Optimal Policy Rules in HANK

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Banco Central de Chile, November 2022

Inequality & stabilization policy

Does **inequality** change optimal **stabilization policy**? If so, how?

- Recently: increased policy interest & fast-growing academic literature.
  E.g.: Acharya et al. (2020), Bhandari et al. (2021), LeGrand et al. (2021), Davila-Schaab (2022), ...

  a) **Transmission**: how do instruments affect any given target? (e.g., output & inflation)

  b) **Objectives**: desire to dampen distributional effects of business cycle

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- **This paper**: linear-quadratic approximation to optimal policy problem
  - Derive **optimal policy rules** as forecast target criteria, applicable for all shocks
  - Main benefits of our approach:
    1. **Conceptual**: separate role of inequality through transmission vs. objectives
    2. **Practical**: write optimal rules as IRFs to estimable **policy shocks** [McKay-Wolf (2022)]

Main results

a) **Dual mandate** $[\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{\lambda_\pi \widehat{\pi}_t^2 + \lambda_y \widehat{y}_t^2\right\}]$

- Find: optimal rule $\perp$ demand block. Optimal $\{y, \pi\}$ paths are unaffected by inequality.
- In principle r could be different. But scope for inequality to change optimal r is limited by aggregate evidence on the transmission of monetary policy shocks.

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b) **Ramsey policy** $[\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{\lambda_\pi \widehat{\pi}_t^2 + \lambda_y \widehat{y}_t^2 + \text{inequality term}\}]$

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  E.g.: MP is progressive in Bhandari et al. (2022) vs. distributionally neutral in Werning (2015).
- Our strategy: infer distributional incidence from rich quantitative model. Lessons:
  (i) Gains from easy monetary policy are rather evenly distributed. Thus do not find it optimal to deviate much dual mandate prescriptions. Illustrate for shock to bottom of hh distribution.
  (ii) Stimulus checks have *monotone* incidence profile. Highly complementary to monetary policy.

Background

Classical optimal monetary policy literature

RANK literature: **optimal policy design** as linear-quadratic control problem

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- **Policy problem**
  - Second-order approximation to **social welfare function** around efficient steady state:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \lambda_\pi \widehat{\pi}_t^2 + \lambda_y \widehat{y}_t^2 \right\}$$

  - Linear constraint set:

$$\widehat{y}_t = \mathbb{E}_t\left[\widehat{y}_{t+1}\right] - \sigma\left(\widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]\right) + \varepsilon_t^d \qquad \text{(IS)}$$

$$\widehat{\pi}_t = \kappa \widehat{y}_t + \beta \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right] + \varepsilon_t^s \qquad \text{(NKPC)}$$

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- Solution: **optimal policy rule** [implicit = forecast target criterion]

$$\mathbb{E}_0 \left[\lambda_\pi \widehat{\pi}_t + \frac{\lambda_y}{\kappa} \left(\widehat{y}_t - \widehat{y}_{t-1} \right) \right] = 0, \quad \forall t = 0, 1, \ldots$$

The distributional effects of monetary policy

- How does MP affect household balance sheets?
  - Duration: household rate exposure depends on duration of assets vs. liabilities
    Theory in Auclert (2019). Measurement in many recent contributions.
  - Labor income:
    - expansionary MP leaves labor share unchanged/lowers it somewhat
      See Christiano et al. (1997), Cantore et al. (2021). Standard model predicts labor share $\uparrow$.
    - low-income households have more cyclical labor income. See Guvenen et al. (2014)

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- Our **model** will have some non-standard features to speak to these channels:
  1. Households trade capital, long-term bonds, and short-term bonds
     Asset prices will jump in response to shocks, including policy interventions
     $\Rightarrow$ capital gains and losses depend on duration of portfolios
  2. Monopoly profits are re-distributed to ensure a constant labor share

Model Environment

Households

Unit continuum of ex-ante identical households $i \in [0, 1]$

- **Consumption-savings problem**
  - Standard preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_{it}) - \nu\left(\ell_{it}\right)\right]$$

  - Idiosyncratic earnings:

$$e_{it} = \Phi(\zeta_{it}, m_t, e_t), \quad \int_0^1 e_{it} di = e_t$$

where m_t is an "inequality shock" (= demand shock) & e_t is total payments to hh's

  - Budget constraint [a_{it} is value of portfolio]:

$$c_{it} + [\text{cost of asset purchases}] = a_{it} + (1 - \tau_y + \tau_{e,t})e_{it} + \boldsymbol{\tau_{x,t}}, \quad a_{it} \geq \underline{a}$$

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- **Labor supply**: intermediated by labor unions

Production & wage-setting

- **Supply side structure**

  a) **Production**

  - Intermediate goods are produced using capital and labor: $y_{jt} = Ak_{jt}^{\alpha}\ell_{jt}^{1-\alpha}$
  - Subject to nominal rigidities. Pay labor & capital, and earn pure profits. A share $1-\alpha$ of profits goes to labor. From before: hard-wiring of labor share responses.
  - Capital is fixed at $\bar{k}$ with maintenance expenses $\delta \times \bar{k}$

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  1. Standard **NKPC**: $\widehat{\pi}_t = \kappa\widehat{y}_t + \beta\widehat{\pi}_{t+1} + \psi\varepsilon_t$, where ε_t is a cost-push shock.
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⇒ assumptions are chosen to have standard NK supply side but with data-consistent income incidence

Assets

- Households in our economy can trade in **three assets**

  1. Claims to the aggregate capital stock

  $$\frac{\alpha y_{t+1}/\bar{k} - \delta + q^k_{t+1}}{q^k_t}$$

  2. Short-term nominal bonds

  $$\frac{1 + i_t}{1 + \pi_{t+1}}$$

  3. Long-term nominal bonds [coupons decline geometrically at rate $1 - \sigma_b$]

  $$\frac{(\bar{r} + \sigma_b) + (1 - \sigma_b) q^b_{t+1}}{q^b_t (1 + \pi_{t+1})}$$

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- **Returns**
  - In our perfect-foresight economy all assets give the same return r_t at $t = 1, 2, \ldots$
  - But returns can differ at $t = 0$. Asset revaluation effects.

Modeling portfolios

- Budget constraint:

$$c_{it} + \frac{1}{1+t} a_{it+1} = a_{it} + (1 - \tau_y + \tau_{e,t}) e_{it} + \tau_{x,t}, \quad a_{it} \geq \underline{a}$$

- Don't need to model portfolio choice (all assets pay same return for $t = 1, 2, \cdots$)
- Only need existing date-0 portfolios when asset prices respond to news
    - We will impute these using data on portfolio composition across net worth levels Related to approach in Auclert-Rognlie (2020)

Government & eq'm characterization

- Policymaker sets two **policy instruments**
  1. Short-term nominal rate $\boldsymbol{i_t}$ [main focus of the talk]
  2. Uniform lump-sum transfers $\boldsymbol{\tau_{x,t}}$ [for joint monetary-fiscal policy]

  Background: taxes/transfers $\tau_{e,t}$ adjust to keep long-term budget balance.

