A Macro Financial Model for the Chilean Economy

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The Macrofinancial Model (MAFIN) is an ongoing project led by the Monetary Policy and Financial Policy Divisions of the Central Bank of Chile. This minute sketches the first version of the model. As the project progresses, and more results are available, new versions of this document will become available.

1 Introduction

The Global Financial Crisis of 2008-2009 pushed central banks to introduce rich financial sector and detailed financial frictions into the models they used to make forecasting and monetary policy analysis. During the crisis, central banks had to rely on unconventional policy and, although these policies had expansionary effects, the causal quantitative impact remained an open question. The crisis also made indisputable that the financial sector has a prominent role in propagating economic shocks, and is the source of some financial shocks.

These questions led to the advance of DSGE models by introducing a more prominent role for financial frictions and the financial system. As such, Christiano et al. (2010) presented one of the first models in which a medium-scale DSGE model, in the style of Smets and Wouters (2003), is populated with a financial system and financial frictions in the style of Bernanke et al. (1999). Similarly, Gertler and Karadi (2011) developed a quantitative monetary model with constrained financial intermediaries, which is later used to evaluate the effects of unconventional monetary policy during the financial crisis. In the same avenue, Christiano et al. (2015) using an NK model, argued that most of the real economy movements during the great recession were due to financial frictions interacting with the zero lower bound.

Motivated by the need to answer these questions, the Central Bank of Chile introduced the Macro-Financial model, the MaFin model, a DSGE model with financial intermediaries and financial frictions. In this new model, the real sector of this model is a slightly simplified version of the model presented in Garcia et al. (2019)[1] The model is expanded by the introduction of a financial system with financial frictions in the spirit of Clerc et al. (2014), long term bonds in the spirit of Woodford (2001), preferred habitat theory of the term structure as in Vayanos and Vila (2009) and imperfect asset substitution as in Andres et al. (2004).

The decision to develop a model instead of using one of the models from the literature has to do with the notion that these models do not fit the structure of the Chilean economy and do not answer the questions that need to be answered. In particular, Chile is a small open economy with an important commodity-exporting sector that plays a prominent role in government revenues. In addition, the Chilean financial system is mostly formed by a highly regulated classic banking sector which is the primary source of financing to firms in the economy. In addition, the model has to include both short-term and long-term financing in nominal and real terms. These characteristics are not found in the literature and are a crucial component of the domestic financial market.

The Central Bank of Chile is not alone in its quest to introduce a rich financial sector along with financial frictions in a DSGE model. Other central banks have also included these advances into the battery of models they use constantly. Among other uses, these institutions use these models to understand the effects of shocks that originated in the financial sector and the role of financial frictions in the propagation of shocks. Central banks also use these models to understand the role of the financial market in the transmission of monetary policy and to assess the effect of non-conventional policies from a structural perspective. In addition, these models are being used to analyze the financial system's stability and for macro-prudential decision-making, calibration of instruments, and stress testing.

For the Eurozone, the ECB uses the New Area-Wide Model II (NAWM II), Coenen et al. (2018), an extension of the original NAWM that incorporates a rich financial sector, financial frictions, and long term loans. For Norway, the Norges Bank uses the Norwegian Economy Model (NEMO), Motzfeldt Kravik and Mimir (2019), is DSGE

[1]This is DSGE model currently used by the Central Bank of Chile to produce macroeconomic forecasts, alternative scenarios, and for monetary policy analysis.

model with a banking sector, a role for housing services, and house prices, and long term debt. The Banque de France use, among the group of models used to calibrate their macroprudential policy, Clerc et al. (2014) and Gerali et al. (2008) both are DSGE models with a banking sector that gives a central role to capital banking in the transmission of economic shocks. For Switzerland, the Riksbank developed the RAMSES II model, Adolfson et al. (2013) an extension of the original RAMSES model, which now includes financial friction in the style of Bernanke et al. (1999).

The document is structured as follows. In Section 2 we present a detailed description of the theoretical structure of MaFin. Section 3 describes the Bayesian estimation of the model, the calibration, the choice of priors and presents the results. Section 4 concludes.

2 A Small Open Economy Model with Financial Frictions

In the following section, we augment a standard New Keynesian small open economy model with financial frictions in the economy's entrepreneurial, banking, and housing sectors. To do this, we introduce new agents taking Clerc et al. (2014) as starting point: entrepreneurs and bankers. The former are the sole owners of capital, who finance their capital investment through banking loans, while the latter are the owners of the banks who lend resources for capital investment and housing investment.

Households are divided between patients, who save using the financial market, and impatients, who borrow using the financial market. We also introduce the segmented financial markets concept in the spirit of Vayanos and Vila (2009). Following Andres et al. (2004) and Chen et al. (2012), saving households can be unrestricted, who can save in short or long term financial assets, or unrestricted, who can save only in short term assets. All households derive utility from a consumption good, leisure, and housing stock.

From the production side, we use a simplified version of Garcia et al. (2019) in which a final good is produced using capital and labor and facing prices a la Calvo and a labor market facing quadratic adjustment cost in the style of Lechthaler and Snower (2011). In addition, we introduce three kinds of firms (capital producers, housing producers, and banks). Concerning debt, we include not only short-term deposits but also long-term government and bank bonds as perpetuities that pay exponentially decaying coupons introduced by Woodford (2001)

Nota: agregar cosas del default de bancos y empresas

2.1 Households

There are two continuums of households of measure one, risk-averse and infinitely lived. These agents differ in their discount factor: β_I for impatient households (I), and β_P for patient households (P), with $\beta_P > \beta_I$. In equilibrium, impatient households borrow from banks and are ex-ante identical in asset endowments and preferences to others of their same patience.

In terms of patient households, following Andres et al. (2004) and Chen et al. (2012), we allow for a distinction between two types of patient households: Restricted (R) and Unrestricted (U) depending on which assets they can access for saving purposes. While Unrestricted households can buy both long and short-term assets with a transaction cost, Restricted households can only buy long-term bonds but do not face any transaction cost. Their combined measure is of size one.

Restricted and Unrestricted households' preferences depend on consumption of a final good C_t relative to external habits $\tilde{C}_{t-1}$, their stock of housing from last period H_{t-1} relative to external habits $\tilde{H}_{t-2}$, and labor supplied (hours worked) n_t in each period. The consumption of the aggregate good $\hat{C}_t^i \equiv \hat{C}(C_t^i, \tilde{C}_{t-1}^i, H_{t-1}^i, \tilde{H}_{t-2}^i)$ for households of type $i = \{U, R, I\}$ is assumed to be a constant elasticity of substitution (CES) as shown in (1):

$$\hat{C}_t^i = \left[\left(1 - o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(C_t^i - \phi_c \tilde{C}_{t-1}^i\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(H_{t-1}^i - \phi_{hh} \tilde{H}_{t-2}^i\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{1}$$

where $o_{\tilde{C}} \in (0, 1)$ is the weight on housing in the aggregate consumption basket, $\eta_{\tilde{C}}$ is the elasticity of substitution between the final good and the housing good, ξ_t^h is an exogenous preference shifter shock and $\phi_c, \phi_{hh} \geq 0$ are parameters guiding the strength of habits in consumption and housing respectively. Households of type $i = \{U, R, I\}$ maximize the following expected utility

$$\max_{\{\hat{C}_t^i, H_t^i\}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta_i^t \varrho_t \left[\frac{1}{1-\sigma} \left(\hat{C}_t^i\right)^{1-\sigma} - \Theta_t^i A_t^{1-\sigma} \xi_t^n \frac{\left(n_t^i\right)^{1+\varphi}}{1+\varphi} \right] \tag{2}$$

where $\beta_i \in (0,1)$ is the respective discount factor, ϱ_t is an exogenous shock to intertemporal preferences, ξ_t^n is a preference shock that affects the (dis)utility from labor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\varphi \geq 0$ is the inverse elasticity of labor supply.

As in Galí et al. (2012), we introduce an endogenous preference shifter Θ_t, that satisfies the following conditions

$$\Theta_t^i = \tilde{\chi}_t^i A_t^\sigma \left(\hat{C}\left(\tilde{C}_t^i, \tilde{C}_{t-1}^i, \tilde{H}_{t-1}^i, \tilde{H}_{t-2}^i\right)\right)^{-\sigma} \tag{3}$$

and

$$\tilde{\chi}_t^i = \left(\tilde{\chi}_{t-1}^i\right)^{1-v} A_t^{-\sigma v} \left(\hat{C}\left(\tilde{C}_t^i, \tilde{C}_{t-1}^i, \tilde{H}_{t-1}^i, \tilde{H}_{t-2}^i\right)\right)^{\sigma v} \tag{4}$$

where the parameter $v \in [0,1]$ regulates the strength of the wealth effect, and $\tilde{C}_t^i$ and $\tilde{H}_{t-1}^i$ are taken as given by the households. In equilibrium $C_t^i = \tilde{C}_t^i$ and $H_t^i = \tilde{H}_t^i$.

2.1.1 Patient Households

Unrestricted Households. This group is formed by fraction $\wp_U$ of the patient households. In equilibrium, they save in one-period government bond, BS_t^U, long-term government bonds, BL_t^U, short-term bank deposits D_t^U, long-term bank-issued bonds, BB_t^U, and one-period foreign bonds quoted in foreign currency $B_t^{\star U}$. All these assets being non-state-contingent.

The structure of long term financial assets follows Woodford (2001), in this framework, long-term instruments are perpetuities, each paying a coupon of unitary value (in units of final goods) in the period after issuance, and a geometrically declining series of coupons (with a decaying factor $\kappa < 1$) thereafter. That is, a bond issued in period-t implies a series of coupon payments starting in $t+1$: $\{1, \kappa, \kappa^2, \ldots\}$. Also, let B_{t-1}, where $B_{t-1} = \left\{BL_{t-1}^U, BB_{t-1}^U\right\}$ represent the total liabilities due in period t from all past bond issues up to period $t-1$. That is

$$B_{t-1} = CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3} + \ldots,$$

thus, $CI_{t-1} = B_{t-1} - \kappa B_{t-2}$. Let Q_t^B denote the period-t price of a new issue, then Q_t^B summarizes the prices at all maturities. For instance, $Q_{t|t-1}^B = \kappa Q_t^B$ is the price in t of a perpetuity issued in period $t-1$. Importantly, note that B_{t-1} denotes both, total liabilities in period-t from previous debt, and –because of the particular coupon structure– the total number of outstanding bonds. Then, the total value of financial asset debt in period t is given by $Q_t B_t$. Finally, the yield to maturity of holding long term assets at period t, R_t^B, as,

$$R_t^B = \frac{P_t}{Q_t^B} + \kappa$$

Unrestricted households must pay a transaction cost ζ_t^L per unit of long-term bond purchased. This costs is paid to a financial intermediary as a fee. This financial intermediary distributes its nominal value profits Π^{FI}, as dividends to its shareholders. Then, unrestricted patient households' period budget constraint is

$$\begin{aligned} BS_t^U + \left(1+\zeta_t^L\right) Q_t^{BL} BL_t^U + D_t^U + \left(1+\zeta_t^L\right) Q_t^{BB} BB_t^U + S_t B_t^{\star U} + P_t C_t^U + Q_t^H H_t^U = \\ R_{t-1} BS_{t-1}^U + Q_t^{BL} R_t^{BL} BL_{t-1}^U + \tilde{R}_t^D D_{t-1}^U + \tilde{R}_t^{BB} Q_t^{BB} BB_{t-1}^U + S_t B_{t-1}^{\star U} R_{t-1}^\star + W_t n_t^U \\ + Q_t^H \left(1-\delta_H\right) H_{t-1}^U + \Psi_t \end{aligned} \tag{5}$$

where R_t^{BL} and R_t^{BB} are the gross yield to maturity for long-term government and bank-issued bonds at time t, P_t denotes the price of the consumption good, Q_t^H denotes the price of housing good, δ_H is the depreciation rate of housing, S_t denotes the nominal exchange rate (units of domestic currency per unit of foreign currency), and $R_t^\star$ denotes the the foreign one-period bond and R_t denotes de short term nominal government bond.

Further, $\widetilde{R}_t^D = R_{t-1}^D(1-\gamma_D PD_t^B)$, $\widetilde{R}_t^{BB} = R_t^{BB}(1-\gamma_{BB} PD_t^B)$ denote the net return on resources loaned to banks in the form of deposits and bank-issued bonds, R_t^D is the gross interest rate received at t on the bank deposits at $t-1$, and R_t^{BB} is the gross return of saving on long term bank bonds, PD_t^B denotes the fraction of resources in banks that fail in period t and $\gamma_D(\gamma_{BB})$ is a linear transaction cost that households must pay in order to recover their funds. Finally, W_t denotes the nominal wage and, Ψ_t denotes lump-sum payments that include taxes T_t, dividend income from entrepreneurs C_t^e, bankers C_t^b, rents from ownership of foreign firms REN_t^* profits from ownership of domestic firms and profits from the financial intermediary in the long-term bond transactions, $\Pi^F = \zeta_t^L(Q_t^{BL} BL_t^U + Q_t^{BB} BB_t^U)$.

Chen et al. (2012) show that the discounted value of future transaction costs implies a term premium. We assume that the period transaction cost is a function of the ratio of the aggregate market value of long-term to short-term assets and a disturbance term. Further, households do not internalize the effect of their choices on this transaction cost, yet in equilibrium $\widetilde{BL}_t^U = BL_t^U$ and $\widetilde{BS}_t^U = BS_t^U$. This ratio captures the idea that holding long-term debt implies a loss of liquidity that households hedge by increasing the amount of short-term debt. Specifically, the functional form is given by

$$\zeta_t^L = \left(\frac{Q_t^{BL}\widetilde{BL}_t^U + Q_t^{BB}\widetilde{BB}_t^U}{\widetilde{BS}_t^U + S_t\widetilde{B}_t^{*U} + \widetilde{D}_t^U} \right)^{\eta_{\zeta_L}} \epsilon_t^{L,S} \tag{6}$$

Households supply differentiated labor services to a continuum of unions which act as wage setters on behalf of the households in monopolistically competitive markets. The unions pool the wage income of all households and then distribute the aggregate wage income in equal proportions among households, hence, they are insured against variations in household-specific wage income. [2] Defining for convenience the multiplier on the budget constraint as $\frac{\lambda_t^U A_t^{-\sigma}}{P_t}$, then, Unrestricted Households solve (2) subject to (1), (3), (4), and (5). From this problem we obtain the following first-order conditions:

$$[C_t^U]: \quad \lambda_t^U A_t^{-\sigma} = \left(\hat{C}_t^U\right)^{-\sigma} \left(\frac{(1-o_{\hat{C}})\,\hat{C}_t^U}{\left(C_t^U - \phi_c \tilde{C}_{t-1}^U\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{7}$$

$$[H_t^P]: \quad \varrho_t \frac{\lambda_t^U A_t^{-\sigma} Q_t^H}{P_t} = \beta_U \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{C}_{t+1}^U\right)^{-\sigma} \xi_{t+1}^h \left(\frac{o_{\hat{C}} \hat{C}_{t+1}^U}{\xi_{t+1}^h \left(H_t^U - \phi_{hh}\tilde{H}_{t-1}^U\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} + (1-\delta_H)\frac{\lambda_{t+1}^U A_{t+1}^{-\sigma} Q_{t+1}^H}{P_{t+1}} \right\} \tag{8}$$

$$[BS_t^U]: \quad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U R_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda_{t+1}^U}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \tag{9}$$

$$[BL_t^U]: \quad \varrho_t \lambda_t^U A_t^{-\sigma} \left(\frac{1+\zeta_t^L}{R_t^{BL} - \kappa_B} \right) = \beta_U \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda_{t+1}^U}{\pi_{t+1}} \left(\frac{R_{t+1}^{BL}}{R_{t+1}^{BL} - \kappa_B} \right) A_{t+1}^{-\sigma} \right\} \tag{10}$$

$$[B_t^{\star U}]: \quad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U R_t^{\star} \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda_{t+1}^U \pi_{t+1}^s}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \tag{11}$$

$$[D_t^U]: \quad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda_{t+1}^U}{\pi_{t+1}} \tilde{R}_{t+1}^D A_{t+1}^{-\sigma} \right\} \tag{12}$$

$$[BB_t^U]: \quad \varrho_t \lambda_t^U A_t^{-\sigma}(1+\zeta_t^L) Q_t^{BB} = \beta_U \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda_{t+1}^U}{\pi_{t+1}} \tilde{R}_{t+1}^{BB} A_{t+1}^{-\sigma} Q_{t+1}^{BB} \right\} \tag{13}$$

In equilibrium, we have that $\tilde{C}_t^P = C_t^P$ and $\tilde{H}_t^P = H_t^P$, which applies for impatient households as well. The implied discount factor for nominal claims is, by iterating upon (9):

$$r_{t,t+s} = \frac{1}{\prod_{i=0}^{s-1} R_{t+i}} = \beta_U^s \frac{\varrho_{t+s}\lambda_{t+s}^U A_{t+s}^{-\sigma} P_t}{\varrho_t \lambda_t^U A_t^{-\sigma} P_{t+s}} \tag{14}$$

Restricted households. This group of households have a mass $\wp_R$ which complements the mass of unrestricted households $\wp_U$, then $\wp_R = 1 - \wp_U$. The main difference with Unrestricted Household is that can only access long-term financial instruments, and thus save and borrow by purchasing domestic currency denominated long-term government bonds, BL_t^R, and bank bonds, BB_t^R. In addition, Restricted Patient Household do not face transaction costs. They are subject to the period-by-period budget constraint

$$P_t C_t^R + Q_t^H H_t^R + Q_t^{BL} BL_t^R + Q_t^{BB} BB_t^R = W_t n_t^R + Q_t^H (1-\delta_H) H_{t-1}^R + Q_t^{BL} R_t^{BL} BL_{t-1}^R + Q_t^{BB} R_t^{BB} BB_{t-1}^R \tag{15}$$

[2] The explanation of how households decide how much labor to supply and is reserved for section 2.4.9.

