# OPTIMAL INFLATION TARGETING UNDER ALTERNATIVE FISCAL REGIMES

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Inflation targeting has become an increasingly popular approach to the conduct of monetary policy worldwide since the early 1990s. Most of the countries that have adopted inflation targeting judge the experiment favorably, at least thus far. In many countries, the adoption of inflation targeting has been associated with reductions in both the average level and the volatility of inflation. Inflation targeting has been especially successful in stabilizing inflation expectations. This stems from inflation-targeting central banks' emphasis on a clear medium-term commitment for inflation (while temporary departures from the inflation target are allowed) and their increased communication with regard to the outlook for inflation over the next few years.

This approach to monetary policy, however, may not be equally suitable for all countries, regardless of their existing institutions, the disturbances to which their economy is subject, and the other policies pursued by the government. One question worthy of discussion is how a country's fiscal policies might affect the suitability of inflation targeting as an approach to the conduct of monetary policy.

The theoretical literature that develops the case for inflation targeting largely neglects the fiscal consequences of a commitment to

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1. Levin, Natalucci, and Piger (2004) compare inflation expectations in inflation-targeting and non-inflation-targeting countries.

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an inflation target.<sup>2</sup> The models used to analyze monetary stabilization policy usually abstract from the government's budget and the dynamics of the public debt, so that any fiscal effects of monetary policy decisions are tacitly assumed to be irrelevant. This may be an acceptable simplification if one is choosing a policy for an economy with sound government finances, by which we mean one with relatively nondistorting sources of revenue and an unquestionable political will to maintain government solvency. The degree to which such an idealization of the circumstances of fiscal policy is realistic varies across countries. As inflation targeting becomes popular in developing countries that have recently had serious problems with inflation precisely because of their precarious government finances, one may wonder how safe it is to ignore the interrelation between monetary and fiscal policy choices.

A number of authors suggest that the appropriateness of inflation targeting as a policy recommendation may depend critically on the nature of fiscal policy. For example, Fraga, Goldfajn, and Minella (2004), in their discussion of inflation targeting for developing countries, remark that

"...the success of inflation targeting (...) requires the absence of fiscal dominance" (p. 383).

They go on to stress that not only must fiscal policy be sound in this respect, but its continued soundness must be credible. Their intent is not to suggest developing countries ought not adopt inflation targeting, but rather to emphasize the importance of enacting credible fiscal reforms, as well. Nevertheless, their insistence on the need for fiscal commitments that are not obviously present in many developing countries raises the question of whether inflation targeting is illadvised in such countries.

Sims (2005) enunciates exactly this view. He argues that some countries' fiscal policies may make the achievement of a target rate of inflation by the central bank impossible, in the sense that there is no possible rational-expectations equilibrium in which the target is fulfilled, regardless of the conduct of monetary policy. He further asserts that in such a case, attempting to target inflation may not only be doomed to frustration, but could even be harmful, by leading to less stability (even less stability of the inflation rate) than might

<sup>2.</sup> See, for example, King (1997), Svensson (1997, 1999, 2003), Woodford (2003, chaps. 7–8), Walsh (2003, chap. 11), or Svensson and Woodford (2005) for the theoretical case for some version of inflation targeting as an optimal policy.

have been achieved through other policies. His essential argument is that if the fiscal regime ensures that primary budget surpluses are not (sufficiently) increased in response to a monetary tightening, then a policy intended to contain inflation—namely, raising nominal interest rates sharply when inflation rises above the inflation target—may cause an explosion of the public debt, which ultimately requires even larger price increases than would have been necessary had the debt not grown. Loyo (1999) and Blanchard (2005) provide examples of models in which "orthodox" monetary policies of this kind lead to explosive debt dynamics.

Our goal in this paper is to analyze the character of an optimal monetary policy commitment under alternative assumptions about the character of fiscal policy, in order to determine the conditions under which an optimal policy will be similar to inflation targeting and the extent to which the form of an optimal monetary policy rule depends on the nature of fiscal policy. To address these issues, we extend the framework used to analyze optimal monetary stabilization policy in Benigno and Woodford (2005a), which allows us to explicitly model debt dynamics and the conditions required for intertemporal government solvency and also to treat the effects of tax distortions. We consider a variety of assumptions regarding the character of fiscal policy, including the kind of fiscal regime—under which the real primary budget surplus is not adjusted to prevent explosion of the public debt as a result of an interest rate hike—that is at the heart of the Loyo (1999) and Blanchard (2005) examples of possible perverse effects of tight-money policies.

# 1. A Model with Nontrivial Monetary and Fiscal Policy Choices

We use a standard new Keynesian model of the trade-offs involved in monetary stabilization policy, augmented to take account of tax distortions.<sup>3</sup>

#### 1.1 The Model

The goal of policy is assumed to be the maximization of the level of expected utility of a representative household. In our model, each household seeks to maximize

3. Further details of the derivation of the structural equations of our model of nominal price rigidity can be found in Woodford (2003, chap. 3).

$$U_{t_0} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \tilde{u}\left(C_t; \xi_t\right) - \int_0^1 \tilde{v}\left(H_t\left(j\right); \xi_t\right) dj \right], \tag{1}$$

where  $C_t$  is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods,

$$C_{t} \equiv \left| \int_{0}^{1} c_{t} \left( i \right)^{\theta/(\theta-1)} di \right|^{(\theta-1)/\theta}, \tag{2}$$

with an elasticity of substitution equal to  $\theta > 1$ , and  $H_t(j)$  is the quantity supplied of labor of type j. Each differentiated good is supplied by a single monopolistically competitive producer. We assume that there are many goods in each of an infinite number of industries; the goods in each industry, j, are produced using a type of labor that is specific to that industry, and their prices are also changed at the same time. The representative household supplies all types of labor and consumes all types of goods. To simplify the algebraic form of our results, we restrict attention in this paper to the case of isoelastic functional forms,

$$\tilde{u}(C_t; \xi_t) \equiv \frac{C_t^{1-\tilde{\sigma}^{-1}} \overline{C}_t^{\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}}$$
 and

$$\tilde{v}(H_t; \xi_t) \equiv \frac{\lambda}{1+\nu} H_t^{1+\nu} \bar{H}_t^{-\nu},$$

where  $\tilde{\sigma}, \nu > 0$  and  $\{\bar{C}_t, \bar{H}_t\}$  are bounded exogenous disturbance processes. (We use the notation  $\xi_t$  to refer to the complete vector of exogenous disturbances, including  $\bar{C}_t$  and  $\bar{H}_t$ .)

We assume a common technology for the production of all goods, in which (industry-specific) labor is the only variable input,

$$y_{t}(i) = A_{t}f(h_{t}(i)) = A_{t}h_{t}(i)^{1/\phi},$$

where  $A_t$  is an exogenously varying technology factor, and  $\phi > 1$ . We invert the production function to write the demand for each type of labor as a function of the quantities produced of the various differentiated goods, and we use the identity

$$Y_t = C_t + G_t$$

to substitute for  $C_t$ , where  $G_t$  is exogenous government demand for the composite good. We can then write the utility of the representative household as a function of the expected production plan,  $\{y_t(i)\}$ .

The producers in each industry fix the prices of their goods in monetary units for a random interval of time, as in the model of staggered pricing introduced by Calvo (1983). We let  $0 \le \alpha < 1$  be the fraction of prices that remain unchanged in any period. A supplier that changes its price in period t chooses its new price  $p_t(t)$  to maximize

$$E_t \left[ \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi(p_t(i), p_T^j, P_T; Y_T, \tau_T, \boldsymbol{\xi}_T) \right], \tag{3}$$

where  $Q_{t,T}$  is the stochastic discount factor by which financial markets discount random nominal income in period T to determine the nominal value of a claim to such income in period t, and  $\alpha^{T-t}$  is the probability that a price chosen in period t will not have been revised by period T. In equilibrium, this discount factor is given by

$$Q_{t,T} = \beta^{T-t} \frac{\tilde{u}_c \left( C_T; \xi_T \right) P_t}{\tilde{u}_c \left( C_t; \xi_t \right) P_T} \,. \tag{4}$$

The function,

$$\begin{split} \Pi\left(p,p^{j},P;Y,\tau,\xi\right) &\equiv (1-\tau)pY\left(p/P\right)^{-\theta} \\ &-\mu_{t}^{w} \frac{\tilde{v}_{h}\bigg(f^{-1}\bigg(Y\left(p^{I}/P\right)^{-\theta}/A\bigg);\xi\bigg)}{\tilde{u}_{c}\left(Y-G;\xi\right)} p \cdot f^{-1}\bigg(Y\left(p/P\right)^{-\theta}/A\bigg) \end{split}$$

indicates the after-tax nominal profits of a supplier with price p, in an industry with common price  $p^j$ , when the aggregate price index is equal to P, aggregate demand is equal to Y, and sales revenues are taxed at rate  $\tau$ . Profits are equal to after-tax sales revenues net of the wage bill. The real wage demanded for labor of type j is assumed to be given by an exogenous markup factor  $\mu_t^w$  (which is allowed to vary over time, but is assumed common to all labor markets) times the marginal rate

<sup>4.</sup> We assume that the government needs to obtain an exogenously given quantity of the Dixit-Stiglitz aggregate in each period, in a cost-minimizing fashion. The government thus allocates its purchases across the suppliers of differentiated goods in the same proportion as do households, and the index of aggregate demand,  $Y_p$  is the same function of the individual quantities,  $\{y_t(i)\}$ , as  $C_t$  is of the individual quantities consumed,  $\{c_t(i)\}$ , defined in equation (2).

of substitution between work of type j and consumption, and firms are assumed to be wage takers. We allow for wage markup variations in order to include the possibility of a pure cost-push shock that affects equilibrium pricing behavior while implying no change in the efficient allocation of resources. Variation in the tax rate,  $\tau_t$ , has a similar effect on this pricing problem (and hence on supply behavior), so when the evolution of the tax rate is treated as an exogenous political constraint, variations in the tax rate also act as pure cost-push shocks.

