Productivity and Trade Dynamics in Sudden Stops

Felipe Benguria

University of Kentucky

Hidehiko Matsumoto

GRIPS - Japan

Felipe E. Saffie

University of Virginia

CBC-IDB-JIE Conference

May 2021

Introduction

- Persistent negative effects of sudden stops on output.
 - Meza and Quintin (2007), Pratap and Urrutia (2012)
- Distortions in firm's dynamism can dampen productivity.
 - Ates and Saffie (2016), Schmitz (2021)
- Sudden stops are accompanied by RER depreciation.
 - Alessandria, Pratap, and Yue (2014), Alfaro et al. (2018)
- ► To export a product generates firm-level productivity gain.
 - Bernard and Jensen (2004), Garcia-Marin and Voigtlander (2019)

Introduction

- Persistent negative effects of sudden stops on output.
 - Meza and Quintin (2007), Pratap and Urrutia (2012)
- Distortions in firm's dynamism can dampen productivity.
 - Ates and Saffie (2016), Schmitz (2021)
- Sudden stops are accompanied by RER depreciation.
 - Alessandria, Pratap, and Yue (2014), Alfaro et al. (2018)
- ► To export a product generates firm-level productivity gain.
 - Bernard and Jensen (2004), Garcia-Marin and Voigtlander (2019)

Questions

- 1. What role do exporters play in aggregate productivity dynamics following sudden stops?
- 2. Do firms adjust their product entry decisions to local and foreign markets during sudden stops?

This Paper

Quantitative model:

- Endogenous financial crises.
- Endogenous productivity and product dynamics.
- Endogenous entry and exit from exporting at the product level.

This Paper

Quantitative model:

- Endogenous financial crises.
- Endogenous productivity and product dynamics.
- Endogenous entry and exit from exporting at the product level.

Chilean product-level data 1996-1999:

- Unique data on Chilean manufacturing firms' product portfolios.
- Model can replicate product dynamics.
- ► Firms shift their portfolio towards exporting products.

This Paper

Quantitative model:

- Endogenous financial crises.
- Endogenous productivity and product dynamics.
- Endogenous entry and exit from exporting at the product level.

Chilean product-level data 1996-1999:

- Unique data on Chilean manufacturing firms' product portfolios.
- Model can replicate product dynamics.
- ► Firms shift their portfolio towards exporting products.

► Main results:

- The collapse of domestic product entry drives the short-run impact of the crisis.
- The persistence of the crisis is mostly driven by the product level exporting dynamics.
- ► 30% of welfare effects are attributed to endogenous firm dynamics.

Literature

Sudden stops with occasionally binding constraints:

Mendoza (2010), Bianchi (2011), Benigno et al. (2013), Bianchi and Mendoza (2018), Jeanne and Korinek (2020), etc.

► IRBC models with endogenous growth:

Comin and Gertler (2006), Gornemann (2014), Ates and Saffie (2016), Benigno and Fornaro (2018), Guerron-Quintana and Jinnai (2019), Ma (2020), Matsumoto (2021), Queralto (2020), etc.

Trade and endogenous growth:

Sampson (2015), Alfaro et al. (2018), Akcigit, Ates, and Impullitti (2018), Buera and Oberfield (2020), Perla, Tonetti, and Waugh (2021), etc.

Literature

Sudden stops with occasionally binding constraints:

Mendoza (2010), Bianchi (2011), Benigno et al. (2013), Bianchi and Mendoza (2018), Jeanne and Korinek (2020), etc.

► IRBC models with endogenous growth:

Comin and Gertler (2006), Gornemann (2014), Ates and Saffie (2016), Benigno and Fornaro (2018), Guerron-Quintana and Jinnai (2019), Ma (2020), Matsumoto (2021), Queralto (2020), etc.

Trade and endogenous growth:

Sampson (2015), Alfaro et al. (2018), Akcigit, Ates, and Impullitti (2018), Buera and Oberfield (2020), Perla, Tonetti, and Waugh (2021), etc.

Literature

Sudden stops with occasionally binding constraints:

Mendoza (2010), Bianchi (2011), Benigno et al. (2013), Bianchi and Mendoza (2018), Jeanne and Korinek (2020), etc.

