Distress Dependence
and Financial Stability

Miguel A. Segoviano
Comisión Nacional Bancaria y de Valores, México

Charles A.E. Goodhart
London School of Economics

The proper estimation of distress dependence amongst the banks in a system is key to monitoring the stability of the banking system. Financial supervisors recognize the importance of assessing not only the risk of distress, i.e. large losses and possible defaults by a specific bank, but also the impact that such an event would have on other banks in the system. Clearly, an event involving simultaneous, large losses in several banks would affect the stability of the whole system, and thus represents a major concern for supervisors. Banks' distress dependence is based on the fact that banks are usually linked, either directly, through the inter-bank deposit market and participation in syndicated loans, or indirectly, through lending to the same sectors and proprietary trades. Their distress dependence varies throughout the economic cycle and tends to rise in times of distress, since the fortunes of banks decline concurrently through either direct links, that is, contagion after idiosyncratic shocks, affecting inter-bank deposit markets and participation in syndicated loans, or indirect links, that is, negative systemic shocks, affecting lending to common sectors and proprietary trades. At such times, the banking system's

joint probability of distress (JPoD, defined as the probability that all banks in the system will experience large losses simultaneously or banks' distress dependence), may experience larger, nonlinear increases than those experienced by the probabilities of distress (PoDs) of individual banks. Consequently, it becomes essential for the proper estimation of the banking system's stability to incorporate banks' distress dependence and its changes across the economic cycle.

Based on Segoviano and Goodhart (2009), in this paper we estimate a set of banking stability measures (BSMs) that express the interdependent structure of bank distress, capturing both linear (correlation) and nonlinear distress dependencies among the banks in the system. Moreover, the structure of linear and nonlinear distress dependencies shifts as banks' probabilities of distress (PoDs) change; hence, the proposed stability measures incorporate changes in distress dependence consistent with the economic cycle. This is a key advantage over traditional risk models, most of which incorporate only linear dependence (correlation structure), assuming it remains constant throughout the economic cycle.[1]

The proposed BSMs represent a set of tools to analyze (define) stability from three different, yet, complementary perspectives, as they quantify: (i) "tail risk" in the banks within a system, (ii) distress between specific banks, and (iii) cascade effects, defined as distress throughout the associated system, triggered by the distress of a specific bank.

As described below, the authors conceptualize the banking system as a portfolio of banks comprising the core banks of systemic importance in any country. We then estimate the banking system portfolio's multivariate density (BSMD), based on which we construct a set of banking stability measures (BSMs). We show how these BSMs can be constructed from a very limited data set, for example, empirical measurements of individual bank distress. Generally speaking, alternative approaches are used, according to data availability. In this case, the authors have opted for a data set that is available in most

---

1. In contrast to correlation, which only captures linear dependence, copula functions characterize the whole dependence structure; i.e., linear and non-linear dependence, embedded in multivariate densities (Nelsen, 1999). Thus, in order to characterize banks' distress dependence we employ a novel, non-parametric copula approach, the CIMDO copula (Segoviano, 2009), described below. Compared to traditional methodologies used to model parametric copula functions, the CIMDO copula avoids the difficulties of explicitly choosing the parametric form of the copula function to be used, and calibrating its parameters, since CIMDO copula functions are inferred directly (implicitly) from the joint (simultaneous) movements of individual bank PoDs.

countries to estimate BSMs. Consequently, such measures can be developed for a wide range of developing and developed countries.

In this paper, we also incorporate non-bank financial institutions, whether corporate or sovereign, to facilitate analysis of distress dependence between the banking sector and other sectors. Being able to establish a set of measures with a minimum of basic components facilitates a broader range of comparative analysis, involving both time series and cross-sections. The flexibility of using these measures is relevant to monitoring banking stability, as cross-border financial linkages are becoming increasingly significant, as has been illustrated by the financial market turmoil of recent months. Thus, monitoring banking stability cannot stop at national borders. Section 1 describes how Segoviano and Goodhart (2009) model distress dependence. Section 2 provides a summary of the Banking Stability Measures proposed by the authors. Section 3 shows how these measures can be employed to analyze stability from different perspectives. Finally, section 4 offers our conclusions.

1. DISTRESS DEPENDENCE IN THE FINANCIAL SYSTEM

Quantitative estimation of distress dependence among banks and/or other financial institutions is a difficult task. Information restrictions and difficulties in modeling distress dependence arise due to the fact that distress is an extreme event, which can be viewed as a tail event defined in the distress region of the probability distribution that describes a bank's implied asset price movements (figure 1).

Figure 1. The Probability of Distress
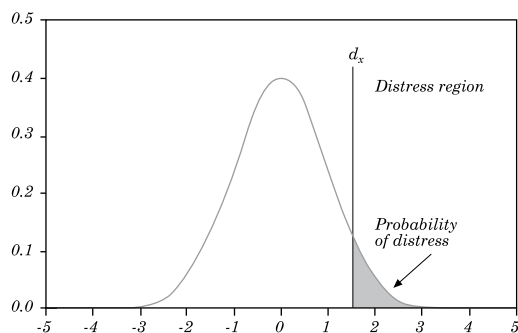
Source: Segoviano and Goodhart (2009).

The fact that distress is a tail event makes the often used correlation coefficient inadequate to capture bank distress dependence and the standard approach to model parametric copula functions difficult to implement. In our modeling of banking systems' stability and distress dependence, we replicate Segoviano and Goodhart (2009) and proceed as follows (figure 2):

Step 1: We conceptualize the banking system as a portfolio of banks.
Step 2: For each of the banks included in the portfolio, we obtain empirical measurements of probabilities of distress (PoDs).

Step 3: Using the Consistent Information Multivariate Density Optimizing (CIMDO) methodology, presented in Segoviano (2006) and summarized below, and taking as input variables the individual banks' PoDs, developed in the previous step, we estimate the banking system's (portfolio) multivariate density (BSMD).
Step 4: Based on the BSMD, we estimate the proposed banking stability measures (BSMs).

The banking system multivariate density (BSMD) characterizes both the individual and joint asset value movements of the portfolio of banks representing the banking system (figure 2).

Figure 2. The Banking System's Multivariate Density

1.1 The Importance of Time-Varying Distress Dependence

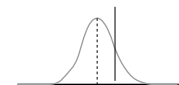We recover the BSMD using the Consistent Information Multivariate Density Optimizing (CIMDO) methodology (Segoviano, 2006b). This offers key technical improvements over traditional risk models that, generally speaking, only account for linear dependence (correlations) assumed to remain constant throughout the cycle or a fixed period of time. The BSMD captures bank distress dependence structure, as characterized by the CIMDO copula function (Segoviano, 2009), in terms of both linear and nonlinear distress dependencies among banks in the system, and allows for these to change throughout the economic cycle, reflecting the fact that distress dependence increases in periods of distress. This implies that systemic risks rise faster than individual risks.

To illustrate this point, for a portfolio of globally active banks we estimate the average probability of default, and the joint probability of default using alternative assumptions to describe the BSMD (a multivariate t-density, t-JPoD), and the CIMDO density (JPoD).[2] The joint probability of default represents the probability of all the banks included in the portfolio becoming distressed. Accordingly, this is estimated by integrating the alternative BSMD across the region of default of each of the marginal densities that compose them.

Daily percentage changes in the JPoD are larger than daily percentage changes in the average for individual PoDs and the t-JPoD. This empirical fact provides evidence that in times of distress, not only do individual PoDs increase (as captured by the three alternative measures), but so does distress dependence (as captured by the JPoD). Therefore, systemic risk may experience larger and nonlinear increases than those for individual bank probabilities of distress (PoDs) and those suggested by a density distribution with fixed correlation parameters. Consequently, measures for financial stability based on averages, or indexes that assume fixed correlation parameters over time could be misleading.

The CIMDO method involves a reduced-form or non-parametric approach to model copulas that seems to adequately capture default dependence and its changes at different points in the economic cycle. This method is easily implementable within the data constraints

2. The degrees of freedom and correlation parameters that characterize the multivariate t-density are estimated using empirically observed data.

affecting bank distress dependence modeling and produces robust estimates under the probability integral transformation (PIT) criterion.[3] To show improvements in modeling distress dependence (and therefore in our proposed measures for stability), in the next sections, we (i) model the BSMD using the CIMDO methodology, and (ii) illustrate the advantages embedded in the CIMDO copula as used to characterize distress dependence among banks in the banking system.

1.2 The CIMDO Approach: Modeling Banking System Multivariate Density

We estimate the BSMD using the CIMDO methodology and empirical measures of probabilities of distress (PoDs) for individual banks. There are alternative approaches to estimating individual banks' probabilities of distress. For example, (i) the structural approach, (ii) credit default swaps, and (iii) out-of-the-money option prices (OOM). It is important to emphasize the fact that in the CIMDO framework, the PoDs for individual banks are exogenous variables and can therefore be calculated using any alternative for estimating PoDs. This makes estimating BSMD very flexible.

The CIMDO methodology is based on the minimum cross-entropy approach (Kullback, 1959). Under this approach, a posterior multivariate distribution p (the CIMDO density) is recovered using an optimization procedure, by which a prior density q is updated with empirical information, using a set of constraints. Thus, the posterior density satisfies the constraints imposed on the prior density. In this case, the banks' empirically estimated PoDs represent the information used to formulate the constraint set. Accordingly, the CIMDO density (the BSMD) is the posterior density that is closest to the prior distribution and that is consistent with the empirically estimated PoDs of banks in the system.

To formalize these ideas, we proceed by defining a banking system (portfolio of banks) composed of two banks, X and Y, whose logarithmic returns are characterized by the random variables x and y. Hence, we define the CIMDO objective function as:[4]

---

3. The PIT criterion for multivariate density's evaluation is presented in Diebold and others (1999).

4. A detailed definition and development of the CIMDO objective function and constraint set and the optimization procedure that is followed to solve the CIMDO function is presented in Segoviano (2006b).