- **Perfect-foresight eq'm** [notation: boldface = time paths]

Equilibrium

Given paths of shocks $\{m_t, \varepsilon_t\}_{t=0}^{\infty}$ and government policy instruments $\{i_t, \tau_{x,t}\}_{t=0}^{\infty}$, paths of aggregate output and inflation $\{y_t, \pi_t\}_{t=0}^{\infty}$ are part of a linearized equilibrium if and only if

$$\widehat{\boldsymbol{\pi}} = \kappa\widehat{\boldsymbol{y}} + \beta\widehat{\boldsymbol{\pi}}_{+1} + \psi\widehat{\boldsymbol{\varepsilon}} \quad \text{(NKPC)}$$

$$\widehat{\boldsymbol{y}} = \tilde{\mathcal{C}}_y\widehat{\boldsymbol{y}} + \tilde{\mathcal{C}}_\pi\widehat{\boldsymbol{\pi}} + \tilde{\mathcal{C}}_i\widehat{\boldsymbol{i}} + \tilde{\mathcal{C}}_\tau\widehat{\boldsymbol{\tau}}_x + \mathcal{C}_m\widehat{\boldsymbol{m}} \quad \text{(IS*)}$$

Dual Mandate

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  Why? Relevant in practice & allows us to disentangle role of loss function vs. transmission.

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  - **Loss function** [exogenously assumed]

$$\mathcal{L}^{DM} \equiv \sum_{t=0}^{\infty} \beta^t \left[\lambda_\pi \widehat{\pi}_t^2 + \lambda_y \widehat{y}_t^2\right] = \lambda_\pi \widehat{\boldsymbol{\pi}}' W \widehat{\boldsymbol{\pi}} + \lambda_y \widehat{\boldsymbol{y}}' W \widehat{\boldsymbol{y}} \tag{1}$$

where $W = \text{diag}(1, \beta, \beta^2, \dots)$

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    where $W = \text{diag}(1, \beta, \beta^2, \dots)$
  - **Constraint set** [follows from eq'm characterization]
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- **Computation**: trivial, since $\{\tilde{\mathcal{C}}_y, \tilde{\mathcal{C}}_\pi, \tilde{\mathcal{C}}_i, \tilde{\mathcal{C}}_x, \mathcal{C}_m\}$ are easy to get [sequence-space methods]

Optimal policy rule

Proposition

The optimal monetary policy rule for a dual mandate policymaker can be written as the ***forecast target criterion***

$$\lambda_\pi \widehat{\pi}_t + \frac{\lambda_y}{\kappa} \left(\widehat{y}_t - \widehat{y}_{t-1} \right) = 0, \quad \forall t = 0, 1, \ldots \tag{2}$$

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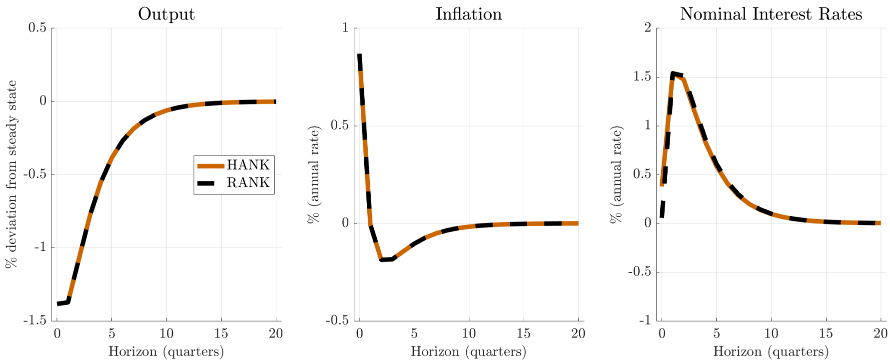
- **Implications**
  - Inequality does not shape *optimal* $\{y, \pi\}$ paths in response to *any* non-policy shock. Thus no difference here between HANK and RANK.
  - Demand block only matters *residually* for i—what sequence of interest rates is needed to achieve the optimal $\{y, \pi\}$ paths?

Quantitative illustration: supply shock



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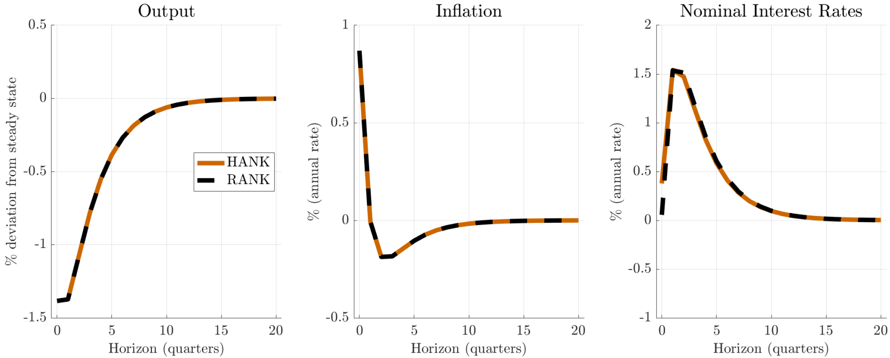

- $\{y, \pi\}$ paths agree exactly. What about **interest rates**?
  - Could in principle disagree substantially. But we have emp. evidence on $i \to \{y, \pi\}$
  - Limiting th'm [McKay-Wolf]: optimal i path can in principle be fully characterized using empirical evidence on the propagation of monetary policy shocks

Ramsey Problem

Social welfare function

- We consider a **social welfare function** with Pareto weights

$$\mathcal{V}^{HA} = \sum_{t=0}^{\infty} \beta^t \int \varphi(\zeta) \left[u\big(\omega_t(\zeta) c_t\big) - \nu\left(\ell_t\right) \right] d\Gamma(\zeta) \tag{3}$$

  - ζ is the idiosyncratic history of a household, $\varphi(\zeta)$ is a Pareto weight on the utility of households with history ζ, and $\omega_t(\zeta)$ is the time-t consumption share of such households

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- Our approach: ensure **efficient steady state** [as in Woodford (2003)] ▸ Formal Discussion
  - Assumptions: production subsidy + back out weights $\varphi(\zeta)$
  - Our SWF will capture cyclical insurance motive, not long-run redistribution

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- **Constraints**
  - (NKPC) and (IS*) are exactly as in the dual mandate problem
  - Asset pricing equations
  - Evolution of consumption shares

$$\widehat{\boldsymbol{\omega}}(\zeta) = \Omega_{\omega(\zeta)} \times \widehat{\boldsymbol{x}} \ \ \forall \zeta, \qquad \boldsymbol{x} \equiv (\boldsymbol{y}, \boldsymbol{i}, \boldsymbol{\pi}, \boldsymbol{\tau}_x, \boldsymbol{\tau_e}, \boldsymbol{m}, q_0^k, q_0^b)$$

