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The choice of sovereign-debt maturity in countries at risk of default represents a complex set of competing forces. The tradeoffs reflect the underlying frictions present in international sovereign-debt markets.

The primary frictions are the lack of state contingency in debt contracts and the inability of the government to commit to future actions. These generate two forces in terms of maturity choice. The first is that long-term bonds may be a useful tool for a government to hedge shocks to the cost of funds, say arising from business cycle fluctuations. However, the lack of contingency opens the door to default occurring in equilibrium. Because of the government's inability to commit to future fiscal decisions, bondholders are subject to future dilution of their claims. This generates an opposite force: short-term bonds provide protection from future dilution and, as we shall see, provide better incentives to the government to minimize the costs of default.

This trade-off between insurance and incentives is fundamental to the maturity choice but misses another element. The presence of a

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significant stock of debt in short-maturity by itself generates another potential risk: it leaves the government vulnerable to self-fulfilling rollover crises. This is probably the main drawback of short-term debt —and perhaps the reason why so many restructurings involve lengthening the maturity structure.

In this paper, we explore the advantage to a country of issuing long-maturity debt with a floating-rate coupon. Through both a simple analytical framework, as well as in a richer quantitative framework, we explore the relative benefits of such bonds. We show that having a coupon on a long-term bond indexed to one-period-ahead default probabilities provides all the incentive properties of one-period bonds, without the vulnerability to rollover risk. This can be implemented by indexing the coupon to the auction price of a small amount of one-period bonds.

The framework we explore has both dilution and rollover risk. Dilution risk is well-known in the literature.[1] Aguiar and others (2019) argue that when default risk is high, it is optimal for the government to issue only short-term bonds. This is the case in many real-world crises, as originally documented by Broner and others (2013). Indeed, Bocola and Dovis (2019) argue that the observed shortening of maturity of new issuances of Italian bonds implies a limited role for rollover risk in the European debt crisis. This runs counter to the conventional wisdom that developed in the wake of Mario Draghi's "Whatever it takes" speech in the summer of 2012.

That wisdom holds that the crisis was a self-fulfilling run by creditors that was solved by the European Central Bank stepping in as the lender of last resort.

Rollover risk was a prominent theme after Mexico's 1994–95 crisis. Cole and Kehoe (1996) and Cole and Kehoe (2000) used that crisis as a launching point for their model of rollover risk. Alesina and Tabellini (1990) provide an earlier analysis of self-fulfilling failed auctions. In fact, our discussion of dilution versus rollover risk mirrors that of Alesina and Tabellini (1990), who discuss the experience of floating-rate Italian nominal bonds as the best response to weak inflation credibility and rollover risk.

Aguiar and Amador (2023) provide some evidence of the presence of rollover risk. In particular, they analyze market swaps that involve issuing long-term bonds to repurchase short-maturity bonds. For a case

1. Chatterjee and Eyigungor (2012), Hatchondo and Martínez (2009), Arellano and Ramanarayanan (2012).

involving the Dominican Republic in 2020, they show that the price of *all* bonds increases at the time of the swap, including those of the long-term bonds being issued. They use an analytical framework similar to the one used below to argue that this is evidence that rollover risk is a prominent feature of the data. The environments we study hew fairly close to the quantitative sovereign-debt literature. The main source of risk is endowment risk, to which we add the possibility of a self-fulfilling failed auction. The calibration is based on the benchmark long-term bond paper, Chatterjee and Eyigungor (2012). We find that issuing floating-rate bonds eliminates the risk of a self-fulfilling run while preserving the incentives of one-period bonds. In particular, the government's welfare in the floating-rate bond model in the presence of rollover risk is similar to that of a government with one-period bonds and zero chance of a rollover crisis. Moreover, the floating-rate model dominates the fixed-rate long-term bond model. Welfare gains of switching to floating-rate bonds at zero debt are roughly one percent of consumption. A few caveats are in order to temper these conclusions. One is that we assume the government can auction small amounts of one-period bonds in order to index the coupon payments on the long-term floating-rate bond. This abstracts from liquidity issues in bond markets. Moreover, Alesina and Tabellini (1990) argue that there is evidence that the Italian benchmark-bond auctions may have been manipulated by the government, a possibility we omit from the analysis. Finally, we incorporate the hedging benefits of long-maturity bonds by having persistent income shocks. However, this omits other sources of risk that can be hedged by long-term bonds, such as shocks to risk premia or the risk-free rate.

While we focus on floating-rate bonds, other bond covenants can be used to deal with both dilution and rollover risk. Floating-rate debt is subject to its own source of multiplicity, as studied by Calvo (1988) and, more recently, Ayres and others (2018). Calvo argues that refusing to issue at a high interest rate can help select the best equilibrium. In this spirit, a cap on the coupon can mitigate the risk of this multiplicity, something we also discuss and incorporate in our analysis. Hatchondo and others (2016) discuss covenants that compensate legacy lenders for capital losses as a solution to dilution.

Finally, beyond contract covenants, fiscal rules[2] have been proposed as the solution to dilution, and alternative auction protocols[3] have

2. For example, Hatchondo and others (2012).
3. For example, Chamon (2007).

been proposed to remove rollover risk. The advantage of floating-rate bonds is that they do not require a commitment to enforce fiscal rules or other nonmarket mechanisms; instead, they rely only on competitive markets to deliver the beneficial features.

The paper is organized as follows. Section 1 introduces the general framework absent rollover risk, section 2 provides some analytical results on the efficiency of one-period bonds, section 3 introduces rollover risk, section 4 presents the results of the quantitative exercises, and section 5 concludes.

1. A General Framework

Our framework is based on the standard environment popular in the quantitative sovereign-debt literature.[4] We extend this framework slightly by allowing for floating-rate-coupon bonds. We also alter the model to allow for rollover risk. For expositional reasons, we hold off on the rollover risk extension until after discussing key properties of the baseline model.

Consider a discrete-time, small open economy model. Time is indexed by $t = 0,1,2...$ and the state of nature in time t is given by $s_t \in \mathbb{S}$. The state will index output, default penalties, and, in the extension, include a sunspot that coordinates lenders' beliefs. The state s_t follows a first-order Markov process. In each period, the economy receives a stochastic endowment $y_t = y(s_t)$ that takes values in some discrete, strictly positive, bounded set.

