

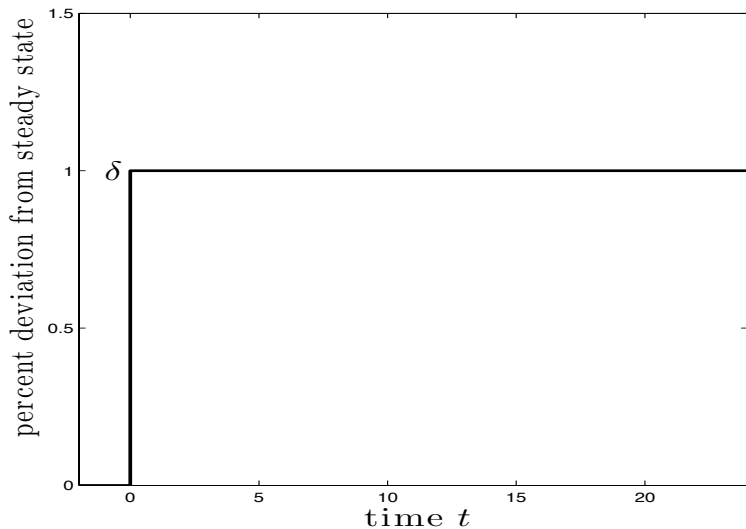
# Small and large price changes and the propagation of monetary shocks

Fernando Alvarez , Hervé Le Bihan , Francesco Lippi

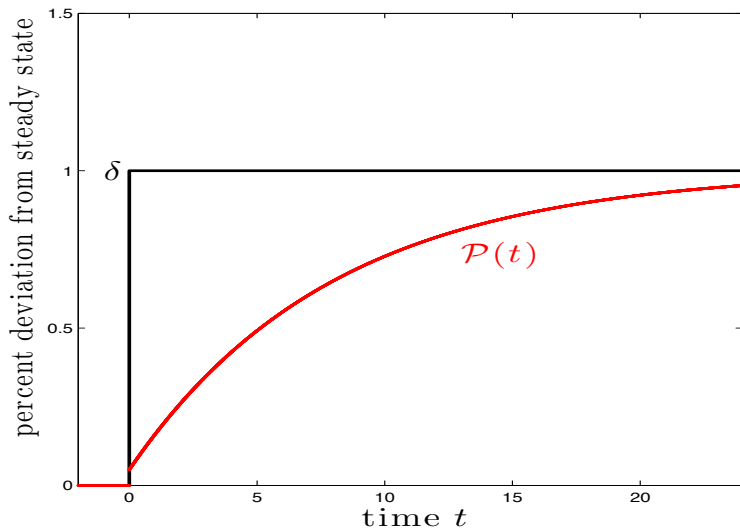
University of Chicago & NBER , Banque de France , EIEF & University of Sassari

Banco Central Chile – August 2014

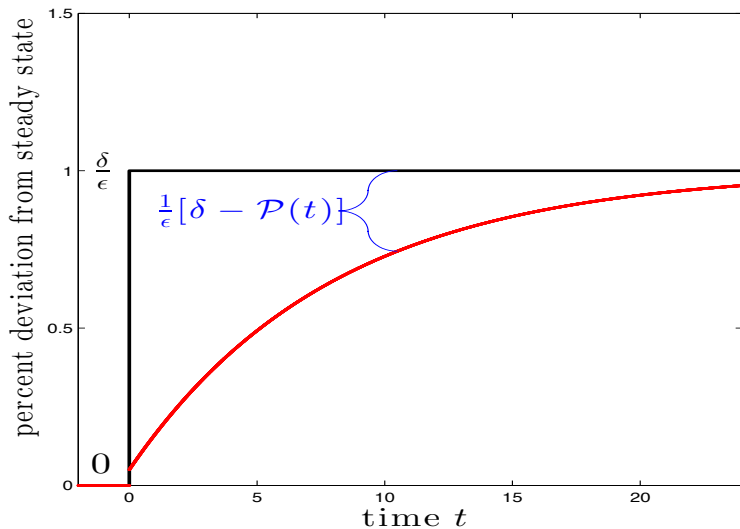
Once and for all monetary shock at  $t = 0$ .



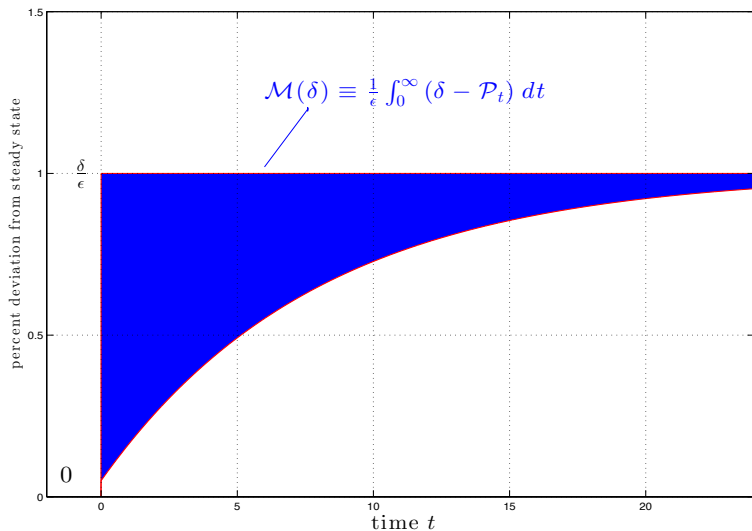
# Impulse response of Price Level



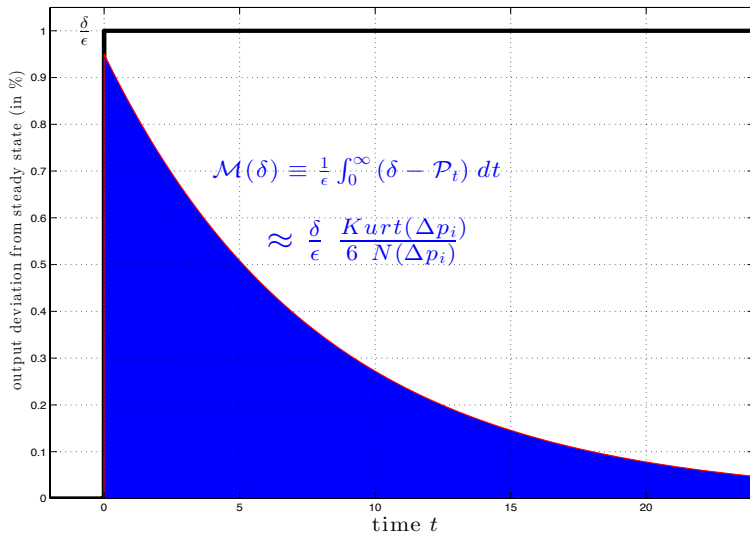
# Impulse response of output $d \log c_t = \frac{1}{\epsilon} d \log(M_t/P_t)$



