Sectoral Credit Allocation, Capital Requirements and Financial Stability

Maximiliano San Millán

Banco Central de Chile
Workshop on Banking and Financial Stability

September 2024

Disclaimer: The views presented here are my own and do not necessarily reflect those of the Central Bank of Chile.

Introduction

- Recent empirical evidence:
  - **Exuberant credit growth in real estate sectors** (firms and households) $\rightarrow$ **higher probability of systemic banking crises** (Muller and Verner, 2023; Jorda et al, 2014). ▸ Evidence
- Bank capital regulation (limits to bank leverage):
  - Mortgages are *individually* safer than other loans (collateral+typically lower default rates) $\rightarrow$ **lower** capital requirements for mortgages (roughly one half of requirements for corporate loans). ▸ More on capital requirements

Introduction

- What are the consequences for financial stability of this micro-prudential design of capital requirements?

- Quantitative analysis for Euro Area: interaction between regulatory design and sectoral risks.

- Focus on **banking crises**.
- Episodes characterized by:
  1. **Exogenous** increases in systematic risk (higher correlation of defaults).
  2. **Endogenous** reallocation of portfolios towards mortgage lending.

Financial Frictions

- Scarce net worth of banks and entrepreneurs (as in Gertler and Kiyotaki, 2010).
- **Bankruptcy costs** associated to entrepreneurs, households and banks.
- Limited liability.
- Insured deposit funding of banks.

Why do capital requirements matter in this context?

- **Microprudential role**: limit the frequency of bank defaults.
- **Macroprudential role**: Preserve the banking system's capacity to extend credit + correct incentives to take systemic risk.

Plan

Mechanism

- Increase in systematic risk:
  - Increase in the probability of **simultaneous** defaults in portfolios.
  - Potentially **asymmetric** across sectors.

- Calibrated model: **mortgages** more strongly affected by increases in **systematic risk**.

Mechanism

- **Endogenous** increase in risk:
  - Banks with **limited liability** + **insured deposits**.

  $\Longrightarrow$ Incentives to invest in portfolios with **greater** systematic risk.

  $\Longrightarrow$ Leverage more attractive as systematic risk increases.

- **Capital requirements and increases in systematic risk**:
  - Incentives to reallocate portfolios towards exposures with lower requirements (thus increasing leverage).
  - **Stronger** incentives when differences in capital charges are large and the overall levels of requirements is low.

Mechanism - Static Example

- Bankers with total net worth N.
- Continuum of islands indexed by j.
- Continuum of firms and housing units in each island indexed by i in each sector $s = h, f$.
- Two specialized banks in each island.

Mechanism - Static Example

- Insured deposits and $R_d = 1$
- Capital charges ϕ_s.
- Expected ROE in each sector s (per unit of credit):

$$ROE_s = \frac{\mathbb{E}\max[\tilde{R}_s(\omega_s^j) - (1-\phi_s), 0]}{\phi_s} \tag{1}$$

- $\tilde{R}_s(\omega_s^j) = \int_0^\infty r(\omega_s^j, \omega_s^i) dF(\omega_s^i)$: Average asset return (conditional on the realization of ω_s^j)
- ω_s^j: **non diversifiable** factor in the portfolio of banks in sector s.
- Assume ω_s^j, ω_s^i have cross sectional dispersion σ_s^j,σ_s^i, respectively.

Bank returns | Distribution of Returns

Mechanism - Static Example

- In equilibrium:

$$ROE_f = ROE_h, \quad (2)$$

$$N = B_h\phi_h + B_f\phi_f. \quad (3)$$

- **Experiment**:
  - Change the composition of risk:

$$\rho_s = \frac{\sigma_s^j}{\sigma_s^j + \sigma_s^i}. \quad (4)$$

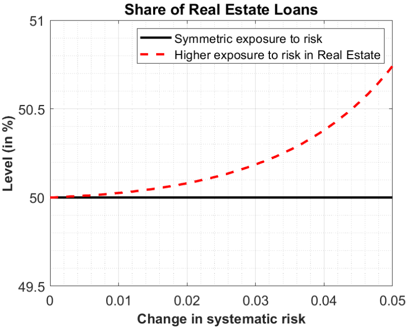
  - Increase in systematic risk (holding total risk constant):

$$\rho_s' = \bar{\rho} + \varsigma_s \epsilon \quad (5)$$

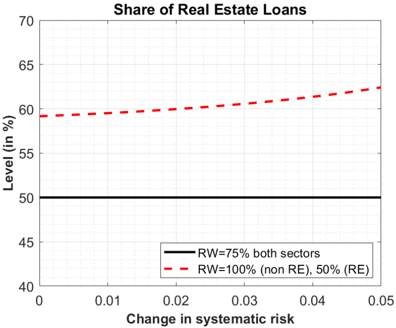
  - Show how the effect on credit allocation changes with different requirements.

Bank returns | Distribution of Returns

Mechanism - Systematic risk and credit allocation



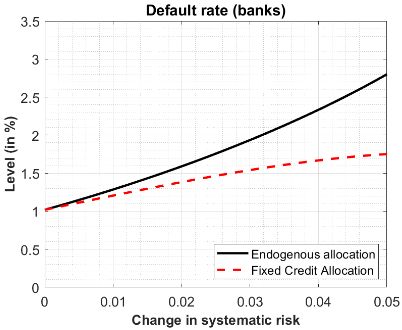
(a)



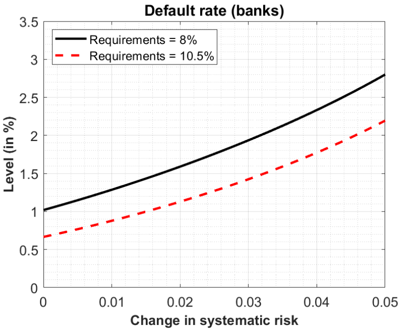
(b)

Figure: Share of mortgages in total bank credit

Mechanism - Effects on financial stability



(a)



(b)

Figure: (a) Endogenous increase in bank failure due to reallocation. (b) Effects of higher capital requirements.

Mechanism

- Two forces pushing in the same direction:

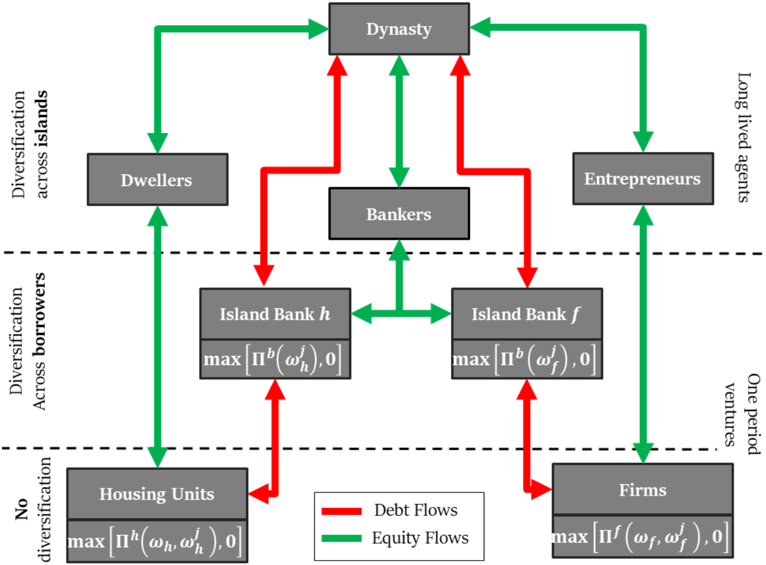
Mortgages more strongly affected by systematic risk + sector with lower capital requirements.

