Part 1: Introduction and Models of Firm Dynamics

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Misallocation and Aggregate Productivity
Central Bank of Chile

Motivation

- TFP differences account for large fraction of Y/L differences:

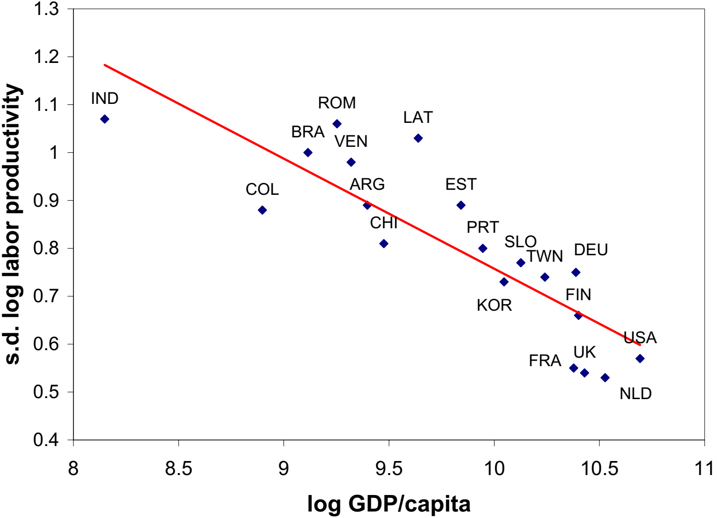
$$\frac{Y}{L} = AF\left(\frac{K}{L}, \frac{H}{L}\right)$$

- Ratio of rich (90th percentile) to poor (10th percentile):
  - Y/L: 24
  - K/L and H/L: 6 - 8
  - Implies ratio of A: 3 - 4
- Poor countries TFP today = Rich countries' TFP 100 years ago
- Difficult to blame on slow technology diffusion
- One possibility: much poorer allocation of resources: *misallocation*.

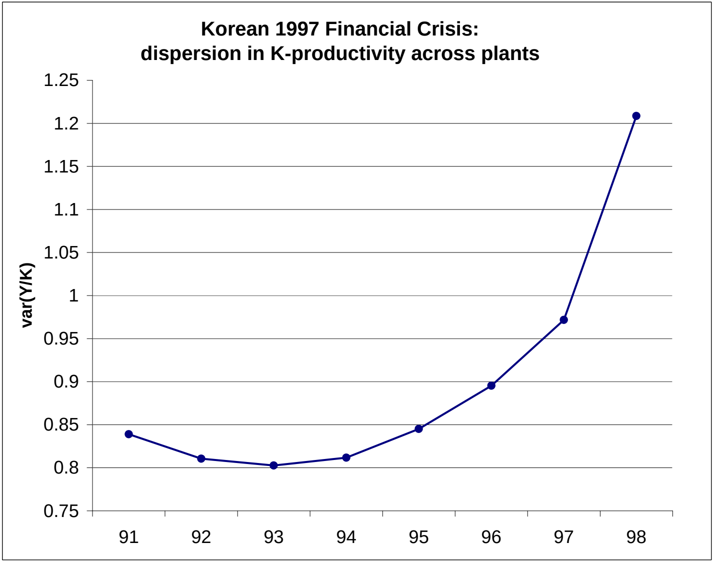
Misallocation

- Misallocation = Dispersion in Marginal Product of K, L across firms
- Basic example with 2 firms
  - Firm A with MPL = 10, firm B with MPL = 5.
  - Allocation inefficient:
    - Moving 1 unit of L from B to A increases output by 5 units.
    - Continue moving labor until $\mathrm{MPL_A} = \mathrm{MPL_B}$

MPL Dispersion in Cross Section



MPK Dispersion in Time Series



Sources of Misallocation

1. Government policies
   - differential rates of taxation/subsidies, access to credit
   - size-dependent policies, e.g. subsidies to smaller firms
   - promoting certain industries at the expense of others
   - labor market policies (e.g. discourage worker turnover)
2. Credit constraints
3. Markup dispersion

Goals

- Measure misallocation
- Understand its sources
  - Obvious implications for development policy