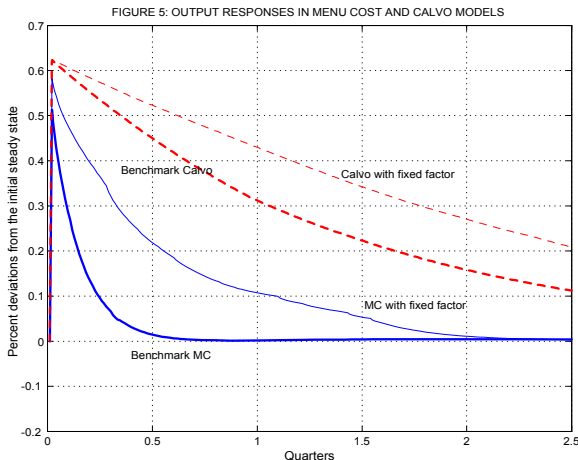
# Area under impulse response of output: $\mathcal{M}$

[▶ back](#)


# Area under impulse response of output: $\mathcal{M}$



Range of effects:  $\mathcal{M}_{Calvo} / \mathcal{M}_{GL} \approx 6$



source: Golosov-Lucas 2008

More Models: Reis / Taylor 1980, Nakamura-Steinsson 2010, Midrigan 2011

All models same  $N(\Delta p_i)$  ; differences due to “selection” effects (Kurtosis)

# Our contribution

- Class of stylized menu-cost models:  
(random) menu cost, multi-product, rational inattentiveness.
- $\mathcal{M}(\delta) \equiv$  cumulative IRF of output to a (small) monetary shock  $\delta$

$$\mathcal{M}(\delta) \approx \frac{\delta}{\epsilon} \frac{\text{Kurt}(\Delta p_i)}{6 N(\Delta p_i)}$$

- Frequency of price changes  $N(\Delta p_i)$  has a first order effect,
- Kurtosis price changes  $\text{Kurt}(\Delta p_i)$  has a first order effect.
- Result in the spirit of “Sufficient Statistic Approach” (useful to macro)
- Gather and evaluate empirical measures of  $\text{Kurt}(\Delta p_i) \approx 4.5$
- Evaluate magnitude of implied adjustment/menu cost.

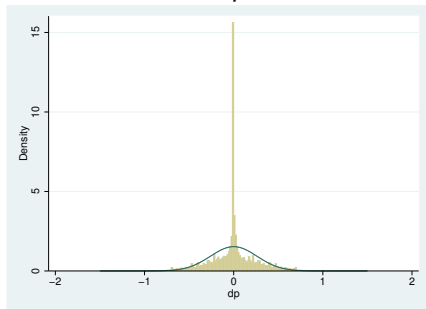


# Micro evidence: overview

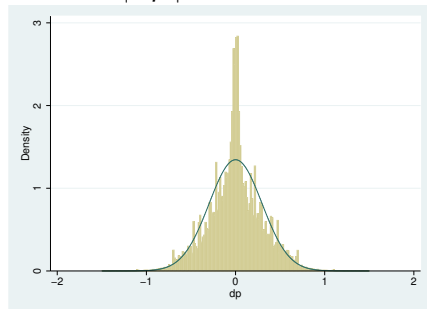
- We explore several datasets  
CPI (French, US, Norway), scanner price (US), Internet scraped data
- Excess Kurtosis due to small and large  $\Delta p_{i,j,t}$  (Kashyap, 1995)
- Excess Kurtosis attenuated but still positive after correcting for:
  - (1) heterogeneity  $\rightarrow$  standardize price changes:  $\frac{\Delta p_{i,j,t} - m_{i,j}}{\sigma_{i,j}}$   
standardization at the good- $i \times$  store- $j$  level
  - (2) measurement error  $\rightarrow$  compare datasets and trimming  
CPI vs scanner data , or trim  $\Delta p_i < 1 \text{ ct}$

# Same scanner data as Eichenbaum et al (2011)

Size distribution of non-zero  $\Delta p_i$  (based on average weekly prices)  
 All  $\Delta p_i$       trim  $|\Delta p_i| < 1$  dollar cent



kurtosis = 4.9 ; 11 million obs.

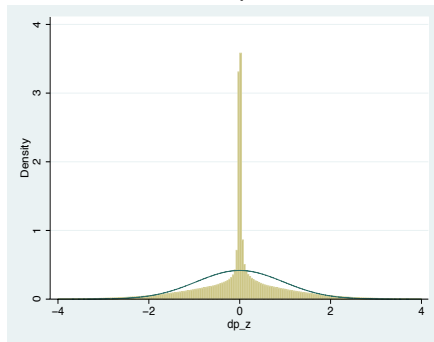


kurtosis = 3.8 ; 8.5 million obs.

- Based upon: Average Weekly prices.
- Lots of price changes smaller than 1 cent! (left panel)

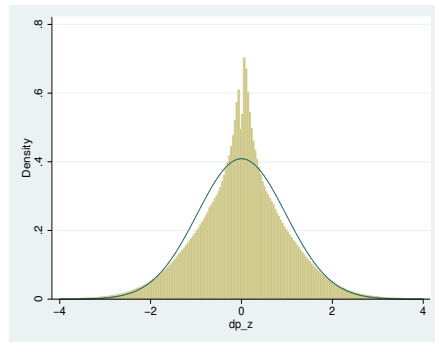
# US: IRI-Symphony scanner data

All  $\Delta p_i$



kurtosis = 34.3 ; 820 million obs.

Standardized & trimmed

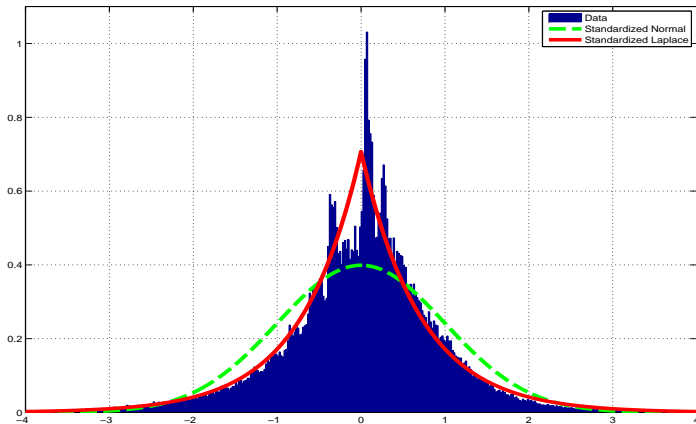


kurtosis = 4.3 ; 610 million obs.

- Large data set: 3,000 stores x 40,000 products x 10 years
- Average weekly prices

# French CPI data 2003-2011

size distribution; Standardized and trimmed; 1.5 million obs.



$Std(\Delta p_i) = 16\%$  ; kurtosis = 8.9

# Measurement error: CPI vs Scraped Data (France)

- CPI very broad, but subject to measurement error:

Eichenbaum et al: spurious small price changes in CPI and scanner data.

- Our approach:
  - Compare “error free” data for selected products with CPI.
  - Use internet scrapped data for France.
  - Match type of goods/outlet with CPI.
  - Use simple measurement error model to extrapolate to whole CPI.
- Conclusion: kurtosis is about half.