- Macroprudential regulation: Dynamically correct systemic risk taking.

Quantitative Exercise

- DSGE model calibrated to match Euro Area targets (Two-sector extension of Mendicino et al, forthcoming JOF)).
- Mortgages more strongly affected by increases in non diversifiable risk.
- Analyze episodes of systemic banking crises (costs of bank default $> 3\%$ of GDP).

Model



Dynasty
Dwellers
Bankers
Entrepreneurs
Island Bank h
$\max\left[\Pi^b\left(\omega_h^j\right), 0\right]$
Island Bank f
$\max\left[\Pi^b\left(\omega_f^j\right), 0\right]$
Housing Units
$\max\left[\Pi^h\left(\omega_h, \omega_h^j\right), 0\right]$
Firms
$\max\left[\Pi^f\left(\omega_f, \omega_f^j\right), 0\right]$
Debt Flows
Equity Flows
Diversification across **islands**
Diversification Across **borrowers**
No diversification
Long lived agents
One period ventures

Calibration

- Calibration is done in two steps:
  - First step: pre-set parameters from the literature.
  - Second step: Internally calibrate parameters using SMM.
- Real and financial data from Euro Area in the period 2003:Q1-2013:Q4.
- Baseline calibration fits Basel II period.
- Model matches credit exuberance in the housing sector in the buildup of financial crises and the subsequent busts.

Targeted Moments | Parameters

Calibration of capital requirements

- Regulators set capital requirements following Basel Committee on Banking Supervision standards.

Details

Calibration of capital requirements

- Regulators set capital requirements following Basel Committee on Banking Supervision standards.
- Regimes
  - Basel I: $\phi^*_{f,t} = 0.08$, $\phi^*_{h,t} = 0.04$.
  - Basel II: $\phi^*_{s,t} = G(PD_{s,t})$, with $G' > 0$
  - Basel III: $\phi^*_{s,t} = M_{s,t} \times G(PD_{s,t})$, where $M_{s,t}$ is the impact of additional buffers (capital conservation and counter-cyclical buffers).

Details

Results

- Study **banking crises** episodes: Periods with gross deposit insurance outlays greater than 3% of GDP (Laeven and Valencia, 2013).
- Understand the role of capital regulation in the buildup and aftermath of crises.

Results

- Study **banking crises** episodes: Periods with gross deposit insurance outlays greater than 3% of GDP (Laeven and Valencia, 2013).
- Understand the role of capital regulation in the buildup and aftermath of crises.

1. Characterization of banking crises (reallocation towards real estate lending).

Results

- Study **banking crises** episodes: Periods with gross deposit insurance outlays greater than 3% of GDP (Laeven and Valencia, 2013).
- Understand the role of capital regulation in the buildup and aftermath of crises.

1. Characterization of banking crises (reallocation towards real estate lending).
2. Relevance of sector-specific capital buffers.

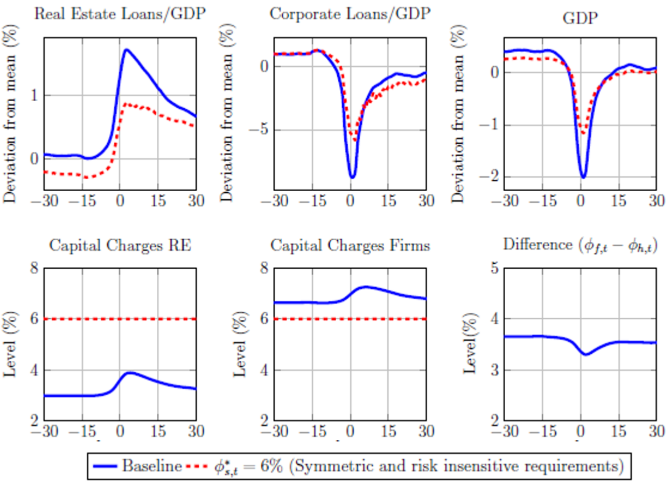
Results

- Study **banking crises** episodes: Periods with gross deposit insurance outlays greater than 3% of GDP (Laeven and Valencia, 2013).
- Understand the role of capital regulation in the buildup and aftermath of crises.

1. Characterization of banking crises (reallocation towards real estate lending).
2. Relevance of sector-specific capital buffers.
3. Probability of banking crises under different regimes.

Risk sensitive capital requirements and the path to crises

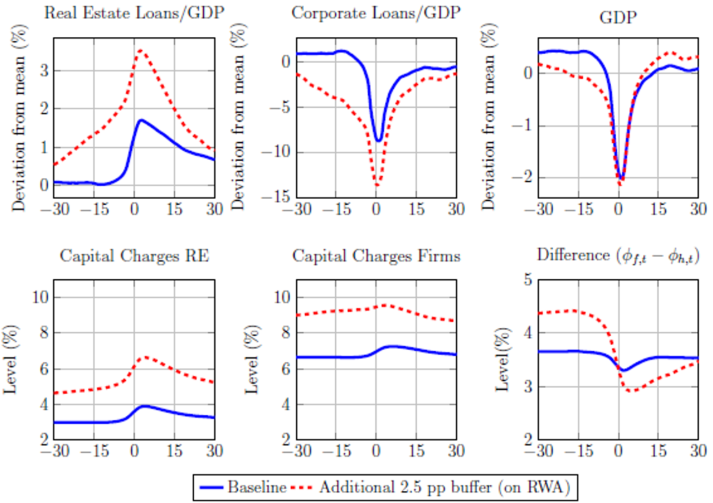
Figure 3: Role of Capital Requirements



Notes: Solid lines correspond to the baseline model. Dashed lines correspond to a version of the model where capital charges are set to 6% and are equal across sectors. Sample paths correspond to shock realizations that generate banking crises in the baseline model.

