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THE IMPLICATIONS OF EXHAUSTIBLE RESOURCES AND SECTORAL COMPOSITION FOR GROWTH ACCOUNTING: AN APPLICATION TO CHILE*

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Abstract
Standard growth accounting overlooks the role of exhaustible resources. This omission leads to overstating physical capital shares and to misleading total factor productivity (TFP). We study an application to Chile, a country dependent on mining production. First, we quantify the sources of economic growth. Second, we study TFP gains arising from changes on the economy's sectoral composition. Our results are as follows. Mining value added grows at an average annual rate of 0.69%. Productivity, physical and human capital contribute 3.75%. The exhaustible resource (ore grade) contributes -2.96%. At the aggregate level, omitting ore grade overstates the contribution of capital, 0.55%, and understates TFP growth, 0.96%. We document a composition gain of -0.53% between the mining and non-mining sectors. We obtain a 0.68% composition gain within the non-mining sector. We show that mining countries are exposed to similar sources of sectoral productivity growth as the Chilean economy.

Resumen
La contabilidad del crecimiento estándar del PIB pasa por alto el papel de los recursos agotables. Esta omisión conduce a exagerar la participación del capital físico y cómputos erróneos de productividad total de los factores (PTF). Estudiamos una aplicación a Chile, un país dependiente de la producción minera. Primero, cuantificamos sus fuentes del crecimiento económico. Segundo, estudiamos las ganancias de la PTF derivadas de cambios en la composición sectorial de la economía. Nuestros resultados son los siguientes. El valor agregado de la minería crece a una tasa promedio anual de 0,69%. Productividad, capital físico y humano aportan 3,75%. El recurso agotable (ley minera) contribuye -2,96%. A nivel agregado, omitir la ley minera sobreestima la contribución del capital, 0,55%, y subestima el crecimiento de la PTF, 0,96%. Documentamos una ganancia de composición de -0,53% entre los sectores minero y no minero. Se obtiene una ganancia de 0,68% dentro del sector no minero. Mostramos que países mineros están expuestos a fuentes de crecimiento de productividad sectorial similares a los de la economía chilena.

* We thank Victor Garay from the Chilean Copper Commission for helping us with data related issues. The views expressed in here are those of the authors and do not necessarily reflect the views of the Central Bank of Chile. Emails: cdelahuerta@bcentral.cl and eluttini@bcentral.cl.

1. Introduction

Growth accounting exercises are a traditional workhorse of studies on economic growth. This methodology decomposes aggregate economic growth into contributions of factors of production: physical and human capital. Total factor productivity (TFP) is computed as the remaining component of growth not accounted by the factors of production. This approach is appealing because of its simplicity to quantify the sources of economic growth. However, simplicity comes at the cost of overlooking some issues.[1] In this article, we focus on two of them. The first one relates to the omission of exhaustible natural resources. This is relevant for countries where natural resources are an important factor of production.[2] Failing to incorporate this factor leads to incorrectly compute factors of production and TFP contributions to economic growth. In particular, omitting natural resources leads to overstating the contribution of physical capital.[3] Moreover, the exhaustion of natural resources is attributed to TFP. A second methodological aspect arises when we incorporate economic sectors into the analysis. Changes on the economy's sectoral composition are expected to take place over time. Whenever the shares of physical and human capital differ across sectors, these changes cause TFP gains.[4] The sign of these gains, however, has to be empirically determined. Decomposing the economy's value added between exhaustible and non-exhaustible resource sectors, we expect the sign of the latter to be negative. Specifically, once natural resources are taken into account, the shares of physical and human capital are lower in the exhaustible than non-exhaustible resource sector. Hence, if sectoral physical and human capital are growing at the same rate, the lower participation of physical and human capital generates a negative TFP gain.

This paper offers an empirical analysis on these methodological issues. Our empirical application focuses on Chile, a country heavily dependent on mining production. The time period, 2000-2016, encompasses the metal and oil prices expansive cycle of 2002-2016 (Figure 1 shaded area). Our contribution is twofold. First, we quantify the sources of economic growth for an economy where exhaustible natural resources sectors contribute to an important share of aggregate value added.

[1]Barro (2000) provides a thourough discussion on the strengths and weaknesses of growth accounting exercises.

[2]Caselli and Feyrer (2007) and Monge-Naranjo et al. (2015) document the extent of the relevance of natural resources for developed and emerging economies.

[3]The contribution of physical capital is calculated as one minus the participation of human capital on aggregate value added.

[4]This result is implied by Bernard and Jones (1996) productivity growth decomposition.

Second, we compare the sources of productivity growth when the economy is decomposed into exhaustible and non-exhaustible resource sectors, in contrast to non-exhaustible resource sectors. We pay special attention to characterizing composition effects between these sectors.

Our analysis takes a bottom-up approach. First, we present growth accounting exercises at the sectoral level. We consider an economy constituted of two sectors: Depletable (mining) and non-depletable (non-mining) resource sectors. We quantify the contributions of productivities, factor of productions, and a key depletable resource (ore grade) to value added growth.[5] We compare these contributions to motivate the relevance of ore grade as a source of economic growth. Next, we consider the aggregate economy. Aggregate value added growth can be decomposed into *pure productivity*, *composition*, and *factors of production* contributions. The first term accounts for TFP gains coming from productivity gains within mining and non-mining sectors. The second term explains TFP gains arising from changes in the economy's sectoral composition. The sum of these terms is an approximation to TFP growth. The last term is the standard growth accounting contribution attributed to factors of production, augmented by ore grade. Finally, we decompose productivity growth within the non-mining sector into *pure productivity* and *composition* terms. We compare the *composition* terms between the aggregate and non-mining sectors.

Our main results are as follows. Mining value added grows at an average annual rate of 0.69%. Productivity and the sum of physical and human capital add 1.41% and 2.24%, respectively. Ore grade massively contributes to value added growth, −2.96%. As for the non-mining sector, value added grows at 4.63%. The bulk of the difference between mining and non-mining value added growth is accounted for by ore grade (75%). The leftover is mostly accounted for by a higher human capital contribution within the non-mining sector. In regard to the aggregate economy, omitting ore grade overstates the actual contribution of capital and understates TFP growth by 0.55% and 0.96%, respectively. Hence, for economies where exhaustible resources play an important role, neglecting them delivers a misleading account on the sources of economic growth. Finally, we document a negative *composition* term between the mining and non-mining sectors, −0.53%, and a positive one within the non-mining sector, 0.68%. Thus, the contribution of the *composition* term between the mining and non-mining sectors is quite different than within the non-mining sector.

[5] Ore grade is the concentration of metal that can be extracted from rocks.

Additionally we discuss the relevance of our work for mining countries. We document that countries with important depletable natural resources are exposed to the same sectoral source of productivity growth as the Chilean economy. In particular, our results suggest that incorporating depletable resources into growth accouting exercises is critical to properly assess their sources of economic growth.

Our paper is part of the literature documenting the role of ore grade on productivity. Looking at the Canadian mining sector, Wedge (1973) is the first to document the role of ore grade on mining value added. More recently, Zheng and Bloch (2014) compute a measure of productivity, corrected by depletion of ore grade, on the Australian mining sector. Both articles document the role of ore grade as a key source of growth on mining value added. Arias and Rodríguez (2008) provide related evidence for the coal sector in Spain. In addition, Lasserre and Ouellette (1988) examine the differences that arise when computing productivities of extractive and non-extractive sectors, focusing on two specific Canadian industrial sectors (textile and asbestos). Our article considers the mining sector in Chile. We present evidence that ore grade has been a crucial factor depressing mining value added in the past seventeen years. We compare the different sources of value added growth between mining and non-mining sectors. Our main contribution to the literature is twofold. First, to the best of our knowledge, we are the first to extend the analysis from the sectoral to the aggregate level. In particular, we incorporate depletable resources into the standard growth accounting framework. Second, we arrive to very different results computing the *composition* term between the mining and non-mining sectors, as opposed to within the non-mining sector.

The methodology of our paper resembles to Bernard and Jones (1996). In fact, our TFP growth decomposition is an approximation to their decomposition. However, the advantage of our approximation is that it can be placed into the standard Solow's growth accounting framework. In addition, our empirical application evidences the productivity paradox documented by Fox (2012). In particular, due to a negative *composition* term between mining and non-mining sectors, aggregate productivity is lower that any convex combination of mining and non-mining productivities.

Finally, our paper relates to the recent literature that includes natural resources as input of aggregate production functions. Caselli and Feyrer (2007) and Monge-Naranjo et al. (2015) show natural resources' contribution to aggregate value added is fairly important. For this reason, ignoring exhaustible resources leads to overestimate the marginal product of physical capital. The latter occurs because non-labor income is incorrectly imputed to physical capital. Our findings suggest natural resources are important to accurately compute the sources of economic growth.

2. Methodology

In this section we discuss the methodological aspects of our growth accounting exercise. Following a bottom-up approach, we introduce value added functions for the mining and non-mining sectors. We incorporate ore grade as an additional input to mining value added. Sectoral value added growth is decomposed into inputs and sector specific productivity contributions. Aggregating the sectoral contributions of inputs and sector specific productivity to value added, value added growth can be approximated by the sum of *pure productivity*, *composition*, and *factors of production* terms. Finally, the sum of the terms *pure productivity* and *composition* approximates to TFP growth.

2.1. Sectoral Value Added Production Functions

To begin with, we describe the mining sector value added function. Following Aguirregaviria and Luengo (2016), we posit a Cobb-Douglas value added function.[6] These authors provide evidence that mining production depends crucially on ore grade. In addition to physical and human capital, ore grade is considered as an additional input. A general concern in regard to sectoral value added functions is letting them to depend on a limited number of inputs: Physical and human capital, and ore grade. If such limited number of inputs are separable with respect to the omitted ones, Sato (1976) shows that value added functions can be expressed depending on this limited number of inputs.[7] Thus, we consider a mining value added function of the form

$$Y_{min\,t} = STFP_{min\,t}\, K_{min\,t}^{1-\alpha_{l\,min}-\alpha_{o\,min}}\, L_{min\,t}^{\alpha_{l\,min}}\, O_{min\,t}^{\alpha_{o\,min}} \tag{1}$$

[6]We rely on Aguirregaviria and Luengo study due to their large coverage of the mining sector. In particular, their dataset covers roughly 85% of worldwide copper production over 1992-2010.

[7]Recently, Herrendorf et al. (2015) use a similar argument to model sectoral value added functions.

where $Y_{min\,t}$, $STFP_{min\,t}$, $K_{min\,t}$, $L_{min\,t}$, and $O_{min\,t}$ denote mining value added, productivity, physical capital, human capital (hours and quality adjusted employment) and ore grade. $\alpha_{l\,min}$ and $\alpha_{o\,min}$ are the mining labor and natural resources shares on mining value added, respectively.

Proceeding in the same manner for the non-mining sector, we consider a Cobb-Douglas value added function, but excluding ore grade. That is,

$$Y_{no\ min\,t} = STFP_{no\ min\,t}\, K_{no\ min\,t}^{1-\alpha_{lno\ min}}\, L_{no\ min\,t}^{\alpha_{lno\ min}} \tag{2}$$

where $STFP_{no\ min\,t}$, $K_{no\ min\,t}$, $L_{no\ min\,t}$, and $\alpha_{\alpha_{lno\ min}}$ are sector specific productivity, physical and human capital, and the non-mining labor share, respectively.