► IRBC models with endogenous growth:

Comin and Gertler (2006), Gornemann (2014), Ates and Saffie (2016), Benigno and Fornaro (2018), Guerron-Quintana and Jinnai (2019), Ma (2020), Matsumoto (2021), Queralto (2020), etc.

Trade and endogenous growth:

Sampson (2015), Alfaro et al. (2018), Akcigit, Ates, and Impullitti (2018), Buera and Oberfield (2020), Perla, Tonetti, and Waugh (2021), etc.

Contributions

- 1. Model: Endogenous crises, firms and trade dynamics.
- 2. **Data:** Firm's product portfolio dynamics during crises and product level validation of Klette and Kortum (2004).

Model

Model Economy:



Final Goods Sector: Mendoza (2010)

Production function:

$$Y_t = \exp\left(\varepsilon_t^A\right)\exp\left[\int_0^1\ln(y_t(i))di
ight]$$

Final Goods Sector: Mendoza (2010)

Production function:

$$Y_t = \exp\left(\varepsilon_t^A\right)\exp\left[\int_0^1\ln(y_t(i))di
ight]$$

Borrow from abroad to finance working capital.

Subject to occasionally binding collateral constraint.

$$-B_t + \phi\left[\int_0^1 p_t(i)y_t(i)di\right] \le \kappa Q_t L$$

Final Goods Sector: Mendoza (2010)

Production function:

$$Y_t = \exp\left(\varepsilon_t^A\right)\exp\left[\int_0^1\ln(y_t(i))di
ight]$$

► Borrow from abroad to finance working capital.

Subject to occasionally binding collateral constraint.

$$-B_t + \phi\left[\int_0^1 p_t(i)y_t(i)di\right] \le \kappa Q_t L$$

Demand for each type of intermediate good *i*:

$$y_t(i) = \frac{Y_t}{p_t(i)} \frac{1}{1 + \phi \frac{\mu_t}{\lambda_t}}$$

• μ_t : Lagrange multiplier on the borrowing constraint.

FOC

Intermediate Sector: Klette and Kortum (2004)



- Each product is produced by the firm with the highest productivity.
- Each firm is a collection of domestic (D) and export (X) lines.

Intermediate Sector: Export Entry



• Firms invest in their own domestic lines to start exporting.

Products are sold domestically before being exported.

Intermediate Sector: Domestic Product Entry



► Firms invest in other firms' products to replace other firms.

► Firms' product portfolios endogenously expand and shrink.

Intermediate Sector: Foreign Innovation



• Foreign innovation pushes export lines back to domestic lines.

Intermediate Sector: Foreign Innovation



► Foreign innovation pushes export lines back to domestic lines.

► Foreign innovation can also steal a domestic product.

Intermediate Sector: Domestic Entry



- ► New entrants steal products.
- ► There is also direct entry into exporting.

• Production uses capital and labor: $y_t(i) = a_t(i) (\ell_t(i))^{\alpha} (h_t(i))^{1-\alpha}$

- Production uses capital and labor: $y_t(i) = a_t(i) (\ell_t(i))^{\alpha} (h_t(i))^{1-\alpha}$
- Profit in the domestic market (s = D, X):

$$\pi_t^s(i) = y_t(i) \left[p_t(i) - \operatorname{Mg} \operatorname{Cost}_t(i) \right]$$

- Production uses capital and labor: $y_t(i) = a_t(i) (\ell_t(i))^{\alpha} (h_t(i))^{1-\alpha}$
- Profit in the domestic market (s = D, X):

$$\pi_t^{s}(i) = \frac{Y_t}{p_t(i)} \frac{1}{1 + \phi \frac{\mu_t}{\lambda_t}} \left[p_t(i) - \operatorname{Mg} \operatorname{Cost}_t(i) \right]$$

- Production uses capital and labor: $y_t(i) = a_t(i) (\ell_t(i))^{\alpha} (h_t(i))^{1-\alpha}$
- Profit in the domestic market (s = D, X):

$$\pi_t^s(i) = \frac{Y_t}{1 + \phi \frac{\mu_t}{\lambda_t}} \left[1 - \frac{\operatorname{Mg} \operatorname{Cost}_t(i)}{p_t(i)} \right]$$