Let us define, for convenience, the multiplier on the budget constraint as $\frac{\lambda_t^R A_t^{-\sigma}}{P_t}$. Then, Restricted Households solve (2) subject to (1), (3), (4), and (15), from which we obtain the following first-order conditions:

$$[C_t^R]: \quad \lambda_t^R A_t^{-\sigma} = \left(\hat{C}_t^R\right)^{-\sigma} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{C}_t^R}{\left(C_t^R - \phi_c \tilde{C}_{t-1}^R\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{16}$$

$$[H_t^P]: \quad \varrho_t \frac{\lambda_t^R A_t^{-\sigma} Q_t^H}{P_t} = \beta_R \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{C}_{t+1}^R\right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{C}_{t+1}^R}{\xi_{t+1}^h \left(H_t^R - \phi_{hh} \tilde{H}_{t-1}^R\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h \right. \tag{17}$$

$$\left. + \left(1 - \delta_H\right) \frac{\lambda_{t+1}^R A_{t+1}^{-\sigma} Q_{t+1}^H}{P_{t+1}} \right\}$$

$$[BL_t^R]: \quad \varrho_t \lambda_t^R A_t^{-\sigma} Q_t^{BL} = \beta_R \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^R}{\pi_{t+1}} R_{t+1}^{BL} Q_{t+1}^{BL} A_{t+1}^{-\sigma} \right\} \tag{18}$$

$$[BB_t^R]: \quad \varrho_t \lambda_t^R A_t^{-\sigma} Q_t^{BB} = \beta_R \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^R}{\pi_{t+1}} R_{t+1}^{BB} Q_{t+1}^{BB} A_{t+1}^{-\sigma} \right\} \tag{19}$$

2.1.2 Impatient Households

Impatient households work, consume, and purchase housing goods. In addition, they take long-term loans in equilibrium from banks to finance their purchases of housing goods, which we model using the same structure presented in the previous section.

We follow the Clerc et al. (2014) by assuming that these mortgage loans are non-recourse and limited liability contracts, which enables the possibility of default for households. For the household, the only consequence of default is losing the housing good on which the mortgage is secured, therefore default is optimal when the value of the total outstanding debt is higher than the value of the assets, $R_t^I Q_t^L L_{t-1}^H > \omega_t^I Q_t^H \left(1 - \delta_H\right) H_{t-1}^I$. limited-liability. Then the impatient household budget constraint is given by:

$$P_t C_t^I + Q_t^H H_t^I - Q_t^L L_t^H = W_t n_t^I + \int_0^\infty \max\left\{\omega_t^I Q_t^H (1 - \delta_H) H_{t-1}^I - R_t^I Q_t^L L_{t-1}^H, 0\right\} dF_I(\omega_t^I) \tag{20}$$

Define ω_t^I as an idiosyncratic shock to the efficiency units of housing of impatient households, which can be interpreted as a reduced-form representation of any shock to the value of houses. The shock ω_t^I is i.i.d. across households and follows a log-normal distribution with pdf $f_I\left(\omega_t^I\right)$ and cdf $F_I\left(\omega_t^I\right)$.

After the realization of aggregate and idiosyncratic shocks individual households decide whether to default, and then the resulting net worth is distributed evenly across members of this type, which optimally decide to choose the same debt, consumption, housing and hours worked. Let

$$R_t^H = \frac{Q_t^H \left(1 - \delta_H\right)}{Q_{t-1}^H}.$$

Then, in order for the impatient household to pay for its loan, the idiosyncratic shock ω_t^I must exceed the threshold

$$\bar{\omega}_t^I = \frac{R_t^I Q_t^L L_{t-1}^H}{R_t^H Q_{t-1}^H H_{t-1}^I} = \frac{x_t^I}{R_t^H}$$

If $\omega_t^I \geq \bar{\omega}_t^I$ the household pays liabilities due in the period t in the amount $R_t^I Q_t^L L_{t-1}^H$, and rolls over remaining outstanding value of debt, $\kappa Q_t^L L_{t-1}^H$, to obtain positive net worth, $(\omega_t^I - \bar{\omega}_t^I) Q_t^H (1 - \delta_H) H_{t-1}^I$. Otherwise, the household debt becomes non-perming, defaults and receives nothing. On the other hand, the bank receives $R_t^I Q_t^L L_{t-1}^H$ from performing loans, but it only recovers $(1 - \mu_I)\omega_t^I R_t^H Q_{t-1}^H H_{t-1}^I$ from non performing loans. With the definition of the $\bar{\omega}_t^I$ threshold, we can define $PD_t^I = F_I\left(\bar{\omega}_t^I\right)$ as the default rate of impatient households on their housing loans. Note that these defaults are over the value of all loans outstanding, $Q_t^L L_{t-1}^H$.

Out of all the loans, the share of the gross return that goes to the bank is denoted as $\Gamma_I\left(\bar{\omega}_t^I\right)$ whereas the share of gross return that goes to the impatient household is $\left(1-\Gamma_I\left(\bar{\omega}_t^I\right)\right)$ where:

$$\Gamma_I\left(\bar{\omega}_t^I\right)=\int_0^{\bar{\omega}_t^I}\omega_t^I f_I\left(\omega_t^I\right)d\omega_t^I+\bar{\omega}_t^I\int_{\bar{\omega}_t^I}^{\infty}f_I\left(\omega_t^I\right)d\omega_t^I$$

The first integral on the right denotes the share of the return that is defaulted while the second integral denotes the share of return that is paid in full. This allows us to rewrite the budget condition from (20) as

$$P_tC_t^I+Q_t^HH_t^I-Q_t^LL_t^H=W_tn_t^I+\left[1-\Gamma_I\left(\bar{\omega}_t^I\right)\right]R_t^HQ_{t-1}^HH_{t-1}^I \tag{21}$$

Also, let

$$G_I\left(\bar{\omega}_t^I\right)=\int_0^{\bar{\omega}_t^I}\omega_t^I f_I\left(\omega_t^I\right)d\omega_t^I$$

denote the part of those returns that comes from the defaulted loans. Taking into consideration the share of the return that is lost due to verification cost as $\mu_IG_I\left(\bar{\omega}_t^I\right)$, then the net share of return that goes to the bank is

$$\Gamma_I\left(\bar{\omega}_t^I\right)-\mu_IG_I\left(\bar{\omega}_t^I\right).$$

The terms of the loan must imply the net expected profits of the bank must equal its alternative use of funds, therefore it must satisfy a participation constraint:

$$\mathbb{E}_t\left\{\left[1-\Gamma^H\left(\bar{\omega}_{t+1}^H\right)\right]\left[\Gamma_I\left(\bar{\omega}_{t+1}^I\right)-\mu_IG_I\left(\bar{\omega}_{t+1}^I\right)\right]R_{t+1}^HQ_t^HH_t^I\right\}\geq\rho_{t+1}^H\phi_HQ_t^LL_t^H \tag{22}$$

Where $\Gamma^H\left(\bar{\omega}_{t+1}^H\right)$ is the fraction of bank gross returns that is used to pay depositors or is lost due to bank defaults when their own idiosyncratic shock ω_{t+1}^H is too low. The rest of the left hand side expression is the total amount of returns on the housing project that goes to the lender bank. The right hand side indicates the opportunity cost, which is investing an amount of equity $\phi_HQ_t^LL_t^H$ at a market-determined rate of return of $\tilde{\rho}_{t+1}^H$, where ϕ_H is a regulatory capital constraint. We elaborate on the bank's problem on subsection 2.3, for now note that we can write (22) with equality without loss of generality.

Thus, following the timing described above, the impatient household's optimization problem can be written as maximizing (2) for $i=I$ subject to their budget constraint (21) and the bank participation constraint (22). For this, define for convenience $\frac{\lambda_t^IA_t^{-\sigma}}{P_t}$ and $\frac{\lambda_t^HA_t^{-\sigma}}{P_t}$ as the multipliers for each constraint respectively. Define also $x_t^I\equiv\frac{R_t^IL_t^H}{Q_t^HH_t^I}$, a measure of household leverage. This yields the following FOC's:

$$[C_t^I]:\quad\lambda_t^IA_t^{-\sigma}=\left\{\left(\hat{C}_t^I\right)^{-\sigma}\right\}\left(\frac{\left(1-o_{\hat{C}}\right)\hat{C}_t^I}{\left(C_t^I-\phi_c\tilde{C}_{t-1}^I\right)}\right)^{\frac{1}{\eta_{\hat{C}}}} \tag{23}$$

$$[H_t^I]:\quad\varrho_t\frac{\lambda_t^IA_t^{-\sigma}Q_t^H}{P_t}=\mathbb{E}_t\left\{\begin{array}{l}\beta_I\varrho_{t+1}\left(\left(\hat{C}_{t+1}^I\right)^{-\sigma}\left(\frac{o_{\hat{C}}\hat{C}_{t+1}^I}{\xi_{t+1}^h\left(H_t^I-\phi_{hh}\tilde{H}_{t-1}^I\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}\xi_{t+1}^h\right.\\ \left.+\frac{\lambda_{t+1}^IA_{t+1}^{-\sigma}}{P_{t+1}}\left[1-\Gamma_I\left(\bar{\omega}_{t+1}^I\right)\right]R_{t+1}^HQ_t^H\right)\\ +\frac{\varrho_t\lambda_t^HA_t^{-\sigma}}{P_t}\left[1-\Gamma^H\left(\bar{\omega}_{t+1}^H\right)\right]\left[\Gamma_I\left(\bar{\omega}_{t+1}^I\right)-\mu_IG_I\left(\bar{\omega}_{t+1}^I\right)\right]R_{t+1}^HQ_t^H\end{array}\right\} \tag{24}$$

$$[L_t^H]:\quad\lambda_t^I=\lambda_t^H\tilde{\rho}_{t+1}^H\phi_H \tag{25}$$

$$[x_t^I]:\quad\frac{\varrho_t\lambda_t^HA_t^{-\sigma}}{P_t}\mathbb{E}_t\left\{\left[1-\Gamma^H\left(\bar{\omega}_{t+1}^H\right)\right]\left[\Gamma_I'\left(\bar{\omega}_{t+1}^I\right)-\mu_IG_I'\left(\bar{\omega}_{t+1}^I\right)\right]\right\}=\beta_I\mathbb{E}_t\left\{\frac{\varrho_{t+1}\lambda_{t+1}^IA_{t+1}^{-\sigma}}{P_{t+1}}\Gamma_I'\left(\bar{\omega}_{t+1}^I\right)\right\} \tag{26}$$

Regarding the idiosyncratic shock, we assume that $\ln\left(\omega_t^I\right)\sim N\left(-\frac{1}{2}\left(\sigma_t^I\right)^2,\left(\sigma_t^I\right)^2\right)$, therefore its unconditional expectation is $\mathbb{E}\left\{\omega_t^I\right\}=1$, and its average conditional on truncation is

$$\mathbb{E}_t\left\{\omega_t^I|\omega_t^I\geq\bar{\omega}_t^I\right\}=\frac{1-\Phi\left(z_t^I-\sigma_t^I\right)}{1-\Phi\left(z_t^I\right)},$$

where Φ is the c.d.f. of the standard normal and z_t^I is an auxiliary variable defined as $z_t^I \equiv \frac{\left(\ln\left(\bar{\omega}_t^I\right)+0.5\left(\sigma_t^I\right)^2\right)}{\sigma_t^I}$. Then, we can obtain the following functional forms:

$$\Gamma_I\left(\bar{\omega}_t^I\right) = \Phi\left(z_t^I - \sigma_t^I\right) + \bar{\omega}_t^I\left(1 - \Phi\left(z_t^I\right)\right)$$

and

$$\Gamma_I\left(\bar{\omega}_t^I\right) - \mu_I G_I\left(\bar{\omega}_t^I\right) = \left(1-\mu_I\right)\Phi\left(z_t^I - \sigma_t^I\right) + \bar{\omega}_t^I\left(1 - \Phi\left(z_t^I\right)\right)$$

Finally, we allow for fluctuations in the variance of the idiosyncratic shock, as σ_t^I is modeled as an exogenous process.

2.2 Entrepreneurs

As in Clerc et al. (2014), we introduce risk-neutral entrepreneurs that follow an overlapping generations structure, where each generation lives across two consecutive periods. The entrepreneurs are the sole owners of productive capital, which is bought from capital producers to be, in turn, rented to the firms that produce different varieties of the home good.

Entrepreneurs born in period t draw utility in $t+1$ from transferring part of final wealth as dividends, C_{t+1}^e, to unrestricted patient households and from leaving the rest as bequests, N_{t+1}^e, to the next generation of entrepreneurs in the form:

$$\max_{C_{t+1}^e, N_{t+1}^e} \left(C_{t+1}^e\right)^{\xi_{\chi_e}\chi_e}\left(N_{t+1}^e\right)^{1-\xi_{\chi_e}\chi_e} \text{ subject to}$$

$$C_{t+1}^e + N_{t+1}^e = \Psi_{t+1}^e$$

where Ψ_{t+1}^e is entrepreneurial wealth at $t+1$, explained below, and ξ_{χ_e} is a stochastic shock to ttheir preferences all nominal variables. The first order conditions to this problem may be written as:

$$[C_{t+1}^e]: \ \xi_{\chi_e}\chi_e(C_{t+1}^e)^{(\xi_{\chi_e}\chi_e-1)}\left(N_{t+1}^e\right)^{1-\xi_{\chi_e}\chi_e} - \lambda_t^{\chi_e} = 0$$

$$[N_{t+1}^e]: \ (1-\xi_{\chi_e}\chi_e)(C_{t+1}^e)^{\xi_{\chi_e}\chi_e}\left(N_{t+1}^e\right)^{-\xi_{\chi_e}\chi_e} - \lambda_t^{\chi_e} = 0$$

$$[\lambda_t^{\chi_e}]: \ C_{t+1}^e + N_{t+1}^e - \Psi_{t+1}^e = 0$$

From first order conditions we get the following optimal rules

$$C_{t+1}^e = \chi_e \Psi_{t+1}^e$$
$$N_{t+1}^e = \left(1-\chi_e\right)\Psi_{t+1}^e$$

In their first period, entrepreneurs will try to maximize expected second period wealth, Ψ_{t+1}^e, by purchasing capital at nominal price Q_t^K, which will be productive (and rented) in the next period. These purchases are financed using the resources left as bequests by the previous generation of entrepreneurs and borrowing an amount L_t^F at nominal rate R_t^L from from F banks . In borrowing from banks, entrepreneurs also face an agency problem of the type faced by impatient households i.e. in $t+1$ entrepreneurs receive an idiosincratic shock to the efficieny units of housing that will ultimately determine their ability to pay their liabilities to banks. Banks cannot observe these shock, but households can. Depreciated capital is sold in the next period to capital producers at Q_{t+1}^K. Entrepreneurial leverage, as measured by assets over equity, is $lev_t^e = \frac{Q_t^K K_t}{N_t^e}$.

In this setting, entrepreneurs solve in their first period

$$\max_{K_t, L_t^F} \mathbb{E}_t\left(\Psi_{t+1}^e\right) \text{ subject to}$$

$$Q_t^K K_t - L_t^F = N_t^e$$

$$\Psi_{t+1}^e = \max\left[\omega_{t+1}^e\left(R_{t+1}^k + \left(1-\delta_K\right)Q_{t+1}^K\right)K_t - R_t^L L_t^F, \ 0\right]$$

and a bank participation condition, which will be explained later. The factor ω_{t+1}^e represents the idiosyncratic shock to the entrepreneurs efficiency units of capital. This shock takes place after the loan with the bank has taken place but before renting capital to consumption goods producers. It is assumed that this shock is independently and identically distributed across entrepreneurs and follows a log-normal distribution with an expected value of one. Let

$$R_{t+1}^{e}=\left[\frac{R_{t+1}^{k}+(1-\delta_{K})\,Q_{t+1}^{K}}{Q_{t}^{K}}\right] \tag{27}$$

be the gross nominal return per efficiency unit of capital obtained in period $t+1$ from capital obtained in period t. Then in order for the entrepreneur to pay for its loan the efficiency shock, ω_{t+1}^{e},noam tit must exceed the threshold

$$\bar{\omega}_{t+1}^{e}=\frac{R_{t}^{L}L_{t}^{F}}{R_{t+1}^{e}Q_{t}^{K}K_{t}}$$

If $\omega_{t+1}^{e}\geq\bar{\omega}_{t+1}^{e}$ the entrepreneurs pays $R_{t}^{L}L_{t}^{F}$ to the bank and gets $(\omega_{t+1}^{e}-\bar{\omega}_{t+1}^{e})R_{t+1}^{e}Q_{t}^{K}K_{t}$. Otherwise, the entrepreneurs defaults and receives nothing. While F-banks only recover $(1-\mu_{e})\omega_{t+1}^{e}R_{t+1}^{e}Q_{t}^{K}K_{t}$ from non performing loans, and $R_{t}^{L}L_{t}^{F}$ from performing loans. With the threshold, we can define $PD_{t}^{e}=F_{e}\left(\bar{\omega}_{t}^{e}\right)$ as the default rate of entrepreneurs on their loans.

