Production, Investment and Wealth Dynamics under Financial Frictions: A General Equilibrium Application to Chile*

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1 Introduction

An extensive literature has studied the importance of financial frictions as determinants of cross-country differences in aggregate income, investment, and productivity. In an economy in where access to external financing is restricted and subject to credit constraints, productive firms with low levels of wealth might be unable to operate at their optimal scale. This can lead to misallocation, reducing aggregate productivity. However, these adverse aggregate effects can be dampened over time and become less relevant, as firms might choose to endogenously grow out of their financial constraints by accumulating wealth and building up collateral -the so-called self financing channel. Under this argument, the elasticities of investment and wealth accumulation by firms to productivity shocks, and how they relate to collateral, play a central role. Therefore, the determinants of these elasticties, such as the parameters governing preferences and technology and the persistence and volatility of the firm-level productivity process, are crucial for the potential strength of self financing as a factor that can alleviate misallocation.[1]

*This note borrows heavily from Aguirre, Tapia and Villaccorta (2021), "Production, Investment and Wealth Dynamics under Financial Frictions: An Empirical Investigation of the Selffinancing Channel", so the same acknowledgments apply. The views expressed in this paper are exclusively those of the authors and do not necessarily reflect the position of the Central Bank of Chile or its Board members. Any errors or omissions are responsibility of the authors. Emails: aaguirre@bcentral.cl, mtapia@bcentral.cl, lvillacorta@bcentral.cl.

[1] For example, Buera and Shin [2011] and Moll [2014] discuss how the degree of persistence of productivity shocks affects the strength of the self-financing channel.

Thus, an empirical analysis of the wealth and productivity processes at firms is essential to understand the quantitative effects of financial frictions at the aggregate level . However, an analysis of these objects using micro data has been absent from the literature. Moreover, the standard approaches to estimate the parameters of the firm's production function and the productivity process are invalid in the presence of financial frictions. In Aguirre et al. [2021], we explore empirically the strength of the self-financing channel by developing a novel empirical framework, robust to the presence of financial constraints, that jointly estimates the firm's production function, the productivity process at the firm process, and the wealth accumulation and investment policy functions. We implement this new estimation method on a sample of firms in the manufacturing sector, using administrative tax records for the universe of Chilean firms between 2006 and 2016. The dataset provides information on the firm's output, inputs and its investment decisions, as well as measures of the firm's net worth. Our results provide empirical evidence of the presence of financial frictions and the self-financing channel. We identify crucial parameters—such as the response of wealth accumulation to productivity shocks—that can discipline quantitative macro models of firm dynamics and financial frictions and quantify the strength of the self-financing channel. This note takes the main insights and results from the paper, and uses them to calibrate a stylized macroeconomic model for the Chilean economy, to get a quantitative measure of the importance of financial frictions for output and productivity.

2 Model

The quantitative model is an extended version of the stylized model presented in Aguirre et al. [2021]. A novelty of our quantitative analysis is the use of moments coming from the estimated policy functions in the paper to parameterize the model. The existence of these additional moments allows us to be more flexible in the specification of the model, particularly in defining preferences and the collateral constraint.

Our model follows Midrigan and Xu [2014], although we only have one sector. We consider an economy with a mass of firms F_t that grows over time at a constant rate γ. We allow for two versions of the model. The first one one has exogenous exit, such that firms face a probability of death 1-q every period. Firms that disappear are replaced by new firms. We also consider an economy with endogenous exit given the existence of an outside option. New firms draw their productivity from the unconditional distribution, and start with K_0 and A_0 stocks of capital and net assets. Operating firms face stochastic shocks to productivity, and must invest in physical capital before observing the productivity shock realization of the current period. Investment is subject to adjustment costs and a collateral constraint that limits debt. In line with the self-financing channel, firms will optimally accumulate wealth to relax the collateral constraint. Firms also hire labor in a frictionless competitive market.

