Quantile Response and Panel Data

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Introduction

- In this lecture I provide an introduction to quantile regression and discuss three empirical applications of quantile techniques to panel data.
- Quantile regression is a useful tool for studying conditional distributions.
- The application of quantile techniques to panel data is interesting because it offers opportunities for identifying nonlinear models with unobserved heterogeneity and relaxing exogeneity assumptions.
- Importantly it also offers the opportunity to consider conceptual experiments richer than a static cross-sectional treatment, such as dynamic responses.

Introduction (continued)

- The first application looks at the effect of child maturity on academic achievement using group data on students and their schools.

- The second application examines the effect of smoking during pregnancy on the birthweight of children.

- The third application examines the persistence of permanent income shocks in a nonlinear model of household income dynamics.

- The applications are based on the results of joint research:
  - Arellano and Weidner (2015)
  - Arellano and Bonhomme (2015)
  - Arellano, Blundell, and Bonhomme (2015).

Part 1

Quantile regression

Conditional quantile function

- Econometrics deals with relationships between variables involving unobservables.
- Consider an empirical relationship between two variables Y and X.
- Suppose that X takes on K different values $x_1, x_2, ..., x_K$ and that for each of those values we have M_k observations of Y: $y_{k1}, ..., y_{kM_k}$.
- If the relationship between Y and X is exact, the values of Y for a given value of X will all coincide, so that we could write

$$Y = q(X).$$

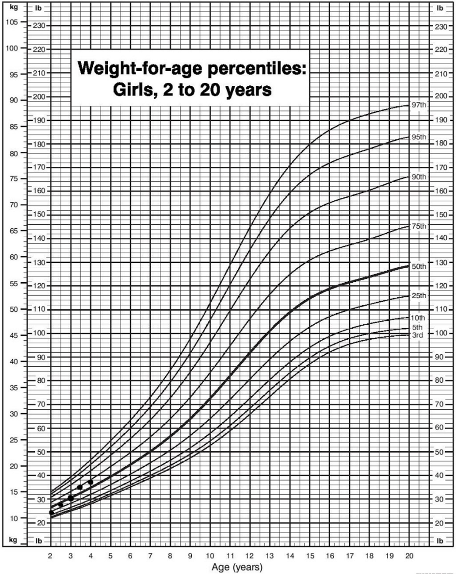
- However, in general units having the same value of X will have different values of Y.
- Suppose that $y_{k1} \leq y_{k2} \leq ... \leq y_{kM_k}$, so the fraction of observations that are less than or equal to y_{km} is $u_{km} = m/M_k$.
- It can then be said that a value of Y does not only depend on the value of X but also on the rank u_{km} of the observation in the distribution of Y given $X = x_k$.
- Generalizing the argument:

$$Y = q(X, U)$$

Conditional quantile function (continued)

- The distribution of the ranks U is always the same regardless of the value of X, so that X and U are statistically independent.
- Also note that $q(x, u)$ is an increasing function in u for every value of x.
- An example is a growth chart where Y is body weight and X is age (Figure 1).
- In this example U is a normalized unobservable scalar variable that captures the determinants of body weight other than age, such as diet or genes.
- The function $q(x, u)$ is called a conditional quantile function.
- It contains the same information as the conditional cdf (it is its inverse), but is in the form of a statistical equation for outcomes that may be related to economic models.
- $Y = q(X, U)$ is just a statistical statement: e.g. for $X = 15$ and $U = 0.5$, Y is the weight of the median girl aged 15, but one that can be given substantive content.

CDC Growth Charts: United States



Published May 30, 2000.
SOURCE: Developed by the National Center for Health Statistics in collaboration with the National Center for Chronic Disease Prevention and Health Promotion (2000).

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Quantile function of normal linear regression

- If the distribution of Y conditioned on X is the normal linear regression model of elementary econometrics:

$$Y = \alpha + \beta X + V \text{ with } V \mid X \sim \mathcal{N}\left(0, \sigma^2\right),$$

  the variable U is the rank of V and it is easily seen that

$$q(x, u) = \alpha + \beta x + \sigma \Phi^{-1}(u)$$

  where $\Phi(.)$ is the standard normal cdf.