We abstract here from any monetary frictions that would account for a demand for central bank liabilities that earn a substandard rate of return; we nonetheless assume that the central bank can control the riskless short-term nominal interest rate  $i_t$ . This, in turn, is related to other financial asset prices through the arbitrage relation,

$$1 + i_t = (E_t Q_{t,t+1})^{-1}. (5)$$

We assume that the zero lower bound on nominal interest rates never binds under the optimal policies considered below. We therefore do not need to introduce any additional constraints on the possible paths of output and prices associated with the need for the chosen evolution of prices to be consistent with a nonnegative nominal interest rate.

Our abstraction from monetary frictions, and hence from the existence of seignorage revenues, does not mean that monetary policy has no fiscal consequences, since interest rate policy and the equilibrium inflation that results from it have implications for the real burden of government debt. In our baseline analysis, we assume that all public debt consists of riskless nominal one-period bonds. The nominal value,  $B_{\ell}$ , of end-of-period public debt then evolves according to a law of motion.

$$B_t = (1 + i_{t-1})B_{t-1} - P_t s_t, (6)$$

where the real primary budget surplus is given by

$$s_t \equiv \tau_t Y_t - G_t - \zeta_t, \tag{7}$$

where  $\zeta_t$  represents the real value of (lump-sum) government transfers. Rational-expectations equilibrium requires that the expected path

<sup>5.</sup> For a discussion of how this is possible even in a cashless economy of the kind assumed here, see Woodford (2003, chap. 2).

<sup>6.</sup> The consequences of longer-maturity public debt are discussed in section 3.3.

of government surpluses must satisfy an intertemporal solvency condition,

$$b_{t-1} \frac{P_{t-1}}{P_t} = E_t \sum_{T=t}^{\infty} R_{t,T} s_T , \qquad (8)$$

in each state of the world that may be realized at date t, where  $R_{t,T} \equiv Q_{t,T} P_T / P_t$  is the stochastic discount factor for a real income stream.

We consider alternative assumptions about the degree of endogeneity of the various contributions to the government budget in equation (7). In the conventional literature on optimal monetary stabilization policy, both  $G_t$  and  $\tau_t$  are exogenous processes (among the real disturbances to which monetary policy may respond), but  $\zeta_t$  can be adjusted endogenously to ensure intertemporal solvency in a way that creates no deadweight loss, so that the fiscal consequences of monetary policy are not significant for welfare. We also consider a more realistic case in which  $G_t$  and  $\zeta_t$  are exogenous disturbances, and additional government revenue has a positive shadow value, but  $\tau_t$  can be varied endogenously to minimize deadweight loss. In the most constrained case, where the concerns stressed by Sims (2005) arise,  $G_t$   $\tau_t$  and  $\zeta_t$  are all exogenous processes determined by political constraints.

## 1.2 An Associated Linear-Quadratic Policy Problem

We approximate the solution to our optimal policy problem by the solution to an associated linear-quadratic (LQ) problem; the derivation of the approximations is presented in detail in Benigno and Woodford (2004). We show that we can define an LQ problem with the property that the solution to the LQ problem is a linear approximation to optimal policy in the exact model when the exogenous disturbances are small enough.

First, we show that maximization of expected utility is (locally) equivalent to minimization of a discounted loss function of the form

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{1}{2} q_y \left( \widehat{Y}_t - \widehat{Y}_t^* \right)^2 + \frac{1}{2} q_\pi \pi_t^2 \right], \tag{9}$$

where the target output level,  $\hat{Y}_t^*$ , is a function of exogenous disturbances. If steady-state tax distortions are not too extreme, then  $q_y$ ,  $q_\pi > 0$  and the loss function is convex, as assumed in conventional accounts of the goals of monetary stabilization policy.

The constraints on possible equilibrium outcomes are given by log-linear approximations to the structural equations of the model described above. Here we omit derivations and proceed directly to the log-linear forms. First, there is an aggregate supply relation between current inflation and real activity,

$$\pi_t = \kappa \left( \hat{Y}_t + \psi \hat{\tau}_t + c_{\xi}' \xi_t \right) + \beta E_t \pi_{t+1}, \tag{10}$$

where  $\kappa$ ,  $\psi > 0$ . This is the familiar new Keynesian Phillips curve, augmented to include the cost-push effects of variations in the sales tax. We can write the constraint in terms of the welfare-relevant output gap,

$$y_t \equiv \widehat{Y}_t - \widehat{Y}_t^*$$

in which case equation (10) becomes

$$\pi_t = \kappa (y_t + \psi \hat{\tau}_t + u_t) + \beta E_t \pi_{t+1},$$

where  $u_t$  is a composite cost-push term associated with exogenous disturbances other than variations in the tax rate.<sup>7</sup> In other words,

$$\pi_t = \kappa \left[ y_t + \psi \left( \hat{\tau}_t - \hat{\tau}_t^* \right) \right] + \beta E_t \pi_{t+1}, \tag{11}$$

where  $\hat{\tau}_t^*$  is a function of exogenous disturbances that indicates the tax change needed to offset the other cost-push terms.

Another constraint on the possible equilibrium paths of inflation, output, and tax rates is the condition for intertemporal government solvency (equation 8).<sup>8</sup> A log-linear approximation to equation (8) takes the form

7. An obvious source of such disturbances would be variations in the wage markup,  $\mu_t^w$ . This is the only source of variations in  $u_t$  when the steady-state involves no distortions. In the case of a distorted steady state, however, most other kinds of real disturbances also have cost-push effects (as shown in Benigno and Woodford, 2004), since they do not move the flexible-price equilibrium level of output to precisely the same extent (in percentage terms) as they move the efficient level of output. The latter sources of cost-push terms become more important as the magnitude of the steady-state distortions increases.

8. This does not amount to requiring that fiscal policy be Ricardian; we consider below the consequences of non-Ricardian fiscal policies of the kind assumed in the warnings of Sims (2005). Instead, equation (8) is a condition that must hold *in equilibrium* under any policy, and it constrains the possible outcomes that can be achieved in determining the best equilibrium under certain constraints on possible policies.

$$\hat{b}_{t-1} - \pi_t - \sigma^{-1} y_t = -f_t + (1 - \beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ b_y y_T + b_\tau \left( \hat{\tau}_T - \hat{\tau}_T^* \right) \right], \tag{12}$$

where  $f_t$  is a composite of the various exogenous disturbances that we refer to as fiscal stress. We have written the constraint in terms of the output gap and the tax gap,  $\hat{\tau}_T - \hat{\tau}_T^*$ , which indicates departures of the tax rate from the level consistent with complete stabilization of both inflation and the output gap. Therefore, the term  $f_t$  (or, more precisely, the sum  $\hat{b}_{t-1} + f_t$ ) measures the extent to which intertemporal solvency prevents complete achievement of the stabilization goals represented in equation (9).

Here we have substituted equation (4) for the stochastic discount factor (and replaced  $C_t$  by  $Y_t - G_t$ ) to obtain a relation that involves only the initial public debt and the paths of inflation, output, taxes, and the various exogenous variables. The effects of interest rate policy on debt dynamics are the key to the scenarios of Loyo, 1999, and Blanchard, 2005, under which tight money can be inflationary. We take these effects into account through the presence of the stochastic discount factor in equation (8), which is linked to the interest rate controlled by the central bank through equation (5). Interest rates do not appear in equation (12) because we have already substituted for them using the connection between interest rates and the paths of output and inflation that must hold in equilibrium, but the effect of tight money on the burden of the public debt is nonetheless taken into account in this equation.

In writing equation (12) in the form given, we have treated  $\zeta_t$  (real net transfers) as one of the exogenous disturbances that affects the fiscal stress term. For the case in which net transfers are endogenous and can be varied to ensure solvency, we need to separate out the  $\zeta_t$  term from the other (exogenous) determinants of  $f_t$ . The solvency constraint ceases to bind, however, given that the level of transfers affects neither the aggregate supply trade-off (equation 11) nor the loss function (equation 9), so that policymakers are free to vary  $\zeta_t$  as necessary to satisfy equation (12). Thus we do not need to write the solvency constraint, except for the case in which  $\zeta_t$  is exogenous.