The economy is run by a government with preferences:

$$E\sum_t \beta^t u(c_t),$$

where c_t is consumption of a freely traded good. We assume u is strictly increasing and strictly concave.

The government trades financial assets with competitive, risk-neutral lenders who discount at rate $R^{-1} = (1 + r)^{-1}$. We assume $\beta R \leq 1$. Financial trade is restricted to a noncontingent bond. A bond is characterized by a maturity and a coupon. Each unit of debt matures with probability $\lambda \in [0,1]$, which is *iid* across units. In any

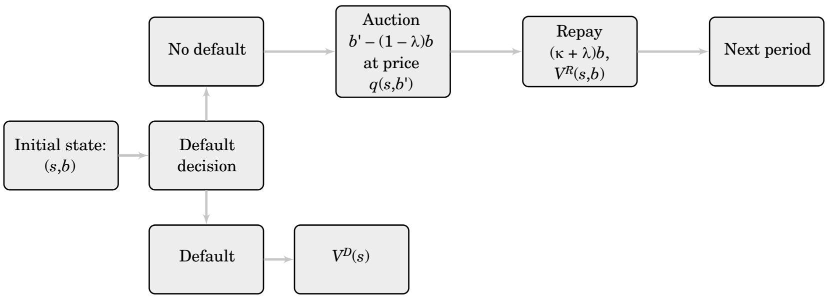
4. See Aguiar and Amador (2021) for a textbook treatment.

nontrivial portfolio, we therefore assume the fraction λ matures and the fraction $1 - \lambda$ remains. The expected maturity is $1/\lambda$. When $\lambda = 1$, we have one-period bonds, and when $\lambda = 0$, we have a perpetuity. Such "perpetual-youth" bonds are a tractable approach to handling bonds of long maturity and have been used by Leland (1994), Hatchondo and Martínez (2009), and Chatterjee and Eyigungor (2012) among others.

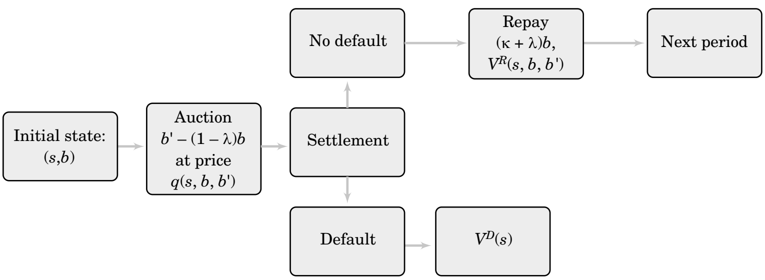
Let $b = b_{t+1}$ be the face value of debt at the end of period t and κ the promised coupon. In $t + 1$ the government owes payments of $(\kappa + \lambda)b$ in every state s_{t+1}. To rule out Ponzi schemes, let $\overline{B}$ denote some arbitrary upper bound on debt issuance and restrict $b \in \mathbb{B} = (-\infty, \overline{B}]$. By making $\overline{B}$ such that promised payments are never greater than the natural debt limit, we ensure it never binds along the equilibrium path, and we will suppress the constraint from the notation going forward.

Figure 1. Within-Period Timing

(a) Eaton-Gersovitz timing



(b) Cole-Kehoe timing



We focus on Markov equilibria, in which equilibrium objects are functions of the exogenous state s_t as well as the government's indebtedness. Let $q : \mathbb{S} \times \mathbb{B} \to [0,1]$ denote the price schedule, and $\mathcal{K} : \mathbb{S} \times \mathbb{B} \to [0, \overline{\kappa}]$ denote the coupon schedule. The coupon is bounded above by a maximum $\overline{\kappa}$, which will be discussed in detail below. For both q and $\mathcal{K}$, the first argument refers to the date on which debt is issued and the coupon is promised, respectively. There is no ex-post contingency in the coupon payment once the state of the next period is realized.

We consider two timing conventions. The first is the "Eaton-Gersovitz" (EG) timing, which is the standard in the literature since Aguiar and Gopinath (2006), Arellano (2008), and Hamann (2002). Under EG timing, depicted in panel (a) of figure 1, the government first observes nature's draw of s, then commits to either repay or default on outstanding debt and then auctions off new bonds. In the alternative, "Cole-Kehoe" (CK) timing, the government, after observing s, first auctions new debt and then decides whether to repay or default on outstanding debt. The key distinction is whether the result of the auction plays a role in the repayment decision. In EG timing, repayment is independent of the realized auction price, while in CK repayment is contingent on the success or failure of a bond auction. We begin by discussing the equilibrium under EG timing.

1.1 The Government's Problem

If the government defaults at time t in state s, we assume it receives value $V^D(s)$. In particular,

$$V^D(s) = u\left(y^D(s), s\right) + \beta \mathbb{E}_s\left[\theta V(s', 0) + (1-\theta) V^D(s')\right]. \tag{1}$$

The term $y^D(s)$ is the endowment received in default when the state is $s \in \mathbb{S}$. This captures any punishment in terms of loss of endowment due to default as well as the fact that the government must consume hand-to-mouth while excluded from financial markets. With probability θ, the government regains access to bond markets and starts anew with zero debt and value $V(s', 0)$ in state s'. With probability $1 - \theta$, the government remains in the default state.

If the government has opted to repay, the government's value satisfies the following Bellman equation:

$$V^R(s,b,\kappa) = \max_{b'} \left\{ u\left(y(s) - (\kappa+\lambda)b + q(s,b')\left(b' - (1-\lambda)b\right)\right) + \beta\mathbb{E}\max\left\{V^R\left(s',b',\mathcal{K}(s,b')\right), V^D(s')\right\}\right. \tag{2}$$

Here, the government takes the schedules q and $\mathcal{K}$ as given and optimally chooses b'. The continuation value reflects that the government has the option to default next period after observing s'. Given that $\kappa = \mathcal{K}(s,b)$ is pinned down in equilibrium by the states, we can redefine the government's value as a function of (s, b) and the lagged state, (s_{-1}). Henceforth, we write $V^R(s_{-1}, s, b)$, with $\kappa = \mathcal{K}(s_{-1}, b)$ being the coupon that is due in the current period.

