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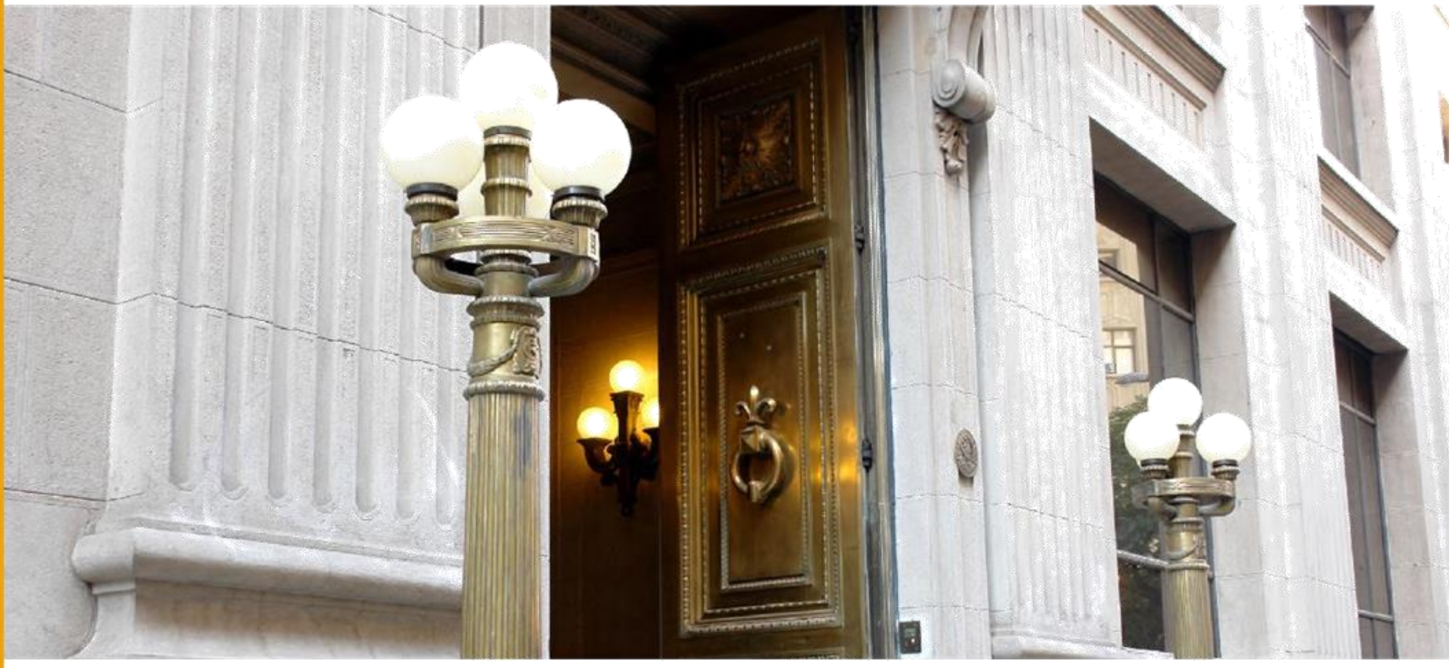

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Regulating Vertical Markets through Delegation *

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Resumen

Estudiamos la regulación óptima de un monopolio en un mercado vertical, como el de la infraestructura de transporte, donde el regulador solo puede contratar con la empresa aguas arriba (un esquema que denominamos delegación). En nuestro modelo, la empresa aguas arriba posee información privada sobre los costos de inversión, mientras que la empresa aguas abajo tiene información privada sobre la demanda. Comparamos los resultados bajo delegación con un modelo centralizado, en el que el regulador contrata directamente con ambas empresas. Desde el punto de vista teórico, demostramos que la delegación puede replicar el resultado centralizado bajo condiciones restrictivas, lo que implica que, en general, conlleva una pérdida de bienestar. Por ello, analizamos una alternativa práctica: un contrato lineal simple que otorga a la empresa aguas arriba un subsidio por unidad en función del nivel de producción aguas abajo. Nuestro principal hallazgo es que este contrato lineal resulta notablemente eficaz. Mostramos que su desempeño mejora con el tamaño del mercado y, a continuación, cuantificamos la pérdida de bienestar mediante la calibración de nuestro modelo al mercado Aero del Área de la Bahía de San Francisco. Los resultados indican una pérdida de bienestar asociada a este contrato simple de alrededor del 2 % en comparación con el contrato centralizado, mientras que una política sin subsidios genera una pérdida de 27-33%. Nuestro análisis sugiere que un subsidio simple constituye una herramienta poderosa para regular mercados verticales bajo información asimétrica.