- In this case all quantiles are linear and parallel, a situation that is at odds with the growth chart example.

Linear quantile regression (QR)

- The linear QR model postulates linear dependence on X but allows for a different slope and intercept at each quantile $u \in (0, 1)$
$$q(x, u) = \alpha(u) + \beta(u)x \tag{1}$$
- In the normal linear regression $\beta(u) = \beta$ and $\alpha(u) = \alpha + \sigma\Phi^{-1}(u)$.
- In linear regression one estimates α and β by minimizing the sum of squares of the residuals $Y_i - a - bX_i$ $(i = 1, ..., n)$.
- In QR one estimates $\alpha(u)$ and $\beta(u)$ for fixed u by minimizing a sum of absolute residuals where $(+)$ residuals are weighted by u and (-) residuals by $1 - u$.
- Its rationale is that a quantile minimizes expected asymmetric absolute value loss.
- For the median $u = 0.5$, so estimates of $\alpha(0.5)$, $\beta(0.5)$ are least absolute deviations.
- All observations are involved in determining the estimates of $\alpha(u)$, $\beta(u)$ for each u.
- Under random sampling and standard regularity conditions, sample QR coefficients are $\sqrt{n}$-consistent and asymptotically normal.
- Standard errors can be easily obtained via analytic or bootstrap calculations.
- The popularity of linear QR is due to its computational simplicity: computing a QR is a linear programming problem (Koenker 2005).

Linear quantile regression (QR) (continued)

- One use of QR is as a technique for describing a conditional distribution. For example, QR is a popular tool in wage decomposition studies.
- However, a linear QR can also be seen as a semiparametric random coefficient model with a single unobserved factor:

$$Y_i = \alpha(U_i) + \beta(U_i) X_i$$

  where $U_i \sim \mathcal{U}(0,1)$ independent of X_i.
- For example, this model determines log earnings Y_i as a function of years of schooling X_i and ability U_i, where $\beta(U_i)$ represents an ability-specific return to schooling.
- This is a model that can capture interactions between observables and unobservables.
- A special case of model with an interaction between X_i and U_i is the heteroskedastic regression $Y \mid X \sim \mathcal{N}\left[\alpha + \beta X, (\sigma + \gamma X)^2\right]$.
  - In this case $\alpha(u) = \alpha + \sigma\Phi^{-1}(u)$ and $\beta(u) = \beta + \gamma\Phi^{-1}(u)$.
- As a model for causal analysis, linear QR faces similar challenges as ordinary linear regression. Namely, linearity, exogeneity and rank invariance.
- Let us discuss each of these aspects in turn.

Flexible QR

- Linearity is restrictive. It may also be at odds with the monotonicity requirement of $q(x, u)$ in u for every value of x.

- Linear QR may be interpreted as an approximation to the true quantile function (Angrist, Chernozhukov, and Fernández-Val 2006).

- An approach to nonparametric QR is to use series methods:

$$q(x, u) = \theta_0(u) + \theta_1(u) g_1(x) + ... + \theta_P(u) g_P(x).$$

- The g's are anonymous functions without an economic interpretation. Objects of interest are derivative effects and summary measures of them.

- In practice one may use orthogonal polynomials, wavelets or splines (Chen 2007).

- This type of specification may be seen as an approximating model that becomes more accurate as P increases, or simply as a parametric flexible model of the quantile function.

- From the point of view of computation the model is still a linear QR, but the regressors are now functions of X instead of the Xs themselves.

Exogeneity and rank invariance

- To discuss causality it is convenient to use a single $0-1$ binary treatment X_i and a potential outcome notation Y_{0i} and Y_{1i}.
- Let U_{0i}, U_{1i} be ranks of potential outcomes and $q_0(u)$, $q_1(u)$ the quantile functions.
- Note that unit i may be ranked differently in the distributions of the two potential outcomes, so that $U_{0i} \neq U_{1i}$. The causal effect for unit i is given by

$$Y_{1i} - Y_{0i} = q_1(U_{1i}) - q_0(U_{0i}).$$

- Under exogeneity X_i is independent of (Y_{0i}, Y_{1i}).
- The implication is that the quantile function of $Y_i \mid X_i = 0$ coincides with $q_0(u)$ and the quantile function of $Y_i \mid X_i = 1$ coincides with $q_1(u)$, so that

$$\beta(u) = q_1(u) - q_0(u).$$

- This quantity is often called a quantile treatment effect (QTE). In general it is just the difference between the quantiles of two different distributions.
- It will only represent the gain or loss from treatment of a particular unit under a rank invariance condition. i.e. that the ranks of potential outcomes are equal to each other.
- Under rank invariance treatment gains may still be heterogeneous but a single unobservable variable determines the variation in the two potential outcomes.
- Next we introduce IV endogeneity in a quantile model with rank invariance.

