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Abstract

Roughly one third of migrants worldwide reside in developing countries, yet most papers on the macroeconomic effects of immigration focus on advanced economies. We investigate the medium- and long-term effects of immigration in an emerging country, considering a salient feature of this type of economies: the importance of labor informality. We build an overlapping generations model featuring 24 cohorts, an informal sector, and households with heterogeneous skill levels, among other features, that help us match key demographic and economic characteristics of Chile, an emerging country that has recently experienced an important immigration wave. An immigration wave increases the supply of labor, creating downward pressure on wages in the formal sector. Workers respond by reallocating labor effort to the informal sector, which allows them to mitigate the decline in consumption per worker triggered by lower formal-sector wages. Our model, thus, constitutes a framework for the quantitative analysis of immigration in emerging countries.

Resumen

Alrededor de una tercera parte de los migrantes del mundo reside en países en desarrollo. Sin embargo, la mayoría de investigaciones sobre los efectos macroeconómicos de la inmigración se enfocan en países avanzados. En este artículo investigamos los efectos de mediano y larzo plazo de la inmigración en un país emergente, considerando una característica clave de este tipo de economías, la importancia del trabajo informal. Para esto, construimos un modelo de generaciones traslapadas con 24 cohortes,

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un sector informal, y hogares con niveles de habilidad heterogéneos, entre otras características, que nos permiten reproducir características demográficas y económicas clave de Chile, un país emergente que recientemente ha experimentado una ola inmigratoria importante. En nuestras simulaciones, una ola inmigratoria aumenta la oferta de trabajo, generando presión a la baja en los salarios del sector formal. Los trabajadores responden reasignando trabajo hacia el sector informal, lo que les permite mitigar la caída en el consumo por trabajador asociada a los salarios más bajos en el sector formal. De este modo, el modelo constituye un marco para el análisis cuantitativo de la inmigración en países emergentes.

1 Introduction

Roughly one third of migrants worldwide reside in developing countries (OECD/ILO, 2018), yet the overwhelming majority of papers on the macroeconomic effects of immigration focus on advanced economies. In this paper, we investigate the medium- and long-term effects of immigration in an emerging country, considering a salient feature of this type of economies: the importance of labor informality.

To address this, we build an overlapping generations model (OLG) featuring 24 cohorts, an informal sector, and households with heterogeneous skill levels, among other features, that help us match key demographic and economic characteristics of Chile, an emerging country that has recently experienced an important immigration wave. Specifically, we match the income distribution, saving rates, and informality across income quantiles. Following much of the literature, we model informality as self-employment and, more specifically, as home production. In our framework, an hour worked in the informal sector generates more disutility than an hour worked in the formal sector. The additional disutility of working in the informal sector captures, for example, the lower benefits and job security associated to this type of employment. Despite this feature, informal labor allows workers to mitigate the decline in consumption per worker triggered by the immigration wave. In this sense, the informal sector acts as a buffer against income fluctuations, which is consistent with the evidence on informality (see, for example, Loayza and Rigolini, 2011). An immigration wave increases the supply of labor, creating downward pressure on wages in the formal sector, so workers respond by reallocating labor effort to the informal sector.

Motivated by the immigration shock recently experienced by Chile, we furthermore model immigrants that, despite having skills similar to those of natives, experience a temporary underemployment spell—a period during which they cannot fully exercise the productivity associated to their skill level. This underemployment spell amplifies the distributional effects of the immigration shock, exacerbating the decline in wages of low-skilled workers and mitigating the decline in wages of high-skilled workers. Intuitively, this transitory underemployment spell can be justified by the time it takes a foreign worker to adapt to the host country (see Lubotsky, 2007). This transitory phenomenon, related to adaptation, differs from the approach found in some papers which assume immigrants have permanently lower productivity, perhaps due to lower education quality in their home country (see, for example, Canova and Ravn, 2000). In our baseline simulation, therefore, immigration generates only transitory effects on per capita variables and factor prices, though these effects are persistent. Since the productivity of immigrants eventually returns to that associated to their skill level, i.e., the underemployment spell ends, the effects of the immigration shock eventually die out.

In addition to the underemployment spell, our simulation of an immigration wave considers the fact that immigrants are much younger than natives. Taking the difference in the age structure of immigrants into account is quantitatively important, as we show below, highlighting the usefulness of an OLG model in this context. To sum up, our simulations consider immigrants that are mostly young adults, and that experience an underemployment spell upon arrival. Immigrants are otherwise assumed to be indistinguishable from natives. This assumption not only simplifies the model, but is supported by some of the characteristics of immigrants in

Chile. Aldunate, Contreras, de la Huerta, and Tapia (2019b) document that, in addition to having similar education levels to those of natives, immigrants have a similar distribution of employment across industries (they are not concentrated on a handful of industries), and have similar employment status (they are not disproportionately informal). There are, however, special characteristics of immigrants which we ignore, such as the higher unemployment rates they seem to face upon arrival (again, see Aldunate et al., 2019b), or the possibilities they bring substantially less wealth than natives, or remit an important fraction of disposable income to their home countries.

The paper lies at the intersection of two strands of the literature. The first considers the macroeconomic effects of immigration. This literature focuses on advanced economies, and typically studies immigrants with lower skills than the native population (see, for example, Boldrin and Montes, 2015; Izquierdo, Jimeno, and Rojas, 2010; Wilson, 2003; Mandelman and Zlate, 2012; and Holler and Schuster, 2018). The second strand of the literature studies the role of informality in the labor market of emerging countries. It finds that the informal sector acts as a buffer that helps sustain household consumption during bad times, and can explain, for instance, why aggregate employment is more volatile in advanced economies (see, for example, Fernández and Meza, 2015; and Finkelstein Shapiro and Mandelman, 2016).

By bridging the gap between the literature on immigration in advanced economies and the literature on labor informality in emerging countries, this paper is the first comprehensive study of the macroeconomic effects of immigration in emerging countries, where informal labor is pervasive.

The paper is organized as follows. Section 2 presents the evidence that motivates the key features of our OLG model. Section 3 spells out the details of the model. Section 4 discusses the calibration and the design of our simulations. Section 5 presents our baseline results, as well as an analysis of their sensitivity to our assumptions on the duration and intensity of the underemployment spell that affects immigrants. Section 6 concludes.[1]

2 Motivating Evidence

The design of the OLG model and our simulation exercises is motivated by features of the Chilean economy and the recent immigration wave it has experienced.

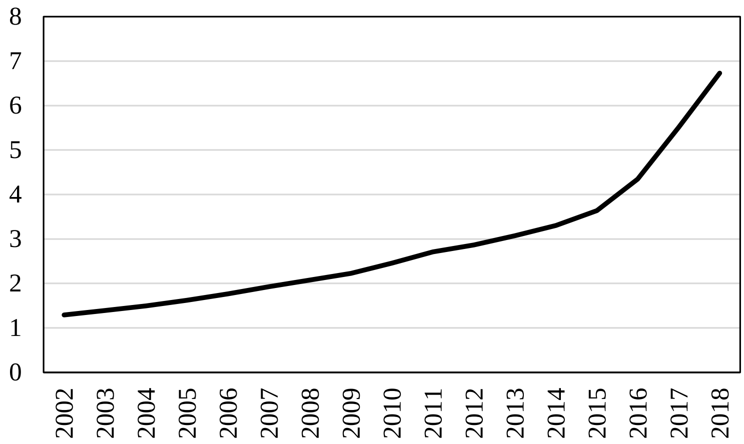
2.1 Chile's Recent Immigration Wave

Chile has experienced a substantial immigration wave. In the last four years, from 2014 to 2018, the share of immigrants in the total population doubled, jumping from 3.3% to 6.7% . We model this immigration wave as exogenous from the perspective of Chile.[2] There is nothing special about Chile in the last few years that could account for such a surge in immigration; certainly not economic growth, which was sluggish during this period. Instead, the inflow of migrants is mainly due to the Venezuelan crisis and the massive emigration it has generated. Therefore, we believe it is reasonable to model this event as exogenous from the perspective of Chile.

[1] One of the appendices shows how the model can be extended to study the effects of a statutory reduction in working hours.

[2] See, e.g., Mandelman and Zlate (2012) for an endogenous analysis of immigration.

Figure 1: Share of immigrants in total population

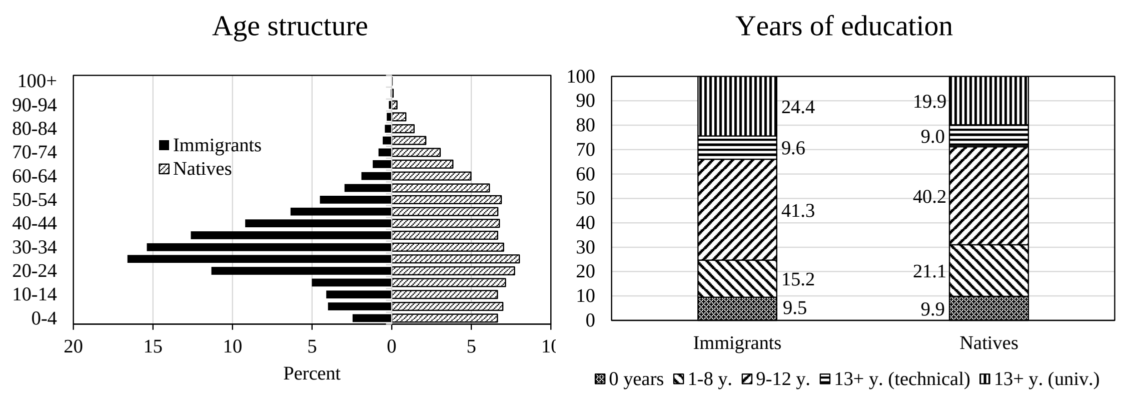


Source: National Statistics Institute.

For the purposes of our analysis, two key features of Chile's immigration wave are that immigrants: (i) are primarily young adults, and (ii) have similar education levels to those of natives. Figure 2 shows the age structure and education level of immigrants and natives. The fact that immigrants are primarily young adults implies that the immigration wave has not only generated higher growth in the total population, but even higher growth in the working-age population. Since immigrants have higher participation rates than natives, as documented in Aldunate et al. (2019b), the immigration-induced growth in the labor force is also higher than that of the total population. Our simulations, therefore, will consider an immigration wave that alters the age structure, reducing the ratio of passive to active individuals.

Following the evidence on the education level of immigrants, our simulations will further assume that immigrants and natives have the same skill distribution. We will, however, allow for an adjustment period in which immigrants are underemployed, as we discuss next.

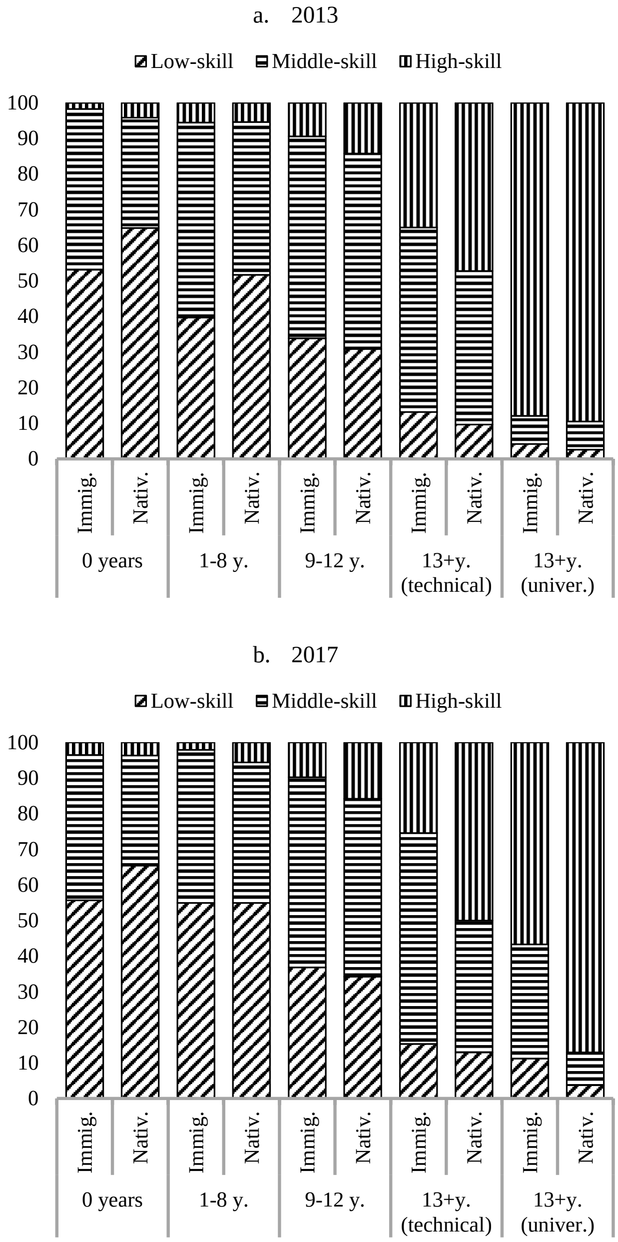
Figure 2: Age and education of immigrants



Source: Aldunate et al. (2019b).

Although immigrants and natives have similar education levels, it seems that immigrants experience an "underemployment spell" upon arrival: a period during which they cannot fully exercise the productivity associated to their skill level. Figure 3 presents some evidence in favor of this conclusion. It shows employment by education and skill level of the occupation, for natives and immigrants, and for the years 2013 (panel a in the top) and 2017 (panel b in the bottom). This evidence is based on data from the CASEN survey, which is conducted every two years in Chile. The recent immigration wave began around 2015, so the evidence from 2013 is prior to that, whereas the evidence from 2017 is the latest that considers the immigrants from this wave. Among individuals of higher education levels, we can see that in 2017 immigrants hold a substantially smaller proportion of high-skill occupations compared to natives, and that this was not the case in 2013. For example, in 2017, only 57% of immigrants with university studies worked in a high-skill occupation, whereas 87% of natives with that education level did. In 2013, prior to the recent immigration wave, immigrants and natives with university studies held almost the same proportion of high-skill occupations. A similar divergence is visible for people with technical education: in 2013, the difference in the proportion of high-skill occupations held by immigrants and natives is 12 percentage points (pp), but in 2017 this difference increases to 25 pp. To sum up, after the immigration wave, immigrants with a similar education level, hold a substantially fewer proportion of high-skill jobs compared to natives. This evidence is imperfect for our purposes, as it only shows a snapshot at two points in time. Unfortunately, there is no data, to the extent of our knowledge, that would allow us to characterize the adaptation *process* of immigrants in the labor market. Nevertheless, we will interpret this evidence as reflecting a transitory underemployment spell experienced by immigrants, and not that immigrants are permanently less productive than natives, perhaps due to lower education quality in their home country, as in, for example, Canova and Ravn (2000).

