanelli and Straub (2021).

UIP deviation $\hat{\Psi}_t$ are necessarily desirable in this case, if sufficiently small.[43] We summarize this discussion in the following proposition:

Proposition 4. (i) *Expected income from FX interventions is weakly negative in the absence of foreign noise trader demand, and thus* $F_t^* = B_t^* - N_t^*$ *to ensure* $\hat{\Psi}_t = 0$ *is optimal in this case.* (ii) *In the presence of foreign noise trade demand, there exist FX interventions* F_t^* *that partially lean against* N_t^* *and generate expected incomes that exceed welfare losses from the induced risk-sharing (UIP) wedge* $\hat{\Psi}_t \neq 0$.

Volatility of the central bank's balance sheet. The policy of FX interventions, whether it results in UIP deviations or not, leads to ex-post income and losses borne by the central bank, even when expected incomes and losses might be zero. In particular, the ex-post income of the central bank is given by $\tilde{R}^*_{t+1}\frac{F_t^*}{R_t^*} = \left[1 - \frac{R_t}{R_t^*}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]F_t^*$ and its variance is given by $\sigma_t^2 \cdot (F_t^*/R_t^*)^2$. Thus, two possible constraints on the central bank's balance sheet may be non-negative foreign reserves $F_t^* \geq 0$ or a value at risk constraint $|F_t^*| \leq \alpha R_t^*/\sigma_t$. Both constraints may limit the ability of the central bank to implement the optimal policies and, in particular, the policy $F_t^* = B_t^* - N_t^*$ from Proposition 1 may be infeasible.

Furthermore, the region of feasibility may not be connected, as there is feedback between policy F_t^* and equilibrium exchange rate volatility σ_t^2. More specifically, limited interventions F_t^* may result in large equilibrium exchange rate volatility σ_t^2, while large interventions, vice versa, limit significantly the equilibrium σ_t^2, thus possibly making the intermediate levels of interventions infeasible.

Finally, in cases when sufficiently large interventions are infeasible and the lowest achievable σ_t^2 with FX interventions is large, a fully fixed exchange rate by means of monetary policy may be superior relative to the output-gap-stabilizing monetary policy and the best feasible level of FX interventions. This can be the case, in particular, even when the divine coincidence of Propositions 2 is not satisfied. Thus, this offers a justification for some exchange rate pegs that are adopted despite the resulting output gaps and suboptimal real exchange rate under the peg.

Capital controls. So far, we have left out capital controls from our considerations. Indeed, Propositions 1 and 2 show that optimal

43. The maximum expected income equals $\frac{1}{4}\tilde{\tau}\omega\sigma_t^2\psi_t^2$, and it is achieved when $\hat{\Psi}_t = -\frac{1}{2}\omega\sigma_t^2\psi_t$, or equivalently $F_t^* = B_t^* - \frac{1}{2}N_t^* = B_t^* - \frac{1}{2}R_t^*\psi_t$. The optimal intervention additionally takes into account the welfare loss from the risk-sharing wedge which is increasing in $\hat{\Psi}_t^2$.

allocations can be attained without any use of capital controls, as long as there are no international transfers ($\tau = 1$ in (19)) and both monetary policy and FX interventions are available and unconstrained. As soon as we consider the full policy problem, which features a general budget constraint (28) with a possibility of transfers, capital controls become useful. The only constraint that cannot be relaxed is the budget constraint; (24) and (29) can be relaxed provided that there are enough policy instruments. Indeed, FX interventions relax the risk-sharing constraint (24), while capital controls on households (or other intertemporal taxes) relax the UIP condition (29). This effectively makes R_t a free choice variable allowing the government to manipulate UIP deviations with both F_t^* and capital controls, thus further maximizing the rents that can be extracted from noise traders.[44] In general, these rents are limited by the intermediation of arbitrageurs, unless separate capital controls can be levied on the arbitrageurs as well.