$C(p,q){=}\int\int p(x,y)\ln\left[\dfrac{p(x,y)}{q(x,y)}\right]dxdy,$ where $q(x,y)$ and $p(x,y)\in\mathbb{R}^2.$

Note that the prior distribution follows a parametric form, q, consistent with economic intuition (for example, default is triggered by a drop in the firm's asset value below a threshold value) and with theoretical models (such as the structural approach to model risk). However, the parametric density, q, is usually inconsistent with the empirically observed measures of distress. Hence, the information provided by the empirical measures of distress of each bank in the system is of prime importance to estimating the posterior distribution. To incorporate this information into the posterior density, we formulate consistency-constraint equations that have to be fulfilled when optimizing the CIMDO objective function. These constraints are imposed on the marginal densities of the multivariate posterior density, and take the form:

$$\int\int p(x,y)\chi\chi_{\left[x_d^x,\infty\right)}dxdy=PoD_t^x,$$
$$\int\int p(x,y)\chi_{\left[x_d^y,\infty\right)}dydx=PoD_t^y,$$

(1)

where $p(x,y)$ is the *posterior* multivariate distribution that represents the unknown to be solved. PoD_t^x and PoD_t^y are the empirically estimated probabilities of distress (PoDs) for each bank in the system, and $\chi_{\left[x_d^x,\infty\right)}$, $\chi_{\left[x_d^y,\infty\right)}$ are indicating functions defined using distress thresholds x_d^x, x_d^y, estimated for each bank in the portfolio. To ensure that the solution for $p(x,y)$ represents a valid density, the conditions that $p(x,y)\geq 0$ and the probability additivity constraint, $\int\int p(x,y)dxdy=1$, must also be satisfied. Once the set of constraints is defined, the CIMDO density is recovered by minimizing the functional:

$$
\begin{aligned}
L(p,q)= &\int\int p(x,y)\ln p(x,y)dxdy-\int\int p(x,y)\ln q(x,y)dxdy\\
&+\lambda_1\left[\int\int p(x,y)\chi_{\left[x_d^x,\infty\right)}dxdy-PoD_t^x\right]\\
&+\lambda_2\left[\int\int p(x,y)\chi_{\left[x_d^y,\infty\right)}dydx-PoD_t^y\right]\\
&+\mu\left[\int\int p(x,y)dxdy-1\right],
\end{aligned}
$$

(2)

where $\lambda_1,$ λ_2 represent the Lagrange multipliers of the consistency constraints and μ represents the Lagrange multiplier of the probability

additivity constraint. By using the calculus of variations, the optimization procedure can be performed. Hence, the optimal solution is represented by a *posterior* multivariate density taking the form:

$$\widehat{p(x,y)} = q(x,y)\exp\left\{-\left[1+\hat{\mu}+(\hat{\lambda}_1\chi_{[x_d^x,\infty)})+(\hat{\lambda}_2\chi_{[x_d^y,\infty)})\right]\right\}. \tag{3}$$

Intuitively, we know that imposing the constraint set on the objective function guarantees that the posterior multivariate distribution (BSMD) contains marginal densities that satisfy the PoDs observed empirically for each bank in the banking portfolio. Thus, in the modeling of portfolio risk, CIMDO-recovered distributions outperform the most commonly used parametric multivariate densities, according to the probability integral transformation (PIT) criterion.[5] This is because using the CIMDO approach to recover multivariate distributions, the available information, embedded in the constraint set, is used to adjust the shape of the multivariate density via the optimization procedure described above. This appears to be a more efficient manner of using the empirically observed information than under parametric approaches, which adjust the shape of parametric distributions via fixed sets of parameters. A detailed development of the PIT criterion and Monte Carlo studies used to evaluate specifications of the CIMDO density are presented in Segoviano (2006b).

1.3 The CIMDO copula: Distress Dependence among Institutions in the System

The BSMD reflects the structure of linear and nonlinear default dependence among banks included in the portfolio that is used to represent the banking system. This dependence structure is characterized by the copula function of the BSMD, that is, the CIMDO-copula, which changes at each period of time, consistent with shifts in the empirically observed PoDs. To illustrate this point, we heuristically introduce the copula approach to characterize dependence structures of random variables and explain the particular advantages of the CIMDO-copula. For further details see Segoviano (2008).

---

5. The standard and conditional normal distributions, the *t*-distribution, and the mixture of normal distributions.

1.3.1 The copula approach

The copula approach is based on the fact that any multivariate density, which characterizes the stochastic behavior of a group of random variables, can be broken into two subsets of information: (i) information about each random variable, that is, the marginal distribution of each variable; and (ii) information about the dependence structure among the random variables. Thus, to recover the latter, the copula approach sterilizes the marginal information for each variable, thereby isolating the dependence structure embedded in the multivariate density. Sterilization of marginal information is done by transforming the marginal distributions into uniform distributions; U(0,1), which are uninformative distributions.[6] For example, let x and y be two random variables with individual distributions $x \sim F$, $y \sim H$ and a joint distribution $(x,y) \sim G$. To transform x and y into two random variables with uniform distributions $\mathbf{U}(0,1)$ we define two new variables as $u=F(x)$, $v=H(y)$, both distributed as $\mathbf{U}(0,1)$ with joint density $c(u,v)$. Under the distribution of transformation of random variables, the copula function $c(u,v)$ is defined as:

$$c(u,v) = \frac{g\left[F^{(-1)}(u), H^{(-1)}(v)\right]}{f\left[F^{(-1)}(u)\right]h\left[H^{(-1)}(v)\right]}, \tag{4}$$

where g, f, and h are defined densities. From equation (4), we see that copula functions are multivariate distributions, whose marginal distributions are uniform on the interval [0,1]. Therefore, since each of the variables is individually (marginally) uniform, i.e. their information content has been sterilized, their joint distribution will only contain dependence information. Rewriting equation (4) in terms of x and y we get:

$$c\left[F(x), H(y)\right] = \frac{g(u,v)}{f(x)h(y)}. \tag{5}$$

Equation (5) tells us that the joint density of u and v is the ratio of the joint density of x and y to the product of the marginal densities. Thus, if the variables are independent, equation (5) is equal to one.

6. For further details, proofs and a comprehensive and didactical exposition of copula theory, see Nelsen (1999), and Embrechts, McNeil, and Straumann (1999) where properties and different types of copula functions are also presented.

The copula approach to model dependence possesses many positive features when compared to correlations. In comparison to correlation, the dependence structure characterized by copula functions, describes linear and nonlinear dependencies of any type of multivariate densities, throughout their entire domain. Moreover, copula functions are invariant under increasing and continuous transformations of marginal distributions. According to standard procedure, first, a given parametric copula is chosen and calibrated to describe the dependence structure among the random variables characterized by a multivariate density. Then, marginal distributions characterizing the individual behavior of random variables, are modeled separately. Lastly, marginal distributions are coupled with the chosen copula function to construct a multivariate distribution. Therefore, dependence modeling using standard parametric copulas involves two important shortcomings:

(i) It requires that modelers deal with the choice, proper specification and calibration of parametric copula functions, that is, the copula choice problem (CCP). In general, the CCP is a challenging task, since results are very sensitive to the functional form and parameter values of the chosen copula functions (Frey and McNeil, 2001). To specify the correct functional form and parameters, it is necessary to have information on the joint distribution of the variables of interest, in this case, joint distributions of distress, which are not available.

(ii) The parametric copula functions commonly employed in portfolio risk measurement require the specification of correlation parameters, which, as usually specified, remain fixed over time (see appendix A). Thus, although it is an improvement on dependence modeling using correlations, the dependence structure characterized using parametric copula functions, still poses the problem of characterizing dependence that remains fixed over time.[7]

1.3.2 The CIMDO copula

Our approach to model multivariate densities is the inverse of the standard copula approach. We first infer the CIMDO density as

---

7. Note that even if correlation parameters are dynamically updated using rolling windows, correlations remain fixed within these rolling windows. Moreover, most of the time, how the length of rolling windows is defined remains subjective.

explained in section 3.1. The CIMDO density portrays the dependence structure among the random variables that it characterizes. Thus, once we have inferred the CIMDO density, we can extract the copula function describing its dependence structure (the CIMDO copula). We do this by estimating marginal densities from the multivariate density and using Sklar's theorem (Sklar, 1959).

The CIMDO copula maintains all the benefits of the copula approach:

(i) It describes linear and nonlinear dependencies among the variables described by the CIMDO density. This dependence structure is invariant under increasing and continuous transformations of marginal distributions.

(ii) It characterizes the dependence structure along the entire domain of the CIMDO density. Nevertheless, the dependence structure characterized by the CIMDO copula appears to be more robust in the tail of the density (see discussion below), where our main interest lies (that is, to characterize distress dependence).

The CIMDO copula, however, avoids the drawbacks implicit in the use of standard parametric copulas:

(i) It circumvents the copula choice problem. The explicit choice and calibration of parametric copula functions is avoided because the CIMDO copula is extracted from the CIMDO density (as explained above). Thus, in contrast to most copula models, the CIMDO copula is recovered without explicitly imposing parametric forms that are difficult to model empirically and frequently wrongly specified, when using restricted data sets. Note that under such information constraints, when, for example, the only information available covers marginal probabilities of distress, the CIMDO copula is not only easily implementable, it outperforms the most common parametric copulas used in portfolio risk modeling under the PIT criterion. This is particularly true for the tail of the copula function, where distress dependence is characterized.

(ii) The CIMDO copula avoids the imposition of constant correlation parameter assumptions. It updates automatically when the probabilities of distress are used to infer the CIMDO density change. Therefore, the CIMDO copula incorporates banks' changing distress dependencies according to the dissimilar effects of shocks on individual banks' probabilities of distress, in a way that is consistent with the economic cycle.

To formalize these ideas, note that if the CIMDO density takes the form presented in equation (3), appendix B shows that the CIMDO copula, $c_c(u,v)$ is represented by

$$
c_c(u,v) = \frac{1}{\displaystyle\int_{-\infty}^{+\infty} q\left[F_c^{-1}(u),y\right]\exp\left[-\hat{\lambda}_2\chi_{x_d^y}(y)\right]dy}
$$

$$
\times \frac{q\left[F_c^{-1}(u),H_c^{-1}(v)\right]\exp\left[-(1+\hat{\mu})\right]}{\displaystyle\int_{-\infty}^{+\infty} q\left[x,H_c^{-1}(v)\right]\exp\left[-\hat{\lambda}_1\chi_{x_d^x}(x)\right]dx},
$$

(6)

where $u=F_c(x) \Leftrightarrow x=F_c^{-1}(u)$, and $v=H_c(y) \Leftrightarrow y=H_c^{-1}(v)$.

Equation (6) shows that the CIMDO copula is a nonlinear function of $\hat{\lambda}_1$, $\hat{\lambda}_2$ and $\hat{\mu}$, the Lagrange multipliers of the CIMDO functional presented in equation (2). As with all optimization problems, the Lagrange multipliers reflect the change in the objective function's value, as a result of a marginal change in the constraint set. Therefore, as the empirical PoDs of individual banks change at each period of time, the Lagrange multipliers change, the values of the constraint set change, and the CIMDO copula changes. Consequently, the default dependence among system banks changes.