# Summary of evidence

Once x-section heterogeneity removed, the data show

- evidence of measurement error (“spurious”  $\Delta p_i$ )  
overestimate Kurtosis on CPI data
- distribution of  $\Delta p_i$  peaked in both the US and France  
between Normal (kurt  $\approx 4$  in US) and Laplace (kurt  $\approx 5$  in FR)
- no big differences between sales vs no-sales size distribution

► Sales-vs-nonsales

► US-vs-France

► CPI-vs-scraped data

► CPI-vs-scanner data

# The model economy

Lifetime Utility : 
$$\int_0^\infty e^{-rt} \left( \frac{c(t)^{1-\epsilon}}{1-\epsilon} - \alpha \ell(t) + \log \frac{M(t)}{P(t)} \right) dt$$

CES aggregate : 
$$c(t) = \left( \int_0^1 \sum_{i=1}^n (Z_{ki}(t) c_{ki}(t))^{1-\frac{1}{\eta}} dk \right)^{\frac{\eta}{\eta-1}}$$

- Intratemporal  $\eta$  (= for firms  $k$  & products  $i$ ).
  - Linear technology  $c_{ki}(t) = \ell_{ki}(t) / Z_{ki}(t)$  and  $Z_{ki}(t) = \exp(\sigma \mathcal{W}_{ki}(t))$ .
  - Household problem:
    - demand for monopolist competitive firms, idiosyncratic pref. shocks  $Z_{ki}(t)$
    - money demand: determination of interest rate (forward looking)
    - Inter-temporal and labor supply elasticity =  $1/\epsilon$
  - Firm's adjustment cost: if  $\psi_\ell$  units of labor paid,  $n$  prices can be changed.
- Random menu cost: w/pr.  $\lambda dt$  adjust without paying  $\psi_\ell$  (Poisson).

- Equilibrium: constant nominal interest rate & wages  $W(t) \propto M(t)$ .

Simplify firm's problem: nominal cost are known.

- Discounted nominal profit of product  $i$  as function of (log) price gap  $p_i$ :

$$\Pi(p_i, c(t)) \propto W(0) e^{-r t} c(t)^{1-\eta\epsilon} e^{-\eta p_i} \left[ e^{p_i \frac{\eta}{\eta-1}} - 1 \right]$$

$$0 = \frac{\partial \Pi(0, c(t))}{\partial p_i} = \frac{\partial^2 \Pi(0, c(t))}{\partial p_i \partial c(t)}$$

- Same reason why consumption and nominal interest rates do not “enter into” the simple NK Phillips curve.
- This means we can consider simple steady state problem for the firm



# Model has 4 primitive parameters $\frac{\psi}{B}$ , $\lambda$ , $\sigma^2$ and $n$

- multi-product: simultaneous adjustment of  $n$  products sold by firm.
- random menu cost:

$$\text{adjust } n \text{ prices paying} = \begin{cases} \psi & \text{with probability } 1 - \lambda dt \text{ or} \\ 0 & \text{with probability } \lambda dt. \end{cases}$$

where menu cost  $\psi$  is measured relative to steady state profits.

- production cost each product:  $\mathcal{W}_{ki}(t)$  random walk w/volatility  $\sigma^2 dt$ .
- $p_i(t)$  (log) percentage deviation from static optimal markup over cost.
- 2nd order approx. to period profit gives:  $-\sum_{i=1}^n B [p_i(t)]^2$   
 $B = (1/2) \eta (\eta - 1)$  where  $\eta$  elasticity of demand.
- Firm maximizes expected net discounted (at rate  $r$ ) profits.

# Simplest case when $n = 1$ and $\lambda \geq 0$

Bellman equation

$$r V(p) = B p^2 + \lambda (V(0) - V(p)) + \frac{\sigma^2}{2} V''(p), \quad \text{for } p \in (-\bar{p}, \bar{p}),$$

value matching and smooth pasting conditions are:

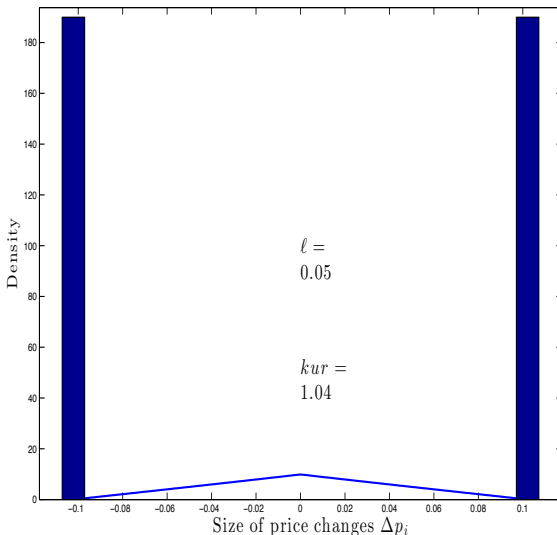
$$V(\bar{p}) = V(0) + \psi, \quad V'(\bar{p}) = 0, \quad \bar{p} \approx \begin{cases} \left( \frac{6\psi\sigma^2}{B} \right)^{\frac{1}{4}} & \text{if } \frac{\psi}{B}\sigma^2(r+\lambda)^2 \text{ is small} \\ \frac{\psi}{B}(r+\lambda) & \text{if } \frac{\psi}{B}\sigma^2(r+\lambda)^2 \text{ is large} \end{cases}$$

Threshold rule  $\bar{p}$  yields cross-sectional model predictions

Frequency of adjustment:  $N(\Delta p_i)$  or  $\ell = \lambda/N$

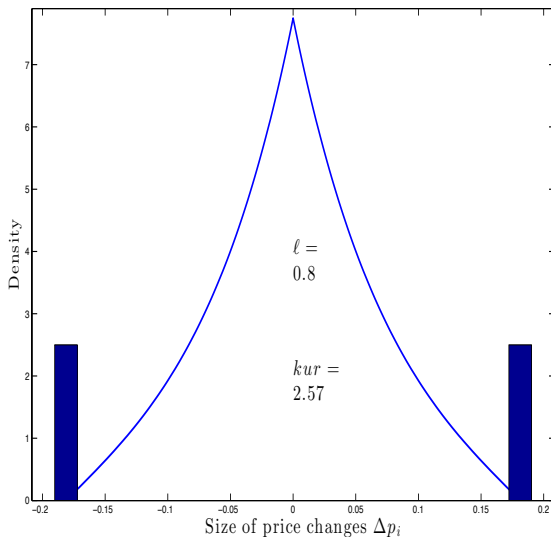
**Shape** of size-distribution of price changes  $w(\Delta p_i)$  depends only on  $\ell$ .