Instrumental variable QR

- The linear instrumental variable (IV) model of elementary econometrics assumes

$$Y_i = \alpha + \beta X_i + V_i$$

where X_i and V_i are correlated, but there is an instrumental variable Z_i that is independent of V_i and a predictor of X_i.

- Potential outcomes are of the form $Y_{x,i} = \alpha + \beta x + V_i$ so that rank invariance holds.

- If x is a $0-1$ binary variable, $Y_{0,i} = \alpha + V_i$ and $Y_{1,i} = \alpha + \beta + V_i$.

- A QR generalization subject to rank invariance is to consider

$$Y_{x,i} = q\left(x, U_i\right).$$

- A linear version of which is

$$Y_{x,i} = \alpha\left(U_i\right) + \beta\left(U_i\right)x.$$

Instrumental variable QR (continued)

- Chernozhukov and Hansen (2006) propose to estimate $\alpha(u)$ and $\beta(u)$ for given u by directly exploiting the IV exclusion restriction.

- Specifically, if we write the model as

$$Y_i = \alpha(U_i) + \beta(U_i) X_i + \gamma(U_i) Z_i,$$

  the IV assumption asserts that Z_i only affects Y_i via X_i so that $\gamma(u) = 0$ for each u.

- Now let $\widehat{\gamma}_u(b)$ be the estimated slope coefficient in a u-quantile regression of $(Y_i - bX_i)$ on Z_i and a constant term.

- The idea, which mimics the operation of 2SLS, is to choose as estimate of $\beta(u)$ the value of b that minimizes $|\widehat{\gamma}_u(b)|$, hence enforcing the exclusion restriction.

- In the absence of rank invariance the treatment effects literature (e.g. Abadie 2003) has focused on QTEs for compliers in the context of a binary treatment that satisfies a monotonicity assumption.

Part 2

QR with fixed effects in large panels

Basics

- The most popular tool in panel data analysis is a linear regression model with common slope parameters and individual specific intercepts:

$$Y_{it} = \beta X_{it} + \alpha_i + V_{it} \qquad (i = 1, ..., N; t = 1, ..., T),$$

  in which $X_i = (X_{i1}, ..., X_{iT})$ is independent of V_{it} but possibly correlated with α_i.

- This is seen as a way of allowing for a special form of non-exogeneity (fixed-effect endogeneity) but also a way of introducing heterogeneity and persistence.

- The estimator of β is OLS including individual dummies, or equivalently OLS of Y on X in deviations from individual-specific means (within-group estimation).

- Observations may be from actual panel data, in which units are followed over time, or from data with a group structure, in which case i denotes groups and T is group size.

- In practice group size will be group specific (T_i) and techniques will be adapted accordingly.

QR with fixed effects

- A QR version of the within-group model specifies
$$Y_{it} = \beta(U_{it}) X_{it} + \alpha_i(U_{it})$$
where $U_{it} \sim \mathcal{U}(0,1)$ independent of X_i and $\alpha_i(.)$.
- The term $\alpha_i(U_{it})$ can be regarded as a function of U_{it} and a vector W_i of unobserved individual effects of unspecified dimension: $\alpha_i(U_{it}) = r(W_i, U_{it})$.
- Thus, the model allows for multiple individual characteristics that affect differently individuals with different error rank U_{it}.
- For example, there may be a multiplicity of school characteristics, some of which are only relevant determinants of academic achievement for high ability students while others are only relevant for low ability students.
- In QR one estimates $\beta(u)$ and $\alpha_1(u), ..., \alpha_N(u)$ for fixed u.
- The large sample properties of these estimates are those of standard QR if T is large in absolute terms and relative to N.
- However, if T is small relative to N or if T and N are of similar size, estimates of the common parameter $\beta(u)$ may be biased or even underidentified.
- The reason is too much sample noise due to estimating too many parameters relative to sample size. This situation is known as the incidental parameter problem.