As mentioned, then, the default dependence gets updated automatically with changes in empirical PoDs for each period in time. This is a relevant improvement over most risk models, which usually account for linear dependence (correlation) only, which, moreover is also assumed to remain constant throughout the cycle or over a fixed period of time.

2. BANKING STABILITY MEASURES

The BSMD characterizes the probability of distress of the individual banks included in the portfolio, their distress dependence, and changes across the economic cycle. This is a rich set of information that allows us to analyze (define) banking stability from three different, yet complementary, perspectives. For this purpose, we define a set of BSMs to quantify:

(i) Tail risk, defined as common distress of the financial institutions in a system;

(ii) Distress among specific institutions;

(iii) Cascade effects, defined as distress in the system associated with distress in a specific institution.

We hope that the complementary perspectives on financial stability offered by the BSMs proposed here constitute a useful tool set to help financial supervisors identify how risks are evolving and where contagion could most easily develop. To illustrate and make it easier to present definitions below, we proceed by defining a banking system (portfolio of banks) consisting of three banks whose asset values are characterized by the random variables *x*, *y* and *r*. We then use the procedure described in section 3.1 to infer the CIMDO density function, which takes the form:

$$\widehat{p(x,y,r)} = q(x,y,r)\exp\left\{-\left[\begin{array}{c}1+\hat{\mu}+(\hat{\lambda}_1\chi_{[x_d^x,\infty)})\\+(\hat{\lambda}_2\;\chi_{[x_d^y,\infty)})+(\hat{\lambda}_3\;\chi_{[x_d^r,\infty)})\end{array}\right]\right\},\tag{7}$$

where *q(x,y,r)* and *p(x,y,r)* $\in \mathbb{R}^3$.

2.1 Perspective 1: Tail Risk

To analyze common distress in the banks comprising the system, we propose the joint probability of distress (JPoD) and the banking stability index.

2.1.1 The joint probability of distress

The joint probability of distress represents the probability of all banks in the system (portfolio) becoming distressed, that is, the tail risk of the system. The JPoD reflects changes in individual bank PoDs and captures changes in distress dependence among the banks. The latter increases in times of financial distress. Therefore, in such periods, the banking system's JPoD may experience larger and nonlinear increases than those experienced by the (average) PoDs of individual banks. For the hypothetical banking system defined in equation (7) the JPoD is defined as *P(X∩Y∩R)* and is estimated by integrating the density (BSMD) as follows:

$$\int_{x_d^r}^{\infty}\int_{x_d^y}^{\infty}\int_{x_d^x}^{\infty}\widehat{p(x,y,r)}\,dxdydr = JPoD.\tag{8}$$

2.1.2 The banking stability index

The banking stability index (BSI) is based on the conditional expectation of default probability measure, developed by Huang (1992).[8] The BSI reflects the expected number of banks becoming distressed, given that at least one bank has become distressed. A higher number signifies increased instability. For example, for a system of two banks, the BSI is defined as follows:

$$\text{BSI} = \frac{P(X \geq x_d^x) + P(Y \geq x_d^y)}{1 - P(X < x_d^x, Y < x_d^y)}. \tag{9}$$

The BSI represents a probability measure based on the condition of any bank, without indicating the specific bank, becoming distressed.[9]

2.2 Perspective 2: Distress Between Specific Banks

2.2.1 Distress dependence matrix

For each period under analysis and for each pair of banks in the portfolio, we estimate the set of pairwise conditional probabilities of distress, presented in the distress dependence matrix (DiDe). This matrix contains the probability of distress of the bank specified in the row, if the bank specified in the column becomes distressed. Although conditional probabilities do not imply causation, this set of pairwise conditional probabilities can provide important insights into interlinkages and the likelihood of contagion between banks in the system. Table 1 provides the DiDE for the hypothetical banking system defined in equation (7), on a given date.

Table 1. Distress Dependence Matrix

	Bank X	*Bank Y*	*Bank R*
Bank X	1	$P(X|Y)$	$P(X|R)$
Bank Y	$P(Y|X)$	1	$P(Y|R)$
Bank R	$P(R|X)$	$P(R|Y)$	1

Source: Segoviano and Goodhart (2009).

8. This function is presented in Huang (1992). For empirical applications, see Hartmann and others (2001).
9. Huang (1992) shows that this measure can also be interpreted as a relative measure of banking linkage. When the BSI=1 in the limit, banking linkage is weak (asymptotic independence). As the value of the BSI increases, banking linkage increases (asymptotic dependence).

Here, for example, the probability of distress in bank X conditional on bank Y becoming distressed is estimated by

$$P(X \geq x_d^x \,|\, Y \geq x_d^y) = \frac{P(X \geq x_d^x, Y \geq x_d^y)}{P(Y \geq x_d^y)}. \tag{10}$$

2.3 Perspective 3: Cascade Effects

2.3.1 The probability of cascade effects (PCE)

This indicator characterizes the likelihood that one, two, or more institutions, up to the total number of Financial Institutions (FIs hereafter) in the system become distressed given that a specific FI becomes distressed. Therefore, this measure quantifies the potential cascade effects in the system, given distress occurring in a specific bank. Consequently, we propose this measure as an indicator to quantify the systemic importance of a specific bank if it becomes distressed. Again, note that conditional probabilities do not imply causation. We do consider, however, that the PCE can provide important insights into systemic interlinkages among the banks in a system. For example, in a system with four banks, X, Y, Z, and R, the PCE can be defined as follows:

$$\begin{aligned} PCE = \ & P(Y|X) + P(Z|X) + P(R|X) \\ & - \big[P(Y \cap R|X) + P(Y \cap Z|X) + P(Z \cap R|X) \big] \\ & + P(Y \cap R \cap Z|X). \end{aligned} \tag{11}$$

3. Banking Stability Measures: Empirical Results

We have used the BSM proposed by Segoviano and Goodhart (2009) to examine relative changes in stability over time in the following cases:

(i) Financial stability and spillovers among country/regions;
(ii) Spillovers between foreign banks (from developed countries) and emerging sovereign markets;
(iii) Spillovers between developed country banks and developed sovereign markets;
(iv) Spillovers between the banking system and corporate sectors.

Our estimations are performed from 2005 to February 2009, using publicly available data, and include major American and European banks, and sovereign banks in Latin America, Eastern Europe, Europe, and Asia. The flexibility inherent in our approach is relevant to monitoring bank stability, since cross-border financial linkages are growing and becoming increasingly significant, as has been underlined by turmoil in financial markets in recent months. Thus, monitoring banking stability cannot stop at national borders.

3.1 Estimating the Probability of Individual Bank Distress

The proposed BSMs can be constructed from a very limited set of data, such as empirical measures of distress for individual banks, which we have labeled probabilities of distress (PoDs). These measures can be estimated using alternative approaches, such as Merton-type models, credit default swaps (CDS), option prices, and bond spreads, depending on the data available. This means that the data set necessary to estimate BSMs is available in most countries. Consequently, such measures can be developed for a wide range of developing and developed countries.

Being able to establish this kind of set of measures with minimal base components, makes a broader range of comparative analysis, including both time series and cross-sections, possible. In the applications below, we used CDS-PoDs, since they seemed the best available distress indicator for the banks under analysis. Note, however, that estimating the proposed BSMs is not intrinsically related to CDS-PoDs. Thus, if we were to find a better approach, replacing the PoD approach selected here to estimate BSMs would be straightforward, since in this framework PoDs are exogenous variables.[10]

10. Arguments against using CDS-PoDs emphasize that CDS spreads may sometimes overshoot. They do not generally stay wrong for long, however. Rating agencies have mentioned that CDS spreads frequently anticipate rating changes. Although the magnitude of shifts may sometimes be unrealistic, the direction is usually a good distress signal. For these reasons, and due to the problems encountered with other approaches (which we consider more serious), we decided to use CDS-PoDs to estimate the proposed BSMs.

3.2 Financial Stability and Spillovers among regions[11]

To analyze financial stability across regions, we included major American, European, and Asian banks, grouped in alternative portfolios, to observe:

3.2.1 Perspective 1: Tail risk

• **FIs are highly interconnected, with distress in one FI associated with high probability of distress elsewhere.** This is clearly indicated by the JPoD and the BSI. Moreover, movements in the JPoD and BSI coincide with events considered relevant by markets on specific dates (figure 3). Note also that risks vary by geographical location and business area of the FI (figure 4).

Figure 3. Tail Risk: January 2007-February 2009

1. Bear Stearns episode, 3/11/08
2. FNM/FRE bailout, 9/6/08
3. Lehman bankruptcy and AIG bailout, 9/15-16/08
4. Short selling ban, 9/19/08
5. TARP bill failure, 9/30/08
6. TARP II passes, TAF increase, 10/5/08
7. Global central bank intervention, 10/8/08

Banking stability index (number of banks, LHS)

JPoD (probability, RHS)
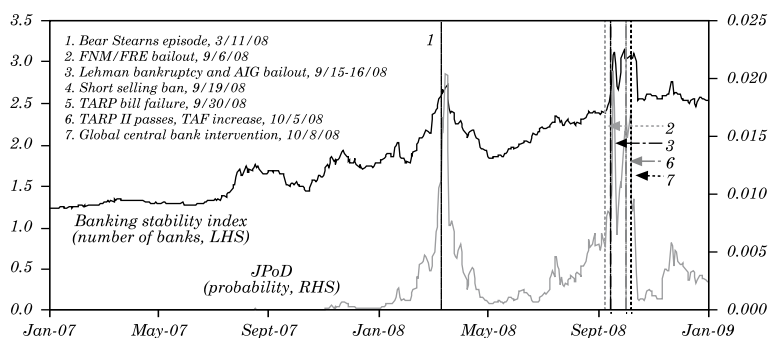
Sources: Bloomberg L.P.; IMF staff estimates.
Note: Global group consists of Bank of America, Citigroup, J.P. Morgan Chase & Co., Wachovia, Goldman Sachs, Lehman Brothers, Merrill Lynch, Morgan Stanley, Deutsche Bank, Royal Bank of Scotland, UBS, HSBC, PMI, AMBAC Financial, AIG, and Swiss Re.