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- **Computation**: integrate out ζ's and then aim to stabilize macro aggregates $\boldsymbol{x}$ Details

Ramsey Problem

Optimal Monetary Policy

Optimal Ramsey monetary policy rule

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The optimal monetary policy rule for a Ramsey planner with loss $\mathcal{L}^{HA}$ can be written as the ***forecast target criterion***

$$\lambda_\pi \Theta'_{\pi,i} W \widehat{\boldsymbol{\pi}} + \lambda_y \Theta'_{y,i} W \widehat{\boldsymbol{y}} + \underbrace{\int \lambda_{\omega(\zeta)} \Theta'_{\omega(\zeta),i} W \widehat{\boldsymbol{\omega}}(\zeta) d\Gamma(\zeta)}_{\textit{effects of instrument on consumption shares}} = \mathbf{0}$$

Notation: column k of $\Theta_{\omega(\zeta),i}$ is the response of type-ζ cons. shares to a shock to interest rates at horizon k.

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- So does **inequality** affect the optimal policy rule?
  - No iff policy does not affect consumption shares ($\Theta_{\omega(\zeta),i} = 0$) [e.g. as in Werning (2015)]
  - Yes in prior work: large distributional effects that can offset effects of business-cycle shocks
    Bhandari et al. (2022): rate cut offsets distributional effects of cost-push shock.

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The optimal monetary policy rule for a Ramsey planner with loss $\mathcal{L}^{HA}$ can be written as the ***forecast target criterion***

$$\lambda_\pi \Theta'_{\pi,i} W\widehat{\boldsymbol{\pi}} + \lambda_y \Theta'_{y,i} W\widehat{\boldsymbol{y}} + \underbrace{\int \lambda_{\omega(\zeta)} \Theta'_{\omega(\zeta),i} W\widehat{\boldsymbol{\omega}}(\zeta) d\Gamma(\zeta)}_{\text{effects of instrument on consumption shares}} = \mathbf{0}$$

Notation: column k of $\Theta_{\omega(\zeta),i}$ is the response of type-ζ cons. shares to a shock to interest rates at horizon k.

- So does **inequality** affect the optimal policy rule?
  - What do we know about $\Theta_{\omega(\zeta),i}$?

⇒ **Our strategy**: use data on household balance sheets—in particular labor income & fin. assets—to discipline distributional effects.

Q: What are the main channels through which monetary policy affects inequality?

Empirical evidence on monetary policy and inequality

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1. Income cyclicality: labor income more cyclical for low-income workers

   Guvenen et al. (2014), Aaronson et al. (2019), Andersen et al. (2020), Patterson (2022), ...

   $e_{it} = \Phi(\zeta_{it}, m_t, (1-\alpha)y_t)$ calibrated as in Guvenen et al. (2022) ▸ Figure

Empirical evidence on monetary policy and inequality

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2. Debt service payments: benefits mortgagors and low-net-worth households
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   Adam-Tzamourani (2016), Andersen et al. (2020), Bartscher et al. (2021), Amberg et al. (2021)
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   Doepke-Schneider (2006)

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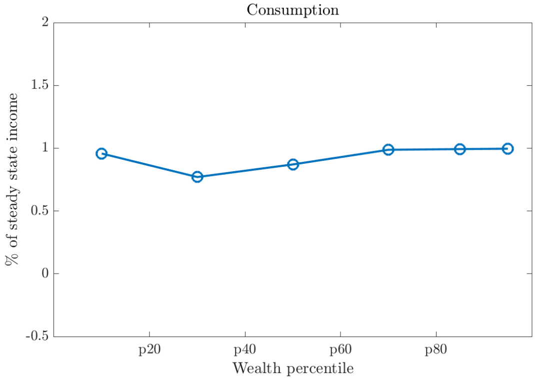
we incorporate all four channels: income via $\Phi(\bullet)$ and financial assets via hh portfolios

Household portfolios

Category	Total	Holdings by net worth group			
		Top 1%	Next 9%	Next 40%	Bottom 50%
Real estate and durables	167	24	48	72	23
Equity and mutual funds	191	101	66	23	2
Currency, deposits, and similar	60	16	23	19	2
Govt. and corp. bonds and similar	29	10	11	7	1
Pension assets	131	6	63	58	4
Mortgage liabilities	49	2	12	24	11
Consumer credit and loans	24	1	2	8	12
Net worth excluding pension assets	374	147	135	89	4
Capital	419	157	135	101	25
Short-term bonds	-12	1	7	-3	-16
Long-term bonds	-33	-11	-8	-9	-5
Total	374	147	135	89	4

▸ More

Consumption inequality in model and data

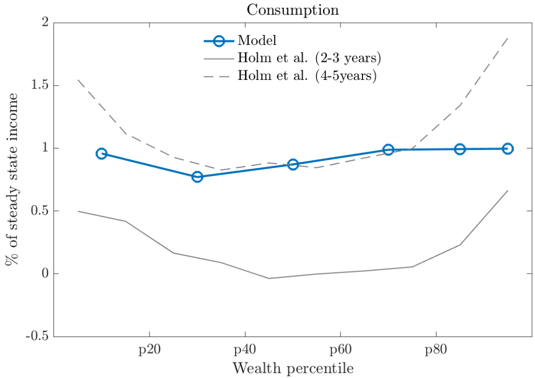


More Calibration Details | Income & wealth distributions

- **Model**
  - Incorporate main distributional channels
  - Then: map into consumption through standard consumption-savings problem

