Workhorse Models of the Small Open Economy

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Objectives and layout of the lecture

- Introduce key issues for analyzing SOE models: stationarity and wealth dynamics; complete v. incomplete markets; role of prec. savings
- Model 1: Deterministic, 1-sector endowment ec.
- Model 2: Deterministic Fisherian GE model
- Model 3: Stochastic variant of model 1 with incomplete markets
- Primer on global solution methods (Model 3)
- Limitations of approximation methods

WORKHORSE MODEL 1: DETERMINISTIC ONE-SECTOR ENDOWMENT ECONOMY MODEL

Key Assumptions

1. SOE with perfect access to world credit market
2. One-period bonds, fixed world real interest rate
3. Perfect foresight OR Complete Markets
4. Credible commitment to repay
5. Frictionless economy, no distortions

– CA supports perfect consumption smoothing

– Long-run NFA is simply annuity value of steady-state trade balance

Intertemporal optimization problem

- Standard social planner form:

$$(I)\sum_{t=0}^{\infty}\beta^t u(c_t)$$

$$(II)\; c_t = y_t - b_{t+1} + b_t R, \quad b_0 \;\; given, \;\; \{y_t\}_{t=0}^{\infty}$$

- Recursive planner's problem:

$$(III)\; V_t(b_t, y_t) = \max_{b_{t+1}} \{u(c_t) + \beta V_{t+1}(b_{t+1}, y_{t+1})\}$$

subject to *(II)*

Equilibrium conditions

- First-order condition of the recursive problem:

$$u'(c_t) = \beta V_{1t+1}(b_{t+1}, y_{t+1})$$

$$B/S \Rightarrow u'(c_t) = \beta R u'(c_{t+1})$$

- Stationarity assumption: $\beta R = 1 \Rightarrow c_t = \bar{c} \quad \forall\, t$

- Closed-form solution (using (II)):

$$\frac{\bar{c}}{(1-\beta)} = \left[\sum_{t=0}^{\infty} \beta^t y_t\right] + b_0 R$$

$$= (1-\beta)W$$

Current account, trade balance and NFA dynamics

- The equilibrium current account is:

$$b_{t+1} - b_t = y_t - \bar{c} + b_t r$$

- Assume output converges:

$$y_t \to \bar{y} \ \ as \ \ t \to \infty$$

- Stationary equilibrium of CA is zero, and steady states of NFA and NX are given by:

$$\bar{b} = -\frac{[\bar{y} - \bar{c}]}{r} = -\frac{\overline{nx}}{r} = -\frac{[\bar{y} - (1-\beta)W]}{r} = \beta W - \frac{\bar{y}}{r}$$

Stationarity and initial conditions

- Stationary equilibrium is unique, but since wealth depends on initial NFA, $\bar{b}$ and $\bar{c}$ depend on b_0 (i.e. steady state depends on initial conditions)
- Borrow when $y_t < \bar{c}$ and save when $y_t > \bar{c}$
  - CA deficit with low y_t
  - CA surplus with high y_t
  - CA is procyclical!
- Is this a good model of actual CA dynamics?

WORKHORSE MODEL 2: DETERMINISTIC GENERAL EQUILIBRIUM MODEL

Irving Fisher's SOE model

- Similar to Model 1 but instead of endowments we have representative household and firm with production and capital accumulation
- Alternative decentralizations work.
  - Example: households max. utility, rent out factors, firms max. value of the firm, hire factors. Both interact in goods and factor markets
- There are no distortions, so social welfare theorems hold, and we can solve for comp. eq. by solving social planner's problem

Recursive social planner's problem

$$V(K, A) = \max_{\{K', A', c\}} \{u(c) + \beta V(K', A')\}$$

$$s.t. \quad c = f(K) - (K' - K) \;\; [1 + \frac{\phi}{2}(K' - K)]$$

$$- \;\; A' + A(1 + r^*))$$

- With a solution characterized by decision rules:

$$\widehat{K'}(K, A), \widehat{A}'(K, A)$$

First-order conditions of the recursive problem

- Capital:

$$\widehat{K'} \,:\; u^{'}(c)[1+\phi(K^{'}-K)] = \beta V_1(K^{'},A^{'})$$

- Assets:

$$\widehat{A'} \,:\; u^{'}(c) = \beta V_2(K^{'},A^{'})$$

Marginal value of capital accumulation

- Differentiate *V*(.) w.r.t. *K*:

$$V_1(K,A) =$$

$$u'(c)\left[f'(K) - \cancel{\widehat{K'}_1} + 1 - \phi(K'-K)\left(\cancel{\widehat{K'}_1} - 1\right) - \cancel{\widehat{A'}_1}\right]$$

$$+\beta\left[V_1\cancel{\widehat{K'}_1} + V_2\cancel{\widehat{A'}_1}\right]$$

- Apply envelope theorem:

$$\Rightarrow V_1(K,A) = u'(c)[f'(K) + 1 + \phi(K'-K)]$$

Marginal value of net foreign assets

- Differentiate *V*(.) w.r.t. *A*:

$$V_2(K,A) =$$

$$u'(c)[-\cancel{\widehat{K'}_2} - \phi(K'-K)\cancel{\widehat{K'}_2} - \cancel{\widehat{A'}_2} + (1+r^*)]$$

$$+\beta[V_1\cancel{\widehat{K'}_2} + V_2\cancel{\widehat{A'}_2}]$$

- Apply envelope theorem:

$$\Rightarrow V_2(K,A) = u'(c)(1+r^*)$$

Optimality conditions

- Euler equations:

$$(I)\quad \widehat{K'}:\quad u'(c)[\overbrace{1+\phi(K'-K)}^{q_t}]$$

$$= \beta u'(c)[\underbrace{f'(K')}_{d_{t+1}} + \underbrace{1+\phi(K''-K')}_{q_{t+1}}]$$

$$(II)\quad \widehat{A'}:\quad u'(c) = \beta u'(c)(1+r^*)$$

- Intertemporal budget constraint & NPG condition:

$$(III)\quad \sum_{t=0}^{\infty}\frac{c_t}{(1+r^*)^t} = \sum_{t=0}^{\infty}\frac{\tilde{y}_t}{(1+r^*)^t} + A_0(1+r^*) \qquad \lim_{t\to\infty}\frac{A_t}{(1+r^*)^t} = 0$$

$$\tilde{y}_t \equiv f(K_t) - (K_{t+1}-K_t)(1+\frac{\phi}{2}(K_{t+1}-K_t))$$

Recursive equilibrium and two properties

- A *recursive competitive equilibrium* is defined by optimal decision rules $\widehat{K'}, \widehat{A'}$, and associated value function *V(.)* that solve the recursive planner's problem (i.e. that satisfy *(I)*-*(III)*)
- Two key properties
  1. Planner's problem produces same allocations as a decentralized comp. equilibrium
  2. Fisherian separation of savings investment

$(I),(II) \Rightarrow$

$$(IV) \quad \frac{d' + q'}{q} \equiv \frac{f'(K') + 1 + \phi(K'' - K')}{1 + \phi(K' - K)} = 1 + r^*$$

Closed-form solutions and dynamics

- Consumption: Use standard stationarity condition

$$\beta(1+r^*) = 1$$

$$\Rightarrow (II) \text{ implies } \; c_t = \bar{c} \;\; \forall \; t = 0, \ldots, \infty$$

$$\Rightarrow (III) \text{ implies } \; \bar{c} = (1-\beta)\left[\sum_{t=0}^{\infty} \beta^t \tilde{y}_t + A_0(1+r^*)\right]$$

$$(V) \qquad \bar{c} = (1-\beta)\left[\sum_{t=0}^{\infty} \beta^t \tilde{y}_t\right] + A_0 r^*$$

Closed-form solutions and dynamics

- Unique steady-state capital stock:

$$(IV) \quad \Rightarrow \quad f'(K_{ss}) = r^*$$

- Optimal investment rule is a standard partial adjustment rule (e.g. Sargent (1979))

$$(VI) \quad \widehat{K'} \cong \lambda K_{ss} + (1-\lambda)K \Rightarrow \widehat{K'} - K \cong \lambda(K_{ss} - K)$$

or from Sargent: $\widehat{K'} - K = (\phi r^*)^{-1}[f'(K) - f'(K_{ss})]$
where λ solves a second-order difference eqn.
and depends on parameters of f(.) and on ϕ and r^*