The share of the gross return that goes to the bank is denoted as $\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)$ whereas the share of gross return that goes to the entrepreneur is $\left(1-\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)\right)$ where:

$$\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)=\int_{0}^{\bar{\omega}_{t+1}^{e}}\omega_{t+1}^{e}f_{e}\left(\omega_{t+1}^{e}\right)d\omega_{t+1}^{e}+\bar{\omega}_{t+1}^{e}\int_{\bar{\omega}_{t+1}^{e}}^{\infty}f_{e}\left(\omega_{t+1}^{e}\right)d\omega_{t+1}^{e}$$

also let

$$G_{e}\left(\bar{\omega}_{t+1}^{e}\right)=\int_{0}^{\bar{\omega}_{t+1}^{e}}\omega_{t+1}^{e}f_{e}\left(\omega_{t+1}^{e}\right)d\omega_{t+1}^{e}$$

denote the part of those returns that come from the defaulted loans. Taking into consideration the share of the return that is lost due to verification cost as $\mu_{e}G_{e}\left(\bar{\omega}_{t+1}^{e}\right)$, then the net share of return that goes to the bank is

$$\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)-\mu_{e}G_{e}\left(\bar{\omega}_{t+1}^{e}\right).$$

Taking this into account then the maximization problem of the entrepreneur can be written as

$$\max_{\bar{\omega}_{t+1}^{e},K_{t}}\mathbb{E}_{t}\left\{\Psi_{t+1}^{e}\right\}=\mathbb{E}_{t}\left\{\left[1-\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)\right]R_{t+1}^{e}Q_{t}^{K}K_{t}\right\},\text{ subject to}$$

$$\mathbb{E}_{t}\left\{\left[1-\Gamma_{F}\left(\bar{\omega}_{t+1}^{F}\right)\right]\left[\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)-\mu_{e}G_{e}\left(\bar{\omega}_{t+1}^{e}\right)\right]R_{t+1}^{e}Q_{t}^{K}K_{t}\right\}=\rho_{t+1}^{F}\phi_{F}L_{t}^{F}, \tag{28}$$

that says that banks will participate in the contract only if its net expected profits equals to the alternative use of funds. This yields the following optimality conditions

$$\left(1-\Gamma_{t+1}^{e}\right)=\lambda_{t}^{e}\left(\frac{\rho_{t+1}^{F}\phi_{t}^{F}}{R_{t+1}^{e}}-\left(1-\Gamma_{t+1}^{F}\right)\left[\Gamma_{t+1}^{e}-\mu^{e}G_{t+1}^{e}\right]\right) \tag{29}$$

$$\Gamma_{t+1}^{e'}=\lambda_{t}^{e}\left(1-\Gamma_{t+1}^{F}\right)\left[\Gamma_{t+1}^{e'}-\mu^{e}G_{t+1}^{e'}\right] \tag{30}$$

Further, it is assumed that $\ln\left(\omega_{t}^{e}\right)\sim N\left(-\frac{1}{2}\left(\sigma_{t}^{e}\right)^{2},\left(\sigma_{t}^{e}\right)^{2}\right)$, leading to analogous properties as with impatient households for $\bar{\omega}_{t}^{e}$, Γ_{e} and G_{e}.

2.3 Bankers and Banks

2.3.1 Bankers

Bankers are modeled as in Clerc et al. (2014) and in a similar way to entrepreneurs: They belong to a sequence of overlapping generations of risk-neutral agents who live 2 periods and have exclusive access to the opportunity of investing their wealth as banks' inside equity capital.

In the first period, the banker receives a bequest N_{t}^{b} from the previous generation of bankers and must distribute it across the two types of existing banks: banks specializing in corporate loans (F banks) and banks specializing in housing loans (H banks). That is, a banker who chooses to invest an amount E_{t}^{F} of inside equity in F banks will invest the rest of her bequest in H banks, $E_{t}^{H}=N_{t}^{b}-E_{t}^{F}$. Then, in the second period bankers receive their returns from both investments, and must choose how to distribute their net worth Ψ^b_{t+1} between transferring dividends C^b_{t+1} to households and leaving bequests N^b_{t+1} to the next generation. Additionally, disturbances to the exogenous variable $\xi^{\chi_b}_t$ capture transitory fluctuations in the banker's dividend policy

Given Ψ^b_{t+1}, the banker will distribute it by solving the following maximization problem:

$$\max_{C^b_{t+1},N^b_{t+1}} \left(C^b_{t+1}\right)^{\xi^{\chi_b}_{t+1}\chi^b} \left(N^b_{t+1}\right)^{1-\xi^{\chi_b}_{t+1}\chi^b} \text{ subject to}$$

$$C^b_{t+1} + N^b_{t+1} = \Psi^b_{t+1}$$

which leads to the following optimal rules

$$C^b_{t+1} = \xi^{\chi_b}_{t+1}\chi^b\Psi^b_{t+1} \tag{31}$$

$$N^b_{t+1} = \left(1 - \xi^{\chi_b}_{t+1}\chi^b\right)\Psi^b_{t+1} \tag{32}$$

In turn, net worth in the second period is determined by the returns on bankers' investments in period-t:

$$\Psi^b_{t+1} = \rho^F_{t+1}E^F_t + \xi^{b,roe}_t\rho^H_{t+1}\left(N^b_t - E^F_t\right)$$

where $\xi^{b,roe}_t$ is a shock to the required returns to equity invested in the different branches, ρ^j_{t+1} is the period $t+1$ ex-post gross return on inside equity E^j_t invested in period t in bank of class j. In order to capture the fact that most of mortgage debt takes the form of non endorsable debt —meaning the issuer bank retains it in its balance sheet to maturity— we assume that the banker $j = H$ invests in the banking project H through a mutual fund which pays the expected average return to housing equity ρ^H_{t+1} every period. Thus, letting $\tilde{\rho}^H_t$ represent the period return on housing portfolio, then $\rho^H_t = \kappa\tilde{\rho}^H_t + (1-\kappa)\rho^H_{t+1}$.

$$\max_{E^F_t} \mathbb{E}_t\left\{\Psi^b_{t+1}\right\} = \mathbb{E}_t\left\{\rho^F_{t+1}E^F_t + \xi^{b,roe}_t\rho^H_{t+1}\left(N^b_t - E^F_t\right)\right\}$$

Then, an interior equilibrium in which both classes of banks receive strictly positive inside equity from bankers will require the following equality to hold:

$$\mathbb{E}_t\left\{\rho^F_{t+1}\right\} = \mathbb{E}_t\left\{\xi^{b,roe}_t\rho^H_{t+1}\right\} = \bar{\rho}_t$$

where $\bar{\rho}_t$ denotes banks' required expected gross rate of return on equity investment undertaken at time t.

2.3.2 Banks

Banks are institutions specialized in extending either corporate or housing loans drawing funds through deposits, and bonds from unconstrained household, and equity from bankers. We assume a continuum of identical banking institutions of j class banks $j = \{F, H\}$. In particular, banks of class j are investment projects created in period-t that in $t+1$ generate profits Π^j_{t+1} before being liquidated with:

$$\Pi^F_{t+1} = \max\left[\omega^F_{t+1}\tilde{R}^F_{t+1}L^F_t - R^D_t D^F_t, 0\right], \quad \Pi^H_{t+1} = \max\left[\omega^H_{t+1}\tilde{R}^H_{t+1}Q^L_t L^H_t - R^{BB}_{t+1}Q^{BB}_{t+1}BB_t, 0\right]$$

where $\tilde{R}^j_{t+1}$ is the realized return on a well-diversified portfolio of loans to entrepreneurs or households and ω^j_{t+1} is an idiosyncratic portfolio return shock, which is i.i.d across banks of class j with a cdf of $F_j(\omega^j_{t+1})$ and pdf $f_j(\omega^j_{t+1})$. Due to limited liability, the equity payoff may not be negative, which defines thresholds $\bar{\omega}^j_{t+1}$:

$$\bar{\omega}^F_{t+1} \equiv \frac{R^D_t D^F_t}{\tilde{R}^F_{t+1}L^F_t}, \qquad \bar{\omega}^H_{t+1} \equiv \frac{R^{BB}_{t+1}Q^{BB}_{t+1}BB_t}{\tilde{R}^H_{t+1}Q^L_t L^H_t}$$

Similar to households and entrepreneurs, $\Gamma_j\left(\bar{\omega}^j_{t+1}\right)$ denotes the share of gross returns to bank j investments which are either paid back to depositors or bond holders, implying that $\left[1 - \Gamma_j\left(\bar{\omega}^j_{t+1}\right)\right]$ is the share that the banks will keep as profits. We also define $G_j\left(\bar{\omega}^j_{t+1}\right)$ as the share of bank j assets which belong to defaulting j banks, and thus $\mu_j G_j\left(\bar{\omega}^j_{t+1}\right)$ is the total cost of bank j defaults expressed as a fraction of total bank j assets.

The balance sheet of banks of class F is given by $L_t^F = E_t^F + D_t^F$, and they face a regulatory capital constraint given by $E_t^F \geq \phi_F L_t^F$, where ϕ_F is the capital-to-asset ratio, and is binding at all times in equilibrium so that the loans can be written as $L_t^F = \frac{E_t^F}{\phi_F}$ and the deposits as $D_t^F = \left(\frac{1-\phi_F}{\phi_F}\right) E_t^F$. Likewise, balance sheet of banks of class H is given by $Q_t^L L_t^H = E_t^H + Q_t^{BB} BB_t$, with binding capital regulation determining $E_t^H = \phi_H Q_t^L L_t^H$, and $Q_t^{BB} BB_t = \frac{(1-\phi_H)}{\phi_H} E_t^H$. Further, using the threshold definitions and the binding capital constraints, we obtain

$$\bar{\omega}_{t+1}^F = (1-\phi_F) \frac{R_t^D}{\tilde{R}_{t+1}^F}$$

$$\bar{\omega}_{t+1}^H = (1-\phi_H) \frac{R_{t+1}^{BB}}{\tilde{R}_{t+1}^H} \left(\frac{Q_{t+1}^{BB}}{Q_t^{BB}}\right)$$

Finally, we define the realized rate of return of equity invested in a bank of class j:

$$\rho_{t+1}^j = \left[1 - \Gamma_j \left(\bar{\omega}_{t+1}^j\right)\right] \frac{\tilde{R}_{t+1}^j}{\phi_j} \tag{33}$$

For completeness, notice that derivations in prior sections imply that following expressions for $\tilde{R}_{t+1}^j$, $j = \{F, H\}$:

$$\tilde{R}_{t+1}^F = \left(\Gamma_e\left(\bar{\omega}_{t+1}^e\right) - \mu_e G_e\left(\bar{\omega}_{t+1}^e\right)\right) \frac{R_{t+1}^e Q_t^K K_t}{L_t^F}$$

$$\tilde{R}_{t+1}^H = \left(\Gamma_I\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I\left(\bar{\omega}_{t+1}^I\right)\right) \frac{R_{t+1}^H Q_t^H H_t^I}{Q_t^L L_t^H}$$

As with households and entrepreneurs, it is assumed that the bank idiosyncratic shock follows a log-normal distribution: $\ln\left(\omega_t^j\right) \sim N\left(-\frac{1}{2}\left(\sigma_t^j\right)^2, \left(\sigma_t^j\right)^2\right)$, leading to analogous properties for $\bar{\omega}_t^j$, Γ_j and G_j.

2.4 Production

The supply side of the economy is composed by different types of firms that are all owned by the households. Monopolistically competitive unions act as wage setters by selling household's differentiated varieties of labor supply n_{it} to a perfectly competitive firm, which packs these varieties into a composite labor service $\widetilde{n}_t$. There is a set of monopolistically competitive firms producing different varieties of a home good, Y_{jt}^H, using wholesale good X_t^Z as input; a set of monopolistically competitive importing firms that import a homogeneous foreign good to transform it into varieties, X_{jt}^F; and three groups of perfectly competitive firms that aggregate products: one packing different varieties of the home good into a composite home good, X_t^H, one packing the imported varieties into a composite foreign good, X_t^F, and, finally, another one that bundles the composite home and foreign goods to create a final good, Y_t^C. This final good is purchased by households (C_t^P,C_t^I), capital and housing producers (I_t^K,I_t^H), and the government (G_t).

Similarly to Clerc et al. (2014) we model perfectly competitive capital-producing and housing-producing firms. Both types of firms are owned by patient households and their technology is subject to an adjustment cost. They produce new units of capital and housing from the final good and sell them to entrepreneurs and households respectively. However, we depart from Clerc et al. (2014) by assuming time-to-build frictions in housing investment. Finally, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad (and which follows an exogenous process). The total mass of firms in each sector is normalized to one.

2.4.1 Capital goods

There is a continuum of competitive capital firm producers who buy an amount I_t^K of final goods at price P_t and use their technology to satisfy the demand for new capital goods not covered by depreciated capital, i.e. $K_t - (1-\delta_K) K_{t-1}$, where new units of capital are sold at price Q_t^K. As is usual in the literature we assume that the aggregate stock of new capital considers investment adjustment costs and evolves according to following law of motion:

$$K_t = (1-\delta_K) K_{t-1} + \left[1 - \frac{\gamma_K}{2}\left(\frac{I_t^K}{I_{t-1}^K} - a\right)^2\right] \xi_t^i I_t^K$$

Where ξ_t^i is a shock to investment efficiency. Therefore a representative capital producer chooses how much to invest in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i} \left\{ Q_{t+i}^K \left[1 - \frac{\gamma_K}{2} \left(\frac{I_{t+i}^K}{I_{t+i-1}^K} - a \right)^2 \right] \xi_{t+i}^i I_{t+i}^K - P_{t+i} I_{t+i}^K \right\}$$

Discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of capital to the level of investment

$$\begin{aligned} P_t &= Q_t^K \left\{ \left(1 - \frac{\gamma_K}{2} \left(\frac{I_t}{I_{t-1}} - a \right)^2 \right) - \gamma_K \left(\frac{I_t}{I_{t-1}} - a \right) \frac{I_t}{I_{t-1}} \right\} \xi_t^i \\ &\quad + E_t \left\{ r_{t,t+1} Q_{t+1}^K \gamma_K \left(\frac{I_{t+1}}{I_t} - a \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \xi_{t+1}^i \right\} \end{aligned} \tag{34}$$

2.4.2 Housing goods

The structure of housing producers is similar to that of capital good producers with the difference that housing goods also face investment adjustment costs in the form of time to build Kydland and Prescott (1982) and Uribe and Yue (2006).As such, there is a continuum of competitive housing firm producers who authorize housing investment projects I_t^{AH} in period t, which will increase housing stock N_H periods later, the time it takes to build.[3] Thus, the law of motion for the aggregate stock of housing in H_t will consider projects authorized N_H periods before, and includes investment adjustment costs,

$$H_t = (1 - \delta_H) H_{t-1} + \left[1 - \frac{\gamma_H}{2} \left(\frac{I_{t-N_H}^{AH}}{I_{t-N_H-1}^{AH}} - a \right)^2 \right] \xi_{t-N_H}^{ih} I_{t-N_H}^{AH}$$

Where ξ_t^{ih} is a shock to housing investment efficiency, and the sector covers all demand for new housing, $H_t - (1 - \delta_H) H_{t-1}$, by selling units at price Q_t^H.

The firm's effective expenditure is spread out during the periods that new housing is being built. In particular, the amount of final goods purchased (at price P_t) by the firm in t to produce housing is given by

$$I_t^H = \sum_{j=0}^{N_H} \varphi_j^H I_{t-j}^{AH}$$

Where φ_j^H (the fraction of projects authorized in period $t-j$ that is outlaid in period t) satisfy $\sum_{j=0}^{N_H} \varphi_j^H = 1$ and $\varphi_j^H = \rho^{\varphi H} \varphi_{j-1}^H$.[4]

Therefore a representative housing producer chooses how much to authorize in new projects I_t^{AH} in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i} \left\{ Q_{t+i}^H \left[1 - \frac{\gamma_H}{2} \left(\frac{I_{t-N_H+i}^{AH}}{I_{t-N_H+i-1}^{AH}} - a \right)^2 \right] \xi_{t-N_H+i}^{ih} I_{t-N_H+i}^{AH} - P_{t+i} I_{t+i}^H \right\}$$

Where discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of housing to the level of housing investment

$$\begin{aligned} E_t \sum_{j=0}^{N_H} r_{t,t+j} \varphi_j^H P_{t+j} &= E_t r_{t,t+N_H} Q_{t+N_H}^H \left\{ \left[1 - \frac{\gamma_H}{2} \left(\frac{I_t^{AH}}{I_{t-1}^{AH}} - a \right)^2 \right] - \gamma_H \left(\frac{I_t^{AH}}{I_{t-1}^{AH}} - a \right) \frac{I_t^{AH}}{I_{t-1}^{AH}} \right\} \xi_t^{ih} \\ &\quad + E_t r_{t,t+N_H+1} Q_{t+N_H+1}^H \left\{ \gamma_H \left(\frac{I_{t+1}^{AH}}{I_t^{AH}} - a \right) \left(\frac{I_{t+1}^{AH}}{I_t^{AH}} \right)^2 \xi_{t+1}^{ih} \right\} \end{aligned} \tag{35}$$

[3] Notice that if $N_H = 0$, the structure is symmetric to the capital producers.