Let $\mathcal{B}: \mathbb{S}\times\mathbb{S}\times\mathbb{B}\rightarrow\mathbb{B}$ denote the optimal policy function of the government. Implicitly in problem (2), we are assuming that there exists a b' such that it is feasible to repay; that is, $y(s) - (\kappa - \lambda)\, b + q\,(s, b')\,(b' - (1 - \lambda)b) \geq 0$ for some $b' \in \mathbb{B}$. If this is not the case, we set $V^R = -\infty$ so that the government defaults whenever repayment is infeasible.

Define $V(s_{-1}, s, b) \equiv max\{V^R(s_{-1}, s, b), V^D(s)\}$ to be the government's value at the start of the period. The government repays if $V^R(s_{-1}, s, b) \geq V^D(s)$ and defaults otherwise. Let $\mathcal{D}: \mathbb{S}\times\mathbb{S}\times\mathbb{B}\rightarrow\{0,1\}$ denote the optimal default policy, with the value one indicating default and zero indicating repayment.

1.2 The Lenders' Break-Even Condition

The restriction on equilibrium prices is that lenders break even in expectation. In particular:

$$q(s,b) = R^{-1}\mathbb{E}\left[\left(1-\mathcal{D}(s,s',b)\right)\left((\kappa+\lambda) + q\left(s',\mathcal{B}(s,s',b)\right)(1-\lambda)\right)\right], \tag{3}$$

where $\kappa = \mathcal{K}\,(s, b)$.

We consider two alternative coupon schedules. The standard approach is a constant coupon. In particular, define the "fixed-rate coupon schedule" as $\mathcal{K}\,(s, b) = \kappa$ for all $(s, b) \in \mathbb{S}\times\mathbb{B}$ for some constant $\kappa \leq \overline{\kappa}$.

The second is a floating-rate coupon. In particular, consider the equilibrium price of a one-period, zero-coupon bond given the equilibrium behavior of the government:

$$q_S\left(s,b\right) \equiv R^{-1}\mathbb{E}_s\left(1 - D\left(s,s',b\right)\right). \tag{4}$$

Note that q_s lies between zero and R^{-1}. Define the "floating-rate coupon schedule" as:

$$K\left(s,b\right) = min\left\{\frac{1}{q_S\left(s,b\right)} - 1, \bar{\kappa}\right\}. \tag{5}$$

This coupon compensates the bondholder for the one-period-ahead risk of default. It is important to keep in mind that the equilibrium behavior is for an environment with a single bond of inverse maturity λ and coupon $\mathcal{K}$; unless $\lambda = 1$, there is no short-term bond actively traded. Nevertheless, given this equilibrium behavior, we can construct a q_s and $\mathcal{K}$. In particular, q_s is the price that would obtain in equilibrium if an infinitesimal amount of one-period bonds were issued along with the benchmark bonds.

The equilibrium in the floating-rate model depends on $\mathcal{K}$, which, in turn, depends on the default policy function. We are looking for a fixed point for this mapping. There may be more than one, as we discuss at the end of this section.

1.3 Equilibrium

We are now ready to define an equilibrium:

Definition: An Eaton-Gersovitz equilibrium is a price schedule q, a coupon schedule $\mathcal{K}$, a value function V^R with associated policies $\mathcal{B}$ and $\mathcal{D}$, and a default value V^D such that: (i) The lenders' break-even condition (3) is satisfied given $\mathcal{B}$, $\mathcal{K}$, and $\mathcal{D}$; (ii) given $\mathcal{D}$, $\mathcal{K}$ is either fixed or determined by equations (4) and (5); (iii) given q and $\mathcal{K}$, V^R solves the government's Bellman equation (2) with optimal policy $\mathcal{B}$, (iv) $\mathcal{D}\ (s, b, \kappa) = 1$ if $V^R(s, b, \kappa) < V^D\ (s)$ and zero otherwise, and (v) given V^R, V^D, solves the recursion (1).

1.4 Prices and Future Fiscal Policies

The two alternative coupon structures have different implications for how future fiscal policy affects bond prices. Under the fixed-rate schedule, equation (3) indicates that for $\lambda < 1$ the debt-issuance policy function $\mathcal{B}(s', b)$ affects the price of the non-maturing bonds next period and hence affects the price of bonds today. This is the standard channel in which lack of commitment to future fiscal policy potentially 'dilutes' existing bondholders and depresses the value of long-term bonds. We shall return to this below.

Now consider the floating-rate coupon. Suppose that in equilibrium $\mathcal{B}$ is such that there is an upper bound on the ergodic distribution of debt, $B_{max} < \overline{B}$. Moreover, suppose that $q_{min} \equiv \min_{s \in \mathbb{S}} q_S(s, B_{max}) > 0$. That is, along the equilibrium path, the government never issues debt to the point that it will default with probability one the next period. Both of these conditions are typically satisfied in standard quantitative sovereign-debt models. Then, if $\overline{\kappa} > 1 / q_{min} - 1$, a valid equilibrium price schedule is $q(s,b) = 1$ for all $s \in \mathbb{S}$ and $b \le B_{max}$. To see this, define the price operator T_q by equation (3):

$$\begin{aligned}
[T_q q](s,b) &= R^{-1}\mathbb{E}\Big[\big(1-\mathcal{D}(s,s',b)\big)\big(\kappa+\lambda+q\big(s',\mathcal{B}(s,s',b)\big)(1-\lambda)\big)\Big] \\
&= R^{-1}\mathbb{E}\left[\big(1-\mathcal{D}(s,s',b)\big)\left(\frac{1}{q_s(s,b)}-1+\lambda+q\big(s',\mathcal{B}(s,s',b)\big)(1-\lambda)\right)\right] \\
&= 1+(1-\lambda)R^{-1}\mathbb{E}_s\big(1-\mathcal{D}(s,s',b)\big)\Big(q\big(s',\mathcal{B}(s,s',b)\big)-1\Big),
\end{aligned}$$

where the last line uses the definition of q_s. This operator maps bounded functions on the domain $\mathbb{S} \times (-\infty, B_{max}]$ into itself, and satisfies the Blackwell conditions for a contraction. For any $\mathcal{B}$ such that $\mathcal{B}(s_{-1}, s, b) \le B_{max}$ on this domain, $q = 1$ is the unique fixed point of the price operator. In this scenario, the price is constant and, more importantly, independent of future fiscal policy.