- Production uses capital and labor: $y_t(i) = a_t(i) (\ell_t(i))^{\alpha} (h_t(i))^{1-\alpha}$
- Bertrand Competition: $p_t(i) = Mg \operatorname{Cost}_t(i) (1 + \sigma^s)$

$$\pi_t^s(i) = \frac{Y_t}{1 + \phi_{\lambda_t}^{\mu_t}} \left[1 - \frac{1}{1 + \sigma^s} \right]$$

- Production uses capital and labor: $y_t(i) = a_t(i) (\ell_t(i))^{\alpha} (h_t(i))^{1-\alpha}$
- ► Bertrand Competition:

$$\pi_t^s(i) = \frac{Y_t}{1 + \phi \frac{\mu_t}{\lambda_t}} \left[1 - \frac{1}{1 + \sigma^s} \right] = \frac{\sigma^s}{1 + \sigma^s} \frac{Y_t}{1 + \phi \frac{\mu_t}{\lambda_t}}$$

- Production uses capital and labor: $y_t(i) = a_t(i) (\ell_t(i))^{\alpha} (h_t(i))^{1-\alpha}$
- Bertrand Competition:

$$\pi_t^s(i) = \frac{Y_t}{1 + \phi \frac{\mu_t}{\lambda_t}} \left[1 - \frac{1}{1 + \sigma^s} \right] = \frac{\sigma^s}{1 + \sigma^s} \frac{Y_t}{1 + \phi \frac{\mu_t}{\lambda_t}}$$

Profits from export lines' foreign sales

$$\pi_t^*(i) = \underbrace{\left(1 - \frac{\left(1 + \xi\right) \left(R_t^L\right)^{\alpha} \left(W_t\right)^{1 - \alpha}}{\left(1 + \sigma^X\right) \left(R_t^{L*}\right)^{\alpha} \left(W_t^*\right)^{1 - \alpha}}\right)}_{1 \text{ - relative marginal cost}} \underbrace{\frac{Y_t^*}{Foreign}}_{\text{demand}}$$

- Production uses capital and labor: $y_t(i) = a_t(i) (\ell_t(i))^{\alpha} (h_t(i))^{1-\alpha}$
- Bertrand Competition:

$$\pi_t^s(i) = \frac{Y_t}{1 + \phi \frac{\mu_t}{\lambda_t}} \left[1 - \frac{1}{1 + \sigma^s} \right] = \frac{\sigma^s}{1 + \sigma^s} \frac{Y_t}{1 + \phi \frac{\mu_t}{\lambda_t}}$$

Profits from export lines' foreign sales

$$\pi_t^*(i) = \underbrace{\left(1 - \frac{\left(1 + \xi\right) \left(R_t^L\right)^{\alpha} \left(W_t\right)^{1 - \alpha}}{\left(1 + \sigma^X\right) \left(R_t^{L*}\right)^{\alpha} \left(W_t^*\right)^{1 - \alpha}}\right)}_{1 \text{ - relative marginal cost}} \underbrace{\frac{Y_t^*}{Foreign}}_{\text{demand}}$$

- ► Two key differences that are relevant during sudden stops:
 - Sudden stop affects domestic demand but not foreign demand.
 - Lower factor prices increase export profits.

Intermediate Sector: Product Dynamics

▶ Firms invest in innovation to acquire new domestic products:

• Invest final goods Z_t^D :



 $i^{D'}(Z^D_t)$

 $\times \quad E_t \left[\Lambda_{t,t+1} V_{t+1}^D \right] = 1$

sum of discounted future profits

• Invest final goods Z_t^X to start exporting a domestic product:

$$\underbrace{(1-d_t)i^{X'}(Z_t^X)}_{t} \quad \times \quad E_t \left[\Lambda_{t,t+1} \left(V_{t+1}^X - V_{t+1}^D \right) \right] = 1$$

marginal increase in success probability

gap in sum of discounted future profits

▶ Direct entry to *X* and *D*.