Challenges

- Measurement: measuring MPL, MPK difficult
- Many government policies not explicit. Poor data in developing countries
- Markups, borrowing constraints difficult to measure
- Data alone thus not sufficient
- Approach: use theory to guide measurement
    - Explicitly model source of misallocation (markups, credit etc.)
    - Estimate model using micro-data
    - Use estimated model to answer questions

Course Outline

1. Conceptual Framework
2. Measuring Misallocation
3. Models with Financial Frictions
4. Models with Variable Markups

Topic 1: Conceptual Framework

- Discuss 4 closely related papers
  - Hopenhayn - Rogerson (1993, JPE)
    - Effect of labor market policies that discourage worker turnover
  - Restuccia - Rogerson (2008, RED)
    - on magnitude of losses from differential rates of taxation
  - Hopenhaun (2014, WP)
    - clarifies several conceptual issues

Hopenhayn Rogerson 1993

- “Job Turnover and Policy Evaluation: A General Equilibrium Analysis”
- Study implication of hiring/firing costs
- Model of firm dynamics a la Hopenhayn (92)
- Firms differ in productivity
- Productivity evolves over time
- Firms makes entry/exit employment decisions

Setup

- Technology of individual producers

$$y = f(z, n) \text{ (e.g., } y = zn^{\eta},\ \eta < 1)$$

  - y output
  - z idiosyncratic productivity, varies over time with $F(z', z)$
  - n employment

- Hiring/Firing costs:

$$g(n_t, n_{t-1})$$

  - E.g., if tax τ per job destroyed, $g = \tau \max(0, n_{t-1} - n_t)$

Producer's problem

- Period dividends:

$$f(z_t, n_t) - w_t n_t - g(n_t, n_{t-1}) - c_f$$

  - producers competitive. price of output normalized to 1
  - wage w_t
  - per-period fixed operating cost c_f

Timing

1. At beginning of period t know z_{t-1} and n_{t-1}
2. Decide whether stay or exit
   - If exit, must pay $g(0, n_{t-1})$
   - If stay, observe z_t, choose n_t, receive

$$p_t f(z_t, n_t) - n_t - g(n_t, n_{t-1}) - p_t c_f$$

Dynamic Program in Ergodic Steady State

- Conditional on staying, value of firm is

$$V(n, z) = \max_{n' \geq 0} f(z, n') - wn' - c_f - g(n', n) +$$
$$\beta \max \left(\int V(z', n') \mathrm{d}F(z', z), -g(0, n') \right)$$

- Entry requires fixed cost c_e. After entry draw z from $\nu(z)$
- Free entry condition (if positive measure of entrants $M > 0$):

$$\int V(0, z) \mathrm{d}\nu(z) = c_e$$

Consumer's Problem

$$\max \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - aN_t\right]$$

subject to

$$C_t = wN_t + D_t$$

where

$$D_t = \text{ sum of dividends of all firms + labor taxes}$$

Gives labor supply:

$$w = \frac{a}{U'(C_t)}$$

Equilibrium

- Let $\mu(n, z)$ be measure of producers, M mass of entrants.
- Let $n'(n, z)$ be the employment choice.
- Let $\mu' = T(\mu, M)$ transition of distribution
- Ergodic steady state: $\mu = T(\mu, M)$

Aggregation

- Total output:

$$Y = \int \left[f(n'(n,z),z) - c_f \right] \mathrm{d}\mu(n,z) + M \int \left[f(n'(0,z),z) - c_e \right] \mathrm{d}\nu(z)$$

- Total employment:

$$N = \int n'(n,z) \mathrm{d}\mu(n,z) + M \int n'(0,z) \mathrm{d}\nu(z)$$

Calibration

- Period = 5 years. Assume $g = 0$ in Benchmark (U.S., simple to solve)
- $y_t = z_t^{1-\eta} n_t^{\eta}$
- Assume $\log z_t = \rho \log z_{t-1} + \varepsilon_t$
  - Use firm's Foc: $\eta y_t / n_t = \eta z_t^{1-\eta} n_t^{\eta - 1} = w$
  - Implies $\log n_t = \text{const.} + \rho \log n_{t-1} + \varepsilon_t$
  - Estimate with LRD data for US. Gives ρ and σ_ε^2
- Set $\eta = 0.64$ (labor share in revenue), $\beta = 0.8$
- Choose c_f, c_e and ν to match
  - 5-year exit rate, mean employment,
  - size distribution of new entrants (0 - 6 yrs.)