4. Conclusion

This paper outlines a simple model of exchange rate determination, which is broadly consistent with the major exchange rate puzzles and uses it to study the optimal exchange rate policy. We emphasize the transmission of monetary and financial shocks via goods and financial markets, which is crucial to explain the PPP, UIP and Mussa puzzles. Sticky prices and financial intermediation frictions imply that there are two wedges in the economy—the output gap and deviations from the optimal risk sharing—and closing them with one policy instrument is only feasible when the optimal real exchange rate is stable. This open economy divine coincidence calls for a fixed nominal exchange rate. More generally, two instruments are required to implement the optimal allocation: while interest-rate policy targets the output gap, FX interventions are used to eliminate UIP deviations, eliminating financial noise but allowing for fundamental exchange rate volatility. When only the monetary instrument is available, the second-best policy balances the two objectives and partially stabilizes the nominal exchange rate, resulting in a partial crawling peg.

While we focus on exchange rate policies, the normative implications are not limited to an open economy environment. It is

44. For further analysis see Itskhoki and Mukhin (2022).

intriguing to study, both theoretically and empirically, the transmission mechanism of monetary policy via financial markets in a closed economy. The ability of a peg to stabilize the risk premium on the carry trade raises the question of whether monetary policy can and should partially stabilize the volatility in the *equity* risk premium by targeting a stock market index. How such policy affects the economy and whether it is desirable are important questions for future research.

References

Adrian, T., E. Etula, and H.S. Shin. 2015. "Risk Appetite and Exchange Rates." Staff Reports 750, Federal Reserve Bank of New York.

Alvarez, F., A. Atkeson, and P.J. Kehoe. 2009. "Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium." *Review of Economic Studie*s 76(3): 851–78.

Amador, M., J. Bianchi, L. Bocola, and F. Perri. 2019. "Exchange Rate Policies at the Zero Lower Bound." *The Review of Economic Studies* 87(4): 1605–45.

Bacchetta, P. and E. van Wincoop. 2006. "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?" *American Economic Review* 96(3): 552–76.

Backus, D.K. and G.W. Smith. 1993. "Consumption and Real Exchange Rates in Dynamic Economies with Non-Traded Goods." *Journal of International Economics* 35(3–4): 297–316.

Basu, S.S., E. Boz, G. Gopinath, F. Roch, and F.D. Unsal. 2020. "A Conceptual Model for the Integrated Policy Framework." IMF Working Papers No. 2020/121.

Baxter, M. and A.C. Stockman. 1989. "Business Cycles and the Exchange rate Regime: Some International Evidence." *Journal of Monetary Economics* 23(3): 377–400.

Benigno, G. and P. Benigno. 2003. "Price Stability in Open Economies." *Review of Economic Studies* 70(4): 743–64.

Berka, M., M.B. Devereux, and C. Engel. 2012. "Real Exchange Rate Adjustment In and Out of the Eurozone." *American Economic Review* 102(3): 179–85.

Berka, M., M.B. Devereux, and C. Engel. 2018. "Real Exchange Rates and Sectoral Productivity in the Eurozone." *American Economic Review* 108(6): 1543–81.

Bianchi, J. 2011. "Overborrowing and Systemic Externalities in the Business Cycle." *American Economic Review* 101(7): 3400–26.

Bianchi, J., S. Bigio, and C. Engel. 2021. "Scrambling for Dollars: International Liquidity, Banks and Exchange Rates." Available at https://www.ssc.wisc.edu/~cengel/.

Blanco, A. and J. Cravino. 2020. "Price Rigidities and Relative PPP." *Journal of Monetary Economics* 116: 104–116.

Camanho, N., H. Hau, and H. Rey. 2018. "Global Portfolio Rebalancing and Exchange Rates." NBER Working Paper No. 24320.

Cavallino, P. 2019. "Capital Flows and Foreign Exchange Intervention." *American Economic Journal: Macroeconomics* 11(2): 127–70.

Chari, V., P.J. Kehoe, and E.R. McGrattan. 2002. "Can Sticky Price Models Generate Volatile and Persistent Exchange Rates?" *Review of Economic Studies* 69(3): 533–63.