for  $n = 1$  ,  $\ell = \lambda/N = 0.05$



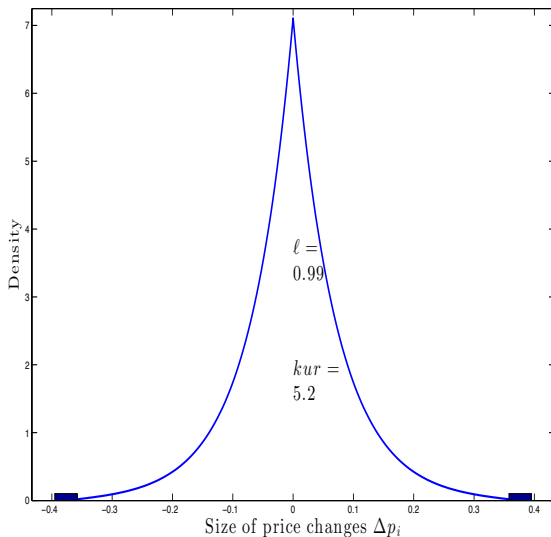
Size distribution  $w(\Delta p_i)$  for  $n = 1$  model ; shape depends ONLY on  $\ell$

for  $n = 1$  ,  $\ell = \lambda/N = 0.80$



Size distribution  $w(\Delta p_i)$  for  $n = 1$  model ; shape depends ONLY on  $\ell$

for  $n = 1$  ,  $\ell = \lambda/N = 0.99$

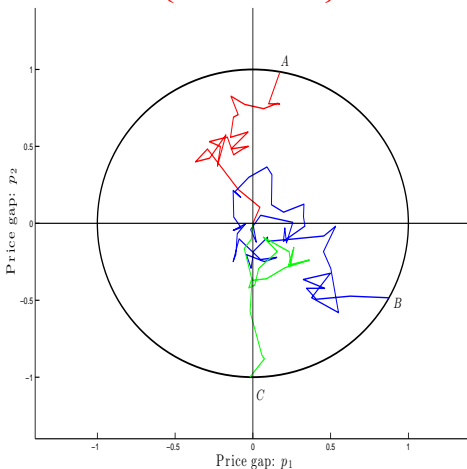


Size distribution  $w(\Delta p_i)$  for  $n = 1$  model ; shape depends ONLY on  $\ell$

# Key for $n > 1$ : summarize state by scalar $y \equiv ||p||^2$

$y \equiv ||p||^2$  square of a **Bessel** process: :  $dy = n \sigma^2 dt + 2 \sigma \sqrt{y} dW$

Inaction region = sphere:  $\mathcal{I} = \{p : ||p||^2 \leq \bar{y}\}$ .



$$v(y) = v(||p||^2) = V(p_1, \dots, p_n)$$

► derivation of  $y$  process

► ODE for  $v(y)$

# Cross section predictions for $n > 1$

All prices change if  $\|p(\tau)\|^2$  hits  $\bar{y}$  or when free-chance arrives.

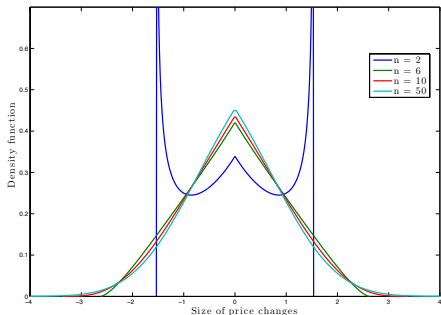
- At such times price changes are  $\Delta p_i(\tau) = -p_i(\tau)$ , i.e. reset gaps to zero.

Use properties of  $\|p(\tau)\|^2 \in [0, \bar{y})$  and statistical tools to characterize:

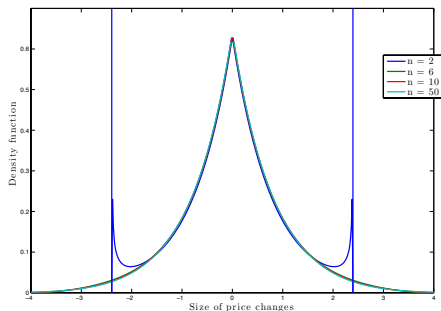
- scale and frequency of  $\Delta p_i$ :  $Std(\Delta p_i)$  ,  $N(\Delta p_i) \implies \ell \equiv \frac{\lambda}{N(\Delta p_i)}$
- size-distribution of  $\Delta p_i$ :  $w(\Delta p_i)$ : **shape** depends ONLY on  $n$  and  $\ell$
- one-to-one mapping  $(n, \lambda, \psi/B, \sigma^2)$  and  $(n, \ell, Std(\Delta p_i), N(\Delta p_i))$ .

# Density of price changes $w(\Delta p_i)$ , for $n > 1$

fraction of free adjustments:  $\ell = 0.2$



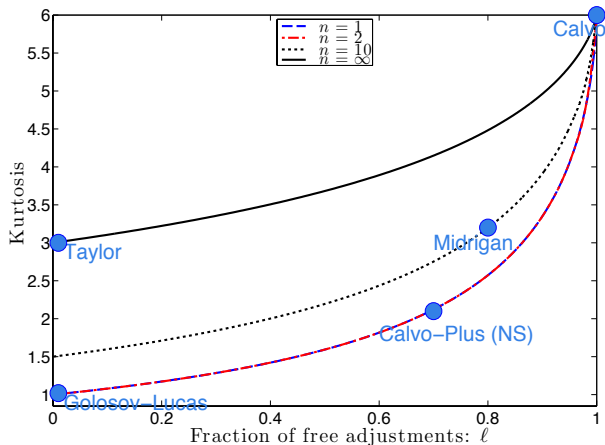
fraction of free adjustments:  $\ell = 0.8$



- $\ell > 0$  and large  $n$ : avoid mass points (or high density) for large  $|\Delta p_i|$ .
- The shape of the distributions only depends on  $n, \ell$ .




# Kurtosis $Kur(\Delta p_i)$ function only of $(n, \ell)$

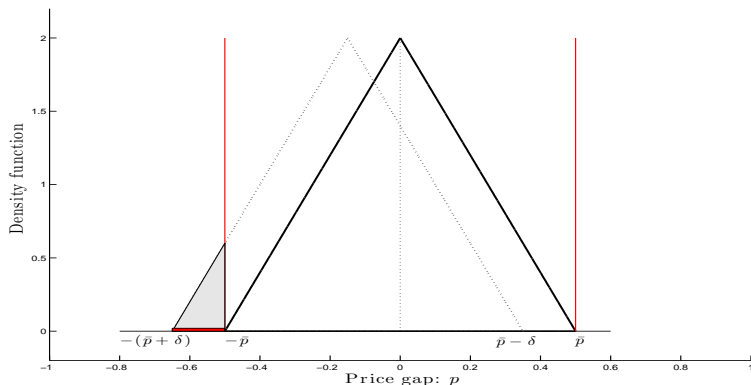


$Kur(\Delta p_i) \in [1, 6]$ , an increasing function of  $(n, \ell)$

# Cumulative Output Response to a Monetary Shock

- Unexpected permanent increase in money of size  $\delta$  starting at steady st.
- Start at steady state with distribution of price gaps across firms.
- Keep decision rules ( $\bar{y}$ ) at steady state (as Caballero-Engel)
  - Numerically small GE feedback effects (Goloso-Lucas )
  - Alvarez-Lippi (2014) prove GE effects are second-order on  $\bar{y}$  for small  $\delta$