Abstract

We study the optimal regulation of an upstream monopoly in a vertical market, such as transportation infrastructure, where the regulator can only contract with the upstream firm (a setup we call delegation). In our model, the upstream firm has private informa-tion on investment costs, and the downstream firm has private information on demand. We compare outcomes under delegation to a centralized benchmark where the regulator contracts directly with both firms. Theoretically, we show that delegation can replicate the centralized outcome only under restrictive conditions, implying it generally entails a welfare loss. We therefore analyze a practical alternative: a simple linear contract providing a per-unit subsidy to the upstream firm based on downstream output. Our main finding is that this linear contract is remarkably effective. We demonstrate that its performance improves with market size and then quantify the welfare loss by cal-ibrating our model to the San Francisco Bay Area airport market. The results show a welfare loss (regret) from this simple contract of around 2% compared to the cen-tralized ideal, while a policy without a subsidy yields a regret of 27-33%. Our analysis suggests that a simple subsidy is a powerful tool for regulating vertical markets under asymmetric information.

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1 Introduction

In many situations, a government is interested in having a regulated firm as the sole producer in a market. Regulated monopolies often yield a high social value when factors such as increasing returns to scale, geographic constraints or other entry barriers are present. Moreover, European and OECD countries have increasingly privatized traditional national monopolies to improve efficiency and diminish the government's burden. Sectors where regulated monopolies are common include public transportation, big terminals like airports and seaports, infrastructure, and water and waste management.

A significant share of these regulated monopolies is on the upstream side of vertical markets. Not only is their investment essential for downstream production, but the interaction with downstream firms and the infrastructure's day-to-day operation may also be the monopoly's critical duties. A relevant example can be found in aviation. The construction of an airport is an essential investment needed for airlines' operations, and the long-term interaction between airports and airlines is critical and must be carried out without day-to-day government supervision.

Furthermore, it is common for governments to allow a firm to become a monopoly by investing in and operating an airport, but regulating its sources of revenue. Since its privatization, Heathrow Airport, one of the world's busiest airports, has been subject to regulation by the Civil Aviation Authority (CAA). The CAA determines a revenue target based on cost and passenger estimates and calculates the maximum allowable yield per passenger. Under this scheme, the airport has "[...] an incentive to invest in better facilities for consumers and outperform the economic settlement by attracting more passengers [...]" (Heathrow Airport Holdings Limited, 2023).[1]

Another example is clinical laboratory services. A case study highlighted by the IDB as a role model for developing economies is the concession of clinical laboratory services in Madrid. The local government signed a contract with a private firm to provide clinical laboratory services to six hospitals. The firm invested in a central laboratory and five satellite ones for the required tests. The key to success was the per-unit payment set in the contract that the firm receives from the government (IDB Invest, 2019). A similar model was adopted for COVID tests in Madrid. For Valencia's La Ribera Hospital, the amortized cost of the infrastructure investment was paid for via an annual per capita payment from the Valencia

[1]Regarding the interaction between the airport and airlines, D'Alfonso and Nastasi (2014) point out that they form various types of vertical relationships. Airport operators enter into contracts for 15 to 30 years, during which signatory airlines obtain the lease of gates on an exclusive-use basis. Among the different contracts between airports and airlines, D'Alfonso and Nastasi (2014) highlight concession revenue sharing agreements, partial ownership by airlines of airport facilities, the issue of facilities revenue bonds to airlines, and price rebates on the input charge as standard practices.

government (Sosa Delgado-Pastor, Brashers, Foong, Montagu, & Feachem, 2016). More examples of vertical markets with an upstream monopoly can be found in water and waste management, as well as transportation terminals, among others.