Clarida, R., J. Galí, and M. Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics* 115(1): 147–180.

Colacito, R. and M.M. Croce. 2013. "International Asset Pricing with Recursive Preferences." *Journal of Finance* 68(6): 2651–86.

Correia, I., E. Farhi, J.P. Nicolini, and P. Teles. 2013. "Unconventional Fiscal Policy at the Zero Bound." *American Economic Review* 103(4): 1172–211.

Corsetti, G., L. Dedola, and S. Leduc. 2010. "Optimal Monetary Policy in Open Economies." In *Handbook of Monetary Economics*, edited by B. M. Friedman, and M. Woodford, vol. 3. Elsevier.

Corsetti, G. and P. Pesenti. 2001. "Welfare and Macroeconomic Interdependence." *Quarterly Journal of Economics* 116(2): 421–45.

Costinot, A., G. Lorenzoni, and I. Werning. 2014. "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation." *Journal of Political Economy* 122(1): 77–128.

Devereux, M.B. and C. Engel. 2002. "Exchange Rate Pass-Through, Exchange Rate Volatility, and Exchange Rate Disconnect." *Journal of Monetary Economics* 49(5): 913–40.

Devereux, M.B. and C. Engel. 2003. "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility." *Review of Economic Studies* 70: 765–84.

Devereux, M.B. and V.V. Hnatkovska. 2020. "Borders and Nominal Exchange Rates in Risk-Sharing." *Journal of the European Economic Association* 18(3): 1238–83.

Egorov, K. and D. Mukhin. 2023. "Optimal Policy under Dollar Pricing." *American Economic Review*, forthcoming.

Eichenbaum, M.S., B.K. Johannsen, and S.T. Rebelo. 2021. "Monetary Policy and the Predictability of Nominal Exchange Rates." *Review of Economic Studies* 88(1): 192–228.

Engel, C. 2011. "Currency Misalignments and Optimal Monetary Policy: A Reexamination." *American Economic Review* 101(6): 2796–822.

Engel, C. and K.D. West. 2005. "Exchange Rates and Fundamentals." *Journal of Political Economy* 113(3): 485–517.

Evans, M.D.D. and R.K. Lyons. 2002. "Order Flow and Exchange Rate Dynamics." *Journal of Political Economy* 110(1): 170–80.

Fama, E.F. 1984. "Forward and Spot Exchange Rates." *Journal of Monetary Economics* 14(3): 319–38.

Fanelli, S. 2017. "Monetary Policy, Capital Controls, and International Portfolios." Unpublished Manuscript.

Fanelli, S. and L. Straub. 2021. "A Theory of Foreign Exchange Interventions." *Review of Economic Studies* 88(6): 2857–85.

Farhi, E. and X. Gabaix. 2016. "Rare Disasters and Exchange Rates." *Quarterly Journal of Economics* 131: 1–52.

Farhi, E., G. Gopinath, and O. Itskhoki. 2014. "Fiscal Devaluations." *Review of Economics Studies* 81(2): 725–60.

Farhi, E. and I. Werning. 2012. "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates." Available at http://scholar.harvard.edu/farhi/publications.

Farhi, E. and I. Werning. 2016. "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities." *Econometrica* 84(5): 1645–704.

Farhi, E. and I. Werning. 2017. "Fiscal Unions." *American Economic Review* 107(12): 3788–834.

Fornaro, L. 2021. "A Theory of Monetary Union and Financial Integration." *Review of Economic Studies*. 89:4, 1911–47.

Friedman, M. 1953. "The Case for Flexible Exchange Rates." *Essays in Positive Economics* 157(203): 33.

Gabaix, X. and M. Maggiori. 2015. "International Liquidity and Exchange Rate Dynamics." *Quarterly Journal of Economics* 130(3): 1369–420.

Gali, J. and T. Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *Review of Economic Studies* 72(3): 707–34.

Goldberg, L. and C. Tille. 2009. "Macroeconomic Interdependence and the International Role of the Dollar." *Journal of Monetary Economics* 56(7): 990–1003.