# 2. Optimal Inflation Targeting: The Conventional Analysis

We begin by using the framework sketched in the previous section to recapitulate well-known arguments for a form of flexible inflation targeting as a way of implementing an optimal state-contingent monetary policy, highlighting the role of (often tacit) assumptions about fiscal policy in deriving these familiar results. The conventional analysis of optimal monetary stabilization policy in a new Keynesian model corresponds to the case of the above model in which the processes  $\{G_t, \tau_t\}$  are both exogenously given as political constraints on what policy can achieve, while the level of net lump-sum transfers,  $\zeta_t$ , is an endogenous policy variable (along with the short-term nominal interest rate). When lump-sum transfers can be chosen to facilitate stabilization policy, the intertemporal solvency constraint ceases to bind, and it can be omitted from our description of the policy problem. We can similarly omit any reference to the path of the public debt. Moreover, when the level of distorting taxes is given exogenously, we can treat the  $\hat{\tau}_t$  term in equation (10) the same as the other cost-push terms.

The problem of optimal stabilization policy is then simply to find paths  $\{\pi_t, y_t\}$  to minimize equation (9) subject to the single constraint,

$$\pi_t = \kappa(y_t + u_t) + \beta E_t \pi_{t+1}, \tag{13}$$

where the definition of  $u_t$  is now modified to include the cost-push effects of variations in  $\tau_t$  (if these are present). This is the optimal policy problem treated, for example, in Clarida, Galí, and Gertler (1999). Here we emphasize how this conception of the goals of monetary stabilization policy provides an argument for inflation targeting.

A first, simple conclusion about optimal policy under these assumptions is that in the absence of cost-push disturbances, optimal policy would involve adjusting interest rates as necessary to maintain zero inflation at all times. This is easily seen from the fact that if  $u_t = 0$  at all times, equation (13) is consistent with maintaining both a zero inflation rate and a zero output gap at all times, and such an outcome obviously minimizes the loss function (equation 9).

This provides one argument for inflation targeting: if cost-push shocks are unimportant (because distortions from market power and taxes are both small, on average, and fairly stable over time), then a low, stable inflation rate is optimal, regardless of the degree of variability in real activity that this may entail (owing to the effects

<sup>9.</sup> See, for example, Clarida, Galí, and Gertler (1999), Svensson (2003), Woodford (2003, chaps. 7–8; 2004), or Svensson and Woodford (2005) for more detailed presentations of the arguments summarized here.

of preference and technology disturbances on  $Y_t^*$ ). It also implies something of more general validity: even when random cost-push shocks of substantial magnitude do occur, optimal policy should involve zero inflation, on average. (This follows from the previous result using the certainty-equivalence property of linear- quadratic optimization problems.) The optimal long-run inflation target is thus quite low (zero, in our simple model), regardless of the degree of distortions in the economy or the degree to which the optimal output level may exceed the level associated with stable prices. Given that any departures from this constant long-run average inflation rate stemming from cost-push shocks should be transitory, expected inflation in the medium term should always be near zero. Therefore, our result justifies a policy that seeks to maintain low and stable medium-term inflation expectations, as at least one criterion that an optimal policy should satisfy.

The conception of optimal stabilization policy just proposed provides an important reason for a central bank to commit itself to an explicit target for inflation, rather than for other variables (such as real activity), even when cost-push shocks are expected to be nontrivial. In the optimal control of a forward-looking system—the kind of problem just posed above—the advance commitment of policy generally offers advantages by influencing expectations at earlier dates in a way that improves the available stabilization outcomes at those dates. But what aspect of future expectations matter? When the only constraint on what policy can achieve is the aggregate supply relation (equation 13), the only aspect of future expectations that affects the inflation and output gap that can be achieved in some period t is expectations regarding future inflation,  $E_t \pi_{t+1}$ . Hence, this type of commitment is directly relevant: committing to achieve a particular inflation rate in the future, which might be different from what would otherwise be chosen later to best achieve one's stabilization goals. Given that the role of a policy commitment should be to anchor the public's inflation expectations, a commitment regarding future inflation and the central bank's communication of the outlook for inflation are straightforward ways to achieve the benefits associated with an optimal policy commitment.

<sup>10.</sup> See Svensson and Woodford (2003) for a discussion of certainty equivalence in the context of policy problems with forward-looking constraints, like the one considered here.

Beyond these general considerations, one can easily characterize the optimal state-contingent evolution of prices and quantities under a particular assumption about the character of the disturbances affecting the economy (though this aspect of our conclusions depends strongly on the precise details of our assumed model of the transmission mechanism of monetary policy). The following first-order conditions are associated with the policy problem stated above:

$$q_{\pi}\pi_{t} = \kappa^{-1} \left( \varphi_{t} - \varphi_{t-1} \right) \text{ and} \tag{14}$$

$$q_{\nu}y_{t} = \varphi_{t}, \tag{15}$$

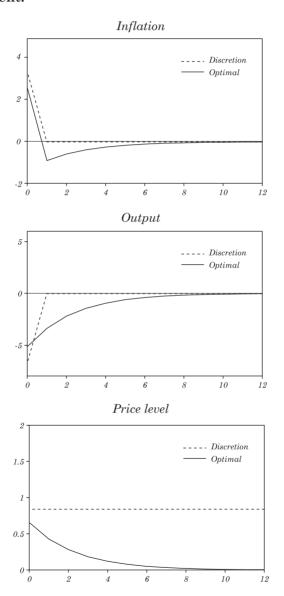
each of which must hold for each  $t \geq 0$ . Here,  $\varphi_t$  is the Lagrange multiplier associated with the aggregate supply constraint (equation 13). We can solve conditions (14) and (15), together with the aggregate supply relation (equation 13), for the optimal evolution of  $\{\pi_t, y_t\}$  given the disturbances  $\{u_t\}$ .

The optimal state-contingent responses can be implemented through commitment to a constant target for the output-gap-adjusted price level:

$$\tilde{p}_t \equiv p_t + \frac{q_y}{\kappa q_x} y_t, \tag{16}$$

where  $\boldsymbol{p}_t$  denotes  $\log P_{t^\prime}$  as discussed in Woodford (2003, chap. 7). A targeting rule of this form determines the optimal trade-off between price increase and output decline that should be selected when the shock occurs; the policy stance should be neither so tight as to cause  $\tilde{p}_t$  to decline (as would be required for there to be no increase in prices) nor so loose as to allow  $\tilde{p}_t$  to rise (as would be required for there to be no reduction in output relative to target output). At the same time, commitment to adhere to such a rule in the future automatically implies invariance of the expected long-run price level and output gap, and it determines the optimal rate of return of both variables to those long-run levels. The output gap should not return to zero too quickly (which would allow prices to remain high and so involve an increase in the gap-adjusted price level) or too slowly (which would cause the gap-adjusted price level to fall once the cost-push disturbance had dissipated). Figure 1 provides an example of the optimal impulse responses of inflation and the output gap to a purely transitory positive cost-push shock (that is, the solution to the first-order conditions listed

Figure 1. Impulse Responses to a Transitory Cost-Push Shock under Discretionary Policy and an Optimal Commitment.



Source: Woodford (2003, chap. 7, fig. 7.3).

above in the case of such a disturbance).<sup>11</sup> The dynamic paths of the log price level and the output gap are perfect mirror images of one another, up to scale, so that  $\tilde{p}_i$  is not allowed to vary.

This is an example of a robustly optimal policy rule in the sense of Giannoni and Woodford (2002): commitment to the same target criterion is optimal, regardless of the statistical properties of the disturbance process. (The optimal dynamic responses shown in figure 1 will be different in the case of a shock that is not completely transitory or not wholly unexpected when it occurs, but the optimal responses of  $p_t$  and  $y_t$  will always mirror one another in the way shown in the figure.) The first-order conditions of equations (14) and (15) can be used directly to show that  $\tilde{p}_t$  must not change over time under an optimal policy, without making any assumptions about the nature of the disturbance.

Such a policy prescription can be viewed as a form of flexible inflation targeting, since the requirement that  $\Delta \, \tilde{p}_t = 0$  can equivalently be written as

$$\pi_t + \frac{q_y}{\kappa q_\pi} \Delta y_t = 0$$

In this form, the rule states that the acceptable rate of inflation at any point in time should vary depending on the rate of change of the output gap. Svensson and Woodford (2005) discuss a more realistic version of this prescription, which incorporates delays in the effects of monetary policy on spending and prices. Here, we are interested in the ways in which this familiar analysis must be complicated under alternative assumptions about fiscal policy.