Figure 3: Employment by years of education and skill level of occupation

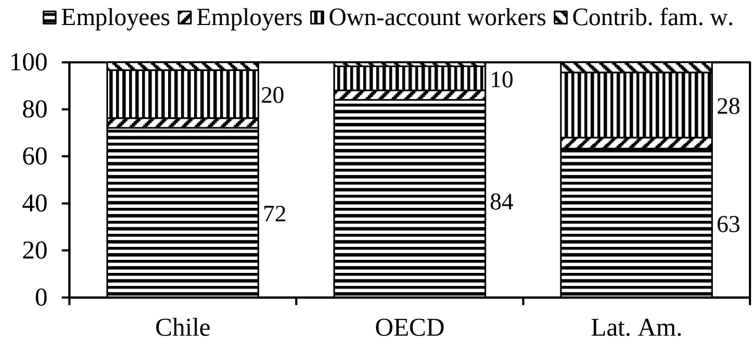


Source: Aldunate et al. (2019b). The underlying data come from the CASEN survey. The classification of occupations by skill level follows Lagakos, Moll, Porzio, Qian, and Schoellman (2018).

2.2 Labor Informality

Following the literature, we focus on self-employment as a proxy measure of labor informality (see, for example, Fernández and Meza, 2015). Figure 4 shows employment by category in Chile, the OECD, and a group of Latin American countries. The proportion of self-employed (own-account workers) is much higher in Chile and the group of Latin American countries (all emerging countries) than in the OECD, a group of mostly advanced economies. This is consistent with the findings of the literature on the importance of labor informality in emerging countries (see also Loayza and Rigolini, 2011).

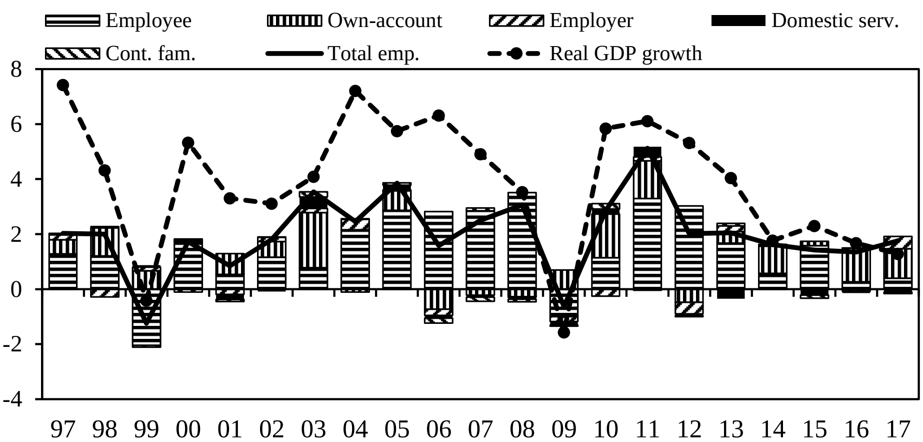
Figure 4: Employment by category



Source: Central Bank of Chile (2018). The underlying data come from the International Labour Organization. Categories of employment are: employees, employers, own-account workers, and contributing family workers.

Labor informality in Chile also conforms to results established in the literature in that it acts as a buffer for income fluctuations. Figure 5 reproduces results from Central Bank of Chile (2018) that show that in good times, employment growth is driven by salaried formal work ("employees"; bars with horizontal lines), whereas in bad times, informal employment sustains job growth ("own-account" workers; bars with vertical lines). In Chile, the economy grew strongly in the mid-to-late 2000s and in the aftermath of the Great Recession (2011-2013), periods during which employment growth was driven by formal work. On the other hand, the Chilean economy struggled in the aftermath of the Asian and Russian crises (up until 2003), and in 2014-17, periods in which informal work sustains job growth.[3]

Figure 5: Employment growth by category



Source: Central Bank of Chile (2018).

[3]See also Parro G. and Reyes R. (2019), who find that in Chile, the share of self-employed increases in periods of low economic growth and vice versa, whereas the share of workers in formal work displays the opposite behavior (it increases in periods of *high* economic growth, and vice versa).

In the next section, we spell out an OLG model that can capture the evidence presented in this section, as well as other demographic and economic features of the Chilean economy.

3 The OLG Model

The model is based on the neoclassical growth models with overlapping generations pioneered by Samuelson (1958) and Diamond (1965). We add several features to an otherwise standard OLG model with twenty four generations (or cohorts).[4] These extensions include, first, endogenous labor supply with heterogeneous productivities, as in Brunner (1996) and Sommacal (2006), among others. This allows us to replicate the observed distribution of income across quintiles. Second, we introduce heterogeneous discount factors, which are decreasing in the income level, and that allow us to match the heterogeneity of saving rates observed in the data. Third, we add an informal sector based on the works of Busato and Chiarini (2004), Busato, Chiarini, and Rey (2012) and Orsi, Raggi, and Turino (2014), with the purpose of capturing reallocation of labor supply between the formal and informal sectors. The informal sector captures possible unemployment situations associated with self-provision of goods, and self-employment, as in Fernández and Meza (2015). Fourth, we consider a financially open economy, where foreigners own a fraction of the domestic capital stock and where national savers invest abroad. Finally, we also add a series of minor extensions throughout the following sections.[5]

3.1 Demographics

Individuals in the model have perfect foresight. They live for 24 periods, and, therefore, in every period there are indivduals of 24 generations alive. A generation born at time t corresponds to the first cohort in period t, the second in $t+1$, and so on. Individuals spend the first 16 periods of their life working and then retire for the remaining 8 periods. Workers spend their labor income on consumption and one-period savings. These savings are transformed into physical capital that is rented to perfectly competitive firms in the formal sector. At the beginning of the following period, the remaining capital, net of depreciation, together with the gained rent is returned back to the individuals, and so wealth is transferred intertemporally in the model.[6] Savings are used to finance consumption during retirement. Workers are divided into 5 different skill groups denoted by the index $i = 1, \ldots, 5$ and ordered from least skilled ($i = 1$) to most skilled ($i = 5$). The size of a new generation that is born into this economy (N_t^1) relative to the size of the previous first generation (N_{t-1}^1) changes at rate $n^r + n_t^m$, i.e., $N_t^1 = N_{t-1}^1 (1 + n^r + n_t^m)$; where n^r denotes a constant growth rate used to accommodate the reproduction rate of the whole population, and n_t^m accounts for time-varying increases of the first cohort due to immigration. Thereafter, we assume that the size of any given cohort may vary over time due to migration only; m_t^g denotes the rate of change of cohort $g > 1$ between periods $t-1$ and t, i.e., $N_t^g = N_{t-1}^{g-1}(1 + m_t^g)$. We will use n_t^m and the m_t^g to simulate the change

[4]We work with a model with twenty four cohorts, since it allows to study medium-term effects (each period is calibrated to last 2.5 years), while retaining reasonable computational efficiency.

[5]We abstract from human capital accumulation; Boldrin and Montes (2015) consider human capital accumulation in a three-period OLG model of immigration for Spain.

[6]Financial openness will alter this setup slightly.

in population size and age structure induced by an immigration wave. The total population in period t is then defined as $N_t = \sum_{g=1}^{24} N_t^g$.[7] We also assume that the distribution of skills for each generation can change over time, such that in period t there are $N_{i,t}^g = \lambda_{i,t}^g N_t^g$ individuals of skill group i in cohort g; $\lambda_{i,t}^g \in (0,1]$ denotes the fraction of agents of skill group i within cohort g in period t, and we have that $\sum_{i=1}^{5} \lambda_{i,t}^g = 1$. Therefore, in the model, both the size of every generation and the size of each skill group within each generation can change over time. We will use the $\lambda_{i,t}^g$ to simulate a change in the skill distribution induced by a wave of immigrants that experience an underemployment spell.

3.2 Firms

A representative firm demands capital, K_t, and labor, L_t, to produce a homogeneous good used for consumption and investment. The firm's production function is

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}. \tag{1}$$

The labor input L_t is a constant-elasticity-of-substitution (CES) bundle of labor of various skills:[8]

$$L_t = \left(\sum_{i=1}^{5} a_i L_{i,t}^{\rho} \right)^{1/\rho}, \tag{2}$$

where $L_{i,t}$ denotes the demand for workers from skill group i, the parameters $a_i > 0$ with $a_1 < a_2 < \cdots < a_5$ measure the productivities of the different worker types, and $\rho < 1$ determines the elasticity of substitution of workers of different skills, which is equal to $1/(1-\rho)$. For $\rho = 1$, workers of different skills are perfect substitutes, and for $\rho \to -\infty$ they are perfect complements. The group of workers from skill group i at time t is formed of individuals of all active generations from that skill group:

$$L_{i,t} = \sum_{g=1}^{16} L_{i,t}^g. \tag{3}$$

The firm maximizes profits subject to (1), (2) and (3) treating the rental rate of capital r_t^k and the skill-specific wage rates, $W_{i,t}^g$ as given. Substituting out the constraints, the firm's optimization problem can be written as:

$$\max_{K_t, L_{i,t}^g} K_t^{\alpha} A_t^{1-\alpha} \left(\sum_{i=1}^{5} a_i \left(\sum_{g=1}^{16} L_{i,t}^g \right)^{\rho} \right)^{\frac{1-\alpha}{\rho}} - r_t^k K_t - \sum_{i=1}^{5} \sum_{g=1}^{16} W_{i,t}^g L_{i,t}^g, \tag{4}$$

yielding the following first-order conditions for capital

$$K_t : \qquad r_t^k = \alpha \frac{Y_t}{K_t}, \tag{5}$$

[7] If in steady state $m_{ss}^g = 0$ for all g, we have that N_{ss}, the total population in steady state, grows at the same rate as the first cohort in steady state, N_{ss}^1, namely $n_{ss}^r + n_{ss}^m$.

[8] Throughout, upper-case letters denote variables that are non-stationary due to population growth, while lower-case letters denote stationary variables.

and for labor

$$L_{i,t}^g: \qquad W_{i,t}^g = a_i W_t \left(\frac{L_t}{L_{i,t}}\right)^{1-\rho}, \tag{6}$$

where $g \leq 16$ and$W_t = \frac{(1-\alpha)Y_t}{L_t}$. These conditions imply that the zero-profit condition

$$Y_t = r_t^k K_t + \sum_{i=1}^{5} \sum_{g=1}^{16} W_{i,t}^g L_{i,t}^g \tag{7}$$

is satisfied in equilibrium.

The law of motion for capital is given by,

$$K_{t+1} = I_t = (1-\gamma) S_t + \gamma^\star N_t^1 \tag{8}$$

where S_t, total savings, is defined below. Note that this equation suggests that each period the capital stock is "created." Each period, a fraction $(1-\gamma)$ of total savings in the economy (the remaining fraction γ is invested abroad), plus an amount $\gamma^\star$ financed by foreigners is invested in capital.[9] This investment is turned into capital that is rented the following period to firms at a rental rater_t^k. At the end of the period, that rent, plus the remaining capital, net of depreciation, δ, is returned to the investors. Gross domestic product (GDP) in the economy equals the market value of the final consumption good, C_t, plus the market value of capital to be carried into the following period, which is K_{t+1}. Therefore,

$$GDP_t = C_t + K_{t+1} \tag{9}$$

3.3 Workers of Type i

Each worker's utility is a function of consumption, $C_{i,t}^g$, both when active ($g \leq 16$) and when retired ($g > 16$), and of labor supplied to the formal, $l_{i,t}^g$, and informal, $h_{i,t}^g$, sectors, when active. The present valued lifetime utility of a worker of type i is given by:

$$\begin{aligned} U_{i,t} = & \sum_{g=1}^{24} \beta_i^{g-1} \frac{\left(C_{i,t+g-1}^g\right)^{1-\theta}}{1-\theta} \\ & - \sum_{g=1}^{16} \beta_i^{g-1} \Theta_{i,t+g-1}^g \frac{\left(l_{i,t+g-1}^g + h_{i,t+g-1}^g\right)^\phi + \kappa \left(h_{i,t+g-1}^g\right)^\phi}{\phi}, \end{aligned} \tag{10}$$

where $\theta > 0$ stands for the inverse of the elasticity of intertemporal substitution, $\phi \geq 1$ determines the elasticity of labor supply, $\beta_i \in (0,1)$ is the subjective discount factor of each group type, and $\kappa > 0$ is a specific utility cost of working at home.[10] The variable $\Theta_{i,t}^g$ is an endogenous preference shifter based on Gali, Smets and Wouters (2012) that is taken as given by the workers

[9] We let the amount invested by foreigners in the domestic economy $\gamma^\star$ grow with N_t^1 to maintain a balanced growth path.

[10] This specific cost can be justified by the lack of health insurance, or by job insecurity, for instance.

and satisfies

$$\Theta^{g}_{i,t+g-1} = \chi^{g}_{i}\left[\left(C^{g}_{i,t+g-1}\right)^{-\theta}\right]^{\upsilon}, \tag{11}$$

with $\upsilon \in \{0,1\}$ and where $\chi^{g}_{i} > 0$ determines the disutility of work for skill group i and age group g. The purpose of this preference shifter is to allow for a zero wealth effect on labor supply. When $\upsilon = 0$, we obtain $\Theta^{g}_{i,t+g-1} = \chi^{g}_{i}$ and thus the standard constant relative risk aversion (CRRA) utility function implying a non-zero wealth effect, while there is no wealth effect when $\upsilon = 1$.

The informal sector is modelled as self-employment or home production, which is conducted through a linear production function, with labor as the only input, and productivity $b_i a_i$, with $b_1 < b_2 < \cdots < b_5$.

Letting $S^{g}_{i,t}$ denote assets of generation g in period t, a worker's sequence of budget constraints for periods t, $t+1$, ... $t+24-1$ are, respectively:

$$C^{1}_{i,t} = W^{1}_{i,t}l^{1}_{i,t} + b_i a_i h^{1}_{i,t} - S^{1}_{i,t}, \tag{12}$$

$$C^{2}_{i,t+1} = W^{2}_{i,t+1}l^{2}_{i,t+1} + b_i a_i h^{2}_{i,t+1} + r_{t+1}S^{1}_{i,t} - S^{2}_{i,t+1}, \tag{13}$$

$$\cdots$$

$$\begin{aligned} C^{16}_{i,t+16-1} = W^{16}_{i,t+16-1}l^{16}_{i,t+16-1} + b_i a_i h^{16}_{i,t+16-1} \\ + r_{t+16-1}S^{15}_{i,t+16-2} - S^{16}_{i,t+16-1}, \end{aligned} \tag{14}$$

$$C^{17}_{i,t+17-1} = r_{t+17-1}S^{16}_{i,t+17-2} - S^{17}_{i,t+17-1}, \tag{15}$$

$$\cdots$$

$$C^{24}_{i,t+24-1} = r_{t+24-1}S^{23}_{i,t+24-2} \tag{16}$$

where (15) and (16) describe the simpler constraints faced by retirees. Note also that (16) incorporates the simplifying assumption that individuals make no bequests, so assets are zero when they die.