Evaluation of a uniform increase of capital requirements

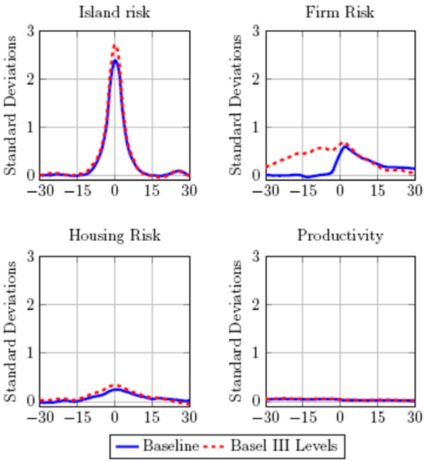
Figure 4: Assessment of Uniform Increase in Capital Requirements



Notes: Solid lines correspond to the baseline model. Dashed lines correspond to capital requirements equal to 10.5% of risk weighted assets (introducing a 2.5% buffer as in the Capital Conservation Buffer of Basel III). Sample paths correspond to shock realizations that generate banking crises in each model

Shocks leading to crises

Figure 5: Shocks leading to a crisis in Basel III



Note: Solid lines correspond to the baseline model. Dashed lines correspond to capital requirements equal to 10.5% of risk weighted assets. Sample paths correspond to shock realizations that generate banking crises in each model.

- Crises characterized by sharp increases in non diversifiable risk.

Evaluation of counter-cyclical rules

- Evaluate generic counter cyclical buffers

$$M_{s,t} = CCyB(\text{Total Credit/GDP gap at time t}) \tag{6}$$

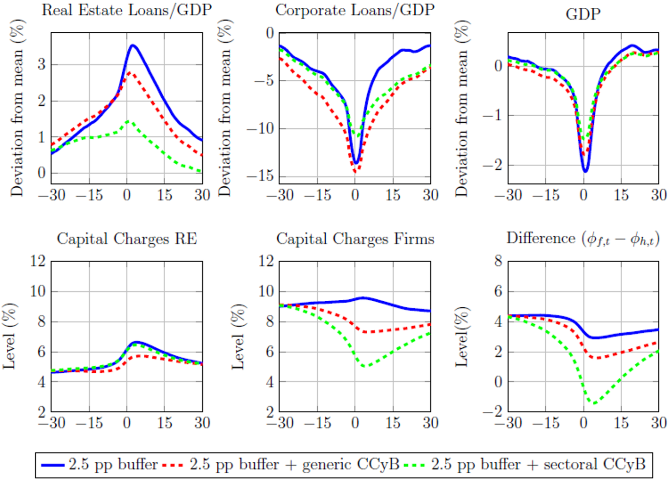
- Sector specific buffers

$$M_{s,t} = CCyB(\text{Sector } s \text{ Credit/GDP gap at time t}) \tag{7}$$

- Parameters in this function are chosen to match existing Basel guidelines on the CCyB.

Evaluation of counter-cyclical rules

Figure 6: Evaluation of Basel III buffers



Notes: Solid lines correspond to Basel III levels of capital requirements. Red dashed lines correspond to a generic CCyB. Green dashed lines correspond to a sectoral CCyB. Sample paths correspond to shock realizations that generate banking crises in each model.

Comparison across regimes

Table 3: Comparison across Regulatory Designs

Outcome Variable	Baseline	Basel III (extra 2.5 pp buffer)	Generic CCyB	Sectoral CCyB
Frequency of Banking Crises	3.024	1.352	1.4229	1.35
Output Losses in Crises	-13	-14.3	-14.08	-11.28
Capital Charge (Firms)	6.68	8.81	8.99	9.08
Capital Charge (Households)	3.02	4.57	4.68	4.64
Default Rate Banks	1.04	0.96	0.96	0.96
Default Rate Firms	1.54	1.55	1.55	1.54
Default Rate Households	0.81	1	1.02	1.02
Welfare	–	0.055	0.01	0.085

Notes: Output losses are reported in cumulative percentage points of GDP in the three years following a banking crisis. Welfare is reported as the percentage change in permanent consumption that would leave consumers as well off in the Baseline scenario as in each of the different regimes. Default rates are reported in annualized percentage points. Each column corresponds to simulations of the model for 500,000 periods, under each different regulatory regime.

Conclusions

- Analysis of macroprudential regulation in the presence of sectoral heterogeneity: reallocation of portfolios matters.

Conclusions

- Analysis of macroprudential regulation in the presence of sectoral heterogeneity: reallocation of portfolios matters.
- Purely microprudential requirements fail to correct endogenous risk taking through reallocation and can exacerbate it.

Conclusions

- Analysis of macroprudential regulation in the presence of sectoral heterogeneity: reallocation of portfolios matters.
- Purely microprudential requirements fail to correct endogenous risk taking through reallocation and can exacerbate it.
- Generic buffers achieve moderate stabilization.

Conclusions

- Analysis of macroprudential regulation in the presence of sectoral heterogeneity: reallocation of portfolios matters.
- Purely microprudential requirements fail to correct endogenous risk taking through reallocation and can exacerbate it.
- Generic buffers achieve moderate stabilization.
- Sectoral specific buffers perform best at mitigating risks.

Appendix

Dynasty

- Dynasty's utility function:

$$V_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\log(C_s) + \lambda_h \log(H_s) - \lambda_L \frac{L_s^{1+\varphi}}{1+\varphi} \right], \quad (8)$$

  where

  - C_t: consumption of the final good.
  - H_t: consumption of housing units.
  - L_t: labor supplied by the dynasty.
  - β is the subjective discount factor; λ_H measures preference for housing; λ_L measures the disutility of labor and φ is the Frisch elasticity of labor.

Dynasty

- Dynasty's budget constraint:

$$C_t + D_t + q_{k,t}K_{h,t} \leq W_t - EQ_t^h, \tag{9}$$

where

$$W_t = w_t L_t + R_{d,t-1}D_{t-1} + q_{k,t}(1-\delta)K_{h,t-1} + K_{h,t-1}^{\alpha_h} + \Pi_t^h + \Upsilon_t \tag{10}$$

Dynasty

- Dynasty's budget constraint:

$$C_t + D_t + q_{k,t} K_{h,t} \leq W_t - EQ_t^h, \tag{9}$$

where

$$W_t = w_t L_t + R_{d,t-1} D_{t-1} + q_{k,t}(1-\delta) K_{h,t-1} + K_{h,t-1}^{\alpha_h} + \Pi_t^h + \Upsilon_t \tag{10}$$

- Housing units financing constraint:

$$q_{h,t} H_t \leq B_{h,t} + EQ_t^h, \tag{11}$$

where $EQ_t^h = \chi_h W_t$.

D_t: bank deposits; $R_{d,t}$: gross rate on deposits; $q_{k,t}$: price of physical capital; $K_{h,t-1}$: physical capital directly held by the dynasty; α_h: household backyard technology; $q_{h,t}$: price of housing units; χ_h: housing equity as fraction of total wealth. Υ_t: net transfers from entrepreneurs, bankers and producers of capital and housing units.

Dynasty's problem and mortgage loans

- Housing purchases are chosen by the dynasty jointly with consumption and investment.