As noted above, in the floating-rate case $\mathcal{K}$ is defined by q_s, which in turn depends on equilibrium behavior. The latter depends on $\mathcal{K}$. There may be multiple fixed points of this mapping. This is multiplicity in the spirit of Calvo (1988). In particular, without the upper bound $\overline{\kappa}$, there is an equilibrium with zero borrowing. To see this, posit the schedule $\mathcal{K}(s, b) = \infty$ for all $s \in S$ and $b > 0$. For any $b > 0$, it is infeasible for the government to repay, and hence the government will default with probability one, validating $q_s = 0$ and $\mathcal{K} = \infty$. For this reason, we introduce the cap on coupons to rule out this extreme equilibrium. At this stage, we do not have sufficient conditions to ensure that there is a unique floating-rate equilibrium.

2. One-Period Bonds as a Planning Problem

With long-term fixed-rate bonds, the existing bondholders are at the mercy of future fiscal policy. One-period fixed-rate bonds do not feature this risk. A useful way to see this advantage of one-period bonds is to consider the dual of problem (2), as done in Aguiar and Amador (2019).

Specifically, consider problem (2) for the case of $\lambda = 1$ and normalize $\kappa = 0$. Then (2) can be written as:

$$V^R(s,b) = \max_{c,b'} \left\{ u(c) + \beta \mathbb{E}_s \max\left\{ V^R(s',b'), V^D(s') \right\} \right\}$$
$$\text{subject to } c \leq y(s) - b + q(s,b')b'.$$

Because $\kappa = 0$, we can drop the lagged s_{-1} as an argument for this exercise. As shown by Aguiar and Amador (2019), on the relevant domain for bonds,[5] $V^R(s, b)$ is strictly decreasing in b for each s. Let $B(s, v)$ denote the inverse of V^R. That is,

$$V^R(s, B(s, v)) = v.$$

Given the strict monotonicity of V^R, B solves the dual problem:

$$B(s,v) = \max_{c,b'} \left\{ y(s) - c + R^{-1} b' \mathbb{E}_s \mathbf{1}_{\left\{V^R(s,b') \geq V^D(s')\right\}} \right\}$$
$$\text{subject to } v = u(c) + \beta \max\left\{ V^R(s,b'), V^D(s') \right\},$$

5. By relevant domain, we mean the domain on which the government can feasibly repay. See Aguiar and Amador (2019) for more details.

where 1_x is the indicator function that equals one when x is true and zero otherwise, and where we have used the equilibrium condition $q = R^{-1} 1_{\{V^R \geq V^D\}}$. As $V^R(s, b')$ is strictly decreasing, the choice of b' is also the choice of the government's continuation value. In particular, we can think of adding $v(s')$ as a choice variable subject to the constraint that $v(s') = V^R(s', b')$ for all s' such that $V^R(s', b') \geq V^D(s')$. This constraint is equivalent to $B(s', v(s')) = b'$ for all s' such that $v(s') \geq V^D(s')$. This leads to the following problem:

$$B(s,v) = \max_{c,b',\{v(s')\}} y(s) - c + R^{-1} b' \mathbb{E}_s 1_{\{v(s') \geq V^D(s')\}} \tag{6}$$

subject to:

$$v = u(c) + \beta \mathbb{E}_s \max\{v(s'), V^D(s')\}$$

$$b' = B(s', v(s')) \text{ for all } s' \text{ such that } v(s') \geq V^D(s').$$

Problem (6) is similar to an optimal contracting problem. The principal (lender) chooses a sequence of consumption and continuation values for the agent (the government) subject to a promise-keeping constraint and the 'spanning' condition $b' = B(s', v(s'))$. This last condition restricts the span of continuation values and reflects that the one-period bond is noncontingent.

The spanning constraint contains an equilibrium object (the inverse value function). An alternative maturity structure would involve a different restriction on spanning. It may be the case that long-term bonds allow for better hedging of risk, and a true planning problem will not be constrained from implementing such an allocation.

Aguiar and Amador (2019) note that equation (6) defines an operator that maps B in the spanning constraint into the B that equals the maximized payoff to lenders. They show that this mapping is a contraction and therefore there is a unique equilibrium in the one-period bond model.[6]

Note that the Principal cannot prevent the government from walking away from the contract and taking the outside option V^D.

6.To do this, it is first necessary to relax the spanning condition to an inequality. See that paper for details. In addition, the result requires that there is no re-entry to financial markets after a default, that is, $\theta = 0$; so that v^D is exogenously given. For an alternative contraction mapping approach, see also Bloise and Vailakis (2022).

Nevertheless, absent default, the choice of c and b' maximizes the joint surplus conditional on the spanning condition. In particular, the equilibrium is the same regardless of whether the government or the lenders set fiscal policy, reflecting that incentives are aligned with one-period bonds.

This alignment of incentives is not true for long-term bonds, and we cannot write the long-term bond equilibrium as a pseudo-planning problem like (6). One way to see why not mechanically, is that there are three relevant variables for long-term bonds: the face value of debt b, the government's value v, and the market value of debt $q \times b$. With long-term bonds, the equilibrium q depends on future policies that are beyond the control of current actors (either lenders or the incumbent government). In the one-period bond model, absent default, the market value and face value coincide at the start of the period.

2.1 An Example

To provide a little more insight into why incentives are aligned regarding fiscal policy in the one-period bond model, we shut down the endowment fluctuation; that is, $y(s) = y$ for all $s \in S$. The only risk is the value of default $V^D(s)$, which we allow to vary with the state. Let s be *iid* over time and be such that $V^D(s) = v^D$ is drawn from a continuous distribution with CDF $F(v^D)$ and support $[\underline{V}, \overline{V}]$.