Closed-form solutions and dynamics

- Without capital adjustment costs, model lacks transitional dynamics and jumps to K_{ss} .
  - From (*IV*), $\phi = 0 \Rightarrow f'(K') = r^*$
- Solution for "net output" follows from (VI):

$$(VII) \quad \hat{\hat{y}}(K) = f(K) - \lambda(K_{ss} - K)[1 + \frac{\phi}{2}\lambda(K_{ss} - K)]$$

- Plug (VII) into (V) to obtain closed-form solution for consumption:

$$(VIII) \quad \hat{c} = (1-\beta)\left[\sum_{t=0}^{\infty} \beta^t \hat{\hat{y}}_t(K_t)\right] + A_0 r^*$$

Closed-form solutions and dynamics

- NFA, current account, and trade balance follow from resource constraint:

$$(IX)\quad \widehat{A'} = f(K) - \left(\widehat{K'} - K\right)\left[1 + \frac{\phi}{2}\left(\widehat{K'} - K\right)\right] - \hat{\hat{c}} + A(1 + r^*)$$

- Stationary equilibria of NFA:

$$A_{ss} = f(K_{ss}) - \hat{\hat{c}} + A_{ss}(1 + r^*)$$

$$A_{ss} = -\left[\frac{f(K_{ss}) - \hat{\hat{c}}}{r^*}\right] = -\frac{nx}{r^*}$$

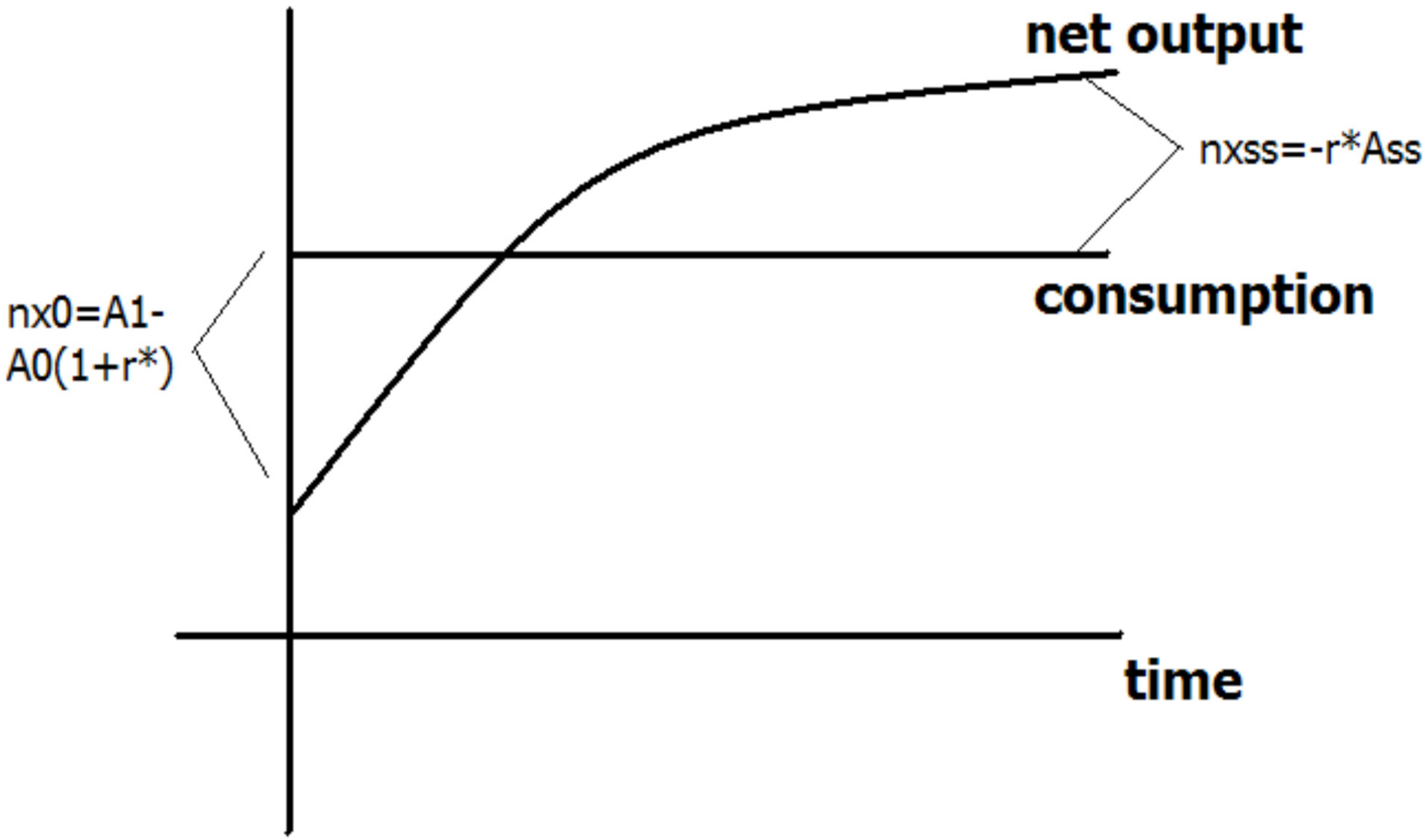
Three additional key properties

3. K_{ss} is unique and independent of initial conditions, but $\hat{\hat{c}}$, $\widehat{A'}$ and A_{ss} depend on A_0

4. Given A_0, $\widehat{A'}$ features well-defined dynamics around unique steady-state A_{ss}
   - But steady-state Euler equations do not produce a closed-form solution for A_{ss}. Instead, we solved for it by solving the model's dynamics

5. Linear approx. methods around det. steady states are not useful for solving these models
   - Even temporary shocks have permanent effects
   - But shooting methods do work

Graphical illustration
(and a gains from trade argument)



Effects of Shocks

1. Additive (e.g. government expenditures)
   - Permanent: No effect on debt or capital dynamics, equal effects on $\hat{\hat{y}}$ and $\hat{\hat{c}}$.
   - Transitory: No effect on investment dynamics but affects debt dynamics through the effect on permanent income as well as A_{ss}.

2. Multiplicative (e.g. productivity, terms of trade)
   - Permanent or transitory: Affect both investment and debt dynamics and A_{ss}, but only permanent shocks affect K_{ss}.

WORKHORSE MODEL 3: STOCHASTIC MODEL WITH INCOMPLETE MARKETS

Uncertainty and Incomplete Markets

- NFA are non-state-contingent, one-period “real” bonds chosen from a discrete grid:

$$B = [b_1 < b_2 < \ldots < b_z]$$

- Income and world interest rate are exogenous

- Income follows exogenous Markov process with “*m*” states and known transition prob. matrix:

$$\bar{y} = [y_1 < y_2 < \cdots < y_m] \qquad P(y_i, y_j)$$

- Asset markets are incomplete: *B* cannot provide full insurance against income fluctuations

Planner's Problem

- Choose $\{b_{t+1}\}_{t=0}^{\infty}$ so as to

$$max \quad E_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

s.t.

$$c_t = y_t - b_{t+1} + b_t R$$

$$b_{t+1} \in B \qquad P(y_t, y_{t+1}) \; known$$

$$(b_0, y_0) \;\; given,$$

…looks very similar to Model 1, but it has very different implications!

Aiyagari's natural debt limit

- *u(.)* is twice differentiable, concave and satisfies the Inada condition:

$$\lim_{c \downarrow 0} u'(c) = \infty$$

- Implies that consumption must be positive at all times, and hence the budget constraint implies:

$$b' \geq -\left[\frac{y_{min}}{R-1}\right]$$

  – Debt cannot exceed the annuity value of the lowest income realization, otherwise the agent is exposed to the risk of zero consumption with positive probability
  – Already highlights "global" nature of decision-making in incomplete markets models

Recursive planner's problem

$$V(b_n, y_i) = \max_{b' \in B} \left\{ u(y - b' + b_n R) + \beta \sum_{j=1} P(y_i, y_j) V(b', y_j) \right\}$$

for each of the *mxz* pairs (b_n, y_i).