Taking first differences to the logarithm of Equations 1 and 2, we aproximate mining and non-mining value added growth by

$$\Delta y_{min\,t} = \Delta stfp_{min\,t} + (1-\alpha_{l\,min}-\alpha_{o\,min})\,\Delta k_{min\,t} + \alpha_{l\,min}\Delta l_{min\,t} + \alpha_{o\,min}\Delta o_{min\,t}\,, \tag{3}$$

$$\Delta y_{no\,min\,t} = \Delta stfp_{no\,min\,t} + (1-\alpha_{l\,no\,min})\,\Delta k_{no\,min\,t} + \alpha_{l\,no\,min}\Delta l_{no\,min\,t}\ , \tag{4}$$

where lowercases are logarithm of level variables and Δx_t means $x_t - x_{t-1}$.

2.2. Growth Accounting

Aggregate value added growth can be approximated by the sum of *pure productivity*, *composition*, and *factors of production* terms. Defining the economy value added, Y_t, as

$$Y_t = TFP_t K_t^{1-\alpha_l-\alpha_o} L_t^{\alpha_l} O_t^{\alpha_0}\,. \tag{5}$$

In Appendix C we show value added growth can be approximated by the following expression

$$
\begin{aligned}
\frac{\Delta Y_t}{Y_{t-1}} \approx\; & \underbrace{\frac{Y_{min\,t-1}}{Y_{t-1}}\Delta stfp_{min\,t} + \frac{Y_{no\,min\,t-1}}{Y_{t-1}}\Delta stfp_{no\,min\,t}}_{\textit{pure productivity}} + \\
& + \underbrace{\frac{Y_{min\,t-1}}{Y_{t-1}}\Bigg((1-\alpha_{l\,min}-\alpha_{o\,min})\,\Delta k_{min\,t} - (1-\alpha_{\text{l}}-\alpha_{\text{o}})\,\Delta k_t + \alpha_{l\,min}\Delta l_{min\,t} - \alpha_{\text{l}}\Delta l_t}\cdots \\
& \cdots\underbrace{+\,\alpha_{o\,min}\Delta o_{min\,t} - \alpha_{\text{o}}\frac{Y_{t-1}}{Y_{min\,t-1}}\Delta o_t\Bigg)}_{\textit{composition}}+\cdots \\
& + \cdots\underbrace{\frac{Y_{t-1\,\text{No Min}}}{Y_{t-1}}\Bigg((1-\alpha_{lno\;min})\,\Delta k_{no\,min\,t} - (1-\alpha_{\text{l}}-\alpha_{\text{o}})\,\Delta k_t + \alpha_{lno\;min}\Delta l_{no\,min\,t} - \alpha_{\text{l}}\Delta l_t\Bigg)} + \\
& + \underbrace{(1-\alpha_{\text{l}}-\alpha_{\text{o}})\,\Delta k_t + \alpha_{\text{l}}\Delta l_t + \alpha_{\text{o}}\Delta o_t}_{\textit{factors of production}}\,.
\end{aligned}
\tag{6}
$$

Equation 6 decomposes value added growth into three distinct terms: *pure productivity*, *composition*, and *factors of production*. The first term accounts for productivity gains within the mining and non-mining sectors. The *composition* term arises due to heterogeneous contributions of factors of production across sectors. An interesting case emerges when factors of production grow at the same rate across sectors. In particular, the heterogeneous contributions of factors of production implies the *composition* term will differ from zero. The *composition* term captures TFP gains due to changes on the economy's sectoral composition. Providing structural reasons to explain the size and sign of this term are beyond the aim of this article. Moreover, the sum of *pure productivity* and *composition* terms is an approximation to Bernard and Jones (1996) TFP growth decomposition.[8] Finally, *factors of production* account for the standard growth accounting contributions of physical and human capital, augmented by ore grade, to value added growth.[9]

[8] The *pure productivity* and *composition* terms are approximations to Bernard and Jones' Productivity Growth and Share Effects.

[9] The term $\alpha_{o\,min}\Delta o_{min\,t} - \alpha_{\text{o}}\frac{Y_{t-1}}{Y_{min\,t-1}}\Delta o_t$ is close to zero. In particular, $\Delta o_{min\,t} = \Delta o_t$ and (in Section 3.1) α_{o}

We compare the sources of economic growth between the aggregate and non-mining sectors. To do so we decompose non-mining economic growth according to Equation 6. In particular,

$$
\begin{aligned}
\frac{\Delta Y_{no\,min\,t}}{Y_{no\,min\,t-1}} \approx & \sum_j \frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} \Delta stfp_{j,t} + \sum_j \frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} \Big(\left(1-\alpha_j\right) \Delta k_{j\,t} - \\
& - \left(1-\alpha_{lno\;min}\right) \Delta k_{no\,min\,t} \Big) + \sum_j \frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} \left(\alpha_j \Delta l_{j\,t} - \alpha_{lno\;min} \Delta l_{no\,min\,t}\right) + \\
& + \left(1-\alpha_{lno\;min}\right) \Delta k_{no\,min\,t} + \alpha_{lno\;min} \Delta l_{no\,min\,t} \,,
\end{aligned}
$$

where j are economic sectors adding up to the non-mining sector. Moreover, each economic sector value added was assumed to follow a Cobb-Douglas function, $Y_{j\,t} = STFP_{j\,t}\, K_{j\,t}^{1-\alpha_j}\, L_{j\,t}^{\alpha_j}$.[10] [11] This is an appealing exercise because, once natural resources are taken into account, the shares of physical and human capital are lower in the mining than non-mining sector (see Table 2). Hence, for economies where exhaustible natural resources are important, the size of the *composition* term is expected to be quite different between the aggregate and non-mining sectors.

3. Results

We apply the methodology discussed in Section 2 to Chile.[12] Our motivation is twofold. First, Chile is the major producer of copper worldwide.[13] Second, Chile's mining sector represents 14% of the economy's value added.[14] Thus, Chilean data is suitable to underscore the relevance of

is calibrated by $\alpha_{o\,min} \overline{\frac{Y_{min\,t}}{Y_t}}$ rendering $\alpha_o \frac{Y_{t-1}}{Y_{min\,t-1}} \approx \alpha_{o\,min}$.

[10] Though Cobb-Douglas sectoral production functions may raise concerns in regard to being too restrictive, for our empirical application seems a good approach. For the Chilean economy, Appendix B shows for the period 2008-2014 that sectoral labor shares are relatively stable. Moreover, Bernard and Jones (1996) assume as well Cobb-Douglas sectoral value added functions to carry out an exercise like ours for a set of OECD countries. Finally, in a three sector (agriculture, manufacturing, and services) model Herrendorf et al. (2015) show that Cobb-Douglas sectoral production functions capture low frequency trends of the US economy as well as a constant elasticity of substitution production function.

[11] The following economic sectors were considered: Aggriculture, Manufacture, Energy, water, and gas, Construction, Retail and wholesales, Transport and communications, Business services, and Comunity services.

[12] Data sources are described in Appendix A.

[13] See U.S. Geological Survey, (2017).

[14] Average for the period 2008-2014. Value added was computed excluding Housing services.

accounting for exhaustible resources. After calibrating sectoral and aggregate value added functions, we document that the difference between mining and non-mining value added growth is mostly explained by ore grade. We later decompose aggregate value added growth into *pure productivity*, *composition*, and *factors of production* contributions. Remarkably, ore grade contracted aggregate activity by an staggering 0.41% on annual basis. We document a negative *composition* term between mining and non-mining sectors, while the *composition* term is positive within the non-mining sector. The latter accounts for the bulk of non-mining productivity growth.

3.1. Production Functions Calibration

We start calibrating the mining sector value added function, Equation 1. First, following the literature, we calibrate $\alpha_{l\,min}$ as the (time series average) wage bill participation on mining value added. Ore grade's exponent, $\alpha_{o\,min}$, is a non-standard parameter. Aguirregaviria and Luengo (2016) carry out structural estimations over Equation 1's exponents. To calibrate $\alpha_{o\,min}$, we use their estimations.[15] In particular, we follow a three steps procedure. First, we take $\alpha_{o\,min}$ as the average across their estimations, obtaining $1 - \alpha_{l\,min} - \alpha_{o\,min}$ as residual. Second, we proceed in the opposite manner, we obtain $1 - \alpha_{l\,min} - \alpha_{o\,min}$ as the average across their estimations, obtaining $\alpha_{o\,min}$ as residual. Finally, $\alpha_{o\,min}$ and $1 - \alpha_{l\,min} - \alpha_{o\,min}$ are the average values from the previous steps.[16] As for the non-mining sector (Equation 2) and all sectors within the non-mining sector, we calibrate $\alpha_{l\,no\,min}$ and α_j as the wage bill participation on sectoral value added. Table 2 shows the results of our calibration. The lower shares of physical and human capital in the mining than non-mining sector are noteworthy. This result is explained by the high contribution of ore grade to the mining sector.

The exponents associated to physical capital and ore grade on the Cobb-Douglas aggregate production function deserve some explanations. To calibrate the capital share on a production function that accounts for non-reproducible physical capital, our approach resembles to Monge-Naranjo et

[15] Under different specifications their estimates for $\alpha_{o\,min}$ are 0.59, 0.61, 0.66, 0.7, 0.74 and 0.77 (these values are close to Young (1991) estimates). As for $1 - \alpha_{l\,min} - \alpha_{o\,min}$ their estimates are 0.13, 0.22, 0.24, 0.24, 0.33, and 0.37.

[16] An alternative to our calibration is to consider Aguirregaviria and Luengo (2016) preferred estimate. However, since we constrain $\alpha_{l\,min}$ to be equal to the wage bill participation on value added, it is no longer clear what is the best estimate for $\alpha_{o\,min}$. Yet, our results are similar to consider Aguirregaviria and Luengo (2016) preferred estimates. These results are available upon request.

al. (2015). Assuming sectors operate in a competitive environment under optimal conditions, the exponents on the Cobb-Douglas aggregate production functions may be expressed as weighted (by nominal sectoral value added share) average of each factor contribution to sectoral value added. Since ore grade affects aggregate output through mining, we can compute the share of ore grade weighting $\alpha_{o\,min}$ by the average nominal share of mining on value added. Finally, calibrating the labor share following the standard procedure in the literature, i.e. the ratio between the aggregate wage bill and value added, the capital share is obtained as residual.[17]

Our calibration for the aggregate production function is similar to adjust the contributions of physical and human capital in the non-mining sector by the contribution of natural resources. Expressing Equation 5 as $Y_t = TFP_t \left(K_t^{\frac{1-\alpha_l-\alpha_o}{1-\alpha_0}} L_t^{\frac{\alpha_l}{1-\alpha_0}} \right)^{1-\alpha_0} O_t^{\alpha_0}$, $\frac{\alpha_l}{1-\alpha_0}$ and $\frac{1-\alpha_l-\alpha_o}{1-\alpha_0}$ turn out to be 0.48 and 0.52, numbers remarkably similar to our calibration for $\alpha_{lno\ min}$ and $1-\alpha_{lno\ min}$ (0.47 and 0.53).