Under exogenous exit, operating firms solve the following maximization problem

$$
\begin{aligned}
V(A_{it}, K_{it}, Z_{it}) &= \max_{A_{it+1}, I_{it}, L_{it}} U(D_{it} + \bar{C};\ \sigma) + \beta q E\left[V(A_{it+1}, K_{it+1}, Z_{it+1}) | Z_{it}\right] \\
st. \quad D_{it} + A_{it+1} &= (1-\tau)(Y_{it} - W L_{it} - (r+\delta) K_{it} + (1+r) A_{it} - \psi(I_t)) + A_{it} \\
Y_{it} &= Z_t K_t^{\beta_k} L_t^{\beta_l} \\
K_{it+1} &= I_{it} + (1-\delta) K_{it} \\
K_{it+1} &\leq \kappa(A_{it}, Z_{it}) \\
D_t + \bar{C} &\geq 0
\end{aligned}
$$

Firms maximize a CRRA utility function $U(\cdot)$ with coefficient of risk aversion σ, and remove the convex function $g(\cdot)$. The argument of this function is dividends D_{it} plus a constant $\bar{C} > 0$. This flexible functional form for preferences takes advantage of the fact that we can use the estimates of the policy functions for calibration. Note that $\bar{C}$ not only affects the curvature of the function at low levels of dividends, but also determines the strength of the limited liability constraint since D_{it} might be negative, but not lower than $-\bar{C}$. This last constraint implies a lower-bound for net-wealth which, due to adjustment costs to capital, is increasing in this last variable. Hence, even if the collateral constraint allows the firm to maintain a high level of capital and productivity is high enough to obtain positive profits, the firm may not be able to invest at that level because a sudden reduction in productivity may generate losses that cannot be absorbed with available assets.

As mentioned, firms face adjustment costs to capital. Again, we try to make the model as flexible as possible. We assume the following functional form

$$\psi(I_{it}) = \eta \left(\frac{I_{it}}{K_{it}}\right)^2 K_{it}$$

where $\eta > 0$ is a constant.

Finally we assume the following functional form for the collateral constraint:

$$\kappa(A_{it}, Z_{it}) = (\lambda + \lambda_z(z_{it} - \bar{z}))A_{it}$$

where λ and λ_z are constants, z_{it} is the log of Z_{it} and $\bar{z}$ its mean, and we impose $\lambda + \lambda_z(\min(z_{it}) - \bar{z}) \geq 1$.

Therefore, credit access does not only depend on assets A_{it}, but also on the productivity of the firm relative to the economy's mean productivity. All else equal, more productive firms can borrow more for a given level of assets.

The productivity process is given by

$$\log(Z_{it+1}) = \mu_z + \rho \log(Z_{it}) + \eta_{it}$$

, where

$$\eta_{it} \sim N(0, \sigma_\eta^2].$$

When exit is endogenous, the incumbent's firm problem is very similar, and can be written as:

$$V(A_{it}, K_{it}, Z_{it}) \quad = \quad \max_{A_{it+1}, I_{it}, L_{it}} \; U(D_{it} + \bar{C};\ \sigma) + \beta E\left[V^*(A_{it+1}, K_{it+1}, Z_{it+1})|Z_{it}\right]$$

, where the continuation value now includes the choice of liquidation.

If the firm continues to operate, it is subject to

$$\begin{aligned}
D_{it} + A_{it+1} &= (1-\tau)(\quad Y_{it} - WL_{it} - (r+\delta)K_{it} + (1+r)A_{it} - \psi(I_t) - \Theta) + A_{it} \\
Y_{it} &= \quad Z_t K_t^{\beta_k} L_t^{\beta_l} \\
K_{it+1} &= \quad I_{it} + (1-\delta)K_{it} \\
K_{it+1} &\leq \quad \kappa(A_{it}, Z_{it}) \\
D_t + \bar{C} &\geq \quad 0
\end{aligned}$$

,

, where operation of the firm now includes a per-period fixed cost Θ.