Figure Housing Payoffs

Dynasty's problem and mortgage loans

- Housing purchases are chosen by the dynasty jointly with consumption and investment.
- Dynasty chooses $H_t, B_{h,t}, R^l_{h,t}, C_t, L_t, K_{h,t}$ such that

$$(H_t, B_{h,t}, R^l_{h,t}, C_t, L_t, K_{h,t}) = \text{argmax } V_t, \ \textbf{s.t.} \tag{12}$$

$$C_t + D_t + q_{k,t}K_{h,t} \leq W_t - EQ^h_t, \tag{13}$$

$$q_{h,t}H_t \leq B_{h,t} + EQ^h_t, \tag{14}$$

and the real estate banks' participation constraint

$$\mathbb{E}_t \underbrace{\Lambda_{t+1}(1-\theta_b+\theta_b v^b_{t+1})}_{\text{Disc. factor bankers}} \underbrace{\Pi^b_{h,t+1}}_{\text{Aggregate payoff on loan portfolios } h} \geq \phi_{h,t} B_{h,t} \underbrace{v^b_t}_{\text{shadow value of bank equity}} . \tag{15}$$

▸ Figure ▸ Housing Payoffs

Firms

- Firms produce the final consumption good according to

$$Y_{t+1} = A_{t+1} K_{f,t}^{\alpha} L_t^{1-\alpha}, \tag{16}$$

Entrepreneurs

Firms

- Firms produce the final consumption good according to

$$Y_{t+1} = A_{t+1} K_{f,t}^{\alpha} L_t^{1-\alpha}, \tag{16}$$

- Firms' financing constraint: $q_{k,t} K_{f,t} + w_t L_t = B_{f,t} + EQ_{f,t}$

Entrepreneurs

Firms

- Firms produce the final consumption good according to

$$Y_{t+1} = A_{t+1} K_{f,t}^{\alpha} L_t^{1-\alpha}, \tag{16}$$

- Firms' financing constraint: $q_{k,t} K_{f,t} + w_t L_t = B_{f,t} + EQ_{f,t}$
- The terminal net worth of a firm f in island j is given by

$$\Pi_{t+1}^{f}(\omega_f, \omega_f^j) = \omega_f \omega_f^j \left[Y_{t+1} + q_{k,t+1}(1-\delta) K_{f,t} \right] - R_{f,t}^{l} B_{f,t}, \tag{17}$$

Entrepreneurs

Firms

- Firms produce the final consumption good according to

$$Y_{t+1} = A_{t+1} K_{f,t}^{\alpha} L_t^{1-\alpha}, \tag{16}$$

- Firms' financing constraint: $q_{k,t} K_{f,t} + w_t L_t = B_{f,t} + EQ_{f,t}$
- The terminal net worth of a firm f in island j is given by

$$\Pi_{t+1}^{f}(\omega_f, \omega_f^j) = \omega_f \omega_f^j \left[Y_{t+1} + q_{k,t+1}(1-\delta) K_{f,t} \right] - R_{f,t}^{l} B_{f,t}, \tag{17}$$

- Aggregate firm payoffs

$$\Pi_{t+1}^{f} = \int_0^{\infty} \int_0^{\infty} \max \left[\Pi_{t+1}^{f}(\omega_f, \omega_f^j), 0 \right] dF_{f,t+1}(\omega_f) dF_{f,t+1}^{j}(\omega_f^j)$$

Entrepreneurs

Firms' problem and Corporate loans

- Corporate loans: choose $K_t, L_t, B_{f,t}, R^l_{f,t}$ such that

$$(K_t, L_t, B_{f,t}, R^l_{f,t}) = \text{argmax } \mathbb{E}_t \underbrace{\Lambda_{t+1}(1-\theta_f+\theta_f v^f_{t+1})}_{\substack{\text{Disc. factor} \\ \text{entrepreneurs}}} \Pi^f_{t+1}, \quad \textbf{s.t.} \tag{18}$$

$$q_{k,t}K_t + w_t L_t \leq B_{f,t} + EQ_{f,t}, \tag{19}$$

and the banks' participation constraint

$$\mathbb{E}_t \underbrace{\Lambda_{t+1}(1-\theta_b+\theta_b v^b_{t+1})}_{\substack{\text{Disc. factor} \\ \text{bankers}}} \underbrace{\Pi^b_{f,t+1}}_{\substack{\text{Aggregate payoff on} \\ \text{loan portfolios } f}} \geq \phi_{f,t} B_{f,t} \underbrace{v^b_t}_{\substack{\text{shadow value of} \\ \text{bank equity}}} . \tag{20}$$

Entrepreneurs Figure

Banks

- Specialized island-sector bank subsidiaries.
- Issue equity and fully insured deposits.
- Invest in diversified portfolio of loans.

Bankers Return on loans

Banks

- Specialized island-sector bank subsidiaries.
- Issue equity and fully insured deposits.
- Invest in diversified portfolio of loans.
- Capital requirements $\phi_{s,t}$ (binding in equilibrium).

Bankers Return on loans

Banks

- Specialized island-sector bank subsidiaries.
- Issue equity and fully insured deposits.
- Invest in diversified portfolio of loans.
- Capital requirements $\phi_{s,t}$ (binding in equilibrium).
- Bank i payoffs

$$\Pi^b_{s,t+1}(\omega^j_s) = \tilde{R}^l_{s,t+1}(\omega^j_s) B_{s,t} - R_{d,t} D_{s,t}, \tag{21}$$

Bankers Return on loans

Banks

- Specialized island-sector bank subsidiaries.
- Issue equity and fully insured deposits.
- Invest in diversified portfolio of loans.
- Capital requirements $\phi_{s,t}$ (binding in equilibrium).
- Bank i payoffs

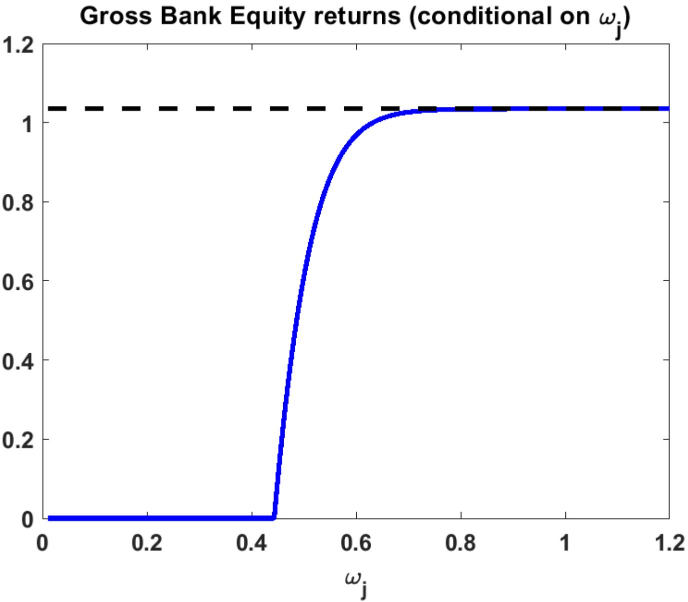
$$\Pi^b_{s,t+1}(\omega^j_s) = \tilde{R}^l_{s,t+1}(\omega^j_s)B_{s,t} - R_{d,t}D_{s,t}, \tag{21}$$