3.2. Growth Accounting

3.2.1. Mining

Over the period, depletion of ore grade keeps mining value added stagnant. We decompose mining value added into productivity, physical and human capital, and ore grade contributions. Table 3 reports (time series average) of each component. Over the past seventeen years mining value added grows at a low 0.69% annual average rate. Rapid ore grade decline explains this result. Ore grade average annual growth is -4.56% (factors of production period average are reported in Table 1), rendering a 2.96% lower annual average value added growth.[18] In spite of ore grade, high levels of physical capital accumulation have prevented the decline of mining production. Physical capital contributes to 2.09% higher value added growth, partially offseting the exhaustion of ore grade. High investment taking place during the commodity cycle of 2004-2013 explains a remarkable physical

[17]Monje-Naranjo et al.(2015) obtain similar values of factor shares for the aggregate economy (labor share 0.45, capital share 0.46, and natural resources share 0.09). Casselli and Feyrer (2007) also estimate factor's contribution. Yet, their methodology overstates the contribution of natural resources (see Monje-Naranjo et al.(2015)). They obtain a labor share of 0.59, capital share 0.16, and natural resources share of 0.25.

[18]Using a different methodology, Zheng and Bloch (2014) provide similar evidence for the Australian mining industry.

capital average annual growth of 9.50%. In contrast, human capital hardly accounts for mining activity. The latter is expected due to the low sectoral labor share. Finally, mining productivity grows at 1.41% average annual rate. Figure 2 depicts the annual contribution of each component to value added growth. Over the period, the sum of physical capital and productivity growth outweigh the loss of production due to exhaustion of ore grade.

We quantify the implications of omitting ore grade to mining productivity. To do so, we use the standard growth accounting framework. In particular, we assume value added is generated from physical and human capital according to $Y_{min\,t} = ASTFP_{min\,t}\, K_{min\,t}^{1-\alpha_{l\,min}}\, L_{min\,t}^{\alpha_{l\,min}}$, where $ASTFP_{min\,t}$ is the Solow's residual. The logarithm of $ASTFP_{min\,t}$ can be expressed as $stfp_{min\,t} - \alpha_{o\,min}\,(k_{min\,t} - o_{min\,t})$. The latter implies that $astfp_{min\,t}$ distorts $stfp_{min\,t}$ by overstating the physical capital share and omitting ore grade.

Our results show that overstating the physical capital share and omitting ore grade are distortions quantitatively important. Figure 3 shows the evolution of $astfp_{min\,t}$ over time. The Solow's residual absorbs the misspecification of mining value added. The largest distortion to $stfp_{min\,t}$, 57%, comes from overstating the physical capital share. The omission of ore grade brings out the remaining 43% of the cumulative gap between $astfp_{min\,t}$ and $stfp_{min\,t}$.

3.2.2. Non-Mining

The stark difference between mining and non-mining value added growth is accounted for by ore grade. To put this result into perspective, if ore grade had remained constant since 1999, mining value added would have grown an average of 3.75% much closer to the 4.63% of non-mining value added.

Physical capital is the main source of growth for the non-mining sector. Physical capital accounts for as much as 54% of non-mining value added growth. Comparing Figures 2 and 4, mining and non-mining physical capital exhibit similar dynamics since 2004. The latter is consistent with spillovers from mining to non-mining sectors.[19]

[19]For a causal link on the spillovers from mining to non-mining sectors, see Fornero et al. (2015).

Though physical capital grows less than for the mining sector (5.31% against 9.50%), physical capital contributes relatively more to non-mining value added growth. This result is accounted for by the higher physical capital share in the non-mining sector. Similarly, human capital is a more important source of growth in the non-mining than the mining sector. As with physical capital, the contrast is explained by the higher contribution of labor to non-mining than mining value added.

Finally, non-mining productivity grows at a similar rate than the mining sector. Despite this result, in Section 3.3 we show that the underlying factors behind the dynamics of mining and non-mining TFP growth are different.

3.2.3. Aggregate

We carry out an aggregate growth accounting exercise augmenting the factors of production by ore grade. Caselli and Feyrer (2007) and Monge-Naranjo et al. (2015) stress that accounting for natural resources is relevant to calibrate the economy's physical capital share. This observation is especially pertinent for Chile, given the economy's heavy dependence on mining production. Taking first differences to the logarithm of Equation 5, we consider the following accounting framework,

$$\Delta y_t = \Delta tfp_t + (1 - \alpha_{\text{l}} - \alpha_0)\,\Delta k_t + \alpha_{\text{l}}\Delta l_t + \alpha_{\text{o}}\Delta o_t\,.$$

Our results show ore grade is quantitatively important to explain value added growth in Chile. Table 3 shows that depletion of ore grade contracted activity by an staggering 0.41%. Figure 5 shows that ore grade is an important factor explaining the decline of Chile's value added growth. The latter is evident during the aforementioned commodity cycle. Since 2012 ore grade is less relevant to explain the evolution of aggregate value added. Lower physical capital and TFP growth take the leading role. This result is similar to other studies documenting the decline and stagnation of Chilean productivity over the past decade.[20]

Table 3 shows the contributions of physical and human capital to aggregate value added growth

[20] Corbo and Gonzalez (2014), Fuentes and García (2014), and Central Bank of Chile (2016) document the recent Chilean productivity slowdown.

are similar to those of the non-mining sector. We estimate a 4.04% average growth rate of value added. Notably, physical capital has been the key source of Chile's ability to keep value added growing. Human capital and productivity growth explain together near to 43% of Chilean growth. Interestingly, TFP growth is lower than any convex combination of mining and non-mining productivity growth.[21] In the next section we explain more extensively the rationale of this result.

To document the implications of omitting ore grade for aggregate growth accounting, we proceed as in Section 3.2.2 and compute the sources of economic growth according to $Y_t = ATFP_t \, K_t^{1-\alpha_1} \, L_t^{\alpha_1}$, where $ATFP_t$ is the Solow's residual. Figure 6 shows the two margins (overstatement of physical capital and omission of ore grade) through the omission of ore grade distorts TFP. Quantitatively, omitting ore grade leads to overstate the actual contribution of physical capital by 0.55% while understates TFP growth by 0.96%.

3.3. Accounting For Changes in Sectoral Composition

How do we reconcile a lower aggregate productivity growth than any convex combination of mining and non-mining sectoral productivity growth? Table 3 shows there is a negative composition effect (-0.53%) between mining and non-mining sectors. In fact, we trace back the negative composition effect to the low contribution of human capital to mining value added.[22] Equation 6 shows that there will be a composition effect when mining (non-mining) physical or human capital (weighted by their respective share) grow at a different rate than the aggregate. On the one hand, Table 3 shows a large gap between the contribution of human capital to mining relative to the contribution to the aggregate economy.[23] On the other hand, the contribution of physical capital to economic growth is similar across sectors. Hence, the low contribution of human capital to mining value added drives the negative composition effect.

At the aggregate level, the rise of mining productivity growth falls behind the negative *composition* term. Aggregate productivity growth is the sum of *pure productivity* and *composition* terms.

[21] This productivity paradox is extensively discussed in Fox (2012).

[22] Sectoral capital stock within the non-mining sector is available until the year 2014. For this reason, the results of this Section are reported for the 2000-2014 period.

[23] A low labor share of the mining sector (0.13%), relative to the aggregate (0.47%), implies that the composition effect is negative even in the case human capital growth rates were the same for both sectors.

Average (weighted by $\frac{Y_{min\,t-1}}{Y_{t-1}}$) productivity gains within the mining sector is smaller than the composition effect's absolute value. Hence, aggregate TFP growth turns out to be driven by non-mining productivity growth. This motivates us to further assess the channels of productivity growth within the non-mining sector.

The *composition* term explains most of productivity growth within the non-mining sector. Non-mining TFP growth is mostly driven by *composition* gains, 0.68%, and to a lesser degree by *pure productivity* gains, 0.54%. Though mining and non-mining productivity are quantitatively similar, the *composition* term is the main driver of the latter, where, by definition, the *pure productivity* component drives the former. Figures 7 and 8 show the contribution of sectoral physical and human capital to non-mining sector *composition* gains.[24] These Figures show the relevance of capital and labor as drivers of the *composition* term. Capital plays a preponderant role. Figure 7 illustrates that sectors with highest physical capital share (Business services, Transport and communications, and Energy, water and gas) are driving the *composition* term's physical capital component. Figure 8 shows sectors with highest human capital share (Personal services, Business services, and Retail and wholesale) determine the *composition* term's human capital component.

To summarize, decomposing productivity growth into *pure productivity* and *composition* terms, lead us to conclude that, for the Chilean economy, non-mining productivity is the source of aggregate productivity growth. Within the latter, the *composition* term accounts for the bulk of non-mining productivity growth. Our empirical application highlights that changes on the economy's sectoral composition is an important factor to account for the drivers of economic growth.[25]

4. Discussion

We show that mining countries are exposed to the same sources of sectoral productivity growth as the Chilean economy.[26] The results of the growth accounting exercise are relevant for other mining

[24] For instance, the contribution of physical and human capital in sector j are $\frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}}\left((1-\alpha_j)\,\Delta k_{j\,t}-(1-\alpha_{lno\ min})\,\Delta k_{no\,min\,t}\right)$ and $\frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}}\left(\alpha_j\Delta l_{j\,t}-\alpha_{lno\ min}\Delta l_{no\,min\,t}\right)$, respectively.

[25] In Appendix D we carry out a related exercise for labor productivity growht. As for the *composition* term the results turn out to be qualitatively similar.

[26] We focus on countries with at least a 5% mining value added share (the countries include Australia, Canada, Colombia, Ecuador, Indonesia, Malaysia, Mexico, Norway, Peru, and South Africa).

countries. To assess the role of depletable natural resources we look at the mining, non-mining, and economy's productivity. However, due to lack of data on depletable natural resources, we compute productivity as the Solow's residual. As discussed above, depletion of natural resources will show up in this residual productivity measure.

Mining Solow's residual growth is largely negative, while for the non-mining sector is positive. Table 5 shows mining, non-mining, and the economy's productivity growth.[27] In particular, mining countries exhibit the same patterns of sectoral productivity as the Chilean economy. Mining productivity growth dramatically falls, whereas the non-mining sector exhibit strong productivity growth. Due to overstating the contribution of physical capital and imputing depletion of natural resources to the Solow's residual, productivity growth is lower for the economy than non-mining sector.

The results for the Solow's residual carry through to labor productivity. In addition to the Solow's residual we look at labor productivity. Labor productivity is affected by depletion of natural resources but avoids overstating the contribution of physical capital. Table 6 shows the results for labor productivity. Within the mining sector across countries labor productivity falls as much as the Solow's residual. The latter highlights the role of depletable natural resources. However, for Canada and South Africa the Solow's residual falls significantly more than labor productivity. As with Chile, the latter might be attributed to overstating the true contribution of physical capital.

Taken together, these results suggest that incorporating depletable resources is relevant to assess the sources of economic growth on mining countries.