Calibration Targets

A. Estimates Derived from the LRD

Serial correlation in log employment (5-year interval, survivors)	.93
Variance in growth rates (log difference, 5-year interval, survivors)	.53
Mean employment	61.7
Exit rate (5-year interval)	37%

B. Size Distribution for Firms Aged 0–6 Years

Employees	Share of Total Firms
1–19	.74
20–99	.18
100–499	.08
500+	.01

Statistics from the Model

A. Summary Statistics for Benchmark Model

Average firm size	61.2
Co-worker mean	747
Variance of growth rates (survivors)	.55
Serial correlation in log n (survivors)	.92
Exit rate of firms	.39
Turnover rate of jobs	.30
Fraction of hiring by new firms	.15
Average size of new firm	7.5
Average size of existing firm	4.9

B. Size Distribution

	1–19	20–99	100–499	500+
Firms	.52	.37	.10	.01
Employment	.06	.24	.37	.33
Hiring	.05	.35	.41	.19
Firing	.12	.19	.34	.35
By cohort:				
1 period	.88	.12	.00	.00
2 periods	.54	.45	.01	.00
5 periods	.29	.58	.12	.01
10 periods	.20	.54	.20	.05
Hazard rates by cohort:				
1 period	.75			
2 periods	.32			
5 periods	.15			
10 periods	.10			

Evaluate Effect of Firing Costs

$$g(n_t, n_{t-1}) = \tau \max(0, n_{t-1} - n_t)$$

- Compare $\tau = 0$ economy (Benchmark) with
  - $\tau = 0.1$: (severance pay = 6 months of wages)
  - $\tau = 0.2$: (severance pay = 12 months of wages)

Effect of Firing Costs

EFFECT OF CHANGES IN τ (Benchmark Model)

	$\tau = 0$	$\tau = .1$	$\tau = .2$
Price	1.00	1.026	1.048
Consumption (output)	100	97.5	95.4
Average productivity	100	99.2	97.9
Total employment	100	98.3	97.5
Utility-adjusted consumption	100	98.7	97.2
Average firm size	61.2	61.8	65.1
Layoff costs/wage bill	0	.026	.044
Job turnover rate	.30	.26	.22
Serial correlation in log(n)	.92	.94	.94
Variance in growth rates	.55	.45	.39

Intuition

- With firing costs, $\log(n)$ no longer linear in $\log(z)$
- (S, s) decision rules: adjust if n_{t-1} outside bands
- Do not react to large range of Δz
- Inefficient allocation of resources
- Dispersion in marginal product of labor
- Illustrate next with simplified version with $\log z' = \log z + \varepsilon'$

Intuition with random walk z

- Suppose no fixed costs, no exit. Firm dividends HD(1) in (z, n):

$$z^{1-\eta}\left(n'\right)^{\eta} - wn' - \tau \max(0, n - n')$$

- Let $\tilde{n}' = n'/z$ and $\tilde{n} = n/z$. Value of the firm:

$$V^{\mathrm{up}}(\tilde{n}) = \max_{\tilde{n}' \geq \tilde{n}} \left(\tilde{n}'\right)^{\eta} - w\tilde{n}' + \beta W(\tilde{n}')$$

$$V^{\mathrm{dn}}(\tilde{n}) = \max_{\tilde{n}' < \tilde{n}} \left(\tilde{n}'\right)^{\eta} - w\tilde{n}' - \tau\left(\tilde{n} - \tilde{n}'\right) + \beta W(\tilde{n}')$$

$$W(\tilde{n}') = \int \max\left(V^{\mathrm{up}}\left(\frac{\tilde{n}'}{\exp(\varepsilon)}\right), V^{\mathrm{dn}}\left(\frac{\tilde{n}'}{\exp(\varepsilon)}\right)\right) \exp(\varepsilon) \mathrm{d}F(\varepsilon)$$