[4] Notice that $\rho^{\varphi H} > 1$ implies that expenditure for any authorized project is back-loaded (increasing over time), while the converse is true for $\rho^{\varphi H} < 1$.

2.4.3 Final goods

A representative final goods firm demands composite home and foreign goods in the amounts X_t^H and X_t^F, respectively, and combines them according to the following technology:

$$Y_t^C = \left[\omega^{1/\eta}\left(X_t^H\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(X_t^F\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}} \tag{36}$$

where $\omega \in (0,1)$ is inversely related to the degree of home bias and $\eta > 0$ measures the substitutability between domestic and foreign goods. The selling price of this final good is denoted by P_t, while the prices of the domestic and foreign inputs are P_t^H and P_t^F, respectively. Subject to the technology constraint (36), the firm maximizes its profits over the inputs, taking prices as given:

$$\max_{X_t^H, X_t^F} \; P_t\left[\omega^{1/\eta}\left(X_t^H\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(X_t^F\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}} - P_t^H X_t^H - P_t^F X_t^F$$

The first-order conditions of this problem determine the optimal input demands:

$$X_t^H = \omega\left(\frac{P_t^H}{P_t}\right)^{-\eta} Y_t^C \tag{37}$$

$$X_t^F = (1-\omega)\left(\frac{P_t^F}{P_t}\right)^{-\eta} Y_t^C \tag{38}$$

Combining these optimality conditions and using that zero profits hold in equilibrium, we can write

$$P_t = \left[\omega\left(P_t^H\right)^{1-\eta} + (1-\omega)\left(P_t^F\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{39}$$

2.4.4 Home composite goods

A representative home composite goods firm demands home goods of all varieties $j \in [0,1]$ in amounts X_{jt}^H and combines them according to the technology

$$Y_t^H = \left[\int_0^1 \left(X_{jt}^H\right)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H - 1}} \tag{40}$$

with $\epsilon_H > 0$. Let P_{jt}^H denote the price of the home good of variety j. Subject to the technology constraint (40), the firm maximizes its profits $\Pi_t^H = P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj$ over the input demands X_{jt}^H taking prices as given:

$$\max_{X_{jt}^H} \; P_t^H \left[\int_0^1 \left(X_{jt}^H\right)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H - 1}} - \int_0^1 P_{jt}^H X_{jt}^H dj$$

This implies the following first-order conditions for all j:

$$\partial X_{jt}^H : \; P_t^H \left(Y_t^H\right)^{1/\epsilon_H} \left(X_{jt}^H\right)^{-1/\epsilon_H} - P_{jt}^H = 0$$

such that the input demand functions are

$$X_{jt}^H = \left(\frac{P_{jt}^H}{P_t^H}\right)^{-\epsilon_H} Y_t^H \tag{41}$$

Substituting (41) into (40) yields the price of home composite goods:

$$P_t^H = \left[\int_0^1 \left(P_{jt}^H\right)^{1-\epsilon_H} dj\right]^{\frac{1}{1-\epsilon_H}} \tag{42}$$

2.4.5 Home goods of variety j

There is a continuum of j's firms, with measure one, that demand a domestic wholesale good X_t^Z and differentiate into home goods varieties Y_{jt}^H. To produce one unit of variety j, firms need one unit of input according to

$$\int_0^1 Y_{jt}^H dj = X_t^Z \tag{43}$$

The firm producing variety j satisfies the demand given by (41) but it has monopoly power for its variety. For varieties, the nominal marginal cost in terms of the composite good price is given by $P_t^H mc_{jt}^H$. Given that, every firm buys their input from the same wholesale market. It implies that all of them face the same nominal marginal costs

$$P_t^H mc_{jt}^H = P_t^H mc_t^H = P_t^Z \tag{44}$$

Given nominal marginal costs $P_t^H mc_{jt}^H$, firm j chooses its price P_{jt}^H to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period the firm can change its price optimally with probability $1-\theta_H$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_H \in [0,1]$ and $1-\kappa_H$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^H$ that maximizes the current market value of the profits generated until it can reoptimize again. [5] As the firms are owned by the households, profits are discounted using the households' stochastic discount factor for nominal payoffs, $r_{t,t+s}$. A reoptimizing firm, therefore, solves the following problem:

$$\max_{\tilde{P}_{jt}^H} E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left(P_{jt+s}^H - P_{t+s}^H mc_{jt+s}^H\right) Y_{jt+s}^H \quad \text{s.t.} \quad Y_{jt+s}^H = X_{jt+s}^H = \left(\frac{\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H}}{P_{t+s}^H}\right)^{-\epsilon_H} Y_{t+s}^H$$

which can be rewritten as

$$\max_{\tilde{P}_{jt}^H} E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left[\left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(P_{t+s}^H\right)^{\epsilon_H} - mc_{jt+s}^H \left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(P_{t+s}^H\right)^{1+\epsilon_H}\right] Y_{t+s}^H$$

The first-order conditions determining the optimal price $\tilde{P}_t^H$ can be written as follows:[6]

$$\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[(1-\epsilon_H)\left(\tilde{P}_t^H\right)^{-\epsilon_H} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(P_{t+s}^H\right)^{\epsilon_H} \\
&\qquad + \epsilon_H mc_{t+s}^H \left(\tilde{P}_t^H\right)^{-\epsilon_H - 1} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(P_{t+s}^H\right)^{1+\epsilon_H}\Bigg] Y_{t+s}^H \\
\Leftrightarrow \quad 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \frac{\left(P_{t+s}^H\right)^{\epsilon_H}}{P_t^H} \\
&\qquad - mc_{t+s}^H \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \frac{\left(P_{t+s}^H\right)^{1+\epsilon_H}}{P_t^H}\Bigg] Y_{t+s}^H \\
\Leftrightarrow \quad 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H} \\
&\qquad - mc_{t+s}^H \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{1+\epsilon_H}\Bigg] Y_{t+s}^H
\end{aligned}$$

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[5] Therefore, the following relation holds:

$$P_{jt+s}^H = \tilde{P}_{jt}^H \pi_{t+1}^{I,H} \dots \pi_{t+s}^{I,H}$$

where

$$\pi_t^{I,H} = \left(\pi_{t-1}^H\right)^{\kappa_H} \left(\pi_t^T\right)^{1-\kappa_H}$$

and, in turn, $\pi_t^H = P_t^H / P_{t-1}^H$ and π_t^T denotes the inflation target in period t.

[6] Notice that the subscript j has been removed from $\tilde{P}_t^H$; this simplifies notation and underlines that the prices chosen by all firms j that reset prices optimally in a given period are equal as they face the same problem by (44).

where the second step follows from multiplying both sides by $-\tilde{P}_t^H/(P_t^H\epsilon_H)$, and the third by defining $\tilde{p}_t^H = \tilde{P}_t^H/P_t^H$. The first-order condition can be rewritten in recursive form as follows, defining $F_t^{H_1}$ as

$$\begin{aligned}
F_t^{H_1} &= \frac{\epsilon_H-1}{\epsilon_H}\left(\tilde{p}_t^H\right)^{1-\epsilon_H}Y_t^H + E_t\sum_{s=1}^{\infty}\theta_H^s r_{t,t+s}\frac{\epsilon_H-1}{\epsilon_H}\left(\tilde{p}_t^H\Pi_{i=1}^s\pi_{t+i}^{I,H}\right)^{1-\epsilon_H}\left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H}Y_{t+s}^H \\
&= \frac{\epsilon_H-1}{\epsilon_H}\left(\tilde{p}_t^H\right)^{1-\epsilon_H}Y_t^H + E_t\sum_{s=0}^{\infty}\theta_H^{s+1} r_{t,t+s+1}\frac{\epsilon_H-1}{\epsilon_H}\left(\tilde{p}_t^H\Pi_{i=1}^{s+1}\pi_{t+i}^{I,H}\right)^{1-\epsilon_H}\left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{\epsilon_H}Y_{t+s+1}^H \\
&= \frac{\epsilon_H-1}{\epsilon_H}\left(\tilde{p}_t^H\right)^{1-\epsilon_H}Y_t^H + \theta_H E_t\left\{r_{t,t+1}\left(\frac{\tilde{p}_t^H\pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H}\left(\pi_{t+1}^H\right)^{\epsilon_H}\sum_{s=0}^{\infty}\theta_H^s r_{t+1,t+s+1}\frac{\epsilon_H-1}{\epsilon_H}\right. \\
&\qquad\left.\times\left(\tilde{p}_{t+1}^H\Pi_{i=1}^s\pi_{t+1+i}^{I,H}\right)^{1-\epsilon_H}\left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{\epsilon_H}Y_{t+s+1}^H\right\} \\
&= \frac{\epsilon_H-1}{\epsilon_H}\left(\tilde{p}_t^H\right)^{1-\epsilon_H}Y_t^H + \theta_H E_t\left\{r_{t,t+1}\left(\frac{\tilde{p}_t^H\pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H}\left(\pi_{t+1}^H\right)^{\epsilon_H}F_{t+1}^{H_1}\right\}
\end{aligned} \tag{45}$$

and, analogously, $F_t^{H_2}$ as

$$\begin{aligned}
F_t^{H_2} &= \left(\tilde{p}_t^H\right)^{-\epsilon_H}mc_t^H Y_t^H + E_t\sum_{s=1}^{\infty}\theta_H^s r_{t,t+s}mc_{t+s}^H\left(\tilde{p}_t^H\Pi_{i=1}^s\pi_{t+i}^{I,H}\right)^{-\epsilon_H}\left(\frac{P_{t+s}^H}{P_t^H}\right)^{1+\epsilon_H}Y_{t+s}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H}mc_t^H Y_t^H + E_t\sum_{s=0}^{\infty}\theta_H^{s+1} r_{t,t+s+1}mc_{t+s+1}^H\left(\tilde{p}_t^H\Pi_{i=1}^{s+1}\pi_{t+i}^{I,H}\right)^{-\epsilon_H}\left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{1+\epsilon_H}Y_{t+s+1}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H}mc_t^H Y_t^H + \theta_H E_t\left\{r_{t,t+1}\left(\frac{\tilde{p}_t^H\pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H}\left(\pi_{t+1}^H\right)^{1+\epsilon_H}\sum_{s=0}^{\infty}\theta_H^s r_{t+1,t+s+1}mc_{t+s+1}^H\right. \\
&\qquad\left.\times\left(\tilde{p}_{t+1}^H\Pi_{i=1}^s\pi_{t+1+i}^{I,H}\right)^{-\epsilon_H}\left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{1+\epsilon_H}Y_{t+s+1}^H\right\} \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H}mc_t^H Y_t^H + \theta_H E_t\left\{r_{t,t+1}\left(\frac{\tilde{p}_t^H\pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H}\left(\pi_{t+1}^H\right)^{1+\epsilon_H}F_{t+1}^{H_2}\right\}
\end{aligned} \tag{46}$$

such that

$$F_t^{H_1} = F_t^{H_2} = F_t^H \tag{47}$$

Using (42), we have

$$\begin{aligned}
1 &= \int_0^1\left(\frac{P_{jt}^H}{P_t^H}\right)^{1-\epsilon_H}dj \\
&= (1-\theta_H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H\left(\frac{P_{t-1}^H\pi_t^{I,H}}{P_t^H}\right)^{1-\epsilon_H} \\
&= (1-\theta_H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H\left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{1-\epsilon_H}
\end{aligned} \tag{48}$$

The second equality above follows from the fact that, under Calvo pricing, the distribution of prices among firms not reoptimizing in period t corresponds to the distribution of aggregate prices in period $t-1$, though with total mass reduced to θ_H.

2.4.6 Wholesale Domestic Goods

There is a representative firm producing a homogeneous wholesale home good, combining capital and labor according to the following technology:

$$Y_t^Z = z_t K_{t-1}^{\alpha}\left(A_t\widetilde{n}_t\right)^{1-\alpha} \tag{49}$$

with capital share $\alpha \in (0,1)$, an exogenous stationary technology shock z_t and a non-stationary technology A_t. Production of the wholesale good composite labor services $\widetilde{n}_t$ and capital K_{t-1}. Additionally, following Lechthaler and Snower (2010), the firm faces a quadratic adjustment costs of labor which is a function of parameter γ_n, and of aggregate wholesale domestic goods $\widetilde{Y_t}^Z$, which in equilibrium are equal to Y_t^Z and which the representative firm takes as given. In a first stage, the firm hires composite labor and rents capital to solve the following problem:

$$\min_{\widetilde{n}_{t+s},K_{t+s-1}} \sum_{s=0}^{\infty} r_{t,t+s} \left\{ W_{t+s}\widetilde{n}_{t+s} + \frac{\gamma_n}{2}\left(\frac{\widetilde{n}_{t+s}}{\widetilde{n}_{t+s-1}} - 1\right)^2 \widetilde{Y_{t+s}}^Z P_t^Z + R_t K_{t+s-1} \right\}$$
$$s.t. \quad Y_{t+s}^Z = X_{t+s}^Z = z_{t+s} K_{t+s-1}^{\alpha} \left(A_{t+s}\widetilde{n}_{t+s}\right)^{1-\alpha}$$

Then, the optimal capital and labor demands are given by:

$$\widetilde{n}_t = (1-\alpha)\left\{ \frac{mc_t^Z Y_t^Z}{W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right)\widetilde{Y}_t^Z P_t^Z - r_{t,t+1}\gamma_n \mathbb{E}_t\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2}\right)\widetilde{Y}_{t+1}^Z P_{t+1}^Z} \right\} \tag{50}$$

$$K_{t-1} = \alpha \left(\frac{mc_t^Z}{R_t^k}\right) Y_t^Z \tag{51}$$

Where mc_t^Z is the lagrangian multiplier on the production function and $r_{t,t+1}$ the households' stochastic discount factor between periods t and $t+1$. The, combining both optimality conditions:

$$\frac{K_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha)R_t^k}\left\{ W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right)\widetilde{Y}_t^Z P_t^Z - r_{t,t+1}\gamma_n \mathbb{E}_t\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2}\right)\widetilde{Y}_{t+1}^Z P_{t+1}^Z \right\}$$

Substituting (50) and (51) into (49) we obtain an expression for the real marginal cost in units of the wholesale domestic good:

$$mc_t^Z = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{\left(R_t^k\right)^{\alpha}}{z_t A_t^{1-\alpha}} \left\{ W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right)\widetilde{Y}_t^Z P_t^Z \right.$$
$$\left. - r_{t,t+1}\gamma_n \mathbb{E}_t\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2}\right)\widetilde{Y}_{t+1}^Z P_{t+1}^Z \right\}^{1-\alpha}$$

In a second stage, the wholesale firm maximize its profits from the production of Y_t^Z, which is sold as X_t^Z at P_t^Z. The problem is:

$$\max_{Y_t^Z} \left(P_t^Z - mc_t^Z\right) Y_t^Z$$

The first-order condition implies that

$$P_t^Z = mc_t^Z.$$

2.4.7 Foreign composite goods

As in the case of home composite goods, a representative foreign composite goods firm demands foreign goods of all varieties $j \in [0,1]$ in amounts X_{jt}^F and combines them according to the technology

$$Y_t^F = \left[\int_0^1 \left(X_{jt}^F\right)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj\right]^{\frac{\epsilon_F}{\epsilon_F - 1}} \tag{52}$$

with $\epsilon_F > 0$. Let P_{jt}^F denote the price of the foreign good of variety j. Analogously to the case of home composite goods, profit maximization yields the input demand functions

$$X_{jt}^F = \left(\frac{P_{jt}^F}{P_t^F}\right)^{-\epsilon_F} Y_t^F \tag{53}$$

for all j, and substituting (53) into (52) yields the price of foreign composite goods:

$$P_t^F = \left[\int_0^1 \left(P_{jt}^F \right)^{1-\epsilon_F} dj \right]^{\frac{1}{1-\epsilon_F}} \tag{54}$$

2.4.8 Foreign goods of variety j

Importing firms buy an amount M_t of a homogeneous foreign good at the price $P_t^{M\star}$ abroad and convert this good into varieties Y_{jt}^F that are sold domestically, and where total imports are $\int_0^1 Y_{jt}^F dj$. We assume that the import price level $P_t^{M\star}$ cointegrates with the foreign producer price level $P_t^\star$, i.e., $P_t^{M\star} = P_t^\star \xi_t^m$, where ξ_t^m is a stationary exogenous process. The firm producing variety j satisfies the demand given by (53) but it has monopoly power for its variety. As it takes one unit of the foreign good to produce one unit of variety j, nominal marginal costs in terms of composite goods prices are

$$P_t^F mc_{jt}^F = P_t^F mc_t^F = S_t P_t^{M\star} = S_t P_t^\star \xi_t^m \tag{55}$$