$\Rightarrow$ **find rather small distr. effects**

Consumption inequality in model and data



More Calibration Details Income & wealth distributions

- **Model**
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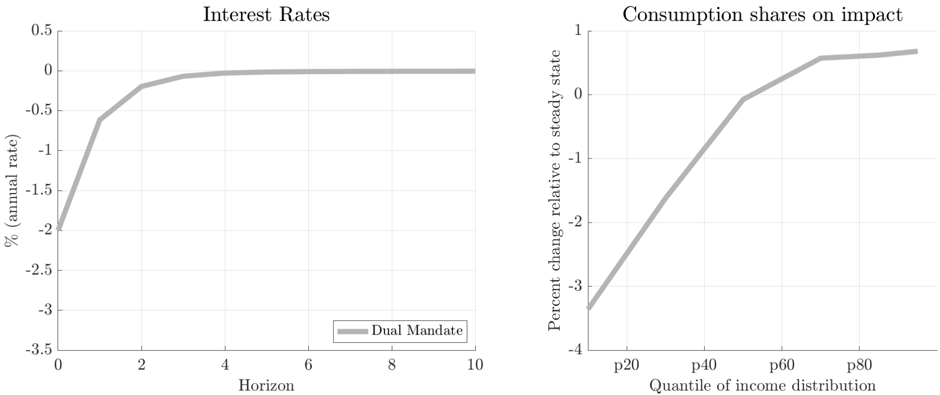
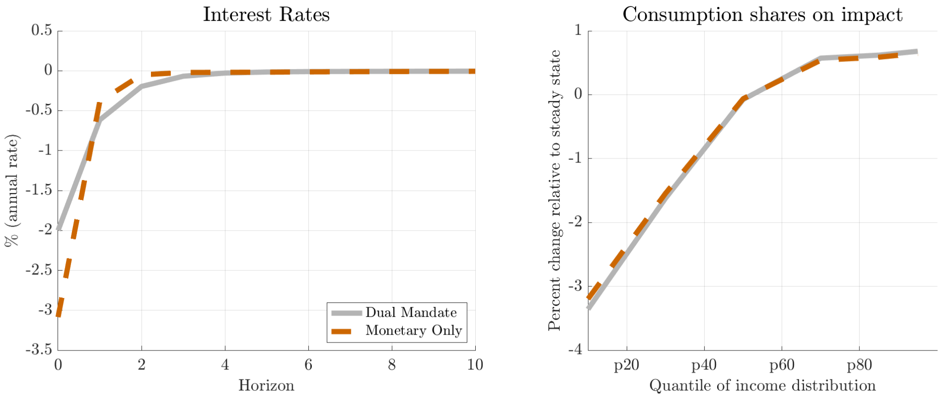
- **Empirical evidence** [Holm-Paul-Tischbirek]

Application: distributional shock



- **Dual mandate**: cut rates to perfectly stabilize aggregate demand and so $\{y, \pi\}$

Application: distributional shock



- **Ramsey policy**: similar, since monetary policy is ill-suited to offset the distr. incidence

  $\Rightarrow$ stabilizing consumption at the bottom would imply large overshooting of y and π ▸ Details

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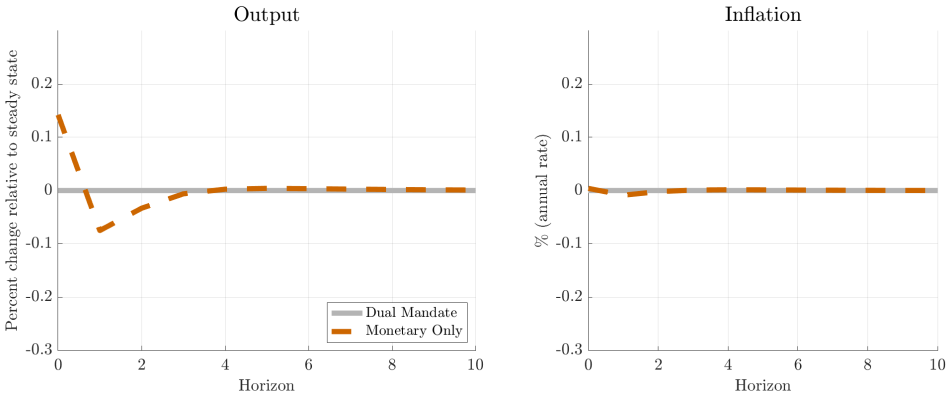

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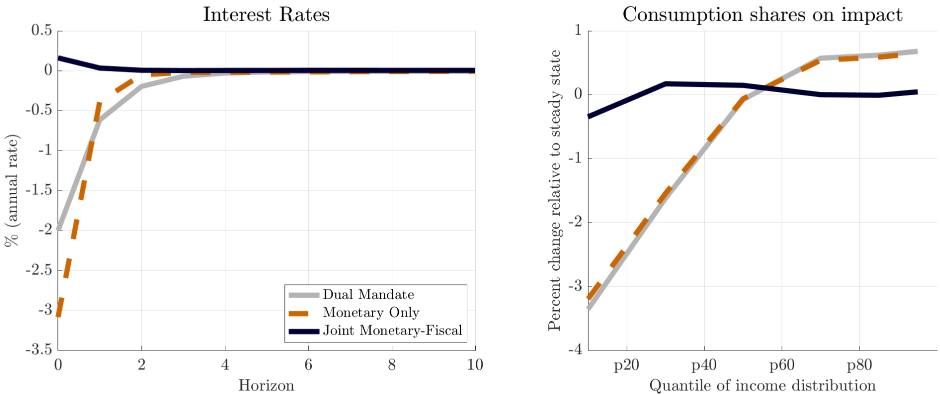
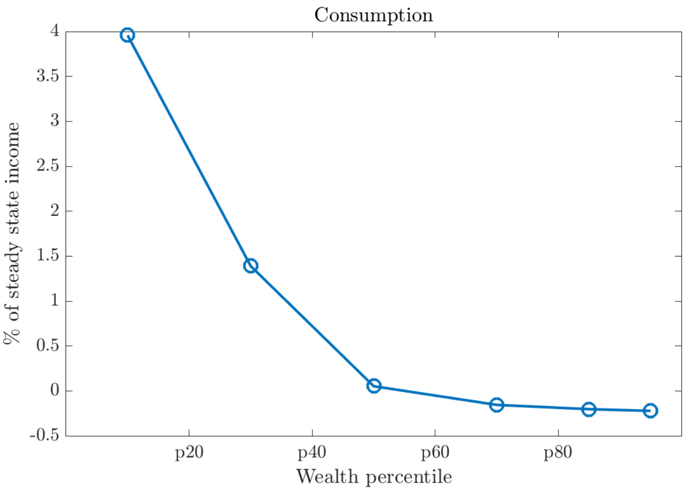
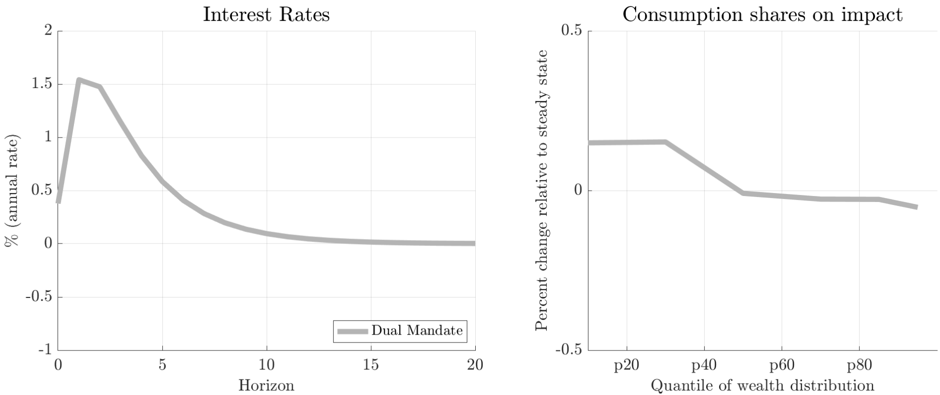