► Final Good:

$$Y_t = \exp\left(\epsilon_t^A\right) \exp\left[\int_0^1 \ln y_t(i) di\right] =$$

► **Final Good:** (after some algebra...)

$$Y_t = \exp\left(\epsilon_t^{\mathcal{A}}\right) \exp\left[\int_0^1 \ln y_t(i) di\right] = A_t \frac{\exp\left(\epsilon_t^{\mathcal{A}}\right)}{\theta_{t-1}^D} \left(\frac{1+\sigma^D}{1+\sigma^X}\right)^{1-\theta_{t-1}^D} \left(L_t^D\right)^{\alpha} \left(H_t^D\right)^{1-\alpha}$$

► Final Good:

$$Y_t = \exp\left(\epsilon_t^A\right) \exp\left[\int_0^1 \ln y_t(i) di\right] = A_t \frac{\exp\left(\epsilon_t^A\right)}{\theta_{t-1}^D} \left(\frac{1+\sigma^D}{1+\sigma^X}\right)^{1-\theta_{t-1}^D} \left(L_t^D\right)^{\alpha} \left(H_t^D\right)^{1-\alpha}$$

Productivity index:

$$A_t = \exp\left[\int_0^1 \ln a_t(i) di\right]$$

► Final Good:

$$Y_t = \exp\left(\epsilon_t^A\right) \exp\left[\int_0^1 \ln y_t(i) di\right] = A_t \frac{\exp\left(\epsilon_t^A\right)}{\theta_{t-1}^D} \left(\frac{1+\sigma^D}{1+\sigma^X}\right)^{1-\theta_{t-1}^D} \left(L_t^D\right)^{\alpha} \left(H_t^D\right)^{1-\alpha}$$

Productivity index:

$$A_t = \exp\left[\int_0^1 \ln a_t(i) di\right]$$

► Trend:

$$\frac{A_{t+1}}{A_t} = \underbrace{(1+\sigma^D)^{e_t^D + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D}}_{\text{domestic}}\underbrace{(1+\sigma^X)^{e_t^X + \theta_{t-1}^D (1-d_t)i_t^X}}_{\text{exporting}}\underbrace{(1+\sigma^X)^{i^{FD}}}_{\text{foreign}}$$

Trade Balance and Current Account

$$TB_{t} = \underbrace{Y_{t} - C_{t} - Z_{t}^{ED} - Z_{t}^{EX} - \theta_{t-1}^{D} \left(Z_{t}^{D} + Z_{t}^{X}\right) - \theta_{t-1}^{X} Z_{t}^{D}}_{\text{final tradable output - absorption}} + \underbrace{\theta_{t-1}^{X} Y_{t}^{*}}_{\text{export of intermediate goods}} - \underbrace{\left(1 - \theta_{t-1}^{D} - \theta_{t-1}^{X}\right) \frac{Y_{t}}{1 + \phi \mu_{t} / \lambda_{t}}}_{\text{import of intermediate goods}}$$

Trade Balance and Current Account

$$TB_{t} = \underbrace{Y_{t} - C_{t} - Z_{t}^{ED} - Z_{t}^{EX} - \theta_{t-1}^{D} \left(Z_{t}^{D} + Z_{t}^{X}\right) - \theta_{t-1}^{X} Z_{t}^{D}}_{\text{final tradable output - absorption}} + \underbrace{\theta_{t-1}^{X} Y_{t}^{*}}_{\text{export of intermediate goods}} - \underbrace{\left(1 - \theta_{t-1}^{D} - \theta_{t-1}^{X}\right) \frac{Y_{t}}{1 + \phi \mu_{t} / \lambda_{t}}}_{\text{import of intermediate goods}}$$

$$CA_{t} = TB_{t} + \left(\exp(\varepsilon_{t-1}^{R})R - 1\right)B_{t-1} = B_{t} - B_{t-1}$$

Quantitative Analysis

Chile as an application:

Chile as an application:

1. Small open economy.

Chile as an application:

- 1. Small open economy.
- 2. Plant level data (ENIA).
 - ► All manufacturing plants that employ at least ten individuals.
 - Unique product level data for each firm during 1996-1999, distinguishing between the domestic and export markets.