- If adjust up, envelope condition:

$$\eta\left(\tilde{n}'\right)^{\eta-1} = w - \beta W'\left(\tilde{n}'\right) \text{ if } \tilde{n}' \geq \tilde{n}$$

  - Let n^{up} denote unconstrained solution. Independent of $\tilde{n}$

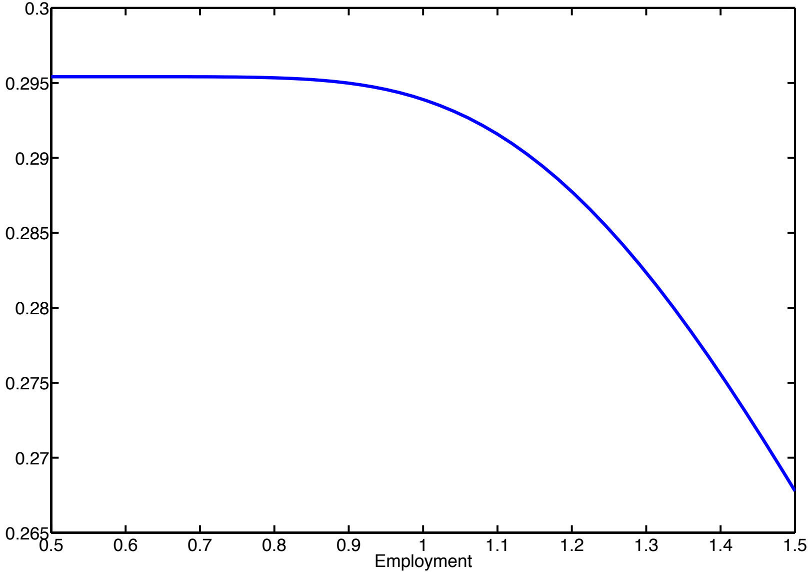
- If adjust down, envelope condition:

$$\eta\left(\tilde{n}'\right)^{\eta-1} = w - \tau - \beta W'\left(\tilde{n}'\right) \text{ if } \tilde{n}' < \tilde{n}$$

  - Let n^{dn} denote unconstrained solution. Independent of $\tilde{n}$

- By concavity of production function, $n^{\text{dn}} > n^{\text{up}}$

Continuation Value, $W'(\tilde{n}) \leq 0$



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$$\eta\left(\tilde{n}'\right)^{\eta-1} = w - \tau - \beta W'\left(\tilde{n}'\right) \text{ if } \tilde{n}' < \tilde{n}$$

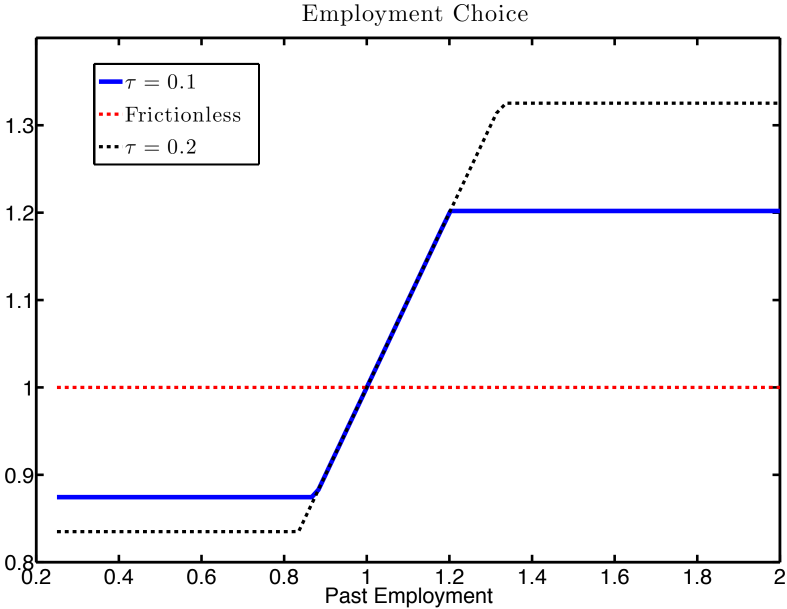
  - Let n^{dn} denote unconstrained solution. Independent of $\tilde{n}$