# 3. OPTIMAL POLICY WHEN ONLY DISTORTING TAXES ARE AVAILABLE: THE CASE OF OPTIMAL TAX SMOOTHING

It is more realistic to assume that lump-sum taxes are not available to offset the fiscal consequences of monetary policy decisions. When we assume the process  $\{\zeta_t\}$  to be exogenously given, the intertemporal solvency condition represents an additional binding constraint on the set of possible equilibrium paths for inflation and output. In Benigno

and Woodford (2004), we consider optimal monetary policy in such an environment, under the assumption that the path of the distorting tax rate,  $\{\tau_t\}$ , is chosen optimally in response to the various types of real disturbances considered in the model. Here we recapitulate the main conclusions of that analysis, before turning to cases in which fiscal policy is assumed to be less flexible or not optimally determined.

In this case, we view monetary and fiscal policy decisions as being jointly determined in a coordinated fashion, so as to solve a single social welfare problem. The planning problem is to find state-contingent paths  $\{\pi_t, y_t, \hat{\tau}_t\}$  to minimize equation (9) subject to the two constraints of equations (11) and (12). An especially simple version of this problem is the limiting case in which prices are perfectly flexible. This case clearly illustrates why the absence of lump-sum taxes can make it optimal for the inflation rate to be highly responsive to fiscal developments, contrary to what inflation targeting is generally assumed to imply. Some authors argue that this kind of analysis is relevant to the choice of monetary institutions in Latin America (Sims, 2002).

### 3.1 Optimal Policy If Prices Are Flexible

In the flexible-price limit of the above model, the coefficient  $q_{\pi}$  in equation (9) is equal to zero, and  $\kappa$  in equation (11) is also zero (that is, the aggregate supply relation is completely vertical). The policy problem reduces to the minimization of

$$\frac{1}{2}q_{y}E_{t_{0}}\sum_{t=t_{0}}^{\infty}\beta^{t-t_{0}}y_{t}^{2},$$
(17)

subject to the constraints

$$y_t + \psi \left( \hat{\tau}_t - \hat{\tau}_t^* \right) = 0 \tag{18}$$

and equation (12). Using equation (18) to substitute for  $y_t$  in equation (17) allows us to equivalently write the stabilization objective as

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( \hat{ au}_t - \hat{ au}_t^* 
ight)^2$$
 ,

in which case the policy objective can be thought of as tax smoothing, as in Barro's (1979) classic analysis. 12

12. Thus our stabilization objective (equation 9) does not omit the concerns of the literature on optimal tax smoothing; the welfare losses associated with a failure to optimally time the collection of taxes are implicit in the output-gap stabilization objective.

The solution will involve  $y_t = 0$  at all times, since it is feasible to achieve this if the monetary and fiscal authorities cooperate. The fiscal authority must choose  $\hat{\tau}_t = \hat{\tau}_t^*$  at all times to ensure this, while the monetary authority must vary the inflation rate,  $\pi_t$ , to ensure government solvency. Equation (12) requires that in such an equilibrium,

$$\pi_t = \hat{b}_{t-1} + f_t \,.$$

Thus unexpected changes in the fiscal stress term must be accommodated entirely by surprise variations in the inflation rate, as in Chari and Kehoe (1999). The tax rate should fluctuate only to the extent that  $\hat{\tau}_t^*$  fluctuates; that is, only to the extent that variations in the tax rate are useful as a supply-side policy, to offset inefficient supply disturbances.<sup>13</sup>

This conclusion implies that an optimal policy will involve highly volatile inflation and extreme sensitivity of inflation to fiscal shocks. This is the basis of Sims' (2002) critique of dollarization as a policy prescription for Mexico; at least a strict form of inflation targeting would presumably be rejected on the same grounds. This analysis, however, neglects the welfare costs of volatile inflation, which are stressed in the literature on inflation targeting. Here we consider the importance of the Chari-Kehoe argument in the presence of a realistic degree of price stickiness.

# 3.2 Optimal Policy If Prices Are Sticky

In the more general case of our model (with some degree of price stickiness), the first-order conditions for the optimal policy problem stated above are

$$q_{\pi}\pi_{t} = \kappa^{-1} \left( \varphi_{1,t} - \varphi_{1,t-1} \right) - \left( \varphi_{2,t} - \varphi_{2,t-1} \right), \tag{19}$$

$$q_{y}y_{t} = \varphi_{1,t} - \left[ (1 - \beta)b_{y} + \sigma^{-1} \right] \varphi_{2,t} + \sigma^{-1} \varphi_{2,t-1}, \qquad (20)$$

$$\varphi_{2,t} = E_t \varphi_{2,t+1}, \text{ and}$$
(21)

13. As shown in Benigno and Woodford (2004), a wide range of inefficient supply disturbances may require such an offset, if the steady state is sufficiently distorted as a result of either market power or a large public debt.

$$\psi \varphi_{1,t} = (1 - \beta)b_{\tau} \varphi_{2,t}, \qquad (22)$$

where now  $\varphi_{1,t}$  is the Lagrange multiplier associated with the aggregate supply relation and  $\varphi_{2,t}$  is the multiplier associated with the intertemporal solvency condition. Conditions (19)–(22), together with the two structural equations (11) and (12), are to be solved for the paths of the endogenous variables,  $\{\pi_t, y_t, \hat{\tau}_t, \hat{b}_t, \varphi_{1,t}, \varphi_{2,t}\}$ , given an exogenous process for  $\{f_t\}$ .

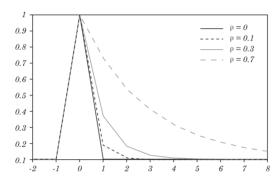
The type of response to shocks implied by these equations can be illustrated using a numerical example. We adopt the same numerical parameter values as in Benigno and Woodford (2004), implying that  $\beta=0.99,\,\sigma^{-1}=0.157,\,\kappa=0.0236,\,\psi=0.397,\,b_\tau=8.33,$  and that the relative weight on output-gap stabilization is  $q_y/q_\pi=0.0024.^{14}$  As in that paper, we examine the effects of an exogenous increase in transfer programs,  $\hat{\zeta}_t$ , equal to one percent of steady-state GDP. Here, however, we consider the consequences of alternative degrees of persistence of such a disturbance; we assume that the value of  $\hat{\zeta}_t$  following the shock is expected to decay at the rate  $\rho^t$ , where the coefficient of serial correlation,  $\rho_t$  is allowed to take values between zero (the case shown in the earlier paper) and 0.7.

Figure 2 shows the impulse response of the shock,  $\hat{\zeta}_{l}$ , for the different values of  $\rho$  considered. Figure 3 then shows the impulse response of the public debt,  $\hat{b}_{l}$ , in response to a pure fiscal shock of this kind under the optimal policy, for each of the alternative values of  $\rho$ . Figure 4 shows the corresponding responses of the tax rate,  $\hat{\tau}_{l}$ , under the optimal policy, and figure 5 the associated responses of the inflation rate. In contrast to the optimal policy in the case of flexible prices (discussed further in Benigno and Woodford, 2004), it is optimal to respond to a pure fiscal shock of this kind by permanently increasing the level of real public debt and planning a corresponding permanent

14. Here the interest-sensitivity of expenditure and the slope of the Phillips curve are calibrated to agree with the econometric estimates of Rotemberg and Woodford (1997) for the US economy, and the fiscal parameters are calibrated to imply that steady-state tax revenues are 20 percent of GDP and that the steady-state public debt is 60 percent of annual GDP. The assumed weights on the two stabilization objectives in the loss function (9) are the ones that correspond to maximization of expected utility, given the parameters of the model, as explained in Benigno and Woodford (2004). Note that in our present notation,  $\pi_\ell$  is a quarterly inflation rate; if we instead write the loss function in terms of an annualized inflation rate, the relative weight on output-gap stabilization would instead be 0.038. This is slightly smaller than the value quoted in Rotemberg and Woodford (1997), mainly as a consequence of the tax distortions assumed here, but abstracted from in that paper.

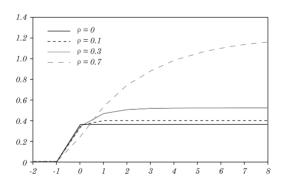
increase in the tax rate. (The increase in the level of real public debt under the optimal policy is more gradual the more persistent the fiscal shock, whereas it was immediate in the case of the purely transitory shock considered in our previous paper.) Optimal policy does involve some unanticipated inflation at the time of the shock, as in the Chari-Kehoe analysis, but it is not nearly large enough to completely offset the fiscal stress, which is why future taxes are also increased.

Figure 2: Alternative Fiscal Shocks



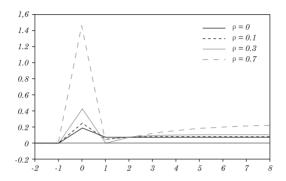
Source: Authors' computations.

Figure 3: Impulse Response of the Public Debt to a Pure Fiscal Shock, for Alternative Degrees of Persistence



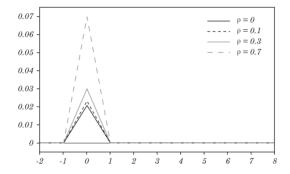
Source: Authors' computations.