Gourinchas, P.-O., W. Ray, and D. Vayanos. 2019. "A Preferred-Habitat Model of Term Premia and Currency Risk." Discussion paper, mimeo.

Gourinchas, P.-O. and A. Tornell. 2004. "Exchange Rate Puzzles and Distorted Beliefs." *Journal of International Economics* 64(2): 303–33.

Greenwood, R., S.G. Hanson, J.C. Stein, and A. Sunderam. 2020. "A Quantity-Driven Theory of Term Premia and Exchange Rates." Discussion paper, National Bureau of Economic Research.

Hassan, T.A. and R.C. Mano. 2018. "Forward and Spot Exchange Rates in a Multi-Currency World." *Quarterly Journal of Economics* 134(1): 397–450.

Itskhoki, O. 2021. "The Story of the Real Exchange Rate." *Annual Review of Economics* 13: 423–55.

Itskhoki, O. and D. Mukhin. 2021a. "Exchange Rate Disconnect in General Equilibrium." *Journal of Political Economy* 129: 2183–232.

Itskhoki, O. and D. Mukhin. 2021b. "Mussa Puzzle Redux." Available at https://itskhoki.com/papers/Mussa.pdf.

Itskhoki, O. and D. Mukhin. 2022. "Optimal Exchange Rate Policy." Available at https://itskhoki.com/papers/ERpolicy.pdf.

Jeanne, O. 2012. "Capital Account Policies and the Real Exchange Rate." In *NBER International Seminar on Macroeconomics*: 7–42. University of Chicago Press.

Jeanne, O. and A. Korinek. 2010. "Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach." *American Economic Review* 100(2): 403–07.

Jeanne, O. and A.K. Rose. 2002. "Noise Trading and Exchange Rate Regimes." *Quarterly Journal of Economics* 117(2): 537–69.

Jiang, Z., A. Krishnamurthy, and H.N. Lustig. 2021. "Foreign Safe Asset Demand and the Dollar Exchange Rate." *Journal of Finance* 76(3): 1049–89.

Kalemli-Özcan, S. and L. Varela. 2021. "Five Facts about the UIP Premium." NBER Working Paper No. 28923.

Kekre, R. and M. Lenel. 2021. "The Flight to Safety and International Risk Sharing." Discussion paper, National Bureau of Economic Research.

Kollmann, R. 1995. "Consumption, Real Exchange Rates and the Structure of International Asset Markets." *Journal of International Money and Finance* 14(2): 191–211.

Kollmann, R. 2005. "Macroeconomic Effects of Nominal Exchange Rate Regimes: New Insights into the Role of Price Dynamics." *Journal of International Money and Finance* 24(2): 275–92.

Kouri, P. 1976. "Capital Flows and the Dynamics of the Exchange Rate." Stockholm, Institute for International Economic Studies, Seminar Paper No. 67.

Kouri, P. 1983. "Balance of Payments and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model." In *Economic Interdependence and Flexible Exchange Rates*, edited by J.S. Bhandari and B.H. Putnam. MIT Press.

Marcet, A. and J.P. Nicolini. 2003. "Recurrent Hyperinflations and Learning." *American Economic Review* 93(5): 1476–98.

Meese, R. and K. Rogoff. 1983. "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?" *Journal of International Economics* 14(1): 3–24.

Mussa, M.L. 1986. "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications." *Carnegie-Rochester Conference Series on Public Policy* 25(1): 117–214.

Obstfeld, M. and K.S. Rogoff. 1996. *Foundations of International Macroeconomics*. The MIT Press.

Obstfeld, M. and K.S. Rogoff. 2001. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" In *NBER Macroeconomics Annual* 2000 15: 339–90.

Rey, H. 2013. "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence." Discussion paper, National Bureau of Economic Research.

Rogoff, K. 1996. "The Purchasing Power Parity Puzzle." *Journal of Economic Literature* 34: 647–68.

Schmitt-Grohé, S. and M. Uribe. 2016. "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment." *Journal of Political Economy* 124(5): 1466–514.