Figure 4. Tail Risk by Regions: January 2007-February 2009
(BSI: number of banks, LHS; JPoD: probability, RHS)

Asia

Euro area

Non Euro area
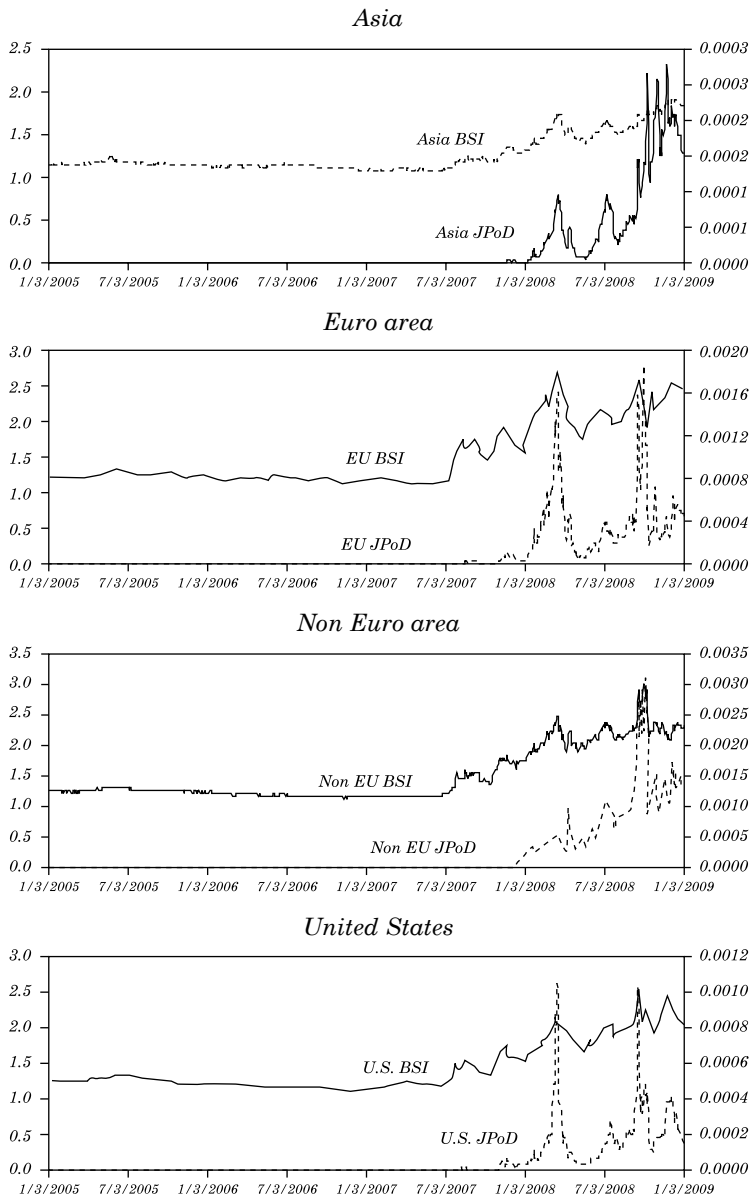
United States

3.2.2 Perspective 2: Distress between specific institutions

Table 2 illustrates the following:
- **Distress dependence across major American FIs has greatly increased.** This is clearly shown by the conditional PoDs presented in the DiDe. On average, if any of the U.S. FIs fell into distress, the average probability of this affecting other FIs increased from 23 percent on 1 July 2007 to 41 percent on 12 September 2008.
- **By September, Lehman and AIG vulnerability had increased significantly.** This is revealed by Lehman's and AIG's large PoDs conditional on any other FI experiencing distress, which increased from 30 and 15 percent, respectively, on 1 July 2007 to 52 and 44 percent on 15 August, and 56 and 55 percent, on average, on 12 September 2008 (row-average Lehman and AIG). Moreover, a Lehman default was estimated on 12 September to raise the chances of distress elsewhere by 46 percent In other words, the PoD of any other bank conditional on Lehman experiencing distress went from 25 percent on 1 July 2007 to 37 percent on 12 September 2008 (column-average Lehman).

   Note that a similar effect in the system would have been caused by the distress of AIG, since the PoD of any other bank dependent on AIG experiencing distress went from 20 percent on 1 July 2007 to 34 percent on 12 September 2008 (column-average AIG).

- **Lehman's connections to the other major U.S. banks were similar to AIG's.** This can be seen by comparing the chances of each one of the U.S. banks being affected by distress in Lehman and AIG (column Lehman versus column AIG) on 12 September. Links were particularly close between Lehman, AIG, Washington Mutual, and Wachovia, all of which were particularly exposed to housing. On 12 September, a Lehman bankruptcy implied an 88, 43, and 27 percent likelihood that Washington Mutual, AIG, and Wachovia, respectively, would fall into distress.
- **Distress dependence appears to be an early warning sign.** It is also very interesting to note that up to a month earlier than the Lehman event, distress dependence was already signaling that a default of Lehman or AIG would have caused significant disruptions in the system. This is revealed by the PoD for any other bank dependent on Lehman or AIG experiencing distress, which increased significantly to 41 and 39 percent, respectively,

Table 2. Distress Dependence Matrix

1 July 2007	Citi	BAC	JPM	Wacho.	WaMu	GS	LB	ML	MS	AIG	Row average
Citibank	1.00	0.09	0.08	0.08	0.05	0.06	0.06	0.06	0.06	0.05	0.16
Bank of America	0.08	1.00	0.22	0.21	0.08	0.08	0.07	0.09	0.09	0.11	0.20
J.P. Morgan Chase	0.10	0.33	1.00	0.23	0.09	0.14	0.12	0.14	0.12	0.11	0.24
Wachovia Bank	0.08	0.27	0.2	1.00	0.08	0.08	0.07	0.08	0.08	0.10	0.20
Washington Mutual	0.14	0.25	0.18	0.20	1.00	0.10	0.10	0.13	0.11	0.12	0.23
Goldman Sachs	0.13	0.20	0.23	0.16	0.08	1.00	0.27	0.23	0.26	0.13	0.27
Lehman Brothers	0.16	0.24	0.25	0.19	0.11	0.35	1.00	0.29	0.26	0.14	0.30
Merrill Lynch	0.15	0.26	0.27	0.19	0.13	0.28	0.26	1.00	0.26	0.15	0.30
Morgan Stanley	0.15	0.25	0.23	0.19	0.10	0.30	0.23	0.25	1.00	0.12	0.28
AIG	0.05	0.11	0.07	0.08	0.04	0.05	0.04	0.05	0.04	1.00	0.15
Column average	0.20	0.30	0.27	0.25	0.17	0.24	0.22	0.23	0.23	0.20	0.23

15 August 2008	Citi	BAC	JPM	Wacho.	WaMu	GS	LB	ML	MS	AIG	Row average
Citibank	1.00	0.32	0.32	0.23	0.13	0.28	0.23	0.23	0.25	0.21	0.32
Bank of America	0.20	1.00	0.41	0.24	0.09	0.24	0.17	0.19	0.21	0.19	0.30
J.P. Morgan Chase	0.18	0.37	1.00	0.20	0.07	0.25	0.17	0.18	0.20	0.15	0.28
Wachovia Bank	0.41	0.69	0.65	1.00	0.23	0.45	0.37	0.39	0.41	0.39	0.5
Washington Mutual	0.83	0.92	0.89	0.85	1.00	0.80	0.77	0.82	0.80	0.78	0.85
Goldman Sachs	0.21	0.28	0.34	0.19	0.09	1.00	0.28	0.26	0.32	0.18	0.31
Lehman Brothers	0.42	0.51	0.56	0.39	0.22	0.69	1.00	0.52	0.54	0.36	0.52
Merrill Lynch	0.39	0.52	0.58	0.37	0.21	0.61	0.48	1.00	0.53	0.36	0.50
Morgan Stanley	0.31	0.41	0.44	0.28	0.15	0.52	0.35	0.37	1.00	0.24	0.41
AIG	0.36	0.52	0.48	0.38	0.20	0.41	0.32	0.35	0.34	1.00	0.44
Column average	0.43	0.55	0.57	0.41	0.24	0.53	0.41	0.43	0.46	0.39	0.44

Table 2. (continued)

| 12 September 2008	Citi	BAC	JPM	Wacho.	WaMu	GS	LB	ML	MS	AIG	Row average
Citibank	1.00	0.20	0.19	0.14	0.07	0.17	0.13	0.14	0.16	0.11	0.23
Bank of America	0.14	1.00	0.31	0.18	0.05	0.16	0.10	0.13	0.15	0.11	0.23
J.P. Morgan Chase	0.13	0.29	1.00	0.16	0.05	0.19	0.11	0.14	0.16	0.09	0.23
Wachovia Bank	0.34	0.60	0.55	1.00	0.17	0.36	0.27	0.31	0.34	0.29	0.42
Washington Mutual	0.93	0.97	0.95	0.94	1.00	0.91	0.88	0.92	0.91	0.89	0.93
Goldman Sachs	0.15	0.19	0.24	0.13	0.06	1.00	0.18	0.20	0.27	0.11	0.25
Lehman Brothers	0.47	0.53	0.58	0.43	0.25	0.75	1.00	0.59	0.62	0.37	0.56
Merrill Lynch	0.32	0.41	0.47	0.30	0.16	0.53	0.37	1.00	0.48	0.26	0.43
Morgan Stanley	0.21	0.28	0.29	0.19	0.09	0.40	0.22	0.27	1.00	0.14	0.31
AIG	0.50	0.66	0.59	0.53	0.29	0.54	0.43	0.49	0.47	1.00	0.55
Column average	0.42	0.51	0.52	0.40	0.22	0.50	0.37	0.42	0.46	0.34	0.41

Source: Authors' calculations.

on 15 August 2008 (column-average Lehman and AIG). Moreover, On 15 August, a Lehman bankruptcy implied a 77, 32, and 37 percent likelihood that Washington Mutual, AIG, and Wachovia, respectively, would experience distress. The Lehman bankruptcy seems to have sealed the fate of AIG and Washington Mutual, while boosting the pressure on Wachovia, as indicated by the DiDe. Even though distress dependence does not imply causation, these results show that the analysis of distress dependence, even several weeks prior to a distress event, can provide useful insights of how distress in a specific institution can affect other institutions and ultimately the stability of the system.

3.2.3 Perspective 3: Cascade effects

• **The probability of cascade effects (PCE) signaled major impacts on markets if Lehman or AIG became distressed.** The PCE for these institutions reached 97 and 95 percent, respectively, on 12 September 2008. Thus, the PCE also signaled the possible domino effect, observed in the days after Lehman's collapse (figure 5). Note that the PCE for both institutions had already increased by August 2008. This analysis is in line with the insights brought by the DiDe in perspective 2, which indicated Lehman's distress would be associated with distress in several institutions.