- The solution is characterized by:
  1. Decision rule $b' = g(b, y)$
  2. Value function $V(b_n, y_i)$
  3. Unconditional stationary distribution of *(b,y)*

$$\lambda(b, y) \;\; = Prob(b_t = b, y_t = y)$$

Law of motion of conditional probabilities

- $P(y_t, y_{t+1})$ and $b' = g(b, y)$ induce a law of motion for conditional transition probabilities from date-t states *(b,y)* to date-t+1 states *(b',y')*:

$$\lambda_{t+1}(b', y') = Prob(b_{t+1} = b', y_{t+1} = y_t)$$

$$= \sum_{b_t \in B} \sum_{y_t \in \bar{y}} Prob(b_{t+1} = b' | b_t = b, y_t = y) \times$$

$$Prob(y_{t+1} = y' | y_t = y) \times Prob(b_t = b, y_t = y)$$

Equilibrium Transition Probabilities

- But since $b' = g(b,y)$ is a unique recursive function of *(b,y)*, the law of motion becomes:

$$\lambda_{t+1}(b',y') = \sum_b \sum_y \lambda_t(b,y)\,Prob(y_{t+1} = y' | y_t = y)\Upsilon(b',b,y)$$

$$\Upsilon(b',b,y) = \begin{cases} 1 \Leftrightarrow b' = g(b,y) \\ 0 \qquad otherwise \end{cases}$$

  – Which can be rewritten as:

$$\lambda_{t+1}(b',y') = \sum_y \sum_{\{b:b'=g(b,y)\}} \lambda_t(b,y)\,P(y,y')$$

Stationary Distribution of Net Foreign Assets

- The stochastic steady state is a joint stationary distribution of NFA and income, which is the fixed point $\lambda(b, y)$ of the law of motion

$$\lambda_{t+1}(b', y') = \sum_{y} \sum_{\{b : b' = g(b,y)\}} \lambda_t(b, y)\, P(y, y')$$

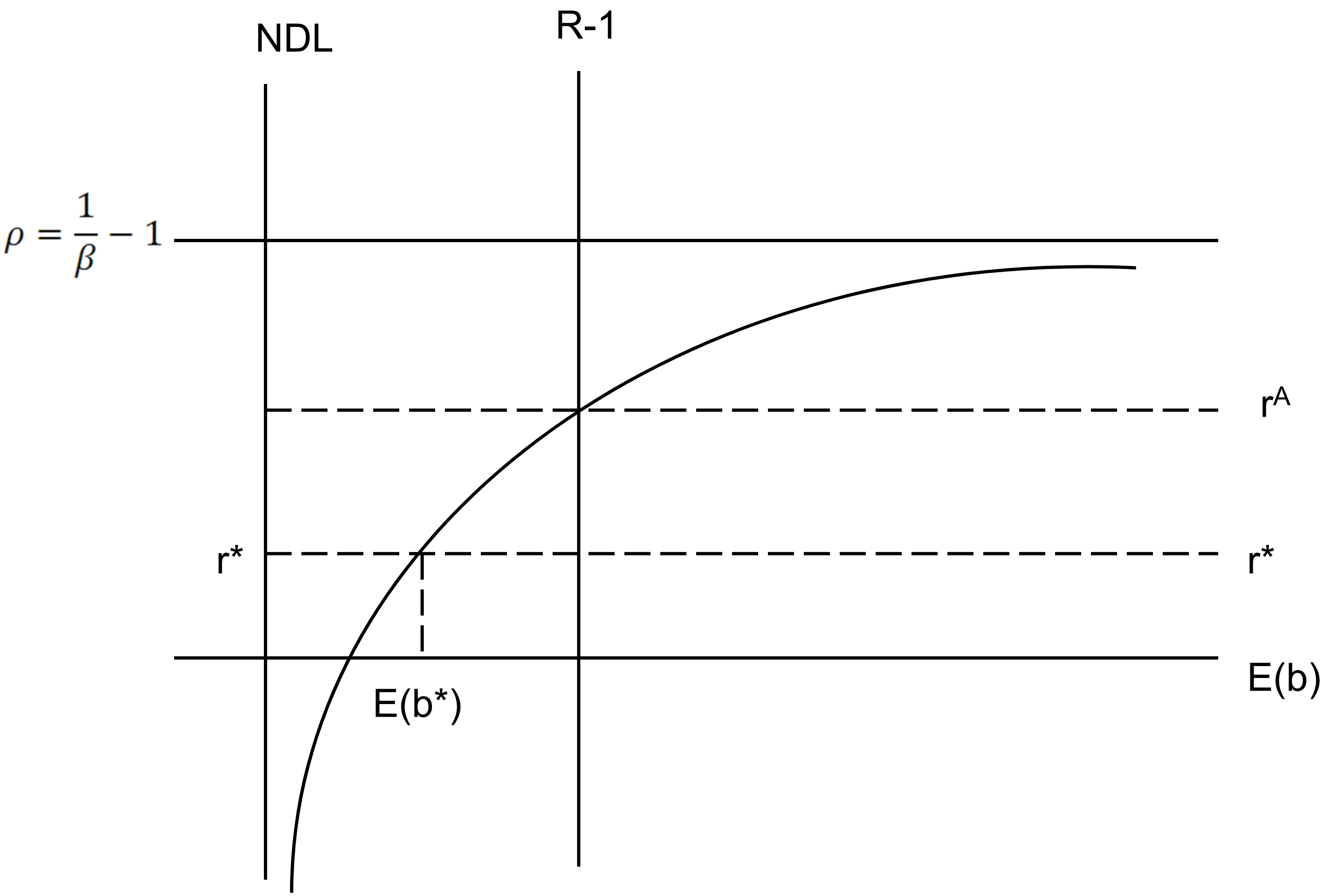
- Methods to solve for $\lambda(b, y)$:
  - Iterating to convergence in the law of motion
  - Computing Eigen values of the *(mxz) x (mxz)* state transition probability matrix
  - Powering to convergence state transition prob. matrix

Precautionary savings (and the failure of the standard stationarity condition)

- Standard stationarity assumption $\beta(1+r^*)=1$ fails
  - Euler eq. implies "constant consumption," but income is always stochastic and NFA is non-state-contingent.
  - Formally, marginal utility follows a Supermartingale process, and Supermartingales converge, but to converge in this case requires $b' \to \infty$

- Agents self insure, build precautionary savings
  - If $\beta R < 1$, force pushing to borrow and force pushing for prec. savings support stationary distribution
  - Natural Debt Limit imposes lower bound on NFA
  - But the deterministic st. state is always the debt limit!

Graphical illustration



Remarks about incomplete markets

- Solving these models generally requires global methods that can track dynamics of wealth dist.
- Certainty equivalence of DSGE models solved w. 1st order approx. fails (e.g. higher variance or persistence of shocks increases average NFA)
- Higher order approx. deviates from certainty equivalence but differs from global solution
- Prec. savings also affects portfolio structure (wealthier agents tolerate more risk, hold larger shares of risky assets at lower premia)

A PRIMER ON GLOBAL SOLUTION METHODS FOR SOLVING INCOMPLETE MARKETS MODELS

Consider again Model 3 planner's problem

$$V(b_n, y_i) = \max_{b' \in B} \left\{ u(y - b' + b_n R) + \beta \sum_{j=1} P(y_i, y_j) V(b', y_j) \right\}$$

for each of the *mxz* pairs (b_n, y_i).

- We need to solve for three objects:
  1. Decision rule $b' = g(b, y)$
  2. Value function $V(b_n, y_i)$
  3. Unconditional stationary distribution of *(b,y)*

$$\lambda(b, y) = Prob(b_t = b, y_t = y)$$

Solution methods

- Classic: discrete state space with "on the nodes" decision rules
  1) Discrete value function iteration
  2) Howard's improvement (policy function) method
  3) Hybrid method of 1) and 2)
  -- There is also "guess-and-verify" the value function
- Advanced: approx. continuous decision rules
  1) VFI and/or PFI with interpolated decision rules
  2) Time-elimination or Euler equation methods
  3) Approximation methods (polynomial, spline, parameterized expectations)
  4) Occasionally binding constraints

Value function iteration method

1) Set initial VF (a "good guess" is best):

$$V^0(b,y) = 0 \quad \forall \quad (b,y) \in B \times Y$$

2) Perform first *maximization step*:

– For each possible pair of "initial states" $(b_n, y_i) \in B \times Y$ find the optimal choice of bonds that solves:

$$V^1(b_n, y_i) = \max_{b' \in B} \left\{ u(y_i - b' + b_n R) + 0 \right\}$$

– By construction, since initial VF is zero in all the domain, the decision rule is:

$$b^0(b,y) = b_1 \ \ (the\ lower\ bound\ of\ B)\ \forall (b,y) \in B \times Y$$

3) Set new value function using result from 2)

$$V^1(b_n, y_i) = u(y_i - b_1 + b_n R) \; for\, each\, (b_n, y_i) \in B \times Y$$