5. Conclusions

Our paper contributes to the economic growth literature by discussing the role of exhaustible natural resources into growth accounting exercises. We present a unified framework that incorporates exhaustible natural resources, and the role of changes in the economy's sectoral composition as

[27] Data availability constrains our exercise to Australia, Canada, Malaysia, Mexico, Norway, Peru, and South Africa. Appendix B describes the labor shares to calculate sectoral productivities.

sources of economic growth. In first place, we provide a methodology to calibrate the contribution of exhaustible natural resources to the economy value added. We provide an application to Chile, a country heavily dependent on copper production. Our results show that ore grade is an important factor explaining the exiguous growth rate of the mining sector's value added for the period 2000-2016. In fact, due to the high participation of the mining sector in the aggregate economy, ore grade depletion explains a 0.41% decline in aggregate economy's value added. Not including natural resources exhaustion leads to overstating the contribution of capital to value added by 0.55%. Despite this, capital accumulation is in fact the main driver explaining value added growth.

In second place, we decompose TFP and non-mining productivity into *pure productivity* and *composition* gains. When considering the aggregate economy we find a negative *composition* term between the mining and non-mining sectors. The low contribution of human capital to mining, relative to the aggregate economy, explains this result. As for the non-mining sector the *composition* term plays the main role explaining 75% of productivity growth. The leftover is attributed to the *pure productivity* effect. Hence, the main source of the country's productivity growth comes from the contribution of *composition* gains within the non-mining sector. Taken together, the non-mining *composition* term is an important determinant of aggregate economic growth.

Finally, we show evidence that mining countries are exposed to the same sources of sectoral productivity growth as the Chilean economy; meaning that accounting for depletion of natural resources is relevant for them. Further understanding on the drivers of productivity growth within the mining sector appears as an important area of reaserach.

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Table 1: Average Growth of Components of Value Added (2000-2016)

Sectors	Physical Capital	Human Capital	Ore Grade
Mining	9.50	1.12	-4.56
Non-Mining	5.31	1.63	–
Economy	6.16	1.82	-4.56

Notes: Average growth rate, period 2000-2016, of sectoral (Mining, Non-Mining, and Aggregate Economy, first, second, and last row, respectively) physical and human capital, and ore grade.

Table 2: Sectoral Production Function Shares: Non-Mining Sectors (2008-2014)

Sectors	Physical Capital	Human Capital	Ore Grade
Panel A: Sectoral			
Aggriculture	0.58	0.42	–
Manufacture	0.62	0.38	–
Energy, Water, and Gas	0.86	0.14	–
Construction	0.43	0.57	–
Retail and Wholesales	0.42	0.58	–
Transport and Communications	0.60	0.40	–
Business Services	0.55	0.45	–
Personal Services	0.19	0.81	–
Mining	0.22	0.13	0.65
Panel B: Aggregate			
Non-Mining	0.47	0.53	–
Economy	0.44	0.47	0.09

Notes: Shares of physical and human capital, and ore grade on economic sectors and aggregate economy (first, second, and last column, respectively). Panel A corresponds to sectoral shares. Panel B corresponds to shares of non-mining sector and aggregate economy. The sum of physical and human capital, and ore grade share is one. The participaction of human capital is computed as ratios between sectoral labor remunerations to value added. The participation of ore grade for the mining sector is computed in three steps. First, we obtain the average across ore grade parameters presented by Aguirregaviria and Luengo (2016) (0.59, 0.61, 0.66, 0.7, 0.74 and 0.77). Second, we compute the participation of capital as the average across Aguirre-gaviria and Luengo estimations (0.13, 0.22, 0.24, 0.24, 0.33, and 0.37), and obtain ore grade as a residual. The participation of ore grade of the mining sector is the average between these two steps. The participaction of ore grade of the aggregate economy is computed as the product of the ore grade participation of the mining sector and the share of mining into aggregate value added. Physical capital share is computed as residual.

Table 3: Growth Accounting (2000-2016)

Sectors	Δy (1)	$\Delta stfp$ (2)	Δk (3)	Δl (4)	Δo (5)
Mining	0.69	1.41	2.09	0.15	-2.96
Non-Mining	4.63	1.27	2.49	0.86	–
Economy	4.04	0.88	2.71	0.85	-0.41

Notes: Decomposition of sectoral value added average 2000-2016 growth (Δy) into productivity ($\Delta stfp$), capital (Δk), labor (Δl), and ore grade (Δo) contributions.

First row, Mining value added. The contributions of each factor are calculated from $y_{min\,t} = stfp_{min\,t} + (1-\alpha_{l\,min}-\alpha_{o\,min})\,k_{min\,t} + \alpha_{l\,min}l_{min\,t} + \alpha_{o\,min}o_{min\,t}$. Columns (2), (3), (4), and (5) are time series averages of $\Delta stfp_{min\,t}$, $(1-\alpha_{l\,min}-\alpha_{o\,min})\,\Delta k_{min\,t}$, $\alpha_{l\,min}\Delta l_{min\,t}$, and $\alpha_{o\,min}\Delta o_{min\,t}$ respectively.

Second row, Non-Mining value added. The contributions of each factor are calculated from $y_{no\,min\,t} = stfp_{no\,min\,t} + (1-\alpha_{l\,no\,min})\,k_{no\,min\,t} + \alpha_{l\,no\,min}l_{no\,min\,t}$. Columns (2), (3), and (4), are time series averages of $\Delta stfp_{min\,t}$, $(1-\alpha_{l\,min})\,\Delta k_{min\,t}$, and $\alpha_{l\,min}\Delta l_{min\,t}$, respectively.

Third row, Economy value added. The contributions of each factor are calculated from $y_t = tfp_t + (1-\alpha_l-\alpha_o)\,k_t + \alpha_l l_t + \alpha_o o_t$. Columns (2), (3), (4), and (5) are time series averages of Δtfp_t, $(1-\alpha_l-\alpha_o)\,\Delta k_t$, $\alpha_l\Delta l_t$, and $\alpha_o\Delta o_t$ respectively.

Δx_t is $x_t - x_{t-1}$. Columns (2), (3), (4), and (5) add up to column (1).

Table 4: TFP Growth Decomposition (2000-2014)

Sectors	Productivity Growth (1)	Pure Productivity (2)	Composition (3)
Non-Mining	1.22	0.54	0.68
Economy	0.75	1.28	−0.53

Notes: Decomposition of average sectoral productivity growth 2000-2014 into *pure productivity* and *composition* contributions. First row, Non-Mining productivity. Columns *pure productivity* and *composition* are time series averages of $\sum_j \frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} \Delta stfp_{j,t}$ and $\sum_j \frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} \Big((1-\alpha_j)\,\Delta k_{j\,t} - (1-\alpha_{lno\ min})\,\Delta k_{no\,min\,t} \Big) + \sum_j \frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} \left(\alpha_j \Delta l_{j\,t} - \alpha_{lno\ min} \Delta l_{no\,min\,t} \right)$, respectively.

Second row, Economy productivity. Columns *pure productivity* and *composition* are time series averages of $\frac{Y_{min\,t-1}}{Y_{t-1}} \Delta stfp_{min\,t} + \frac{Y_{no\,min\,t-1}}{Y_{t-1}} \Delta stfp_{no\,min\,t}$ and $\frac{Y_{min\,t-1}}{Y_{t-1}} \Big((1-\alpha_{l\,min}-\alpha_{o\,min})\,\Delta k_{min\,t} - (1-\alpha_{\mathrm{l}}-\alpha_{\mathrm{o}})\,\Delta k_t + \alpha_{l\,min} \Delta l_{min\,t} - \alpha_{\mathrm{l}} \Delta l_t + \alpha_{o\,min} \Delta o_{min\,t} - \alpha_{\mathrm{o}} \frac{Y_{t-1}}{Y_{min\,t-1}} \Delta o_t \Big) + \frac{Y_{no\,min\,t-1}}{Y_{t-1}} \Big((1-\alpha_{lno\ min})\,\Delta k_{no\,min\,t} - (1-\alpha_{\mathrm{l}}-\alpha_{\mathrm{o}})\,\Delta k_t + \alpha_{lno\ min} \Delta l_{no\,min\,t} - \alpha_{\mathrm{l}} \Delta l_t \Big)$, respectively.

Δx_t is $x_t - x_{t-1}$. Columns (2) and (3) add up to column (1).

Table 5: Solow's Residual Growth in Mining Countries (2002-2015)

Country	Mining (1)	Non-Mininig (2)	Economy (3)
Australia	-3.52	0.50	-0.19
Canada	-3.92	0.37	-0.10
Chile	-8.67	1.38	-0.14
Malaysia	-9.39	2.26	1.16
Mexico	-2.93	0.85	0.53
Norway	-5.65	0.49	-0.64
Peru	-6.57	0.83	0.28
South Africa	-3.63	1.01	0.54
Average	-5.54	1.00	0.21

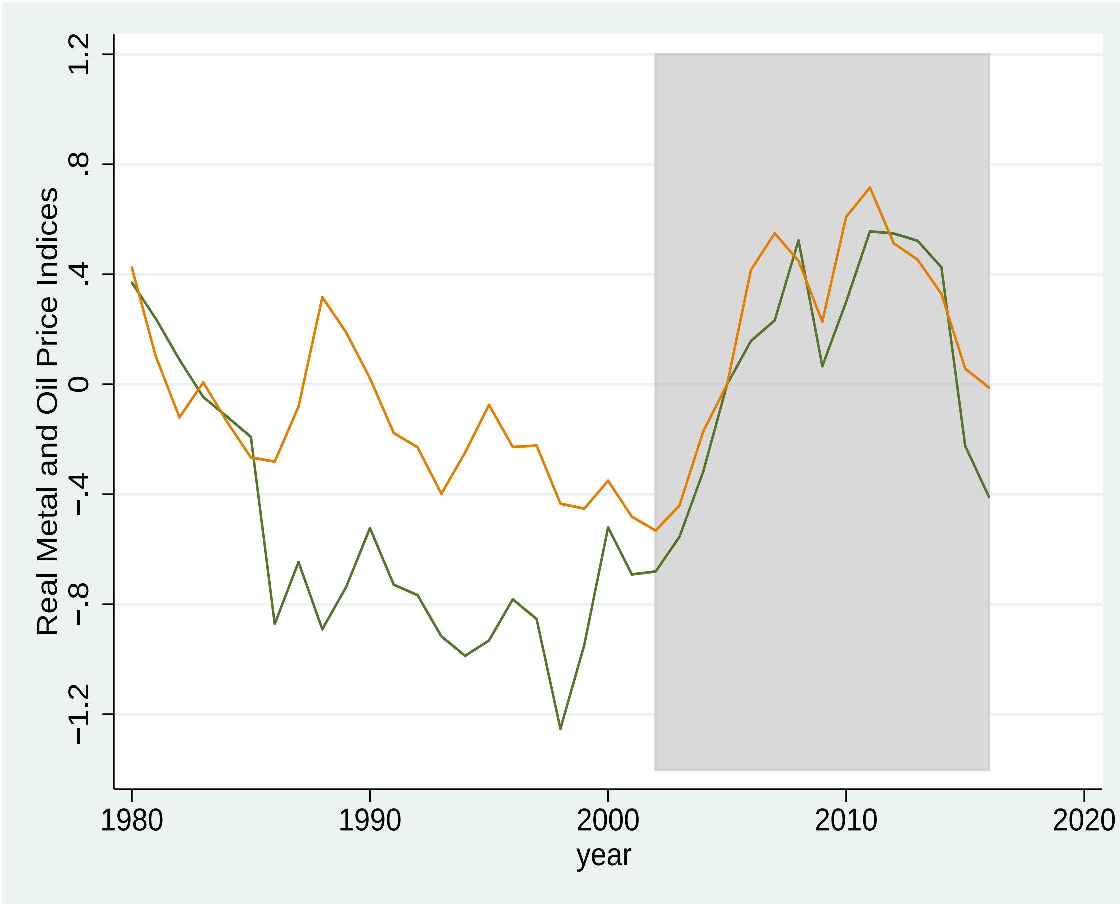
Notes: Average Solow's residual growth 2002-2015. The Solow's residual is calculated from $ASTFP_{s\,t} = \frac{Y_{s\,t}}{K_{s\,t}^{1-\alpha_{l\,s}} L_{s\,t}^{\alpha_{l\,s}}}$ where $Y_{s\,t}$, $K_{s\,t}$, $L_{s\,t}$, and $\alpha_{l\,s}$ denote value added, physical and human capital, and labor share, respectively, and s is Mining, Non-Mining, and Economy sectors (columns 1, 2, and 3, respectively). Malaysia's and South Africa's average spans 2002-2014 and 2004-2014.