Chile as an application:

- 1. Small open economy.
- 2. Plant level data (ENIA).
 - ► All manufacturing plants that employ at least ten individuals.
 - Unique product level data for each firm during 1996-1999, distinguishing between the domestic and export markets.
- 3. Exogeneous sudden stop.
 - August 1998: Russia defaulted on domestic debt and declared a moratorium on foreign creditors.
 - ▶ Interest rate spread rose by 270 bp the week after the default.
 - ▶ Non-FDI financial flows decreased by more than 40%.

Externally Calibrated Parameters

	Variable	Value	Source
β	Discount factor	0.96	Standard
R	Foreign bond interest rate	1.05	Standard
ω	Frisch elasticity $1/(\omega - 1)$	1.455	Mendoza (1991)
α	Asset share in production	0.08	Targets Capital to Output ratio (Chile)
ξ	Iceberg trade cost	0.21	Anderson & van Wincoop (2004)
φ	Fraction of input subject to working cap. req.	0.2	Targets Total credit to GDP ratio (Chile)
κ	Coefficient on borrowing constraint	0.2	Mendoza (2010)
L	Amount of productive asset	0.6	Targets Frequency of Sudden Stops
ρ	Concavity of innovation investment	1.5	Median value from literature

► Standard SOE RBC literature.

	Variable	Value	Target	Model	Data
σ^D	Domestic innovation step size	0.06	Aggregate growth rate	2.5%	2.5%
σ^X	Export innovation step size	0.30	Relative profit of non-exporters to exporters	27.8%	26.2%
η^{ED}	Non-exporter entry coefficient	1.46	Share of single-product non-exporters	37.1%	38.3%
η^{EX}	Exporter entry coefficient	0.31	Share of exporters in single-product firms	20.8%	21%
η^D	Domestic innovation coefficient	2.97	Average number of products by non-exporters	2.24	2.56
η^X	Export innovation coefficient	0.52	Average number of exported products by exporters	1.05	1.7
<i>y</i> *	Foreign demand	0.74	Export revenue share for exporters	30.5%	35.9%
i ^{FX}	Foreign innovation rate on X lines	0.23	Domestic innovation rate by domestic	firms	
i ^{FD}	Foreign innovation rate on D lines	0.01	Export innovation rate by domestic fi	rms	

► Klette and Kortum (2004) style model calibrated to product data.

Product Dynamics in Normal Times

Relationship between existing product portfolio (1996) and transitions (product entry/exit between 1996/1997):

$$Y_f = \beta_1 \cdot X_f + \beta_2 \cdot \operatorname{Age}_f + \phi_s + \epsilon_f$$

Product Dynamics in Normal Times

Relationship between existing product portfolio (1996) and transitions (product entry/exit between 1996/1997):

	(1)	(2)	(3)	(4)
	Not Produced	Domestic	Domestic	Exported
	to	to	to	to
	Domestic	Exported	Not Produced	Domestic
Panel A: Data				
Number of Products	0.015***			
	(0.002)			
Number of Domestic Products		0.006***	0.066***	
		0.001	0.002	
Number of Exported Products				0.048***
				(0.011)
Observations	3996	3996	3996	870
Unconditional Prob.	0.15	0.05	0.16	0.05
Panel B: Model				
Number of Products	0.080***			
	(0.003)			
Number of Domestic Products		0.010***	0.121***	
		0.001	0.003	
Number of Exported Products				0.103**
				(0.051)
Observations	4498	4498	4498	1118

$$Y_f = \beta_1 \cdot X_f + \beta_2 \cdot \operatorname{Age}_f + \phi_s + \epsilon_f$$

Product Dynamics in Normal Times

Relationship between existing product portfolio (1996) and transitions (product entry/exit between 1996/1997):

		-		
	(1)	(2)	(3)	(4)
	Not Produced	Domestic	Domestic	Exported
	to	to	to	to
	Domestic	Exported	Not Produced	Domestic
Panel A: Data				
Number of Products	0.015***			
	(0.002)			
Number of Domestic Products		0.006***	0.066***	
		0.001	0.002	
Number of Exported Products				0.048***
				(0.011)
Observations	3996	3996	3996	870
Unconditional Prob.	0.15	0.05	0.16	0.05
Panel B: Model				
Number of Products	0.000###			
Number of Froducts	(0.000)			
Number of Demostic Products	(0.003)	0.010###	0 101888	
Number of Domestic Products		0.010	0.121	
Number of Exported Products		0.001	0.005	0.102**
Number of Exported Products				(0.051.)
Obsomutions	4408	4408	4409	1119
Observations	++90	++90	++90	1110

$$Y_f = \beta_1 \cdot X_f + \beta_2 \cdot \operatorname{Age}_f + \phi_s + \epsilon_f$$

► The BGP replicates the 1996-1997 product dynamics.