Given marginal costs, the firm producing variety j chooses its price P_{jt}^F to maximize profits. In setting prices, the firm faces a Calvo-type problem similar to domestic firms, whereby each period the firm can change its price optimally with probability $1-\theta_F$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_F \in [0,1]$ and $1-\kappa_F$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^F$ that maximizes the current market value of the profits generated until it can reoptimize.[7] The solution to this problem is analogous to the case of domestic varieties, implying the first-order condition

$$F_t^{F_1} = F_t^{F_2} = F_t^F \tag{56}$$

where, defining $\tilde{p}_t^F = \tilde{P}_t^F / P_t^F$,

$$F_t^{F_1} = \frac{\epsilon_F - 1}{\epsilon_F} \left(\tilde{p}_t^F \right)^{1-\epsilon_F} Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} \left(\pi_{t+1}^F \right)^{\epsilon_F} F_{t+1}^{F_1} \right\}$$

and

$$F_t^{F_2} = \left(\tilde{p}_t^F \right)^{-\epsilon_F} mc_t^F Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\pi_{t+1}^F \right)^{1+\epsilon_F} F_{t+1}^{F_2} \right\}$$

Using (54), we further have

$$1 = (1-\theta_F) \left(\tilde{p}_t^F \right)^{1-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F} \right)^{1-\epsilon_F} \tag{57}$$

2.4.9 Wages

Recall that demand for productive labor is satisfied by perfectly competitive packing firms that demand all varieties $i \in [0,1]$ of labor services in amounts $n_t(i)$ and combine them in order to produce composite labor services $\widetilde{n}_t$. The production function, variety i demand, and aggregate nominal wage are respectively given by:

$$\widetilde{n}_t = \left[\int_0^1 n_t(i)^{\frac{\epsilon_W - 1}{\epsilon_W}} di \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \qquad \epsilon_W > 0. \tag{58}$$

$$n_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\epsilon_W} \widetilde{n}_t \tag{59}$$

[7] As in the home varieties case, the following relation holds:

$$P_{jt+s}^F = \tilde{P}_{jt}^F \pi_{t+1}^{I,F} \ldots \pi_{t+s}^{I,F}$$

where

$$\pi_t^{I,F} = \left(\pi_{t-1}^F \right)^{\kappa_F} \left(\pi_t^T \right)^{1-\kappa_F}$$

and, in turn, $\pi_t^F = P_t^F / P_{t-1}^F$.

$$W_t = \left[\int_0^1 W_t(i)^{1-\epsilon_W} di \right]^{\frac{1}{1-\epsilon_W}}. \tag{60}$$

Regarding the supply of differentiated labor, as in Erceg et al. (2010), there is a continuum of monopolistically competitive unions indexed by $i \in [0,1]$, which act as wage setters for the differentiated labor services supplied by households. These unions allocate labor demand uniformly across patient and impatient households, so $n_t^P(i) = n_t^I(i)$ and $n_t^P(i) + n_t^I(i) = n_t(i)\ \forall i, t$, with $n_t^P(i) = \wp_U n_t^U(i) + (1-\wp_U)\, n_t^R(i)$, which also holds for the aggregate n_t^P, n_t^I and n_t.

The union supplying variety i satisfies the demand given by (59) but it has monopoly power for its variety. Wage setting is subject to a Calvo-type problem, whereby each period a union can set its nominal wage optimally with probability $1-\theta_W$. The wages of unions that cannot optimally adjust, are indexed to a weighted average of past and steady state productivity and inflation, with a gross growth rate of

$$\pi_t^{I,W} \equiv a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W}$$

Where $\Gamma_{t,s}^W = \Pi_{i=1}^s \pi_{t+i}^{I,W}$ is the growth of indexed wages s periods ahead of t. A union reoptimizing in period t chooses the wage $\widetilde{W}_t$ (equal for patient and impatient households) that maximizes the households' discounted lifetime utility. This union weights the benefits of wage income by considering the agents' marginal utility of consumption –which will usually differ between patient and impatient households– and weighs each household equally by considering a lagrangian multiplier of $\lambda_t^W = \left(\lambda_t^P + \lambda_t^I\right)/2$, with $\lambda_t^P = \wp_U \lambda_t^U + (1-\wp_U)\, \lambda_t^R$. We assume, for the sake of simplicity, that $\beta_W = (\beta_P + \beta_I)/2$ with $\beta_P = \wp_U \beta_U + (1-\wp_U)\, \beta_R$, and $\Theta_t = \left(\Theta_t^P + \Theta_t^I\right)/2$ with $\Theta_t^P = \wp_U \Theta_t^U + (1-\wp_U)\, \Theta_t^R$.

All things considered, taking the aggregate nominal wage as given, the union i's maximization problem can be expressed as

$$\begin{aligned}
&\max_{\widetilde{W}_t(i)} E_t \sum_{s=0}^{\infty} (\beta_U \theta_W)^s \varrho_{t+s} \left(\frac{\lambda_{t+s}^U A_{t+s}^{-\sigma}}{P_{t+s}} \widetilde{W}_t \Gamma_{t,s}^W n_{t+s}(i) - \Theta_{t+s} (A_{t+s})^{1-\sigma} \xi_{t+s}^n \frac{n_{t+s}(i)^{1+\varphi}}{1+\varphi} \right), \\
&\text{s.t.} \quad n_{t+s}(i) = \left(\frac{\widetilde{W}_t \Gamma_{t,s}^W}{W_{t+s}} \right)^{-\epsilon_W} \widetilde{n}_{t+s},
\end{aligned}$$

Which, after some derivation, results in the FOCs in a recursive formulation:

$$f_t^{W1} = \tilde{w}_t^{1-\epsilon_W} \left(\frac{\epsilon_W - 1}{\epsilon_W} \right) \widetilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W - 1} f_{t+1}^{W1} \right\}$$

$$f_t^{W2} = \tilde{w}_t^{-\epsilon_W(1+\varphi)} mc_t^W \widetilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W(1+\varphi)} f_{t+1}^{W2} \right\}$$

Where $f_t^{W1} = f_t^{W2} = f_t^W$ are the LHS and RHS of the FOC respectively, $mc_t^W = \frac{-U_n/U_C}{W_t/A_t P_t} = \frac{\xi_t^n (\widetilde{n}_t)^\varphi}{\lambda_t^U} \left(\frac{A_t P_t}{W_t} \right) \Theta_t$, is the gap with the efficient allocation when wages are flexible[8], $\pi_{t+1}^W = \frac{W_{t+1}}{W_t}$, $\pi_{t+1}^{\widetilde{W}} = \frac{\widetilde{W}_{t+1}}{\widetilde{W}_t}$ and $\tilde{w}_t = \tilde{W}_t / W_t$.

Further, let $\Psi^W(t)$ denote the set of labor markets in which wages are not reoptimized in period t. By (60), the aggregate wage index W_t evolves as follows:

$$\begin{aligned}
(W_t)^{1-\epsilon_W} = \int_0^1 W_t(i)^{1-\epsilon_W} di &= (1-\theta_W) \left(\widetilde{W}_t\right)^{1-\epsilon_W} + \int_{\Psi^W(t)} \left[W_{t-1}(i)\, \pi_t^{I,W} \right]^{1-\epsilon_W} di, \\
&= (1-\theta_W) \left(\widetilde{W}_t\right)^{1-\epsilon_W} + \theta_W \left[W_{t-1} \pi_t^{I,W} \right]^{1-\epsilon_W},
\end{aligned}$$

or, dividing both sides by $(W_t)^{1-\epsilon_W}$:

$$1 = (1-\theta_W) \tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W} \right)^{1-\epsilon_W}.$$

[8] U_n and U_C are the first derivatives of the utility function with respect to labor and consumption respectively.

The third equality above follows from the fact that the distribution of wages that are not reoptimized in period t corresponds to the distribution of effective wages in period $t-1$, though with total mass reduced to θ_W.

Finally, the clearing condition for the labor market is

$$n_t = \int_0^1 n_t(i)\,di = \tilde{n}_t \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_W} di = \tilde{n}_t \Xi_t^W,$$

Where Ξ_t^W is a wage dispersion term that satisfies

$$\Xi_t^W = (1-\theta_W)\tilde{w}_t^{-\epsilon_W} + \theta_w \left(\frac{\pi_t^{I,W}}{\pi_t^W}\right)^{-\epsilon_W} \Xi_{t-1}^W.$$

2.4.10 Commodities

We assume the country receives an exogenous and stochastic endowment of commodities Y_t^{Co}. Moreover, these commodities are not consumed domestically but entirely exported. Therefore, the entire production is sold at a given international price $P_t^{Co\star}$, which is assumed to evolve exogenously. We further assume that the government receives a share $\chi \in [0,1]$ of this income and the remaining share goes to foreign agents.

2.5 Fiscal and monetary policy

The government consumes an exogenous stream of final goods G_t, levies lump-sum taxes T_t, and issues one-period bonds B_t and long-term bonds $B_t^{L,G}$. Hence, the government satisfies the following period-by-period constraint,

$$T_t - BS_t^G - Q_t^{BL} BL_t^G + \chi S_t P_t^{Co\star} Y_t^{Co} = P_t G_t - R_{t-1} BS_{t-1}^G - R_t^{BL} Q_t^{BL} BL_{t-1}^G + DIA_t \tag{61}$$

where

$$T_t = \alpha^T GDPN_t + \epsilon_t \left(BS_{SS}^G - BS_t^G + Q_{SS}^{BL} BL_{SS}^G - Q_t^{BL} BL_t^G\right) \tag{62}$$

As in Chen et al. (2012), we assume that the government control the supply of long-term bonds according to a simple rule given by an exogenous AR(1) process on BL_t^G. In turn, monetary policy is carried out according to a Taylor-type rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E)\pi_t + \alpha_E \mathbb{E}_t\{\pi_{t+4}\}}{\pi_t^T}\right)^{\alpha_\pi} \left(\frac{GDP_t/GDP_{t-1}}{a}\right)^{\alpha_y}\right]^{1-\alpha_R} e_t^m \tag{63}$$

where $\alpha_R \in [0,1)$, $\alpha_\pi > 1$, $\alpha_y \geq 0$, $\alpha_E \in [0,1]$ and where π_t^T is an exogenous inflation target and e_t^m an i.i.d. shock that captures deviations from the rule.[9]

2.6 Rest of the world

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the domestic economy is assumed to be small relative to the foreign economy. The latter implies that the foreign producer price level $P_t^\star$ is identical to the foreign consumption-based price index. Further, let $P_t^{H\star}$ denote the price of home composite goods expressed in foreign currency. Given full tradability and competitive export pricing, the law of one price holds separately for home composite goods and the commodity good, i.e. $P_t^H = S_t P_t^{H\star}$ and $P_t^{Co} = S_t P_t^{Co\star}$. That is, domestic and foreign prices of both goods are identical when expressed in the same currency. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods, i.e., $P_t^F mc_t^F = S_t P_t^\star \xi_t^m$ from (55). The real exchange rate rer_t therefore satisfies

$$rer_t = \frac{S_t P_t^\star}{P_t} = \frac{P_t^F}{P_t}\frac{mc_t^F}{\xi_t^m} \tag{64}$$

We also have the following relation

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{65}$$

[9]We do not need a time-varying target, so we will set it to a constant.

where $\pi_t^s = S_t/S_{t-1}$. Foreign demand for the home composite good $X_t^{H\star}$ is given by

$$X_t^{H\star} = \left(\frac{P_t^H}{S_t P_t^\star}\right)^{-\eta^\star} Y_t^\star \tag{66}$$

with $\eta^\star > 0$ and where $Y_t^\star$ denotes foreign aggregate demand or GDP. Both $Y_t^\star$ and $\pi_t^\star$ evolve exogenously. The relevant foreign nominal interest rate is composed by an exogenous risk-free world interest rate R_t^W plus a country premium that decreases with the economy's net foreign asset position (expressed as a ratio of nominal GDP):

$$R_t^\star = R_t^W \exp\left\{-\frac{\phi^\star}{100}\left(\frac{S_t B_t^\star}{GDPN_t} - \bar{b}\right)\right\} \xi_t^R z_t^R \tag{67}$$

with $\phi^\star > 0$ and where ξ_t^R is an exogenous shock to the country premium.

2.7 Aggregation and Market Clearing

2.7.1 Aggregation across patient households

Aggregate variables add up the per-capita amounts from unrestricted and restricted patient households, according to their respective mass $\wp_U$ and $1 - \wp_U$:

$$C_t^P = \wp_U C_t^U + (1 - \wp_U) C_t^R$$

$$H_t^P = \wp_U H_t^U + (1 - \wp_U) H_t^R$$

$$n_t^P = \wp_U n_t^U + (1 - \wp_U) n_t^R$$

$$n_t^U = n_t^R$$

$$D_t^{Tot} = \wp_U D_t^U$$

$$B_t^{*,Tot} = \wp_U B_t^{\star,U}$$

$$BS_t^{Pr} = \wp_U BS_t^U$$

$$BL_t^{Pr} = \wp_U BL_t^U + (1 - \wp_U) BL_t^R$$

$$BB_t^{Pr} = \wp_U BB_t^U + (1 - \wp_U) BB_t^R$$

2.7.2 Goods market clearing

In the market for the final good, the clearing condition is

$$Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + \Upsilon_t/P_t \tag{68}$$

where Υ_t includes final goods used in default costs: the resources lost by households recovering deposits at failed banks, the resources lost by the banks to recover the proceeds from defaulted bank loans by the recovery of deposits by the deposit insurance agency and the cost of adjusting labor.

$$\Upsilon_t = \begin{array}{c} \gamma_D PD_t^D R_{t-1}^D D_{t-1}^{Tot} + \gamma_D PD_t^D Q_t^{BB} R_t^{BB} BB_{t-1}^{Pr} + \mu_e G_e(\bar{\omega}_t^e) R_t^e Q_{t-1}^K K_{t-1} + \mu_I G_I(\bar{\omega}_t^I) R_t^H Q_{t-1}^H H_{t-1}^I \\ + \mu_H G_H(\bar{\omega}_t^H) \tilde{R}_t^H Q_{t-1}^L L_{t-1}^H + \mu_F G_F(\bar{\omega}_t^F) \tilde{R}_t^F L_{t-1}^F + \frac{\gamma_n}{2}\left(\frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1\right)^2 Y_t^Z \end{array}$$

In the market for the home and foreign composite goods we have, respectively,

$$Y_t^H = X_t^H + X_t^{H\star} \tag{69}$$

and

$$Y_t^F = X_t^F \tag{70}$$

while in the market for home and foreign varieties we have, respectively,

$$Y_{jt}^H = X_{jt}^H \tag{71}$$

and

$$Y_{jt}^F = X_{jt}^F \tag{72}$$

for all j.

In the market for the wholesale domestic good, we have

$$Y_t^Z = X_t^Z \tag{73}$$

Finally, in the market for housing, demand from both households must equal supply from housing producers:

$$H_t = H_t^P + H_t^I$$

2.7.3 Factor market clearing

In the market for labor, the clearing conditions are:

$$n_t^P + n_t^I = n_t = \widetilde{n}_t \Xi_t^W \tag{74}$$

$$n_t^P = n_t^I = \frac{n_t}{2} \tag{75}$$

Combining (51) and (50), the capital-labor ratio satisfies:

$$\frac{K_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) R_t^k} \left\{ W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) Y_t^Z P_t^Z - r_{t,t+1} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) Y_{t+1}^Z P_{t+1}^Z \right\}$$

2.7.4 Deposits clearing

Bank F takes deposits, and its demand must equal the supply from unrestricted households:

$$D_t^F = D_t^{Tot}$$

2.7.5 Domestic bonds clearing

The aggregate net holding of participating agents in bond markets are in zero net supply:

$$BL_t^{Pr} + BL_t^{CB} + BL_t^G = 0$$

$$BS_t^{Pr} + BS_t^G = 0$$

Where BL_t^{CB} is an exogenous process that represents the long-term government bond purchases done by the Central Bank.