# Example: canonical menu cost example $n = 1, \ell = 0$



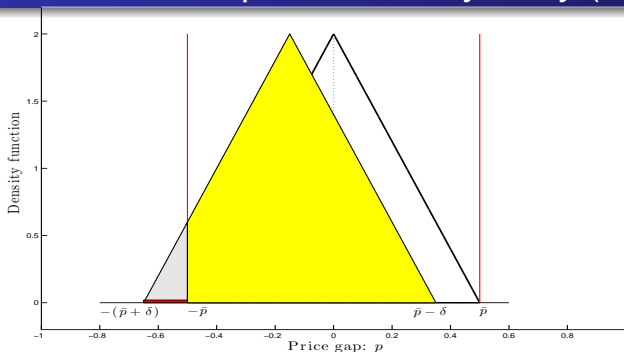
- Start from Steady State distribution of desired price adjustments (p. gaps)
- size of shock matters
- selection effect: in size and time

# Two useful, general, simplifying results:

- No need to characterize whole evolution of  $p$  distribution after the shock  
 Only compute the firms' effect on CPI until *first time they adjust prices*,  
 Result due to symmetry after adjustment.
- The decision rule in the transition  $\bar{y}$  is the same one as in the st. st.  
 No GE effect due to multiplicative profit function:  $\Pi(p, c) = \hat{\Pi}(p) c$ ,  
 Interest rates and wages solved independently of prices.

► Gol-Lucas figure

# Key idea to solve the problem analytically ( $n = 1$ )



$$m(p) = \mathbb{E} \left[ \int_0^\tau -p(t) dt \mid p_0 = p \right] \quad \text{then} \quad \mathcal{M}(\delta) = \frac{1}{\epsilon} \int_{-\bar{p}}^{\bar{p}} m(p - \delta) g(p) dp$$

useful because we have ODE for  $m(p)$  :  $\lambda m(p) = -p + m''(p) \frac{\sigma^2}{2}$

with boundary conditions  $m(0) = m(\bar{p}) = 0$  and known  $g(p)$

# Example: two famous models ( $n = 1$ )

From the definition  $\mathcal{M}(\delta) = \frac{1}{\epsilon} \int_{-\bar{p}}^{\bar{p}} m(p - \delta) g(p) dp$

Using the approximation  $\mathcal{M}(\delta) \cong \delta \mathcal{M}'(0) = -\frac{\delta}{\epsilon} \int_{-\bar{p}}^{\bar{p}} m'(p) g(p) dp$

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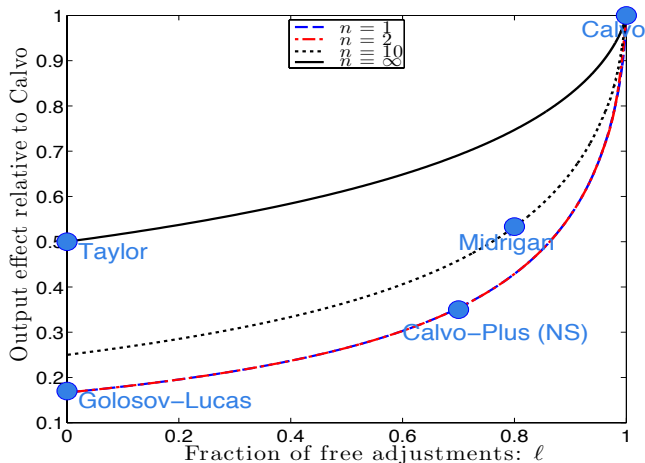
Using the approximation  $\mathcal{M}(\delta) \cong \delta \mathcal{M}'(0) = -\frac{\delta}{\epsilon} \int_{-\bar{p}}^{\bar{p}} m'(p) g(p) dp$

and the closed form expressions for  $m(p)$  and  $g(p)$  we get

$$\delta \mathcal{M}'(0) = \begin{cases} \frac{\delta}{\epsilon} \frac{1}{6 N(\Delta p_i)} = \frac{\delta}{\epsilon} \frac{\text{kur}(\Delta p_i, \ell)}{6 N(\Delta p_i)} & \text{if } \ell = 0 \quad (\text{Gol-Luc}) \\ \frac{\delta}{\epsilon} \frac{6}{6 N(\Delta p_i)} = \frac{\delta}{\epsilon} \frac{\text{kur}(\Delta p_i, \ell)}{6 N(\Delta p_i)} & \text{if } \ell \rightarrow 1 \quad (\text{Calvo}) \end{cases}$$

More analysis shows this generalizes to any  $n \geq 1$  and  $\ell \in [0, 1]$

# Cumulated output effect relative to Calvo

[details](#)


- One curve for each  $n$ , no other parameters; infinite slope at  $\ell = 1$
- Differences are due to *selection* on “time” and “size” of adjustments.



# Robustness and extensions

- Non-zero but small inflation, same result. [▶ details](#)
- Free vs Cheap random cost, same result [▶ details](#)
- Rational Inattentiveness: (observation cost), same result. [▶ details](#)
- Aggregation across sectors with different  $Kur(\Delta p_i)$ ,  $N(\Delta p_i)$  [▶ details](#)
- Implied menu costs, reasonable for  $\ell < 0.9$  and large  $n$ . [▶ details](#)
- Fat-tailed (discontinuous) shocks, different result (identification?) [▶ details](#)
- Large Monetary Shocks, different results. [▶ details](#)

# Lack of sensitivity to Inflation $\mu$

- Inflation has only second order effect around  $\mu = 0$  on
  - entire hazard rate  $h$  and frequency of price changes  $N(\Delta p_i)$ ,
  - marginal distribution of absolute value of price changes  $w(|\Delta p_i|)$ ,
  - all centered even moments of marginal price changes (e.g.  $Kurt(\Delta p_i)$ ).
  - area under output IRF small monetary shock:  $\mathcal{M}'(0)$
  - results on  $\mathcal{M}'(0)$  due to symmetry of  $\mu$  around zero & of  $(\mu, \delta)$  around  $(0,0)$ .
- Thus expression for holds for small inflation rates:

$$\mathcal{M}(\delta; \mu) \approx \frac{\delta}{\epsilon} \frac{Kurt(\Delta p_i; \mu)}{N(\Delta p_i; \mu)} \frac{1}{6}$$

# Lack of sensitivity to Inflation $\mu$

- Static “target” prices have drift  $\mu$ , all price gaps drift down

$$p_i(t) = -\mu t + \sigma W_i(t) + \sum_{j:\tau_j < t} \Delta p_i(\tau_j) \text{ all } t \geq 0, i = 1, \dots, n.$$

- Optimal decision rule are different (no closed form)
  - State is entire vector  $p$ , not just  $y = ||p||^2$ .
  - Prices are not reset to static target at adjustment.
  - Inaction set  $\mathcal{I}$  is *not* a hyper-sphere.
  - GE model: nominal rate  $r + \mu$ , wages grow at  $\mu$ .

# Cheap vs free adjustment cost: Nakumura-Steinsson

- Take the case of  $n = 1$  product.