Table 6: Labor Productivity Growth in Mining Countries (2002-2015)

Country	Mining (1)	Non-Mininig (2)	Economy (3)
Australia	−3.04	1.14	0.35
Canada	−2.22	0.80	0.75
Chile	−1.97	2.34	1.75
Colombia	2.45	1.98	1.98
Ecuador	−3.03	0.02	0.00
Indonesia	−2.51	3.94	3.50
Malaysia	−9.36	2.55	1.92
Mexico	−3.12	1.29	0.99
Norway	−6.44	1.15	0.17
Peru	−6.88	3.55	3.23
South Africa	−1.57	2.54	2.11
Average	−3.43	1.94	1.52

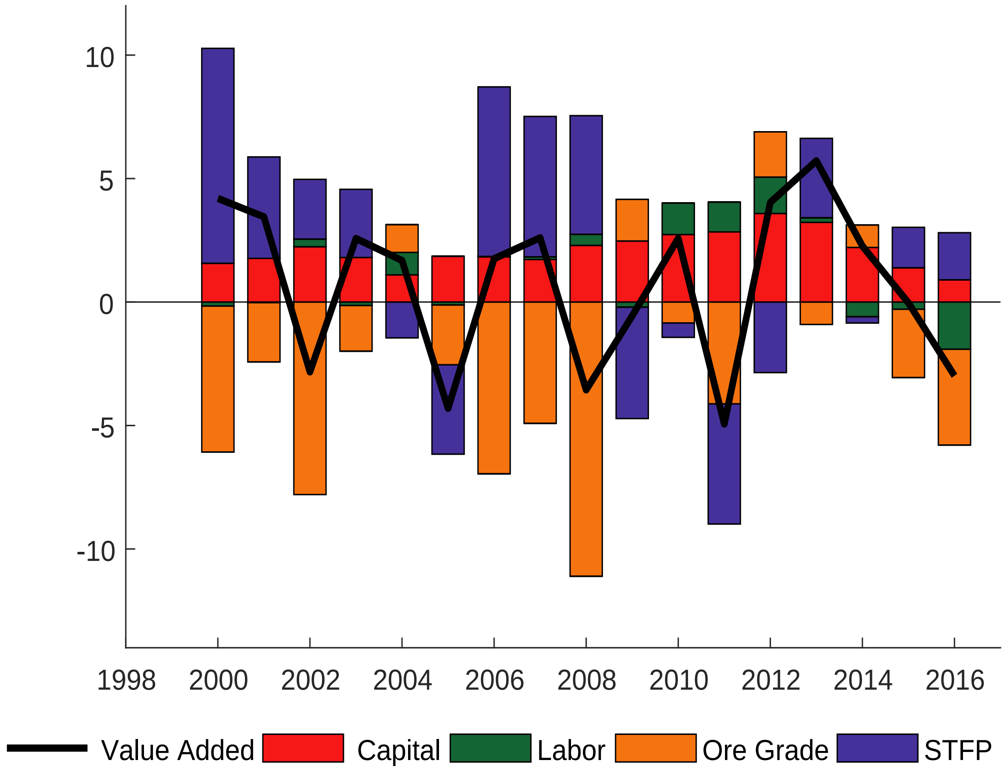
Notes: Average sectoral labor productivity growth 2002-2015. Labor productivity is calculated from $\frac{Y_{s\,t}}{N_{s\,t}}$ where $Y_{s\,t}$ and $N_{s\,t}$ denote value added and number of workers and s is Mining, Non-Mining, and Economy sectors (columns 1, 2, and 3, respectively). Ecuador, Indonesia, Malaysia, South Africa's average spans 2007-2015, 2002-2014, 2002-2014, and 2004-2014, respectively.

Figure 1: Real Metal and Oil Price Indices (1980-2016)



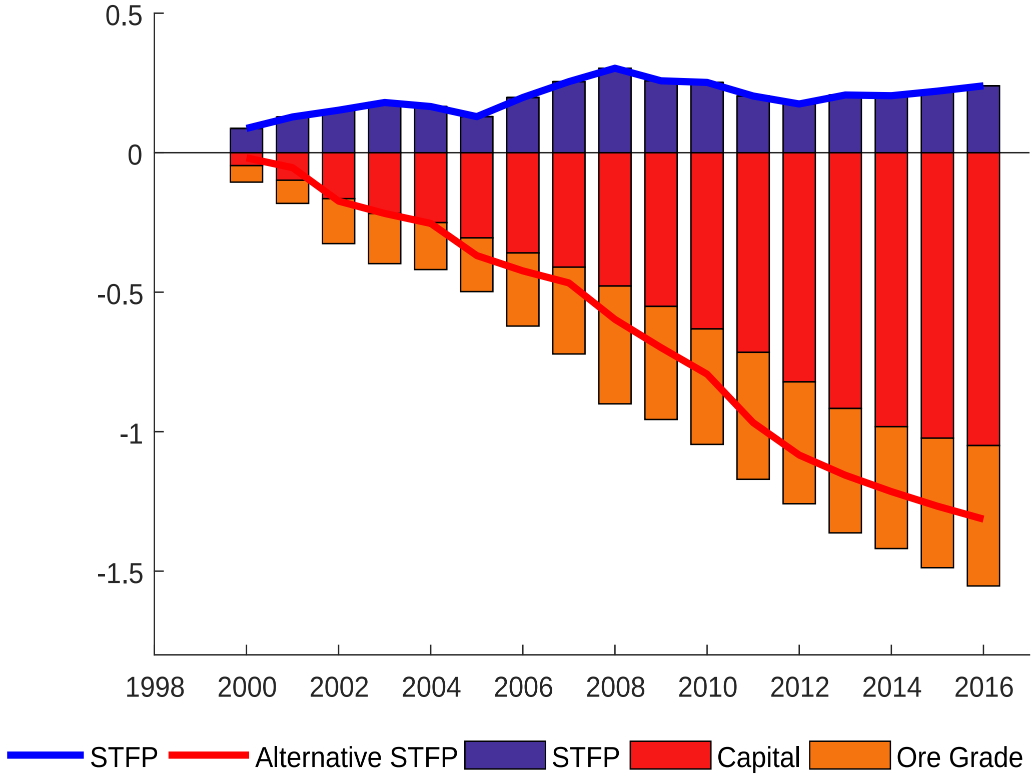
Notes: Real metal (solid yellow line) and oil (solid green line) price indices are logarithms of nominal metal and oil price indices deflated by the United States GDP deflator. The shaded area highlights the period 2002-2016.
Source: Authors' own calculation based on International Monetary Fund Primary Commodity Prices and United States Bureau of Economic Analysis.

Figure 2: Growth Accounting: Mining Sector (2000-2016)



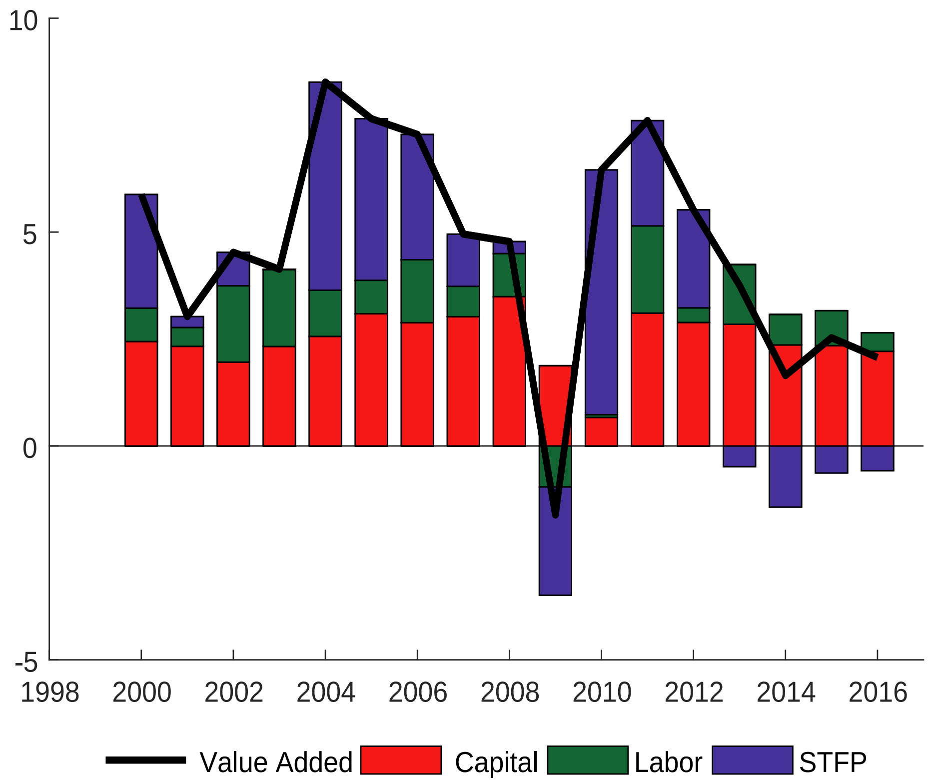
Notes: Decomposition of mining value added growth into productivity, physical and human capital, and ore grade contributions. Each factor contribution is calculated from $y_{min\,t} = stfp_{min\,t} + (1 - \alpha_{l\,min} - \alpha_{o\,min})\,k_{min\,t} + \alpha_{l\,min} l_{min\,t} + \alpha_{o\,min} o_{min\,t}$, where each term corresponds to the contribution of each factor (y, $stfp$, k, l, and o are logarithm of value added, productivity, physical and human capital, and ore grade, respectively). The red, green, orange, and blue bars are physical and human capital, ore grade, and productivity contributions. The bars add up to value added (solid black line).