The period-by-period budget constraints (12) through (16) can be combined to obtain an intertemporal budget constraint (IBC):

$$\begin{aligned} \sum_{g=1}^{24}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1} C^{g}_{i,t+g-1} = \sum_{g=1}^{16}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1} \\ \left[W^{g}_{i,t+g-1}l^{g}_{i,t+g-1} + b_i a_i h^{g}_{i,t+g-1}\right], \end{aligned} \tag{17}$$

where

$$r_t = 1 + (1-\gamma)\left(R^{Tk}_{t} - 1\right) + \gamma r^{\star} \tag{18}$$

denotes the gross return on assets. We introduce financial openness in a simple way assuming that an exogenous fraction $\gamma \in [0,1]$ of assets is invested abroad at a fixed net interest rate $r^{\star}$, while the remaining savings are invested in domestic capital with total gross return R_t^{Tk}.

The first generation at time t, then, faces the problem of maximizing utility (10) subject to the IBC (17), taking $W^g_{i,t+g-1}$ and r_{t+g-1} as given. This problem can be represented by the following Lagrangian:

$$\begin{aligned}\mathcal{L}_{i,t} = &\sum_{g=1}^{24} \beta_i^{g-1} \frac{\left(C^g_{i,t+g-1}\right)^{1-\theta}}{1-\theta} \\ &- \sum_{g=1}^{16} \beta_i^{g-1} \Theta^g_{i,t+g-1} \frac{\left(l^g_{i,t+g-1} + h^g_{i,t+g-1}\right)^{\phi} + \kappa \left(h^g_{i,t+g-1}\right)^{\phi}}{\phi} \\ &+ \Psi_{i,t} \left(\sum_{g=1}^{16} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1} \left[W^g_{i,t+g-1} l^g_{i,t+g-1} + b_i a_i h^g_{i,t+g-1} \right] \right. \\ &\left. - \sum_{g=1}^{24} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1} C^g_{i,t+g-1} \right) \end{aligned} \tag{19}$$

where $\Psi_{i,t}$ denotes the Lagrange multiplier associated with (17). Then, for $g = 1, ..., 24$ the first-order conditions are:

$$C^g_{i,t+g-1}: \quad 0 = \beta_i^{g-1} \left(C^g_{i,t+g-1}\right)^{-\theta} - \Psi_{i,t} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1}. \tag{20}$$

For $g = 1, ..., 16$, i.e., when individuals are in the workforce,

$$\begin{aligned} l^g_{i,t+g-1}: \quad 0 = &-\beta_i^{g-1} \Theta^g_{i,t+g-1} \left(l^g_{i,t+g-1} + h^g_{i,t+g-1} \right)^{\phi-1} \\ &+ \Psi_{i,t} W^g_{i,t+g-1} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1}, \end{aligned} \tag{21}$$

$$\begin{aligned} h^g_{i,t+g-1}: \quad 0 = &-\beta_i^{g-1} \Theta^g_{i,t+g-1} \left[\left(l^g_{i,t+g-1} + h^g_{i,t+g-1} \right)^{\phi-1} + \kappa \left(h^g_{i,t+g-1} \right)^{\phi-1} \right] \\ &+ \Psi_{i,t} b_i a_i \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1}. \end{aligned} \tag{22}$$

From (20), we obtain the set of Euler equations for consumption for any two different periods, i.e. for $g_1 < g_2 \leq 24$:

$$\left(C^{g_1}_{i,t+g_1-1}\right)^{-\theta} = \beta_i^{g_2-g_1} \left(\prod_{n_g=g_1}^{g_2-1} r_{t+n_g} \right) \left(C^{g_2}_{i,t+g_2-1}\right)^{-\theta}. \tag{23}$$

Solving for $\Psi_{i,t}$ in (20) and using the result in (21) yields the intratemporal formal-sector labor supply schedules, $g \leq 16$:

$$\Theta^g_{i,t+g-1}\left(l^g_{i,t+g-1}+h^g_{i,t+g-1}\right)^{\phi-1}=\left(C^g_{i,t+g-1}\right)^{-\theta}W^g_{i,t+g-1}. \tag{24}$$

Similarly, from (22) we obtain the informal-sector labor supply schedules, $g \leq 16$:

$$\Theta^g_{i,t+g-1}\left[\left(l^g_{i,t+g-1}+h^g_{i,t+g-1}\right)^{\phi-1}+\kappa\left(h^g_{i,t+g-1}\right)^{\phi-1}\right]=\left(C^g_{i,t+g-1}\right)^{-\theta}b_i a_i. \tag{25}$$

To gain intuition, it is useful to combine (24) with (25), and to solve the resulting equation for $h^g_{i,t+g-1}$, which yields:

$$h^g_{i,t+g-1}=\begin{cases}\left[\left(C^g_{i,t+g-1}\right)^{-\theta}\frac{\left(b_i a_i-W^g_{i,t+g-1}\right)}{\Theta^g_{i,t+g-1}\kappa}\right]^{\frac{1}{\phi-1}} & \text{if } \; b_i a_i > W^g_{i,t+g-1},\\ 0 & \text{otherwise.}\end{cases}$$

These equations show that households supply labor to home production as long as the marginal income they earn from this activity exceeds the marginal income that they earn by working in the formal labor market. This is necessary so that households are compensated for the additional utility cost that informal work generates. This is a limitation of our framework, since the evidence suggests wages in the formal sector are higher than the income obtained in the informal sector; see, e.g., Joubert (2015) for the case of Chile.

We can use the intertemporal budget contraint to re-write the consumption equations as follows (noting $S^0_{i,t-1}=0$):

for $g_3 \leq 16$

$$C^{g_3}_{i,t+g_3-1}=\left(\sum_{g=g_3}^{24}\left(\prod_{n_g=g_3}^{g-1}r_{t+n_g}\right)^{\frac{1-\theta}{\theta}}\left(\beta_i^{g-g_3}\right)^{\frac{1}{\theta}}\right)^{-1} \tag{26}$$
$$\left[r_{t+g_3-1}S^{g_3-1}_{i,t+g_3-2}+\sum_{g=g_3}^{16}\left(\prod_{n_g=g_3}^{g-1}r_{t+n_g}\right)^{-1}\left[W^g_{i,t+g-1}l^g_{i,t+g-1}+b_i a_i h^g_{i,t+g-1}\right]\right],$$

for $g_3 > 16$

$$C^{g_3}_{i,t+g_3-1}=\left(\sum_{g=g_3}^{24}\left(\prod_{n_g=g_3}^{g-1}r_{t+n_g}\right)^{\frac{1-\theta}{\theta}}\left(\beta_i^{g-g_3}\right)^{\frac{1}{\theta}}\right)^{-1}r_{t+g_3-1}S^{g_3-1}_{i,t+g_3-2} \tag{27}$$

The model is non-stationary due to population growth. The appendix contains the complete set of equilibrium conditions, as well as the detrended version of the model, and the derivation of the steady state.

4 Calibration and Simulation Design

The model is calibrated to the Chilean economy. Table 1 summarizes the baseline calibration. The duration of a period, given by parameter T, is set to 10 quarters. This implies that individuals in the model are retired for 20 years following a working life of 40 years. The 5 different skill groups in the economy represent income quintiles.[11] The subjective discount factors of the different skill groups, β_i for $i = 1, ..., 5$, in turn, are calibrated to replicate the average saving rates for each income quintile, which are computed from the 2012 Family Budget Survey (Encuesta de Presupuestos Familiares; EPF), using the methodology proposed by Madeira (2015, 2016). The matched saving rates are 9%, -0.1%, 4.2%, 8.2% and 17% for the first to fifth quintile, respectively. These rates imply an aggregate saving rate, weighted by income, of 7.7%.[12] The parameter that determines the disutility of work, χ_i^g, is calibrated so as to set total labor supply, both formal and informal, equal across skill groups in the initial steady state.

The annual population growth rate, n^r, is set to 0.5%, in line with estimates and forecasts of trend growth obtained by Albagli, Contreras, de la Huerta, Luttini, Naudon, and Pinto (2015) for Chile for the period 2016-2050. Following the same study, the labor share in production, $1 - \alpha$, is set to 52%, which corresponds to the 2008-2013 average of the ratio of salaries paid by the corporate sector to the value added of that sector, net of taxes, according to national accounts data.[13] The capital depreciation rate, δ, is set to 4% per year.

The parameter υ, which determines the size of the wealth effect on labor supply is set to 1. This specification favors the class of preferences that have a zero wealth effect, similar to the preferences proposed by Greenwood, Hercowitz, and Huffman (1988). Consequently, changes in the quantity of labor supplied are only due to fluctuations in wages. Correia, Neves, and Rebelo (1995) show that these preferences are useful for real small open economy models to match features of aggregate fluctuations.

The productivity parameters in the formal sector, a_i for $i = 1, ..., 5$, are calibrated to replicate the distribution of labor income (percentage of aggregate income corresponding to each quintile) as reported by the 2015 Socioeconomic Characterization Survey (Encuesta de Caracterización Socioeconómica Nacional; CASEN).[14] The parameter that determines the specific utility cost of working at home, κ, is set to 324 following Orsi et al. (2014). In turn, the parameters that determine the efficiency of home production, b_i, are calibrated so as to obtain participations in the informal sector of 57%, 18%, 18%, 17% and 14% for quintiles 1 to 5, respectively, which implies an aggregate participation of 25% in the informal sector. These statistics were computed using data from the 2012 New Supplementary Income Survey (Nueva Encuesta Suplementaria de Ingresos; NESI). We consider the following as informal workers: employees without a contract, and self-employed individuals that work at home or on the street (excluding those with a university education, which are considered formal workers). The parameter ρ, which determines the elasticity of substitution between the different skill groups, is set to 0.33, following Sommacal

[11] Quintile number 1 represents the lowest income group, and, therefore, the least skilled group.

[12] This number is close to the aggregate saving rate for Chile that was estimated to be between 8.3% and 9.8% since 2010 by the OECD (https://data.oecd.org/hha/household-savings.htm).

[13] The source of these data is the Central Bank of Chile.

[14] According to this survey, the fraction of aggregate income corresponding to each quintile is 4.2%, 9.7%, 14.5%, 21.1% and 50.5%.

(2006). This value implies an elasticity of substitution between the different skill types of 1.5, in line with the empirical evidence for the U.S. reported by Ciccone and Peri (2005).

Table 1: Baseline calibration

Parameter	Description	Value/to match	Source
T	Duration of a period	10 quarters	-
$\beta_i^{-1/T} - 1$	Annual discount rates	saving rates	2012 Family Budget Survey
$\chi_i^g = \chi_i$	Disutility of labor	Total labor supply constant $\forall i$	-
n	Annual population growth rate	0.5%	Albagli et al. (2015)
θ	Intertemp. elasticity of subs., inv	1	Literature
ϕ	Labor supply elasticity, inverse	3	Literature
α	Capital share	0.48	Albagli et al. (2015)
$1-(1-\delta)^{1/T}$	Annual depreciation	4%	Literature
κ	Disutility of informality	324	Orsi et al (2014)
υ	Wealth effect	1	Greenwood et al. (1998)
ρ	Elasticity of subs. across skills	0.33	Ciccone and Peri (2005)
a_5	Productivity, Form. sector	1	Calibration targets:
a_4	Productivity, Form. sector	0.28	Labor income quintiles
a_3	Productivity, Form. sector	0.17	(2015 CASEN)
a_2	Productivity, Form. sector	0.11	
a_1	Productivity, Form. sector	0.002	$[4.2, 9.7, 14.5, 21.1, 50.5]^*$
$a_5 b_5$	Product. Inf. sector	32.5	Calibration targets: Aggregate
$a_4 b_4$	Product. Inf. sector	13.9	participation in informal sector
$a_3 b_3$	Product. Inf. sector	9.01	and distribution across quintiles.
$a_2 b_2$	Product. Inf. sector	5.87	(2012 NESI)
$a_1 b_1$	Product. Inf. sector	1.38	$[57, 18, 18, 17, 14]^*$

Note: *income quintiles 1 through 5 are ordered from lowest to highest income.

The fraction of domestic savings invested abroad, γ, is set to 40%. This number corresponds to the value of all foreign investment by the Pension Fund Administrators (Administradoras de Fondos de Pensiones), as a fraction of total assets, according to the data from the Chilean Superintendency of Pensions from August 2016. The foreign interest rate, r^*, is set to 3% annually, which corresponds to the global real return rate assumed by the OECD (2015). In turn, the fraction of foreign investment in Chile, γ^*, is set to 145% of GDP at the initial steady state. This number corresponds to the gross investment position of foreign entities in Chilean banks, other financial institutions, non-financial firms, and households, expressed as a ratio of GDP, according to National Accounts.

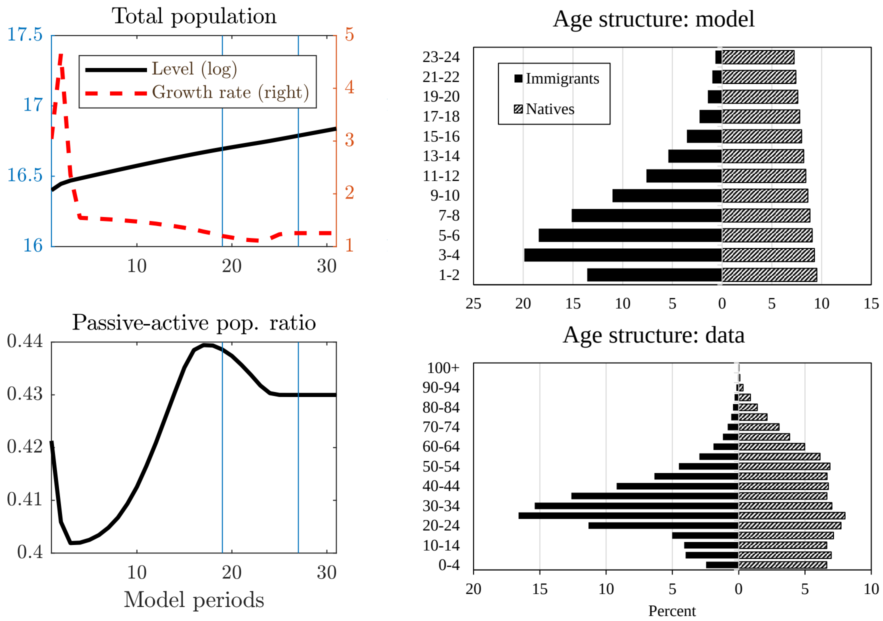
We now describe the design of our simulation exercises. This design is based on the recent experience of Chile. We consider an immigration wave that takes place over 3 model periods (7.5 years) and represents 8.7% of the total labor force prior to the beginning of the wave (about 750,000 immigrants and a labor force of 8.5 million). These numbers are set to match data and projections for 2015-2022. Of the total number of immigrants, 29% arrives during the first period, 56% during the second period, and the final 15% during the last period.[15] The upper-left

[15]The dynamics of the labor force are based on estimates and projections for the total population, the working-age population, and participation rates. The population data come from the National Statistics Institute (December 2018 release). The participation rates used to pin down the labor force come from Aldunate, Bullano, Canales, Contreras, Fernández, Fornero, García, García, Pena, Tapia, and Zúniga (2019a), who construct forecasts of the labor force as an input for forecasts of trend growth in Chile.

graph in figure 6 depicts the higher growth rate of the total population generated by these waves of immigrants entering the model economy. The first vertical line in that graph marks the first period in which all immigrant workers are retired, and the second vertical line marks the first period in which all immigrants are deceased. In addition, as shown in the lower-right graph of figure 6, the age structure of immigrants is skewed towards younger working-age adults. We capture this in the simulations as shown in the upper-right graph of figure 6. Since individuals in the model are active for 40 years (16 periods, each with duration of 10 quarters) and retired for 20 years (8 periods), and since we abstract from human capital accumulation, our simulations match the age distribution of immigrants in the 20-80 range of age. In the model, the native population is assumed to grow at a constant rate, so the age distribution of natives is smoother than in the data. The lower-left graph of Figure 5 shows that the ratio of retired to active individuals declines due to the immigration of mostly younger workers.