The exit decision is given by

$$V^*(A_{it}, K_{it}, Z_{it}) = \max(V(A_{it}, K_{it}, Z_{it}), V^o(A_{it}, K_{it}))$$

where $\mathrm{V}^o(A_{it}, K_{it}) = \max_{A_{it+1}}(D_{it} + \bar{C}; \sigma) + \beta V^o(A_{it+1}, 0)$

If the firm exits, the entrepreneur faces the adjustment cost of liquidating the remaining capital, and after that period simply earns the market returns on its assets

$$D_{it} + A_{it+1} = -\delta K_{it} - \psi((1-\delta)K_{it}) + r_{it}(A_{it} - K_{it}) + A_{it}$$

and $\mathrm{D}_t + \bar{C} \geq 0$

3 Calibration

The first group of parameter relates to the production function, and has a clear link with the estimates from the exercises in Aguirre et al. [2021]. In particular we set $\beta_k = 0.43$ and $\beta_l = 0.47$. This implies a span of control parameter of 0.9. In the case of the productivity process, we set $\rho = 0.82$ and σ_μ such that $\sigma_z = 0.49$, consistent with our previous estimations.

We set a second group of parameters, like the depreciation rate, the interest rate, the net entry rate and the exogenous exit rate, from related literature or directly from the data. In particular, we set the depreciation rate $\delta = 0.1$, the annual real interest rate $\mathring{=}0.04$, the net entry rate rate $\gamma = 5.5\%$, corresponding to the average growth rate of the manufacturing sector over the last 20 years, and the exogenous exit rate = 9.8%. Parameters for both groups are presented in Table1.

We parameterize the rest of the parameters in both versions of the model by requiring that the models account for some relevant features of the data and the first moments of our estimated policy functions. Table 2 lists the moments used to calibrate these parameters.

Table 3 summarizes the parameter values used to obtain the selected moments in both models. Instead of fixing their values ex ante, we use the preference parameters β and σ to search for the best fit. For the discount factor β we find a values that are slightly smaller that those typically assumed in previous papers, while for the risk aversion coefficient σ we obtain values that are very close to 2, consistent with standard measures of households preferences.[2]

[2] Buera and Shin fix the interest rate at 4.5% and, since theirs is a general equilibrium model, must set $\beta = 0.904$ to obtain that value. They set $\sigma = 1.5$. Midrigan and Xu set the discount factor at 0.92 and assume log utility,so σ=1.

Table 1: Estimated and Assigned Parameters

		Value
Estimated parameters:		
Capital elasticity	β_K	0.43
Labor elasticity	β_L	0.47
AR(1) coeff. productivity	ρ	0.82
Std. Dev. productivity	σ_z	0.49
Assigned parameters:		
Depreciation rate	δ	0.1
Interest rate (returns)	r	0.04
Net entry	γ	5.5
Exit (exog)	$1-q$	9.8

Table 2: Targeted Moments

	Target	Model 1	Model 2
From assets policy function estimation:			
AR(1) coeff.	0.60	0.48	0.60
Productivity elasticity	0.41	0.53	0.47
Capital elasticity	0.25	0.19	0.15
Other moments:			
Capital to output K/Y	1.69	1.62	1.61
Net assets to output A/Y	0.89	0.89	0.93
SD Output	1.54	1.16	1.19
Exit rate	9.8		9.8

Table 3: Calibrated Parameters

		Model 1	Model 2
Discount factor	β	0.881	0.852
Coefficient of risk aversion	σ	2.192	2.060
Constant in utility function	$\bar{C}$	0.410	0.347
Collateral constraint	λ	2.506	2.311
Collateral constraint	λ_Z	1.163	1.005
Constant in adjustment costs	η	0.315	0.509
Interest rate spread	ψ	6.515	5.373
Fixed cost (% wage)	Θ		0.270