4) Perform 2nd maximization step by solving:

$$V^2(b_n, y_i) = \max_{b' \in B} \left\{ u(y_i - b' + b_n R) + \beta \sum_{j=1}^{m} P(y_i, y_j) V^1(b', y_j) \right\}$$

$$which\ yields\ \ b^{1\prime}(b_n, y_i)\ for\ each\, (b_n, y_i) \in B \times Y$$

5) Continue iterating by solving:

$$V^{j+1}(b_n, y_i) = \max_{b' \in B} \left\{ u(y_i - b' + b_n R) + \beta \sum_{j=1}^{m} P(y_i, y_j) V^j(b', y_j) \right\}$$

$$which\ yields\ b^{j\prime}(b_n, y_i)\ for\, each\, (b_n, y_i) \in B \times Y$$

6) After each iteration check convergence criteria:

– PF convergence (much faster!):

$$\max \mid b^{j+1\prime}(b,y) - b^{j\prime}(b,y) \mid \leq \varepsilon^{b} \; \forall (b,y) \in B \times Y$$

– VF convergence (slower, but needed for welfare/policy analysis)

$$\max \mid V^{j+1\prime}(b,y) - V^{j\prime}(b,y) \mid \leq \varepsilon^{V} \; \forall (b,y) \in B \times Y$$

7) If 6) fails, return to 5) if not go to 8)

8) Find ergodic dist. by iterating to convergence:

$$\lambda_{t+1}(b', y') = \sum_{y} \sum_{\{b : b' = g(b,y)\}} \lambda_t(b,y) \, P(y,y')$$

– Each iteration is a date-t transitional distribution

– If dist. is invariant, any initial cond. should work

9) Use ergodic dist., decision rule and resource constraint to compute unconditional moments:

$$E[b] = \sum_{(b,y)\in B\times Y} \lambda(b,y)b \qquad E[c] = \sum_{(b,y)\in B\times Y} \lambda(b,y)(y - b'(b,y) + Rb)$$

$$\mathrm{var}[b] = \sum_{(b,y)\in B\times Y} \lambda(b,y)(b - E[b])^2 \qquad \mathrm{var}[c] = \sum_{(b,y)\in B\times Y} \lambda(b,y)(y - b'(b,y) + Rb - E[c])^2$$

10) Use trans. distributions to create forecasting functions (akin to impulse response functions)

$$E_t[b] = \sum_{(b,y)\in B\times Y} \lambda_t(b,y)b \qquad E_t[c] = \sum_{(b,y)\in B\times Y} \lambda_t(b,y)(y - b'(b,y) + Rb)$$

11) Solve under alternative parameterizations for sensitivity, welfare & policy experiments.

Policy function iteration method

1) Value-computation step: choose a feasible decision rule and compute its "value" by iterating to convergence on this recursive sum:

$$b^{j\prime}(b,y) \rightarrow V^{j,t+1}(b,y) = \left\{ u(y - b^{j\prime}(b,y) + bR) + \beta \sum_{j=1}^{m} P(y,y_j) V^{j,t}(b^{j\prime}(b,y), y_j) \right\}$$

$which\ converges\ to\ V^j(b,y)$

2) Find a new candidate decision rule by performing a max. step once using the above VF:

$$\max_{b' \in B} \left\{ u(y - b' + bR) + \beta \sum_{j=1}^{m} P(y,y_j) V^j(b', y_j) \right\}$$

$$which\ yields\ b^{j+1\prime}(b_n, y_i)\ for\, each\, (b_n, y_i) \in B \times Y$$

3) Check for convergence using same criteria as before. If it fails, return to 1) using result from 2) as new candidate rule, if not go to 4)

4) Once convergence is attained, one can proceed as in VFI method
   – Compute ergodic dist., unconditional moments
   – Compute forecasting functions using transition dists.
   – Do sensitivity, welfare, policy experiments

• Hybrid method: Split iterations so that every T iterations, J<T are done using a policy function and T-J perform max. step and VFI (much faster than either one alone!)

A few words about advanced methods

- Decision rules in classic methods are rough approximations of true solutions, because they are set to values <u>on</u> nodes of discrete grids
- Advanced methods use "approximately continuous" decision rules by interpolating or approximating value functions using various tools
  - If the problem allows it, approx. tools are applied on the FOC of the Bellman equation to solve dec. rule
- If single Bellman eq. cannot fully represent equilibrium, we need more steps to clear markets
  - Compare workhorse SOE model w. Hugget's heterogeneous agents closed-economy model

An example: One good model from Durdu, Mendoza & Terrones (2008)

- SOE with exogenous Markov endowment:

$$V(b,\varepsilon) = \max_{b'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \exp(-v(c)) E[V(b',\varepsilon')] \right\}$$

$$s.t. \qquad c = \varepsilon y - b' + bR + A$$

$$b_{t+1} \geq \phi \geq -\min(\varepsilon_t y + A)/r$$

- Allows for 2 formulations of rate of time pref.:
  1. Uzawa-Epstein endogenous rate of time preference
  2. Bewley-Aiyagari-Hugget setup with $\beta R < 1$

$$v(c\ ) = \rho^{UE} \ln(1+c) \;\; or \;\; \ln(1+\rho^{BAH})$$

Calibration

- Discrete state space:

$$(b, b') \in B = \{b_1 < b_2 < \ldots < b_n\} \qquad n{=}1000$$

$$\varepsilon \in \mathrm{E} = \left\{\varepsilon_1 < \varepsilon_2 < \ldots < \varepsilon_j\right\} \qquad \pi(\varepsilon_{t+1} \mid \varepsilon_t)$$

- Income process (set to Mexico's detrended GDP)

$$y_t = \rho_y y_{t-1} + e_t \quad \sigma_y = 3.301\% \quad \rho_y = 0.597$$

$$\sigma_e = \sqrt{\sigma_y^2(1-\rho_y^2)} = 2.648 \text{ percent}$$

  – Discretized using Tauchen-Hussey quadrature method with j=5 (yields process with 3.28% s.d. and AR=0.55)
  – T-H code available at: http://public.econ.duke.edu/~get/
  – Can also use canonical Markov chains (e.g. “simple persistence” rule) to discretize time-series processes

- E[y] = 1 for simplicity (variables are GDP ratios)
- E[b] = -0.44 Mexico's average NFA/GDP 1985-2004 in Lane & Milesi Ferretti (06)
- E[c] = 69.2 Mexico's average C/GDP 1965-2005
- R = 1.059 Mexico's country real interest rate from Uribe and Yue (06)
- It follows that $A = y + b(R\text{-}1)\text{-}c = 0.282.$
- Discount factors and rates of time preference:
  - UE: $\rho^{UE} = \ln(R)/\ln(1+c) = 0.109 \quad (1+c)^{-0.109} = 0.944$
  - BAH: $\rho^{BAH} = 0.064$ set by searching for values of ad-hoc debt limit & discount factor that match E[b]=-0.44 and sd(c)=3.28% (ϕ =-0.51 β=0.94)

Calibrated state space

- Vector of income realizations

1	-0.075642
2	-0.035892
3	0.0
4	0.035892
5	0.075642

- Transition prob. matrix of income shocks

	COL 1	COL 2	COL 3	COL 4	COL 5
ROW 1	0.34500	0.52508	0.12475	0.00513915	2.0099D-05
ROW 2	0.081986	0.47956	0.38426	0.053385	0.00080242
ROW 3	0.011257	0.22208	0.53333	0.22208	0.011257
ROW 4	0.00080242	0.053385	0.38426	0.47956	0.081986
ROW 5	2.0099D-05	0.00513915	0.12475	0.52508	0.34500