Firm-Size Distribution

► Firm-size distribution based on total number of products sold.



Firm-Size Distribution

► Firm-size distribution based on total number of products sold.



The model replicates non-targeted firm-size distribution.

Sudden Stop Dynamics: Trade Margins



 Production cost for exporters drops, increasing profits on impact.

Product Transitions During Crises

	(1)	(2)	(3)	(4)		
	Data		Мо	Model		
	Not Sold	Domestic	Not Sold	Domestic		
	to	to	to	to		
	Domestic	Exported	Domestic	Exported		
1[Sudden Stop _t]	-0.083*** (0.006)	-0.018*** (0.003)	-0.075*** (0.008)	-0.003 (0.003)		
Firm Fixed Effects	Yes	Yes	Yes	Yes		
Observations	15523	15523	12785	12785		

 $Y_{ft} = \beta_1 \times \mathbf{1}[\text{Sudden Stop}_t] + \phi_f + \epsilon_{ft}$

The probability of starting to export decreases less than the probability of adding a new domestic product.

Benguria, Matsumoto and Saffie

Productivity and Trade Dynamics

 $\log{(\mathbf{Y}_{ft})} = \boldsymbol{\beta}_1 \times \mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exporter}_{ft}] + \phi_f + \delta_t + \epsilon_{ft}$

	(1)	(2)	(3)	(4)
	Data		Model	
	Revenue	Revenue Profits		Profits
$1[\text{Sudden Stop}_t] \times 1[\text{Exporter}_{ft}]$	0.066*** (0.010)	0.107*** (0.017)	0.095*** (0.011)	0.118*** (0.011)
Year Fixed Effects	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes
Observations	21213	20797	19022	19022

• Exporters are less affected by sudden stop than non-exporters

Industry Variation: Financial Dependence

$$\begin{split} \log\left(\text{Rev}_{\textit{fpmt}}\right) &= \beta_1 \times \mathbf{1}[\text{SS}_t] \times \mathbf{1}[\text{Exp}_{\cdot m}] + \beta_2 \times \mathbf{1}[\text{SS}_t] \times \text{Fin. Dep}_{\cdot p} + \beta_3 \times \mathbf{1}[\text{Exp}_{\cdot m}] \times \text{Fin. Dep}_{\cdot p} + \\ & \beta_4 \times \mathbf{1}[\text{SS}_t] \times \mathbf{1}[\text{Exp}_{\cdot m}] \times \text{Fin. Dep}_{\cdot p} + \phi_f + \rho_p + \delta_t + \nu_m + \epsilon_{\textit{fpmt}} \end{split}$$

	(1)	(2)
$1[Sudden\;Stop_t] \times 1[Exported_m] \times Fin.\;Dep._p$	0.507*** (0.152)	0.539*** (0.130)
Observations	50048	46981

- The differential revenue from exporting a product during the crisis is larger in more financially dependent industries (high ϕ).
- Column 2: Robust to product-firm fixed effects.

Productivity Loss



- Effect on impact is driven by domestic innovation.
- Recovery is driven by export innovation.

Welfare

Compensating Variation:

$$V_1^{noss} = \ln\left((1 + v^{cf})c_1 - A_1 \frac{H_1^{\omega}}{\omega}\right) + E_1[V_2^{cf}]$$

Economy	Welfare loss (%, relative to baseline)		
Total Cost of Sudden Stop	4.62%		
No Effect of SS on Productivity Growth g_t	3.24% (70.1%)		
No Effect of SS on Domestic Innovation Rate i_t^D	3.49% (75.5%)		
No Effect of SS on Export Innovation Rate i_t^X	5.04% (109.1%)		

- ▶ 30% of the welfare cost comes from endogenous growth.
- Exporter's innovation shields the economy from further welfare losses.