Figure 5. Probability of Cascade Effects if Lehman/AIG fall into Distress
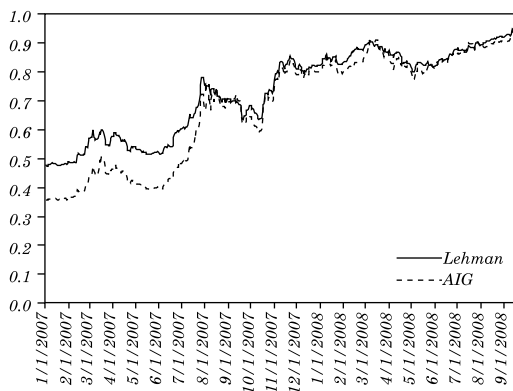
Source: Authors' calculations.

3.3 Spillovers Between Foreign Banks and Emerging Sovereign Markets

In this section, we extend our methodology to analyze how rising problems in advanced country banking systems are linked with increasing risks in emerging markets. For this purpose, we use CDS spreads based on sovereign and bank bonds to derive probabilities of sovereign and bank distress. These PoDs, then, represent markets' views on the risk of distress for these banks and countries. While absolute risks are discussed, the focus is largely on cross distress dependence of risks and what this can tell us about emerging vulnerabilities (perspective 2). Specifically, using publicly available data we estimate cross vulnerabilities between Latin American, Eastern European, and Asian emerging markets and the advanced country banks with the most presence in these regions. Countries and banks analyzed are:

- **Latin America**. Countries: Mexico, Colombia, Brazil and Chile. Banks: BBVA, Santander, Citigroup, and HSBC.
- **Eastern Europe**. Countries: Bulgaria, Croatia, Hungary and Slovakia. Banks: Intesa, Unicredito, Erste, Société Générale, and Citigroup.
- **Asia**. Countries: China, South Korea, Thailand, Malaysia, the Philippines, and Indonesia. Banks: Citigroup, J.P. Morgan Chase, HSBC, Standard and Chartered, BNP, Deutsche Bank, and DBS.

The key observation from this analysis is that concerns about bank solvency and emerging market instability appear to be highly interlinked. To illustrate these interlinkages, we present distress dependence matrices estimated for each region in table 3.[12] To analyze how distress dependence has evolved over time, we also estimate the time series of the conditional probabilities of distress of banks/countries if other banks/countries default.[13]

---

12. These matrices can be estimated for each day. They report links across countries (bottom right, quadrant 4), and across banks (top left, quadrant 1). The bottom left (quadrant 3) reports how sovereign distress is conditional on bank problems, while the top right (quadrant 2) indicates the opposite.

13. Note that there is a daily time series for each of the quadrants described in the previous footnote. Each observation in the time series corresponds to the average of the conditional probabilities in each quadrant, at each day.

3.3.1 Distress between foreign banks and emerging sovereign markets

- **The analysis shows that risk in sovereign markets and banks increased markedly after October 2008.** In the run-up to the crisis, there was little concern about risks to sovereigns and parent banks in Eastern Europe, and risk perceptions in Latin America and Asia were falling. From July 2007 to September 2008, both sovereign risk and bank risk increased and moved in tandem, but from October 2008, risk in sovereigns has been significantly higher than in banks (figure 6). This may reflect the deepening downturn in emerging economies in late 2008 and the support received by banks in developed countries from their sovereigns.
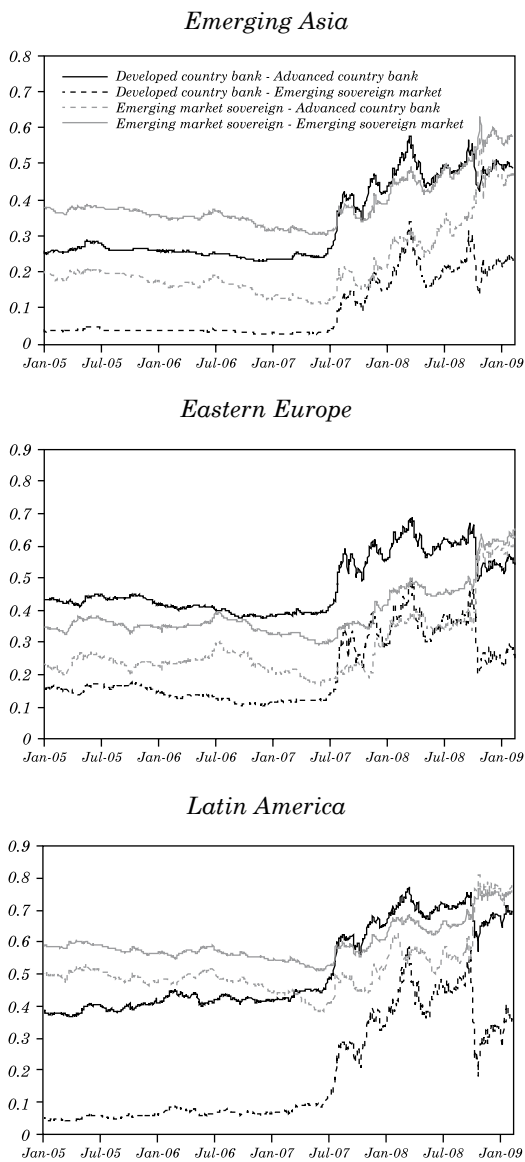- **Bank problems appear to have a significant impact on sovereign distress.** This is seen by comparing the probability of distress of the emerging sovereign Markets conditional on distress in the mature market banks in July 2007 and in September 2008. In the last quarter of 2008, sovereign risk conditional on bank risk has increased further (figure 7).
- **Bank location is important to sovereign distress.** Quadrant 3 of the distress dependence matrices (table 3) reveals that distress among Spanish banks is associated with the most distress in Latin America, while distress among Italian banks has the most impact on Eastern Europe. Distress of standard chartered banks is associated with significant stress in Asia (quadrant 3, column-average). These results suggest that location matters, since these banks have a substantial presence in the respective regions under analysis.
- **Direct links between banks and countries matter.** Distress in countries with a particularly large foreign bank presence, such as Mexico and the Czech Republic, is most strongly associated with potential banking distress (quadrant 2). Direct links between individual banks and countries also matter, for example, distress at Citigroup, Intesa, and DBS were more important to Mexico, Hungary, and Indonesia than other countries (quadrant 3).
- **The results also illustrate the influence of systemic risk, which constitutes an indirect link to Asia, over and above direct regional and bilateral links.** Direct ownership and lending by foreign banks is generally lower

Figure 6. Probabilities of Distress

Emerging Asia

Average for advanced country
banks exposed to the region

Average for emerging
sovereigns market in the region
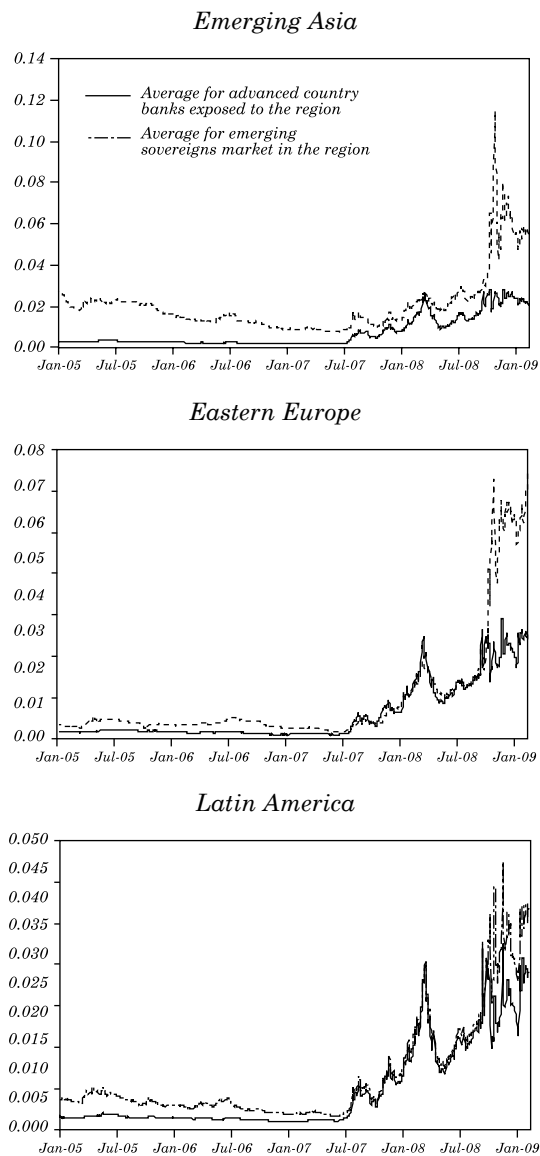
Eastern Europe

Latin America

Figure 7. Distress Dependence over Time
(average conditional probabilities for the region)

Emerging Asia

Legend:
- Developed country bank - Advanced country bank
- Developed country bank - Emerging sovereign market
- Emerging market sovereign - Advanced country bank
- Emerging market sovereign - Emerging sovereign market

Eastern Europe

Latin America

Source: Authors' calculations.

in Asia than in Eastern Europe or Latin America, insulating banking systems somewhat from these direct links, and increasing the relative importance of indirect links involving bank and/or sovereign distress. In addition, links between banks may be somewhat less important for emerging Asia, as borrowing through debt markets tends to play a larger role in local financial systems. Indirect effects are particularly evident in South Korea and Indonesia. An important strength of our approach is that market prices reflect the perception that direct and indirect links exist. For the former, market presence might be an important element, as in Latin America and Eastern Europe. For the latter, however, liquidity pressures and systemic banking distress/macroeconomic spillovers might play an important role. This feature of our approach appears to be particularly relevant in Asia.

- **Overall, the results indicate that systemic bank risks and emerging market vulnerabilities appear to be highly dependent.** This probably reflects the fact that distress in individual banks acts as a bellwether for the state of the financial system overall, through direct or indirect links.