- **Joint fiscal-monetary**: stimulus checks provide agg. & cross-sectional stabilization
  - ⇒ monetary policy at the Ramsey optimum barely responds

Stimulus check incidence

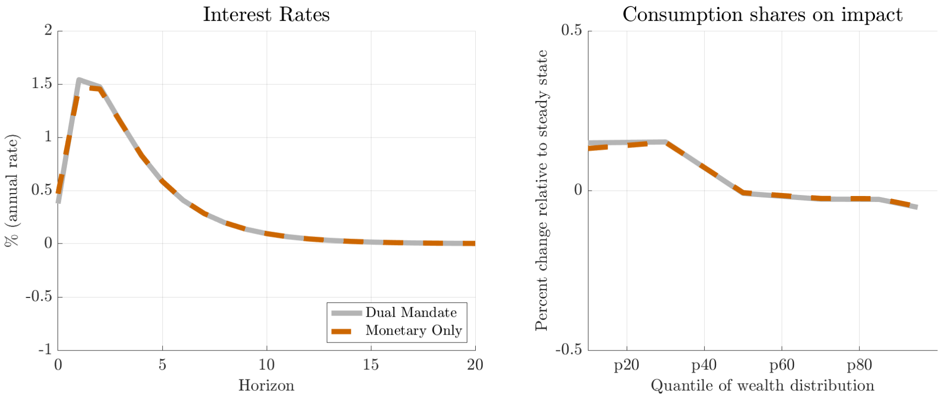


Application: cost-push shock



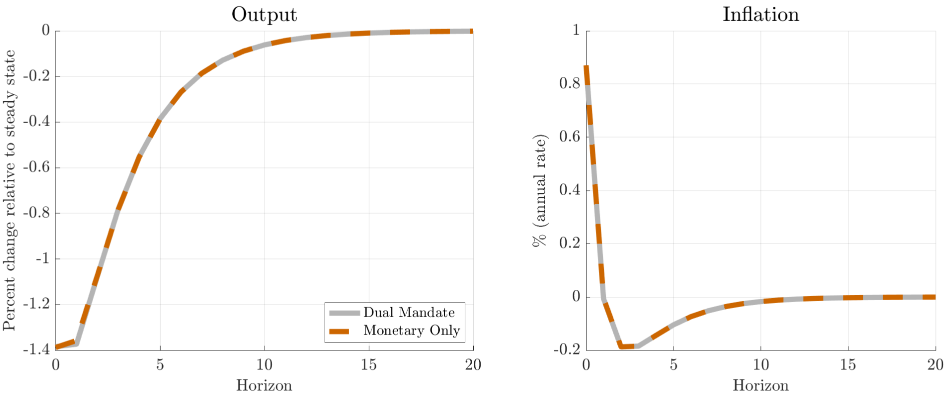
- **Dual mandate**: aggressively lean against cost-push shock to moderate inflation increase

Application: cost-push shock



- **Ramsey policy**: essentially no change—neither the shock nor MP is distributional

Application: cost-push shock



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Conclusions

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  a) **Dual mandate**
     - Same y & π outcomes, possibly different r outcomes (but limited by empirical evidence)
     - For HANK to affect eq'm y & π: loss function or supply side

Conclusions

- How does **inequality** affect **optimal stabilization policy**?

  a) **Dual mandate**
     - Same y & π outcomes, possibly different r outcomes (but limited by empirical evidence)
     - For HANK to affect eq'm y & π: loss function or supply side

  b) **Ramsey policy**
     - Deviate from dual mandate prescription iff monetary policy has (meaningful) distributional effects. That's ultimately an empirical question.
     - Our reading of evidence + model: gains from expansionary MP are quite broad-based
     - Suggests that fiscal policy is much better-suited for targeted cross-sectional stabilization

Appendix

Production block

- Unit continuum of unions k, demand ℓ_{ikt} units from household i. Total union labor supply is $\ell_{kt} \equiv \int_0^1 e_{it}\ell_{ikt} di$.
- Total output is

$$y_t = \left(\int_k \ell_{kt}^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dk \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$$

- The price index of the labor aggregate is

$$w_t = \left(\int w_{kt}^{1-\varepsilon_t} dk \right)^{1/(1-\varepsilon_t)}$$

and demand for labor from union k is

$$\ell_{kt} = \left(\frac{w_{kt}}{w_t} \right)^{-\varepsilon_t} y_t.$$

back

Production block

- Union problem: choose the reset wage w^* and ℓ_{kt} to maximize

$$\sum_{s\geq 0}\beta^s\theta^s\left[u_c(c_{t+s})(1-\tau_y)\frac{\bar{\varepsilon}\Xi}{(\bar{\varepsilon}-1)(1-\tau_y)}\frac{w^*}{p_{t+s}}\ell_{kt}-\nu_\ell\left(\ell_{t+s}\right)\ell_{kt}\right]$$

subject to labor demand constraint

Ξ is subsidy-related steady-state wedge, see loss function proof.

- This gives

$$\widehat{\pi}_t=\kappa\widehat{y}_t+\beta\widehat{\pi}_{t+1}+\psi\widehat{\varepsilon}_t$$

where $\kappa\equiv(\phi+\gamma)\frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\phi\equiv\frac{\nu_{\ell\ell}(\bar{\ell})\bar{\ell}}{\nu_\ell(\bar{\ell})}$ and $\psi\equiv-\frac{\kappa}{(\phi+\gamma)(\epsilon-1)}$

- Aggregating production gives $y_t=\frac{\ell_t}{d_t}$ where $\ell_t\equiv\int_0^1\int_0^1 e_{it}\ell_{ikt}didk$ and d_t captures efficiency losses

back

Equilibrium characterization

- NKPC is as in original optimality condition. Proof combines all other optimality and market-clearing conditions to get (IS*)
- Consumption-savings problem gives aggregate consumption function. Using output market-clearing, $e_{it} w_t \ell_{it} = e_{it} y_t$, we get

$$\widehat{\boldsymbol{y}} = \mathcal{C}_y \widehat{\boldsymbol{y}} + \mathcal{C}_r \widehat{\boldsymbol{r}} + \mathcal{C}_x \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_e \widehat{\boldsymbol{\tau}}_e + \mathcal{C}_m \widehat{\boldsymbol{m}}$$

- Write relationships between asset prices and rates of return as

$$\widehat{r}_0 = r_0(\widehat{\pi}_0, \widehat{y}_0, \widehat{q}_0), \;\; \widehat{\boldsymbol{r}}_{+1} = r_{+1}(\widehat{\boldsymbol{i}}, \widehat{\boldsymbol{\pi}}), \;\; \widehat{\boldsymbol{q}} = q(\widehat{\boldsymbol{\pi}}_{+1}, \widehat{\boldsymbol{y}}_{+1}, \widehat{\boldsymbol{r}}_{+1})$$