Figure 4: Impulse Response of the Tax Rate to a Pure Fiscal Shock, for Alternative Degrees of Persistence



Source: Authors' computations.

Figure 5: Impulse Response of the Inflation Rate to a Pure Fiscal Shock, for Alternative Degrees of Persistence



Source: Authors' computations

As shown in figure 5, the inflationary impact of a fiscal shock under the optimal policy regime is quite small. In the case of a purely transitory (one-quarter) increase in the size of transfer programs by an amount equal to one percent of GDP, optimal policy allows an increase in the inflation rate that quarter of only two basis points (at an annualized rate). <sup>15</sup> Moreover, the increase in inflation is

<sup>15.</sup> The log price level is thus allowed to increase that quarter by only half a basis point.

limited to the quarter of the shock. This compares with an increase in the inflation rate of nearly two percentage points under the optimal policy in the case of flexible prices. This conclusion that the optimal inflation response is small does not depend on an extreme calibration of the degree of price stickiness. In Benigno and Woodford (2004), we show that the optimal response to a purely transitory fiscal shock is similarly small even if prices are assumed to be much less sticky than under the calibration used here; there is a dramatic difference between optimal policy under fully flexible prices and under even slightly sticky prices (that is, the short-run aggregate supply trade-off is not completely vertical). The optimal inflation response is larger if the shock is more persistent, since in this case the cumulative cost of the increased transfers—and thus the total increase in fiscal stress—is several times as large. Even when  $\rho = 0.7$ , however, the optimal increase in the inflation rate is only about seven basis points. Finally, the effect on inflation is purely transitory under optimal policy, regardless of the degree of persistence of the fiscal shock itself.

This last conclusion—that variations in inflation should be purely transitory under the optimal policy, so that the expected rate of inflation never varies at all—is quite robust to the type of shock considered. The conclusion follows directly from the first-order conditions that characterize optimal policy. Condition (19) implies that forecastable variations in the inflation rate should be allowed only to the extent that there are forecastable variations in one or the other of the Lagrange multipliers. Condition (21) implies that there are no forecastable variations in the multiplier associated with the solvency constraint, while condition (22) implies that the two multipliers should covary perfectly with one another, so that there are no forecastable variations in the multiplier associated with the aggregate supply constraint either, under an optimal policy.

The fiscal consequences of monetary policy thus matter if all sources of government revenue are distorting. This creates additional reasons for departures from strict price stability to be optimal. It is now optimal for the inflation rate to vary, at least to some extent, in response to disturbances (such as a change in the size of government transfer programs) that are irrelevant in the classic analysis reviewed in the previous section. Even so, optimal policy continues to possess important features of an inflation targeting regime. The rate of inflation that is forecastable for the

future should never vary, regardless of the kind of disturbances hitting the economy, and the unforecastable variations in inflation that should be allowed are quite small.

It is no longer optimal to target a constant value for the output-gap-adjusted price level,  $\tilde{p}_t$ . In fact, the optimal policy now involves some degree of base drift in the price level, since the transitory inflation shown in figure 5 permanently shifts the price level. Nonetheless, optimal monetary policy can be characterized by commitment to a target criterion that is only a slight generalization of the one presented above for the case of lump-sum taxes. We return to this topic in section 6 below.

### 3.3 Consequences of Additional Fiscal Instruments

The analysis of Benigno and Woodford (2004) assumes that a small and quite specific set of policy instruments are available to the fiscal authority: the only source of government revenue is a proportional sales tax, and the only kind of government debt that may be issued is a very short-term (one-period) riskless nominal bond. Here we briefly discuss the consequences of allowing for additional instruments and, hence, a broader range of possible fiscal policies.

Not surprisingly, additional fiscal instruments, if used skillfully enough, can allow a better equilibrium to be achieved. This can make it simpler to characterize optimal monetary policy, since we no longer have to rely on a limited set of instruments to simultaneously serve multiple stabilization objectives. Suppose, for example, that it is possible to independently vary the level of several different types of distorting taxes. With two distinct tax rates, the cost-push term,  $\psi \hat{\tau}_t$ , in equation (10) becomes  $\psi_1 \hat{\tau}_{1,t} + \psi_2 \hat{\tau}_{2,t}$ , while the term  $b_{\tau} \hat{\tau}_t$  in equation (12) becomes  $b_1 \hat{\tau}_{1,t} + b_2 \hat{\tau}_{2,t}$ . In general, not only will there be different elasticities in the case of different taxes, but the ratios of the elasticities will not be the same in the two equations; the fact that a given percentage increase in one tax rate results in a 20 percent larger increase in revenues than that resulting from a similar increase in a second tax rate does not imply that it also results in a 20 percent larger cost-push effect. The existence of multiple taxes that can be independently varied (and are not at some boundary value under an optimal policy) thus allows the fiscal authority to independently shift the aggregate supply relation and affect the government's budget.

If this is possible, then a lump-sum tax is essentially possible, as some combination of tax increases and decreases will be able to increase tax revenues without any net effect on the aggregate supply relation. 16 We are not, however, returning to the classic situation analyzed in section 2. This setup actually simplifies matters, for tax policy can now be used to offset the cost-push effects of other disturbances, without any consequences for government solvency. Constraint (12) therefore ceases to bind, as in section 2, but tax policy can be used to shift the aggregate supply relation, as in sections 3.1 and 3.2. Optimal policy then involves using taxes to offset the cost-push term,  $u_{\nu}$ , entirely and then applying monetary policy to completely stabilize both inflation and the output gap. (Taxes are also used to ensure that this equilibrium is consistent with intertemporal government solvency.) In such a case, the optimal monetary policy will be a strict inflation target that maintains  $\pi_t = 0$  at all times, regardless of the shocks to which the economy may be subject.<sup>17</sup>

The case for inflation targeting is thus quite strong indeed when tax policy can be varied in any of a range of directions and the fiscal authority can be expected to exercise its power skillfully. This may not be of the greatest practical interest, however. For instance, if the tax rates are each required to be nonnegative, then it may be optimal to raise all revenue using only one tax—namely, the one with the lowest ratio of  $\psi_j$  to  $b_j$  (and thus with the least distortion created per dollar of revenue raised). The optimal policy problem would then end up being similar to the one treated above, in which there is assumed to be only a single type of distorting tax.

Allowing for the possibility of issuing other forms of government debt would also increase the flexibility of fiscal policy and reduce the constraints on what monetary policy can achieve. For example, if it were possible to issue arbitrary kinds of state-contingent debt, then in principle it would be possible to arrange for  $\hat{b}_{t-1}$  to vary with the state that is realized at date t in such a way that  $\hat{b}_{t-1} + f_t$  never varies, regardless of the exogenous disturbances. Complete stabilization of both inflation and the output gap would again be possible, and the optimal monetary policy would be a strict inflation target of zero.

<sup>16.</sup> Here we assume that the various taxes in question affect all sectors of the economy identically, as in the presence of both a sales tax and a wage income tax. Under this assumption, taxes create no distortions other than the effect indicated by the cost-push term in the aggregate supply relation.

<sup>17.</sup> Our ability to achieve the first-best outcome with a sufficient number of taxes is reminiscent of the conclusion of Correia, Nicolini, and Teles (2003) in the context of a model with a different kind of price stickiness.

However, the supposition that state-contingent payoffs on government debt can be arranged with such sophistication is hardly realistic.

One possibility is for countries to use maturity to vary the kind of debt that they issue. If government debt does not all mature in one period, then  $\hat{b}_{t-1}$  is no longer a predetermined state variable; instead, it depends on the market valuation of bonds in period t, which generally depends on the shocks that occur at that date. Since the prices of bonds with different maturities respond distinctly to shocks occurring at date t, different maturity structures of the public debt will have varying effects on the state contingency of  $\hat{b}_{t-\!1}$  . With a sufficient number of maturities available, it may well be possible once again to bring about the kind of state contingency that makes  $\hat{b}_{t-1} + f_t$  independent of shocks, thereby eliminating the need for state-contingent debt, as proposed by Angeletos (2001). Both inflation and the output gap can thus be fully stabilized, and a strict inflation target would again be the optimal monetary policy. To develop these points in more detail, we extend out analysis to allow for the existence of longer-maturity nominal government debt. In the most general case, the intertemporal budget constraint (equation 8) takes the form

$$E_t\!\left(\sum_{T=t}^{\infty}R_{t,T}s_T\right)\!=E_t\!\left(\sum_{T=t}^{\infty}R_{t,T}b_{t-1,T}\,\frac{P_{t-1}}{P_T}\right)\!,$$

where for any  $T \geq t$ ,  $b_{t-1,T}$  denotes the real value at time t-1 of the debt that matures at time T. A log-linear approximation can be computed as before, yielding

$$\hat{b}_{t-1} - E_t \sum_{T=t}^{\infty} d_{T-t+1} \left( \sigma^{-1} y_T + \sum_{s=t}^{T} \pi_s \right) = \\
-f_t + (1-\beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ b_y y_T + b_\tau \left( \hat{\tau}_T - \hat{\tau}_T^* \right) \right].$$
(23)