- cheap random menu cost:

$$\text{adjust the price paying} = \begin{cases} \psi & \text{with probability } 1 - \lambda \, dt \text{ or} \\ b\psi & \text{with probability } \lambda \, dt. \end{cases}$$

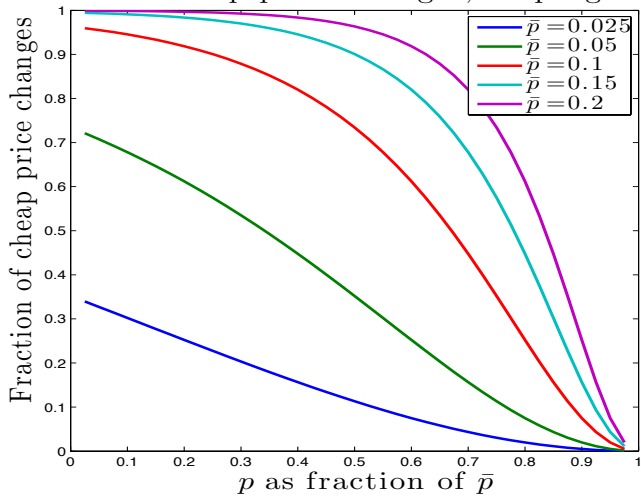
- Cheap adjustment fraction  $b \in [0, 1)$  of normal cost  $\psi$ .
- Optimal policy: two thresholds  $0 < \underline{p} < \bar{p}$ , adjust price if

$$\text{abs. value price gap} = |p(t)| \geq \begin{cases} \bar{p} & \text{and cost } \psi \\ \underline{p} & \text{and cost } b\psi \end{cases}$$

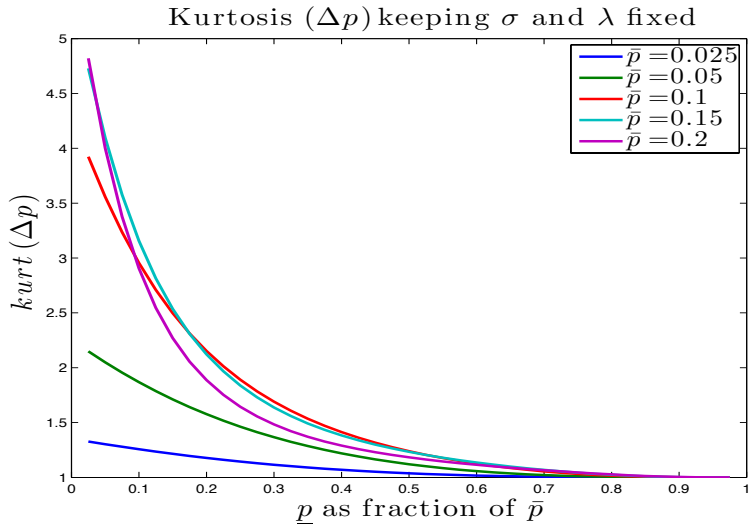
- Eliminates price changes smaller than  $\underline{p}$ , & hence decreases  $Kurt(\Delta p_i)$ .
- Result  $\mathcal{M} \approx \frac{\delta}{\epsilon} \frac{Kurt(\Delta p_i)}{6 N(\Delta p_i)}$  still holds.

# Fraction of cheap price changes for several $\bar{p}$ and $\underline{p}$

Fraction of cheap price changes, keeping  $\sigma$  and  $\lambda$



# Kurtosis for several $\bar{p}$ and $\underline{p}$



# Rational Inattentiveness (no menu cost):

- Caballero, Reis, Carvalho-Schwartzman, Alvarez-Lippi-Paciello.
- Firms observe price gap only if they pay a *random observation costs*.
- Optimal decision rules:
  - set times  $\tau$  until next review = and price adjustment
  - decision cannot depend on (unobserved) current price gaps.
- Randomness of expected observation cost implies random times  $\tau$ .
- Models with  $n = \infty$  have similar formal properties.

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- Randomness of expected observation cost implies random times  $\tau$ .
- Models with  $n = \infty$  have similar formal properties.
- Monetary shock  $\delta$  learned only at times when firms observe their cost.
- Allows for any value of  $Kurt(\Delta p_i)$ , even larger than 6.
- Result  $\mathcal{M} \approx \frac{\delta}{\epsilon} \frac{Kurt(\Delta p_i)}{6 N(\Delta p_i)}$  still holds.



# Implied menu costs (for $r \downarrow 0$ )

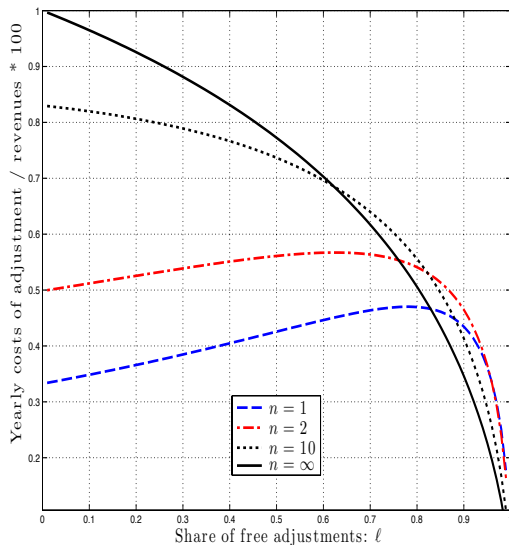
- Fixing  $n$  and  $\ell \in (0, 1)$ , the menu cost  $\psi \geq 0$  :

$$\text{Menu cost per product} = \frac{\psi}{n} = B \frac{\text{Var}(\Delta p_i)}{N(\Delta p_i)} \Psi(n, \ell)$$

- $\Psi(n, \ell)$  is only a function of  $(n, \ell)$ .
- Total cost:  $B = \eta(\eta - 1)/2$  or markup  $m \equiv 1/(\eta - 1)$  then

$$\frac{\text{Yearly costs of price adjustment}}{\text{Yearly revenues}} = \frac{1}{2} \frac{\text{Var}(\Delta p_i)}{m} (1 - \ell) \Psi(n, \ell)$$

# The cost of price adjustment



Levy et al (QJE, 1997) avg. cost per year around 0.7% of revenues.

# Aggregation across heterogeneous sectors

- Let  $e(s)$  the expenditure share of sector  $s$  with different parameters.
- Allow different,  $N(s)$ ,  $Std(s)$  and  $Kurt(s)$  by sector  $s$
- For small  $\delta$  shocks aggregation across sectors yields:

$$\mathcal{M}(\delta) \approx \frac{1}{6} \frac{\delta}{\epsilon} \sum_{s \in S} \frac{e(s)}{N(s)} Kurt(s)$$

- If  $Kurt(s)$  the same across sectors: the model aggregates using expenditure weighted by duration.
- If  $Kurt(s)$  varies across sectors: needs to consider its covariation. In French data we found about 15% higher effect due to this effect.

# Model with fat tailed shocks

- Process for cost: BM  $W$  + Poisson counter  $N$  w/ intensity  $\lambda$

$$dp_i(t) = \sigma dW_i(t) + \xi_i(t) dN(t) \quad \text{for } i = 1, \dots, n$$

- distribution of fat-tailed shock:

$$0 < \underline{\xi} \equiv \inf ||\xi|| \quad \text{with } \Gamma \text{ probability} = 1$$

- Results:

- 1 If the  $\underline{\xi}$  large enough, then threshold  $\bar{y}$  is the same as in baseline model.
- 2 Fat tails contributes to kurtosis by mostly adding large price changes.
- 3 "Lack of identification". (almost) anything goes if  $\Gamma$  unrestricted and  $\psi$  small .