With this *iid* shock process, once the government decides to repay, the realized value of s is irrelevant, and we can drop it as a state variable. That is, $V^R(s, b)$ can be written $V^R(b)$, and its inverse is $B(v)$. In the dual problem, there is a single continuation value v' and the spanning condition becomes $b' = B(v')$. In this case, we can substitute the spanning condition into the objective and use the fact that the government repays if $v^D \leq v'$ to write the dual problem as:

$$B(v) = \max_{c,v'} y - c + R^{-1} F(v') B(v')$$

$$\text{subject to: } v = u(c) + \beta\left(F(v')v' + \int_{v'}^{\overline{V}} v^D dF(v^D) \right).$$

This is a true planning problem, subject to limited participation of the government. The key distinction between this problem and the original (6) is that, without income fluctuations or persistence in the outside option, there is no risk that can be hedged. Bonds of any maturity will either be defaulted on or will have a price that is invariant to v^D conditional on repayment.

The planner's inverse Euler equation for this problem (assuming $v' \in (\underline{V}, \overline{V})$) is:

$$\frac{1}{u'(c')} = \frac{\beta R}{u'(c)} + \frac{f(v')B(v')}{F(v')}, \tag{7}$$

where $f = dF/dv$ and c' is next period's consumption conditional on repayment and the optimal choice v'. To gain some intuition, set $\beta R = 1$ and let $u(c) = log(c)$. We then have

$$c' = c + \frac{f(v')B(v')}{F(v')}.$$

The second term on the right-hand side is the marginal probability of default times the amount of debt. If this is strictly positive, then the optimal plan sets $c < c'$. That is, the optimal plan is to save. And the rate of saving is determined by the marginal decline in default probability. The greater $f(v')/F(v')$, the stronger the incentive to save at the margin. This reflects that the risk to the lender is the amount of debt outstanding times the probability of default. The optimal contract internalizes that saving reduces this risk.

Now recall that the optimal contracting problem is just an alternative view of the equilibrium in which the government makes all decisions. Why does the government want to reduce the risk of default? Keep in mind that the government strategically defaults, so at the moment of default, it captures an increase in value. Why not just wait for a high v^D (say a bailout or forgivable default) and then default?

In equilibrium, it is the price schedule that aligns incentives. Specifically, $q(b') = R^{-1}F(V^R(b'))$. Differentiating:

$$q'(b') = R^{-1} f(v')V^{R'}(b'),$$

where $v' = V^R(b')$. From the envelope condition, $V^R(b') = -u'(c')$ Substituting in, equation (7) becomes:

$$u'(c)\left(1 + \frac{q'(b')b'}{q(b')}\right) = \beta R u'(c').$$

In the equilibrium, the government saves because $q'(b') < 0$, and it understands that, by saving, it will issue/roll over its bonds at a higher price. In particular, the government captures the *entire* benefit of reducing default risk via high prices, and therefore incentives are aligned between borrower and lender to minimize the risk of default.

Now, it is also the case that $q'(b) < 0$ with long-term bonds. However, the government is not rolling over its entire stock of debt. Thus, it does not internalize the entire cost of default to the lender, which involves new bonds as well as legacy bonds, and hence does not capture the entire benefit of reductions in default risk. At the extreme of a perpetuity, the government does not have to roll over any debt and has no incentive to reduce the risk of default. This is the sense that fiscal policy is inefficient with long-term bonds.

2.2 Floating Rate Bonds

With these insights in hand, we can now see one of the advantages of floating-rate bonds. In particular, if the coupon on the entire stock of debt reflects the default probability, the government has the same incentive to save as in the case of one-period bonds.

More formally, consider the case discussed in the previous section in which $q(\mathrm{s}, b) = 1$ in a floating-rate equilibrium for a domain that encompasses the ergodic support, $b \leq B_{max}$. The government's value conditional on repayment is $V^R(s_{-1}, s, b)$. Recall that the original value function was written $V^R(s, b, \kappa)$. For an equilibrium $\mathcal{K}$, we replaced κ with s_{-1}. To construct a pseudo-planning problem, we do not substitute out $\mathcal{K}$ but include it explicitly as a constraint in the dual problem. Specifically, let $B(s, v, \kappa)$ be the inverse of $V^R(s, b, \kappa)$. The government's budget constraint (with $q(\mathrm{s}, b') = 1$) is:

$$c = y(s) - (\kappa + \lambda)b + b' - (1 - \lambda)b.$$

Let $\tilde{B}(s,v,\kappa) = (1+\kappa)B(s,v,\kappa)$ and $\tilde{b} = (1+\kappa)b$. The dual problem becomes:

$$\tilde{B}(s,v,\kappa) = \max_{c,b',\kappa',\{v(s')\}} \left\{ y(s) - c + \frac{\tilde{b}'}{1+\kappa'} \right\} \tag{8}$$

subject to:

$$v = u(c) + \beta \mathbb{E}_s max\left\{v(s'), V^D(s')\right\}$$

$$\tilde{b}' = \tilde{B}(s', v(s'), \kappa') \text{ for all } s' \text{ such that } v(s') \geq V^D(s')$$

$$\kappa' = \mathcal{K}(s, b'),$$

where we have suppressed the ergodic set constraint that $v(s')$ must be such that $b' = B(s', v(s'), \kappa') \leq B_{max}$, as it should not bind in this case.

If we allow the pseudo-planner to "see through" the equilibrium $\mathcal{K}$, we can characterize the best equilibrium with a planning problem that is isomorphic to (6). This resolves the Calvo multiplicity in favor of the efficient outcome. Specifically, recall that

$$\frac{1}{1+\mathcal{K}(s,b')} = R^{-1}\mathbb{E}_s \mathbf{1}_{\{V^R(s,b',\kappa) \geq V^D(s')\}}.$$

Replacing $V^R(s', b', \kappa)$ with $v(s')$, we obtain:

$$\frac{1}{1+\mathcal{K}(s,b')} = R^{-1}\mathbb{E}_s \mathbf{1}_{\{v(s') \geq V^D(s')\}}.$$

Inspection of the value function (8) shows that we can drop κ as an argument of $\tilde{B}$. Let $\overline{B}\,(s, v)$ represent the best possible equilibrium, then we have that $\overline{B}$ solves:

$$\tilde{B}(s,v) = \max_{c,b',\{v(s')\}} \left\{ y(s) - c + R^{-1}b'\mathbb{E}_s \mathbf{1}_{\{v(s') \geq V^D(s')\}} \right\} \tag{9}$$

subject to:

$$v = u(c) + \beta\mathbb{E}_s max\left\{v(s'), V^D(s')\right\}$$

$$\tilde{b}' = \overline{B}(s', v(s')) \text{ for all } s' \text{ such that } v(s') \geq V^D(s').$$

This is the same as problem (6). Thus, conditional on selecting the best equilibrium, the floating-rate bond provides all the same incentive and spanning features as the one-period bonds. The one caveat about the mapping from floating-rate to one-period bonds is the potential for Calvo multiplicity.