- Aggregate banks payoff (diversification across islands)

$$\Pi^b_{s,t+1} = \int_0^\infty \max\left[\Pi^b_{s,t+1}(\omega^j_s), 0\right] dF^j_{s,t+1}(\omega^j_s), \tag{22}$$

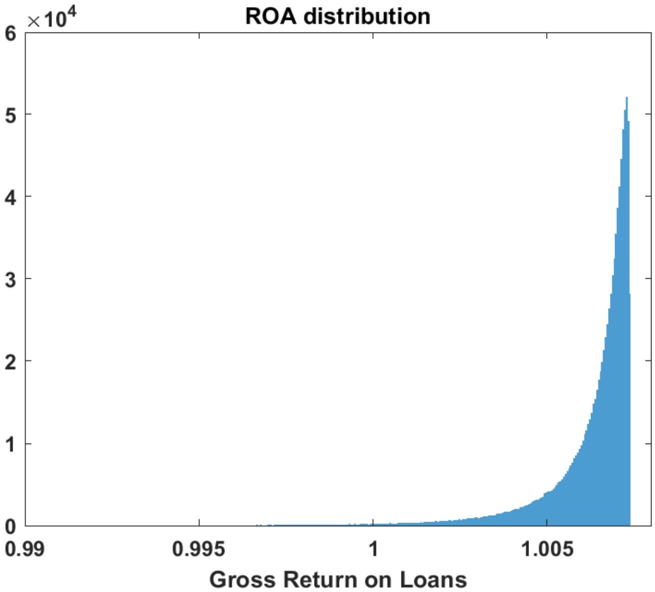
Bankers Return on loans

Shape of Returns



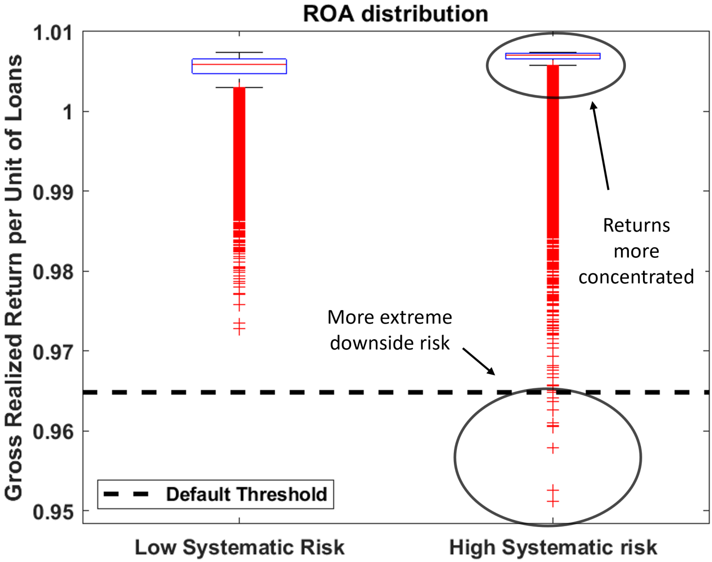
Back to mechanism

Distribution of Returns



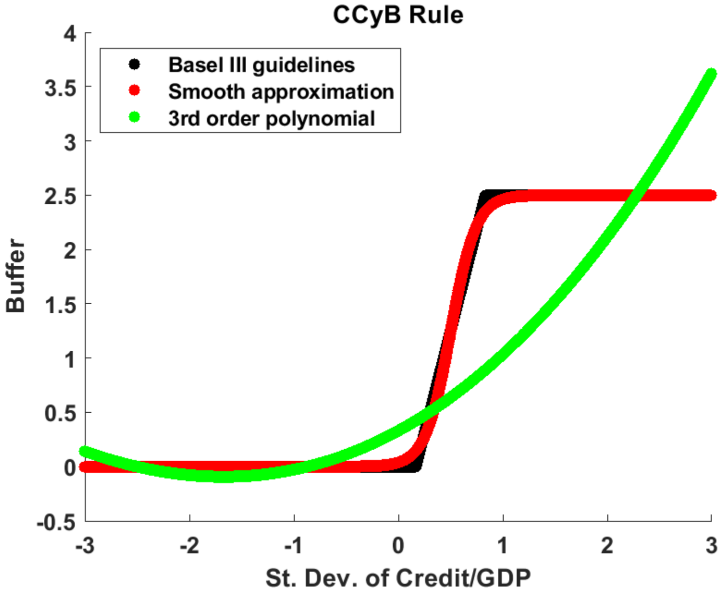
Back to mechanism

Distribution of Returns II



Back to mechanism

CCyB Rule



▸ Back

Calibration - Targeted Moments

Table 2: Targeted Moments

Moment	Data	Model	Moment	Data	Model
Mean NFC Loans/GDP	1.785	2.046	Std. NFC Loans/GDP	0.128	0.237
Mean HH Loans/GDP	2.014	2.638	Std. HH Loans/GDP	0.053	0.059
Mean Spread NFC Loans	2.279	1.494	Std. Spread NFC Loans	0.493	0.907
Mean Spread HH Loans	1.331	0.5457	Std. Spread HH Loans	0.376	0.344
Mean write-off rate NFC Loans	0.543	0.584	Std. write-off NFC Loans	0.334	0.455
Mean write-off rate HH Loans	0.126	0.277	Std. write-off HH Loans	0.057	0.162
Mean Housing Wealth/ Total Non Financial wealth	0.947	0.633	Std. GDP	0.023	0.022
Mean ROE NFCs	4.706	4.223	Std. ROE NFCs	8.148	3.238
Mean ROE Banks	4.619	11.21	Std. ROE Banks	12.201	11.017
Mean Capital held by Households	0.185	0.153	Std. Investment/GDP	0.008	0.004
			Std. Housing prices	0.054	0.033

Notes: This table displays the targeted moments in the calibration and their model counterparts. Spreads, write-off rates and returns on equity are reported in annualized percentage points. The standard deviation of GDP corresponds to the standard deviation of the log of GDP, in quarterly terms. All variables are linearly detrended before computing standard deviations.