Verdelhan, A. 2010. "A Habit-Based Explanation of the Exchange Rate Risk Premium." *Journal of Finance* 65(1): 123–46.

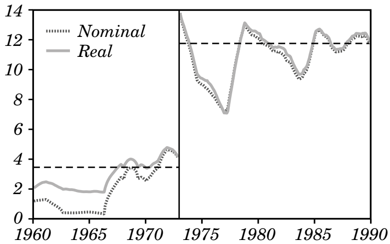
Wallace, N. 1981. "A Modigliani-Miller theorem for open-market operations." *American Economic Review* 71(3): 267–74.

APPENDICES

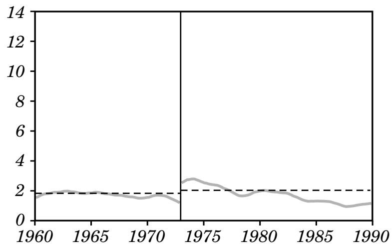
A. Additional Figures

Figure A1. Macroeconomic Volatility Over Time

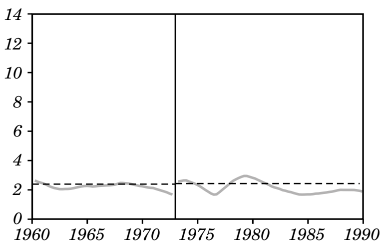
(*a*) *Exchange rates changes, e_t and q_t*



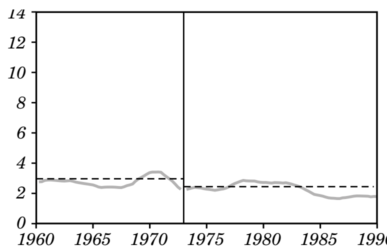
(*b*) *Relative inflation, $\pi_t - \pi_t^*$*



(c) Relative consumption growth, $c_t - c_t^*$



(*d*) Relative GDP growth, $y_t - y_t^*$



Source: Itskhoki and Mukhin (2021b).
Note: Annualized standard deviations (in log points) for G7 countries (plus Spain, without Canada) relative to the U.S., estimated as moving averages with a window over 18 months (for exchange rates and inflation) or ten quarters (for consumption and real GDP growth) before and after, treating 1973.1 as the end point for the two regimes; the dashed lines correspond to the average standard deviations under the two regimes. Note that under a full peg ($e_t = 0$), by definition $q_t = \pi_t - \pi_t^*$; under a float, the empirical correlation between e_t and q_t is close to 1 and between q_t (or e_t) and $\pi_t - \pi_t^*$ is close to 0. See figure 1 for raw data series for q_t and c_t.

B. Derivations and Proofs

Proof of Lemma 1. First, we use market clearing for home-currency bond, $B_t + N_t + D_t + F_t = 0$, and the zero capital (carry trade) portfolios of noise traders and arbitrageurs, $D_t / R_t + \mathcal{E}_t D_t^* / R_t^* = 0$, and $N_t / R_t + \mathcal{E}_t N_t^* / R_t^* = 0$, to obtain:

$$\frac{B_t + F_t}{R_t} - \frac{\mathcal{E}_t \left(N_t^* + D_t^* \right)}{R_t^*} = 0.$$

Then using the definition of the country's NFA position,

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} = \frac{B_t + F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*},$$

to express out $B_t + F_t$ and dividing through by $\mathcal{E}_t / R_t^*$ results in $B_t^* = F_t^* + N_t^* + D_t^*$, as stated in the lemma.