This paper studies the optimal regulation mechanism in a vertical market. Following our motivating examples, we assume that the upstream firm, privately informed about its construction costs, chooses an investment that decreases the downstream firm's marginal costs. The downstream firm, privately informed about its demand, determines how much of the final good to produce and sell to consumers. In the aviation example, a better airport reduces, among other things, congestion costs and delays. Moreover, the firm that builds and operates the airport is privately informed about construction costs, while airlines are privately informed about demand.

We focus on delegation contracts as an alternative to centralized mechanisms. A centralized mechanism assumes that the principal can deal directly with both the upstream and downstream firms. Both firms report to the principal their private information, who chooses the investment and production of the final good, which the firms carry out. Payments are realized directly between the principal and each firm. On the other hand, delegation implies that the principal deals only with the upstream firm and delegates some decisions to it. Delegation entails offering a menu of contracts to the upstream firm contingent on the private information of the upstream firm and the observable actions of the upstream and downstream firms. A key feature is that the contract between the upstream and downstream firms is private.

We start by identifying the set of allocations that can be enforced through delegation contracts. We then show that any implementable allocation can achieve the same level of (expected) welfare as a centralized mechanism when the planner does not value downstream profits. We also identify the conditions under which delegation can enforce the centralized allocation and find that these conditions are rarely satisfied in a procurement environment. However, these conditions are naturally met in organizational hierarchies where owners delegate production responsibility to managers, or general managers delegate to regional managers (N. Melumad, Mookherjee, & Reichelstein, 1992, N. D. Melumad, Mookherjee, & Reichelstein, 1995). This means that centralized allocations can consistently be implemented through delegation in such cases. We further extend the analysis to a case with multiple downstream firms.

We then analyze the performance of linear delegation contracts, which are often used in practice. These contracts involve a payment from the regulator to the upstream firm directly proportional to the downstream output. To assess their effectiveness, we consider expected regret, which measures the delegation's expected relative loss of social welfare compared to a centralized contract. We provide theoretical bounds for the delegation regret of a linear contract within a linear-quadratic environment and show that the regret decreases as the market size increases, and it remains quite small. However, if the upstream firm's investment significantly impacts the downstream firm's operational costs, the delegation regret tends to be higher.

To shed more light on how significantly delegation underperforms relative to the centralized mechanism design and what economic primitives matter the most for the gap, we study the performance of a linear delegation contract using data from a complex market: aviation. Our empirical application is based on calibrating a model to reflect the characteristics of an airport in the San Francisco Bay Area as closely as possible. We use functional forms estimated for these airports and parameters based on observed passenger volumes and airport size, mostly from Yan and Winston (2014), who estimate a model of airport competition in the San Francisco Bay Area. The model's quantitative predictions also align well with real-world observations. The predicted airport size ranges from 2 to 3 runways, with an expected annual traffic of between 7 and 10 million passengers. These figures are comparable to those of airports in the San Francisco Bay Area, with San Francisco International (SFO) and Oakland (OAK) being larger, while San José (SJC) is slightly smaller.

Our main finding is that a simple linear delegation contract performs remarkably well in our empirical application, yielding only minimal welfare losses. The regret, $R(s^*)$, remains consistently small across all specifications, with a maximum value of 2%. For comparison, a delegation contract without per-passenger payments is significantly inferior, generating regrets ranging from 27% to 33%.

Our article provides three main contributions. First, we characterize the optimal delegation contract and the complete set of allocations implementable with this regulatory regime. The optimal delegation contract is a payment rule that depends on the type declared by the upstream firm and observed (ex-post) quantities, including downstream production. The payment can be decomposed into two terms: an ex-ante transfer from the monopoly to the regulator and an ex-post payment to the monopoly. The first can be interpreted as a franchise fee that ensures incentive compatibility. The non-linear payment aligns the upstream monopoly profit maximization, including its interaction with the downstream firm, with surplus maximization while providing (informational) rents.

Second, we demonstrate that a delegation contract can achieve the benchmark outcome, where the planner contracts with both firms separately, only under very restrictive conditions. Therefore, delegation generally entails a cost in the context of regulation. However, in all cases, the delegation contract, conditional on the implemented allocation, involves the same usage of public funds as the centralized mechanism. This result can be a significant issue when the cost of public funds is high, as it commonly is in developing countries. Third, we show theoretically and through simulations that a simple delegation contract, in which the ex-post payment rule depends linearly on the downstream quantity, performs well. Using typical functional forms, we show that the simple contract achieves at least 60 percent of the benchmark's welfare and surpasses 75% when the principal's goal is to maximize only the consumer surplus.