# Effect depends on size of the monetary shock $\delta$

- $\underline{\delta}$  smallest once-and-for-all monetary shock that gives full price flexibility.
- Depends on  $Std(\Delta p_i)$  and  $\ell$ :

$$\underline{\delta} = 2 \sqrt{\frac{\bar{y}}{n}} = 2 Std(\Delta p_i) \sqrt{\frac{\mathcal{L}^{-1}(\ell; n)}{\ell}},$$

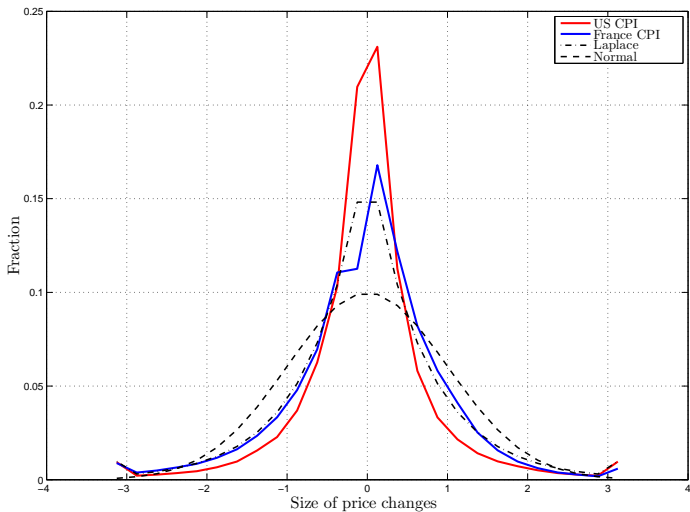
where  $\mathcal{L}(\cdot; n)$ .

- Fixing any  $n \geq 1$ , the ratio

$\frac{\underline{\delta}}{2 Std(\Delta p_i)}$  is a strictly increasing function of  $\ell$

ranges from 1 to  $\infty$  as  $\ell$  goes from 0 to 1.





Pooled Standardized  $\Delta p$ : US (Klenow Krystov) vs France CPI

# Measurement error: CPI vs Scraped Data (France)

- CPI very broad, but subject to measurement error:

Eichenbaum et al: spurious small price changes in CPI and scanner data.

- Our approach:

- Compare “error free” data for selected products with CPI.
- Use internet scraped data for France.
- Match type of goods/outlet with CPI.
- Use simple measurement error model to extrapolate to whole CPI.

- Conclusion: kurtosis is about half.



# Measurement error: CPI vs Scraped Data (France)

Table : Matching CPI vs. the BPP by store x products

| Statistic  | BPP<br>retailer 1 | BPP<br>retailer 5 | CPI<br>Hypermarkets | BPP<br>retailer 4 | CPI<br>Large ret. electr. |
|--|-------------------|-------------------|---------------------|-------------------|---------------------------|
| duration   | 8.58              | 8.06              | 4.82                | 6.44              | 7.24                      |
| <b>Statistics for standardized price changes: <math>z</math></b> |                   |                   |                     |                   |                           |
| mean $ z $   | 0.71              | 0.70              | 0.65                | 0.78              | 0.70                      |
| % below 0.5 mean $ z $   | 37.85             | 40.93             | 45.48               | 29.17             | 41.69                     |
| % below 0.25 mean $ z $  | 17.46             | 25.26             | 26.19               | 15.33             | 23.10                     |
| kurtosis of $z$  | 5.50              | 4.30              | 10.15               | 2.82              | 6.33                      |

# Measurement error: CPI vs Scanner data

|  | France    |                        | US (scanner data) |            |                     |
|--|-----------|------------------------|-------------------|------------|---------------------|
|  | —<br>CPI  | Cavallo-Rigobon<br>BPP | EJR<br>Safe-way   | —<br>IRI   | Midrigan<br>Nielsen |
| Statistics for standardized price changes: $z$ |           |                        |                   |            |                     |
| mean of $ z $                                  | 0.65      | 0.70                   | 0.78              | 0.73       | -                   |
| % below 0.50 mean $ z $                        | 45        | 39                     | 33                | 39         | 29                  |
| % below 0.25 mean $ z $                        | 24        | 21                     | 23                | 25         | 13                  |
| <b>kurtosis of <math>z</math>:</b>             | <b>10</b> | <b>5</b>               | <b>3.0</b>        | <b>4.3</b> | <b>3.5</b>          |

Measurement error model: measured changes = true ( $u$ ) + error ( $\epsilon$ )

true  $u$  w/ pr  $\theta$  and std  $\sigma_u$ , error  $\epsilon$  w/ pr  $1 - \theta$  and std  $\sigma_\epsilon$ :  $\lim_{\sigma_\epsilon \downarrow 0} kurt = \frac{kurt_u}{\theta}$

bottom-line: comparison of BPP vs CPI suggests  $\theta \cong 1/2$  and  $kurt \in (3, 5)$

# French CPI data 2003-2011 for $\Delta p_i$

|   | CPI Data    |           | Benchmarks |         |
|---|-------------|-----------|------------|---------|
|   | all records | exc.sales | Normal     | Laplace |
| Frequency of price changes  | 17.09       | 14.70     |            |         |
| Moments of standardized price changes: $z$                          |             |           |            |         |
| Kurtosis  | 8.89        | 10.40     | 3          | 6       |
| Moments for the absolute value of standardized price changes: $ z $ |             |           |            |         |
| Average: $\mathbb{E}( z )$  | 0.70        | 0.69      | 0.80       | 0.70    |
| Fraction of observations $< 0.25 \cdot \mathbb{E}( z )$             | 22.2        | 20.7      | 16         | 22      |
| Fraction of observations $> 2 \cdot \mathbb{E}( z )$                | 12.9        | 12.5      | 11         | 13      |
| Number of obs. with $\Delta p \neq 0$                               | 1,544,829   | 1,080,183 |            |         |

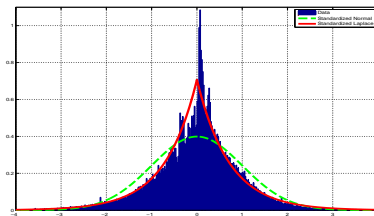
cross-section heterogeneity  $\rightarrow$  standardized price changes

$$z_{i,j,t} = \frac{\Delta p_{i,j,t} - m_{ij}}{\sigma_{ij}}$$

$i$ -good category (270),  $j$ -outlet type (11),  $t$ -time (120 months)

bottom-line: distribution of the standardized  $\Delta p$  closer to Laplace than Normal