Second, substitute firm profits $\Pi_t = P_{Tt} Y_{Tt} + P_{Nt} Y_{Nt} - W_t L_t$ and household consumption expenditure $P_t C_t = P_{Nt} C_{Nt} + P_{Tt} C_{Tt}$ into the household budget constraint (2) and use market clearing $C_{Nt} = Y_{Nt}$ to obtain:

$$\frac{B_t}{R_t} - B_{t-1} = NX_t + T_t,$$

where $NX_t = P_{Tt} Y_{Tt} + P_{Tt} C_{Tt} = \mathcal{E}_t (Y_{Tt} - C_{Tt})$ using the law of one price with $P_{Tt}^* = 1$. Next combine this with the government budget constraint (in the text) to obtain:

$$\frac{B_t + F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*} - B_{t-1} - F_{t-1} - \mathcal{E}_t F_{t-1}^* = NX_t + \tau \mathcal{E}_t \pi_t^*.$$

Using the definition of NFA B_t^* above and the market clearing $B_t + D_t + N_t + F_t = 0$, as well as the result above that $B_t^* = D_t^* + N_t^* + F_t^*$, we rewrite:

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} - \mathcal{E}_t B_{t-1}^* + \mathcal{E}_t \left(D_{t-1}^* + N_{t-1}^* \right) + \left(D_{t-1} + N_{t-1} \right) = NX_t + \tau \mathcal{E}_t \pi_t^*.$$

Finally, recall that $\pi_t^* = \tilde{R}_t^* \frac{D_{t-1}^* + N_{t-1}^*}{R_{t-1}^*} = \left[1 - \frac{R_{t-1}}{R_{t-1}^*}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right]\left(D_{t-1}^* + N_{t-1}^*\right)$. Subtract $\mathcal{E}_t \pi_t^*$ on both sides:

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} - \mathcal{E}_t B_{t-1}^* = NX_t - (1-\tau)\tilde{R}_t^* \frac{\mathcal{E}_t\left(D_{t-1}^* + N_{t-1}^*\right)}{R_{t-1}^*},$$

where we used the fact that zero-capital portfolios of noise traders and arbitrageurs imply: $\left(D_{t-1} + N_{t-1}\right) + \frac{R_{t-1}}{R_{t-1}^*}\mathcal{E}_{t-1}\left(D_{t-1}^* + N_{t-1}^*\right) = 0$.

Divide through by ε_t, use the fact that $NX_t/\mathcal{E}_t = Y_{Tt} - C_{Tt}$, and the fact above that $D_{t-1}^* + N_{t-1}^* = B_{t-1}^* - F_{t-1}^*$ to rewrite:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = \left(Y_{Tt} - C_{Tt}\right) - (1-\tau)\tilde{R}_t^* \frac{\mathcal{E}_t\left(B_{t-1}^* - F_{t-1}^*\right)}{R_{t-1}^*},$$

resulting in (19) in the lemma.

Finally, (20) in the lemma follows directly from the optimal portfolio of the arbitrageurs (in the text), which we rewrite expanding the expressions for Θ_{t+1} and $\tilde{R}_{t+1}^*$ as:

$$\omega\sigma_t^2 \frac{D_t^*}{R_t^*} = \mathbb{E}_t\left\{\Theta_{t+1}\tilde{R}_{t+1}^*\right\} = \mathbb{E}_t\left\{\beta \frac{C_{Tt}}{C_{T,t+1}} \cdot \left[R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]\right\}.$$

Subtracting the household Euler equation (3), after noting that optimal household expenditure $\gamma P_{Nt} C_{Nt} = (1-\gamma)\mathcal{E}_t C_{Tt}$ and substituting for $D_t^* = B_{t-1}^* - N_{t-1}^* - F_{t-1}^*$ finishes the proof.

Proof of Proposition 3. Consider the policy problem when F_t^* is constrained, for concreteness $F_t^* = 0$:

$$\max_{\left\{C_{Tt}, C_{Nt}, B_t^*, \mathcal{E}_t, R_t, \sigma_t^2\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} + (1-\gamma)\left(\log C_{Nt} - \frac{C_{Nt}}{A_t}\right)\right],$$

subject to

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},$$

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} = 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^*}{R_t^*},$$

$$\beta R_t^* \mathbb{E}_t \frac{C_{Nt}}{C_{N,t+1}} = 1,$$

$$\mathcal{E}_t = \frac{\gamma}{1-\gamma}\frac{C_{Nt}}{C_{Tt}},$$

$$\sigma_t^2 = R_t^2 . \operatorname{var}_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right).$$