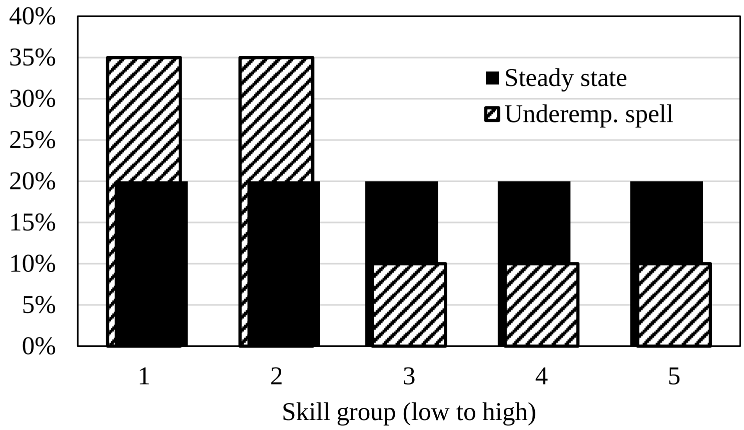
Finally, as mentioned in subsection 2.1, we assume that the skill distribution of immigrants is similar to that of the native population. However, in line with our assumption that immigrants experience an underemployment spell before they can fully integrate into the labor market, we impose a temporary *de facto* distribution of skills on immigrants during the first two periods they live in this economy. This transitory shock to the skill distribution of immigrants lowers the effective productivity of a fraction of them. Recall that in steady state, the population is distributed evenly among the five skill groups. We consider two scenarios, first a baseline case in which half of each of the three higher-skilled groups of immigrants entering the economy are reassigned to the two groups of lower skills. Figure 7 describes this scenario; the steady state distribution of skills is shown in solid bars, whereas the transitory skill distribution of immigrants is shown in dashed bars. Note that in the baseilne scenario, 30% of immigrants entering the economy experience the underemployment spell for two periods (10 percentage points (pp) of each of the three higher-skilled groups). An alternative scenario is also considered: a case in which all immigrants entering the economy are grouped evenly in the two groups of lower skills for the first two periods after entering the economy. In this alternative scenario, 60% of immigrants experience the underemployment spell for two periods (20pp coming from each of the three higher-skilled groups). After the underemployment spell is over, agents recover their productivity immediately and converge to the steady state distribution.

Figure 6: Population dynamics in simulations



Source: Aldunate et al. (2019b) and authors' calculations.
Note: The vertical lines in the left-hand-side graphs mark the first period in which: (i) all immigrant workers are retired, and (ii) all immigrant workers are deceased. Each period in the model lasts 2.5 years.

Figure 7: Distribution of skills



Source: Authors' calculations.

5 Results

This section presents the distributional, sectoral, and aggregate results of our baseline simulation of an immigration wave. We highlight the role of the informal sector, and study the sensitivity of the results to our assumptions on the duration and intensity of the underemployment spell that affects immigrants upon arrival.

5.1 Baseline Simulation

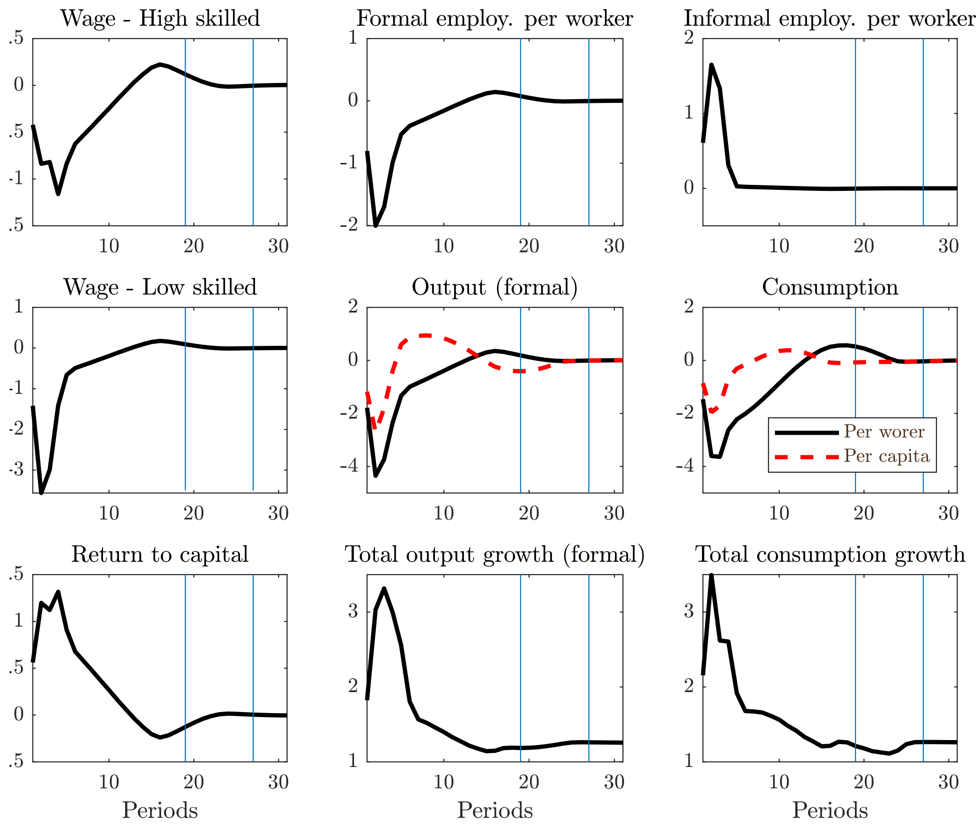
Figure 8 shows the results of our baseline simulation of an immigration wave. As mentioned in the previous section, the three key ingredients of the baseline simulation are shocks to population growth and the age structure, and the underemployment spell that immigrants experience. The immigration wave leads to an increase in labor supply of workers of all skill levels, which generates a generalized decline in wages. The decline in wages of low-skilled workers, however, is about three times larger than that of high-skilled workers. This distributional effect of the shock is primarily due to the underemployment spell that affects a fraction of immigrants, which is modeled as a transitory decline in their skill level. On impact, therefore, the increase in supply of low-skilled labor is larger than that of high-skilled labor, so low-skilled wages fall proportionately more. We will elaborate on this point below. Capital is predetermined, so the economy-wide ratio of capital to (formal) labor declines. Consequently, the return to capital increases.[16]

The generalized decline in formal sector wages induces active workers to reallocate labor effort to the informal sector: the average active worker devotes less hours to the formal sector and more hours to the informal sector. Although workers of all skill levels reallocate to the informal sector to some extent, this effect is larger among low-skilled workers, whose formal-sector wages are the most affected.

Regarding the aggregate effects of the immigration wave, note that output (only produced in the formal sector) and consumption (of formal output and home production) per worker both decline. But note that consumption per worker falls less than output per worker. As we show below, this is due to the informal sector, which acts as a buffer that mitigates the decline in consumption. In per capita terms, i.e. with respect to the total population, output and consumption fall less than in terms of workers, because the total population increases less than the labor force in the simulation, as in the data (recall immigrants are mostly young adults).

[16]Note that capital is only used in the formal sector, so the relevant measure of scarcity relates it to formal labor.

Figure 8: Baseline simulation

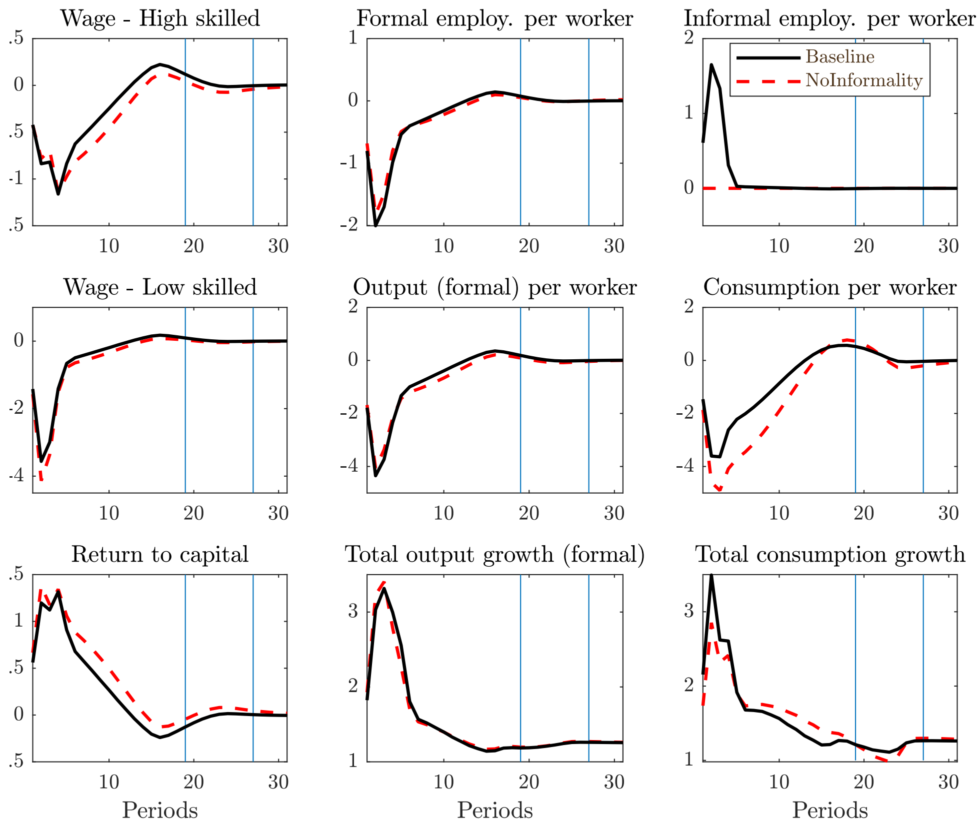


Note: Percent deviation from steady state, except for total output and consumption growth rates. The two vertical lines in each graph mark the first period in which: (i) all immigrant workers are retired, and (ii) all immigrant workers are deceased. Each period lasts 2.5 years. Individuals live for 24 periods (16 active; 8 retired). Simulations conducted under perferct foresight.

To quantify the role of the informal sector, we shut it down and compare the effects of an immigration wave to the baseline results.[17] Figure 9 shows that the main effect of the informal sector is that it mitigates the decline in consumption per worker, which falls about one third more when the informal sector is shut down.

[17]To shut down the informal sector, we lower its productivity (using the parameters b_i) up to the point in which it is no longer a relevant option for workers.

Figure 9: Baseline simulation: The role of the informal sector

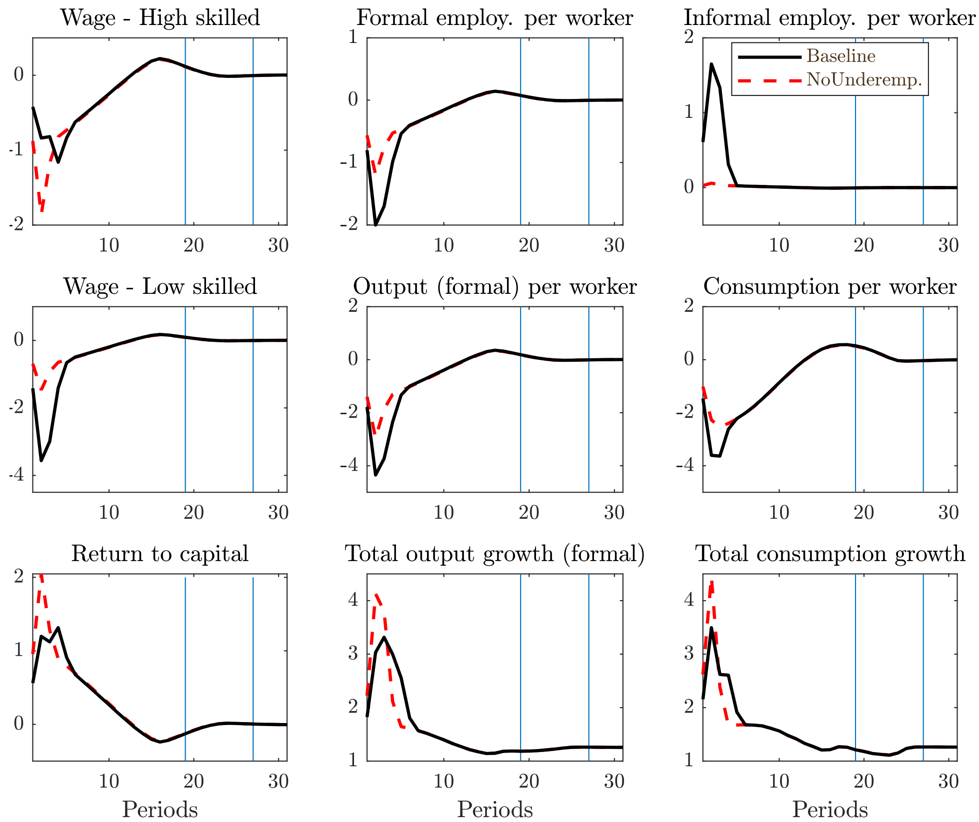


Note: The red lines show results from a simulation with the informal sector shut down. Percent deviation from steady state, except for total output and consumption growth rates. The two vertical lines in each graph mark the first period in which: (i) all immigrant workers are retired, and (ii) all immigrant workers are deceased.