Here we have defined

$$\hat{b}_{t-1} = \sum_{T=t}^{\infty} \beta^{T-t} \, rac{b_{t-1,T} - ar{b}_{T+1-t}}{ar{b}} \, ,$$

where  $\bar{b}_i$  is the steady-state real value of *i*-period debt, and  $\bar{b}$  is the steady-state real value of all outstanding government liabilities, given by

$$\overline{b} = \sum_{i=1}^{\infty} \beta^{i-1} \overline{b}_i \; .$$

The weights,  $d_i$ , are defined as

$$d_i = rac{eta^{i-1} \overline{b_i}}{\overline{b}}$$

for each  $i \geq 1.$  Finally, the composite fiscal stress term,  $f_{\nu}$  is now defined by

$$f_t = E_t \sum_{T=t}^{\infty} d_{T-t+1} \bigg[ \sigma^{-1} \bigg[ g_T - \hat{Y}_T^* \bigg] \bigg] - (1-\beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} \bigg[ b_{\mathbf{y}} \hat{Y}_T^* + b_{\mathbf{\tau}} \hat{\tau}_T^* + b_{\boldsymbol{\xi}}' \boldsymbol{\xi}_T \bigg],$$

which can be written more compactly as

$$f_{t} = E_{t} \sum_{T=t}^{\infty} d_{T-t+1} h_{\xi}' \xi_{T} + (1-\beta) E_{t} \sum_{T=t}^{\infty} \beta^{T-t} f_{\xi}' \xi_{T} , \qquad (24)$$

again using the notation defined in Benigno and Woodford (2004).

The planning problem is to find state-contingent paths  $\{\pi_t, y_t, \hat{\tau}_t\}$  to minimize equation (9) subject to constraints (11) and (23). As before, the composite disturbance,  $f_t$ , completely summarizes the information at date t about the exogenous disturbances that interfere with complete stabilization of inflation and the output gap. In contrast to the case of one-period debt, output and inflation can now be stabilized at their optimal level even when prices are sticky by appropriately choosing the steady-state structure of maturity. This is because the stochastic properties of the fiscal stress term now depend on the maturity structure. With an appropriate choice of the maturity structure, one can even ensure that  $f_t$  is identically equal to zero at all times, in which case complete achievement of both stabilization objectives will be possible.

Let government debt have a maximum maturity of N periods and let J be the number of stochastic disturbances of the model. Let us further suppose (purely for illustrative purposes, for our argument could easily be generalized) that the disturbances are all first-order autoregressive, or AR(1), processes,

$$\xi_t^j = \rho_i \xi_{t-1}^j + \varepsilon_t^j,$$

where  $\varepsilon_t^j$  is a white-noise process and  $|\rho_j| < 1$  for each disturbance j. In this case, equation (24) takes the form

$$f_t = \sum_{i=1}^{N} d_i \sum_{j=1}^{J} \rho_j^i h_{\xi_j} \xi_t^j + (1-\beta) \sum_{j=1}^{J} (1-\rho_j \beta)^{-1} f_{\xi_j} \xi_t^j ,$$

where  $h_{\xi_j}$  and  $f_{\xi_j}$  are the jth components of the vectors  $h_{\xi}$  and  $f_{\xi}$ , respectively.

It now follows (generically) that for  $\boldsymbol{f}_t$  to be zero at all times, it is necessary and sufficient that

$$\sum_{i=1}^{N} \rho_j^i d_i = z_j, \tag{25}$$

where  $z_i$  is defined by

$$z_{j} = (1-\beta)(1-\rho_{j}\beta)^{-1}h_{\xi_{j}}^{-1}f_{\xi_{j}}$$

for each j. Then the set of J equations (25) together with the identity

$$\sum_{i=1}^{N} d_i = 1, (26)$$

forms a set of J+1 equations in the N unknowns,  $\{d_i\}$ . We can write this system of linear equations using matrix notation. To this end, we define the matrix

$$\mathbf{A} \equiv \begin{bmatrix} 1 & \rho_1 & \dots & \rho_1^{N-1} \\ 1 & \rho_2 & \cdots & \rho_2^{N-1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \rho_J & \cdots & \rho_J^{N-1} \\ 1 & 1 & \cdots & 1 \end{bmatrix},$$

and let  $\mathbf{z}$  be the vector whose first J elements are the  $z_j$ , and whose final element is 1. We can then write the system of linear equations in the compact form,

$$\mathbf{Ad} = \mathbf{z} \,, \tag{27}$$

where  ${\bf d}$  is the vector of coefficients  $d_i$ . Standard results ensure that there is a solution of equation (27) as long as  ${\bf A}$  is of full rank. In this case, there is at least one vector  ${\bf d}$ —that is, at least one steady-state maturity structure—such that  $f_t=0$ , so that complete stabilization of both inflation and the output gap can be achieved.

In particular, if N=J+1, there is exactly one solution for any given  $\mathbf{z}$ , when  $\mathbf{A}$  is of full rank. For example, in the case of a single stochastic disturbance (J=1), the matrix  $\mathbf{A}$  is always of full rank, and the first-best outcome can be achieved simply by issuing nominal debt with one- and two-period maturities. The optimal maturity structure in this case depends on the persistence of the shock, as well as on its contribution to movements in the fiscal stress measure,  $f_t$ . If J>1,  $\mathbf{A}$  is of full rank if and only if  $\rho_i\neq\rho_j$  for each i and j. (Otherwise there generally is no solution.)

Angeletos (2001) shows in a flexible-price model that to complete the markets, it is necessary and sufficient to issue nominal debt that has at least N-period maturity, where N is the number of states of nature in the model. Here we establish that in a log-linear model, what matters is not the number of distinct states of nature, but only the number of stochastic disturbances, as Angeletos conjectured on the basis of his numerical results. As long as debt can be issued in moderately long maturities, it will generally be possible, at least in principle, to choose a maturity structure that achieves the first-best outcome. The optimal monetary policy will simply aim at complete price stability, while the distorting tax rate will be used to offset cost-push disturbances, so that zero inflation is compatible with a zero output gap.

As Buera and Nicolini (2004) note in a related context, however, the maturity structure required for such an outcome may be implausible, involving very large long and short positions in different maturities. They also show that the optimal maturity structure may be extremely sensitive to small changes in model parameters, such as small changes in the serial correlation of disturbance processes. Here again, while in principle the opportunity to increase the flexibility of fiscal policy in this way can greatly facilitate monetary stabilization policy, the practical relevance of this case is open to question. We accordingly restrict the remainder of our analysis to the case of a single maturity of government debt, specifically, one very short-term (single-period) debt. In fact, most

<sup>18.</sup> This can be seen from our analysis above, since a small change in these parameters can cause the rank condition to fail.

countries with serious fiscal imbalances issue almost exclusively short-maturity debt, so our assumption seems likely to represent the most relevant case for the countries facing the concerns addressed in this paper. This emphasis is also consistent with our desire to consider the cases in which possible constraints on fiscal policy are most likely to create problems for inflation targeting. The presence of a larger number of fiscal instruments, or fewer constraints on how they are used, will generally strengthen the case for inflation targeting. Our interest, however, is in the extent to which a form of inflation targeting continues to be desirable even when fiscal policy is much less helpful.

# 4. Optimal Monetary Policy When Fiscal Policy Is Exogenous

This section explores a still more constrained case, in which  $\{G_t, \zeta_t, \tau_t\}$  are all assumed to be exogenous processes, determined by political factors that the central bank cannot influence. This is the type of fiscal policy assumed by Loyo (1999), which Sims (2005) uses in his critique of inflation targeting. In a flexible-price model such as Loyo's, this policy implies a purely exogenous evolution of the real primary government budget surplus,  $\{s_t\}$ . The central bank must beware that a tight-money policy does not cause explosive growth of the public debt, for it is assumed that neither taxes nor government spending will be adjusted to prevent such dynamics.

In this case, the intertemporal solvency condition (equation 12) constrains the possible paths for inflation and output that can be achieved by any monetary policy, and no endogenous fiscal instruments are available to adjust this constraint. At the same time, the possible paths for inflation and output are constrained by the aggregate supply trade-off (equation 11), and there is no endogenous fiscal instrument that can shift this relation either, in contrast with the assumption in the previous section. The central bank's ability to achieve its inflation and output-gap stabilization objectives is accordingly more tightly constrained.