2.7.6 The no-arbitrage condition

The no-arbitrage condition implies the following relation between short and long-tem interest rates:

$$R_t \left(\frac{1+\zeta_t^L}{R_t^{L,G} - \kappa_B} \right) = \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} \left(\frac{R_{t+1}^{L,G}}{R_{t+1}^{L,G} - \kappa_B} \right) A_{t+1}^{-\sigma} \right\} \left(\mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \right)^{-1}$$

which can be further rearranged (up to a first order) by using the definition of R_t^L

$$R_t \left(1 + \zeta_t^L\right) \quad \approx \quad \mathbb{E}_t \left\{ \left(\frac{Q_{t+1}^{L,B}}{Q_t^{L,B}} R_{t+1}^{L,G} \right) \right\} \tag{76}$$

2.7.7 Inflation and relative prices

The following holds for $j = H, F$:

$$p_t^j = \frac{P_t^j}{P_t}$$

and, also,

$$\frac{p_t^j}{p_{t-1}^j} = \frac{\pi_t^j}{\pi_t}$$

2.7.8 Aggregate supply

Using the productions of different varieties of home goods (43)

$$\int_0^1 Y_{jt}^H dj = X_t^Z$$

Integrating (71) over j and using (41) then yields aggregate output of home goods as

$$\int_0^1 Y_{jt}^H dj = \int_0^1 X_{jt}^H dj = Y_t^H \int_0^1 \left(p_{jt}^H\right)^{-\epsilon_H} dj$$

or, combining the previous two equations,

$$Y_t^H \Xi_t^H = X_t^Z$$

where Ξ_t^H is a price dispersion term satisfying

$$\begin{aligned}\Xi_t^H &= \int_0^1 \left(\frac{P_{jt}^H}{P_t^H}\right)^{-\epsilon_H} dj \\ &= (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{-\epsilon_H} \Xi_{t-1}^H\end{aligned}$$

2.7.9 Aggregate demand

Aggregate demand or GDP is defined as the sum of domestic absorption and the trade balance. Domestic absorption is equal to $Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + \Upsilon_t$. The nominal trade balance is defined as

$$TB_t = P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \tag{77}$$

Integrating (72) over j and using (53) shows that imports satisfy

$$M_t = \int_0^1 Y_{jt}^F dj = \int_0^1 X_{jt}^F dj = Y_t^F \int_0^1 \left(\frac{P_{jt}^F}{P_t^F}\right)^{-\epsilon_F} dj = Y_t^F \Xi_t^F$$

where Ξ_t^F is a price dispersion term satisfying

$$\Xi_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F} \Xi_{t-1}^F$$

We then define real and nominal GDP, respectively, as

$$GDP_t = C_t^P + C_t^I + I_t + I_t^H + G_t + X_t^{H\star} + Y_t^{Co} - M_t$$

and

$$GDPN_t = P_t\left(C_t^P + C_t^I + I_t + I_t^H + G_t\right) + TB_t \tag{78}$$

Note that by combining (78) with the zero profit condition in the final goods sector, i.e., $P_t Y_t^C = P_t^H X_t^H + P_t^F X_t^F$, and using the market clearing conditions for final and composite goods, (68)-(69), GDP is seen to be equal to total value added (useful for the steady state):

$$\begin{aligned}GDPN_t &= P_t Y_t^C - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H X_t^H + P_t^F X_t^F - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H Y_t^H + S_t P_t^{Co\star} Y_t^{Co} + P_t^F X_t^F - S_t P_t^{M\star} M_t - \Upsilon_t\end{aligned}$$

2.7.10 Balance of payments

Aggregate nominal profits, dividends, rents and taxes are given by

$$\begin{aligned}
\Psi_t &= \underbrace{P_t Y_t^C - P_t^H X_t^H - P_t^F X_t^F}_{\Pi_t^C} + \underbrace{P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj}_{\Pi_t^H} + \underbrace{P_t^F Y_t^F - \int_0^1 P_{jt}^F X_{jt}^F dj}_{\Pi_t^F} \\
&\quad + \underbrace{\int_0^1 Y_{jt}^H \left(P_{jt}^H - P_t^Z\right) dj}_{\int_0^1 \Pi_{jt}^H dj} + \underbrace{\int_0^1 \left(P_{jt}^F Y_{jt}^F - S_t P_t^{M\star} Y_{jt}^F\right) dj}_{\int_0^1 \Pi_{jt}^F dj} \\
&\quad + \underbrace{Q_t^K \left(K_t - (1-\delta_K) K_{t-1}\right) - P_t I_t}_{\Pi_t^I} + \underbrace{Q_t^H \left(H_t - (1-\delta_H) H_{t-1}\right) - P_t I_t^H}_{\Pi_t^{I^H}} + \underbrace{\left(P_t^Z - mc_t^Z\right) Y_t^Z}_{\Pi_t^Z} + \underbrace{\zeta_t^L \left(\frac{1}{R_t^{L,G} - \kappa_B}\right) B_t^{L,UP}}_{\Pi_t^F} \\
&\quad + C_t^e + C_t^b + S_t REN_t^* - T_t \\
&= P_t \left(C_t + G_t\right) + \Upsilon_t + P_t^H X_t^{H\star} - S_t P_t^{M\star} M_t - W_t n_t - R_t^k K_{t-1} \\
&\quad + Q_t^K \left(K_t - (1-\delta_K) K_{t-1}\right) + Q_t^H \left(H_t - (1-\delta_H) H_{t-1}\right) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left(\frac{1}{R_t^{L,G} - \kappa_B}\right) B_t^{L,UP} \\
&= P_t \left(C_t + G_t\right) + \Upsilon_t + TB_t - S_t P_t^{Co\star} Y_t^{Co} - W_t n_t - R_t^k K_{t-1} \\
&\quad + Q_t^K \left(K_t - (1-\delta_K) K_{t-1}\right) + Q_t^H \left(H_t - (1-\delta_H) H_{t-1}\right) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left(\frac{1}{R_t^{L,G} - \kappa_B}\right) B_t^{L,UP}
\end{aligned}$$

Where the second equality uses the market clearing conditions (68)-(75), and the third equality uses the definition of the trade balance, (77). Substituting out Ψ_t in the households' budget constraint (5) and using the government's budget constraint (61) to substitute out taxes T_t shows that the net foreign asset position evolves according to

$$S_t B_t^\star = S_t B_{t-1}^\star R_{t-1}^\star + TB_t + S_t REN_t^* - (1-\chi) S_t P_t^{Co\star} Y_t^{Co}$$

Table 1: Calibration of Parameters of the Real Sector

Parameter	Description	Value	Source
α	Labor share of 66%	0.34	Garcia et al. (2019)
α_E	Expected Inflation weight in Taylor Rule	0.5	Garcia et al. (2019)
β_U	Unrestricted Patient HH Utility Discount Factor	0.99997	Garcia et al. (2019)
β_R	Restricted Patient HH Utility Discount Factor	0.99997	Garcia et al. (2019)
α^{BSG}	Short-term govt. bonds as percentage of GDP	-0.4	Data: 2009-2019
α^{BLG}	Long-term govt. bonds as percentage of GDP	-4.5	Data: 2009-2019
β_I	Impatient Utility HH Discount Factor	0.98	Clerc et al. (2014)
χ	Codelco production as percentage of GDP	0.33	Garcia et al. (2019)
δ_H	Housing Annual Depreciation rate	0.01	Assumption: same as capital depreciation
δ_K	Capital Annual depreciation rate	0.01	Adolfson et al. (2013)
ϵ_F	Elasticity of subsititution among foreign goods	11	Garcia et al. (2019)
ϵ_H	Elasticity of subsititution among home goods	11	Garcia et al. (2019)
ϵ_W	Elasticity of subsititution of types of workers	11	Garcia et al. (2019)
ω	home bias in domestic demand	0.79	Garcia et al. (2019)
N_H	Time-to-build periods in housing goods	6	IEF 2018 S2
π_t^T	Annual inflation target of 3%	$1.03^{1/4}$	Garcia et al. (2019)
$\rho_{\varphi h}$	Spending profile for long term housing investment	1	Even investment distribution asumption
σ	Log Utility	1	Garcia et al. (2019)
υ	Strength of wealth Effect	0	No wealth effect
ω_U	Fraction of unrestricted patient households	0.7	Chen et al. (2012)
ω_{BL}	Ratio of long term assets to short assets	0.822	Chen et al. (2012)
ϵ_τ	Convergence speed towards SS Gov debt	0.1	Normalization
κ	Coupon discount in housing loans	0.975	Parameter implies a duration of 10 years
κ_{BL}	Coupon discount in long term government bonds	0.975	Parameter implies a duration of 10 years
κ_{BB}	Coupon discount in long term banking bonds	0.95	Parameter implies a duration of 5 years

3 Calibration and Estimation

As mentioned previously, this model takes as a starting point a reduced version of the model presented in Garcia et al. (2019), its real sector, which includes production of final and intermediate goods, an open economy structure, a government and consuming households. As such, for the calibration, most of the parameters related to the real sector use the same values used in Garcia et al. (2019). On the other hand, the financial sector was modeled after Clerc et al. (2014), so we take several parameters values from that work that are difficult to estimate from the data. Finally, the set of parameters that models the term premium of interest rates comes from Chen et al. (2012).

Table 2: Calibration of financial sector parameters

Parameter	**Description**	**Value**	**Source**
χ_b	Banks dividend policy	0.05	Clerc et al. (2015)
χ_e	Entrepreneurs dividen policy	0.05	Clerc et al. (2015)
γ_{bh}	Household cost bank bonds default	0.1	Clerc et al. (2015)
γ_d	Cost of recovering defaulted bank deposits	0.1	Clerc et al. (2015)
μ_e	Entrepreneurs bankruptcy cost	0.3	Clerc et al. (2015)
μ_F	Corporate bank bankruptcy cost	0.3	Clerc et al. (2015)
μ_H	Housing bank bankruptcy cost	0.3	Clerc et al. (2015)
μ_I	Impatient Household bankruptcy cost	0.3	Clerc et al. (2015)
ϕ_F	Bank Capital Requirement (RWA)	0.123	Data (2000-2020)
ϕ_H	Bank Capital Requirement (RWA)	0.091	Data (2000-2020)

The rest of the parameters either come directly from the data or are estimated using Bayesian methods. The parameters that set the steady state value of short term and long term government bonds as a percentage of GDP, α^{BSG} and α^{BLG}, respectively, were obtained from DCV. [10] Regarding the housing depreciation rate, δ_H, we assume that it has the same depreciation rate as productive capital. The value used is in line with the one used in Clerc et al. (2014). The value used for the time that takes a house to be built, N_H is taken from the second semester of 2018 IEF.[11] The value of the parameter that determines the strength of the wealth effect, υ, produces some problems if it is not calibrated to zero. This value also is in line with the value obtained in the estimation of Garcia et al. (2019). Finally, the parameters that determine the geometric decline of the long term housing debt, κ, and government bonds, κ_{BL}, are set so their duration is 10 years, while the duration of the bank bonds, κ_{BB}, is set to 5 years.

We compute the model solution by a linear approximation around the deterministic steady state. The parameters that are not calibrated are estimated by Bayesian methods using quarterly data from 2001q3 to 2019q3. Data for the real Chilean sector is obtained from the Central Bank of Chile, while prices and labor statistics are obtained from the National Statistics Institute (INE). Finally, financial data is obtained from the Financial Markets Committee (CMF) and foreign data is obtained from Bloomberg. A list of the data used can be found in 3. The results of the estimation appear in tables 4 and 5.

[10] DCV is an entity that processes and registers transfer operations that take place in several exchange markets.

[11] IEF stands for Financial Stability Report published twice a year by the Central Bank of Chile.

Table 3: Observable Data

Real Data		Financial Data	
$\Delta \log Y_t^{NoCo}$	Non mining real GDP	R_t^L	Comercial Loans interest rate
$\Delta \log Y_t^{Co}$	Copper real GDP	R_t^I	Housing Loans Interest Rate
$\Delta \log C_t$	Total Consumption	R_t^D	Nominal Interest Rate on Deposits
$\Delta \log G_t$	Goverment Consumption	R_t^{LG}	10 Year BCP Rate
$\Delta \log I_t^K$	Real Capital Investment	$\Delta \log(L_t)$	Housing and Corporate Loand
$\Delta \log I_t^H$	Real Housing Investment	ROE_t	Banks ROE
$TB_t/GDPN_t$	Trade Balance-GDP Ratio		
$\Delta \log N_t$	Total Employment		
$\Delta \log WN_t$	Nominal Cost of labor		
π_t	CPI w/o volatiles		
R_t	Nominal MPR		
rer_t	Real Exchange Rate		
$\Delta \log y_t^*$	Real External GDP		
π_t^*	Foreign Price Index		
π_t^M	Imports Deflactor		
π_t^{Co*}	Nominal Copper Price		
R_t^*	LIBOR		
Ξ_t^R	EMBI Chile		
π_t^H	Housing Price Index		

Sources: INE, BCCh, CMF and Bloomberg.

Table 4: Estimated Deep Parameters

Parameter	**Description**	**prior mean**	**mode**	**s.d.**	**prior dist**	**pstdev**
α_π	Inflation weight in Taylor Rule	1.7	2.2256	0.071	norm	0.1
α_R	Previous interest rate weight in Taylor Rule	0.85	0.7329	0.016	beta	0.025
α_W	Weight on past productivity on wage indexation	0.25	0.2136	0.0753	beta	0.075
α_y	Output weight in Taylor Rule	0.125	0.1718	0.0607	norm	0.075
η	Elasticity of substitution between home and foreign goods	1	1.7674	0.2675	gamm	0.25
$\eta_{\hat{C}}$	CES Calibration	1	0.8728	0.0473	gamm	0.25
η^*	Foreign elasticity of substitution between home and foreign goods	0.25	0.1459	0.0458	gamm	0.075
γ_H	Housing investment adjustment cost parameter	3	2.6307	0.2281	gamm	0.25
γ_K	Capital investment adjustment cost parameter	3	2.7775	0.2288	gamm	0.25
γ_n	labor adjustment cost parameter	3	1.55	0.1476	gamm	0.25
κ_F	Weight on past inflation on foreign good indexation	0.5	0.5576	0.065	beta	0.075
κ_H	Weight on past inflation on home good indexation	0.5	0.6992	0.068	beta	0.075
κ_W	Weight on past inflation on wages indexation	0.85	0.837	0.0268	beta	0.025
ϕ^*	Country premium parameter in the foreign interest rate	1	0.2341	0.0388	invg	Inf
ϕ_c	Habit formation in good consumption	0.85	0.743	0.0282	beta	0.025
ϕ_{hh}	Habit formation in housing consumption	0.85	0.856	0.015	beta	0.025
θ_F	Probability of foreign goods producer to not adjust prices	0.5	0.7859	0.0207	beta	0.075
θ_H	Probability of domestic goods producer to not adjust prices	0.5	0.8208	0.0105	beta	0.025
θ_W	Probability of wage setter to not adjust prices	0.5	0.7573	0.0265	beta	0.075
φ	Labor elasticty	7.5	6.6759	1.3552	gamm	1.5
η_{ζ_L}	Sensibility of term premium to changes in portfolio	0.15	0.1431	0.0292	gamm	0.03

Table 5: Estimated Parameters of Shock

Shock Description	**Autocorrelation of Shocks**					**Variance of Shocks**				
	Par.	**pr. mean**	**mode**	**s.d.**	**pr. dist**	**Par.**	**pr. mean**	**mode**	**s.d.**	**pr. dist**
Non stationary productivity	ρ_a	0.25	0.2906	0.0831	beta	σ_a	0.5	0.2288	0.0461	invg
Monetary Policy	ρ_{e^m}	0.15	0.0746	0.0438	beta	σ_{e^m}	0.5	0.132	0.0114	invg
Goverment spending	ρ_g	0.75	0.7356	0.0725	beta	σ_g	0.5	1.275	0.081	invg
Copper price	$\rho_{p^{co}}$	0.75	0.8374	0.0246	beta	$\sigma_{p^{co}}$	0.5	12.9031	0.753	invg
Foreign Inflation	ρ_{π^*}	0.75	0.3854	0.0291	beta	σ_{π^*}	0.5	2.3444	0.1335	invg
Rest of the world interest rate	ρ_{R^W}	0.75	0.8353	0.0244	beta	σ_{R^W}	0.5	0.1288	0.0125	invg
Entrepreneurs risk	$\rho_{\sigma^{ee}}$	0.75	0.8983	0.0356	beta	$\rho_{\sigma^{ee}}$	0.5	0.1529	0.0228	invg
Corporate bank risk	$\rho_{\sigma^{ff}}$	0.75	0.5173	0.068	beta	$\sigma_{\sigma^{ff}}$	0.5	0.9171	0.1471	invg
Housing bank risk	$\rho_{\sigma^{hh}}$	0.75	0.7634	0.0772	beta	$\sigma_{\sigma^{hh}}$	0.5	0.2272	0.0903	invg
Impatient risk	$\rho_{\sigma^{II}}$	0.75	0.7677	0.077	beta	$\sigma_{\sigma^{II}}$	0.5	0.2326	0.0968	invg
Preference	ρ_{ϱ}	0.75	0.5573	0.0731	beta	σ_{ϱ}	0.5	3.325	0.4937	invg
Housing preference	ρ_{ξ^h}	0.75	0.9311	0.0153	beta	σ_{ξ^h}	0.5	6.7633	3.1614	invg
Capital investment efficiency	ρ_{ξ^I}	0.75	0.4435	0.0559	beta	σ_{ξ^I}	0.5	5.6263	0.8781	invg
Housing investment efficiency	$\rho_{\xi^{ih}}$	0.75	0.5459	0.0602	beta	$\sigma_{\xi^{ih}}$	0.5	11.7034	2.4834	invg
Foreign producer price	ρ_{ξ^m}	0.75	0.6872	0.0571	beta	σ_{ξ^m}	0.5	2.0835	0.2418	invg
Labor disutility	ρ_{ξ^n}	0.75	0.4242	0.0622	beta	σ_{ξ^n}	0.5	16.5326	5.732	invg
Country premium	ρ_{ξ^R}	0.75	0.7	0.0415	beta	σ_{ξ^R}	0.5	0.0686	0.0046	invg
Banker dividend	$\rho_{\xi^{\chi b}}$	0.75	0.4036	0.0851	beta	$\sigma_{\xi^{\chi b}}$	0.5	0.6284	0.1691	invg
Entrepreneur dividend	$\rho_{\xi^{\chi e}}$	0.75	0.7296	0.0697	beta	$\sigma_{\xi^{\chi e}}$	0.5	0.3465	0.1785	invg
Banker return requirement	$\rho_{\xi^{roe}}$	0.75	0.7552	0.0497	beta	$\sigma_{\xi^{roe}}$	0.5	0.3897	0.0651	invg
Foreign output	$\rho_{\xi^{y*}}$	0.85	0.9026	0.0475	beta	$\sigma_{\xi^{y*}}$	0.5	0.372	0.04	invg
Copper Production	$\rho_{\xi^{yco}}$	0.85	0.7905	0.08	beta	$\sigma_{\xi^{yco}}$	0.5	2.5429	0.1907	invg
Stationary productivity	ρ_z	0.85	0.9491	0.0166	beta	σ_z	0.5	0.3452	0.0624	invg
Unobservable country premium	ρ_{z_τ}	0.75	0.6508	0.0518	beta	σ_{z_τ}	0.5	0.6401	0.1334	invg
Transaction costs	ρ_{ϵ^L}	0.75	0.9396	0.0176	beta	σ_{ϵ^L}	0.5	2.7202	0.8794	invg

4 Conclusion

This document presents the MaFin model, a large scales estimated macroeconomic DSGE model for the Chilean economy. The main characteristic of the model is that it incorporates into a large scale DSGE monetary model with financial frictions, defaults and a rich financial sector. The model is based on the Garcia et al. (2019) for the real sector and Clerc et al. (2014) for the financial sector.