To quantify the role of the underemployment spell, figure 10 compares the baseline results to a simulation that ignores this feature, i.e., a simulation that includes only demographic shocks that alter the size and age structure of the population, so that immigrants and natives have the same skill distribution. We can see that the main effect of the underemployment spell is distributional: in its absence, the decline in wages across workers of different skill levels is fairly similar at around 1% from steady state. When a fraction of immigrants are underemployed, as if they had lower skills, low-skilled labor is relatively more abundant, so low-skill wages fall much more and high-skill wages fall much less. Furthermore, in the absence of an underemployment spell, there is substantially less reallocation of labor from the formal to the informal sector. This is due to the smaller decline in low-skill wages (recall low-skilled workers are more suceptible to reallocation). Since formal employment per worker declines less than in the baseline simulation, output per worker also declines less than in the baseline results. Finally, and as one would expect, consumption per worker is also less sensitive to the immigration wave when immigrants are not affected by the underemployment spell.

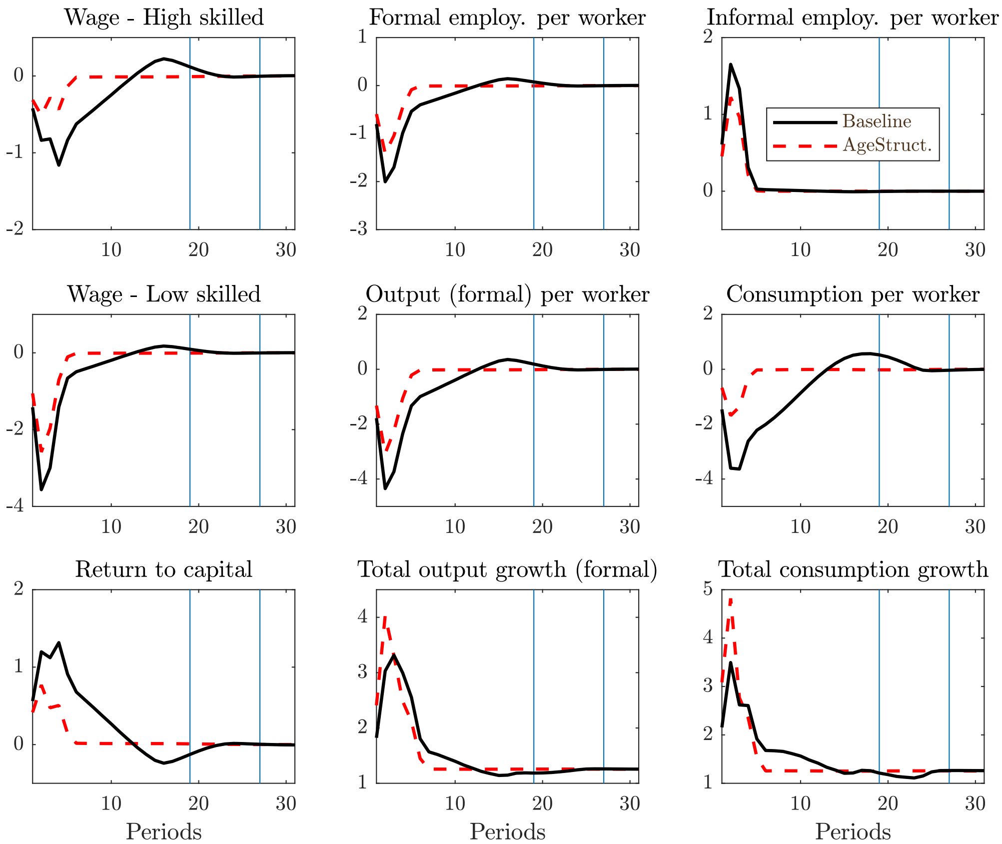
To conclude this subsection, we study the importance of considering that immigrants have a very different age structure than natives, i.e., the importance of using a model with multiple generations. Figure 11 compares the baseline results with a simulation in which immigrants

Figure 10: Baseline simulation: The role of the underemployment spell



Note: The red lines show results from a simulation in which immigrants do not experience an underemployment spell, i.e., one that includes only demographic shocks that alter the size and age structure of the population. Percent deviation from steady state, except for total output and consumption growth rates. The two vertical lines in each graph mark the first period in which: (i) all immigrant workers are retired, and (ii) all immigrant workers are deceased.

Figure 11: Baseline simulation: The role of age structure



Note: The dashed red lines show results from a simulation in which the age structure of immigrants is identical to that of natives (they still experience an underemployment spell). Percent deviation from steady state, except for total output and consumption growth rates. The two vertical lines in each graph mark the first period in which: (i) all immigrant workers are retired, and (ii) all immigrant workers are deceased.

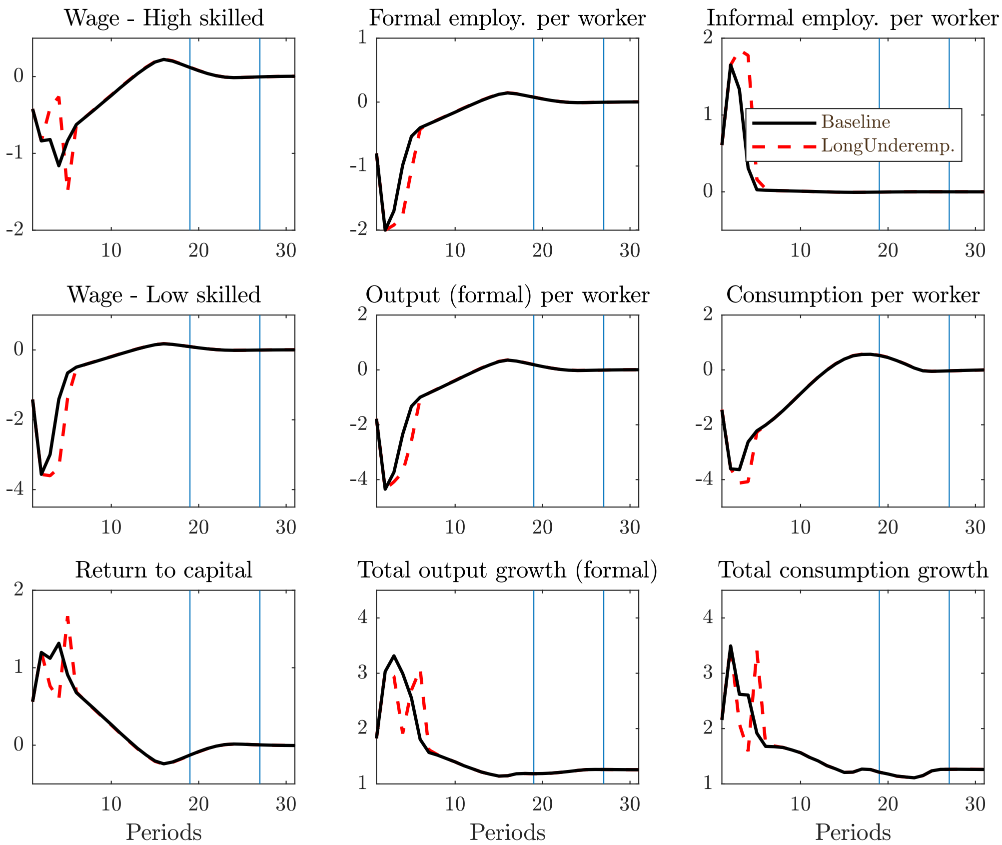
and natives have the same age structure. In this case, the magnitude of the effects of an immigration wave are substantially smaller and less persistent. Since the immigrant population is older than in the baseline simulation, active immigrants reach the retirement age sooner, and retired immigrants represent a larger fraction of the total immigrant population. The increase in labor supply is, thus, smaller and less persistent, and so is the downward pressure on wages. Reallocation to the informal sector is also smaller than in the baseline simulation. Naturally, the decline in output and consumption per worker is much smaller, reaching a trough of about half of that in the baseline simulation. These results support the use of an OLG model that is able to accomodate heterogeneous age structures.

5.2 Sensitivity Analysis

We now study the sensitivity of our results to the duration and intensity of the underemployment spell experienced by a fraction of immigrants. Figure 12 considers the case in which immigrants experience a longer underemployment spell, whereas figure 13 considers the case in which the underemployment spell affects a larger fraction of immigrants.

In our baseline simulation, 30% of immigrants experience an underemployment spell of 2 model periods (5 years). In figure 12, we compare these results with a scenario in which the underemployment spell lasts twice as long: 4 model periods (10 years). Under a longer underemployment spell, the decline in low-skill wages is more persistent, which induces a larger reallocation of labor effort from the formal to the informal sector. Consequently, output and consumption per worker decline more persistently than in the baseline simulation.

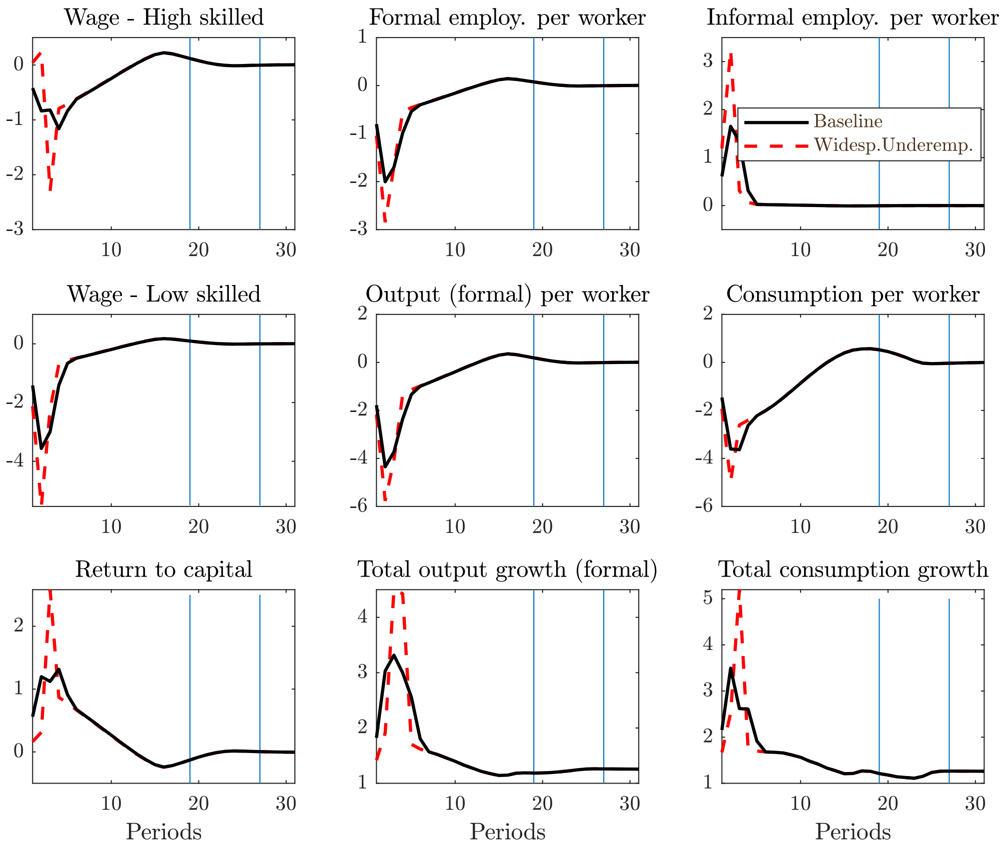
Figure 12: Longer underemployment spell



Note: The red lines show results from a simulation in which immigrants experience an underemployment spell of 4 periods (10 years); 2 periods in the baseline results. Percent deviation from steady state, except for total output and consumption growth rates. The two vertical lines in each graph mark the first period in which: (i) all immigrant workers are retired, and (ii) all immigrant workers are deceased.

Figure 13 studies the sensitivity of our results to our assumption on the fraction of immigrants that experience the underemployment spell. In the baseline results, 30% of immigrants are affected; the figure compares these results with a simulation that assumes 60% of immigrants experience the underemployment spell. The distributional effects of the shock are larger in this scenario, with the low-skill wage falling more, and the high-skill wage falling less, than in the baseline simulation. Since low-skilled workers are more susceptible to reallocation, informal employment per worker expands more than in the baseline simulation, formal employment per worker contracts more than in the baseline simulation, and thus, output and consumption per worker are more adversely affected.

Figure 13: Widespread underemployment spell



Note: The red lines show results from a simulation in which 60% of immigrants experience an underemployment spell; 30% in the baseline results. Percent deviation from steady state, except for total output and consumption growth rates. The two vertical lines in each graph mark the first period in which: (i) all immigrant workers are retired, and (ii) all immigrant workers are deceased.

6 Conclusion

Emerging countries receive substantial immigration flows, yet the literature on the macroeconomic effects of immigration has largely ignored them. This paper fills this gap, considering the importance of labor informality in emerging countries. The informal sector acts as a buffer that allows individuals to mitigate the decline in consumption per worker generated by an immigration wave. An overlapping generations model (OLG) calibrated to Chilean demographic and economic data suggests that immigration increases labor supply, putting downward pressure on formal-sector wages, and triggering a reallocation of labor effort to the informal sector, which helps workers, especially those with lower skills, sustain income and consumption.

Our simulations highlight the importance of modeling a rich demographic structure that allows for immigrants with a different skill distribution and age structure than those of natives. The model then provides important quantitative references for the macroeconomic effects of immigration in an emerging country.

Our analysis of the macroeconomic effects of immigration in an emerging country is based on simulations of an OLG model calibrated to match Chilean data. While it would be desirable to complement these results with those of an empirical identification of the effects of an immigration shock, as in Furlanetto and Robstad (2019), who estimate Bayesian vector autoregressions on data for Norway, data limitations prevent this type of analysis for an economy such as Chile, which lacks quarterly data on net immigration flows. Moreover, substantial immigration is a very recent phenomenon; it began around 2015. Even if better data were available, it would be challenging to observe enough variation in immigration to identify the effect of shocks.

Finally, the OLG model we build in this paper could be extended to analyze other issues. For example, including a fiscal block would allow an analysis of the effect of immigration on public finances and pension systems.