Conclusion

- Unified framework of endogenous sudden stops, trade and growth.
- Calibrated to product-firm level data.
- ► Replicates non-targeted firm-product portfolios dynamics.
- Lower product innovation rate explains 30% of welfare cost of sudden stops.
- Boosted export entry helps recovery and reduces welfare cost by 9%.
- Important to consider firm and trade dynamics in policy analysis.

Appendix

Final Tradable Sector **back**

Maximization problem:

$$\max_{\{\{y_t(i)\}_{i=0}^{1}, B_t, L_t\}} E_0 \sum_{t=0}^{\infty} \left[\beta^t \lambda_t \Pi_t^T\right]$$

$$\Pi_t = \underbrace{Y_t - \int_0^1 p_t(i)y_t(i)di}_{\text{output - cost}} \underbrace{-B_t + \exp(\varepsilon_{t-1}^R)RB_{t-1} - Q_tL_t + (Q_t + R_t^L)L_{t-1}}_{\text{net foreign asset}} \underbrace{-Q_tL_t + (Q_t + R_t^L)L_{t-1}}_{\text{capital holding and return}}$$

$$-B_t + \phi \left[\int_0^1 p_t(i)y_t(i)di\right] \le \kappa Q_t L_{t-1}$$

$$FOCs:$$

$$y_t(i) = \frac{Y_t}{p_t(i)} \frac{1}{1 + \phi\mu_t / \lambda_t}$$

$$\lambda_t - \mu_t = \beta R \exp(\varepsilon_t^R)E_t [\lambda_{t+1}]$$

$$Q_t = \frac{\beta E_t \left[\lambda_{t+1} \left(Q_{t+1} + R_t^L\right) + \kappa \mu_{t+1}Q_{t+1}\right]}{\lambda_t}$$

Productivity and Trade Dynamics

Intermediate Firms' Profit

Marginal cost for production:

$$MC_t(i) = \frac{1}{a_t(i)} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \left(R_t^L\right)^{\alpha} (W_t)^{1-\alpha}$$



$$\pi_t^s(i) = p_t(i)y_t(i) - R_t^L \ell_t(i) - W_t h_t(i)$$

• Using optimal price $p_t(i) = \widetilde{MC}_t(i)$ and demand function $y_t(i) = Y_t/p_t(i)$

$$\begin{aligned} \pi_t^s(i) &= p_t(i)y_t(i) - MC_t(i)y_t(i) = Y_t - MC_t(i)\frac{Y_t}{p_t(i)} \\ &= \left(1 - \frac{MC_t(i)}{\widetilde{MC}_t(i)}\right)Y_t \end{aligned}$$

Domestic Product Line 🛛 🗠

► Value of a firm satisfies:

$$V_t(n^D, n^X) = n^D V_t(1, 0) + n^X V_t(0, 1)$$

► Value of a domestic product line

$$\begin{split} V_t(1,0) &= \max_{Z_t^D, Z_t^X} \pi_t^D - Z_t^D - Z_t^X \\ &+ \left[i^D(Z_t^D) + (1 - d_t) \left(1 - i^X(Z_t^X) \right) \right] E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] \\ &+ \left[(1 - d_t) i^X(Z_t^X) \right] E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] \end{split}$$

Domestic Expansion by Domestic:

$$\eta^{D} \frac{1}{\rho^{D}} \left(\frac{Z_{t}^{D}}{A_{t}} \right)^{1/\rho - 1} \frac{1}{A_{t}} E_{t} \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] = 1$$

• Exporting Innovation:

$$(1-d_t)\eta^X \frac{1}{\rho} \left(\frac{Z_t^X}{A_t}\right)^{1/\rho-1} \frac{1}{A_t} \left(E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] - E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] \right) = 1$$

Benguria, Matsumoto and Saffie

Export Product Line

Value of an export product line

$$V_t(0,1) = \max_{Z_t^D} \pi_t^X + \pi_t^* - Z_t^D + \left(i^D(Z_t^D) + i^{FX} \right) E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] + (1 - i^{FX}) E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right]$$