- Grid of bonds: spacing=0.001514, nodes=1000, lower bound=-0.5123

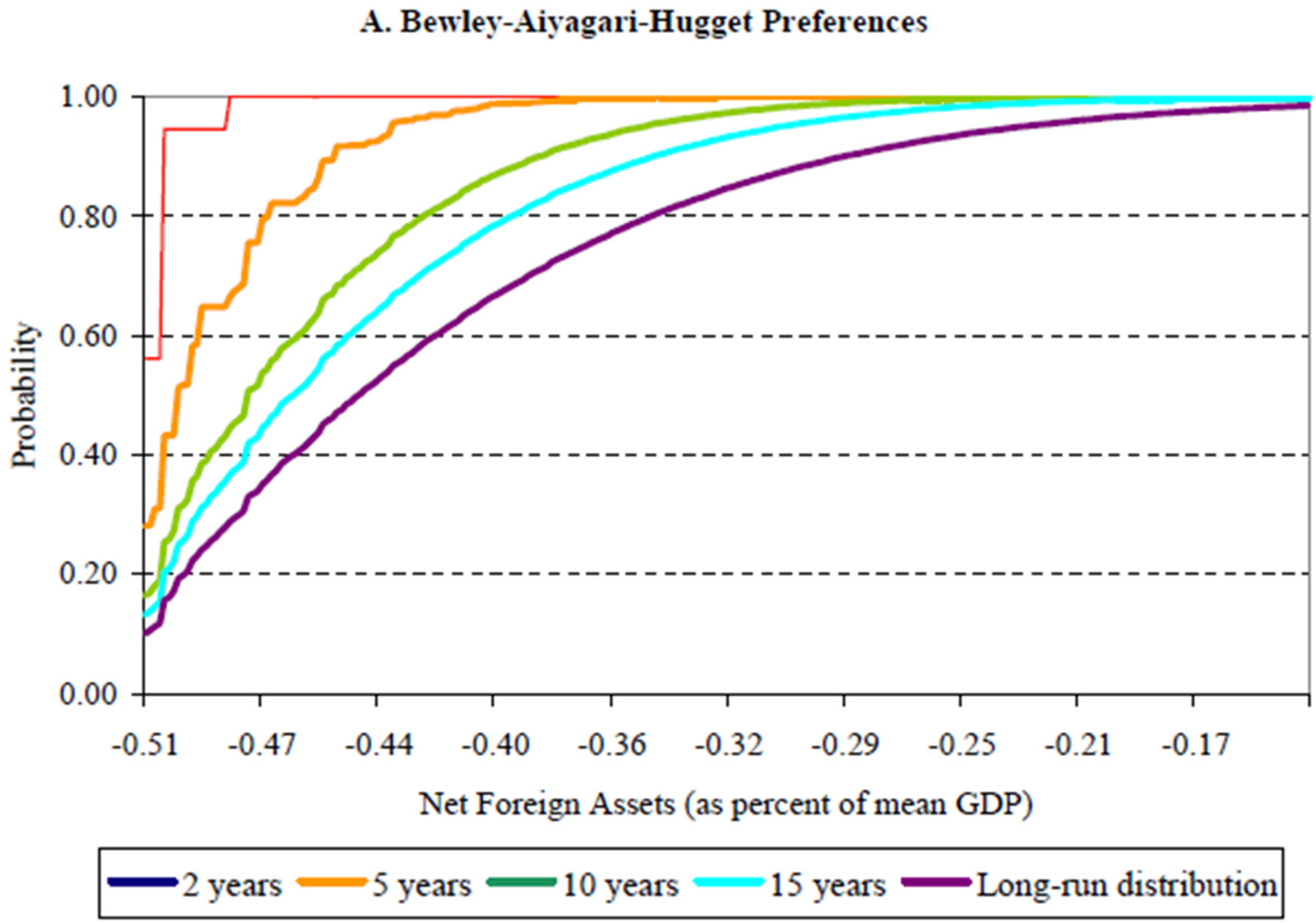
Calibrated parameter values

ρ^{BAH}	Rate of time preference in the BAH setup	0.064
ρ^{UE}	Rate of time preference elasticity in the UE setup	0.109
γ	Coefficient of relative risk aversion	2.000
ϕ	Ad-hoc debt limit	-0.510
R	Gross world interest rate	1.059
y	Mean output	1.000
c	Consumption-output ratio	0.692
b	Net foreign assets-output ratio	-0.440
σ_e	Standard deviation of output innovations	0.026
ρ	Autocorrelation of output	0.597
A	Lump-sum absorption	0.282

Computer codes

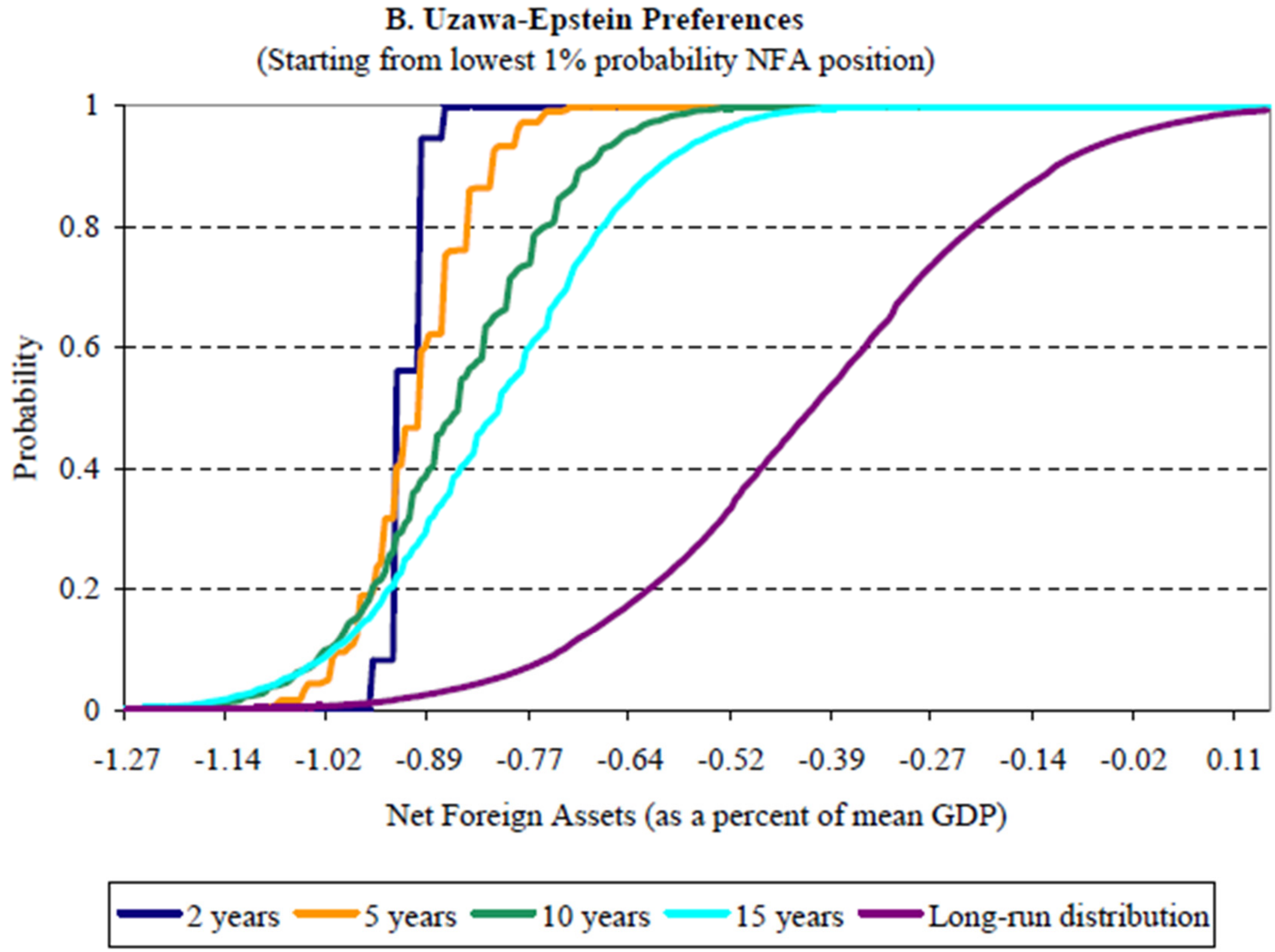
- Due to intensive looping, number-crunching, and nonlinearities, it is best to program global methods in core languages (Phyton, Java, C++, Fortran) instead of software (Matlab, GAUSS)
- Fortran codes (VFI, Tauchen) available from me (free compiler http://gcc.gnu.org/fortran/)
- Matlab codes available in Hamann Toolbox (see also code from Ljungqvist & Sargent section 17.7 at http://dge.repec.org/codes/sargent/bewley/)
- http://ideas.repec.org/s/dge/qmrbcd.html has code for many applications in diff. languages