The existence of the financial sector comes motivated by the need of entrepreneurs and households to finance capital and housing investment, respectively. The financial sector, in turn, obtains resources for these loans from households in the form of deposits and banking bonds. The model also incorporates long term bonds for housing, government and banking financing whose rates deviates from the expectation hypothesis by introducing preferred habitat theory of investments as in Chen et al. (2012).

The rich and microfounded structure of the MaFin model allows it to become a bridge between monetary policy and financial policy. It not only builds a unified framework for the separate analysis of these two policies but also for the analysis of interaction when these policies act in tandem. In particular, it allows for the study of episodes when there is an increase in the default rate or the risk of firms and households.

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A Stationary Equilibrium Conditions

We define $a_t = A_t/A_{t-1}$, $g_t = G_t/A_t$, $y_t^{Co} = Y_t^{Co}/A_t$, $y_t^\star = Y_t^\star/A_t$, $p_t^{Co\star} = P_t^{Co\star}/P_t^*$, $bl_t^{CB} = \frac{BL_t^{CB}}{A_t P_t}$, and $bl_t^G = \frac{BL_t^G}{A_t P_t}$, and we assume that each exogenous variable follows an AR(1) process:

$$\log(x_t/x) = \rho_x \log(x_{t-1}/x) + u_t^x$$

for $x = \left\{a, e^m, g, p^{Co\star}, \pi^\star, R^W, \sigma^e, \sigma^F, \sigma^H, \sigma^I, \varrho, \xi^h, \xi^i, \xi^{ih}, \xi^m, \xi^n, \xi^R, y^\star, y^{Co}, z, bl_t^{CB}, bl_t^G, \epsilon_t^{L,S}\right\}$, and where all disturbances u_t^x are white noise.

Using the above definitions, in this section the model is brought into stationary form. For this, the following variables are defined: $w_t = \frac{W_t}{A_t P_t}$, $r_t^k = \frac{R_t^k}{P_t}$, $tb_t = \frac{TB_t}{A_t P_t}$, $b_t^\star = \frac{B_t^\star}{A_t P_t^\star}$, $bs_t^U = \frac{BS_t^U}{A_t P_t}$, $q_t^K = \frac{Q_t^K}{P_t}$, $q_t^H = \frac{Q_t^H}{P_t}$, $q_t^{BL} = \frac{Q_t^{BL}}{P_t}$, $c_t^i = \frac{C_t^i}{A_t P_t}$, $n_t^i = \frac{N_t^i}{A_t P_t}$, $\psi_t^i = \frac{\Psi_t^i}{A_t P_t}$, $l_t^j = \frac{L_t^j}{A_t P_t}$, $d_t^j = \frac{D_t^j}{A_t P_t}$, $e_t^j = \frac{E_t^j}{A_t P_t}$, $d_t = \frac{D_t}{A_t P_t}$, $\upsilon_t = \frac{\Upsilon_t}{A_t P_t}$, $gdpn_t = \frac{GDPN_t}{A_t P_t}$ and the constant $ren^* = \frac{REN_t^*}{A_t P_t^*}$ for $i = \{e, b\}$ and $j = \{F, H\}$. In addition, all other upper case variables with a unit root are divided by A_t (including $bl_t = \frac{BL_t}{A_t}$, $bl_t^U = \frac{BL_t^U}{A_t}$, $bl_t^R = \frac{BL_t^R}{A_t}$,) and written as lower case variables.

The rational expectations equilibrium of the stationary version of the model is then the set of sequences for the endogenous variables such that for a given set of initial values and exogenous processes the following conditions are satisfied:

A.1 Patient Households

A.1.1 Unrestricted (UP)

$$\hat{c}_t^U = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c_t^U - \phi_c \frac{c_{t-1}^U}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(\frac{h_{t-1}^U}{a_t} - \phi_{hh} \frac{h_{t-2}^U}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{1}$$

$$\lambda_t^U = \left(\hat{c}_t^U\right)^{-\sigma} \left(\frac{(1 - o_{\hat{C}})\, \hat{c}_t^U}{\left(c_t^U - \phi_c \frac{c_{t-1}^U}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{2}$$

$$\varrho_t \lambda_t^U q_t^H = \beta_U \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{c}_{t+1}^U a_{t+1}\right)^{-\sigma} \xi_{t+1}^h \left(\frac{o_{\hat{C}} \hat{c}_{t+1}^U a_{t+1}}{\xi_{t+1}^h \left(h_t^U - \phi_{hh} \frac{h_{t-1}^U}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} + (1 - \delta_H)\, \lambda_{t+1}^U a_{t+1}^{-\sigma} q_{t+1}^H \right\} \tag{3}$$

$$\varrho_t \lambda_t^U = \beta_U R_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \tag{4}$$

$$\varrho_t \lambda_t^U = \beta_U \mathbb{E}_t \left\{ \frac{\tilde{R}_{t+1}^D}{\pi_{t+1}} \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} \right\} \tag{5}$$

$$\varrho_t \lambda_t^U = \beta_U R_t^\star \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U \pi_{t+1}^s}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \tag{6}$$

$$\varrho_t \lambda_t^U \left(1 + \zeta_t^L\right) q_t^{BL} = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} R_{t+1}^{BL} q_{t+1}^{BL} \right\} \tag{7}$$

$$\varrho_t \lambda_t^U \left(1 + \zeta_t^L\right) q_t^{BB} = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} \tilde{R}_{t+1}^{BB} q_{t+1}^{BB} \right\} \tag{8}$$

A.1.2 Restricted (RP)

$$\hat{c}_t^R = \left[\left(1-o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(c_t^R - \phi_c \frac{c_{t-1}^R}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(\frac{h_{t-1}^R}{a_t} - \phi_{hh} \frac{h_{t-2}^R}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{9}$$

$$\lambda_t^R = \left(\hat{c}_t^R\right)^{-\sigma} \left(\frac{\left(1-o_{\hat{C}}\right) \hat{c}_t^R}{\left(c_t^R - \phi_c \frac{c_{t-1}^R}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{10}$$

$$\varrho_t \lambda_t^R q_t^H = \beta_R \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{c}_{t+1}^R a_{t+1}\right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{c}_{t+1}^R a_{t+1}}{\xi_{t+1}^h \left(h_t^R - \phi_{hh} \frac{h_{t-1}^R}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h + \left(1-\delta_H\right) \lambda_{t+1}^R a_{t+1}^{-\sigma} q_{t+1}^H \right\} \tag{11}$$

$$\varrho_t \lambda_t^R q_t^{BL} = \beta_R \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^R q_{t+1}^{BL} R_{t+1}^{BL} a_{t+1}^{-\sigma} \right\} \tag{12}$$

$$\varrho_t \lambda_t^R q_t^{BB} = \beta_R \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^R q_{t+1}^{BB} R_{t+1}^{BB} a_{t+1}^{-\sigma} \right\} \tag{13}$$

$$q_t^{BL} bl_t^R + q_t^{BB} bb_t^R + c_t^R + q_t^H h_t^R = q_t^{BL} R_t^{BL} \frac{bl_{t-1}^R}{a_t} + q_t^{BB} R_t^{BB} \frac{bb_{t-1}^R}{a_t} + w_t n_t^R + q_t^H \left(1-\delta_H\right) \frac{h_{t-1}^R}{a_t} \tag{14}$$

A.2 Impatient Households

$$\frac{R_t^H}{\pi_t} = \frac{q_t^H \left(1-\delta_H\right)}{q_{t-1}^H} \tag{15}$$

$$\hat{c}_t^I = \left[\left(1-o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(c_t^I - \phi_c \frac{c_{t-1}^I}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(\frac{h_{t-1}^I}{a_t} - \phi_{hh} \frac{h_{t-2}^I}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{16}$$

$$\lambda_t^I = \left(\hat{c}_t^I\right)^{-\sigma} \left(\frac{\left(1-o_{\hat{C}}\right) \hat{c}_t^I}{\left(c_t^I - \phi_c \frac{c_{t-1}^I}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{17}$$

$$\bar{\omega}_t^I = \frac{R_t^I q_t^L l_{t-1}^H}{R_t^H q_{t-1}^H h_{t-1}^I} \pi_t \tag{18}$$

$$R_t^I = \frac{1}{q_{t-1}^L} + \kappa \tag{19}$$

$$\varrho_t \lambda_t^I q_t^H = \mathbb{E}_t \left\{ \begin{array}{c} \beta_I \varrho_{t+1} \left(\left(\hat{c}_{t+1}^I a_{t+1}\right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{c}_{t+1}^I a_{t+1}}{\xi_{t+1}^h \left(h_t^I - \phi_{hh} \frac{h_{t-1}^I}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h + \lambda_{t+1}^I a_{t+1}^{-\sigma} \left[1 - \Gamma_I\left(\bar{\omega}_{t+1}^I\right)\right] \frac{R_{t+1}^H}{\pi_{t+1}} q_t^H \right) + \\ \varrho_t \lambda_t^I \left[1 - \Gamma_H\left(\bar{\omega}_{t+1}^H\right)\right] \left[\Gamma_I\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I\left(\bar{\omega}_{t+1}^I\right)\right] \frac{R_{t+1}^H q_t^H}{\tilde{\rho}_{t+1}^H \phi_H} \end{array} \right\} \tag{20}$$

$$\beta_I \mathbb{E}_t \left\{ \tilde{\rho}^H_{t+1} \right\} = \mathbb{E}_t \left\{ \frac{\varrho_t \lambda^I_t \pi_{t+1}}{\phi_H \varrho_{t+1} \lambda^I_{t+1} a^{-\sigma}_{t+1}} \left[1 - \Gamma_H \left(\bar{\omega}^H_{t+1} \right) \right] \frac{\left[\Gamma'_I \left(\bar{\omega}^I_{t+1} \right) - \mu_I G'_I \left(\bar{\omega}^I_{t+1} \right) \right]}{\Gamma'_I \left(\bar{\omega}^I_{t+1} \right)} \right\} \tag{21}$$

$$c^I_t + q^H_t h^I_t - q^L_t l^H_t = \frac{w_t n_t}{2} + \left[1 - \Gamma_I \left(\bar{\omega}^I_t \right) \right] \frac{R^H_t q^H_{t-1} h^I_{t-1}}{a_t \pi_t} \tag{22}$$

$$PD^I_t = F_I \left(\bar{\omega}^I_t \right) \tag{23}$$

A.3 Entrepreneurs

$$q^K_t k_t = n^e_t + l^F_t \tag{24}$$

$$\frac{R^e_t}{\pi_t} = \frac{r^k_t + (1 - \delta_K) q^K_t}{q^K_{t-1}} \tag{25}$$

$$\bar{\omega}^e_t = \frac{R^L_{t-1} l^F_{t-1}}{R^e_t q^K_{t-1} k_{t-1}} \tag{26}$$

$$c^e_t = \xi_{\chi_e} \chi_e \psi^e_t \tag{27}$$

$$n^e_t = (1 - \xi_{\chi_e} \chi_e) \psi^e_t \tag{28}$$

$$\psi^e_t a_t \pi_t = \left[1 - \Gamma_e \left(\bar{\omega}^e_t \right) \right] R^e_t q^K_{t-1} k_{t-1} \tag{29}$$

$$\left(1 - \Gamma^e_{t+1} \right) = \lambda^e_t \left(\frac{\rho^F_{t+1} \phi^F_t}{R^e_{t+1}} - \left(1 - \Gamma^F_{t+1} \right) \left[\Gamma^e_{t+1} - \mu^e G^e_{t+1} \right] \right) \tag{30}$$

$$\Gamma^{e'}_{t+1} = \lambda^e_t \left(1 - \Gamma^F_{t+1} \right) \left[\Gamma^{e'}_{t+1} - \mu^e G^{e'}_{t+1} \right] \tag{31}$$

$$PD^e_t = F_e \left(\bar{\omega}^e_t \right) \tag{32}$$

A.4 F Banks

$$d^F_t + e^F_t = l^F_t \tag{33}$$

$$\bar{\omega}^F_t = (1 - \phi_F) \frac{R^D_{t-1}}{\tilde{R}^F_t} \tag{34}$$

$$e^F_t = \phi_F l^F_t \tag{35}$$

$$\tilde{\rho}^F_t = \left[1 - \Gamma_F \left(\bar{\omega}^F_t \right) \right] \frac{\tilde{R}^F_t}{\phi_F} \tag{36}$$

$$\tilde{R}^F_t = \left[\Gamma_e \left(\bar{\omega}^e_t \right) - \mu_e G_e \left(\bar{\omega}^e_t \right) \right] \frac{R^e_t q^K_{t-1} k_{t-1}}{l^F_{t-1}} \tag{37}$$

$$PD^F_t = F_F \left(\bar{\omega}^F_t \right) \tag{38}$$

A.5 H Banks

$$q_t^{BB}bb_t^{Pr}+e_t^H=q_t^Ll_t^H \tag{39}$$

$$\bar{\omega}_t^H=(1-\phi_H)\,\frac{R_t^{BB}q_t^{BB}}{\widetilde{R}_t^H q_{t-1}^{BB}}\pi_t \tag{40}$$

$$e_t^H=\phi_H q_t^L l_t^H \tag{41}$$

$$\rho_t^H=\left[1-\Gamma_H\left(\bar{\omega}_t^H\right)\right]\frac{\tilde{R}_t^H}{\phi_H} \tag{42}$$

$$\tilde{R}_t^H=\left[\Gamma_I\left(\bar{\omega}_t^I\right)-\mu_I G_I\left(\bar{\omega}_t^I\right)\right]\frac{R_t^H q_{t-1}^H h_{t-1}^I}{q_{t-1}^L l_{t-1}^H} \tag{43}$$

$$PD_t^H=F_H\left(\bar{\omega}_t^H\right) \tag{44}$$

A.6 Bankers and Banking System

$$\tilde{\rho}_t^H=(1-\kappa)\,\rho_t^H+\kappa\mathbb{E}\left[\tilde{\rho}_{t+1}^H\right] \tag{45}$$

$$\mathbb{E}\left[\rho_{t+1}^F\right]=\xi_t^{b,roe}\mathbb{E}\left[\rho_{t+1}^H\right] \tag{46}$$

$$c_t^b=\xi_t^{\chi_b}\chi_b\psi_t^b \tag{47}$$

$$n_t^b=\left(1-\xi_t^{\chi_b}\chi_b\right)\psi_t^b \tag{48}$$

$$\psi_t^b a_t\pi_t=\rho_t^F e_{t-1}^F+\tilde{\rho}_t^H e_{t-1}^H \tag{49}$$

$$n_t^b=e_t^F+e_t^H \tag{50}$$

$$PD_t^D=\frac{Q_{t-1}^{BB}BB_{t-1}PD_t^H+d_{t-1}^{Tot}PD_t^F}{Q_{t-1}^{BB}BB_{t-1}+d_{t-1}^{Tot}} \tag{51}$$

A.7 Capital and Housing Goods

$$k_t=(1-\delta_K)\frac{k_{t-1}}{a_t}+\left[1-\frac{\gamma_K}{2}\left(\frac{i_t^K}{i_{t-1}^K}a_t-a\right)^2\right]\xi_t^i i_t^K \tag{52}$$

$$\begin{aligned}1 \quad &= \quad q_t^K\left[1-\frac{\gamma_K}{2}\left(\frac{i_t^K}{i_{t-1}^K}a_t-a\right)^2-\gamma_K\left(\frac{i_t^K}{i_{t-1}^K}a_t-a\right)\frac{i_t^K}{i_{t-1}^K}a_t\right]\xi_t^i \\ &+ \quad \beta_P\mathbb{E}_t\left\{\frac{\varrho_{t+1}\lambda_{t+1}^P}{\varrho_t\lambda_t^P}a_{t+1}^{-\sigma}q_{t+1}^K\gamma_K\left(\frac{i_{t+1}^K}{i_t^K}a_{t+1}-a\right)\left(\frac{i_{t+1}^K}{i_t^K}a_{t+1}\right)^2\xi_{t+1}^i\right\}\end{aligned} \tag{53}$$

$$h_t=(1-\delta_H)\frac{h_{t-1}}{a_t}+\left[1-\frac{\gamma_H}{2}\left(\frac{i_{t-N_H}^{AH}}{i_{t-N_H-1}^{AH}}a_t-a\right)^2\right]\xi_{t-N_H}^{ih}\frac{i_{t-N_H}^{AH}}{\prod_{i=0}^{N_H-1}a_{t-j}} \tag{54}$$

$$
\begin{aligned}
0 &= E_t \sum_{j=0}^{N_H} \beta_P^j \varrho_{t+j} \lambda_{t+j}^P \varphi_j^H \prod_{i=j+1}^{N_H} \left(a_{t+i}^{\sigma}\right) \\
&- E_t \beta_P^{N_H} \varrho_{t+N_H} \lambda_{t+N_H}^P q_{t+N_H}^H \left\{ \left[1 - \frac{\gamma_H}{2} \left(\frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right)^2 \right] - \gamma_H \left(\frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right) \frac{i_t^{AH}}{i_{t-1}^{AH}} a_t \right\} \xi_t^{ih} \\
&- E_t \beta_P^{N_H+1} \varrho_{t+N_H+1} \lambda_{t+N_H+1}^P q_{t+N_H+1}^H a_{t+N_H+1}^{-\sigma} \left\{ \gamma_H \left(\frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} - a \right) \left(\frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} \right)^2 \xi_{t+1}^{ih} \right\}
\end{aligned} \tag{55}
$$

$$
i_t^H = \sum_{j=0}^{N_H} \varphi_j^H \frac{i_{t-j}^{AH}}{\prod_{i=0}^{j-1} a_{t-j}} \tag{56}
$$