- Since $W'(\tilde{n}) \leq 0$, $n^{\text{up}} <$ frictionless $= 1$

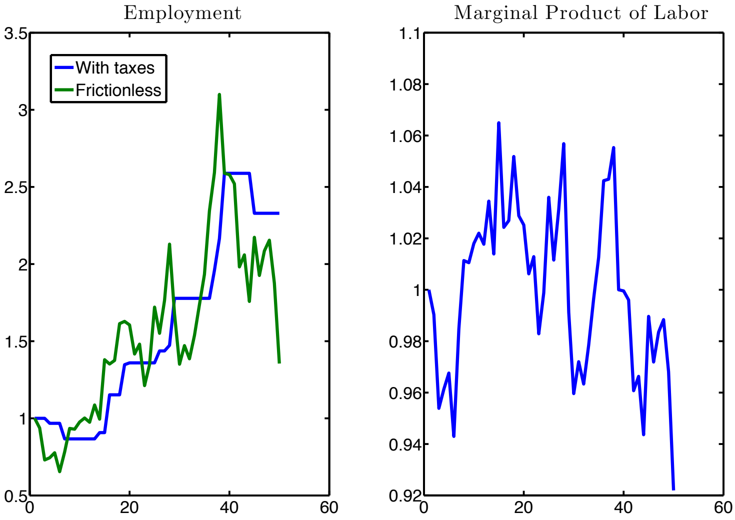
- Since $\tau + \beta W'(\tilde{n}) \geq 0$, $n^{\text{dn}} \geq$ frictionless $= 1$

Employment Choice

Effect of higher firing tax



Sample Paths



Dispersion in MPL

Absolute Deviations from MPL = $1/p$

Size of Deviation (%)	Fraction of Firms within Interval $\tau = .1$	$\tau = .2$
0–3	.30	.00
3–5	.45	.12
5–10	.15	.78
10–15	.00	.05
>15	.00	.05

Why is dispersion in MPL costly?

- Derive implications for TFP
- Let μ_i be MPL of firm i:

$$\eta z_i^{1-\eta} n_i^{\eta-1} = \mu_i$$

- Gives

$$n_i = z_i \left(\frac{\eta}{\mu_i} \right)^{\frac{1}{1-\eta}}$$

- Let $N = \int n_i \mathrm{d}i$

$$N = \int z_i \left(\frac{\eta}{\mu_i} \right)^{\frac{1}{1-\eta}} \mathrm{d}i$$

$$\frac{n_i}{N} = \frac{z_i \mu_i^{\frac{1}{\eta-1}}}{\int z_i \mu_i^{\frac{1}{\eta-1}} \mathrm{d}i}$$

Why is dispersion in MPL costly?

- Let $Y = \int y_i \mathrm{d}i = \int z_i^{1-\eta} n_i^{\eta} \mathrm{d}i$

$$Y = \frac{\int z_i \mu_i^{\frac{\eta}{\eta-1}} \mathrm{d}i}{\left(\int z_i \mu_i^{\frac{1}{\eta-1}} \mathrm{d}i \right)^{\eta}} N^{\eta}$$

- Compare to efficient allocations

$$\max_{n_i} \int z_i^{1-\eta} n_i^{\eta} \quad \text{s.t.} \int n_i \mathrm{d}i = N$$

- Solution:

$$\eta z_i^{1-\eta} n_i^{\eta-1} = \lambda$$

$$Y^{\mathrm{eff}} = \left(\int z_i \mathrm{d}i \right)^{1-\eta} N^{\eta}$$

Log-Normal Approximation

- Suppose (μ_i, z_i) log-Normal with $\sigma_\mu^2, \sigma_z^2, \sigma_{\mu,z}$
- Then

$$\log A - \log A^{\text{eff}} = -\frac{1}{2}\frac{1}{1-\eta}\sigma_\mu^2$$

- TFP losses proportional to variance of MPL
- Covariance MPL, z drops out.
  - Taxing high z firms more inconsequential
- Losses magnified when η closer to 1.