Figure 3: Mining Productivity and Omitting of Ore Grade (2000-2016)



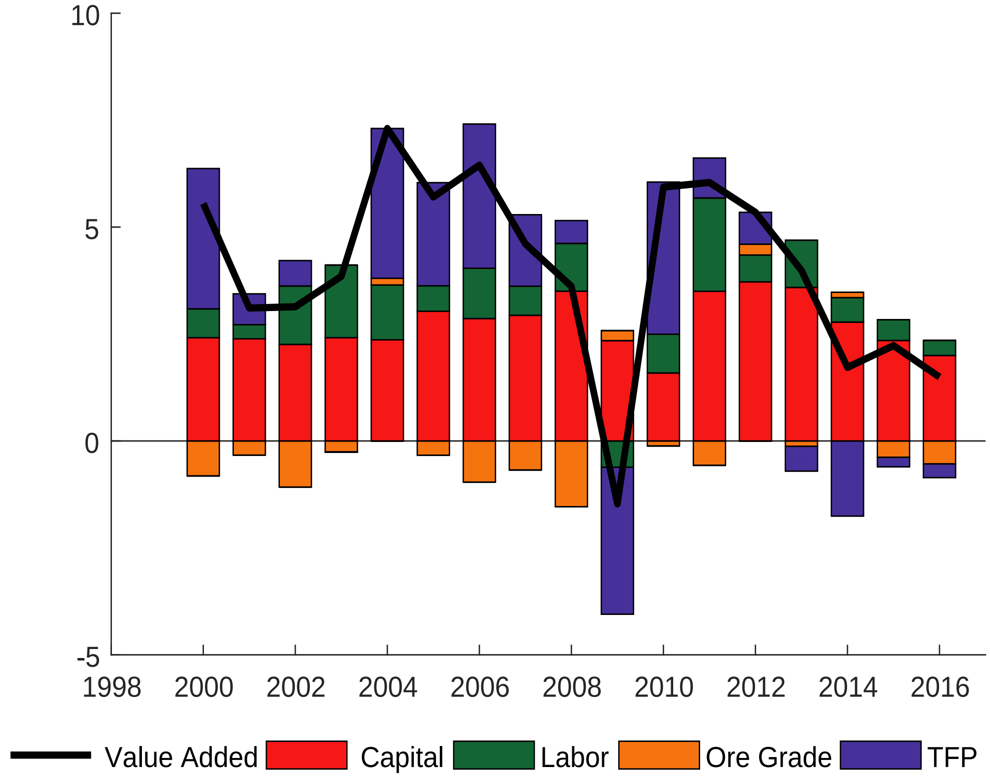
Notes: Ommiting ore grade and mining productivity. The blue line is logarithm of mining productivity. Mining productivity is calculated from $STFP_{min\,t} = \frac{Y_{min\,t}}{K_{min\,t}^{1-\alpha_{l\,min}-\alpha_{o\,min}} L_{min\,t}^{\alpha_{l\,min}} O_{min\,t}^{\alpha_{o\,min}}}$, where $Y_{min\,t}$, $STFP_{min\,t}$, $K_{min\,t}$, $L_{min\,t}$, and $O_{min\,t}$ denote mining value added, productivity, physical and human capital and ore grade, respectively. The red line is the logarithm of mining Solow's residual. The Solow's residual is calculated from $ASTFP_{min\,t} = \frac{Y_{min\,t}}{K_{min\,t}^{1-\alpha_{l\,min}} L_{min\,t}^{\alpha_{l\,min}}}$. The logarithm of $ASTFP_{min\,t}$ is $stfp_{min\,t} - \alpha_{o\,min}\,(k_{min\,t} - o_{min\,t})$, where lowercases are logarithms of level variables. The blue, red, and orange bars are $stfp_{min\,t}$, $-\alpha_{o\,min} k_{min\,t}$, and $\alpha_{o\,min} o_{min\,t}$, respectively, initial values are normalized to zero.

Figure 4: Growth Accounting: Non-Mining Sector (2000-2016)



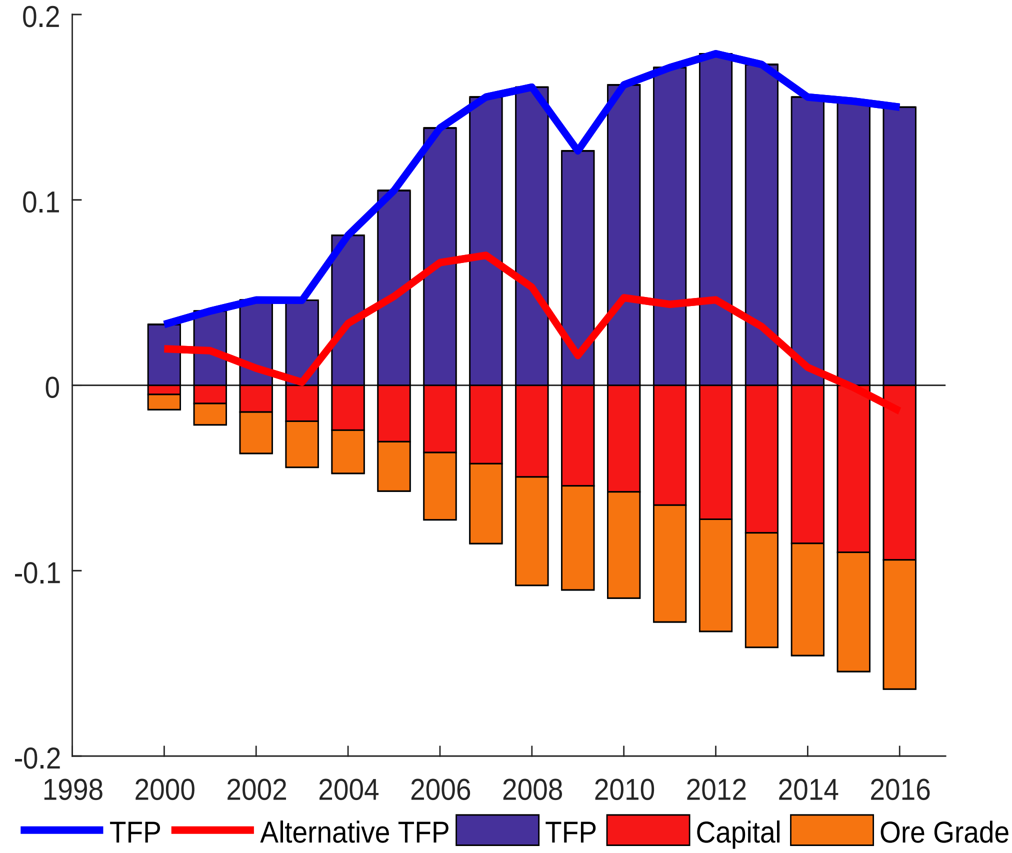
Notes: Decomposition of non-mining value added growth into productivity, physical and human capital contributions. Each factor contribution is calculated from $y_{no\,min\,t} = stfp_{no\,min\,t} + (1 - \alpha_{l\,no\,min})\,k_{no\,min\,t} + \alpha_{l\,no\,min} l_{no\,min\,t}$, where each term corresponds to the contribution of each factor (y, $stfp$, k, and l are logarithm of value added, productivity, physical and human capital, respectively). The red, green, and blue bars are physical and human capital, and productivity contributions. The bars add up to value added (solid black line).

Figure 5: Growth Accounting: Aggregate Economy (2000-2016)



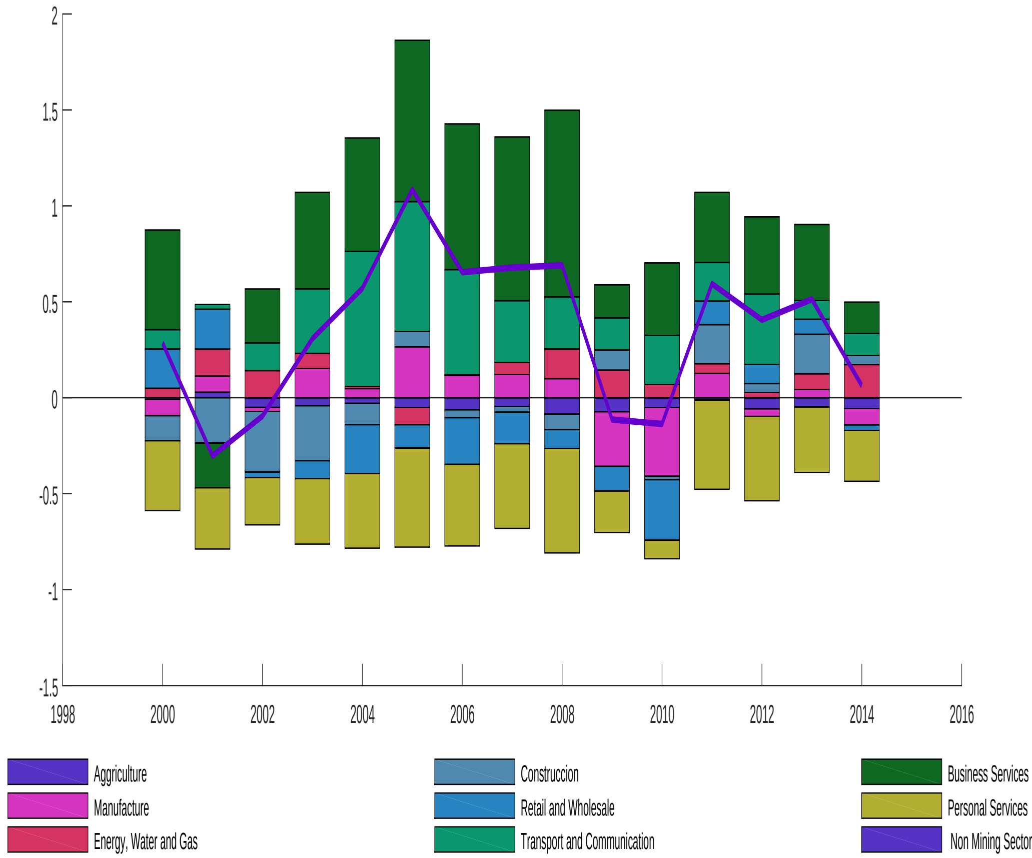
Notes: Decomposition of aggregate value added growth into productivity, physical and human capital, and ore grade contributions. Each factor contribution is calculated from $y_t = tfp_t + (1 - \alpha_l - \alpha_o) k_t + \alpha_l l_t + \alpha_o o_t$, where each term corresponds to the contribution of each factor (y, $stfp$, k, l, and o are logarithm of value added, productivity, physical and human capital, and ore grade, respectively). The red, green, orange, and blue bars are physical and human capital, ore grade, and productivity contributions. The bars add up to value added (solid black line).