Dealing with incidental parameters: fixed T and large T approaches

- In the static linear model, within-group estimates of the slope parameter are free from incidental parameter biases, but in nonlinear models the opposite is true in general.
- In situations where T is very small relative to N one reaction is to consider models and estimators of those models that are fixed-T consistent for large N.
- An example is the second application on the effect of smoking on birthweight, which uses a sample of $N = 12360$ women with $T = 3$ children each.
- There are also panels in which T is not negligible and not negligible relative to N, even if N still is much larger than T.
- An example, is the dataset in our first application that contains $N = 389$ schools with an average of $\overline{T} = 40$ students per school.
- An alternative approach in those situations has been to approximate the sampling distribution of the fixed effects estimator as T/N tends to a constant.
- For smooth objective functions this approach leads to a bias correction that can be easily implemented by analytical or numerical methods.
- A simple implementation is Jackknife bias correction (delete-one Jackknife in Hahn and Newey 2004; split-panel Jackknife in Dhaene and Jochmans 2015).

Bias reduction in QR

- The existing techniques are not applicable to QR due to the non-smoothness of the sample moment conditions of quantile models.
- Arellano and Weidner (2015) characterize the incidental parameter bias of QR and instrumental-variable QR estimators.
- They also find bias correcting moment functions that are first-order unbiased, that is, whose expected value is of order $1/T^2$.
- Moment functions within their class depend on the choice of a weight sequence. Some weight sequences are bias reducing while others are not.
- They uncover a bias-variance trade-off when attempting to correct bias, and provide bias corrected estimators that balance this trade-off.
- Interestingly their discussion of bias correction around choice of weight sequence is similar to bias reduction in nonparametric Kernel regression.
- Arellano and Weidner show that delete-one Jackknife is not first-order bias correcting in QR due to the fact that the second-order bias has a non-standard structure.
- They find that a permutation-invariant version of split-panel Jackknife is bias-correcting and exhibits good variance properties.

Interpreting the incidental parameter bias

- Arellano and Weidner (2015) find that the leading-order bias term vanishes in the special case where $\beta(u) = \beta$ is constant over u.
- This result is of limited interest if the goal is to estimate nonlinear models, although it may be useful in testing for linearity.
- They also provide an approximation to the leading order bias in the case where $\beta(u)$ is almost constant, so that $\beta(u) - \overline{\beta}$ is small.
- Under this approximation the leading order bias can be interpreted as resulting from measuring $\beta(u)$ at the wrong quantile $u + \Delta u$ and from smoothing out $\beta(u)$ around this wrong quantile with a density whose standard deviation shrinks at the rate $T^{-1/2}$.
- The implication is that the incidental parameter bias would tend to average effects across quantiles.

The effect of child maturity on academic achievement

- Arellano and Weidner study the effect of age on academic achievement of school children following Bedard and Dhuey (2006).
- Bedard and Dhuey consider multiple countries and students of different age groups. Their question is whether initial maturity differences in kindergarten and primary school have long-lasting effects.
- Here we only consider data from Canada for third and fourth graders (9 year old in 1995) from the Trends in International Mathematics and Science Study (TIMSS).
- There are 389 schools with an average of 40 students per school. Therefore, it is an unbalanced pseudo-panel or dataset with a group structure.
- The outcome variable is the math test score of student t in school i normalized to haven mean 50 and standard deviation 10 over the whole sample.
- The main regressor is observed age measured in months.
- Age is potentially endogenous because of grade retention and early or late school enrolment (which are not observed).

The effect of child maturity on academic achievement (continued)

- Following Bedard and Dhuey we use age relative to the school cutoff date to instrument for age.
- The school cutoff date in Canada is January 1. So we define relative or assigned age as $z = 0$ for children born in December and $z = 11$ for children born in January.
- Relative age is a strong instrument.
- We only require exogeneity of relative age conditional on school effects, which for example will capture the age distribution at school level.
- Quantile analysis is interesting, because age effects might be different for low- and high-performing students.
- Whether maturity and academic ability are substitutes or complements is an empirical question that may have implications for school policy.
- Controlling for school fixed effects turns out to be important for the results. Age composition may vary across schools, so age is likely fixed-effect endogenous.