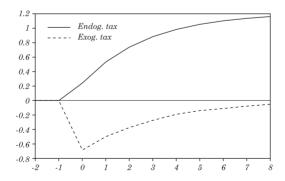
As Sims (2005) notes, full price stability (or even complete stabilization of the inflation rate at some nonzero value) will typically be infeasible under these assumptions—unlike the situation considered in the previous section, where this is a possible, though not quite optimal, monetary policy. Condition (11) allows us to easily derive the unique output-gap process consistent with complete stabilization of the inflation rate. However, the process  $\{y_t\}$  obtained in this way (together with the assumed constant inflation rate and the exogenously given

tax process) will almost surely not satisfy the intertemporal solvency condition (equation 12) for all possible realizations of the disturbances that affect the fiscal stress term,  $f_t$ . This does not mean that monetary policy is powerless to stabilize either nominal or real variables. While one cannot commit to completely stable inflation both immediately and for the indefinite future, policymakers can choose among alternative paths for inflation, some of which involve inflationary spirals of the sort modeled by Loyo, and others of which involve a fairly quick return to price stability. Here we consider the central bank's optimal choice among the set of possible equilibria, given the constraints implied by exogenous fiscal policy.

The optimization problem in this case is to find paths,  $\{\pi_t, y_t\}$ , that minimize equation (9) subject to the constraints in equations (11) and (12), in which we now treat  $\{\hat{\tau}_t\}$  as another exogenous disturbance process. The first-order conditions for this optimization problem are the same as before (conditions 19–21). The only difference is that condition (22) need no longer hold (as the tax rate need not be chosen optimally); this condition is replaced by the exogenously given process,  $\{\hat{\tau}_t\}$ .

Optimal state-contingent responses to exogenous disturbances of various types can easily be derived in this case, using the same methods as in the previous section. For purposes of illustration, we again consider a pure fiscal shock, by which we mean an exogenous increase in the size

Figure 6: Impulse Response of the Public Debt under Optimal Monetary Policy and Two Assumptions about Tax Policy<sup>a</sup>

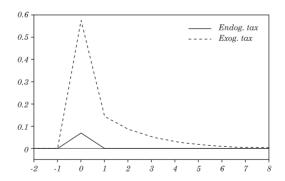


a. The figure shows the impulse response of the real public debt to a pure fiscal shock under optimal monetary policy, both under the assumption that tax policy also responds optimally (solid line; same as in figure 3) and under the assumption that the path of the tax rate does not respond at all (dashed line).

of government transfer programs. To simplify our figures, we present results only for the case of  $\rho=0.7$ . Figure 6 shows the impulse response of the real public debt to such a shock under optimal monetary policy, both under the assumption that tax policy also responds optimally (as in the previous section) and under the assumption that the path of the tax rate does not respond at all. Figure 7 shows the impulse response of the inflation rate under optimal monetary policy, under the same two alternative assumptions about fiscal policy.

As figure 7 indicates, the degree to which it is optimal to allow a fiscal shock to affect the inflation rate is much greater when tax policy cannot be expected to adjust in response to the shock. The optimal immediate effect on the inflation rate is about eight times as large, in our calibrated example, under the exogenously given path for the tax rate; it is also slightly more persistent, so the inflation rate expected over the next few quarters should be allowed to rise slightly in response to such a shock. The larger immediate increase in inflation means that the reduction of the real burden of the public debt through unexpected inflation plays a bigger role in offsetting the fiscal stress in this case. This is necessary because under the assumption of an exogenous path of taxes, the long-run level of the real public debt cannot be increased (as would occur under the optimal fiscal policy); instead, it must continue to equal the unique level consistent with

Figure 7: Impulse Response of the Inflation Rate under Optimal Monetary Policy and Two Assumptions about Tax Policy<sup>a</sup>



a. The figure shows the impulse response of the real public debt to a pure fiscal shock under optimal monetary policy, both under the assumption that tax policy also responds optimally (solid line; same as in figure 3) and under the assumption that the path of the tax rate does not respond at all (dashed line).

intertemporal solvency given the expected long-run tax rate. As shown in figure 6, the level of the real public debt must fall, rather than rise, in response to the fiscal shock so that it can approach its unchanged long-run level from below. (The real public debt must be expected to grow over the quarters in which the size of transfer programs is still temporarily high, but this is no longer a surprise.) This can occur only through a sufficiently large surprise increase in inflation in the quarter in which the shock occurs, just as under the optimal policy for the flexible-price economy analyzed by Chari and Kehoe (1999).

Even under this extreme assumption about the nonresponsiveness of tax policy, an optimal monetary policy does not involve too great an increase in inflation in response to a disturbance that increases fiscal stress. In the case of the shock considered in figure 7, the cumulative increase in the price level is still only about a quarter of a percentage point, whereas the price increase under optimal policy for the flexible-price economy would be about six times as large. Even when tax increases do not contribute to relieving fiscal stress at all, less inflation is required to maintain intertemporal solvency in the case of a sticky-price economy, because inflationary policy stimulates real activity. The resulting higher real incomes imply higher tax revenues, which contribute substantially to government solvency in the equilibrium shown by the dashed lines in figures 6 and 7.

This illustrates an important benefit of an appropriately managed inflation-targeting regime, even when fiscal policy is purely exogenous, as in the pessimistic case considered by Sims (2005). The central bank is able to maintain intertemporal solvency without too much inflation in our example precisely because inflationary expectations are contained even while transitory inflation is allowed to erode the real value of existing nominal claims on the government. If expected inflation does not increase much at the time of the fiscal shock, the aggregate supply trade-off (equation 11) implies a relatively large increase in real output for a given increase in the current inflation rate, so a substantial improvement in government solvency can be obtained without too much inflation. If, instead, the expected future inflation rate were to rise as much as the current inflation rate (or even more), the increase in real activity resulting from inflationary monetary policy would be tiny or nonexistent—or even of the opposite sign. In that case, tax revenues would increase little if at all, and all of the fiscal stress would have to be offset through a reduction in the real value of the public debt owing to unexpected inflation; the required immediate increase in inflation would then be many times larger.

We can illustrate this trade-off quantitatively by considering alternative responses to a disturbance to the fiscal stress. <sup>19</sup> Suppose that in response to such a shock in period t, monetary policy allows the path of inflation to change in such a way that

$$E_t \pi_{t+j} - E_{t-1} \pi_{t+j} = \tilde{\pi}_t \lambda^j,$$

for all  $j \geq 0$ , for some initial inflation response,  $\tilde{\pi}_l$ , and some persistence factor,  $0 \leq \lambda \leq 1$ . In addition, suppose for simplicity that the disturbance does not change the expected path of the tax gap, <sup>20</sup>

$$E_t \Big( \hat{ au}_{t+j} - au_{t+j}^* \Big).$$

For any choice of  $\lambda$ , there exists a unique value of  $\tilde{\pi}_t$  (given the size of the shock at date t) such that this represents a possible equilibrium response under a suitable monetary policy. We can then consider how  $\tilde{\pi}_t$ —and hence the entire path of the inflation response—varies with the choice of  $\lambda$ .

Solving equation (11) for the implied response of the output gap, we find that

$$E_t y_{t+j} - E_{t-1} y_{t+j} = \frac{1 - \beta \lambda}{\kappa} \tilde{\pi}_t \lambda^j,$$

for each  $j \ge 0$ . Substituting this and the conjectured inflation response into the intertemporal solvency condition (equation 12), we find that the condition is satisfied if and only if

$$\tilde{\pi}_t = \frac{\tilde{f}_t}{1 + \sigma^{-1} (1 - \beta \lambda) + (1 - \beta) b_y / \kappa}.$$
(28)

This indicates how the initial effect on inflation relates to the expected degree of persistence of the shock's effect on the inflation rate.

19. This might be the pure fiscal shock considered in the numerical examples presented, but it might also be any other kind of exogenous disturbance that affects the term  $f_t$ .

20. If the path of the tax gap also changes, a derivation like the one sketched below is again possible, except that in the numerator of equation (28), instead of  $\tilde{f}_t$ , one has  $\tilde{f}_t$  plus a multiple of the present value of changes in the expected tax gap. The conclusions obtained below on how  $\tilde{\pi}_t$  depends on the value of  $\lambda$  continue to apply.

A higher value of  $\lambda$  makes the denominator of equation (28) a smaller positive quantity, meaning that  $\tilde{\pi}_t$  must be larger. Thus a policy that makes the shock's effect on inflation more persistent will involve a larger initial effect on inflation, as well as (a fortiori) a larger effect on inflation at all later dates.

Even under the constraints assumed in this section, the central bank should credibly commit itself to restoring low inflation relatively soon after a disturbance that creates fiscal stress. This requires both that monetary policy be clearly focused on inflation control and that the central bank's commitment to an essentially constant medium-term inflation target be unwavering, even when fiscal stress requires a short-run departure from the medium-term target. The credibility of such a commitment will be greater to the extent that the central bank is able to explain why the size of departure that is currently occurring is consistent with the principles to which it is committed, rather than representing an abrogation of those principles or a concession that they are frequently inapplicable. We next consider the formulation of a more flexible form of target criterion that would be suitable for this purpose.