Denote $\Gamma_t \equiv 1/C_{Nt}$ and express out R_t and E_t using the third and fourth constraints:

$$\max_{\{C_{Tt},\Gamma_t,B_t^*,\sigma_t^2\}} \mathbb{E}_0 \sum_{t-0}^{\infty} \beta^t \left[\gamma \log C_{Tt} - (\gamma - 1)\left(\log \Gamma_t + \frac{1}{A_t \Gamma_t} \right)\right]$$

subject to $\dfrac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},$

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 - \omega\sigma_t^2 \frac{N_t^* - B_t^*}{R_t^*},$$

$$\beta^2 C_{Tt}^2 \left(\mathbb{E}_t \Gamma_{t+1}\right)^2 \sigma_t^2 = \mathbb{E}_t \left(\Gamma_{t+1} C_{T,t+1}\right)^2 - \left(\mathbb{E}_t C_{T,t+1}\right)^2 \left(\mathbb{E}_t \Gamma_{t+1}\right)^2.$$

Use Lagrange multipliers (λ_t, μ_t, δ_t) for the three remaining constraints:

$$\begin{aligned}\mathcal{L} = \sum_{t-0}^{\infty} \beta^t \Bigg\{ & \left[\gamma \log C_{Tt} - (1-\gamma)\left(\log \Gamma_t + \frac{1}{A_t \Gamma_t} \right)\right] \\ & + \lambda_t \left[B_{t-1}^* + Y_{Tt} - C_{Tt} - \frac{B_t^*}{R_t^*} \right] + \mu_t \left[1 - \omega\sigma_t^2 \frac{N_t^* - B_t^*}{R_t^*} - \beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} \right] \\ & + \delta_t \left[\beta^2 C_{Tt}^2 \left(\mathbb{E}_t \Gamma_{t+1}\right)^2 \sigma_t^2 + \left(\mathbb{E}_t C_{T,t+1}\right)^2 \left(\mathbb{E}_t \Gamma_{t+1}\right)^2 - \mathbb{E}_t \left(\Gamma_{t+1} C_{T,t+1}\right)^2 \right] \Bigg\}.\end{aligned}$$

Note that μ_t has the same sign as σ_t^2 $(N_t^* - B_t^*)$ so that $\mu_t\, \sigma_t^2\, (N_t^* - B_t^*) \geq 0$ and $\delta_t \geq 0$, with equalities only if $\sigma_t^2\, (N_t^* - B_t^*) = 0$. Also note that $\mathbb{E}_t$ in the Lagrangian stands for $\Sigma s_{t+1}\, \pi_t\, (s_{t+1})$ where $\pi_{t+1} = \pi_t(s_{t+1})$ is the probability of state s_{t+1} at t+1 conditional on state s_t at t. We take FOCs with respect to σ_t^2 and Γ_{t+1} in state s_{t+1}:

$$-\mu_t\omega\frac{N_t^* - B_t^*}{R_t^*} + \delta_t\beta^2 C_{Tt}^2\left(\mathbb{E}_t\Gamma_{t+1}\right)^2 = 0$$

$$\beta\pi_{t+1}\left(1-\gamma\right)\frac{1}{\Gamma_{t+1}}\left(\frac{1}{A_{t+1}\Gamma_{t+1}} - 1\right)$$

$$+ 2\delta_t\pi_{t+1}\left[\left(\beta^2 C_{Tt}^2\sigma_t^2 + \left(\mathbb{E}C_{T,t+1}\right)^2\right)\left(\mathbb{E}_t\Gamma_{t+1}\right) - C_{T,t+1}^2\Gamma_{t+1}\right] = 0.$$

Simplify and rewrite:

$$\delta_t\beta^2 C_{Tt}^2(\mathbb{E}_t\Gamma_{t+1})^2 = \mu_t\omega\frac{N_t^* - B_t^*}{R_t^*},$$

$$\beta\left(1-\gamma\right)\left(\frac{1}{A_{t+1}\Gamma_{t+1}} - 1\right)$$

$$= 2\delta_t\left[\left(\Gamma_{t+1}C_{T,t+1}\right)^2 - \left(\beta^2 C_{Tt}^2\sigma_t^2 + \left(\mathbb{E}_t C_{T,t+1}\right)^2\right)\left(\mathbb{E}_t\,\Gamma_{t+1}\right)\Gamma_{t+1}\right].$$