Table 1
Effect of Age on Math Test Scores at 3rd & 4th Grade
Canadian TIMSS 15549 students $N = 394$ schools

OLS	IV	OLS+FE	IV+FE
0.017	0.184	-0.0332	0.178
(0.010)	(0.026)	(0.009)	(0.0241)
Number in brackets are standard errors			
IV uses assigned age to instrument for observed age			
Controls: sex, grade, rural, mother native, father-native			
both parents, calculator, computer, +100books, hh size			
std(Y)=10, i.e. age effect of 0.18 is a 1.8% st dev			
per month effect or 22% st deviations per year			

- Table 1 reproduces results in Bedard and Dhuey (2006).
- IV estimates with and without school fixed effects are very similar, i.e. the instrument appears to be uncorrelated with school effects.

Table 2
Effect of Age on Math Test Scores at 3rd & 4th Grade
Quantile IV, no fixed effects

$u = 0.1$	$u = 0.3$	$u = 0.5$	$u = 0.7$	$u = 0.9$
0.14	0.16	0.18	0.24	0.19
(0.01)	(0.01)	(0.01)	(0.07)	(0.03)
IV uses assigned age to instrument for observed age				
Controls: sex, grade, rural, mother native, father-native				
both parents, calculator, computer, +100books, hh size				

- Without controlling for school fixed effects, one finds a significant difference in age effects across quantiles.
- Age effects are increasing.
- The results in Table 2 would point to maturity and ability as complements in the production of test scores.

Table 3
Effect of Age on Math Test Scores at 3rd & 4th Grade
Quantile IV with fixed effects, no bias correction

$u = 0.1$	$u = 0.3$	$u = 0.5$	$u = 0.7$	$u = 0.9$
0.18	0.15	0.18	0.19	0.16
(0.05)	(0.03)	(0.03)	(0.04)	(0.04)

IV uses assigned age to instrument for observed age
Controls: sex, grade, rural, mother native, father-native
both parents, calculator, computer, +100books, hh size

- Table 3: Once we control for school fixed effects, we do not find a significant difference in age effects across quantiles.

- Age effects are relatively constant in u. But is this because there is really no effect, or because the incidental parameter bias tends to average effects across quantiles?

Table 4
Effect of Age on Math Test Scores at 3rd & 4th Grade
Quantile IV with fixed effects, bias correction

$u = 0.1$	$u = 0.3$	$u = 0.5$	$u = 0.7$	$u = 0.9$
0.21	0.15	0.18	0.18	0.09
(0.05)	(0.03)	(0.04)	(0.04)	(0.05)

IV uses assigned age to instrument for observed age
Controls: sex, grade, rural, mother native, father-native
both parents, calculator, computer, +100books, hh size

- Table 4: After bias correction age effects are decreasing in u.
- There seems to be evidence that maturity and ability are substitutes in academic achievement.

Part 3

QR with random effects in short panels

- Application of QR with fixed effects is straightforward as it proceeds in a quantile-by-quantile fashion allowing for a different fixed effect at each quantile.

- However, in short panels the incidental parameter problem is a challenge.

- Moreover, while being agnostic about the number of the unobserved group factors affecting outcomes is attractive, sometimes substantive reasons suggest that only a small number of underlying factors play a role.

- Whether one uses a quantile model with a different individual effect at each quantile or a model with a small number of unobserved effects also has implications for identification.

- Rosen (2010) shows that a fixed-effects model for a single quantile may not be point identified.

- Arellano and Bonhomme (2015) show that a QR model with a scalar fixed effect is nonparametrically identified in panel data with $T = 3$ subject to completeness assumptions (Newey and Powell 2003; Hu and Schennach 2008).

Flexible quantile modelling with random effects

- Arellano and Bonhomme aim to estimate models of the form:

$$Y_{it} = \beta\left(U_{it}\right) X_{it} + \gamma\left(U_{it}\right) \eta_i + \alpha\left(U_{it}\right) \tag{2}$$

  where $U_{it} \sim \mathcal{U}\left(0,1\right)$ independent of X_i and η_i, but X_i and η_i may be correlated.