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Appendices

A Equilibrium Conditions

In equilibrium, labor supply of each worker type and cohort equals labor demand:

$$l_{i,t}^{g} N_{i,t}^{g} = L_{i,t}^{g}, \quad i = 1, ..., 5 \;\; g = 1, ..., 16$$

Aggregate consumption is given by

$$C_t^g = \sum_{i=1}^{5} C_{i,t}^{g} N_{i,t}^{g}.$$

Aggregate savings satisfy

$$S_t = \sum_{g=1}^{23} S_t^g,$$

where

$$S_t^g = \sum_{i=1}^{5} S_{i,t}^{g} N_{i,t}^{g}.$$

A fraction $1-\gamma$ of total savings, S_t, is then invested - into capital - to transfer the savings into the following period:

$$I_t = (1-\gamma) S_t + \gamma^{\star} N_t^1, \tag{28}$$

where we allow for the possibility that foreign agents finance some fraction $\gamma^{\star} \in [0,1]$ of the capital stock.[18]

The (non-stationary) competitive equilibrium of the model is then given by the set of sequences

$$\begin{aligned}
\{C_{1,t}^1, ..., C_{5,t}^1, ..., C_{1,t}^{24}, ..., C_{5,t}^{24}, S_{1,t}^1, ..., S_{5,t}^1, ..., S_{1,t}^{23}, ..., S_{5,t}^{23},\\
l_{1,t}^1, ..., l_{5,t}^1, ..., l_{1,t}^{16}, ..., l_{5,t}^{16}, h_{1,t}^1, ..., h_{5,t}^1, ..., h_{1,t}^{16}, ..., h_{5,t}^{16},\\
W_{1,t}^1, ..., W_{5,t}^1, ..., W_{1,t}^{16}, ..., W_{5,t}^{16}, L_{1,t}, ..., L_{5,t}, L_{1,t}^1, ..., L_{5,t}^1,\\
..., L_{1,t}^{16}, ..., L_{5,t}^{16}, N_{1,t}, ..., N_{5,t}, N_t^1, ..., N_t^{24},\\
r_t^k, R_t^{Tk}, r_t, Y_t, GDP_t, L_t,\\
W_t, C_t, C_t^1, ..., C_t^{24}, S_t, S_t^1, ..., S_t^{23}, K_{t+1}, I_t, N_t\}_{t=0}^{\infty},
\end{aligned}$$

such that for given initial values, the following conditions are satisfied for every t. For $0 < g \leq 16$, defining $S_{i,t}^0 = 0$,

$$C_{i,t}^g = W_{i,t}^g l_{i,t}^g + b_i a_i h_{i,t}^g + r_t S_{i,t-1}^{g-1} - S_{i,t}^g,$$

for $16 < g < 24$:

$$C_{i,t}^g = r_t S_{i,t-1}^{g-1} - S_{i,t}^g,$$

[18]That fraction is assumed to grow with the trend N_t^1 so as to ensure a balanced growth path.

and

$$C_{i,t}^{24} = r_t S_{i,t-1}^{23}.$$

For $gg \leq 16$:

$$C_{i,t+gg-1}^{gg} = \left(\sum_{g=gg}^{24} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} \left[r_{t+gg-1} S_{i,t+gg-2}^{gg-1} + \right.$$
$$\left. \sum_{g=gg}^{16} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{-1} \left[W_{i,t+g-1}^{g} l_{i,t+g-1}^{g} + b_i a_i h_{i,t+g-1}^{g} \right] \right],$$

for $16 < gg$:

$$C_{i,t+gg-1}^{gg} = \left(\sum_{g=gg}^{24} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} r_{t+gg-1} S_{i,t+gg-2}^{gg-1},$$

for $g = 1, ..., 24$:

$$C_t^g = \sum_{i=1}^{5} C_{i,t}^g N_{i,t}^g,$$

for $0 < g \leq 16$:

$$\chi_i^g \left(l_{i,t}^g + h_{i,t}^g \right)^{\phi-1} = \left(C_{i,t}^g \right)^{-\theta(1-\upsilon)} W_{i,t}^g,$$

$$h_{i,t}^g = \max \left\{ \left[\left(C_{i,t}^g \right)^{-\theta(1-\upsilon)} \frac{\left(b_i a_i - W_{i,t}^g \right)}{\chi_i^g \kappa} \right]^{\frac{1}{\phi-1}}, 0 \right\},$$

$$W_{i,t}^g = a_i W_t \left(\frac{L_t}{L_{i,t}} \right)^{1-\rho},$$

$$L_{i,t} = \sum_{g=1}^{16} L_{i,t}^g,$$

$$l_{i,t}^g N_{i,t}^g = L_{i,t}^g,$$

$$N_{i,t}^g = \lambda_{i,t}^g N_t^g,$$

$$Y_t = K_t^{\alpha} \left(A_t L_t \right)^{1-\alpha},$$

$$L_t = \left(\sum_{i=1}^{5} a_i L_{i,t}^{\rho} \right)^{1/\rho},$$

$$W_t = \frac{(1-\alpha) Y_t}{L_t},$$

$$r_t^k = \alpha \frac{Y_t}{K_t},$$

$$R_t^{Tk} = r_t^k + (1-\delta),$$

$$r_t = 1 + (1-\gamma)\left(R_t^{Tk} - 1\right) + \gamma r^\star,$$

$$S_t = \sum_{g=1}^{23} S_t^g,$$

for $g \leq 23$:

$$S_t^g = \sum_{i=1}^{5} S_{i,t}^g N_{i,t}^g.$$

$$C_t = \sum_{g=1}^{24} C_t^g,$$

$$I_t = (1-\gamma) S_t + \gamma^\star N_t^1,$$

$$K_{t+1} = I_t,$$

$$GDP_t = C_t + K_{t+1},$$

$$N_t^1 = N_{t-1}^1 (1 + n^r + n_t^m),$$

$$N_t^g = N_{t-1}^g (1 + m_t^g) \quad \forall g > 1.$$

Some complementary variables:

$$H_t = \sum_{g=1}^{16} \sum_{i=1}^{5} h_{i,t}^g N_{i,t}^g, \tag{29}$$

$$L_t^{total} = \sum_{g=1}^{16} \sum_{i=1}^{5} l_{i,t}^g N_{i,t}^g. \tag{30}$$

To rewrite the model in stationary form, we define the stationary per-capita cariables: $c_{i,t}^g = C_{i,t}^g$, $c_t^g = C_t^g/N_t^g$, $s_{i,t}^g = S_{i,t}^g$, $s_t^g = S_t^g/N_t^g$, $s_t = S_t/N_t^1$, $w_{i,t}^g = W_{i,t}^g$, $w_t = W_t$, $l_{i,t} = L_{i,t}/N_t^1$, $l_{i,t}^g = L_{i,t}^g/N_{i,t}^g$, $l_t = L_t/N_t^1$,$k_t = K_t/N_{t-1}^1$, $i_t = I_t/N_t^1$, $\gamma_t^I = \Gamma_t/N_t^1$, $y_t = Y_t/N_t^1$, $c_t = C_t/N_t^1$,$gdp_t = GDP_t/N_t^1$.

The stationary competitive equilibrium of the model is then given by the set of sequences

$$\begin{aligned}
\{c_{1,t}^1, ..., c_{5,t}^1, ..., c_{1,t}^{24}, ..., c_{5,t}^{24}, s_{1,t}^1, ..., s_{5,t}^1, ..., s_{1,t}^{23}, ..., s_{5,t}^{23},\\
l_{1,t}^1, ..., l_{5,t}^1, ..., l_{1,t}^{16}, ..., l_{5,t}^{16}, h_{1,t}^1, ..., h_{5,t}^1, ..., h_{1,t}^{16}, ..., h_{5,t}^{16},\\
w_{1,t}^1, ..., w_{5,t}^1, ..., w_{1,t}^{16}, ..., w_{5,t}^{16}, l_{1,t}, ..., l_{5,t},\\
q_t, r_t^k, R_t^{Tk}, r_t, y_t, gdp_t, l_t, h_t, l_t^{total}\\
w_t, c_t, c_t^1, ..., c_t^{24}, s_t, s_t^1, ..., s_t^{23}, k_{t+1}, i_t, \Gamma_t\}_{t=0}^{\infty},
\end{aligned}$$

such that for given initial values, the following conditions are satisfied for every t:[19]

$$c_{i,t}^{1} = w_{i,t}^{1} l_{i,t}^{1} + b_i a_i h_{i,t}^{1} - s_{i,t}^{1}, \tag{31}$$

for $1 < g \leq 16$

$$c_{i,t}^{g} = w_{i,t}^{g} l_{i,t}^{g} + b_i a_i h_{i,t}^{g} + r_t s_{i,t-1}^{g-1} - s_{i,t}^{g}, \tag{32}$$

for $16 < g \leq 24$

$$c_{i,t}^{g} = r_t s_{i,t-1}^{g-1} - s_{i,t}^{g}, \tag{33}$$

and

$$c_{i,t}^{24} = r_t s_{i,t-1}^{23}, \tag{34}$$

for $gg \leq 16$ and defining $s_{i,t-1}^{0} = 0$:

$$\begin{aligned} c_{i,t+gg-1}^{gg} = & \left(\sum_{g=gg}^{24} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} \\ & \left\{ r_{t+gg-1} s_{i,t+gg-2}^{gg-1} + \sum_{g=gg}^{16} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{-1} \right. \\ & \left. \left[w_{i,t+g-1}^{g} l_{i,t+g-1}^{g} + b_i a_i h_{i,t+g-1}^{g} \right] \right\}, \\ & t = s - gg + 1, \end{aligned} \tag{35}$$

$$\begin{aligned} c_{i,t}^{gg} = & \left(\sum_{g=gg}^{24} \left(\prod_{n_g=1}^{g-gg} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} \\ & \left\{ r_t s_{i,t-1}^{gg-1} + \sum_{g=gg}^{16} \left(\prod_{n_g=1}^{g-gg} r_{t+n_g} \right)^{-1} \right. \\ & \left. \left[w_{i,t+g-gg}^{g} l_{i,t+g-gg}^{g} + b_i a_i h_{i,t+g-gg}^{g} \right] \right\}, \end{aligned}$$

if $16 < gg$:

$$c_{i,t+gg-1}^{gg} = \left(\sum_{g=gg}^{24} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} r_{t+gg-1} s_{i,t+gg-2}^{gg-1}, \tag{36}$$

$$c_{i,t}^{gg} = \left(\sum_{g=gg}^{24} \left(\prod_{n_g=1}^{g-gg} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} r_t s_{i,t-1}^{gg-1},$$

[19] To cast this system into Dynare syntax for predetermined variables, k_{t+1} and k_t need to be written as k and $k\,(-1)$ respectively.

for $g = 1, ..., 24$:

$$c_t^g = \sum_{i=1}^{5} c_{i,t}^g \lambda_{i,t}^g, \tag{37}$$

for $0 < g \leq 16$:

$$\chi_i^g \left(l_{i,t}^g + h_{i,t}^g\right)^{\phi-1} = \left(c_{i,t}^g\right)^{-\theta(1-\upsilon)} w_{i,t}^g, \tag{38}$$

$$h_{i,t}^g = \max\left\{\left[\left(c_{i,t}^g\right)^{-\theta(1-\upsilon)} \frac{\left(b_i a_i - w_{i,t}^g\right)}{\chi_i^g \kappa}\right]^{\frac{1}{\phi-1}}, 0\right\}, \tag{39}$$

$$w_{i,t}^g = a_i w_t \left(\frac{l_t}{l_{i,t}}\right)^{1-\rho}, \tag{40}$$

$$l_{i,t} = \sum_{g=1}^{16} l_{i,t}^g \lambda_{i,t}^g \prod_{j=0}^{g-2} \frac{1 + m_{t-j}^{g-j}}{1 + n^r + n_{t-j}^m}, \tag{41}$$

$$y_t = \left(\frac{k_t}{(1 + n^r + n_t^m)}\right)^{\alpha} (l_t)^{1-\alpha}, \tag{42}$$

$$l_t = \left(\sum_{i=1}^{5} a_i (l_{i,t})^{\rho}\right)^{1/\rho}, \tag{43}$$

$$w_t = \frac{(1-\alpha) y_t}{l_t}, \tag{44}$$

$$r_t^k = \alpha \frac{y_t}{k_t} (1 + n^r + n_t^m), \tag{45}$$

$$R_t^{Tk} = r_t^k + 1 - \delta, \tag{46}$$

$$r_t = 1 + (1-\gamma)\left(R_t^{Tk} - 1\right) + \gamma r^{\star}, \tag{47}$$

$$s_t = \sum_{g=1}^{23} s_t^g \prod_{j=0}^{g-2} \frac{1 + m_{t-j}^{g-j}}{1 + n^r + n_{t-j}^m}, \tag{48}$$

for $0 < g \leq 23$

$$s_t^g = \sum_{i=1}^{5} s_{i,t}^g \lambda_{i,t}^g, \tag{49}$$

$$c_t = \sum_{g=1}^{24} c_t^g \prod_{j=0}^{g-2} \frac{1 + m_{t-j}^{g-j}}{1 + n^r + n_{t-j}^m}, \tag{50}$$

$$i_t = (1-\gamma) s_t + \gamma^{\star}, \tag{51}$$

$$k_{t+1} = (1 - \Gamma_t) i_t, \tag{52}$$

$$gdp_t = c_t + k_{t+1}. \tag{53}$$

Some complementary variables:

$$h_t = \sum_{g=1}^{16} \sum_{i=1}^{5} h_{i,t}^g \lambda_{i,t}^g \prod_{j=0}^{g-2} \frac{1 + m_{t-j}^{g-j}}{1 + n^r + n_{t-j}^m}, \tag{54}$$

$$l_t^{total} = \sum_{g=1}^{16} \sum_{i=1}^{5} l_{i,t}^g \lambda_{i,t}^g \prod_{j=0}^{g-2} \frac{1 + m_{t-j}^{g-j}}{1 + n^r + n_{t-j}^m}, \tag{55}$$

$$0 = y_t - r_t^k k_t \left(1 + n^r + n_t^m\right)^{-1} - \sum_{i=1}^{5} \sum_{g=1}^{16} w_{i,t}^g l_{i,t}^g \lambda_{i,t}^g \prod_{j=0}^{g-2} \frac{1 + m_{t-j}^{g-j}}{1 + n^r + n_{t-j}^m}. \tag{56}$$

Output per capita:

$$\begin{aligned}
\frac{Y_t}{\sum_{g=1}^{24} N_t^g} &= y_t \frac{N_t^1}{\sum_{g=1}^{24} N_t^g} \\
&= y_t \frac{1}{\sum_{g=1}^{24} \frac{N_t^g}{N_t^1}} \\
&= y_t \frac{1}{\sum_{g=1}^{24} \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}} \\
&= \frac{y_t}{1 + \sum_{g=2}^{24} \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}}.
\end{aligned} \tag{57}$$

Capital per capita:

$$\begin{aligned}
\frac{K_t}{\sum_{g=1}^{24} N_t^g} &= k_t \frac{A_{t-1} N_{t-1}^1}{A_t \sum_{g=1}^{24} N_t^g} \\
&= k_t \frac{1}{\sum_{g=1}^{24} \frac{N_t^g}{N_{t-1}^1}} \\
&= k_t \frac{1}{\sum_{g=1}^{24} \left(1 + n^r + n_t^m\right) \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}} \\
&= \frac{k_t \left(1 + n^r + n_t^m\right)^{-1}}{1 + \sum_{g=2}^{24} \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}}.
\end{aligned} \tag{58}$$