3. Rollover Risk

To introduce rollover risk, we alter the timing within a period as in Cole and Kehoe (2000). The government first auctions debt and then decides to repay maturing debt. This timing makes the repayment decision contingent on the outcome of the auction.[7]

We begin with the fixed-rate coupon environment. Working backward through the period, suppose the government has issued $b'-(1-\lambda)$ bonds at price q during the auction. At the time of settlement, the government's value of repayment is:

$$V^R(s,b,b',q) = u\left(y(s) - (\kappa+\lambda)b + q \times \left(b' - (1-\lambda)b\right)\right) + \beta \mathbb{E}_s V(s',b'),$$

where we have repurposed the notation to fit the current environment. We can let s index the price as well, so that $q = q(s, b, b')$ and drop q as an argument of the repayment value.

The default payoff is the same as in the EG benchmark.[8] The government defaults if $V^R(s, b, b') < V^D(s)$. The government's problem at the time of auction is:

$$V(s,b) = \max\left\{\max_{b'} V^R(s,b,b'), V^D(s)\right\}.$$

Note that there is perfect foresight within a period, and hence the government knows what the payoffs to repayment and default are. Let $\mathcal{B}(s,b) = argmax_{b'} V^R(s,b,b')$ denote the debt-issuance policy, and $\mathcal{D}(s,b) = 1$ if $\max_{b'} V^R(s,b,b') < V^D(s)$ and zero otherwise.

To see the indeterminacy in this environment, consider fixing the continuation equilibrium. Specifically, hold the function $\mathbb{E}_s V(s', b')$ constant in the government's problem, as well as future policies. Let $\bar{q}(s, b')$ be the break-even price conditional on repayment in the current period. That is,

$$\bar{q}(s,b') = R^{-1}\mathbb{E}_s\left[\left(1-\mathcal{D}(s',b')\right)\left[\kappa+\lambda+(1-\lambda)q\left(s',b',\mathcal{B}(s',b')\right)\right]\right].$$

Note that this is identical to (3); the only difference is that the policy functions may differ in an environment with rollover risk. This is the 'good' equilibrium.

To see the 'crisis' equilibrium, suppose that $q(s, b, b') = 0$ for all $b' \geq 0$. In this case, for $b' \geq 0$,

$$V^R(s,b,b') = u\left(y(s) - (\kappa+\lambda)b\right) + \beta E V\left(s',(1-\lambda)b\right).$$

7. See panel (b) of figure 1.

8. For simplicity, we assume that if the government auctions debt at a positive price and then defaults, the auction proceeds are lost to both parties. On the equilibrium path, this never occurs.

The government must pay the entire amount of maturing debt plus coupon out of current endowment. It then carries over non-maturing debt into the next period. This is a failed auction. If $u(y(s) - (\kappa + \lambda) b) + \beta \mathbb{E}V(s',(1 - \lambda)b) < V^D(s)$, then a zero price is consistent with the lenders' break-even condition.[9] The lenders see that the government will default at settlement, and refuse to pay a positive price at auction. Such a scenario is possible if $(\kappa + \lambda)b$ is large relative to y.

For pairs of b such that both $q = 0$ and $q = \overline{q}$ are possibilities, following Cole and Kehoe (2000) we let a sunspot coordinate beliefs. That is, s contains a random variable that takes a value of one for a crisis and zero otherwise.

In the case of short-term debt, $\lambda = 1$, the debt burden is particularly painful after a rollover crisis. This is the logic behind why short-term debt makes a government particularly vulnerable to a rollover crisis. Conversely, if $\lambda = 0$, for a given face value the repayment burden is light, and a crisis is possible only for very large b.

This sets up the canonical maturity dilemma. On the one hand, short-term debt provides correct incentives. On the other, it exposes the country to rollover risk, and, perhaps, offers less spanning of income risk. A floating-rate coupon bond provides the same incentives as one-period debt, but defers the maturity payments, mitigating rollover risk. Indeed, if we ignore spanning (as in our simple model without income risk), then the floating-rate perpetuities offer the best of both worlds—correct incentives but limited rollover risk.[10]

The only drawback is that a long-term bond may provide better hedging of income and other potential risks, but this is a quantitative question. In the next section, we therefore turn to a quantitative model that incorporates floating-rate debt and noninsurable income risk.

4. A Quantitative Model

In this section, we introduce income risk as well as rollover risk in a quantitative model. We explore five alternatives: a one-period bond EG model (henceforth EG-ST); a one-period bond model with rollover risk (CK-ST); the same two environments but with long-term

9. We assume the government cannot repurchase long-term bonds at zero price. See Aguiar and Amador (2013) for how this can be supported in equilibrium.

10. A floating-rate equilibrium is constructed in the presence of rollover risk along the same lines as in the benchmark EG model. That is, we price a one-period bond, which now must compensate lenders for rollover risk as well as 'fundamental' default risk and set the coupon to compensate lenders for that risk.

fixed-rate bonds (EG-LT and CK-LT); and finally a long-maturity floating-rate bond (FR) with rollover risk. As we shall see, the long-maturity floating-rate bond eliminates the risk of a rollover crisis, so we do not need to present the floating-rate bond under the Eaton-Gersovitz timing in addition to the Cole-Kehoe timing.