Transitional and stationary distributions



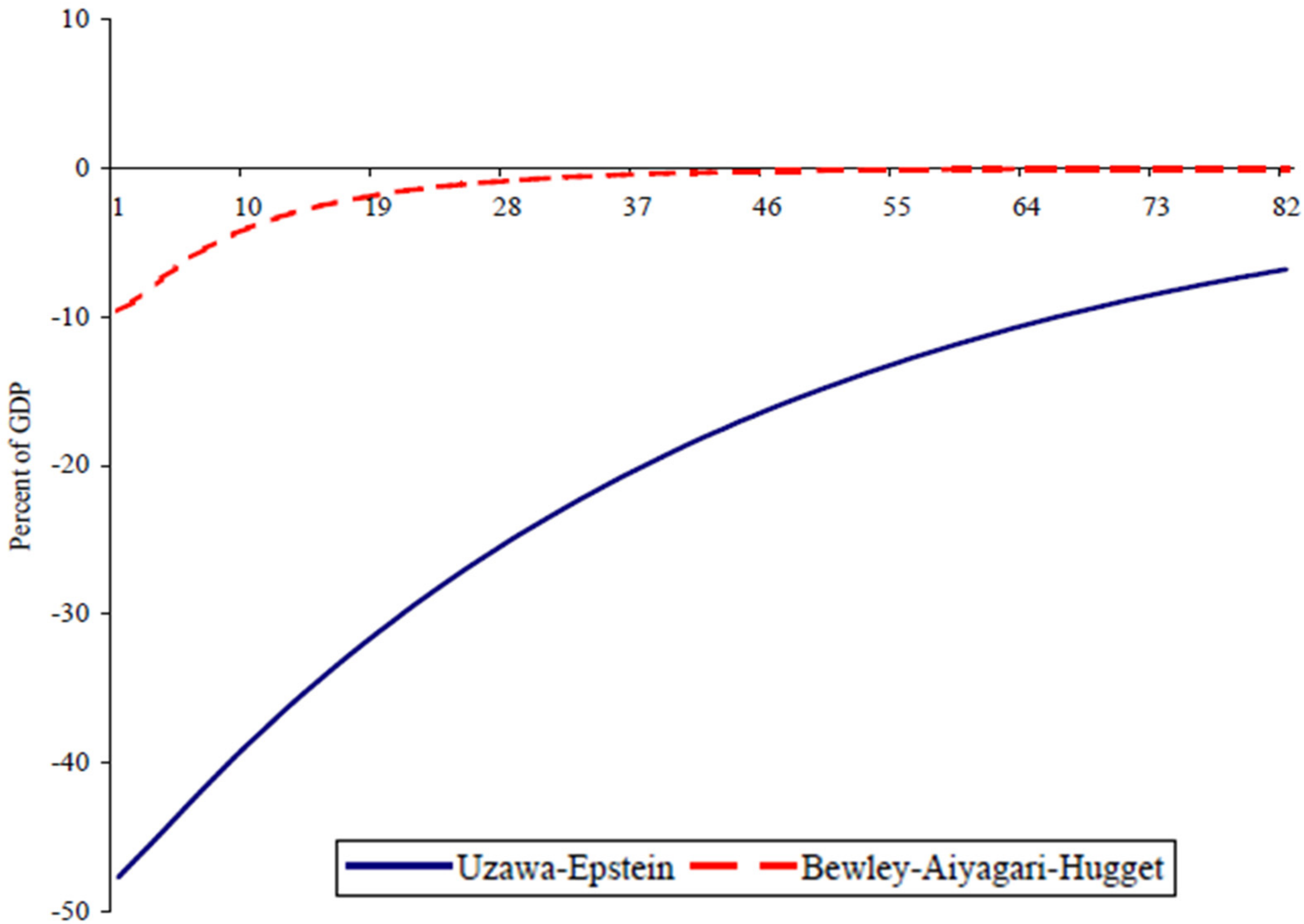
Note: Initial conditions are lowest (b,y) with positive long-run probability

Transitional and stationary distributions



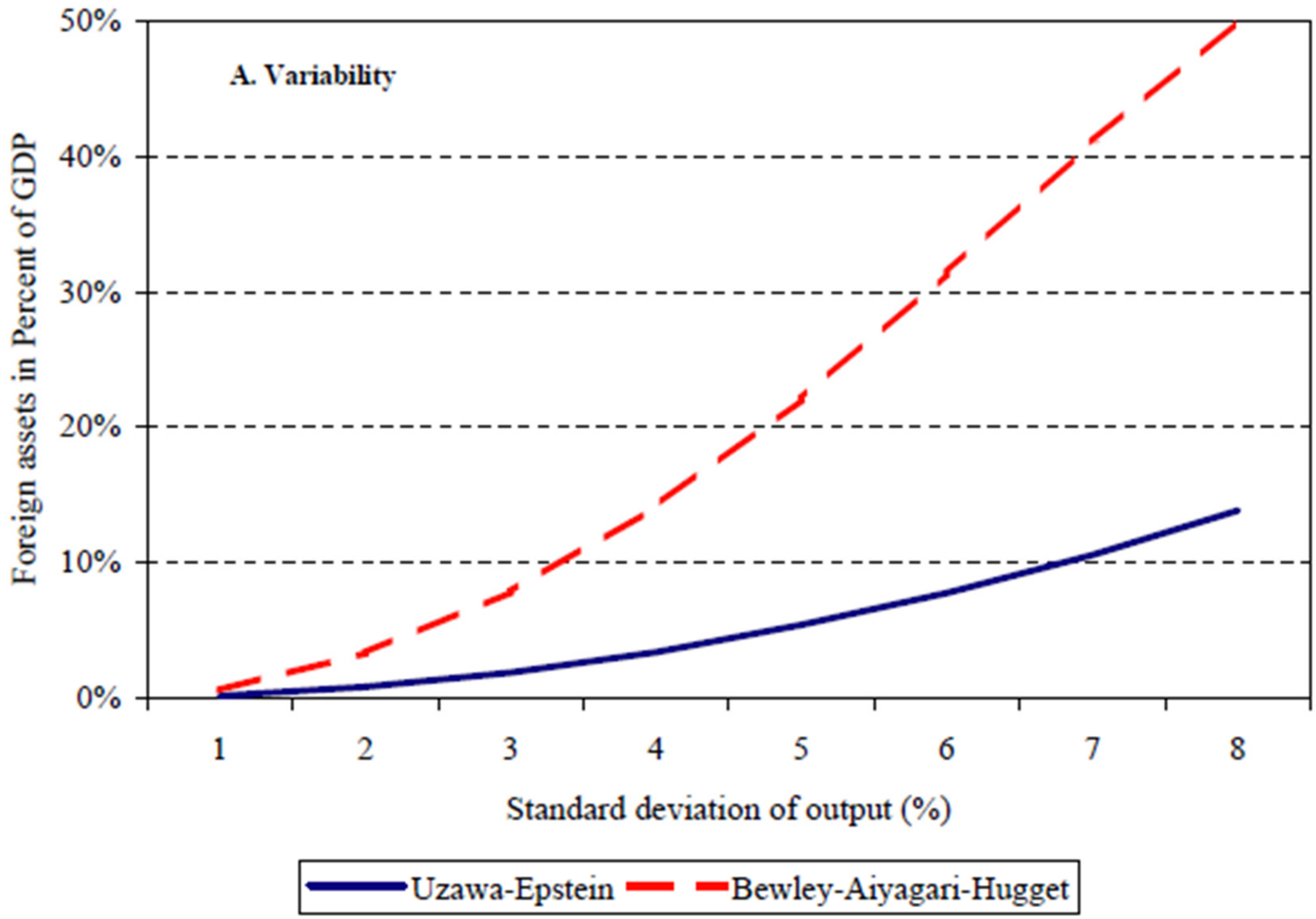
Note: Initial conditions are lowest (b,y) with positive long-run probability

Transitional dynamics of NFA



Note: Dynamics show forecasting function starting from lowest positive prob. B and neutral income shock and plotted as differences relative to long-run averages.