## All data



## Excluding sales

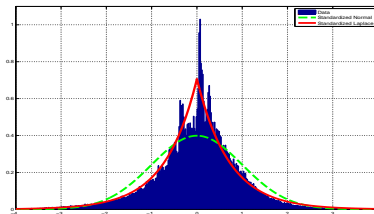


Table : Fraction of small price changes: US and French CPI

|  | Moments for the absolute value of price changes: $ \Delta p $ |           |        |         |
|--|---|-----------|--------|---------|
|  | France  | US        | Normal | Laplace |
| Average $ \Delta p $   | 9.2   | 14.0      |        |         |
| Fraction of $ \Delta p $ below 1%                                      | 11.8  | 12.5      |        |         |
| Fraction of $ \Delta p $ below 2.5%                                    | 32.5  | 24.0      |        |         |
| Fraction of $ \Delta p $ below 5%                                      | 57.1  | 40.6      |        |         |
| Fraction of $ \Delta p $ below $(1/14) \cdot \mathbb{E}( \Delta p )$   | 2.4   | 12.5      | 4.5    | 6.9     |
| Fraction of $ \Delta p $ below $(2.5/14) \cdot \mathbb{E}( \Delta p )$ | 13.5  | 24.0      | 11.3   | 16.4    |
| Fraction of $ \Delta p $ below $(5/14) \cdot \mathbb{E}( \Delta p )$   | 28.7  | 40.6      | 22.4   | 30.0    |
| Number of obs  | 1,542,586   | 1,047,547 |        |         |

Data is NOT standardized

# Useful simplifying assumptions for our problem

## Simplification on firm problem

- unit root shocks (no mean reversion):  $\rightarrow$  state summarized by gaps

## Simplification on eq. structure

- Linear leisure +  $\log(M/P)$  + one-time shock  $\rightarrow$  wages proport. to money

[▶ back to firm-problem](#)

## MENU COSTS AND PHILLIPS CURVES

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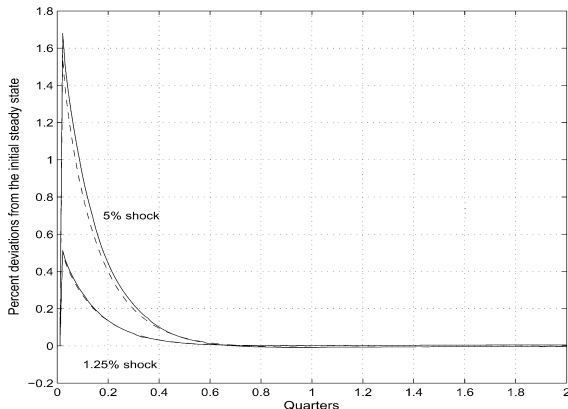
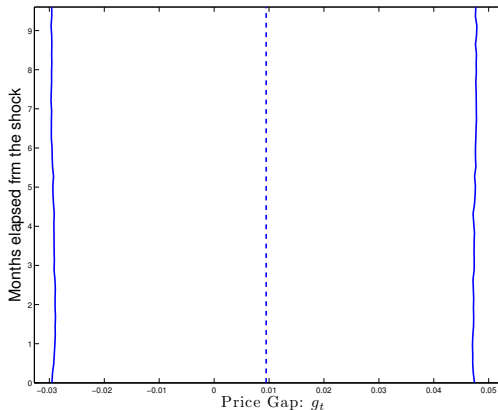


FIG. 7.—Approximate (dashed lines) and exact (solid lines) impulse-response functions: responses of output to a one-time increase (impulse) in the level of money. Initial levels are normalized to one.

- Solid line: fixed point on path of aggregate consumption.
- Dashed line: keeps aggregate consumption at steady state.

# Policy rules in non-linear GE model ( $n = 1, \mu = 0.02$ )

Figure : Threshold rules



Source: Alvarez-Lippi-Paciello (2012)



# Selection on size and time

Compare the average price change of firms that adjust  $t$  periods after shock

- Golosov-Lucas: early on almost all adjustments upwards. [▶ graph](#)
- Selection on size decreases with  $\ell$  and  $n$ .
  - Calvo: adjustments are independent of price gaps.
  - Taylor: adjustments are independent of each price gaps.
  - Any case with  $n = \infty$  avg. price change  $\delta$  every horizon  $t$
  - Difference due to distribution of times to adjust  $\{\tau\}$ .

In general, when there is no selection in size,

$$\mathcal{M}(\delta) = \frac{\delta}{\epsilon} \left[ \frac{1 + CV(\tau)}{2 N(\Delta p_i)} \right]$$

- So higher *variability* of times to adjustments  $\{\tau\}$  increases area under IRF.

# Interpretation as menu cost

- Firms observe profit from  $n$  products, which are proportional to  $\|p\|^2$ .
- Firms don't know profits of each product line separately.
- If they pay  $\psi$  to disentangle profits from each product, and can change prices accordingly.
- In this case  $\psi$  covers other activities than setting new prices.

# Results for Impulse Responses : scaling

$\mathcal{P}_{n,\ell}(\delta, t)$ : aggregate price level  $t$  periods after the monetary shock  $\delta$

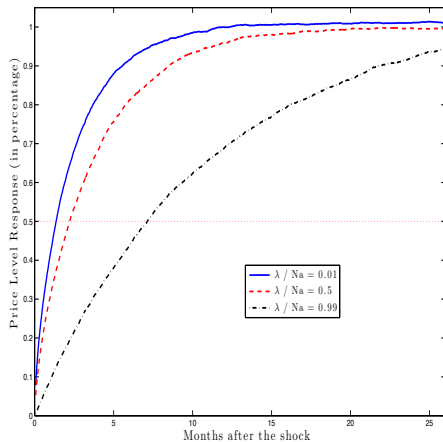
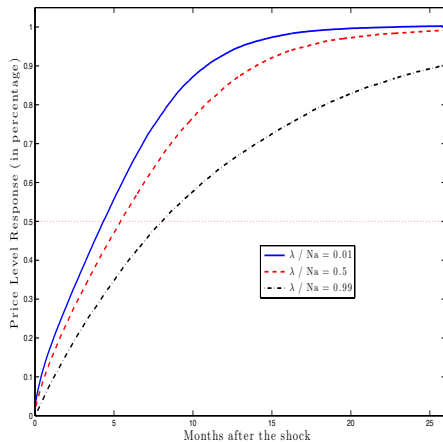
- Scaling and Stretching:

IRF  $\mathcal{P}$  of economy with  $(Std[\Delta p], N(\Delta p_i))$  at  $(\delta, t)$  is

a *scaled* version of one for  $\delta/Std[\Delta p]$  and *stretched horizon*  $N(\Delta p_i) t$ :

$$\begin{aligned} & \mathcal{P}_{n,\ell}(\delta, t; N(\Delta p_i), Std[\Delta p]) \\ &= Std[\Delta p] \mathcal{P}_{n,\ell}\left(\frac{\delta}{Std[\Delta p]}, N(\Delta p_i) t; 1, 1\right) \end{aligned}$$

# $\mathcal{P}_{n,\ell}(\delta, t)$ : Response of CPI to shock $\delta = 1\%$

Economy with  $n = 1$ Economy with  $n = 10$ 

Scales depend on  $N(\Delta p_i) = 2$ ,  $Std(\Delta p) = 0.15$  ; Shape on  $n, \ell$