- From the government budget constraint we get

$$\widehat{\boldsymbol{\tau}}_e = \tau_e(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{\tau}}_x, \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{q}}).$$

back

Equilibrium characterization

- Plugging the asset pricing and gov't budget relations into the consumption function:

$$\widehat{\boldsymbol{y}} = \mathcal{C}_y \widehat{\boldsymbol{y}} + \mathcal{C}_r \widehat{\boldsymbol{r}}(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{i}}) + \mathcal{C}_x \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_e \widehat{\boldsymbol{\tau}}_e(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{i}}, \widehat{\boldsymbol{\tau}}_x) + \mathcal{C}_m \boldsymbol{m}$$

  and so

$$\widehat{\boldsymbol{y}} = \underbrace{[\mathcal{C}_y + \mathcal{C}_r \mathcal{R}_y + \mathcal{C}_e \mathcal{T}_y]}_{\tilde{\mathcal{C}}_y} \widehat{\boldsymbol{y}} + \underbrace{[\mathcal{C}_r \mathcal{R}_\pi + \mathcal{C}_e \mathcal{T}_\pi]}_{\tilde{\mathcal{C}}_\pi} \widehat{\boldsymbol{\pi}} + \underbrace{[\mathcal{C}_r \mathcal{R}_i + \mathcal{C}_e \mathcal{T}_i]}_{\tilde{\mathcal{C}}_i} \widehat{\boldsymbol{i}} + \underbrace{[\mathcal{C}_x + \mathcal{C}_e \mathcal{T}_x]}_{\tilde{\mathcal{C}}_x} \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_m \boldsymbol{m}$$

- This has verified all eq'm relations, giving sufficiency of (NKPC) and (IS*)

▸ back

Optimal dual mandate rule: proof

- FOCs of optimal policy problem are

$$\begin{aligned} \lambda_\pi W \widehat{\boldsymbol{\pi}} + \Pi_\pi' W \boldsymbol{\varphi}_\pi - \tilde{\mathcal{C}}_\pi' W \boldsymbol{\varphi}_y &= \mathbf{0} \\ \lambda_y W \widehat{\boldsymbol{y}} - \Pi_y' W \boldsymbol{\varphi}_\pi + (I - \tilde{\mathcal{C}}_y') W \boldsymbol{\varphi}_y &= \mathbf{0} \\ -\tilde{\mathcal{C}}_i' W \boldsymbol{\varphi}_y &= \mathbf{0}, \end{aligned}$$

- Guess that $\boldsymbol{\varphi}_y = \mathbf{0}$. Then we get

$$\lambda_\pi \widehat{\boldsymbol{\pi}} + \lambda_y W^{-1} \Pi_\pi' (\Pi_y')^{-1} W \widehat{\boldsymbol{y}} = \mathbf{0}$$

which can re-written to give the stated relation

- Remains to verify the guess that $\boldsymbol{\varphi}_y = \mathbf{0}$

back

Optimal dual mandate rule: proof

- Consider some arbitrary $(\boldsymbol{m}, \boldsymbol{\varepsilon})$, and let $(\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*)$ denote the solution of the system (NKPC) + dual mandate rule given $(\boldsymbol{m}, \boldsymbol{\varepsilon})$
- Plugging into the consumption function:

$$\underbrace{\widehat{\boldsymbol{y}}^* - \tilde{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* - \tilde{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \mathcal{C}_m \boldsymbol{m}}_{\text{demand target}} = \tilde{\mathcal{C}}_i \widehat{\boldsymbol{i}}$$

- Remains to show that we can find $\widehat{\boldsymbol{i}}^*$ such that this relation holds

▸ back

Optimal dual mandate rule: proof

- Supply term has NPV

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \bar{y}\widehat{y}_t$$

- Aggregating household budget constraints we get that

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \bar{c}\widehat{c}_t = \sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \{(1+\bar{r})\bar{a}\widehat{r}_t + (1-\tau_y)\bar{y}\widehat{y}_t + \bar{\tau}_x\widehat{\tau}_{xt} + \bar{\tau}_e\widehat{\tau}_{et}\}$$

  Doing the same for the gov't budget constraint:

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^t \{(1+\bar{r})\bar{a}\widehat{r}_t + \bar{\tau}_x\widehat{\tau}_{xt} + \bar{\tau}_e\widehat{\tau}_{et}\} = \sum_{t=0}^{\infty}\tau_y\bar{y}\widehat{y}_t$$

- Thus the two have the same NPV. Then the stated condition is sufficient to ensure implementability.

back

Ramsey loss function

Proposition

To second order, the social welfare function (3) is proportional to $-\mathcal{L}^{HA}$, *given as*

$$\mathcal{L}^{HA} \equiv \sum_{t=0}^{\infty} \beta^t \left[\widehat{\pi}_t^2 + \frac{\kappa}{\bar{\varepsilon}} \widehat{y}_t^2 + \frac{\kappa \gamma}{(\gamma + \phi)\bar{\varepsilon}} \int \frac{\widehat{\omega}_t(\zeta)^2}{\bar{\omega}(\zeta)} d\Gamma(\zeta) \right] \tag{4}$$

where $\widehat{\omega}_t(\zeta) = \omega_t(\zeta) - \bar{\omega}(\zeta)$ *and* $\bar{\omega}(\zeta)$ *is the steady-state consumption share of an individual with history* ζ.

back

Ramsey planner loss function: proof

- Write planner per-period utility flow as

$$U_t = \int \varphi(\zeta) \frac{\left(\bar{c} e^{\widehat{c}_t} \omega_t(\zeta)\right)^{1-\gamma} - 1}{1-\gamma} d\Gamma(\zeta) - \nu\left(\bar{\ell} e^{\widehat{\ell}_t}\right) \tag{5}$$

- Objective: find 2nd-order approximation to U_t that depends only on 2nd-order terms
- Preliminary definitions
  - Steady state needs to equalize marginal utility of consumption across histories:

$$\varphi(\zeta)\bar{c}^{1-\gamma}\bar{\omega}(\zeta)^{-\gamma} = \bar{u}_c \bar{c} \qquad \forall \zeta$$

  - Imposing that consumption shares integrate to 1 yields

$$\int \varphi(\zeta)^{1/\gamma} d\Gamma(\zeta) = \bar{c}\bar{u}_c^{1/\gamma}$$

back

Ramsey planner loss function: proof

- Preliminary definitions
  - Can recover consumption shares as a function of planner weights:

$$\bar{\omega}(\zeta) = \frac{\varphi(\zeta)^{1/\gamma}}{\int \varphi(\zeta)^{1/\gamma} d\Gamma(\zeta)} \qquad \forall \zeta$$

  - For future reference define

$$\Xi \equiv \left(\int \varphi(\zeta)^{1/\gamma} d\Gamma(\zeta) \right)^{\gamma} = \varphi(\zeta)\bar{\omega}(\zeta)^{-\gamma} \qquad \forall \zeta$$