The benchmark parameterization is the same as Chatterjee and Eyigungor (2012) (henceforth, CE12).[11] The model is quarterly. The underlying process for log income follows:

$$\ln y_t = x_t + z_t$$
$$x_t = \rho x_{t-1} + \varepsilon_t$$
$$\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$$
$$z \sim \text{Truncated } N\left(0, \sigma_z^2\right).$$

Following CE12, we set $\rho = 0.95$, $\sigma_\varepsilon = 0.027$ and $\sigma_z = 0.01$. The persistent process x is approximated by Tauchen's method with a span of three standard deviations of the long-run distribution. The *idd* shock z is a truncated Normal with support $[-2\sigma_z, + 2\sigma_z]$, and is included for computational reasons, as discussed by CE12. In default, the endowment is reduced by a quadratic factor. Specifically,

$$\ln y_t^D = x_t^D + z_t$$
$$x_t^D = x_t - \max\left\{0, -0.189 x_t + 0.246 x_t^2\right\}.$$

In the first period of default, we set $z = \underline{z}$, its minimum value.

The government's preferences consist of a constant relative risk aversion felicity with a risk-aversion parameter 2 and a discount factor $\beta = 0.95$. The risk-free interest rate is $R = 1.01$.

The benchmark maturity is $\lambda = 0.05$ or an expected maturity of 20 quarters. For the one-period bond models, we set $\lambda = 1.0$. We set $\kappa = 0.01$ for all models with fixed-rate bonds. And let $\overline{\kappa} = 0.06$ in the baseline specification of the floating-rate bonds.

Finally, in the environments with rollover risk, we set the probability of a sunspot to 10 percent quarterly, although the frequency of crises will be lower in equilibrium. A rollover crisis occurs only if the sunspot is realized and debt is large enough. We assume the

11. The code and additional computational information is available at https://github.com/manuelamador/floating-rate-debt.

probability of a crisis is *iid* over time. See Bocola and Dovis (2019) for a quantitative model in which the probability of a crisis follows a persistent process.

In table 1, we report ergodic moments for the five models, plus an additional floating-rate model in which $\overline{\kappa}$ is set at 0.015, fifty basis points higher than the risk-free net interest rate of 0.01. A few things stand out. One is that with short-term debt, the presence of rollover risk looms large. Comparing EG-ST with CK-ST, debt is much lower in the latter, and one hundred percent of the defaults are due to self-fulfilling runs. With long-term debt, rollover risk is essentially nonexistent, but default is more frequent.

The floating-rate model generates few defaults, with the floating-rate coupon addressing dilution and the long maturity essentially eliminating rollover risk. The corresponding moments for the floating rate and the EG-ST models are very similar.[12] However, in the last column, we report the floating-rate model with $\overline{\kappa} = 0.015$ and the outcome is quite different. The hard cap binds, and this opens the door to dilution risk.

Our focus is on the welfare of the government under alternative arrangements. To evaluate this, we present the value at zero debt for alternative endowments: $V(\cdot, 0)$. In figure 2 panel (a), we plot the value function for the two one-period models (EG-ST and EG-CK) as well as the floating-rate model. The horizontal axis traces out alternative initial endowment states.

The EG-ST and FR values are indistinguishable, while the Cole-Kehoe short-term bond model has a distinctly lower value. The fact that rollover risk lowers welfare is intuitive, particularly with short-maturity bonds. As anticipated by the analytical models, the floating-rate model preserves the good features of the one-period model while eliminating the vulnerability to rollover risk.

In panel (b), we plot the consumption equivalent welfare gain between the CK-ST model and the FR model. For low-endowment states, welfare increases by slightly more than one percent, while for high-endowment states the gain is an order of magnitude less. Recall that welfare is evaluated at zero debt, and hence the likelihood of default (whether fundamental or self-fulfilling) lies well in the future.

12. This result however depends on the value of $\overline{\kappa}$. For example, with a value of 1.0, there are more noted differences between the two models. Therefore, there is an intermediate range of values for $\overline{\kappa}$ for which the environments align.

Table 1. Moments of the Ergodic Distribution

	FR $\bar{\kappa}$ −0.060	*EG-ST*	*CK-ST*	*EG-LT*	*CK-LT*	*FR* $\bar{\kappa}$ −0.015
$\mathbb{E}[\frac{b'}{y}]$	0.82	0.82	0.38	0.94	0.94	0.87
$\mathbb{E}[\frac{q\times b'}{y}]$	0.82	0.82	0.38	0.72	0.72	0.78
Default Rate$^{(*)}$	0.003	0.003	0.002	0.067	0.067	0.033
$\mathbb{E}[r-r^*]^{(*)}$	0.003	0.003	0.002	0.080	0.080	0.038
$StDev[r-r^*]^{(*)}$	0.005	0.004	0.003	0.044	0.044	0.029
$\mathbb{E}\kappa$	0.011	0.010	0.010	0.010	0.010	0.011
$StDev(\kappa)$	0.001	0	0	0	0	0.002
Runs/Defaults	0.087	0	1.000	0	0.003	0.003

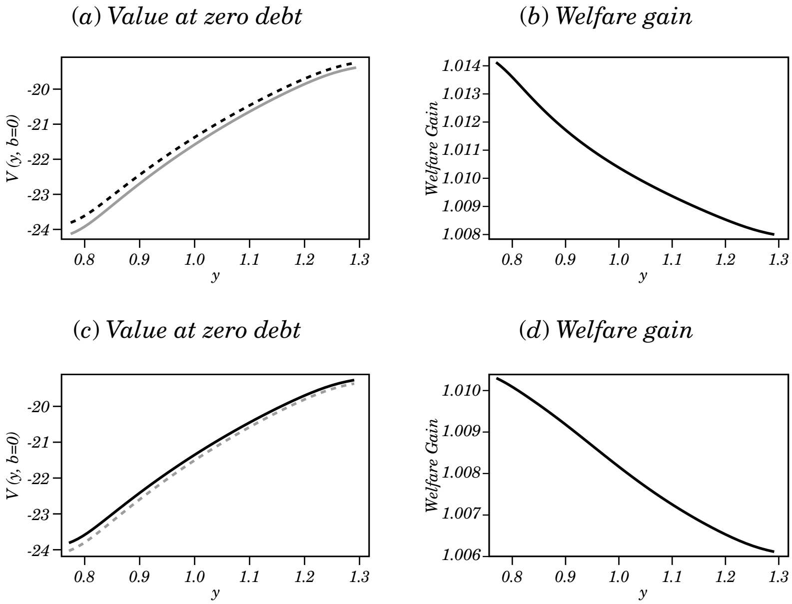
Source: Authors' calculations.
Note: This table reports key moments from the ergodic distribution of each model. All moments are conditional on being in good credit standing for the prior 20 quarters. The first row is the average level of debt issued as a fraction of the endowment. The second row is the average market value of debt issuance, again normalized by the level of endowment. The third row is the annualized frequency of default. The fourth and fifth rows are the mean and standard deviation of implied spreads, respectively. Spreads are computed in annualized form as $(1/q)^4 - R^4$. The sixth and seventh rows are the mean, and standard deviation of the coupon, respectively. The final row is the fraction of defaults that occur due to a self-fulfilling rollover crisis.
(*) Annualized.