▸ Back

Calibration - Resulting Parameters

Table 1: Internally calibrated parameters

Parameter	Symbol	Value	Parameter	Symbol	Value
New bankers' endowment	χ_b	0.682	Std. Island risk (Firms)	ς_f^j	0.055
New entrepreneurs' endowment	χ_f	0.584	Std. Island risk (HH)	ς_h^j	0.035
Housing equity	χ_h	0.388	Std. Firm risk	ς_f	0.075
Mean Island risk shock (Firms)	$\bar{\sigma}_f^j$	0.263	Std. HH risk	ς_h	0.025
Mean Island risk shock (HH)	$\bar{\sigma}_h^j$	0.216	Std. productivity shocks	ς_A	0.003
Mean Firm risk shocks	$\bar{\sigma}_f$	0.304	Pers. Island risk (Firms)	ϱ_f^j	0.705
Mean HH risk shocks	$\bar{\sigma}_h$	0.047	Pers. Island risk (HH)	ϱ_h^j	0.705
Relative housing preference	λ_h	0.109	Pers. Firm risk	ϱ_f	0.906
HH backyard technology	α_{hh}	0.1	Pers. HH risk	ϱ_h	0.926
Investment adjustment costs	ψ	1.99	Pers. Productivity	ϱ_A	0.98
			CR partial adjustment coefficient	η	0.9

- Model assigns higher (relative) importance of non diversifiable risk in the real estate sector.

Back to Calibration

Model I

- DSGE macro-banking framework, two sector extension of MNSS(2021).
- Continuum of **islands** (j) of measure one, each with continua of **firms** (f) and **housing units** (h).
- Island j has two specialized **bank subsidiaries**: corporate and residential real estate.

Model I

- DSGE macro-banking framework, two sector extension of MNSS(2021).
- Continuum of **islands** (j) of measure one, each with continua of **firms** (f) and **housing units** (h).
- Island j has two specialized **bank subsidiaries**: corporate and residential real estate.
- Banks invest in a well diversified portfolio of **risky loans** $B_{s,t}$, $s = f, h$ with promised **gross rate** $R^l_{s,t}$.
- Bank debt is fully insured and bank leverage is limited by **capital requirements**: $EQ^b_{s,t} \geq \phi_{s,t} B_{s,t}$.

Model I

- DSGE macro-banking framework, two sector extension of MNSS(2021).
- Continuum of **islands** (j) of measure one, each with continua of **firms** (f) and **housing units** (h).
- Island j has two specialized **bank subsidiaries**: corporate and residential real estate.
- Banks invest in a well diversified portfolio of **risky loans** $B_{s,t}$, $s = f, h$ with promised **gross rate** $R^l_{s,t}$.
- Bank debt is fully insured and bank leverage is limited by **capital requirements**: $EQ^b_{s,t} \geq \phi_{s,t} B_{s,t}$.
- **Limited participation** in bank and firm equity markets: dynasty with long-lived bankers and entrepreneurs.

Model I

- DSGE macro-banking framework, two sector extension of MNSS(2021).
- Continuum of **islands** (j) of measure one, each with continua of **firms** (f) and **housing units** (h).
- Island j has two specialized **bank subsidiaries**: corporate and residential real estate.
- Banks invest in a well diversified portfolio of **risky loans** $B_{s,t}$, $s = f, h$ with promised **gross rate** $R^l_{s,t}$.
- Bank debt is fully insured and bank leverage is limited by **capital requirements**: $EQ^b_{s,t} \geq \phi_{s,t} B_{s,t}$.
- **Limited participation** in bank and firm equity markets: dynasty with long-lived bankers and entrepreneurs.
- Borrowers and banks are protected by **limited liability**.

Risk layers

- Two layers of **cross sectional risk:**

- **Time variation**:

Risk layers

- Two layers of **cross sectional risk:**
  - Borrower specific shocks: mean one **idiosyncratic shocks** to the terminal value of firm and household assets. Notation: ω_f, ω_h.
  - Non diversifiable **island shock**: mean one shock to all assets of class i in island j. Notation: ω_f^j, ω_h^j.
- **Time variation**:

Risk layers

- Two layers of **cross sectional risk:**
  - Borrower specific shocks: mean one **idiosyncratic shocks** to the terminal value of firm and household assets. Notation: ω_f, ω_h.
  - Non diversifiable **island shock**: mean one shock to all assets of class i in island j. Notation: ω_f^j, ω_h^j.
- **Time variation**:
  - Standard AR(1) aggregate productivity shocks.

Risk layers

- Two layers of **cross sectional risk:**
  - Borrower specific shocks: mean one **idiosyncratic shocks** to the terminal value of firm and household assets. Notation: ω_f, ω_h.
  - Non diversifiable **island shock**: mean one shock to all assets of class i in island j. Notation: ω_f^j, ω_h^j.
- **Time variation**:
  - Standard AR(1) aggregate productivity shocks.
  - Shocks to the dispersion of ω_f, ω_h.

$$\log\left(\frac{\sigma_{s,t+1}}{\bar{\sigma}_s}\right) = \varrho_s \log\left(\frac{\sigma_{s,t}}{\bar{\sigma}_s}\right) + \varsigma_s \varepsilon_{s,t+1}, \tag{23}$$

  - Shock to the dispersion of ω_f^j, ω_h^j.

$$\log\left(\frac{\sigma_{s,t+1}^j}{\bar{\sigma}_s^j}\right) = \varrho_s^j \log\left(\frac{\sigma_{s,t}^j}{\bar{\sigma}_s^j}\right) + \varsigma_s^j \varepsilon_{t+1}, \tag{24}$$

Housing payoffs

- Terminal housing net worth

$$\Pi^{h}_{t+1}(\omega_h, \omega^{j}_{h}) = \omega_h \omega^{j}_{h} R_{h,t+1} q_{h,t} H_t - R^{l}_{h,t} B_{h,t}, \qquad (25)$$

where $R_{h,t+1} = \frac{q_{h,t+1}(1-\delta_h)}{q_{h,t}}$.

Back to dynasty's problem

Housing payoffs

- Terminal housing net worth

$$\Pi^h_{t+1}(\omega_h, \omega^j_h) = \omega_h \omega^j_h R_{h,t+1} q_{h,t} H_t - R^l_{h,t} B_{h,t}, \tag{25}$$

where $R_{h,t+1} = \frac{q_{h,t+1}(1-\delta_h)}{q_{h,t}}$.

- Aggregate returns on housing

$$\Pi^h_{t+1} = \int_0^\infty \int_0^\infty \max\left[\Pi^h_{t+1}(\omega_h, \omega^j_h), 0\right] dF_{h,t+1}(\omega_h) dF^j_{h,t+1}(\omega^j_h), \tag{26}$$

Back to dynasty's problem

Bankers

- Bankers manage their wealth and invest it in corporate and mortgage banks.
- They exit with probability $1-\theta_b$.
- When exiting, they rebate their terminal net worth $n_{b,t+1}$ to the dynasty.
- Value function of individual banker

$$V^b_{t+1}(n^b_t(i)) = \max_{\substack{\mathrm{div}^b_t(i), EQ^b_{h,t}(i), \\ EQ^b_{f,t}(i), n^b_{t+1}(i)}} \left\{ \mathrm{div}^b_t(i) + \mathbb{E}_t \Lambda_{t+1} \left[(1-\theta_b) n^b_{t+1}(i) + \theta_b V^b_{t+1}(n^b_{t+1}(i)) \right] \right. \tag{27}$$

$$\text{s.t. } \mathrm{div}^b_t(i) + EQ^b_{h,t}(i) + EQ^b_{f,t}(i) = n^b_t(i), \tag{28}$$

$$n^b_{t+1}(i) = \rho^b_{h,t+1} EQ^b_{h,t}(i) + \rho^b_{f,t+1} EQ^b_{f,t}(i), \tag{29}$$

$$\mathrm{div}^b_t(i) \geq 0, \tag{30}$$

with $\rho^b_{s,t+1} \equiv \dfrac{\Pi^b_{s,t+1}}{EQ^b_{s,t+1}}$.