B Steady State

Let variables wothout time subscript denote steady state values. We solve or the steady state using numerical methods, using as starting values for the numerical solver the analytical steady state solution for the special case when $\chi_i^y = \chi_i^m$ for all i, $\beta_i = \beta_j$ for all i and j, $\theta = \rho = 1$, $\upsilon = \gamma = \gamma^\star = 0$ and $\lambda_{i,t}^g = \lambda$. We further have that in steady state, $n_{ss}^r = n^r$, and there is no migration, i.e. $n_{ss}^m = 0$ and $m_{ss}^g = 0$. Then, from (17)

$$\sum_{g=1}^{m} r^{m-g} c_i^g = \sum_{g=1}^{16} r^{m-g} \left[w_i^g l_i^g + (1+z)^{g-1} e^{I_{g<g_{old}}} b_i a_i h_i^g \right]$$

and from (23)

$$c_i^{g_1} r^{m-g_1} \sum_{g=1}^{m} \beta_i^{g-g_1} = \sum_{g=1}^{16} r^{m-g} \left[w_i^g l_i^g + (1+z)^{g-1} e^{I_{g<g_{old}}} b_i a_i h_i^g \right]$$

or

$$c_i^{g_1} = \left(\sum_{g=1}^{m} \beta_i^{g-g_1} \right)^{-1} \sum_{g=1}^{16} r^{g_1-g} \left[w_i^g l_i^g + (1+z)^{g-1} e^{I_{g<g_{old}}} b_i a_i h_i^g \right] \tag{59}$$

$$c_i^{g_2} \beta_i^{g_1-g_2} r^{g_1-g_2} = c_i^{g_1} \tag{60}$$

From (40), and since $\rho = 1$,

$$w_i^g = (1+ag)^{g-1} a_i e^{I_{g<g_{old}}} w \tag{61}$$

Thus, if $g_1 < g_2 < g_{old}$ or $g_{old} < g_1 < g_2$

$$w_i^{g_1} = (1+z)^{g_1-g_2} w_i^{g_2}$$

and if $g_1 < g_{old} \leq g_2$

$$w_i^{g_1} = (1+z)^{g_1-g_2} e w_i^{g_2}$$

From (39), and as long as $\chi_i^{g_1} = \chi_i^{g_2}$, for all g_1 and g_2, $\theta = 1$ and $\upsilon = 0$,

$$h_i^g = \max \left\{ \left[\frac{(1+ag)^{g-1} e^{I_{g<g_{old}}} a_i (b_i - w)}{c_i^g \chi_i^g \kappa} \right]^{\frac{1}{\phi-1}}, 0 \right\} \tag{62}$$

if $b_i < w$ we have that for $g_1 < g_2 \leq 16$

$$h_i^{g_1} = 0 = h_i^{g_2}$$

else, using (60), we have,

$$\begin{aligned} h_i^{g_1} &= \left[\frac{(1+z)^{g_1-1} e^{I_{g_1<g_{old}}} a_i (b_i - w)}{c_i^{g_1} \chi_i^{g_1} \kappa} \right]^{\frac{1}{\phi-1}} \\ &= \left((1+z)^{g_1-1} e^{I_{g_1<g_{old}}} \right)^{\frac{1}{\phi-1}} \left[\frac{a_i (b_i - w)}{c_i^{g_2} \beta_i^{g_1-g_2} r^{g_1-g_2} \chi_i^{g_2} \kappa} \right]^{\frac{1}{\phi-1}} \\ &= \left(\frac{\beta_i^{g_2-g_1} r^{g_2-g_1} e^{I_{g_1<g_{old}}}}{(1+z)^{g_2-g_1} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} h_i^{g_2} \end{aligned} \tag{63}$$

We further choose b_i such that

$$l_i^g = \frac{h_i^g}{w} \tag{64}$$

In addition, assuming that such a b_i is constant over i, we have that, departing from (59), using (64) and (61)

$$c_i^{g_1} = \left(\sum_{g=1}^{m} \beta_i^{g-g_1}\right)^{-1} \sum_{g=1}^{16} r^{g_1-g} \left[(1+z)^{g-1} a_i e^{I_{g<g_{old}}} w l_i^g + (1+ag)^{g-1} e^{I_{g<g_{old}}} b_i a_i w l_i^g\right]$$

$\Rightarrow$

$$c_i^{g_1} = \left(\sum_{g=1}^{m} \beta_i^{g-g_1}\right)^{-1} \sum_{g=1}^{16} r^{g_1-g} (1+b_i)(1+z)^{g-1} e^{I_{g<g_{old}}} a_i w l_i^g \tag{65}$$

From (38), using (65), (64)

$$l_i^{g_1} = \frac{1}{1+w}\left[\left(\left(\sum_{g=1}^{m} \beta_i^{g-g_1}\right)^{-1} \sum_{g=1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} l_i^g\right)^{-1} \frac{(1+ag)^{g_1-1} e^{I_{g_1<g_{old}}}}{\chi_i^{g_1}(1+b_i)}\right]^{\frac{1}{\phi-1}}$$

$\Rightarrow$

$$l_i^{g_1}\left(\beta_i^{1-g_1} r^{1-g_1} (1+z)^{g_1-1} e^{I_{g_1<g_{old}}}\right)^{\frac{1}{1-\phi}} = \frac{1}{1+w} \left[\left(\left(\sum_{g=1}^{m} \beta_i^{g-1}\right)^{-1} \sum_{g=1}^{16} r^{1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} l_i^g\right)^{-1} \frac{1}{\chi_i^{g_1}(1+b_i)}\right]^{\frac{1}{\phi-1}}$$

and

$$l_i^{g_2}\left(\beta_i^{1-g_2} r^{1-g_2} (1+z)^{g_2-1} e^{I_{g_2<g_{old}}}\right)^{\frac{1}{1-\phi}} = \frac{1}{1+w} \left[\left(\left(\sum_{g=1}^{m} \beta_i^{g-1}\right)^{-1} \sum_{g=1}^{16} r^{1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} l_i^g\right)^{-1} \frac{1}{\chi_i^{g_2}(1+b_i)}\right]^{\frac{1}{\phi-1}} \tag{66}$$

Then, if $\phi \neq 1$, we have

$$l_i^{g_1} = l_i^{g_2}\left(\frac{\beta_i^{g_2-g_1} r^{g_2-g_1} e^{I_{g_1<g_{old}}}}{(1+z)^{g_2-g_1} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}} \tag{67}$$

Therefore, formal work of two generations satisfy the same proportion as the informal work of the same two generations, see (63) and (67). Using (66) in (67) we have,

$$l_i^{g_2}\left(\sum_{g=1}^{16} r^{1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} l_i^g\right)^{\frac{1}{\phi-1}} = \frac{1}{1+w}\left[\frac{(1+ag)^{g_2-1} e^{I_{g_2<g_{old}}} \sum_{g=1}^{m} \beta_i^{g-1}}{\beta_i^{g_2-1} r^{g_2-1} \chi_i^{g_2}(1+b_i)}\right]^{\frac{1}{\phi-1}}$$

and

$$l_i^g = l_i^{g_2} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}}$$

then

$$l_i^{g_2} = \left[\left(\sum_{g=1}^{16} r^{1-g} (1+ag)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right.$$
$$\left. \frac{(1+w)^{1-\phi} (1+z)^{g_2-1} e^{I_{g_2<g_{old}}} \sum_{g=1}^{m} \beta_i^{g-1}}{\beta_i^{g_2-1} r^{g_2-1} \chi_i^{g_2} (1+b_i)} \right]^{\frac{1}{\phi}} \tag{68}$$

Then, from (41), (67) and (68),

$$\begin{aligned} l_i &= \sum_{g=1}^{16} e^{I_{g<g_{old}}} l_i^g \lambda (1+n)^{1-g} \\ &= l_i^{g_2} \sum_{g=1}^{16} e^{I_{g<g_{old}}} \lambda (1+n)^{1-g} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \\ &= \left[\left(\sum_{g=1}^{16} r^{1-g} (1+ag)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right. \\ &\quad \left. \frac{(1+w)^{1-\phi} (1+z)^{g_2-1} e^{I_{g_2<g_{old}}} \sum_{g=1}^{m} \beta_i^{g-1}}{\beta_i^{g_2-1} r^{g_2-1} \chi_i^{g_2} (1+b_i)} \right]^{\frac{1}{\phi}} \\ &\quad \sum_{g=1}^{16} e^{I_{g<g_{old}}} \lambda (1+n)^{1-g} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \end{aligned} \tag{69}$$

To normalize l_i to 1 - in this steady state - one needs to set, for any gg and $g_2 < gr$,

$$\chi_i^{gg} = \left[\left(\sum_{g=1}^{16} r^{1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right.$$
$$\left. \frac{(1+w)^{1-\phi} (1+z)^{g_2-1} e^{I_{g_2<g_{old}}} \sum_{g=1}^{m} \beta_i^{g-1}}{\beta_i^{g_2-1} r^{g_2-1} (1+b_i)} \right]$$
$$\left(\sum_{g=1}^{16} e^{I_{g<g_{old}}} \lambda (1+n)^{1-g} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right)^{\phi}$$

or

$$\chi_i^{gg} = \left[\left(\sum_{g=1}^{16} r^{-g} (1+z)^g \, e^{I_{g<g_{old}}} \left(\frac{\beta_i^{-g} r^{-g} e^{I_{g<g_{old}}}}{(1+z)^{-g}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right.$$
$$\left. \frac{(1+w)^{1-\phi} \sum_{g=1}^{m} \beta_i^g}{(1+b_i)} \right]$$
$$\left(\sum_{g=1}^{16} e^{I_{g<g_{old}}} \lambda (1+n)^{1-g} \left(\frac{\beta_i^{-g} r^{-g} e^{I_{g<g_{old}}}}{(1+z)^{-g}} \right)^{\frac{1}{\phi-1}} \right)^{\phi}$$

From (43):

$$l = \sum_{i=1}^{5} a_i l_i = \sum_{i=1}^{5} a_i \tag{70}$$

Then, using (67) in (65), we can write

$$c_i^{g_1} = \left(\sum_{g=1}^{m} \beta_i^{g-g_1} \right)^{-1} \sum_{g=1}^{16} r^{g_1-g} \left[w_i^g l_i^g + (1+z)^{g-1} \, e^{I_{g<g_{old}}} b_i a_i h_i^g \right] \tag{71}$$

$$l_i^g = l_i^{g_2} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} \, e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}}$$

$$c_i^{g_1} = a_i w (1+b_i) \, l_i^{g_2} \left(\sum_{g=1}^{m} \beta_i^{g-g_1} \right)^{-1}$$
$$\sum_{g=1}^{16} r^{g_1-g} (1+z)^{g-1} \, e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} \, e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \tag{72}$$

From (26) and (27), if $1 < g_1 \leq 16$

$$c_i^{gg} = \left(\sum_{g=gg}^{m} \beta_i^{g-gg} \right)^{-1} \left\{ r s_i^{gg-1} + \sum_{g=gg}^{16} r^{gg-g} (1+b_i)(1+z)^{g-1} \, e^{I_{g<g_{old}}} a_i w l_i^g \right\}$$

$$c_i^{g_1} = \left(\sum_{g=g_1}^{m} \beta_i^{g-g_1} \right)^{-1} \left[r s_i^{g_1-1} + (1+b_i) \, a_i w \sum_{g=g_1}^{16} r^{g_1-g} (1+z)^{g-1} \, e^{I_{g<g_{old}}} l_i^g \right]$$

$\Rightarrow$

$$s_i^{g_1-1} = \left(\sum_{g=g_1}^{m} \beta_i^{g-g_1} \right) \frac{c_i^{g_1}}{r} - \frac{1}{r} (1+b_i) \, a_i w \sum_{g=g_1}^{16} r^{g_1-g} (1+z)^{g-1} \, e^{I_{g<g_{old}}} l_i^g$$

$\Rightarrow$

$$s_i^{g_1-1} = \left(\sum_{g=g_1}^{m} \beta_i^{g-g_1}\right) \frac{1}{r}$$
$$(1+b_i)\, a_i w l_i^{g_2} \left(\sum_{g=1}^{m} \beta_i^{g-g_1}\right)^{-1}$$
$$\sum_{g=1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}}$$
$$-\frac{1}{r}(1+b_i)\, a_i w \sum_{g=g_1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} l_i^g$$

$$l_i^g = l_i^{g_2} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}}$$

$$s_i^{g_1-1} = \left(\sum_{g=g_1}^{m} \beta_i^{g-g_1}\right) \frac{1}{r}$$
$$(1+b_i)\, a_i w l_i^{g_2} \left(\sum_{g=1}^{m} \beta_i^{g-g_1}\right)^{-1}$$
$$\sum_{g=1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}}$$
$$-\frac{1}{r}(1+b_i)\, a_i w l_i^{g_2} \sum_{g=g_1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}}$$

$\Rightarrow$

$$s_i^{g_1-1} = \left\{ \left(\sum_{g=g_1}^{m} \beta_i^{g-g_1}\right) \left(\sum_{g=1}^{m} \beta_i^{g-g_1}\right)^{-1} \right.$$
$$\sum_{g=1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}}$$
$$\left. -\sum_{g=g_1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}} \right\}$$
$$\frac{1+b_i}{r} a_i w l_i^{g_2} \tag{73}$$