### 5. An Optimal Targeting Rule for Monetary Policy

We have argued that even in the case of severe constraints on the degree to which an optimal adjustment of tax policy can be expected. an optimal monetary policy will involve a commitment not to allow temporary increases in inflation to persist, so that medium-term inflation expectations remain well anchored. This raises the question of what kind of commitment regarding the future conduct of monetary policy would serve this purpose, without appearing to promise different conduct in the future than what is exhibited in the present—a promise that would not easily be made credible. The answer, in our view, is that monetary policy should be conducted in such a way as to seek at all times to conform to an appropriately formulated target criterion. The target criterion should both explain how much inflation can be allowed in the short run, in response to a given type and size of disturbance, and guarantee (if it is expected to be followed in the future, as well) that no significant fluctuations in the inflation rate should be forecasted more than a few quarters into the future.

To identify a criterion that will serve this purpose under each of the assumptions about the fiscal regime considered above and for all the different types of disturbances that might affect the economy, we use the method illustrated in section 2—that is, we use the first-order conditions that characterize optimal policy to derive a target criterion that must be satisfied in an optimal equilibrium. Conditions (19)—(21) must hold if monetary policy is optimal, under all the fiscal regimes considered thus far. Consequently, a target criterion that follows from (and, in turn, guarantees) these conditions will be a criterion for the optimality of monetary policy that will generally be useful. Since the first-order conditions also apply regardless of the nature of the (additive) exogenous disturbances that may perturb the model structural relations, the resulting criterion is also robust to alternative assumptions about the statistical properties of the disturbances, as stressed by Giannoni and Woodford (2002).

A robustly optimal target criterion that is equivalent to demanding the existence of Lagrange multiplier processes  $\{\varphi_{1t}, \varphi_{2t}\}$  that satisfy equations (19)–(21) can be formulated as follows. As in the simpler case treated in section 2, optimal policy can be described in terms of commitment to a target for the output-gap-adjusted price level,  $\tilde{p}_t$ , defined in equation (16). The central bank should use its policy instrument to ensure that each period,  $p_t$  satisfies

$$\tilde{p}_{t} = p_{t-1}^* + (1+\eta)(p_t^* - p_{t-1}^*), \tag{29}$$

where

$$\eta \equiv \frac{\sigma^{-1}}{(1-\beta)b_{\nu} + \kappa} > 0$$

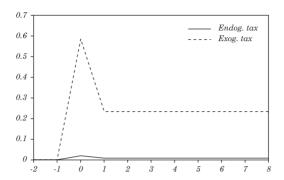
and  $p_t$ \* is the central bank's estimate (conditional on information at t) of the long-run (output-gap-adjusted) price level consistent with intertemporal government solvency.

Implementation of policy in accordance with this criterion would require the central bank to estimate the current value of the long-run price-level target,  $p_t^*$ , as part of each decision cycle. This would be determined, in principle, as follows. Equation (29) implies that

<sup>21.</sup> Further details of the derivation are given in Benigno and Woodford (2005b), where we also discuss the form of targeting rule that is appropriate under a broader class of possible assumptions about fiscal policy.

<sup>22.</sup> These conditions also hold in the case of lump-sum taxes, as assumed in section 2, but with the additional condition that  $\varphi_{2t} = 0$  at all times, which allows the first-order conditions to be reduced to the system of equations (14) and (15).

Figure 8: Optimal Response of the Output-Gap-Adjusted Price Level under the Two Polar Assumptions about Fiscal Policy



Source: Authors' computations.

$$E_t \tilde{p}_T = p_t^*$$

for all  $T \geq t+1$ . A value for  $p_t^*$  thus implies not just a value for  $\tilde{p}_t$ , but a complete expected path,  $\{E_t\tilde{p}_T\}$ , for all  $T \geq t$ . The central bank's model of the economy—including its model of the behavior of the fiscal authority—can then be used to derive the implied forecast paths for the other endogenous variables corresponding to a given current estimate of  $p_t^*$ . The right estimate of  $p_t^*$  is then the one that leads to a set of forecast paths consistent with intertemporal government solvency.

The degree to which  $p_t^*$  will be found to increase in response to a given disturbance depends on the nature of the fiscal regime. Figure 8 shows the optimal responses of the path of the output-gap-adjusted price level for both an endogenous (optimal) and an exogenous path for the tax rate, for the same kind of real disturbance as in figures 6 and 7. In both cases, the shape of the optimal response of this variable is the same; the response is simply scaled in proportion to the different-sized jump in the long-run price level.  $^{23}$ 

The same would be true if we were to plot optimal responses to other types of exogenous disturbances, or if we assumed a different

<sup>23.</sup> The same is true when the tax rate is predetermined for a certain period of time, after which it adjusts optimally (see Benigno and Woodford, 2005b). In such a case, the size of the response is intermediate between the two cases shown in figure 8.

degree of persistence of the disturbance; this is the feature of optimal policy that allows such a simple target criterion to provide a robust guide for policy. The same kind of criterion also applies in the case of lump-sum taxes, as assumed in section 2. In this case, however, there is never any need to vary the long-run price-level target to ensure solvency, so equation (29) applies with  $p_t^*$  equal to a constant  $p^*$ .

Implementing this kind of targeting procedure requires the central bank to make projections not only of the future evolution of prices and real activity, but also of the evolution of the government finances and the public debt, so as to evaluate the consistency of alternative monetary policies with intertemporal government solvency. Some may fear that this sounds like a prescription for exactly the sort of fiscal dominance of monetary policy against which Fraga, Goldfain, and Minella (2004) warn. It is true that we have described a regime under which monetary policy could be conducted in a constrained-optimal way, even if the fiscal authority were understood to be completely unwilling ever to adjust fiscal instruments to maintain intertemporal solvency. However, the knowledge that the central bank reasons in this way should not provide an incentive for the fiscal authority to be profligate, relying on the central bank to adjust monetary policy as necessary to accommodate any degree of spending. Under the regime proposed here, the central bank would make its own judgment regarding the degree of fiscal adjustment that could properly be expected, given the constraints under which fiscal policy is expected to be determined, and then target a path for the output-gap-adjusted price level accordingly. It would be appropriate for the central bank to publicize the projections that serve as the basis for this decision. Among other things, this would inform the fiscal authority of the degree of eventual revenue increases expected by the central bank, which will be necessary to maintain intertemporal solvency given the central bank's target path for the gap-adjusted price level.

#### 6. Conclusions

The nature of fiscal policy has important consequences for the optimal conduct of monetary policy, for two reasons. On the one hand, monetary policy has consequences for the intertemporal solvency of the government under a given fiscal policy, so a change in monetary policy can require corresponding changes in fiscal policy, which will

have welfare consequences if all sources of government revenue are distorting. On the other hand, fiscal policy decisions generally have supply-side consequences that affect the available trade-off between inflation stabilization and the central bank's ability to stabilize the welfare-relevant output gap. Hence, alternative assumptions about the set of instruments available to the fiscal authority and the flexibility and accuracy with which they will be adjusted can greatly change the complexity of the challenges involved in monetary stabilization policy.

Nonetheless, we have argued that it is possible to prescribe an optimal approach to the conduct of monetary policy that is applicable to a range of different assumptions regarding fiscal institutions and the character of fiscal policy. The problem of monetary stabilization policy is likely to be more complex, under realistic assumptions about fiscal policy, than in familiar analyses that abstract altogether from interactions between monetary and fiscal policy decisions. We found, however, that even under considerably more general assumptions, an optimal monetary policy has important aspects of a flexible inflation-targeting regime.

Under all of the regimes considered, optimal monetary policy can be implemented through a commitment to use policy to guarantee fulfillment of a target criterion, which specifies the acceptable level of an output-gap-adjusted price level given the central bank's current projections of the economy's possible future evolution. A credible commitment to such a rule should serve to anchor inflation expectations. As we have seen, commitment to the target criterion implies that there should be no forecastable variation in the growth rate of the output-gap-adjusted price level over any horizons beginning a quarter or further in the future. This means that any variations in the inflation forecast must be fully justifiable in terms of the projected change in the output gap over the same horizon. Moreover, since forecastable changes in the output gap over periods more than a few quarters in the future will always be negligible, this implies that medium-term inflation forecasts must essentially be constant.

Thus an important feature of an optimal policy commitment is a credible commitment by the central bank to return inflation to its long-run target level fairly promptly after any unforeseen disturbance that justifies a temporary departure from that target. When the set of available fiscal instruments is fairly constrained, it is important to allow for temporary variations in the inflation rate in response to exogenous disturbances, and disturbances that affect the economy

mainly through their impact on the government budget are among the types of disturbances that should be allowed to have a transitory effect on the inflation rate. Nevertheless, even while the central bank allows such disturbances to affect the current rate of inflation (and its current target for the gap-adjusted price level), it should stress the fact that the size of the one-time effect on prices is calculated to be consistent with a prompt stabilization of prices. The development of an explicit calculus that can be used to justify temporary departures from the inflation target, which would have been maintained in the absence of the shock, is an important project for adapting the practice of inflation targeting to the circumstances of countries with frequent and urgent fiscal imbalances.

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