Effects of income variability on precautionary NFA demand



Unconditional moments

	Baseline		Auto Corr 0.7		Std Dev. 5%		Std Dev. 2.5%		Risk Aver. 5.0	
	UE	BAH	UE	BAH	UE	BAH	UE	BAH	UE	BAH
Precautionary savings [1/]	0.02	0.10	0.04	0.12	0.05	0.22	0.01	0.05	0.10	0.24
Means										
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.69	0.69	0.69	0.70	0.70	0.70	0.69	0.69	0.70	0.70
Foreign assets	-0.42	-0.42	-0.41	-0.39	-0.39	-0.30	-0.43	-0.46	-0.34	-0.28
Trade balance [2/]	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.02	0.02
Discount factor	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94
Coefficients of variation (in percent)										
Output	3.28	3.28	3.63	3.63	4.97	4.97	2.49	2.49	3.28	3.28
Consumption	3.13	3.26	3.92	3.92	4.72	4.66	2.38	2.59	4.11	3.11
Foreign assets	24.41	10.11	29.73	13.39	36.97	20.28	18.52	6.33	40.92	20.10
Current account [2/]	2.68	2.02	2.77	2.08	4.08	3.42	2.03	1.40	2.81	2.48
Trade balance [2/]	3.04	2.11	3.27	2.23	4.62	3.66	2.30	1.44	3.72	2.78
Discount factor	0.14	0.00	0.18	0.00	0.21	0.00	0.11	0.00	0.18	0.00

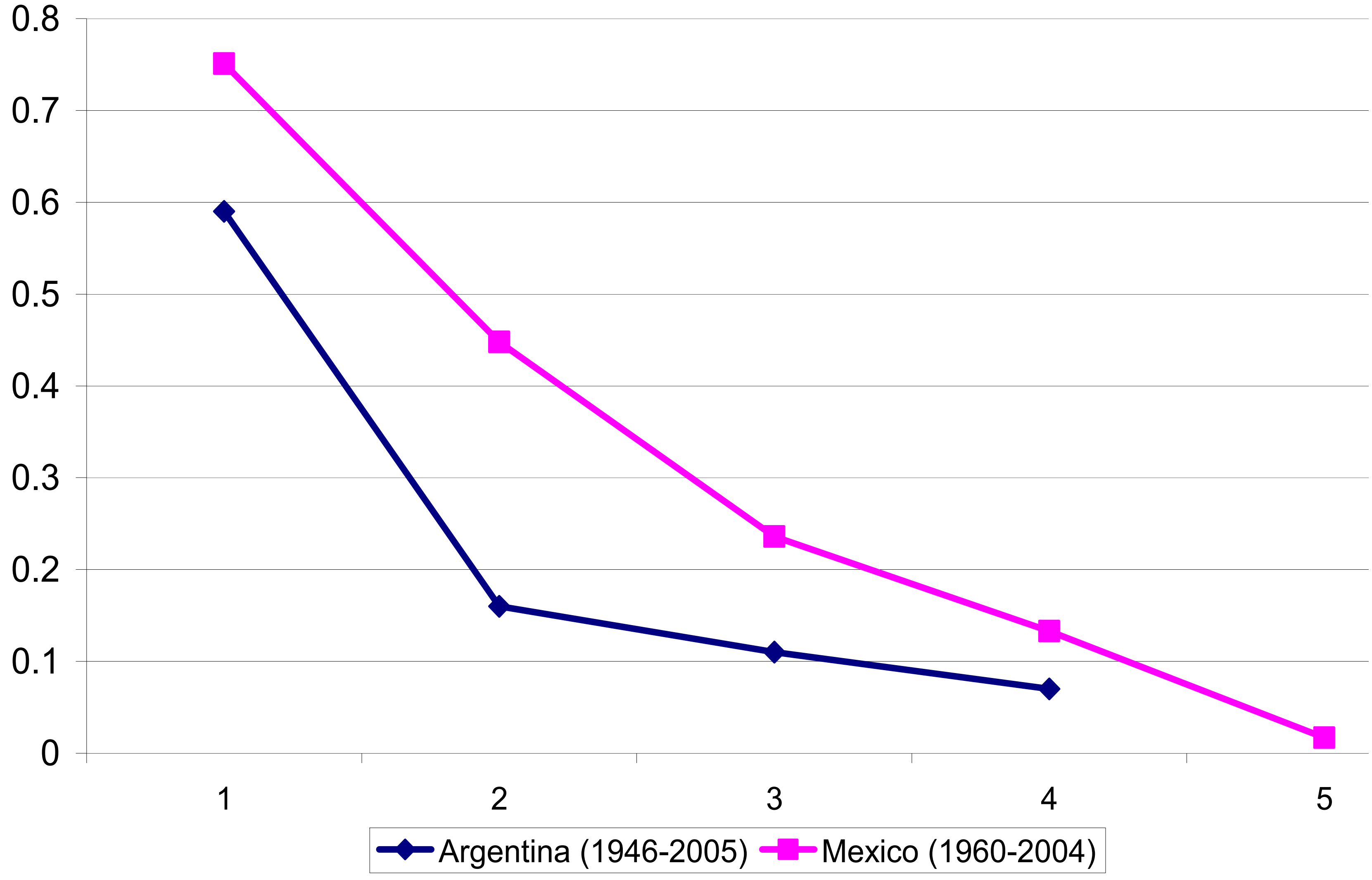
	Baseline		Auto Corr 0.7		Std Dev. 5%		Std Dev. 2.5%		Risk Aver. 5.0	
	UE	BAH	UE	BAH	UE	BAH	UE	BAH	UE	BAH
Normalized coefficients of variation (relative to output)										
Consumption	0.95	0.99	1.08	1.08	0.95	0.94	0.96	1.04	1.25	0.95
Foreign assets	7.43	3.08	8.19	3.69	7.43	4.08	7.44	2.55	12.46	6.12
Current account [2/]	0.82	0.62	0.76	0.57	0.82	0.69	0.82	0.56	0.86	0.75
Trade balance [2/]	0.92	0.64	0.90	0.61	0.93	0.74	0.93	0.58	1.13	0.85
Discount factor	0.04	0.00	0.05	0.00	0.04	0.00	0.04	0.00	0.06	0.00
Output correlations										
Consumption	0.42	0.75	0.48	0.78	0.42	0.67	0.42	0.81	0.26	0.54
Foreign assets	0.32	0.56	0.34	0.53	0.32	0.44	0.32	0.62	0.19	0.33
Current account [2/]	0.97	0.85	0.97	0.83	0.97	0.89	0.97	0.81	0.99	0.93
Trade balance [2/]	0.76	0.70	0.68	0.63	0.76	0.73	0.76	0.67	0.66	0.74
Discount factor	-0.42	0.00	-0.48	0.00	-0.42	0.00	-0.42	0.00	-0.26	0.00
Autocorrelations										
Output	0.59	0.59	0.69	0.69	0.59	0.59	0.59	0.59	0.59	0.59
Consumption	0.97	0.84	0.97	0.88	0.97	0.88	0.97	0.81	0.99	0.93
Foreign assets	0.99	0.96	0.99	0.98	0.99	0.98	0.99	0.94	1.00	0.99
Current account [2/]	0.57	0.51	0.67	0.62	0.57	0.54	0.57	0.49	0.59	0.56
Trade balance [2/]	0.67	0.55	0.76	0.67	0.67	0.59	0.67	0.52	0.76	0.64
Discount factor	0.98	0.00	0.98	0.00	0.98	0.00	0.97	0.00	0.99	0.00

LIMITATIONS OF PERTURBATION METHODS
(“CLOSING SMALL OPEN ECONOMY MODELS” AND THE IMPORTANCE OF INCOMPLETE MARKETS)

Be careful how we "close" SOE models

- Schmitt-Grohe & Uribe (JIE 03) proposed three ad-hoc ways to induce stationarity and apply perturbation methods:
  1. Interest rate function *r(b)*
  2. Resource cost of holding assets *h(b)*
  3. Rate of time pref. depends on "aggregate" C
- They showed these are about equivalent in an RBC moment-matching exercise
- Using this approach, Uribe et al (AER 10) found that RBC-SOE model cannot explain AR behavior of net exports

Autocorrelation functions of TB/Y



Dangers of ad-hoc stationarity methods

- In the data NX is AR(1) but in RBC-SOE model solved with r(b) it is almost a unit root process.
- …but this is ***not*** a property of the "true" solution, it is an implication of the r(b) function
- To show it use this:
  1. Definition of net exports: $tb_t = b_{t+1} - b_t R^*$
  2. Assume AR(1) process for NFA: $b_{t+1} = \rho b_t + \varepsilon_{t+1}$ and notice ad-hoc method imposes the AR coeff when specifying r(b)—Uribe et al. assume it is very close to 1, to avoid r(b) being "essential"