Table 3. Distress Dependence Matrices: Sovereign Markets: and Banks
(as of February 2009)

Latin America

	BBVA	Santander	Citigroup	HSBC	Row average
BBVA	1.000000	0.725706	0.326296	0.637743	0.672436
Santander	0.728609	1.000000	0.315021	0.627339	0.667742
Citigroup	0.751264	0.722414	1.000000	0.776343	0.812505
HSBC	0.586723	0.574852	0.310214	1.000000	0.617947
Column average	0.766649	0.755743	0.487883	0.760356	0.692658
Mexico	0.865202	0.863165	0.809673	0.874189	0.853057
Colombia	0.823127	0.815681	0.646378	0.817612	0.775700
Brazil	0.821372	0.817196	0.700703	0.820338	0.789902
Chile	0.738229	0.730452	0.564896	0.742177	0.693939
Column average	0.811983	0.806623	0.680412	0.813579	0.778149

	Mexico	Colombia	Brazil	Chile	Row average
BBVA	0.275122	0.256586	0.275776	0.361877	0.292340
Santander	0.275571	0.255282	0.275471	0.359497	0.291455
Citigroup	0.592785	0.463909	0.541666	0.637557	0.558979
HSBC	0.255741	0.234477	0.253395	0.334707	0.269580
Column average	0.349805	0.302564	0.336577	0.423410	0.353089
Mexico	1.000000	0.651145	0.803716	0.872234	0.831774
Colombia	0.664230	1.000000	0.660733	0.754643	0.769902
Brazil	0.761189	0.613444	1.000000	0.800777	0.793853
Chile	0.565809	0.479885	0.548478	1.000000	0.648543
Column average	0.747807	0.686119	0.753232	0.856914	0.761018

Table 3. (continued)

Eastern Europe

	Intesa	Unicredito	Erste	Société	Citigroup	Row average
Intesa	1.000000	0.475783	0.301497	0.450473	0.210372	0.487625
Unicredito	0.599617	1.000000	0.365881	0.550721	0.266108	0.556465
Erste	0.557821	0.537140	1.000000	0.574143	0.338382	0.601497
Société Générale	0.384907	0.373383	0.265153	1.000000	0.183779	0.441444
Citigroup	0.518365	0.520285	0.450654	0.529977	1.000000	0.603856
Column average	0.612142	0.581318	0.476637	0.621063	0.399728	0.538178
Bulgaria	0.708755	0.718608	0.715218	0.738093	0.632545	0.702644
Croatia	0.804806	0.793321	0.756003	0.773592	0.663734	0.758291
Hungary	0.831557	0.81284	0.810723	0.784304	0.664342	0.780753
Slovakia	0.352204	0.357517	0.300776	0.351336	0.327638	0.337894
Estonia	0.687141	0.672014	0.637246	0.635676	0.559833	0.638382
Czech Republic	0.610193	0.625443	0.638816	0.593791	0.449097	0.583468
Column average	0.665776	0.6632905	0.64313033	0.646132	0.5495315	0.633572

	Bulgaria	Croatia	Hungary	Slovakia	Estonia	Citigroup / Czech Rep.	Row average
Intesa	0.139906	0.177797	0.192069	0.176178	0.166857	0.242417	0.182537
Unicredito	0.178771	0.220875	0.236603	0.225381	0.205656	0.313147	0.230072
Erste	0.261211	0.309007	0.346445	0.278363	0.286297	0.469552	0.325146
Société Générale	0.124492	0.146027	0.154783	0.150164	0.131893	0.201566	0.151488
Citigroup	0.307667	0.361306	0.378085	0.403829	0.334969	0.439626	0.370914
Column average	0.202409	0.243002	0.261597	0.246783	0.225134	0.333262	0.252031
Bulgaria	1.000000	0.707068	0.668112	0.758508	0.710653	0.777554	0.770316
Croatia	0.631785	1.000000	0.678201	0.703084	0.860053	0.820782	0.782318
Hungary	0.571006	0.648697	1.000000	0.650592	0.640253	0.852863	0.727235
Slovakia	0.299327	0.310517	0.300402	1.000000	0.314773	0.396426	0.436908
Estonia	0.577698	0.628698	0.608991	0.648420	1.000000	0.731451	0.699210
Czech Republic	0.386346	0.456421	0.498531	0.499141	0.447083	1.000000	0.547920
Column average	0.493232	0.608867	0.617225	0.700247	0.652432	0.760304	0.638718

Table 3. (continued)

Asia

	HSBC	SC	Citigroup	Deutche	BNP	DBS	JPM	Row average
HSBC	1.000000	0.400728	0.240106	0.467571	0.594485	0.243707	0.277133	0.460533
Standard and Chartered	0.731765	1.000000	0.373773	0.653803	0.785633	0.399322	0.422132	0.623775
Citigroup	0.600891	0.512247	1.000000	0.680894	0.650505	0.358004	0.847451	0.664285
Deutsche Bank	0.390320	0.298881	0.227122	1.000000	0.570166	0.182848	0.299087	0.424061
BNP	0.349454	0.252899	0.152794	0.401492	1.000000	0.154917	0.194071	0.357947
DBS	0.476818	0.427845	0.279885	0.428549	0.515625	1.000000	0.296335	0.489294
J.P. Morgan Chase	0.274964	0.229358	0.335977	0.355477	0.327567	0.150275	1.000000	0.381945
Column average	0.546316	0.445994	0.372808	0.569684	0.634854	0.355582	0.476601	0.485977
South Korea	0.594517	0.587287	0.398186	0.547021	0.616806	0.709589	0.402420	0.550832
Malaysia	0.419980	0.438035	0.305322	0.447027	0.503945	0.548694	0.312387	0.425056
Thailand	0.410019	0.407681	0.275756	0.374748	0.441744	0.478346	0.280762	0.381294
China	0.413471	0.368967	0.273494	0.363895	0.412435	0.376860	0.295244	0.357767
Philippines	0.472991	0.510020	0.358525	0.493129	0.527604	0.547252	0.334387	0.463415
Indonesia	0.681152	0.693283	0.522499	0.630787	0.690313	0.825260	0.513146	0.650920
Column average	0.498688	0.500879	0.355630	0.476101	0.532141	0.581000	0.356391	0.471547

Table 3. (continued)

	S. Korea	Malaysia	Thailand	China	Philippines	Indonesia	Row average
HSBC	0.200850	0.199495	0.193485	0.218316	0.135680	0.129475	0.179550
Standard and Chartered	0.362310	0.379956	0.351305	0.355753	0.267159	0.240644	0.326188
Citigroup	0.336656	0.362956	0.325657	0.361394	0.258380	0.248554	0.315433
Deutsche Bank	0.154272	0.177260	0.147623	0.160394	0.118085	0.100092	0.142954
BNP	0.122491	0.140713	0.122536	0.128010	0.088965	0.077133	0.113308
DBS	0.469029	0.509939	0.441641	0.389318	0.307138	0.306915	0.403997
J.P. Morgan Chase	0.134889	0.147226	0.131452	0.154671	0.095170	0.096777	0.126697
Column average	0.254357	0.273935	0.244814	0.252551	0.181368	0.171370	0.229732
South Korea	1.000000	0.686822	0.605330	0.614034	0.484226	0.438772	0.638198
Malaysia	0.488483	1.000000	0.460463	0.463792	0.380784	0.334829	0.521392
Thailand	0.433370	0.463506	1.000000	0.377362	0.290046	0.299040	0.477221
China	0.392881	0.417240	0.337257	1.000000	0.275819	0.253024	0.446037
Philippines	0.570289	0.630551	0.477141	0.507694	1.000000	0.415918	0.600265
Indonesia	0.779839	0.836727	0.742386	0.702845	0.627663	1.000000	0.781577
Column average	0.610810	0.672474	0.603763	0.610955	0.509756	0.456931	0.577448

Source: IMF staff estimates.

3.4 Spillovers between Developed Country Banks and their Sovereign Markets

This section applies the proposed model to study the transmission of shocks from banks in developed countries (with large exposure to emerging market, such as Austria, the United Kingdom, France, and Germany) to their own sovereign markets.
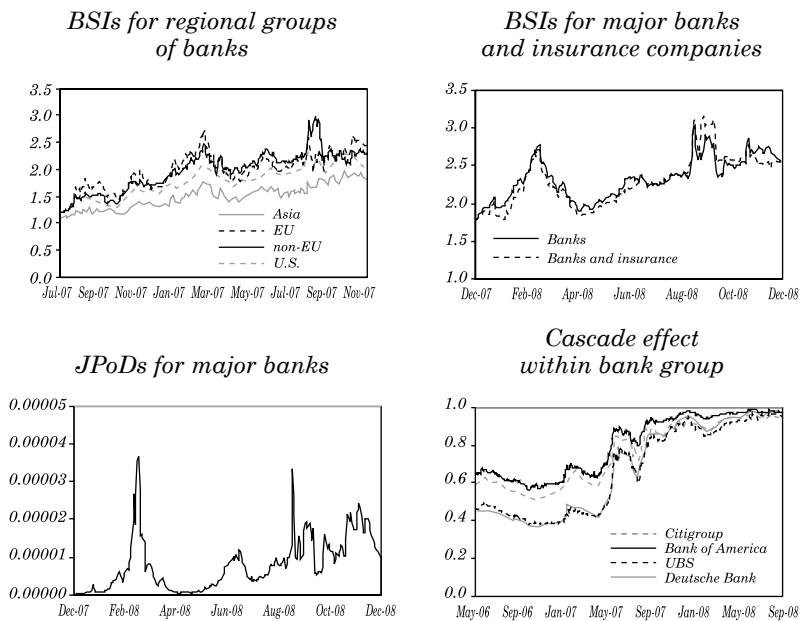
3.4.1 Tail risk and cascade effects

- **Measures of bank interconnectedness started to rise at the onset of the crisis**. The joint probability of distress (JPoD) and banking stability index indicate that systemic tail risk has risen substantially (figure 8).
- **The probability of cascade effects has also increased substantially,** suggesting that future shocks would be transmitted quickly through the financial system (figure 8).