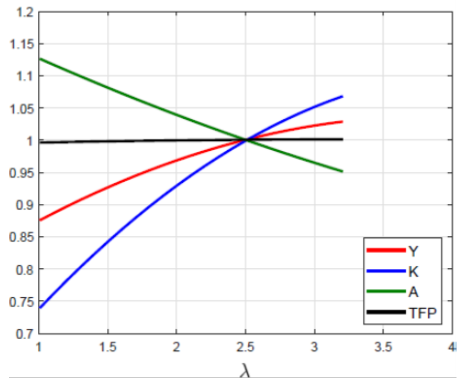
How does the collateral constraint look in the data? We find that a firm that has a productivity level equal to the distribution mean can hold capital that is almost twice the value of its net-assets. This means that the debt to net-assets ratio cannot be larger than 1.31 in thw model with endogenous exit. This is close to the 1.35 imposed by Buera and Shin, and much lower than the value of 7.14 found by Midrigan and Shu for Korea. A novel result from our calibration is that the ratio is dependent on the productivity of the firm. We find a positive and relatively large effect. A firm whose productivity is in the 10th percentile of the stationary distribution can only sustain a debt to net-assets ratio of up to 0.83, while for the one in the 90th percentile this number rises to 1.9.

4 Macroeconomic implications of financial frictions

We conclude this note by using our calibrated models to simulate the effect of changes in the collateral constraints on macroeconomic outcomes. In particular, we simulate the effect of changes in λ, the main multiplier on assets in the collateral constraint, on aggregate output, capital, TFP and total assets. Results are presented in Figures 1 (model with exogenous exit) and 2 (model with endogenous exit). In both figures, we show variations in λ with respect to its calibrated value, and its effect on the four macroeconomic outcomes, measured as deviations from the initial steady state. Increases in λ inambiguously relax the financial constraint, as they allow firms to borrow more for a given level of assets and productivity.

Both figures suggest that changes in the level of financial frictions only

Figure 1: Simulated Effects of Changes in Financial Constraints: Model with Exogenous Exit



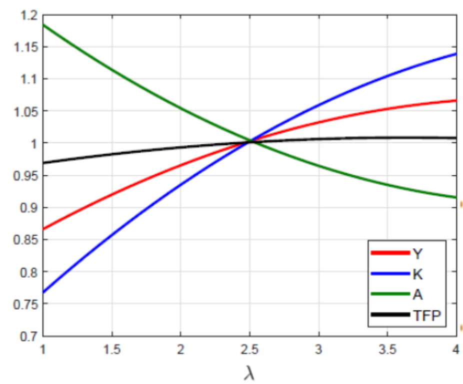
Note: The graph shows the simulated effect of changes in λ (higher λ, higher debt-to-assets ratio) from its calibrated value (2.5) on output (Y), capital (K), assets (A) and aggregate productivity (TFP). All values are defined relative to the initial equilibrium.

have a modest effect on efficiency, as the impacts of changes in λ on TFP is quite small. However, effects on aggregate capital are more substantial, as the effects on the firms' borrowing ability has a relevant impact on investment. By construction, these effects on capital are associated to significant impacts on output. In the model with endogenous exit, a large increase in λ from 2.5 to 3.5 (equivalent to increasing the debt to assets ratio from 1.5 to 2.5) increases steady-state capital by 10%, a significant although not exceedingly large figure. The figures also illustrate the operation of the self-financing channel: in an economy with tighter financial constraints (lower λ), asset accumulation by firms is significantly larger. A reduction in λ from 2.5 to 1.5 (equivalent to decreasing the debt to assets ratio from 1.5 to 0.5) increases assets by almost 10%, even if firms are now significantly smaller in scale. As financial frictions are eased, asset holdings fall accordingly.

5 Conclusion

The effects of collateral constraints on capital accumulation and productivity are likely to be important, although recent research suggests that the impact can be softened by self-financing by firms. This note presents a stylized general equilibrium of financial frictions, and calibrates it for the

Figure 2: Simulated Effects of Changes in Financial Constraints: Model with Endogenous Exit



Note: The graph shows the simulated effect of changes in λ (higher λ, higher debt-to-assets ratio) from its calibrated value (2.5) on output (Y), capital (K), assets (A) and aggregate productivity (TFP). All values are defined relative to the initial equilibrium.

manufacturing sector in Chile using the estimates from microdata in Aguirre et al (2021). Results show that a reduction in financial constraints has a very modest effect on efficiency and overall productivity, but a more substantial effect in the capital stock.

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