Starting from (51) and (52), and since from (**??**) we have

$$\gamma_t^I = 0 \tag{74}$$

we can write

$$
\begin{aligned}
k &= (1-\gamma)\, s \\
&\overbrace{=}^{(48)} (1-\gamma) \sum_{g=1}^{23} s^{g} \, (1+n)^{1-g} \, (1+z)^{1-g} \\
&\overbrace{=}^{(49)} (1-\gamma) \sum_{i=1}^{5} \overbrace{\lambda_i}^{\lambda_i=\lambda} \left(\sum_{g=1}^{23} s_i^{g} \, (1+n)^{1-g} \, (1+z)^{1-g} \right)
\end{aligned}
$$

$$
\overbrace{=}^{(73)} \sum_{i=1}^{5} \lambda \left(\begin{array}{c} (1-\gamma) \sum_{g=1}^{23} (1+n)^{1-g} \, (1+z)^{1-g} \\ \left(\left(\sum_{gg=g+1}^{m} \beta_i^{gg-g-1} \right) \left(\sum_{gg=1}^{m} \beta_i^{gg-g-1} \right)^{-1} \right. \\ \sum_{gg=1}^{16} r^{g+1-gg} \, (1+z)^{gg-1} \, e^{I_{gg<g_{old}}} \left(\frac{\beta_i^{g2-gg} r^{g2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g2-gg} e^{I_{g2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \\ \left. - \sum_{gg=g+1}^{16} r^{g+1-gg} \, (1+z)^{gg-1} \, e^{I_{gg<g_{old}}} \left(\frac{\beta_i^{g2-gg} r^{g2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g2-gg} e^{I_{g2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right) \end{array} \right) \frac{1+b_i}{r} a_i w l_i^{g2}
$$

$$
\overbrace{=}^{\beta_i=\beta_j} \frac{w}{r} \left(\begin{array}{c} (1-\gamma) \sum_{g=1}^{23} (1+n)^{1-g} \, (1+z)^{1-g} \\ \left(\left(\sum_{gg=g+1}^{m} \beta_i^{gg-g-1} \right) \left(\sum_{gg=1}^{m} \beta_i^{gg-g-1} \right)^{-1} \right. \\ \sum_{gg=1}^{16} r^{g+1-gg} \, (1+z)^{gg-1} \, e^{I_{gg<g_{old}}} \left(\frac{\beta_i^{g2-gg} r^{g2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g2-gg} e^{I_{g2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \\ \left. - \sum_{gg=g+1}^{16} r^{g+1-gg} \, (1+z)^{gg-1} \, e^{I_{gg<g_{old}}} \left(\frac{\beta_i^{g2-gg} r^{g2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g2-gg} e^{I_{g2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right) \end{array} \right) \sum_{i=1}^{5} \lambda \, (1+b_i) \, a_i l_i^{g2}
$$

$$
\overbrace{=}^{b_i=b_j} \frac{w}{r} (1+b_i) \left(\begin{array}{c} (1-\gamma) \sum_{g=1}^{23} (1+n)^{1-g} \, (1+z)^{1-g} \\ \left(\left(\sum_{gg=g+1}^{m} \beta_i^{gg-g-1} \right) \left(\sum_{gg=1}^{m} \beta_i^{gg-g-1} \right)^{-1} \right. \\ \sum_{gg=1}^{16} r^{g+1-gg} \, (1+ag)^{gg-1} \, e^{I_{gg<g_{old}}} \left(\frac{\beta_i^{g2-gg} r^{g2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g2-gg} e^{I_{g2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \\ \left. - \sum_{gg=g+1}^{16} r^{g+1-gg} \, (1+ag)^{gg-1} \, e^{I_{gg<g_{old}}} \left(\frac{\beta_i^{g2-gg} r^{g2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g2-gg} e^{I_{g2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right) \end{array} \right) \sum_{i=1}^{5} \lambda a_i l_i^{g2}
$$

$$
\overbrace{=}^{(69)} \frac{w}{r} (1+b_i) \left(\sum_{g=1}^{16} e^{I_{g<g_{old}}} \lambda \, (1+n)^{1-g} \left(\frac{\beta_i^{g2-g} r^{g2-g} e^{I_{g<g_{old}}}}{(1+z)^{g2-g} \, e^{I_{g2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right)^{-1}
$$

$$
\left(\begin{array}{c} (1-\gamma) \sum_{g=1}^{23} (1+n)^{1-g} \, (1+z)^{1-g} \\ \left(\left(\sum_{gg=g+1}^{m} \beta_i^{gg-g-1} \right) \left(\sum_{gg=1}^{m} \beta_i^{gg-g-1} \right)^{-1} \right. \\ \sum_{gg=1}^{16} r^{g+1-gg} \, (1+z)^{gg-1} \, e^{I_{gg<g_{old}}} \left(\frac{\beta_i^{g2-gg} r^{g2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g2-gg} e^{I_{g2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \\ \left. - \sum_{gg=g+1}^{16} r^{g+1-gg} \, (1+z)^{gg-1} \, e^{I_{gg<g_{old}}} \left(\frac{\beta_i^{g2-gg} r^{g2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g2-gg} e^{I_{g2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right) \end{array} \right) \sum_{i=1}^{5} \lambda a_i l_i
$$

$$\underbrace{=}^{(70)} \frac{1+b_i}{r}\left(\sum_{g=1}^{16} e^{I_{g<g_{old}}}(1+n)^{1-g}\left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}}\right)^{-1}$$

$$\left(\begin{array}{c} (1-\gamma)\sum_{g=1}^{23}(1+n)^{1-g}(1+z)^{1-g} \\ \left(\left(\sum_{gg=g+1}^{m}\beta_i^{gg-g-1}\right)\left(\sum_{gg=1}^{m}\beta_i^{gg-g-1}\right)^{-1}\right. \\ \sum_{gg=1}^{16} r^{g+1-gg}(1+z)^{gg-1} e^{I_{gg<g_{old}}}\left(\frac{\beta_i^{g_2-gg} r^{g_2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g_2-gg} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}} \\ \left.-\sum_{gg=g+1}^{16} r^{g+1-gg}(1+z)^{gg-1} e^{I_{gg<g_{old}}}\left(\frac{\beta_i^{g_2-gg} r^{g_2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g_2-gg} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}}\right) \end{array}\right) wl$$

$$\underbrace{=}^{(42),(44)} \frac{1+b_i}{r}\left(\sum_{g=1}^{16} e^{I_{g<g_{old}}}(1+n)^{1-g}\left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}}\right)^{-1}$$

$$\left(\begin{array}{c} (1-\gamma)\sum_{g=1}^{23}(1+n)^{1-g}(1+z)^{1-g} \\ \left(\left(\sum_{gg=g+1}^{m}\beta_i^{gg-g-1}\right)\left(\sum_{gg=1}^{m}\beta_i^{gg-g-1}\right)^{-1}\right. \\ \sum_{gg=1}^{16} r^{g+1-gg}(1+z)^{gg-1} e^{I_{gg<g_{old}}}\left(\frac{\beta_i^{g_2-gg} r^{g_2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g_2-gg} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}} \\ \left.-\sum_{gg=g+1}^{16} r^{g+1-gg}(1+z)^{gg-1} e^{I_{gg<g_{old}}}\left(\frac{\beta_i^{g_2-gg} r^{g_2-gg} e^{I_{gg<g_{old}}}}{(1+z)^{g_2-gg} e^{I_{g_2<g_{old}}}}\right)^{\frac{1}{\phi-1}}\right) \end{array}\right) \frac{(1-\alpha)}{(1+z)^{\alpha}(1+n)^{\alpha}}\left(\frac{k}{l}\right)^{\alpha} l$$

Then, the capital labor ratio is then a non linear function of r, and the model parameters,

$$\frac{k}{l} = \left(\begin{array}{c} \frac{1+b_i}{r}\left(\sum_{g=1}^{16} e^{I_{g<g_{old}}}(1+n)^{1-g}\left(\frac{\beta_i^{-g} r^{-g} e^{I_{g<g_{old}}}}{(1+z)^{-g}}\right)^{\frac{1}{\phi-1}}\right)^{-1} \\ \frac{(1-\alpha)(1-\gamma)}{(1+z)^{\alpha}(1+n)^{\alpha}}\sum_{g=1}^{23}(1+n)^{1-g}(1+z)^{1-g} \\ \left(\left(\sum_{gg=g+1}^{m}\beta_i^{gg-g-1}\right)\left(\sum_{gg=1}^{m}\beta_i^{gg-g-1}\right)^{-1}\right. \\ \sum_{gg=1}^{16} r^{g+1-gg}(1+z)^{gg-1} e^{I_{gg<g_{old}}}\left(\frac{\beta_i^{-gg} r^{-gg} e^{I_{gg<g_{old}}}}{(1+z)^{-gg}}\right)^{\frac{1}{\phi-1}} \\ \left.-\sum_{gg=g+1}^{16} r^{g+1-gg}(1+z)^{gg-1} e^{I_{gg<g_{old}}}\left(\frac{\beta_i^{-gg} r^{-gg} e^{I_{gg<g_{old}}}}{(1+z)^{-gg}}\right)^{\frac{1}{\phi-1}}\right) \end{array}\right)^{\frac{1}{1-\alpha}} \tag{75}$$

and thus, using (70), we can obtain the steady state value for k,

$$k = \frac{k}{l} l \tag{76}$$

From (44) and (42), then,

$$w = \frac{(1-\alpha)}{(1+ag)^{\alpha}(1+n)^{\alpha}}\left(\frac{k}{l}\right)^{\alpha} \tag{77}$$

Next we derive the b_i that ensures that (64) is satisfied, and show that this b_i is common for all i. For this, we depart from equations (64) and use (62), (68) as well as (60), (67) and (72),/()()*()

$$wl_i^g = h_i^g = \left[\frac{(1+z)^{g-1} e^{I_{g<g_{old}}} a_i (b_i - w)}{c_i^g \chi_i^g \kappa}\right]^{\frac{1}{\phi-1}}$$

(72), (67)⇒

$$wl_i^{g_1} = \left[\frac{(1+z)^{g_1-1} e^{I_{g_1<g_{old}}} a_i (b_i - w)}{a_i w (1+b_i) l_i^{g_2} \left(\sum_{g=1}^m \beta_i^{g-g_1}\right)^{-1} \chi_i^{g_1} \kappa} \right.$$

$$\left. \left(\sum_{g=1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right]^{\frac{1}{\phi-1}}$$

$$= \left[\frac{(1+z)^{g_1-1} e^{I_{g_1<g_{old}}} (b_i - w) \left(\sum_{g=1}^m \beta_i^{g-g_1}\right)}{w (1+b_i) l_i^{g_1} \left(\frac{\beta_i^{g_2-g_1} r^{g_2-g_1} e^{I_{g_1<g_{old}}}}{(1+z)^{g_2-g_1} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{1-\phi}} \chi_i^{g_1} \kappa} \right.$$

$$\left. \left(\sum_{g=1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right]^{\frac{1}{\phi-1}}$$

⇒

$$\left(wl_i^{g_1}\right)^{\phi} = \frac{(1+z)^{g_1-1} e^{I_{g_1<g_{old}}} (b_i - w) \left(\sum_{g=1}^m \beta_i^{g-g_1}\right)}{(1+b_i) \left(\frac{\beta_i^{g_2-g_1} r^{g_2-g_1} e^{I_{g_1<g_{old}}}}{(1+z)^{g_2-g_1} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{1-\phi}} \chi_i^{g_1} \kappa}$$

$$\left. \left(\sum_{g=1}^{16} r^{g_1-g} (1+z)^{g-1} e^{I_{g<g_{old}}} \left(\frac{\beta_i^{g_2-g} r^{g_2-g} e^{I_{g<g_{old}}}}{(1+z)^{g_2-g} e^{I_{g_2<g_{old}}}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right]$$

(68)⇒

$$b_i = \kappa w^{\phi} (1+w)^{1-\phi} + w \tag{78}$$

for all i and for all g. Therefore, there exists ab_i that ensures that (64) is satisfied. In addition, one can observe, that since $w > 0$ by definition, and $\kappa > 0$, $b_i > w$. The remaining steady state values are then as follows, from (42),

$$y = \left(\frac{k_t}{(1+z)(1+n)} \right)^{\alpha} l^{1-\alpha} \tag{79}$$

From (52)

$$k = i \tag{80}$$

From (**??**),

$$q = 1 \tag{81}$$

From (45),

$$r^k = \alpha \frac{y}{k} (1+z)(1+n) \tag{82}$$

From (46)

$$R^{Tk} = r^k + 1 - \delta \tag{83}$$

From (47),

$$\begin{aligned} r &= 1 + (1-\gamma)\left(R^{Tk} - 1\right) + \gamma r^{\star} \\ &= 1 + (1-\gamma)\left(\alpha \frac{y}{k}(1+z)(1+n) - \delta\right) + \gamma r^{\star} \end{aligned} \tag{84}$$

Using (77) and (84) to express w and r in terms of the capital labor ratio, (75) and (78) constitute a non-linear system of two equations and two unknowns, b_i and $\frac{\kappa}{l}$. Once we solve for this unknowns, all other variables can be determined. From (49)

$$s^g = \sum_{i=1}^{5} s_i^g \lambda_i \tag{85}$$

From (48),

$$s = \sum_{g=1}^{23} s^g (1+n)^{1-g} (1+z)^{1-g} \tag{86}$$

From (37),

$$c^g = \sum_{i=1}^{5} c_i^g \lambda_i \tag{87}$$

From (50)

$$c = \sum_{g=1}^{m} c^g (1+n)^{1-g} (1+z)^{1-g} \tag{88}$$

From (53)

$$gdp = c + k \tag{89}$$

C Workweek Reduction

This appendix shows one way in which our model could be modified in order to study a situation in which working hours are statutorily reduced, e.g., reducing the workweek from 45 to 40 hours.

A reduction in working hours could be interpreted as a reduction in the labor input of the representative firm. We would introduce the parameter $l_i^{adj} \in (0,1)$ in equation (3), such that $L_{i,t} = \sum_{g=1}^{16} l_i^{adj} L_{i,t}^g$. This would effectively generate a reduction in labor demand, consistent with an increase in labor costs, as the modified equations for profit maximization and optimal labor demand show:

$$\max_{K_t, L_{i,t}^g} K_t^{\alpha} A_t^{1-\alpha} \left(\sum_{i=1}^{5} a_i \left(\sum_{g=1}^{16} l_i^{adj} L_{i,t}^g \right)^{\rho} \right)^{\frac{1-\alpha}{\rho}} - r_t^k K_t - \sum_{i=1}^{5} \sum_{g=1}^{16} W_{i,t}^g L_{i,t}^g,$$

$$L_{i,t}^g : \qquad W_{i,t}^g = a_i W_t l_i^{adj} \left(\frac{L_t}{L_{i,t}} \right)^{1-\rho}.$$

Ignoring changes in population due to immigration, we could reinterpret the problem facing each individual of skill level i, who decides how many hours to work in the formal and informal sectors, as a problem facing each *household* of skill level i, which would decide how many members to allocate to the formal and informal sectors. Under this interpretation, the problem of the household could include the lower disutility that working fewer hours in the formal sector would entail. This extension would allow labor supply to compensate to some extent the decline in hours. We introduce the factor $(l_i^{adj})^{\frac{1}{\phi^{adj}}}$ in the household's optimization problem:

$$
\begin{aligned}
U_{i,t} = & \sum_{g=1}^{24} \beta_i^{g-1} \frac{\left(C_{i,t+g-1}^g\right)^{1-\theta}}{1-\theta} \\
& - \sum_{g=1}^{16} \beta_i^{g-1} \Theta_{i,t+g-1}^g \frac{\left((l_i^{adj})^{\frac{1}{\phi^{adj}}} l_{i,t+g-1}^g + h_{i,t+g-1}^g\right)^{\phi} + \kappa \left(h_{i,t+g-1}^g\right)^{\phi}}{\phi},
\end{aligned}
$$

where the exponent $\frac{1}{\phi^{adj}}$ regulates the extent to which labor supply would compensate the decline in total hours. When $\phi^{adj} = 1$, the compensation of labor supply is complete in the sense that family income is unchanged. When $\phi^{adj} \to \infty$, labor supply does not compensate the decline in hours.

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