Figure 6: TFP and Omitting Ore Grade (2000-2016)



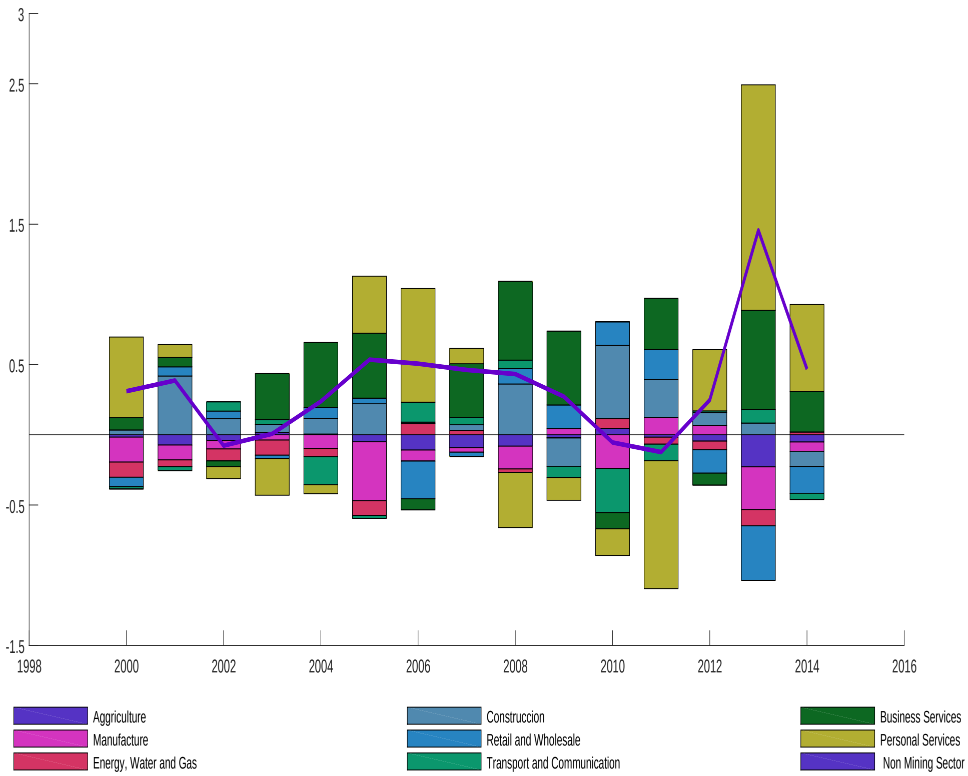
Notes: Ommiting ore grade and TFP. The blue line is logarithm of aggregate productivity. TFP is calculated from $TFP_t = \frac{Y_t}{K_t^{1-\alpha_1-\alpha_o} L_t^{\alpha_1} O_t^{\alpha_0}}$, where Y_t,TFP_t, K_t, L_t, and O_t denote aggregate value added, TFP, physical and human capital and ore grade, respectively. The red line is the logarithm of Solow's residual. Solow's residual is calculated from $ATFP_t = \frac{Y_t}{K_t^{1-\alpha_l} L_t^{\alpha_l}}$. The logarithm of $ATFP_t$ is $tfp_t - \alpha_o (k_t - o_t)$, where lowercases are logarithms of level variables. The blue, red, and orange bars are tfp_t, $-\alpha_o k_t$, and $\alpha_o o_t$, respectively, initial values are normalized to zero.

Figure 7: Composition Effect-Capital Contribution: Non-mining (2000-2014)



Notes: Annual physical capital contribution to Composition Effect defined as $\sum_j \frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} \left((1-\alpha_j)\,\Delta k_{j\,t} - (1-\alpha_{lno\ min})\,\Delta k_{no\,min\,t}\right)$. Sector j contribution is $\frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} \left((1-\alpha_j)\,\Delta k_{j\,t} - (1-\alpha_{lno\ min})\,\Delta k_{no\,min\,t}\right)$, where j are sectors: Aggriculture, Manufacture, Energy, water, and gas, Construction, Retail and wholesales, Transport and communications, Business services, and Personal services. For details on variable definitions see Section 2.

Figure 8: Composition Effect-Human Capital Contribution: Non-mining (2000-2014)



Notes: Annual human capital contribution to Composition Effect defined as $\sum_j \frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} (\alpha_j \Delta l_{j\,t} - \alpha_{lno\ min} \Delta l_{no\,min\,t})$. Sector j contribution is $\frac{Y_{j\,t-1}}{Y_{no\,min\,t-1}} (\alpha_j \Delta l_{j\,t} - \alpha_{lno\ min} \Delta l_{no\,min\,t})$, where j are sectors: Aggriculture, Manufacture, Energy, water, and gas, Construction, Retail and wholesales, Transport and communications, Business services, and Personal services. For details on variable definitions see Section 2.

Appendix A. Data Sources

Tables A1-to-A11 describe the data sources for Australia, Canada, Chile, Colombia, Ecuador, Indonesia, Malaysia, Mexico, Norway, Peru, and South Africa.

Appendix B. Sectoral Labor Shares Over Time and Countries

Figure B1 shows the stability of the Chilean sectoral labor shares over time. Such stability is consitent with our modelling choice of sectoral value added Cobb-Douglas functions. Finally, Table B1 reports sectoral labor shares for Australia, Canada, Malaysia, Mexico, Norway, Peru, and South Africa.

Appendix C. Growth Accounting

We turn to showing that aggregate value added growth can be approximated by the sum of *pure productivity*, *composition*, and *factors of production* terms. Defining the economy value added, Y_t, as the sum of mining and non-mining value added, aggregate value added growth is

$$\frac{\Delta Y_t}{Y_{t-1}} = \frac{Y_{min\,t-1}}{Y_{t-1}} \frac{\Delta Y_{min\,t}}{Y_{min\,t-1}} + \frac{Y_{no\,min\,t-1}}{Y_{t-1}} \frac{\Delta Y_{no\,min\,t}}{Y_{no\,min\,t-1}} \,. \tag{C.1}$$

We approximate $\frac{\Delta Y_{min\,t}}{Y_{min\,t-1}}$ and $\frac{\Delta Y_{no\,min\,t}}{Y_{no\,min\,t-1}}$ using Equations 3 and 4. We substitute them back into Equation C.1. Then, we obtain

$$\begin{aligned} \frac{\Delta Y_t}{Y_{t-1}} \approx \; & \frac{Y_{min\,t-1}}{Y_{t-1}} \left(\Delta stfp_{min\,t} + (1-\alpha_{l\,min}-\alpha_{o\,min})\,\Delta k_{min\,t} + \alpha_{l\,min}\Delta l_{min\,t} + \alpha_{o\,min}\Delta o_{min\,t}\right) + \\ & + \frac{Y_{no\,min\,t-1}}{Y_{t-1}} \left(\Delta stfp_{no\,min\,t} + (1-\alpha_{lno\;min})\,\Delta k_{no\,min\,t} + \alpha_{lno\;min}\Delta l_{no\,min\,t}\right) . \end{aligned} \tag{C.2}$$

Assuming the economy value added is generated according to Equation 5. Adding and substracting $(1-\alpha_{\text{l}}-\alpha_0)\,\Delta k_t$, $\alpha_{\text{l}}\Delta l_t$, and $\alpha_{\text{o}}\Delta o_t$ to Equation C.2 and re-arranging terms, one obtains.

$$\frac{\Delta Y_t}{Y_{t-1}} \approx \underbrace{\frac{Y_{min\,t-1}}{Y_{t-1}}\Delta stfp_{min\,t} + \frac{Y_{no\,min\,t-1}}{Y_{t-1}}\Delta stfp_{no\,min\,t}}_{pure\ productivity} +$$

$$+ \underbrace{\frac{Y_{min\,t-1}}{Y_{t-1}}\Bigg((1-\alpha_{l\,min}-\alpha_{o\,min})\,\Delta k_{min\,t} - (1-\alpha_{\text{l}}-\alpha_{\text{o}})\,\Delta k_t \,+\, \alpha_{l\,min}\Delta l_{min\,t} - \alpha_{\text{l}}\Delta l_t}\ldots$$

$$\ldots\underbrace{+\,\alpha_{o\,min}\Delta o_{min\,t} - \alpha_{\text{o}}\frac{Y_{t-1}}{Y_{min\,t-1}}\Delta o_t\Bigg)}_{composition}\ldots+$$

$$+ \ldots\underbrace{\frac{Y_{t-1\,\text{No Min}}}{Y_{t-1}}\Bigg((1-\alpha_{lno\;min})\,\Delta k_{no\,min\,t} - (1-\alpha_{\text{l}}-\alpha_{\text{o}})\,\Delta k_t \,+\, \alpha_{lno\;min}\Delta l_{no\,min\,t} - \alpha_{\text{l}}\Delta l_t\Bigg)} +$$

$$+ \underbrace{(1-\alpha_{\text{l}}-\alpha_{\text{o}})\,\Delta k_t \,+\, \alpha_{\text{l}}\Delta l_t \,+\, \alpha_{\text{o}}\Delta o_t}_{factors\ of\ production}$$

Appendix D. Labor Productivity Growth Decomposition

In this Appendix we show the results of decomposing labor productivity growth into into *pure productivity*, *composition* and *dynamic* contributions.[28] More concretely, labor productivity growth can be written as

$$\frac{\frac{Y_{2014}}{N_{2014}} - \frac{Y_{1999}}{N_{1999}}}{\frac{Y_{1999}}{N_{1999}}} = \underbrace{\frac{\sum_{j=1}^{J}\frac{N_{j1999}}{N_{1999}}\left(\frac{Y_{j2014}}{N_{j2014}} - \frac{Y_{j1999}}{N_{j1999}}\right)}{\frac{Y_{1999}}{N_{1999}}}}_{pure\ productivity} + \underbrace{\frac{\sum_{j=1}^{J}\frac{Y_{j1999}}{N_{j1999}}\left(\frac{N_{j2014}}{N_{2014}} - \frac{N_{j1999}}{N_{1999}}\right)}{\frac{Y_{1999}}{N_{1999}}}}_{composition} +$$

$$+ \underbrace{\frac{\sum_{j=1}^{J}\left(\frac{Y_{j2014}}{N_{j2014}} - \frac{Y_{j1999}}{N_{j1999}}\right)\left(\frac{N_{j2014}}{N_{2014}} - \frac{N_{j1999}}{N_{1999}}\right)}{\frac{Y_{1999}}{N_{1999}}}}_{dynamic}$$

[28]For a recent application of this decomposition see Fuentes and García (2014) and Üngör (2017).

where Y, N, and j are value added, employment, and economic sectors, respectively. Table D1 shows the result of the decomposition. Stronger composition gains in the non-mining sector as opposed to the economy is robust to this alternative decomposition. However, for the economy the composition term is much lower than in the TFP growth decomposition. The reason is labor productivity does not weigh employment by its true contribution to the production process. Our TFP growth decompostion does so. Once considered the lower contribution of employment into mining activity, the results of both decompostion are consistent. Finally, the dynamic effect, absent in our accounting setting, turn out to be close to zero.

Table A1: Variables and Data Sources for Australia

Variable	Definition	Source
Capital stock	Non-residential capital stock, by industry, chain volume measures.	Table 58, Australian System of National Accounts, Australian Bureau of Statistics.
Compensation of employees	Compensation of Employees, by industry at current prices.	Table 38 and Table 48, Australian System of National Accounts, Australian Bureau of Statistics.
Employment	Employed persons by industry division of main job.	Table 04, Labour Force Survey, Australian Bureau of Statistics.
Hours worked	Hours actually worked in all jobs.	Labour Force Surveys, Australian Bureau of Statistics.
Labor quality index	Human capital index, based on years of schooling and returns to education, PWT9.	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Nominal and real GDP	Non-residential Gross Value Added (GVA) by industry at current and constant prices (chain volume).	Table 38 and Table 5, Australian System of National Accounts, Australian Bureau of Statistics.

Table A2: Variables and Data Sources for Canada

Variable	Definition	Source
Capital stock	Non-residential capital stock, by industry and asset, Canada, provinces and territories (year-to-date (averages)), annual.	Table $031-0005$, Statistics Canada.
Compensation of employees	Compensation of employees, quarterly at current prices.	Table $379-0029$, Table $380-0074$, $384-0037$, Statistics Canada.
Employment	Number of workers in the labor force.	Table $282-0008$, Labour Force Survey, Statistics Canada.
Hours worked	Actual Hours worked, unadjusted for seasonality, annual.	Table $282-0027$ and Table $282-0021$, Labour Force Survey, Statistics Canada.
Labor quality index	Human capital index, based on years of schooling and returns to education, PWT9.	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Nominal and real GDP	Non-residential Gross Domestic Product at current and constant prices.	Table $379-0031$, Table $379-0001$, and Table $384-0037$, Statistics Canada.