- Model (2) is a special case of a series-based specification that allows for nonlinearities and interactions between X_{it} and η_i:

$$Y_{it} = \sum_{k=1}^{K_1} \theta_k\left(U_{it}\right) g_k\left(X_{it}, \eta_i\right) \tag{3}$$

- The dependence of η_i on X_i is also specified as a flexible quantile model:

$$\eta_i = \sum_{k=1}^{K_2} \delta_k\left(V_i\right) h_k\left(X_i\right) \tag{4}$$

  where V_i is a uniform random variable independent of U_{it} and X_{it} for all t.

- This is a correlated random effects approach in the sense that a model for the dependence between η_i and X_i is specified.
- However, it is more flexible than alternative specifications in the literature and can be seen as an approximation to the conditional quantile function as K_2 increases.
- If η_i is a vector of individual effects a triangular structure is assumed in place of (4).

Simulation-based estimation

Basic intuition behind the Arellano and Bonhomme method

- If η_i were observed, one would simply run an ordinary QR of Y_{it} on X_{it} and η_i.
- But since η_i is not observed they construct some imputations, say M imputed values $\eta_i^{(m)}$, $m = 1, ..., M$ for each individual in the panel. Having got those, one can get estimates by computing a QR averaged over imputed values.
- For the imputed values to be valid they have to be draws from the distribution of η_i conditioned on the data, which depends on the parameters to be estimated (θ's and δ's in the flexible model).
- This is therefore an iterative approach.
- They start by selecting initial values for a grid of conditional quantiles of Y_{it} and η_i, which then allows them to generate imputes of η_i, which can be used to update the quantile parameter estimates and so on.
- To deal with the complication that $\theta_k(u)$ and $\delta_k(v)$ are functions, they use a finite-dimensional approximation to those functions based on interpolating splines with L knots (similar to Wei and Carroll 2009).
- The resulting method is a stochastic EM algorithm.

Simulation-based estimation (continued)

Stochastic EM algorithm

- A difference with most applications of EM algorithms is that parameters are not updated in each iteration using maximum likelihood but QR.
- This is important because once imputes for η_i are available, QR estimates can be calculated in a quantile-by-quantile fashion, which together with the convexity of QR minimization make each parameter update fast and reliable.
- Arellano and Bonhomme obtain the asymptotic properties of the estimator based on the stochastic EM algorithm for a fixed number of draws M in the case where the parametric model is assumed correctly specified (extending results in Nielsen 2000).
- That, is K_1, K_2 and L are held fixed as N tends to infinity for fixed T.
- They also establish consistency as K_1, K_2 and L tend to infinity with N in the large-M limit.

Other approaches

- Other recent approaches to quantile panel data models include Chernozhukov, Fernández-Val, Hahn & Newey (2013), and Graham, Hahn, Poirier & Powell (2015).
- These approaches are non-nested with the previous model and will recover different quantile effects.