- Now can begin with first-order terms:
  - For c_t we get

$$\begin{aligned} \frac{\partial U}{\partial \widehat{c}_t} &= \int \varphi(\zeta)(\bar{c}\bar{\omega}(\zeta))^{1-\gamma} d\Gamma(\zeta) \\ &= \bar{c}^{1-\gamma}\Xi \end{aligned}$$

back

Ramsey planner loss function: proof

- Now can begin with first-order terms:
  - For ℓ_t we have
    $$\frac{\partial U}{\partial \widehat{\ell}_t} = -\nu_\ell(\bar{\ell})\bar{\ell}.$$
    Set union subsidy so that $\Xi \bar{c}^{-\gamma} = \nu_\ell$
  - For consumption shares $\omega_t(\zeta)$ we have
    $$\begin{aligned} \frac{\partial U}{\partial \omega_t(\zeta)} &= \varphi(\zeta)\bar{c}^{1-\gamma}\bar{\omega}(\zeta)^{-\gamma} d\Gamma(\zeta) \\ &= \bar{c}^{1-\gamma}\Xi d\Gamma(\zeta) \end{aligned}$$

back

Ramsey planner loss function: proof

- Next consider second-order terms:
  - For level & split of consumption we have

$$
\begin{aligned}
\frac{\partial^2 U_t}{\partial \widehat{c}_t^2} &= (1-\gamma)\Xi \bar{c}^{1-\gamma} \\
\frac{\partial U_t}{\partial \omega_t(\zeta)^2} &= -\gamma \bar{c}^{1-\gamma} \frac{\Xi}{\bar{\omega}(\zeta)} d\Gamma(\zeta) \\
\frac{\partial^2 U_t}{\partial \widehat{c}_t \partial \omega_t(\zeta)} &= (1-\gamma)\Xi \bar{c}^{1-\gamma} d\Gamma(\zeta)
\end{aligned}
$$

  - For hours worked we have

$$
\frac{\partial^2 U}{\partial \widehat{\ell}_t^2} = -\nu_{\ell\ell}(\bar{\ell})\bar{\ell}^2 - \nu_\ell(\bar{\ell})\bar{\ell}
$$

▸ back

Ramsey planner loss function: proof

- We can now put everything together:

$$
\begin{aligned}
U_t \approx\ & \bar{U} + \bar{c}^{1-\gamma}\Xi\widehat{c}_t - \nu_\ell(\bar{\ell})\bar{\ell}\widehat{\ell}_t \\
& + \frac{1}{2}(1-\gamma)\Xi\bar{c}^{1-\gamma}\widehat{c}_t^2 - \frac{1}{2}\left[\nu_{\ell\ell}(\bar{\ell})\bar{\ell}^2 + \nu_\ell(\bar{\ell})\bar{\ell}\right]\widehat{\ell}_t^2 - \frac{1}{2}\gamma\bar{c}^{1-\gamma}\Xi\int\frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)}d\Gamma(\zeta) \\
& + \bar{c}^{1-\gamma}\Xi\int\widehat{\omega}_t(\zeta)d\Gamma(\zeta) + (1-\gamma)\bar{c}^{1-\gamma}\Xi\widehat{c}_t\int\widehat{\omega}_t(\zeta)d\Gamma(\zeta)
\end{aligned}
$$

  Terms in last row are zero.

- Can now write this as

$$
\begin{aligned}
U_t \approx\ & \bar{U} + \bar{c}^{1-\gamma}\Xi\widehat{c}_t - \nu_\ell(\bar{\ell})\bar{\ell}\left(\widehat{c}_t + \widehat{d}_t\right) \\
& + \frac{1}{2}(1-\gamma)\Xi\bar{c}^{1-\gamma}\widehat{c}_t^2 - \frac{1}{2}(\phi+1)\,\nu_\ell(\bar{\ell})\bar{\ell}(\widehat{c}_t + \widehat{d}_t)^2 - \frac{1}{2}\gamma\bar{c}^{1-\gamma}\Xi\int\frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)}d\Gamma(\zeta)
\end{aligned}
$$

back

Ramsey planner loss function: proof

- Set union subsidy so that the $\widehat{c}_t$ terms cancel. We thus have

$$U_t \approx \bar{U} - \nu_\ell(\bar{\ell})\bar{\ell}\widehat{d}_t - \frac{1}{2}\nu_\ell(\bar{\ell})\bar{\ell}(\gamma + \phi)\widehat{y}_t^2 - \frac{1}{2}\gamma\nu_\ell(\bar{\ell})\bar{\ell}\int \frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)}d\Gamma(\zeta)$$

- Finally follow standard steps to express d_t in terms of the history of inflation. After standard steps we get

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t U_t &\approx -\nu_\ell(\bar{\ell})\bar{\ell}\sum_{t=0}^{\infty}\beta^t\left[\frac{\theta\bar{\varepsilon}}{2(1-\theta)(1-\beta\theta)}\widehat{\pi}_t^2 + \frac{1}{2}(\gamma+\phi)\widehat{y}_t^2 + \frac{\gamma}{2}\int\frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)}d\Gamma(\zeta)\right] \\ &= -\frac{\nu_\ell(\bar{\ell})\bar{\ell}\theta\bar{\varepsilon}}{2(1-\theta)(1-\beta\theta)}\sum_{t=0}^{\infty}\beta^t\left[\widehat{\pi}_t^2 + \frac{\kappa}{\bar{\varepsilon}}\widehat{y}_t^2 + \frac{\kappa\gamma}{(\gamma+\phi)\bar{\varepsilon}}\int\frac{\widehat{\omega}(\zeta^t)^2}{\bar{\omega}(\zeta^t)}d\Gamma(\zeta^t)\right],\end{aligned}$$

▸ back

Getting the Ω's: computational details

- Idea: can obtain fluctuations in consumption shares as a function of fluctuations in a small number of inputs to the consumption-savings problem
- Formally, let $\boldsymbol{x} \equiv (\boldsymbol{r}', \boldsymbol{y}', \boldsymbol{\tau}_x', \boldsymbol{\tau}_e', \boldsymbol{m}')'$ be the stacked sequences of inputs to the household problem. Then can show that there is symmetric matrix Q such that

$$\sum_{t=0}^{\infty} \beta^t \int \frac{\widehat{\omega}_t(\zeta, \boldsymbol{x})^2}{\bar{\omega}(\zeta)} d\Gamma(\zeta) = \widehat{\boldsymbol{x}}' Q \widehat{\boldsymbol{x}} + \mathcal{O}(||\widehat{\boldsymbol{x}}||^3)$$