Table A3: Variables and Data Sources for Chile

Variable	Definition	Source
Capital stock	Real capital stock at constant prices.	Central Bank of Chile, reference 2008 and 2013.
Compensation of employees	Total labor remunerations of all sectors (including an imputation for the remuneration of self-employment).	National Accounts, Central Bank of Chile, reference 2008 and 2013.
Employment	Number of workers in the labor force.	National Statistics Institute, old and new Employment Surveys.
Hours worked	Average weekly hours.	National Statistics Institute, old and new Employment Surveys. Series joined formerly by the Central Bank of Chile.
Labor quality index	Human capital index, based on years of schooling and returns to education, PWT9.	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Nominal and real GDP	Gross Domestic Product at current and constant prices (chain volume).	Central Bank of Chile, reference 2008 and 2013.
Ore grade	Average ore grade. Calculated as weighted averages of mineral processing.	Chilean Copper Commission based on information of the main mining companies, which represent $99,6\%$ of total production (year 2015).

Table A4: Variables and Data Sources for Colombia

Variable	Definition	Source
Employment	Number of workers in the labor force.	Great Integrated Household Survey, Administrative National Statistics Department.
Real GDP	Real Gross Domestic Product.	National Accounts, Administrative National Statistics Department.

Table A5: Variables and Data Sources for Ecuador

Variable	Definition	Source
Employment	Number of workers in the labor force.	National Survey of Employment, Unemployment and Subemployment, National Institute of Census and Statistics.
Real GDP	Real Gross Domestic Product.	National Accounts, Central Bank of Ecuador.

Table A6: Variables and Data Sources for Indonesia

Variable	Definition	Source
Employment	Number of workers in the labor force.	Statistics Indonesia.
Real GDP	GDP at constant market prices by industrial origin.	Statistics Indonesia.

Table A7: Variables and Data Sources for Malaysia

Variable	Definition	Source
Capital stock	Net capital stock at constant prices.	Department of Statistics Malaysia.
Hours worked	Average annual hours worked by persons engaged, PWT9.	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Labor quality index	Human capital index, based on years of schooling and returns to education, PWT9.	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Nominal and real GDP	Gross Value Added at current and constant prices.	Department of Statistics Malaysia.

Table A8: Variables and Data Sources for Mexico

Variable	Definition	Source
Capital stock	Net capital stock at constant prices.	National Institute of Statistics and Geografy.
Compensation of employees	Remuneration at Current prices.	National Institute of Statistics and Geografy.
Hours worked	Hours worked per employee.	National Institute of Statistics and Geografy.
Labor quality index	Human capital index, based on years of schooling and returns to education, PWT9.	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Nominal and real GDP	Gross Value Added at current and constant prices.	National Institute of Statistics and Geografy.

Table A9: Variables and Data Sources for Norway

Variable	Definition	Source
Capital stock	Capital stocks, by industry, at constant prices.	Table $9181-12$, Statistics Norway.
Compensation of employees	Wages and salaries at current prices.	Table $9174-1$, Statistics Norway.
Hours worked	Total hours worked for employees and self-employed (million work-hours).	Table $9174-1$, Statistics Norway.
Labor quality index	Human capital index, based on years of schooling and returns to education, PWT9.	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Nominal and real GDP	Value added at current and constant.	Table $9170-3$ and Table $9170-11$, Statistics Norway.

Table A10: Variables and Data Sources for Peru

Variable	Definition	Source
Capital stock	Capital stock imputed through perpetual inventory method using data on gross capital formation at constant prices.	National Institute of Statistics and Information Technology.
Compensation of employees	Total labor remunerations of all sectors at current prices.	Data from National Accounts 2007, Central Reserve Bank of Peru.
Employment	Number of workers in the labor force.	Peru: Evolutions of Employment and Remuneration Indicators by State $2001-2010$ and $2004-2015$, National Institute of Statistics and Information Technology.
Hours worked	Average weekly hours.	Peru: Evolutions of Employment and Remuneration Indicators by State $2001-2010$ and $2004-2015$, National Institute of Statistics and Information Technology.
Labor quality index	Human capital index, based on years of schooling and returns to education, in PWT9.	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Nominal and real GDP	Gross Domestic Product at current and constant prices.	National Institute of Statistics and Information Technology and Central Reserve Bank of Peru.
Returns of physical capital	Depreciation rate and long term rate of growth of Gross Domestic Product to build capital stock.	Cespedes, Nikita., Pablo Lavado and Nelson Ramirez Rondan (2016).

Table A11: Variables and Data Sources for South Africa

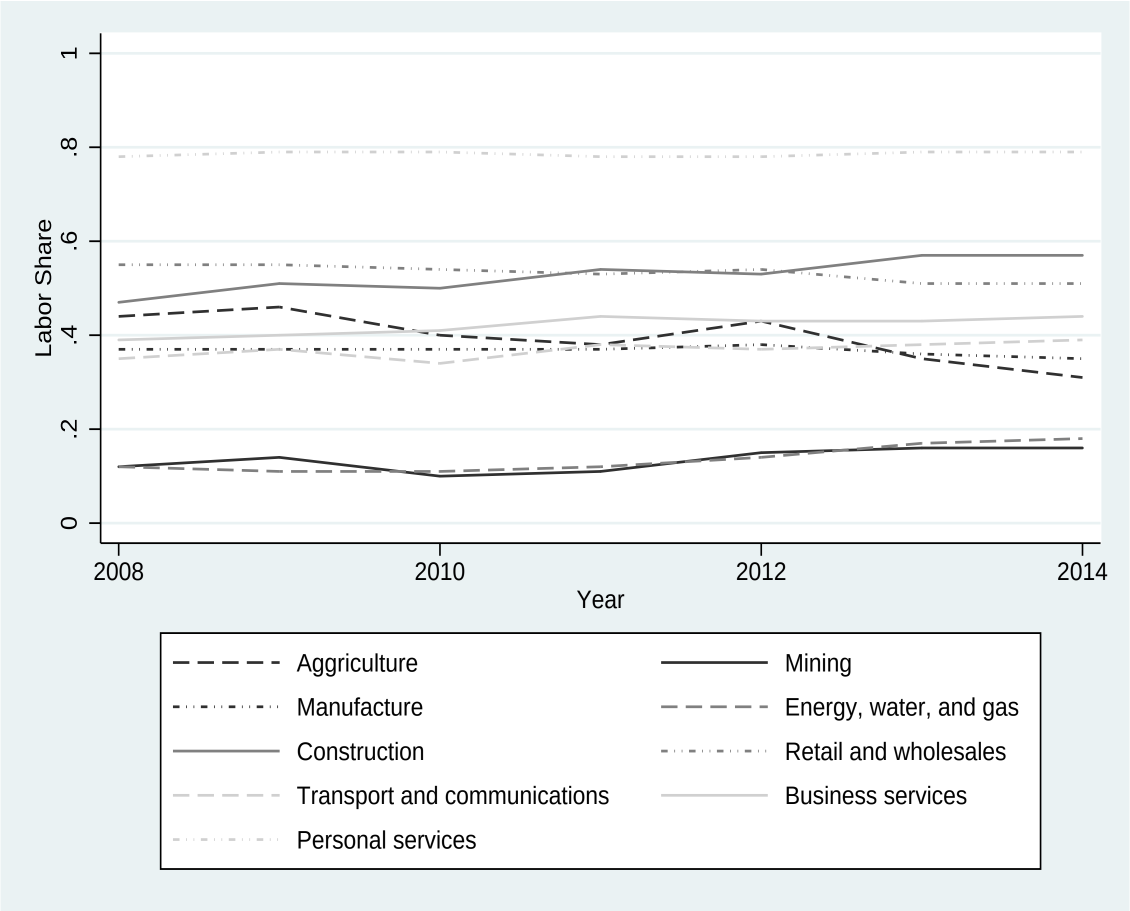
Variable	Definition	Source
Capital stock	Non farms fixed capital stock at constant prices.	National accounts, South African Reserve Bank.
Compensation of employees	Non farms quarterly Compensation of Employees (R millions).	Statistics South Africa.
Employment	Non farms workers in the labor force.	Quarterly Employment Statistics, Statistics South Africa.
Hours worked	Average annual hours worked by persons engaged, PWT9	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Labor quality index	Human capital index, based on years of schooling and returns to education, PWT9.	Feenstra, Robert C., Robert Inklaar and Marcel P.Timmer (2015).
Nominal and real GDP	Quarterly Non farms Gross Domestic Product by industry at current prices and constant prices.	Statistics South Africa.

Table B1: Labor Share in Mining Latin-American Countries

Country	Mining (1)	Non-Mininig (2)	Economy (3)
Australia	0.23	0.54	0.51
Canada	0.21	0.54	0.51
Malaysia	0.05	0.36	0.32
Mexico	0.07	0.31	0.30
Norway	0.15	0.61	0.52
Peru	0.19	0.37	0.34
South Africa	0.44	0.52	0.52
Average	0.19	0.48	0.45

Notes: First, second, and third row show labor shares of the Mining, Non-Mining, and Aggregate Economy respectively. First, second, and third columns present labor shares for Mexico and Peru. Labor shares are calculated as nominal compensation of employees divided by gross value added.

Figure B1: Sectoral Labor Shares Over Time (2008-2014)



Notes: Participactions of human capital on economic sectors. The participaction of human capital is computed as ratios between sectoral labor remunerations to value added.

Table D1: Labor Productivity Growth Decomposition (2000-2014)

Sectors	Productivity Growth (1)	Pure Productivity (2)	Composition (3)	Dynamic (4)
Non-Mining	2.47	1.98	0.58	−0.09
Economy	1.70	1.73	0.00	−0.03

Notes: Decomposition of average sectoral labor productivity growth 2000-2014 into *pure productivity*, *composition* and *dynamic* contributions.

First row corresponds to Non-Mining labor productivity. Second row is the Economy labor productivity. Columns *pure productivity*, *composition* and *dynamic* are time series averages of $\frac{\sum_{j=1}^{J}\frac{N_{j1999}}{N_{1999}}\left(\frac{Y_{j2014}}{N_{j2014}}-\frac{Y_{j1999}}{N_{j1999}}\right)}{\frac{Y_{1999}}{N_{1999}}}$, $\frac{\sum_{j=1}^{J}\frac{Y_{j1999}}{N_{j1999}}\left(\frac{N_{j2014}}{N_{2014}}-\frac{N_{j1999}}{N_{1999}}\right)}{\frac{Y_{1999}}{N_{1999}}}$, and $\frac{\sum_{j=1}^{J}\left(\frac{Y_{j2014}}{N_{j2014}}-\frac{Y_{j1999}}{N_{j1999}}\right)\left(\frac{N_{j2014}}{N_{2014}}-\frac{N_{j1999}}{N_{1999}}\right)}{\frac{Y_{1999}}{N_{1999}}}$, respectively. Columns (2), (3) and (4) add up to column (1).

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