A.8 Final Goods

$$
y_t^C = \left[\omega^{1/\eta} \left(x_t^H\right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(x_t^F\right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} \tag{57}
$$

$$
x_t^F = (1-\omega) \left(p_t^F\right)^{-\eta} y_t^C \tag{58}
$$

$$
x_t^H = \omega \left(p_t^H\right)^{-\eta} y_t^C \tag{59}
$$

A.9 Home Goods

$$
f_t^H = \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} \left(\pi_{t+1}^H\right)^{\epsilon_H} f_{t+1}^H \right\} \tag{60}
$$

$$
f_t^H = \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left(\pi_{t+1}^H\right)^{1+\epsilon_H} f_{t+1}^H \right\} \tag{61}
$$

$$
1 = (1-\theta_H) \left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H} \right)^{1-\epsilon_H} \tag{62}
$$

$$
\pi_t^{I,H} = \left(\pi_{t-1}^H\right)^{\kappa_H} \left(\pi^T\right)^{1-\kappa_H} \tag{63}
$$

$$
mc_t^H = \frac{p_t^Z}{p_t^H} \tag{64}
$$

A.10 Wholesale Domestic Goods

$$
\begin{aligned}
mc_t^Z = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(r_t^k)^\alpha}{z_t} \Bigg\{ & w_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) y_t^Z p_t^Z \\
& - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \Bigg\}^{1-\alpha}
\end{aligned} \tag{65}
$$

$$
\begin{aligned}
\frac{k_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) r_t^k} \Bigg\{ & w_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) y_t^Z p_t^Z \\
& - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \Bigg\} a_t
\end{aligned} \tag{66}
$$

$$
p_t^Z = mc_t^Z \tag{67}
$$

A.11 Foreign Goods

$$p_t^F mc_t^F = rer_t \xi_t^m \tag{68}$$

$$f_t^F = \frac{\epsilon_F - 1}{\epsilon_F} \left(\tilde{p}_t^F\right)^{1-\epsilon_F} y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} \left(\pi_{t+1}^F\right)^{\epsilon_F} f_{t+1}^F \right\} \tag{69}$$

$$f_t^F = \left(\tilde{p}_t^F\right)^{-\epsilon_F} mc_t^F y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\pi_{t+1}^F\right)^{1+\epsilon_F} f_{t+1}^F \right\} \tag{70}$$

$$1 = (1-\theta_F) \left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F} \right)^{1-\epsilon_F} \tag{71}$$

$$\pi_t^{I,F} = \left(\pi_{t-1}^F\right)^{\kappa_F} \left(\pi^T\right)^{1-\kappa_F} \tag{72}$$

A.12 Wages

$$\lambda_t^W = \frac{\lambda_t^P + \lambda_t^I}{2} \tag{73}$$

$$\lambda_t^P = \wp_U \lambda_t^U + (1-\wp_U) \lambda_t^R \tag{74}$$

$$\Theta_t = \frac{\left(\wp_U \Theta_t^U + (1-\wp_U)\Theta_t^R\right) + \Theta_t^I}{2} \tag{75}$$

$$mc_t^W = \Theta_t \frac{\xi_t^n \left(\widetilde{n}_t\right)^{\varphi}}{\lambda_t^U w_t} \tag{76}$$

$$\Theta_t^i = \tilde{\chi}_t^i \left(\hat{c}_t^i\right)^{-\sigma} \quad \forall \quad i = \{U, R, I\} \tag{77}$$

$$\tilde{\chi}_t^i = \left(\tilde{\chi}_{t-1}^i\right)^{1-v} \left(\hat{c}_t^i\right)^{\sigma v} \quad \forall \quad i = \{U, R, I\} \tag{78}$$

$$\begin{aligned} f_t^W =& \left(\frac{\epsilon_W - 1}{\epsilon_W}\right) \tilde{w}_t^{1-\epsilon_W} \widetilde{n}_t \\ &+ \left(\frac{\left(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}\right) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}\lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W - 1} f_{t+1}^W \right\} \end{aligned} \tag{79}$$

$$\begin{aligned} f_t^W =& \tilde{w}_t^{-\epsilon_W(1+\varphi)} mc_t^W \widetilde{n}_t \\ &+ \left(\frac{\left(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}\right) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}\lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W(1+\varphi)} f_{t+1}^W \right\} \end{aligned} \tag{80}$$

$$1 = (1-\theta_W) \tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W} \right)^{1-\epsilon_W} \tag{81}$$

$$\pi_t^{I,W} = a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W} \tag{82}$$

A.13 Monetary Policy and Rest of the World

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E)\,\pi_t + \alpha_E \mathbb{E}_t\{\pi_{t+4}\}}{\pi_t^T} \right)^{\alpha_\pi} \left(\frac{gdp_t}{gdp_{t-1}}\right)^{\alpha_y} \right]^{1-\alpha_R} e_t^m \tag{83}$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{84}$$

$$R_t^\star = R_t^W \exp\left\{ \frac{-\phi^\star}{100} \left(\frac{rer_t b_t^\star}{gdpn_t} - \frac{rer b^\star}{gdpn} \right) \right\} \xi_t^R z_t^\tau \tag{85}$$

$$x_t^{H\star} = \left(\frac{p_t^H}{rer_t}\right)^{-\eta^\star} y_t^\star \tag{86}$$

A.14 Fiscal Policy

$$\begin{aligned} \tau_t + R_{t-1}\frac{bs_{t-1}^G}{a_t\pi_t} + q_t^{BL} R_t^{BL} bl_{t-1}^G \frac{1}{a_t} + \chi s_t p_t^{Co\star} y_t^{Co} =& g_t + bs_t^G + q_t^{BL} bl_t^G + \gamma_D \frac{PD_t^D R_{t-1}^D d_{t-1}^F}{a_t\pi_t} \\ &+ \gamma_{BH}\frac{PD_t^H R_t^{BB} q_t^{BB} bb_{t-1}^{priv}}{a_t} \end{aligned} \tag{87}$$

$$\tau_t = \alpha^T gdpn_t + \epsilon_t \left(bs^G - bs_t^G + q^{BL} bl^G - q_t^{BL} bl_t^G \right) \tag{88}$$

A.15 Aggregation and Market Clearing

$$y_t^C = c_t^P + c_t^I + i_t^K + i_t^H + g_t + \upsilon_t \tag{89}$$

$$c_t^P = \wp_U c_t^U + (1-\wp_U)\, c_t^R \tag{90}$$

$$\begin{aligned} \upsilon_t a_t \pi_t =& \gamma_D PD_t^D R_{t-1}^D d_{t-1}^F + \gamma_{BH} PD_t^H R_t^{BB} q_t^{BB} bb_{t-1}^{Priv} + \mu_e G_e(\bar{\omega}_t^e) R_t^e q_{t-1}^K k_{t-1} + \mu_I G_I(\bar{\omega}_t^I) R_t^H q_{t-1}^H h_{t-1}^I \\ &+ \mu_H G_H(\bar{\omega}_t^H) \tilde{R}_t^H l_{t-1}^H q_{t-1}^L + \mu_F G_F(\bar{\omega}_t^F) \tilde{R}_t^F l_{t-1}^F + \frac{\gamma_n}{2}\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)^2 y_t^Z p_t^Z \end{aligned} \tag{91}$$

$$y_t^H = x_t^H + x_t^{H\star} \tag{92}$$

$$y_t^F = x_t^F \tag{93}$$

$$h_t = h_t^P + h_t^I \tag{94}$$

$$h_t^P = \wp_U h_t^U + (1-\wp_U)\, h_t^R \tag{95}$$

$$bl_t^{Pr} = \wp_U bl_t^U + (1-\wp_U)\, bl_t^R \tag{96}$$

$$bs_t^{Pr} = \wp_U bs_t^U \tag{97}$$

$$bb_t^{Tot} = \wp_U bb_t^U \tag{98}$$

$$b_t^{*Tot} = \wp_U b_t^{*U} \tag{99}$$

$$bl_t^{Pr} + bl_t^{CB} + bl_t^G = 0 \tag{100}$$

$$bs_t^{Pr} + bs_t^G = 0 \tag{101}$$

$$d_t^F = \wp_U d_t^U \tag{102}$$

$$\zeta_t^L = \left(\frac{q_t^{BL} bl_t^U + q_t^{BB} bb_t^U}{bs_t^U + rer_t b_t^{\star,U} + d_t^U} \right)^{\eta_\zeta} \epsilon_t^{L,S} \tag{103}$$

$$\tilde{R}_t^D = R_{t-1}^D \left(1 - \gamma_D PD_t^D\right) \tag{104}$$

$$\tilde{R}_t^{BB} = R_t^{BB} \left(1 - \gamma_{BH} PD_t^H\right) \tag{105}$$

$$R_t^{BL} = \frac{1}{q_t^{BL}} + \kappa_{BL} \tag{106}$$

$$R_t^{BB} = \frac{1}{q_t^{BB}} + \kappa_{BB} \tag{107}$$

$$R_t^{Nom,BL} = R_t^{BL} \pi_t \tag{108}$$

$$\frac{p_t^H}{p_{t-1}^H} = \frac{\pi_t^H}{\pi_t} \tag{109}$$

$$\frac{p_t^F}{p_{t-1}^F} = \frac{\pi_t^F}{\pi_t} \tag{110}$$

$$\pi_t^W = \frac{w_t}{w_{t-1}} a_t \pi_t \tag{111}$$

$$\pi_t^{\widetilde{W}} = \frac{\widetilde{w}_t}{\widetilde{w}_{t-1}} \pi_t^W \tag{112}$$

$$y_t^H \Xi_t^H = x_t^Z \tag{113}$$

$$y_t^Z = z_t \left(\frac{k_{t-1}}{a_t} \right)^{\alpha} \widetilde{n}_t^{1-\alpha} \tag{114}$$

$$y_t^Z = x_t^Z \tag{115}$$

$$\Xi_t^H = (1 - \theta_H) \left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H} \right)^{-\epsilon_H} \Xi_{t-1}^H \tag{116}$$

$$m_t = y_t^F \Xi_t^F \tag{117}$$

$$\Xi_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F\left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F}\Xi_{t-1}^F \tag{118}$$

$$n_t = \widetilde{n}_t \Xi_t^W \tag{119}$$

$$n_t = n_t^P + n_t^I \tag{120}$$

$$n_t^P = n_t^I \tag{121}$$

$$n_t^P = \wp_U n_t^{UP} + (1-\wp_U)\, n_t^R \tag{122}$$

$$n_t^U = n_t^R \tag{123}$$

$$\Xi_t^W = (1-\theta_W)\tilde{w}_t^{-\epsilon_W} + \theta_w\left(\frac{\pi_t^{I,W}}{\pi_t^W}\right)^{-\epsilon_W}\Xi_{t-1}^W \tag{124}$$

$$gdp_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + x_t^{H\star} + y_t^{Co} - m_t \tag{125}$$

$$gdpn_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + tb_t \tag{126}$$

$$tb_t = p_t^H x_t^{H\star} + rer_t p_t^{Co\star} y_t^{Co} - rer_t \xi_t^m m_t \tag{127}$$

$$rer_t b_t^\star = \frac{rer_t}{a_t \pi_t^\star} b_{t-1}^\star R_{t-1}^\star + tb_t + rer_t ren^* - (1-\chi)\, rer_t p_t^{Co\star} y_t^{Co} \tag{128}$$

The exogenous processes are:

$$\log(z_t/z) = \rho_z \log(z_{t-1}/z) + u_t^z$$
$$\log(a_t/a) = \rho_a \log(a_{t-1}/a) + u_t^a$$
$$\log(\xi_t^n/\xi^n) = \rho_{\xi^n} \log(\xi_{t-1}^n/\xi^n) + u_t^{\xi^n}$$
$$\log(\xi_t^h/\xi^h) = \rho_{\xi^h} \log(\xi_{t-1}^h/\xi^h) + u_t^{\xi^h}$$
$$\log(\xi_t^i/\xi^i) = \rho_{\xi^i} \log(\xi_{t-1}^i/\xi^i) + u_t^{\xi^i}$$
$$\log(\xi_t^{ih}/\xi^{ih}) = \rho_{\xi^{ih}} \log(\xi_{t-1}^{ih}/\xi^{ih}) + u_t^{\xi^{ih}}$$
$$\log(\xi_t^R/\xi^R) = \rho_{\xi^R} \log(\xi_{t-1}^R/\xi^R) + u_t^{\xi^R}$$
$$\log(e_t^m/e^m) = \rho_{e^m} \log(e_{t-1}^m/e^m) + u_t^{e^m}$$
$$\log(g_t/g) = \rho_g \log(g_{t-1}/g) + u_t^g$$
$$\log(y_t^{Co}/y^{Co}) = \rho_{y^{Co}} \log(y_{t-1}^{Co}/y^{Co}) + u_t^{y^{Co}}$$
$$\log(\pi_t^\star/\pi^\star) = \rho_{\pi^\star} \log(\pi_{t-1}^\star/\pi^\star) + u_t^{\pi^\star}$$
$$\log(R_t^W/R^W) = \rho_{R^W} \log(R_{t-1}^W/R^W) + u_t^{R^W}$$
$$\log(y_t^\star/y^\star) = \rho_{y^\star} \log(y_{t-1}^\star/y^\star) + u_t^{y^\star}$$
$$\log(p_t^{Co\star}/p^{Co\star}) = \rho_{p^{Co\star}} \log(p_{t-1}^{Co\star}/p^{Co\star}) + u_t^{p^{Co\star}}$$
$$\log(\xi_t^m/\xi^m) = \rho_{\xi^m} \log(\xi_{t-1}^m/\xi^m) + u_t^{\xi^m}$$

$$\log(\sigma_t^I/\sigma^I) = \rho_{\sigma^I} \log(\sigma_{t-1}^I/\sigma^I) + u_t^{\sigma^I}$$
$$\log(\sigma_t^e/\sigma^e) = \rho_{\sigma^e} \log(\sigma_{t-1}^e/\sigma^e) + u_t^{\sigma^e}$$
$$\log(\sigma_t^F/\sigma^F) = \rho_{\sigma^F} \log(\sigma_{t-1}^F/\sigma^F) + u_t^{\sigma^F}$$
$$\log(\sigma_t^H/\sigma^H) = \rho_{\sigma^H} \log(\sigma_{t-1}^H/\sigma^H) + u_t^{\sigma^H}$$
$$\log(\epsilon_t^{L,S}/\epsilon^{L,S}) = \rho_{\epsilon^{L,S}} \log(\epsilon_{t-1}^{L,S}/\epsilon^{L,S}) + u_t^{\epsilon^{L,S}}$$
$$\log(bl_t^G/bl^G) = \rho_{bl^G} \log(bl_{t-1}^G/bl^G) + u_t^{bl^G}$$
$$\log(bl_t^{CB}/bl^{CB}) = \rho_{bl^{CB}} \log(bl_{t-1}^{CB}/bl^{CB}) + u_t^{bl^{CB}}$$
$$\log(\varrho_t/\varrho) = \rho_\varrho \log(\varrho_{t-1}/\varrho) + u_t^\varrho$$
$$\log(\xi_t^{\chi b}/\xi^{\chi b}) = \rho_\xi^{\chi b} \log(\xi_{t-1}^{\chi b}/\xi^{\chi b}) + u_t^{\xi^{\chi b}}$$
$$\log(\xi_t^{\chi e}/\xi^{\chi e}) = \rho_\xi^{\chi e} \log(\xi_{t-1}^{\chi e}/\xi^{\chi e}) + u_t^{\xi^{\chi e}}$$
$$\log(\xi_t^{roe}/\xi^{roe}) = \rho_\xi^{roe} \log(\xi_{t-1}^{roe}/\xi^{roe}) + u_t^{\xi^{roe}}$$
$$\log(z_t^\tau/z^\tau) = \rho_{z^\tau} \log(z_{t-1}^\tau/z^\tau) + u_t^{z^\tau}$$

All disturbances u are white noise.