  - Key step is to show that $\widehat{\omega}_t(\zeta, \boldsymbol{x}) \approx \Omega_t(\zeta)\widehat{\boldsymbol{x}}$ which yields

$$\frac{\widehat{\omega}_t(\zeta^t, \boldsymbol{x})^2}{\bar{\omega}(\zeta^t)} = \widehat{\boldsymbol{x}}' \underbrace{\frac{\Omega_t(\zeta^t)'\Omega_t(\zeta^t)}{\bar{\omega}(\zeta^t)}}_{\equiv Q_t(\zeta^t)} \widehat{\boldsymbol{x}} + \mathcal{O}(||\widehat{\boldsymbol{x}}||^3)$$

  and so

$$\sum_{t=0}^{\infty} \beta^t \int \frac{\widehat{\omega}_t(\zeta^t, \boldsymbol{x})^2}{\bar{\omega}(\zeta^t)} d\Gamma(\zeta^t) = \widehat{\boldsymbol{x}}' \underbrace{\left(\sum_{t=0}^{\infty} \beta^t \int Q_t(\zeta^t) d\Gamma(\zeta^t) \right)}_{\equiv Q} \widehat{\boldsymbol{x}} + \mathcal{O}(||\widehat{\boldsymbol{x}}||^3)$$

▸ back

Getting the Ω's: computational details

- We obtain $\Omega_t(\zeta)$ using sequence-space methods + simulation [see paper for details]
- Given Q, we have a finite-dimensional but non-diagonal LQ problem
  - The objective function can be written as
$$\mathcal{L} \equiv \frac{1}{2}\boldsymbol{x}' P \boldsymbol{x},$$
  - We then get the FOC
$$\Theta_{xz}' P \boldsymbol{x} = 0$$
    and the corresponding optimal instrument path
$$\boldsymbol{z}^* \equiv -\left(\Theta_{x,z}' P \Theta_{x,z}\right)^{-1} \times \left(\Theta_{x,z}' P \Theta_{x,\varepsilon} \cdot \boldsymbol{\varepsilon}\right)$$

back

More on model calibration

γ	CRRA	1.2
ϕ	Frisch elasticity	1
β	Discount factor	0.983
κ	Phillips curve slope	0.022
α	Capital share	36%
δ	Depreciation rate	1%
$\underline{a}/\bar{y}$	Borrowing limit	-0.27
δ	Bond duration	0.025

back

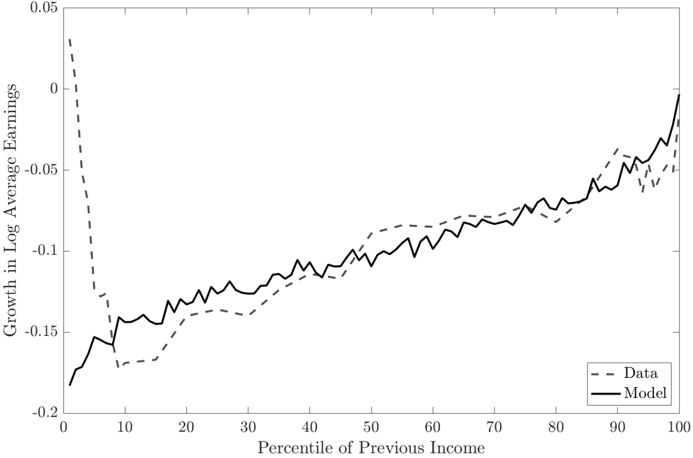
Income and wealth distribution

	Wealth		Income	
	Data	Model	Data	Model
Top 1%	37	30	17	20
Top 5%	65	61	32	37
Top 10%	76	74	43	51
Top 25%	91	93	64	66
Top 50%	99	101	84	82

Table: Shares (%) of wealth and income concentrated in the top x% of the distribution. Data are from the 2019 Survey of Consumer Finance.

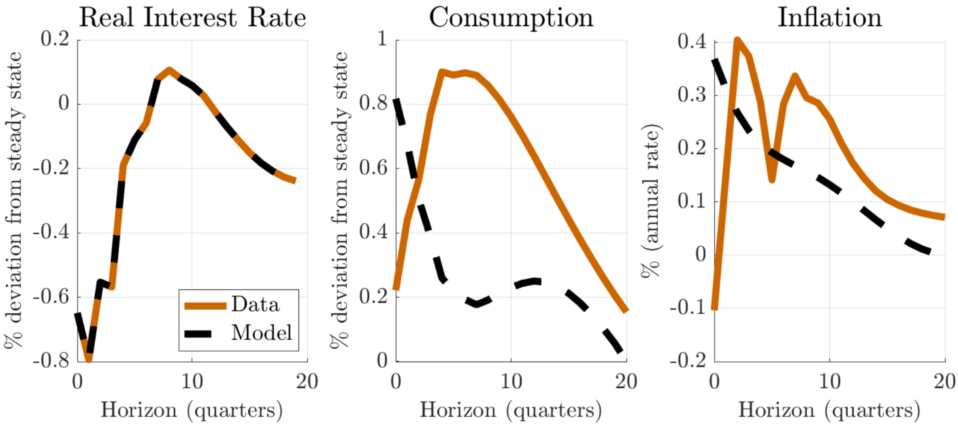
▸ back

Factor structure of Volcker recession



back

Factor structure of Volcker recession



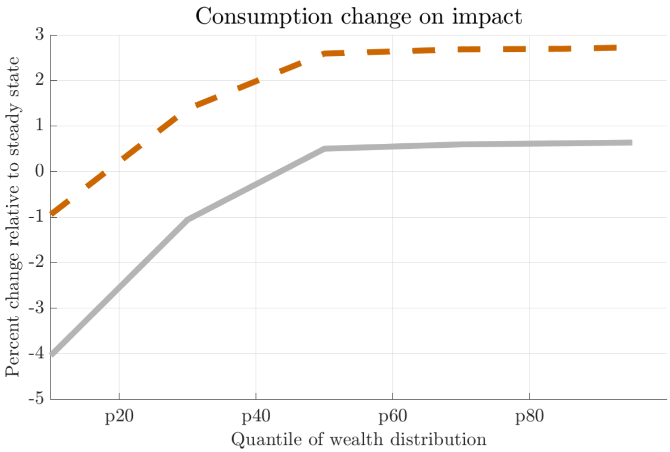
back

Calibration of household portfolios

- **Household portfolios**
  - We classify SCF assets and liabilities into bundles of capital, short-term bonds, and long-term bonds
    - \$1 equity = \$1.32 capital - \$0.20 long-term bonds -\$0.12 short-term bonds
    - \$1 mortgage balance = -\$0.50 long-term bonds -\$0.50 short-term bonds
    - \$1 consumer credit = -\$1 short-term bonds
    - \$1 currency or deposits = \$1 short-term bonds
  - We then impute portfolio for households in our model as a function of their net worth
  - These portfolio positions will matter at date 0, through revaluation effects
- **Pension assets**
  - We treat pensions as part of the government
  - Returns earned on these assets are then paid out slowly through taxes

back

Application: distributional shock



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