Conclusions: Hopenhayn-Rogerson 1993

- Policies limiting job turnover have sizable welfare consequences
  - Increase dispersion MPL, reduce TFP

Restuccia-Rogerson 2008

- Heterogeneous firms a la Hopenhayn (92), Hopenhayn-Rogerson (93)
- Non-degenerate distribution of firm productivity
- Distribution of resources across firms critical object for aggregate TFP
- Policy distortions that induce dispersion of factor prices faced by firms
- Quantify the effect of distortions on TFP
- Claim taxing more productive firms more costly

Technology

$$y = f(z, n, k) = z^{1-\eta}\left(k^{\alpha} n^{1-\alpha}\right)^{\eta}$$

- z is time-invariant
- Fixed costs of entry c_e and operating c_f as in HR '93
- Producer-specific productivity z drawn at birth from $\nu(z)$
- Output tax τ drawn at birth from $P(z, \tau)$
- Firm i profits

$$(1 - \tau_i) z_i^{1-\eta}\left(k_i^{\alpha} n_i^{1-\alpha}\right)^{\eta} - w n_i - r k_i - c_f$$

Dispersion in MPL and MPK

- Optimal choice of k:

$$f_k(z_i, k_i, n_i) = \alpha\eta \frac{y_i}{k_i} = \frac{r}{1-\tau_i}$$

- Optimal choice of n:

$$f_n(z_i, k_i, n_i) = (1-\alpha)\eta \frac{y_i}{n_i} = \frac{w}{1-\tau_i}$$

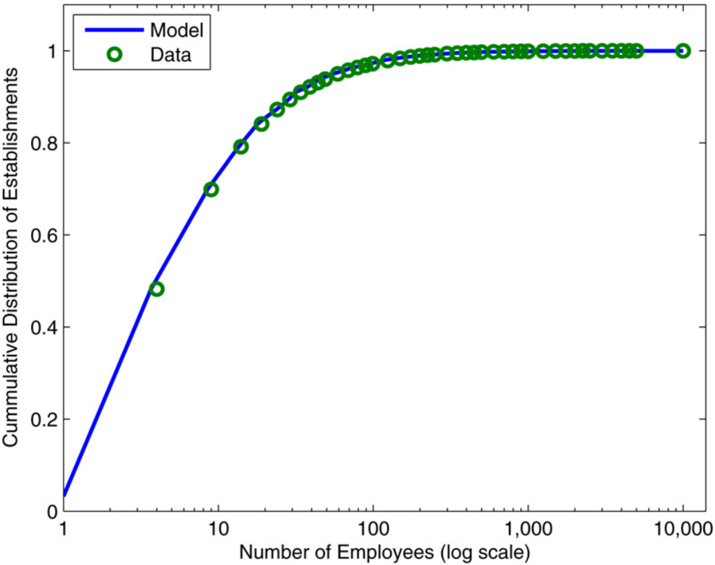
- Implies TFP losses if τ varies across firms. Identical formulas as above.

- Taxes rebated lump-sum to households

Calibration

- As HR do, assume Benchmark economy undistorted
- Choose $\nu(z)$ to match establishment size distribution
- Set $\eta = 0.85$ (estimates of returns to scale U.S.)

Size Distribution of Establishments



Size Distribution of Establishments

Distribution statistics of benchmark economy

	Establishment size (number of employees)		
	< 5	5 to 49	$\geqslant 50$
Share of establishments	0.56	0.39	0.05
Share of output	0.08	0.34	0.58
Share of labor	0.08	0.34	0.58
Share of capital	0.08	0.34	0.58
Average employment	2.4	15.5	183.0

Quantitative analysis of policies

- Two exercises
  - τ uncorrelated with z. Half are taxed, half subsidized.
    - Given τ (e.g. 0.10), choose subsidy so no effect on K, L in aggregate
  - τ correlated with z. Firms above median z taxed, below subsidized

Effect of Uncorrelated Distortions

Effects of idiosyncratic distortions—uncorrelated case

Variable	τ_t			
	0.1	0.2	0.3	0.4
Relative Y	0.98	0.96	0.93	0.92
Relative TFP	0.98	0.96	0.93	0.92
Relative E	1.00	1.00	1.00	1.00
Y_s/Y	0.72	0.85	0.93	0.97
S/Y	0.05	0.08	0.09	0.10
τ_s	0.06	0.09	0.10	0.11