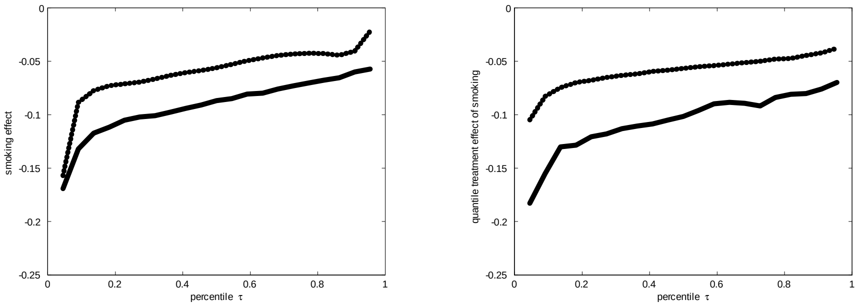
The effect of smoking on birth weight

- We revisit the effect of maternal inputs on children's birth outcomes. Specifically, we study the effect of smoking during pregnancy on children's birthweights.
- Abrevaya (2006) uses a mother-FE approach to address endogeneity of smoking.
- We use QR with mother-specific effects to allow for both unobserved heterogeneity and nonlinearities in the relationship between smoking and birthweight.
- We use a balanced subsample from the US natality data used in Abrevaya (2006), which comprises 12360 women with 3 children each. Our outcome is log-birthweight.
- The main covariate is a binary smoking indicator. Age of the mother and gender of the child are used as additional controls.
- An OLS regression yields a negative point estimate of the smoking coefficient: $-.095$. The fixed-effects estimate is also negative, but it is twice as small: $-.050$ (significant).
- Moreover, running a standard (pooled) QR suggests that the effect of smoking is more negative at lower quantiles of birthweights.
- However, these results might be subject to an endogeneity bias, which may not be constant along the distribution.

The effect of smoking on birth weight (continued)

- The left graph of Figure 2 shows the smoking coefficient in a pooled QR (solid line), and the REQR estimate of the smoking effect (dashed line).
- REQR estimates use $L = 21$ knots. The stochastic EM algorithm is run for 100 iterations, with 100 random walk Metropolis-Hastings draws within each iteration.
- Parameter estimates are averages of the 50 last iterations of the algorithm.
- The smoking effect becomes less negative when correcting for time-invariant endogeneity through the introduction of mother-specific fixed-effects.
- At the same time, the effect remains sizable, and is increasing along the distribution.
- The right graph shows the QTE of smoking as the difference in log-birthweight between a sample of smoking women, and a sample of non-smoking women, keeping all other characteristics (observed, X_i, and unobserved, η_i) constant.
- This calculation illustrates the usefulness of estimating a complete model of the joint distribution of outcomes and unobservables, to compute counterfactual distributions that take unobserved heterogeneity into account.
- The solid line shows the empirical difference between unconditional quantiles, while the dashed line shows the QTE that accounts for both observables and unobservables.
- The results are broadly in line with those reported on the left graph of Figure 2.

Figure 2: QR coefficient of smoking and QTE (difference in potential outcomes)

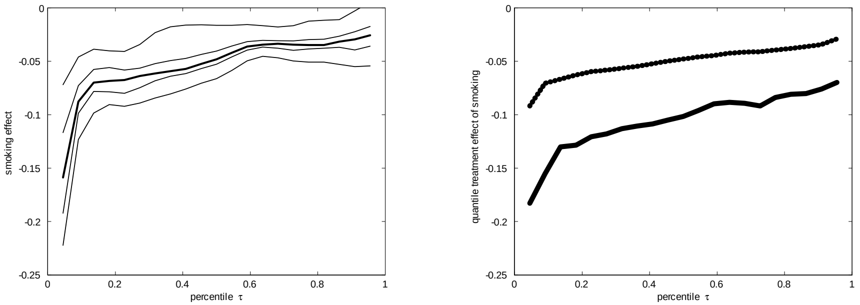


- Data from Abrevaya (2006).
- Left: Solid line is the pooled QR smoking coefficient; dashed line is the panel QR smoking coefficient.
- Right: Solid line is the raw QTE of smoking; dashed line is the QTE estimate based on panel QR.

QR with smoking interacted with mother heterogeneity and baby heterogeneity

- Lastly, we report the results of an interacted quantile model, where the specification allows for all first-order interactions between covariates and the unobserved mother-specific effect.
- In this model the quantile effect of smoking is mother-specific.
- The results on the right graph in Figure 3 show the unconditional QTE of smoking. Results are similar to the ones obtained for the linear specification.
- However, on the left graph we see substantial mother-specific heterogeneity in the conditional quantile treatment effect of smoking.
- For some mothers smoking appears particularly detrimental to children's birthweight, whereas for other mothers the smoking effect, while consistently negative, is much smaller.
- This evidence is in line with the results of a linear random coefficients model reported in Arellano and Bonhomme (2012).

Figure 3: Quantile effects of smoking and QTE (interacted specification)



- Data from Abrevaya (2006).
- Left: lines represent the percentiles .05, .25, .50, .75, and .95 of the heterogeneous smoking effect across mothers, at various percentiles u.
- Right: Solid line is the raw QTE of smoking; dashed line is the QTE estimate based on panel QR with interactions.