3.4.2 Distress between banks and sovereign markets in developed economies

- **Links between advanced country banks and sovereign markets increased markedly after October**. As the fiscal costs of potential bank bailouts have become apparent, banking sector concerns and sovereign risk have become increasingly intertwined. This is significant in Austria and the United Kingdom (figure 9).
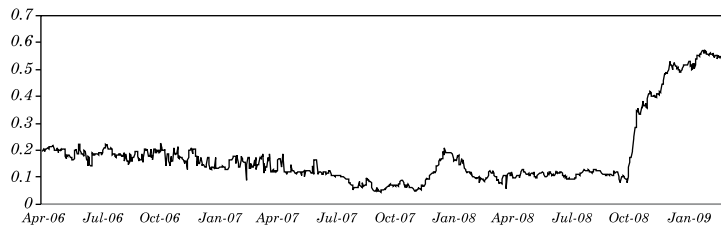
Figure 8. Tail Risk and Cascade Effects

BSIs for regional groups of banks

BSIs for major banks and insurance companies

JPoDs for major banks

Cascade effect within bank group
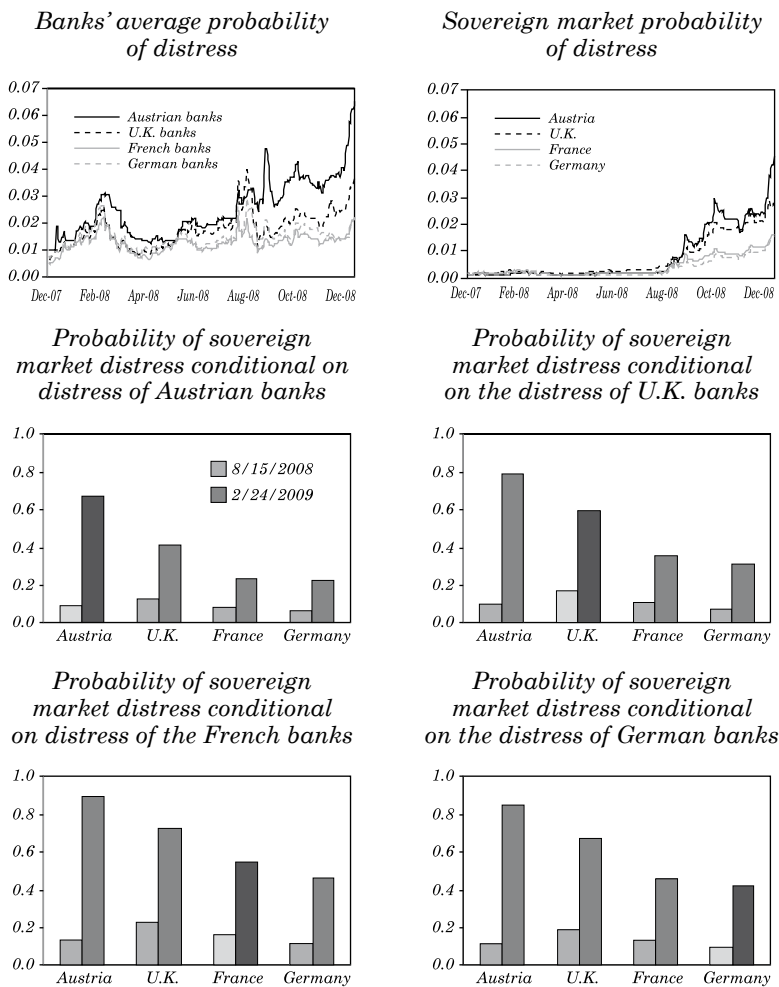

Source: IMF staff estimates.

Figure 9. Distress between Banks and Sovereign Markets in Developed Economies

Probability of distress of an advanced country sovereign market, conditional on an advanced country bank falling into distress (sample average)

Source: IMF staff estimates.

Figure 9. (continued)

Banks' average probability of distress

Sovereign market probability of distress

Probability of sovereign market distress conditional on distress of Austrian banks

Probability of sovereign market distress conditional on the distress of U.K. banks

Probability of sovereign market distress conditional on distress of the French banks

Probability of sovereign market distress conditional on the distress of German banks

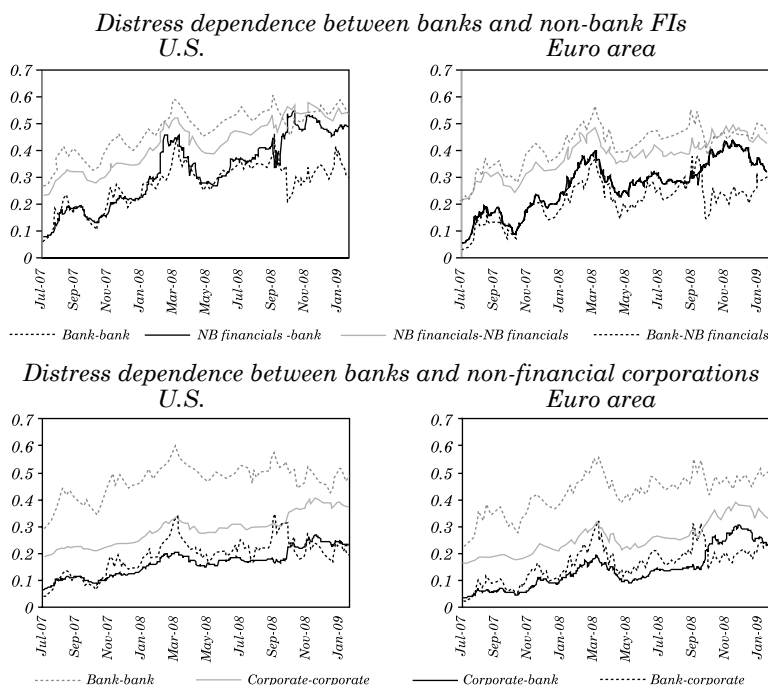Sources: Bloomberg and IMF staff estimates.

3.5 Spillovers between the Banking System and Corporate Sectors

To analyze spillovers between the banking system and the corporate sector, we estimated linkages between non-bank financial companies, other corporations and banks in the U.S. and Europe.

3.5.1 Distress between banks and corporations

- **Banks in developed countries have gradually become more interlinked with non-banks and non-financial corporations**. Banks became less dependent on other corporations in late 2008 (likely due to public support), but spillovers to other corporations continued to rise (figure 10). This constitutes evidence of spillovers from the banking crisis into the real economy.

Figure 10. Distress between Banks and Corporations in Developed Economies

Distress dependence between banks and non-bank FIs

U.S.   Euro area

······ *Bank-bank* —— *NB financials -bank* —— *NB financials-NB financials* ········ *Bank-NB financials*

Distress dependence between banks and non-financial corporations

U.S.   Euro area

······ *Bank-bank* —— *Corporate-corporate* —— *Corporate-bank* ········ *Bank-corporate*

Source: IMF staff estimates.

4. CONCLUSIONS

The purpose of this paper is to estimate the BSM proposed by Segoviano and Goodhart (2009) and use it to analyze the financial stability of the main banks in any country or region, to enable tracking of the relative stability of this portfolio of banks over time and compare cross-sections of comparative groupings. This framework offers several advantages:

• It provides measures for analyzing (defining) stability from three different, but complementary perspectives.
• It can be constructed using a very limited set of data, specifically, the empirical measurements of default probabilities for individual banks. These measures can also be estimated using alternative approaches, depending on data availability. The data set needed for estimates is available in many developed and developing countries, provided there is reasonable data on individual bank PoDs.
• It includes banks' default interdependence structure (copula function), thus capturing linear and nonlinear default dependencies among the main banks in a system.
• It allows quantification of changes in banks' default interdependence structure at specific points in time. Thus, it can be useful to quantify empirically observed increases in dependencies at times of distress and relax the assumption of fixed correlations across time, commonly used in risk measurement models.

In the empirical part of this paper, we discussed the application of this methodology to several country and regional examples, using information available up to February 2009. This flexibility is useful to monitoring bank stability, as cross-border financial linkages increase and become more significant, as apparent during the financial market turmoil of recent months. Thus, monitoring bank stability cannot stop at national borders.

APPENDIX A
Copula Functions

Let x and y be two random variables with individual distributions $x \sim F$, $y \sim H$ and a joint distribution $(x, y) \sim G$. The joint distribution contains three types of information. Individual (marginal) information on the variable x, individual (marginal) information on the variable y and information on the dependence between x and y. To model the dependence structure between the two random variables, the copula approach sterilizes the marginal information on x and y from their joint distribution, isolating the dependence structure as a result. Marginal information is sterilized by transforming the distribution of x and y into a uniform distribution; U(0,1), which is uninformative. Under this distribution the random variables have an equal probability of taking a value between 0 and 1 and a zero probability of taking a value outside [0,1]. Therefore, this distribution is typically thought of as being uninformative. To transform x and y into U(0,1) we use the probability integral transformation (PIT).

Under PIT, two new variables are defined as $u=F(x)$, $v=H(y)$, both distributed as U(0,1) with joint density $c(u,v)$. Under the distribution of transformation of random variables (Cassella and Berger, 1990), the copula function $c(u,v)$ is defined as:

$$c(u,v) = \frac{g\left[F^{(-1)}(u), H^{(-1)}(v)\right]}{f\left[F^{(-1)}(u)\right]h\left[H^{(-1)}(v)\right]}, \tag{A1}$$

where g, f, and h are defined densities.

From equation (A1), we see that copula functions are multivariate distributions, whose marginal distributions are uniform on the interval [0,1]. Therefore, since each of the variables is individually (marginally) uniform (that is, their information content has been sterilized via PIT), their joint distribution will only contain dependence information. Rewriting equation (A1) in terms of x and y we get

$$c\left[F(x), H(y)\right] = \frac{g(x,y)}{f(x)h(y)}. \tag{A2}$$

From equation (A2), we see that the joint density of u and v is the ratio of the joint density of x and y to the product of the marginal densities. Therefore, if the variables are independent, equation (A2) is equal to one.

Sklar's Theorem

The following theorem was developed by Sklar (1959) and is known as Sklar's Theorem. It is relevant to copula functions, and is used in all applications of copulas. If G is a joint distribution function with marginals F and $H,$ then a copula C exists for all x, y in $\mathbb{R}$,

$$G(x,y) = C\big[F(x),H(y)\big]. \tag{A3}$$

If F and H are continuous, then C is unique; otherwise, C is uniquely determined on Ran$F \times$ Ran$H$. Conversely, if C is a copula and F and H are distribution functions, then the multivariate function G defined by equation (A3) is a joint distribution function with univariate margins F and H. Thus, the dependence structure is completely characterized by the copula C (Nelsen, 1999). Nelsen also provides the following corollary to Sklar's theorem.

Corollary: Let G be any joint distribution with continuous marginals F and H. Let $F^{(-1)}(u)$, $H^{(-1)}(v)$ denote the (quasi) inverses of the marginal distributions. Then there exists a unique copula C: [0,1] x [0,1]$\rightarrow$[0,1] such that, $g[F^{(-1)}(u), H^{(-1)}(v)]\forall\in$[0,1] x [0,1]. If the cross partial derivatives of equation (A3) are taken, we obtain:

$$g(x,y) = f(x)h(y)c\big[F(x),H(y)\big]. \tag{A4}$$

The converse of Sklar's theorem implies that we can couple together any marginal distributions, of any family, with any copula function and a valid joint density will be defined. The corollary implies that from any joint distribution we can extract the implied copula and marginal distributions (Nelsen, 1999).

Parametric Copula Functions

In the finance literature, it is common to see the Gaussian copula and the t copula for modeling dependence among financial assets. These are defined as follows (Embrechts, Lindskog, and McNeil, 2003):

Gaussian copula: The copula of the bivariate normal distribution can be written as:

$$C_R^{Ga}(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}} \exp\left[-\frac{s^2 - 2\rho st + t^2}{2(1-\rho^2)}\right] dsdt, \quad \text{(A5)}$$

where ρ is the linear correlation coefficient of the corresponding bivariate normal distribution, and Φ^{-1} denotes the inverse of the distribution function of the univariate standard normal distribution.