Back to banks

Bankers II

- Gertler and Kiyotaki (2010) $\rightarrow$ value function linear in net worth +

$$v_t^b n_t^b = \max_{\text{div}_t^b, EQ_{f,t}^b, EQ_{h,t}^b} \{\text{div}_t^b + \mathbb{E}_t \underbrace{\Lambda_{t+1}\left[(1-\theta_b) + \theta_b v_{t+1}^b\right]}_{\substack{\text{Stoch. disc.} \\ \text{factor bankers } \equiv \Lambda_{t+1}^b}} n_{t+1}^b\} \tag{31}$$

- As long as $v_{t+1}^b > 1$, we have $\text{div}_t^b = 0$.
- No arbitrage type condition: $\mathbb{E}_t \Lambda_{t+1}^b \rho_{f,t+1}^b = \mathbb{E}_t \Lambda_{t+1}^b \rho_{h,t+1}^b$.
- Shadow value of bankers' net worth $v_t^b = \mathbb{E}_t \Lambda_{t+1}^b \rho_{f,t+1}^b = \mathbb{E}_t \Lambda_{t+1}^b \rho_{h,t+1}^b$

Back to banks

Entrepreneurs

- Entrepreneurs manage their wealth and invest it in firms.
- They exit with probability $1-\theta_f$.
- When exiting, they rebate their terminal net worth $n_{f,t+1}$ to the dynasty.
- Value function of individual entrepreneur

$$V_t^f\left(n_t^f(i)\right)=\max_{\substack{\text{div}_t^f(i),\\ n_{t+1}^f(i),EQ_t^f(i)}}\left\{\text{div}_t^f(i)+\mathbb{E}_t\Lambda_{t+1}\left[(1-\theta_f)n_{t+1}^f(i)+\theta_f V_{t+1}^f(n_{t+1}^f(i))\right]\right. \tag{32}$$

$$\text{s.t. } \text{div}_t^f(i)+EQ_t^f(i)=n_t^f(i), \tag{33}$$

$$n_{t+1}^f(i)=\rho_{t+1}^f EQ_t^f(i), \tag{34}$$

$$\text{div}_t^f(i)\geq 0 \tag{35}$$

with $\rho_{t+1}^f \equiv \frac{\Pi_{t+1}^f}{EQ_{t+1}^f}$.

Back to firms

Entrepreneurs II

- Gertler and Kiyotaki (2010) $\rightarrow$ value function linear in net worth then

$$v_t^f n_t^f = \max_{\text{div}_t^f, EQ_t^f} \{\text{div}_t^f + \mathbb{E}_t \underbrace{\Lambda_{t+1}\left[(1-\theta_f) + \theta_f v_{t+1}^f\right]}_{\substack{\text{Stoch. disc.} \\ \text{factor entrepreneurs } \equiv \Lambda_{t+1}^f}} \rho_{t+1}^f n_t^f\} \tag{36}$$

- As long as $v_{t+1}^f > 1$, we have $\text{div}_t^f = 0$.
- Shadow value of entrepreneurial equity $v_{t+1}^f = \mathbb{E}_t \Lambda_{t+1}^f \rho_{t+1}^f$.

Back to firms

Laws of motion

- Law of motion of aggregate bankers' net worth

$$N^b_{t+1} = [\theta_b + (1-\theta_b)\chi_b]\left[\rho^b_{f,t+1} EQ^b_{f,t} + \rho^b_{h,t+1} EQ^b_{h,t}\right] - T_t, \tag{37}$$

  where T_t are taxes levied by the prudential authority.

- Law of motion of aggregate entrepreneurial net worth

$$N^f_{t+1} = [\theta_f + (1-\theta_f)\chi_f]\rho^f_{t+1} EQ^f_t. \tag{38}$$

- Capital and housing stock

$$X_{t+1} = S\left(\frac{I_{x,t}}{X_t}\right) X_t + (1-\delta_x) X_t, \tag{39}$$

  with $X = K, H$ and $S\left(\frac{I_{x,t}}{X_t}\right) = \frac{a_1}{1-1/\psi}(I_{x,t}/X_t)^{1-1/\psi} + a_2$

Market clearing

- Bankers' net worth: $N_t^b = EQ_{f,t}^b + EQ_{h,t}^b$
- Entrepreneurs' net worth: $N_t^f = EQ_t^f$
- Capital: $K_t = K_{f,t} + K_{h,t}$.
- Final consumption good
  $Y_t = C_t + I_{k,t} + I_{h,t} + \Sigma_{f,t} + \Sigma_{h,t} + \Sigma_{b,t}$, where $\Sigma_{s,t}$ are bankruptcy costs associated to firms, households and banks.
- Deposits: $D_t = D_{f,t} + D_{h,t}$.

Realized return on loans

- Realized return on mortgage loans

$$\tilde{R}^{l}_{h,t+1}(\omega_{j_h})B_{h,t} \equiv R^{l}_{h,t}B_{h,t}\left[1 - F_{h,t+1}(\bar{\omega}_{h,t+1}(\omega^{j}_{h}))\right] + (1-\mu_h)\omega^{j}_{h}R_{h,t+1}q_{h,t}H_t\int_{0}^{\bar{\omega}_{h,t+1}(\omega^{j}_{h})}\omega_h dF_{h,t+1}(\omega_h), \quad (40)$$

- Realized return on corporate loans

$$\tilde{R}^{l}_{f,t+1}(\omega^{j}_{f})B_{f,t} \equiv R^{l}_{f,t}B_{f,t}\left[1 - F_{f,t+1}(\bar{\omega}_{f,t+1}(\omega^{j}_{f}))\right] + (1-\mu_f)\omega^{j}_{f}\left[Y_{t+1} + q_{t+1}(1-\delta)K_{f,t}\right]\int_{0}^{\bar{\omega}_{f,t+1}(\omega^{j}_{f})}\omega_f dF_{f,t+1}(\omega_f), \quad (41)$$