- Higher dispersion in taxes: larger TFP losses

Effect of Increasing Fraction of Taxed Firms

Relative TFP—uncorrelated distortions

Fraction of establishments taxed (%):	τ_t			
	0.1	0.2	0.3	0.4
90	0.92	0.84	0.78	0.74
80	0.95	0.89	0.84	0.81
60	0.98	0.94	0.91	0.89
50	0.98	0.96	0.93	0.92
40	0.99	0.97	0.95	0.94
20	1.00	0.99	0.98	0.97
10	1.00	0.99	0.99	0.99

- More firms taxed: larger TFP losses

Effect of Correlated Distortions

Variable	τ_t			
	0.1	0.2	0.3	0.4
Relative Y	0.90	0.80	0.73	0.69
Relative TFP	0.90	0.80	0.73	0.69
Relative E	1.00	1.00	1.00	1.00
Y_s/Y	0.42	0.67	0.83	0.92
S/Y	0.17	0.32	0.43	0.49
τ_s	0.40	0.48	0.52	0.53

- Taxing more productive firms: larger TFP losses

Hopenhayn 2014

- “On the Measure of Distortions”
- Explicitly characterize mapping from distortions to TFP
- Clarify when correlated distortions lead to large losses
- Relationship between distortions and size distribution

Setting

- As in Hopenhayn - Rogerson 1993. Labor only factor
- Economy populated by measure M of firms with technology

$$y = z^{1-\eta} n^{\eta}$$

- Planner's problem:

$$\max_{n_i} \int_0^M z_i^{1-\eta} n_i^{\eta} \mathrm{d}i \quad \text{s.t.} \quad \int_0^M n_i \mathrm{d}i \leq N$$

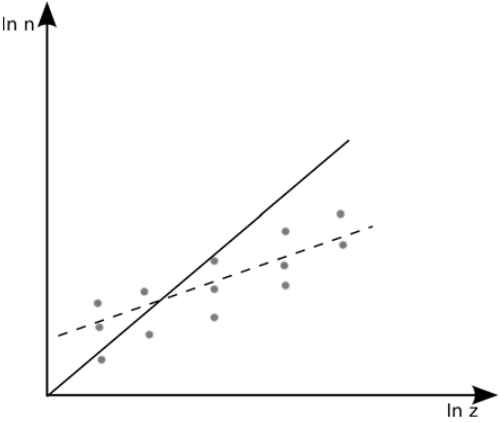
- Solution:

$$n_i = z_i \lambda = \frac{z_i}{\int_0^M z_i \mathrm{d}i} N$$

- Aggregate output: (let $Z =$ mean z_i)

$$Y = \int_0^M z_i^{1-\eta} n_i^{\eta} \mathrm{d}i = \left(\int_0^M z_i \mathrm{d}i \right)^{1-\eta} N^{\eta} = Z^{1-\eta} M^{1-\eta} N^{\eta}$$

Correlated vs. Uncorrelated Distortions



Distortions

- Model as RR do, tax on output:
- Firm's problem:

$$\max_{n_i}(1-\tau_i)z_i^{1-\eta}n_i^{\eta}$$

- Employment choice distorted, reduces aggregate TFP
- TFP of distorted economy:

$$TFP = \frac{\int_0^M z_i(1-\tau_i)^{\frac{\eta}{1-\eta}}\mathrm{d}i}{\left(\int_0^M z_i(1-\tau_i)^{\frac{1}{1-\eta}}\mathrm{d}i\right)^{\eta}}$$

Example

- 2 types, $z_1 = 1$ and $z_2 = 4$. $\eta = 1/2$, $N = 2000$, 16 firms each type
- Optimality requires $n_2 = 100$, $n_1 = 25$ and $Y = 400$
- Consider 3 distortions
  - Uncorrelated for low productivity
    - Destroy 12 low-product, remaining 4 get 100 workers each: $Y = 360$
  - Uncorrelated for high productivity
    - Destroy 3 high-product, one high-product gets 400 workers: $Y = 360$
  - Correlated
    - Destroy 12 low-product, one high-product gets 400 workers: $Y = 360$
- What matters is not correlation, but number workers affected (300)