***t* copula:** The copula of the bivariate t-distribution with υ degrees of freedom and correlation ρ is:

$$C_{\upsilon\rho}^{t}(u,v) = \int_{-\infty}^{t_\upsilon^{-1}(u)} \int_{-\infty}^{t_\upsilon^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}} \left[1 + \frac{s^2 - 2\rho st + t^2}{\upsilon(1-\rho^2)}\right]^{-(\upsilon+2)/2} dsdt, \quad \text{(A6)}$$

where $t_\upsilon^{-1}(v)$ denotes the inverse of the distribution function of the standard univariate t-distribution with υ degrees of freedom. As it can be seen, this copula depends only on ρ and υ.

APPENDIX B
CIMDO copula

To provide a heuristic explanation of the CIMDO copula, we compare the copula of a bivariate CIMDO distribution and a bivariate distribution of the form that the prior density in the entropy functional would take, for example, a *t*-distribution. First, we recall from equation (4) that copula functions are defined as

$$c(u,v) = \frac{g\left[F^{(-1)}(u), H^{(-1)}(v)\right]}{f\left[F^{(-1)}(u)\right]h\left[H^{(-1)}(v)\right]}.$$

We then assume that the prior has a density function $q(x,y)$. Thus, its marginal cumulative distribution functions take the form

$$u = F(x) = \int_{-\infty}^{\bar{x}} \int_{-\infty}^{+\infty} q(x,y)\,dy\,dx \text{ , and}$$

$$v = H(y) = \int_{-\infty}^{\bar{x}} \int_{-\infty}^{+\infty} q(x,y)\,dx\,dy \text{ , where } u = F(x) \Leftrightarrow x = F^{-1}(u),$$

and $v = H(y) \Leftrightarrow y = H^{-1}(v)$.

Therefore, its marginal densities take the form

$$f(x) = \int_{-\infty}^{+\infty} q(x,y)\,dy, \text{ and}$$

$$h(y) = \int_{-\infty}^{+\infty} q(x,y)\,dx.$$

Substituting these into the copula definition, we obtain the copula of the prior,

$$c_q(u,v) = \frac{q\left[F^{-1}(u), H^{-1}(v)\right]}{\int_{-\infty}^{+\infty} q\left[F^{-1}(u), y\right]dy \int_{-\infty}^{+\infty} q\left[x, H^{-1}(v)\right]dx}. \tag{B1}$$

Similarly, we assume that the CIMDO distribution with $q(x,y)$ as the prior takes the form

$$\widehat{p(x,y)} = q(x,y)\exp\left\{-\left[1+\hat{\mu}+(\hat{\lambda}_1\chi_{\left[x_d^x,\infty\right)})+(\hat{\lambda}_2\chi_{\left[x_d^y,\infty\right)})\right]\right\}.$$

We also define $u=F_c(x) \Leftrightarrow x=F_c^{-1}(u)$, and $v=H_c(y) \Leftrightarrow y=H_c^{-1}(v)$. Its marginal densities take the form

$$f_c(x) = \int_{-\infty}^{+\infty} q(x,y)\exp\left\{-\left[1+\hat{\mu}+(\hat{\lambda}_1\chi_{x_d^x}(x))+(\hat{\lambda}_2\chi_{x_d^y}(y))\right]\right\}dy,$$

and

$$h_c(y) = \int_{-\infty}^{+\infty} q(x,y)\exp\left\{-\left[1+\hat{\mu}+(\hat{\lambda}_1\chi_{x_d^x}(x))+(\hat{\lambda}_2\chi_{x_d^y}(y))\right]\right\}dx.$$

Substituting these into the copula definition, we obtain the CIMDO copula,

$$c_c(u,v) = \frac{1}{\int_{-\infty}^{+\infty} q\left[F_c^{-1}(u),y\right]\exp\left[-\hat{\lambda}_2\chi_{x_d^y}(y)\right]dy}$$

$$\times \frac{q\left[F_c^{-1}(u),H_c^{-1}(v)\right]\exp\left[-(1+\hat{\mu})\right]}{\int_{-\infty}^{+\infty} q\left[x,H_c^{-1}(v)\right]\exp\left[-\hat{\lambda}_1\chi_{x_d^x}(x)\right]dx}.$$

(B2)

Equation (B2) shows that the CIMDO copula is a nonlinear function of $\hat{\mu}$, $\hat{\lambda}_1$, and $\hat{\lambda}_2$, which change as the PoDs of the banks under analysis change. Therefore, the CIMDO copula captures changes in PoDs, as these change at different periods of the economic cycle.

REFERENCES

Aspachs, O., C.A.E. Goodhart, M.A. Segoviano, and L. Zicchino. 2006. "Searching for a Metric for Financial Stability." London School of Economics Financial Markets Group, Special Paper Series, SP167.

Athanosopoulou, M., M.A. Segoviano, and A. Tieman. 2009. "Banks' Probability of Default: Which Methodology, When, and Why?" Working paper. Washington: International Monetary Fund.

Bordo, M.D., B. Eichengreen, D. Klingebiel, and M.S. Martínez-Peria. 2001. "Financial Crises: Lessons from the Last 120 Years." *Economic Policy,* April.

Cassela, G. and R. Berger. 2002. *Statistical Inference.* Pacific Grove, Calif.: Duxbury.

Danielsson, J. and C. de Vries. 1997. "Tail Index and Quantile Estimation with Very High Frequency Data." *Journal of Economics and Finance* 4: 241–57.

De Bandt, O. and P. Hartmann. 2001. "Systemic Risk: A Survey." In *Financial Crisis, Contagion and the Lender of Last Resort: A Book of Readings*, eds. C.A.E. Goodhart and G. Illing, pp. 249–298. Oxford University Press.

De Vries, C. 2005. "The Simple Economics of Bank Fragility." *Journal of Banking and Finance* 29: 803–25.

Diebold, F.X., T. Gunther, and A. Tay. 1998. "Evaluating Density Forecasts with Applications to Financial Risk Management." *International Economic Review* 39: 863–83.

Diebold, F. X., J. Hahn, and A. Tay. 1999. "Multivariate Density Forecast Evaluation and Calibration in Financial Risk Management: High-Frequency Returns on Foreign Exchange." *The Review of Economics and Statistics* 81(4): 661–73.

Embrechts, P., A.J. McNeil, and D. Straumann. 2002. "Correlation and Dependence in Risk Management: Properties and Pitfalls." In *Risk Management: Value at Risk and Beyond*, edited by M.A.H. Dempster, pp. 176–224. Cambridge University Press.

Embrechts, P., F. Lindskog, and A.J. McNeil. 2003. "Modeling Dependence with Copulas and Applications to Risk Management." In *Handbook of Heavy Tailed Distributions in Finance*, edited by S.T. Rachev, pp. 329–84. Amsterdam: Elsevier Science.

Frey, R. and A.J. McNeil. 2001. "Correlation and Dependence Properties in Risk Management: Properties and Pitfalls, Department of Mathematics." ETH, Zurich RiskLab.

Glasserman, P., P. Heidelberger, and P. Shahabuddin. 2000. "Portfolio Value-at-Risk with Heavy-Tailed Risk Factors." Columbia University Business School, Money, Economics, and Finance Working Paper.

Goodhart, C.A.E., B. Hofmann, and M.A. Segoviano. 2004. "Bank Regulation and Macroeconomic Fluctuations." *Oxford Review of Economic Policy* 20(4): 591–615.

————. 2006. "Default, Credit Growth, and Asset Prices." Working paper 06/223. Washington: International Monetary Fund.

Goodhart, C.A.E. and M.A. Segoviano. 2004. "Basel and Procyclicality: A Comparison of the Standardized and IRB Approaches to an Improved Credit Risk Method." London School of Economics. Financial Markets Group Discussion Paper Series, DP524.

Goodhart, C.A.E., P. Sunirand, and D.P. Tsomocos. 2006. "A Model to Analyse Financial Fragility." *Economic Theory* 27(1): 107–42.

Hartmann, P., S. Straetmans, and C. De Vries. 2001. "Asset Market Linkages in Crisis Periods." Working paper 71. Frankfurt: European Central Bank

Hoelscher, D. 2006. *Bank Restructuring and Resolution*. Washington: International Monetary Fund, Monetary and Financial Systems Department, 1 October 2006.

Hoskin, J.R.M., G. Bonti, and D. Siegel. 2000. "Beyond the Lognormal." *Risk* 13(5): 59–62.

Huang, X. 1992. "Statistics of Bivariate Extreme Values." Ph.D. thesis No. 22, Tinbergen Institute Research Series, Erasmus University.

Jaynes, E. 1957. "Information Theory and Statistical Mechanics." *Physics Review* 106: 620–30.

Kullback, S. 1959. *Information Theory and Statistics,* New York: John Wiley.

Kullback, S. and R. Leibler, 1951. "On Information and Sufficiency." *Annals of Mathematical Statistics*, 22: 79–86.

Nelsen, R. 1999. *An Introduction to Copulas*. New York: Springer.

Segoviano, M.A. 1998. "A Structural Approach for Portfolio Credit Risk vs. Creditmetrics." Mimeo. London School of Economics.

————. 2006a. "The Conditional Probability of Default Methodology." Financial Markets Group. London School of Economics Discussion Paper 558.

————. 2006b. "The Consistent Information Multivariate Density Optimizing Methodology." Financial Markets Group, London School of Economics Discussion Paper 557.

————. 2009. "The CIMDO Copula. Robust Estimation of Distress Dependence under Data Restrictions." Working paper. Washington: International Monetary Fund.

Segoviano, M.A. and C.A.E. Goodhart. 2009. "Banking Stability Measures." Discussion Paper 627. London: Financial Markets Group, London School of Economics.

Segoviano, M.A. and P. Lowe. 2002. "Internal Ratings, the Business Cycle and Capital Requirements: Some Evidence from an Emerging Market Economy." BIS Working Paper 117.

Segoviano, M.A. and P. Padilla. 2006. "Portfolio Credit Risk and Macroeconomic Shocks: Applications to Stress Testing under Data Restricted Environments." International Monetary Fund Working Paper 06/283.

Sklar, A. 1959. "Fonctions de répartition à n dimensions et leurs marges." *Publications de l'Institut de Statistiques de L'Université de Paris* 8: 229–31.