Back to banks

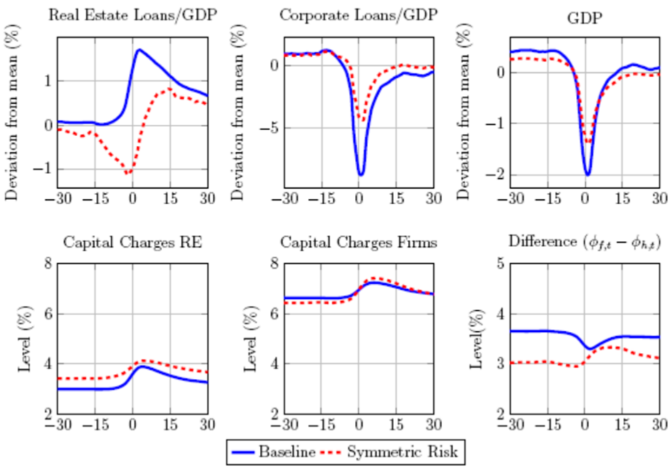
IRB formulae

$$\text{IRB}_{s,t} = \text{LGD}_s \left[\Phi \left(\frac{\Phi^{-1}\left(\text{PD}_{s,t}\right) + \sqrt{\zeta_{s,t}}\Phi^{-1}(0.999)}{\sqrt{1-\zeta_{s,t}}} \right) - \text{PD}_{s,t} \right], \tag{42}$$

where LGD_s is loss given default; $PD_{s,t}$ is the IRB default probability, and $\zeta_{s,t}$ is the portfolio correlation coefficient of each class of loans.

Role of differences in non-diversifiable risk

Figure 2: Role of non-diversifiable risk



Note: Solid lines correspond to the baseline model. Dashed lines correspond to a version of the model where real estate and corporate lending have the identical risk parameters. Sample paths correspond to shock realizations that generate banking crises in the baseline model.

Back to results

More on Capital Requirements

- Limits to bank leverage $EQ_t \geq \phi_t B_t$

Back to intro

More on Capital Requirements

- Limits to bank leverage $EQ_t \geq \phi_t B_t$
- Computed according to the risk characteristics of individual loans (microprudential approach):

Back to intro

More on Capital Requirements

- Limits to bank leverage $EQ_t \geq \phi_t B_t$
- Computed according to the risk characteristics of individual loans (microprudential approach):
  - Probability of default of loans.
  - Loss given default.
  - Loan maturity.
  - Correlation of defaults in a portfolio (to some extent).

Back to intro

More on Capital Requirements

- Limits to bank leverage $EQ_t \geq \phi_t B_t$
- Computed according to the risk characteristics of individual loans (microprudential approach):
  - Probability of default of loans.
  - Loss given default.
  - Loan maturity.
  - Correlation of defaults in a portfolio (to some extent).
- But also more recently, **macroprudential** aspects:

Back to intro

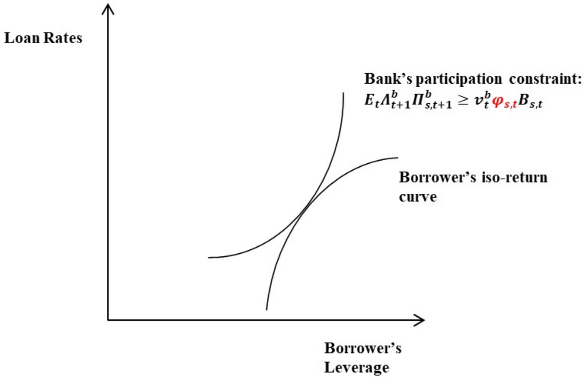
More on Capital Requirements

- Limits to bank leverage $EQ_t \geq \phi_t B_t$
- Computed according to the risk characteristics of individual loans (microprudential approach):
  - Probability of default of loans.
  - Loss given default.
  - Loan maturity.
  - Correlation of defaults in a portfolio (to some extent).
- But also more recently, **macroprudential** aspects:
  - Systemic relevance of banks.
  - Interconnectedness of a banking system.
  - **Evolution of the credit cycle** (Counter-cyclical buffer **CCyB**)
  - Sectoral developments (e.g. rapid credit growth in the real estate sector) (**sectoral CCyB**)

Back to intro

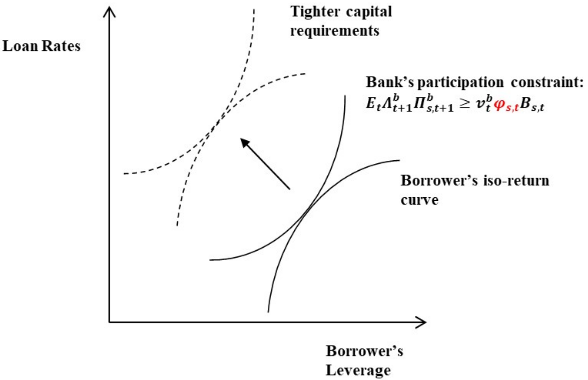
Loan Contracts

- Borrowers (firms and households) decide on investment, consumption and borrowing taking into account the participation constraints of banks.
- Loan terms specify both a promised loan rate and the leverage of the borrower.



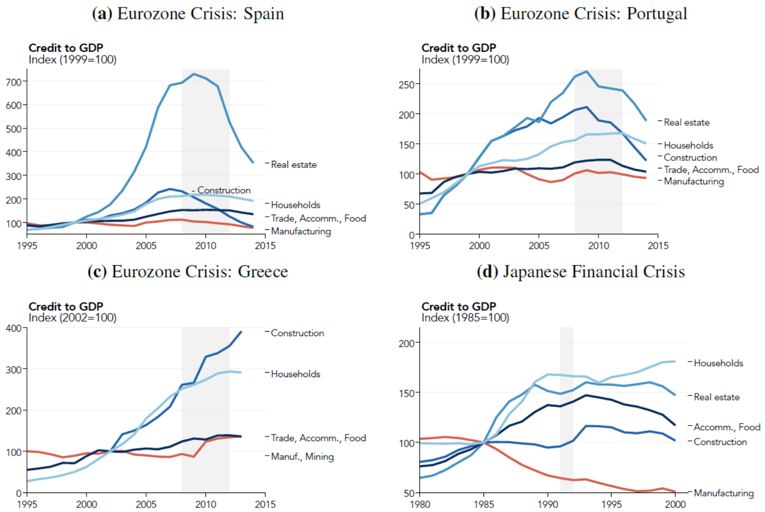
Loan Contracts

- Borrowers (firms and households) decide on investment, consumption and borrowing taking into account the participation constraints of banks.
- Loan terms specify both a promised loan rate and the leverage of the borrower.



Motivation - Credit growth around crises - Muller and Verner (2023)

Figure 3: Case Studies: The Eurozone and Japanese Crises



Back to Intro

Motivation - Credit growth around crises - Muller and Verner (2023)

Figure 7: Credit Dynamics around Systemic Banking Crises

(a) Tradable vs. Non-Tradable Sector