Part 4

Dynamic quantile models

Autoregressive models and predetermined variables

- The Arellano-Bonhomme approach covers dynamic autoregressive models and models with general predetermined variables of the form:

$$Y_{it} = Q_Y \left(Y_{i,t-1}, X_{it}, \eta_i, U_{it} \right)$$

- If the Xs are strictly exogenous variables, the quantile model for the individual effect is as before except for the inclusion of the initial outcome variable:

$$\eta_i = Q_\eta \left(Y_{i1}, X_i, V_i \right)$$

- In the case of general predetermined variables the model is incomplete.

- To complete the specification a Markov feedback process is assumed:

$$X_{it} = Q_X \left(Y_{i,t-1}, X_{i,t-1}, \eta_i, A_{it} \right)$$

  and the quantile model of the effects is conditioned only on initial values:

$$\eta_i = Q_\eta \left(Y_{i1}, X_{i1}, V_i \right)$$

Models with time-varying unobservables

- The framework also extends to models with time-varying unobservables, such as the following nonlinear permanent-transitory model:

$$
\begin{aligned}
Y_{it} &= \eta_{it} + V_{it} &(5)\\
\eta_{it} &= Q_Y\left(\eta_{i,t-1}, U_{it}\right) &(6)
\end{aligned}
$$

  where V_{it} and U_{it} are i.i.d. distributed.

- Arellano, Blundell and Bonhomme (2014) use a quantile-based approach to document nonlinear relationships between earnings shocks to households and their lifetime profiles of earnings and consumption.

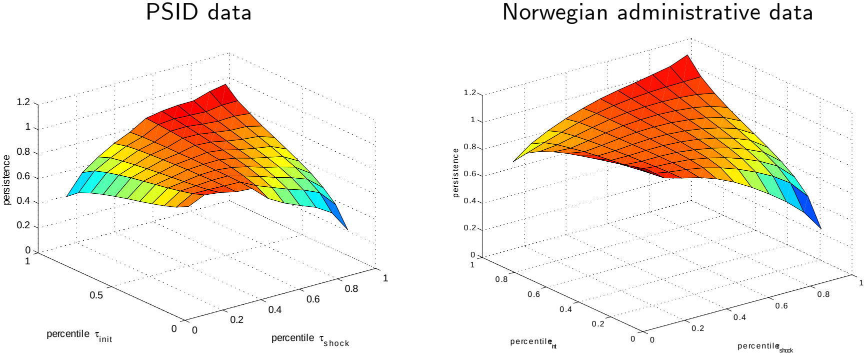
- They estimate model (5)-(6) using PSID household labor income data for the years 1998–2008.

Persistence of permanent income shocks

- Evidence of nonlinearity in the persistence of earnings can be seen from Figure 4.
- This figure plots estimates of the average derivative of the conditional quantile function of current income with respect to lagged income.
- The graphs show strong similarity in the patterns of the nonlinearity of household earnings in the PSID survey data and in the population register data from Norway.
- They also show a clear difference in the impact of past shocks according to the percentile of the shock and the percentile of the past level of income.
- A large positive shock for a low income family or a large negative shock for a high income family appears to reduce the persistence of past shocks.

Figure 4: Quantile autoregressions of log-earnings

$$\frac{\partial Q_{y_t|y_{t-1}}(y_{i,t-1},\tau)}{\partial y}$$



Note: Residuals of log pre-tax household labor earnings, Age 35-65 1999-2009 (US), Age 25-60 2005-2006 (Norway). Estimates of the average derivative of the conditional quantile function of y_{it} given $y_{i,t-1}$ with respect to $y_{i,t-1}$.

Persistence of permanent income shocks (continued)

- Arellano, Blundell, and Bonhomme find that in the central range of the distribution, measured persistence of $\eta_{i,t-1}$ is of similar magnitude and close to unity, so that the unit root model would be an acceptable description for this part of the distribution.

- However, a very negative shock reduces the persistence of a "positive history" (a positive lagged level of η) but preserves the persistence of a negative history.

- At the other end, a very positive shock reduces the persistence of a negative history but preserves the persistence of a good history.

- These results suggest a richer view of persistence, away from the conventional unit root versus mean reversion dichotomy, and help explain household consumption behavior.