General Characterization

- Recall $n(\tau, z) = (1-\tau)^{\frac{1}{1-\eta}} z = (1-\tau)^{\frac{1}{1-\eta}} n(z)$

- Let $\theta = (1-\tau)^{\frac{1}{1-\eta}}$: ratio of actual to undistorted n

- Let n be undistorted employment, θn actual employment

- Let $\mu(\theta, n)$: distribution of distortions. Total employment unchanged so

$$N = \int n \mathrm{d}\mu(\theta, n) = \int \theta n \mathrm{d}\mu(\theta, n)$$

- For every $\hat{\theta}$, let

$$N(\hat{\theta}) = \int_{\theta \leq \hat{\theta}} n \mathrm{d}\mu(\theta, n)$$

- $N(\hat{\theta})$: total undistorted employment affected by a distortion $\theta \leq \hat{\theta}$

General Characterization

- $N(\theta)$: measure on θ with properties:

$$N = \int \mathrm{d}N(\theta)$$

$$N = \int \theta \mathrm{d}N(\theta)$$

- Note distorted output is $y(\theta, z) = z^{1-\eta}\theta^{\eta}n^{\eta} = \theta^{\eta}y(z)$

- But in undistorted economy $\eta y(z)/n(z) = \lambda$ is equal across firms

- So can write $y(\theta, z) = a\theta^{\eta}n(z)$ where $a = \lambda/\eta$

- So aggregate output is

$$Y = \int y(\theta, z) = a\int \theta^{\eta} n \mathrm{d}\mu(\theta, n) = a\int \theta^{\eta}\mathrm{d}N(\theta)$$

General Characterization

- In undistorted economy $\theta = 1$ so ratio of output gives TFP ratio:

$$\frac{Y}{Y^0} = \frac{\text{TFP}}{\text{TFP}^0} = \frac{\int \theta^\eta \mathrm{d}N(\theta)}{\int \mathrm{d}N(\theta)} = \frac{\int \theta^\eta \mathrm{d}N(\theta)}{N}$$

- TFP ratio $= \eta$ moment of the *employment-weighted* distribution of θ

- Since $\int \theta \mathrm{d}N(\theta) = N$ and $\eta < 1$ mean preserving spread reduces TFP

Recall 2 results in Restuccia-Rogerson

% Estab. taxed	Uncorrelated		Correlated	
	τ_t		τ_t	
	0.2	0.4	0.2	0.4
90%	0.84	0.74	0.66	0.51
50%	0.96	0.92	0.80	0.69
10%	0.99	0.99	0.92	0.86

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1. More firms taxed, lower TFP

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2. Taxing more productive firms reduces TFP

1. More firms taxed, lower TFP

- Assume uncorrelated distortion (all firms equal size N at optimum):
  - Suppose α are taxed, $\theta_\tau < 1$ and $(1-\alpha)$ subsidized $(\theta_s > 1)$
  - Need $\alpha(1-\theta_\tau) = (1-\alpha)(\theta_s - 1)$ to maintain constant N
  - Measure of distortions: $(\alpha N, \theta_\tau)$, $((1-\alpha)N, \theta_s)$
  - Increase in α (fraction taxed) = mean preserving spread (higher θ_s)
  - Since TFP $\sim [\eta(< 1)$ moment of $\theta]$, m.p.s reduces TFP

2. Correlated Distortions reduce TFP more

- i.e., taxing more productive firms (high z) is worse than taxing at random
  - (wrong) intuition: move n from high to low z so more damage
  - wrong because the *marginal* productivity matters, not z

- Intuition is that taxing high z firms requires more reallocation of n:

$$N_\tau(1-\theta_\tau) = N_s(\theta_s - 1)$$

- Correlated distortions in Restuccia-Rogerson:
  - Higher N_τ (employment in undistorted economy)
  - So need higher θ_s: a mean preserving spread

- No general theorem about whether correlation increases TFP losses
  - Paper provides proposition for specific pattern of correlation
  - Gives example where more correlation reduces TFP losses
  - Log-normality: correlation has 0 effect (see earlier formula)