Domestic expansion by exporters:

$$\eta^{D} \frac{1}{\rho} \left(\frac{Z_{t}^{D}}{A_{t}} \right)^{1/\rho - 1} \frac{1}{A_{t}} E_{t} \left[\Lambda_{t, t+1} V_{t+1}(1, 0) \right] = 1$$

• Entry to Domestic and Exporting:

$$\begin{split} \eta^{ED} \frac{1}{\rho} \left(\frac{Z_t^{ED}}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] &= 1 \\ \eta^{EX} \frac{1}{\rho} \left(\frac{Z_t^{EX}}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] &= 1 \end{split}$$

Share of domestic lines:

$$\theta_t^D = \theta_{t-1}^D + \underbrace{\left(e_t^D + \left(\theta_{t-1}^D + \theta_{t-1}^X\right)i_t^D\right)\frac{1 - \theta_{t-1}^D - \theta_{t-1}^X}{1 - \theta_{t-1}^X}}_{M \to D} + \underbrace{\theta_{t-1}^X i^{FX}}_{X \to D}$$

$$-\underbrace{\theta_{t-1}^D (1 - d_t)i_t^X}_{D \to X} - \underbrace{\theta_{t-1}^D i^{FD}}_{D \to M} - e_t^X \frac{\theta_{t-1}^D}{1 - \theta_{t-1}^X}}_{M \to D}$$

$$\bullet \text{ Share of export lines (extensive margin of exports):}$$

$$\theta_t^X = \theta_{t-1}^X + \underbrace{\theta_{t-1}^D (1 - d_t)i_t^X}_{D \to X} + e_t^X - \underbrace{\theta_{t-1}^X i^{FX}}_{X \to D}$$

Share of import lines (extensive margin of imports): $1 - \theta_t^D - \theta_t^X$

Aggregation of Intermediate Sector

• Growth in average productivity:

$$\frac{A_{t+1}}{A_t} = \underbrace{(1 + \sigma^D)^{e_t^D + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D}(1 + \sigma^X)^{e_t^X + \theta_{t-1}^D(1 - d_t)i_t^X}(1 + \sigma^X)^{i^{FD}}}_{\text{domestic}}$$

► Replacement rate:

$$d_{t} = \left(e_{t}^{D} + e_{t}^{X} + (\theta_{t-1}^{D} + \theta_{t-1}^{X})i_{t}^{D}\right)\frac{1}{1 - \theta_{t-1}^{X}} + i^{FD}$$

Asset and labor allocations:

$$1 = \theta_{t-1}^{D} \ell_{t}^{D} + \theta_{t-1}^{X} \left(\ell_{t}^{X} + \ell_{t}^{*} \right)$$
$$H_{t} = \theta_{t-1}^{D} h_{t}^{D} + \theta_{t-1}^{X} \left(h_{t}^{X} + h_{t}^{*} \right)$$



Maximization problem:

$$\max_{\{C_t,H_t,Z_t^E,Z_t^{EX}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \ln\left(C_t - A_t \frac{(H_t)^{\omega}}{\omega}\right)$$

subject to

$$C_{t} + Z_{t}^{E} + Z_{t}^{EX} = W_{t}H_{t} + \Pi_{t} + \theta_{t-1}^{D} \left(\pi_{t}^{D} - Z_{t}^{D} - Z_{t}^{X}\right) + \theta_{t-1}^{X} \left(\pi_{t}^{X} + \pi_{t}^{*} - Z_{t}^{D}\right)$$

► FOCs:

$$A_t(H_t)^{\omega-1} = W_t$$

and λ_t is given by:

$$\lambda_t = \frac{1}{C_t - A_t(H_t)^{\omega} / \omega}$$

Number of firms per transition

	Number of Products Added or Dropped			
	Any	1	2	3+
Not Produced to Domestic	0.15	0.10	0.03	0.03
Domestic to Exported	0.05	0.05	0.004	0.004
Domestic to Not Produced	0.16	0.10	0.03	0.03
Exported to Domestic	0.05	0.04	0.005	0.003
Not Produced to Domestic + Exported	0.03	0.02	0.003	0.002
Domestic + Exported to Not Produced	0.03	0.02	0.004	0.001