Dangers of ad-hoc stationarity methods

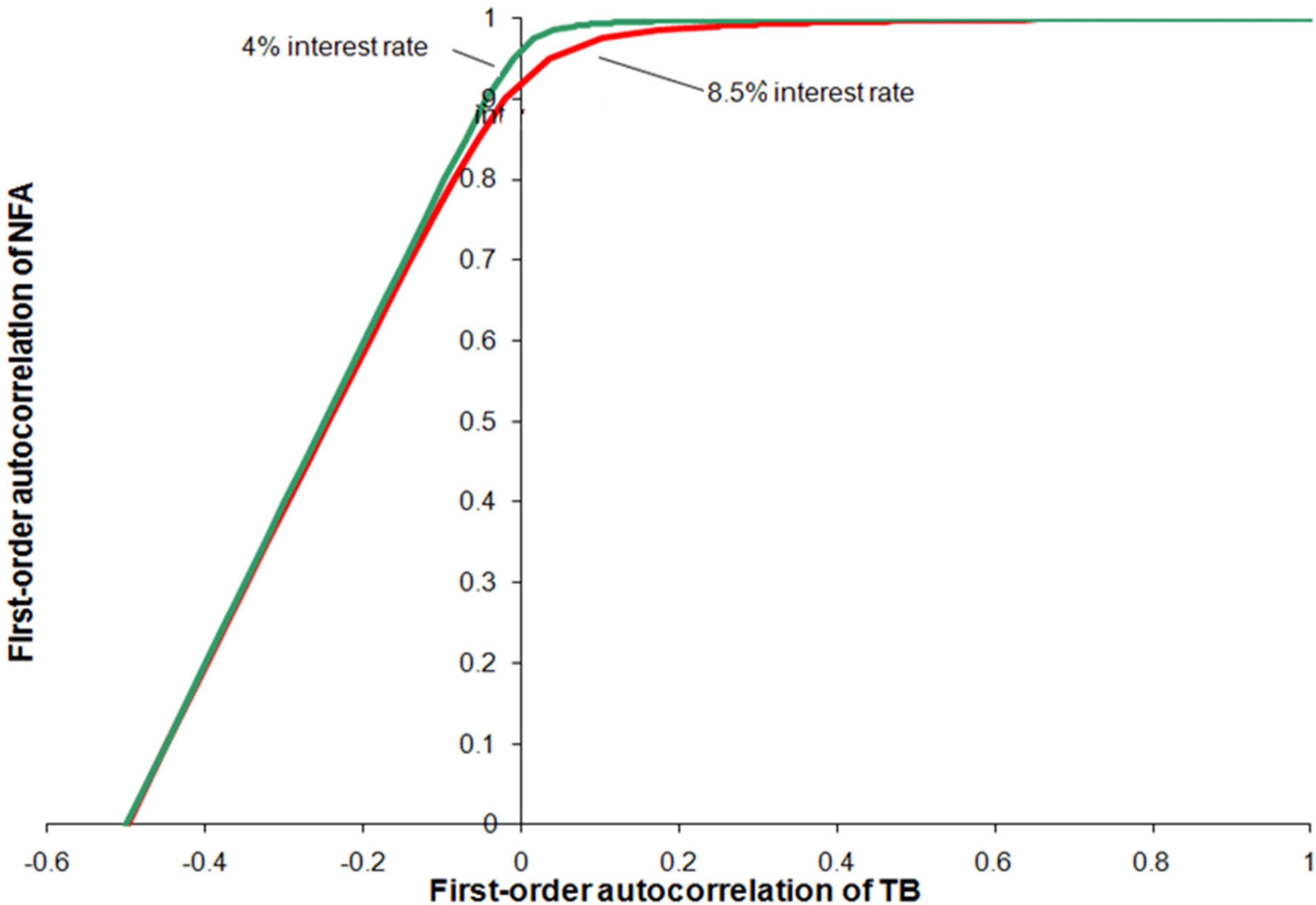
- Combine 1 & 2, solve for AR(1) of net exports:

$$\rho(nx) = \frac{q^2\rho + \rho - q - q\rho^2}{1 + q^2 - 2q\rho}$$

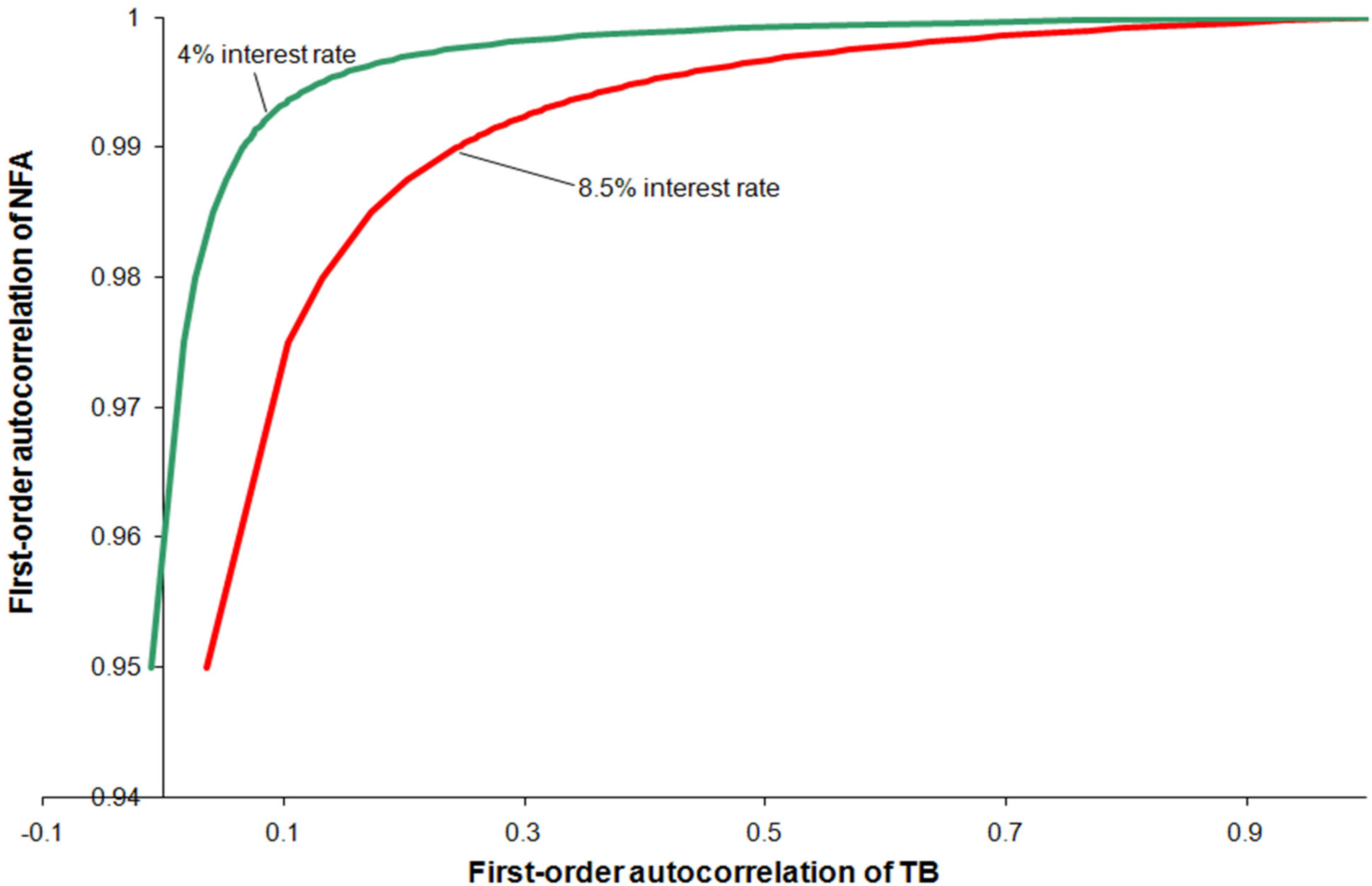
  where $q = 1/R^*$

- AR of NX is a “highly” nonlinear function of AR of NFA, so we need “exact” solution for the latter in order to derive correct results about the former
  - Changing AR of NFA from 0.95 to 0.999 changes the AR of NX from near zero to 0.999!!
  - Knowing true solution of NFA dynamics is critical

Autocorrelations of NFA and NX



Autocorrelations of NFA and NX



Other dangers of ad-hoc approach

- Generally: ad-hoc approach imposes a long-run NFA position instead of solving for it
- “True” global, non-linear solution is not critical for some business cycle moments, but it is critical for those directly related to NFA dynamics, and for other key issues:
  1. Global imbalances (accumulation of reserves)
  2. Financial crises & macro-prudential regulation
  3. Sovereign risk
  4. Financial development