Next take the expectation $\mathbb{E}_t$ of the second condition and use the definition of σ_t^2 to simplify:

$$\beta\left(1-\gamma\right)\mathbb{E}_t\left(\frac{1}{A_{t+1}\Gamma_{t+1}} - 1\right)$$

$$= 2\delta_t[\mathbb{E}_t(\Gamma_{t+1}C_{T,t+1})^2 - \left(\beta^2 C_{Tt}^2\sigma_t^2 + \left(\mathbb{E}_t C_{T,t+1}\right)^2\right)\left(\mathbb{E}_t\,\Gamma_{t+1}\right)^2\Big] = 0$$

as the RHS corresponds to the definition of σ_t^2. Thus, average output gap is zero, $\mathbb{E}_t X_{t+1} = 1$.

Now substitute out δ_t:

$$\beta\left(1-\gamma\right)\left(\frac{1}{A_{t+1}\Gamma_{t+1}} - 1\right) = \frac{2\omega\mu_t}{\beta^2}\frac{N_t^* - B_t^*}{R_t^*}\left[\frac{(\Gamma_{t+1}C_{T,t+1})^2}{C_{Tt}^2(\mathbb{E}_t\Gamma_{t+1})^2} - \frac{\mathbb{E}_t(\Gamma_{t+1}C_{T,t+1})^2}{C_{Tt}^2(\mathbb{E}_t\Gamma_{t+1})^2}\frac{\Gamma_{t+1}}{\mathbb{E}_t\Gamma_{t+1}}\right],$$

where we used:

$$\mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} = \mathbb{E}_t \frac{W_t}{W_{t+1}} = 1/\left(\beta R_t\right).$$

Rewrite in terms of C_{Nt} and $\mathcal{E}_t$:

$$\beta(1-\gamma)\left(\frac{C_{N,t+1}}{A_{t+1}}-1\right) = \frac{2\omega\mu_t}{\beta^2}\frac{\mathcal{E}_t\left(N_t^* - B_t^*\right)}{R_t^*}$$

$$\left[\frac{\left(\frac{1}{\mathcal{E}_{t+1}}\right)^2}{(\mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}})^2} - = \frac{\mathbb{E}_t\left(\frac{1}{\mathcal{E}_{t+1}}\right)^2}{(\mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}})^2}\frac{\left(\frac{1}{\mathcal{E}_{t+1}C_{T,t+1}}\right)}{\mathbb{E}_t\left(\frac{1}{\mathcal{E}_{t+1}C_{T,t+1}}\right)^2}\right],$$

and further simplify by noting that:

$$\mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} = \mathbb{E}_t \frac{W_t}{W_{t+1}} = 1/\left(\beta R_t\right)$$

$$\beta\gamma C_{N,t+1}\left(X_{t+1}-1\right) = \frac{2\mathcal{E}_t R_t^2 \mu_t \;\; \hat{\Psi}_t}{\sigma_t^2}\left[\frac{C_{T,t+1}}{\mathcal{E}_{t+1}} - \frac{\mathbb{E}_t(1/\mathcal{E}_{t+1})^2}{\mathbb{E}_t 1/\left(\mathcal{E}_{t+1}C_{T,t+1}\right)}\right].$$

Therefore, monetary policy uses variation in output gap X_{t+1} around 1 to increase $C_{N,t+1}$ above A_{t+1} when $C_{T,t+1}$ is particularly high, and vice versa, to reduce the volatility of the exchange rate $\mathcal{E}_{t+1} \propto C_{N,t+1}/C_{T,t+1}$, thus bringing down σ_t^2 and the period t risk-sharing wedge $\hat{\Psi}_t$, in particular in periods where UIP deviations are large to begin with.