In panels (c) and (d) of figure 2 we repeat the same exercises for the long-term bond models. In panel (c), the EG-LT and CK-LT models generate the same value for the government. The reason is that the long-term bonds eliminate the vulnerability to rollover risk. However, the FR model dominates in welfare. This is because the long-term fixed-rate models suffer from dilution risk, something not present with a floating-rate coupon. Panel (d) presents the consumption equivalent welfare gain between FR and CK-LT(=EG-LT). We see that, at low-endowment states, the welfare gain is roughly one percent.

Another approach to evaluating the efficiency of alternative debt instruments is to trace out the frontier between lenders' payoffs and the government's value at different levels of debt. Specifically, consider a state (y_{-1}, y, b). The government's value is $V(y_{-1}, y, b)$, where y_{-1} is an irrelevant state in the fixed-rate environments. The lenders' market value at the start of the period is:

$$MV\left(y_{-1},y,b\right)=\left(1-\mathcal{D}\left(y_{-1},y,b\right)\right)\times b\times\left[\kappa+\lambda+\left(1-\lambda\right)q\left(y,\mathcal{B}\left(y_{-1},y,b\right)\right)\right].$$

Figure 2. Government Welfare



Source: Authors' calculations.
Note: Panel (a) depicts the equilibrium value function at zero debt as a function of current endowment. The solid black line represents the floating-rate bond model, the dashed white line represents the short-term EG model, and the solid gray line represents the short-term CK model. Note that the black and the dashed white lines are identical. Panel (b) represents the consumption-equivalent welfare gain for the government between the floating-rate model and the short-term CK model. Panel (c) repeats panel (a) but with the long-term versions of EG and CK. In this case, the EG and CK models are identical. Panel (d) repeats panel (b) comparing the floating-rate model with the long-term CK model.

The value is zero if the government defaults ($\mathcal{D}(\cdot) = 1$). Otherwise, lenders receive the coupon and principal $(\kappa + \lambda)b$, and the market value of non-maturing debt is $q(y, b') \times b$, where b' is the equilibrium debt-issuance policy.

Figure 3 traces out the frontier between *MV* on the vertical axis and *V* on the horizontal axis as we vary b and hold y and y_{-1} at the mean value. Panel (a) contains the short-term fixed-rate models, and panel (b) the long-maturity environments, with both panels containing the floating-rate case as well.

For each frontier, the point furthest to the left on the horizontal axis is the default value for the government. This point represents all b such that the government defaults and lenders receive zero. Note that the default value varies across environments due to the probability of re-entry. Hence, a lower reentry value lowers the default value.

The remaining points represent positive values for the lenders. In panel (a), we see that the one-period EG model (EG-ST) and the floating-rate model lie on top of each other for this parameterization. The floating-rate frontier depends on the coupon, which in the figure depicted is evaluated at the mean endowment (that is, we assume y_{-1} equals the unconditional mean). The CK-ST bond model is clearly dominated by both. The CK-ST model is prone to rollover risk, which depresses the frontier. However, the low default value (due to the low reentry value) of CK-ST enables the government to sustain lower repayment values without defaulting, thus extending the frontier to the left.

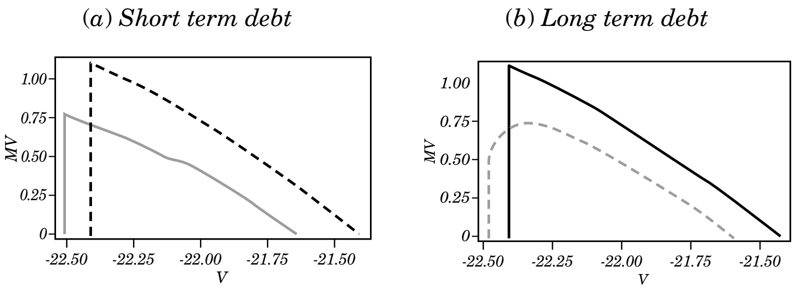
Panel (b) repeats the frontier for the long-term bond model. Recall that in this case, the EG-LT model and the CK-LT model are equivalent, as maturity is such that there is no rollover risk. However, there is the risk of debt dilution. For this reason, the floating-rate frontier dominates the other two. Note that the upward portions of the frontier for the fixed-rate bonds are on the 'wrong' side of the debt Laffer curve. That is, debt forgiveness would increase both lender and government values. This reflects that legacy bondholders are being diluted. Such debt forgiveness is ruled out a priori because it cannot be implemented via voluntary market transactions due to the holdout problem. Hatchondo and others (2014) provide an analysis of negotiated restructurings to alleviate this inefficiency.

5. Conclusion

In this paper we presented analytical and quantitative arguments in favor of long-term bonds with floating-rate coupons. We showed that such bonds combine the incentive properties of one-period bonds with the protection from the rollover risk of a long-term bond. In the presence of rollover and dilution risk, such bonds provide government welfare that dominates both short-term and long-term bonds.

As noted in the introduction, while the analysis includes standard features in the literature, it omits some real-world complications. Perhaps primary among these omissions are the shocks to the global required rate of return.

Figure 3. Pareto Frontiers



Source: Authors' calculations.
Note: This depicts the frontier between lenders' value (vertical axis) and government value (horizontal axis) as b varies, evaluating y and lagged y at the mean. Panel (a) depicts the one-period bond models as well as the floating-rate model. Panel (b) compares the long-term bond models with the floating-rate model. In each panel, the black line is the floating-rate model, the dashed white line is the EG model, and the solid gray line is the CK model.

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