1.1 Contribution to the Literature

The literature studying the regulation of monopolies in the presence of private information is abundant. The seminal contributions of D. Baron and Myerson (1982), D. P. Baron and Besanko (1984), and Laffont and Tirole (1986) consider a monopolist with private information on its cost structure. Subsequent studies, starting with Riordan (1984) and Lewis and Sappington (1988), consider the case of private information on demand. More recent work by Auriol and Picard (2009) incorporates the idea that regulation can occur after the firm has been granted the right to operate, which introduces the technical complication of a type-dependent outside option. Auriol and Picard (2009) discuss under which circumstances it is better for a regulator to manage a firm through public ownership (giving informational rents to the public manager) or to delegate building an operation to a third party. We add to this literature by considering a vertical structure. The regulated firm (or concessionaire) deals with a downstream firm, and the regulator can influence their interaction through a delegation contract.

We also relate to the literature on delegation in hierarchies and organizations. For example, the problem of a principal dealing with multiple agents with private information within a firm has been analyzed in various settings. D. P. Baron and Besanko (1992) compares the centralized case, where the principal deals directly with each agent, treating the agents as a single coalition and delegating to one agent the responsibility of contracting with the others. N. Melumad et al. (1992), N. D. Melumad et al. (1995) establish the conditions and payments under which delegation achieves the optimal assignments. Further extensions include a regulator dealing with two producers of complementary goods (Gilbert & Riordan, 1995); the issue of collusion between agents within an organization (Faure-Grimaud, Laffont, & Martimort, 2003, Laffont & Martimort, 1998, Severinov, 2008) and the role of an intermediary between the principal and agents (Mookherjee & Tsumagari, 2004)

We also contribute to the literature about airport regulation under imperfect information. Czerny (2010) analyzes how to manage congestion when there is uncertainty about passenger benefits and congestion costs. Aravena, Basso, and Figueroa (2019) study the performance of congestion management mechanisms under asymmetric information of marginal costs. We contribute to this strand of literature by analyzing the entire vertical structure, including the process of contracting with airports and airlines. In this sense, Engel, Fischer, and Galetovic (2018) is closer to our framework; however, they study a principal contracting with a firm to build and operate an airport without considering the airport-airline relationship and in the context of moral hazard without adverse selection.

Finally, we add to the literature that assesses the performance of simple, often linear, contracts. Among these, we add to the articles investigating what share of the gains achievable by the fully optimal complex can be attained with simple contracts (Bose, Pal, & Sappington, 2011, Chu & Sappington, 2007, Garrett, 2014, Rogerson, 2003).

2 The model

We consider a market with an upstream firm U that invests in a facility that allows a downstream firm D to produce with increased efficiency and reduced costs. For example, the firm U is an airport that builds the infrastructure to enable airlines to operate with lower congestion and, therefore, lower costs.

The upstream firm chooses the investment a_U at a cost $C^U(a_U, \theta_U)$ where $\theta_U \in \Theta_U \equiv [\underline{\theta}_U, \bar{\theta}_U]$ represents its efficiency, is private information and is distributed according to F_U. The downstream firm chooses a quantity a_D to be sold in the market. Its cost is given by $C^D(a_U, a_D)$, where a_U decreases D's marginal cost. The demand faced by D is given by the *inverse demand function* $P(a_D, \theta_D)$, where $\theta_D \in \Theta_D \equiv [\underline{\theta}_D, \bar{\theta}_D]$ is private information distributed according to F_D.

We make standard assumptions about demand and cost functions to capture the basic features of the market being analyzed.

Assumption 1 *The following are true:*

(i) Firm U has increasing and convex investment costs and lower θ_U represents higher efficiency: $C^U_{a_U} > 0, C^U_{a_U a_U} \geq 0$ and $C^U_{\theta_U a_U} > 0$.

(ii) Firm D's marginal costs are positive and upstream investment decreases downstream costs and marginal costs: $C^D_{a_D} > 0$, $C^D_{a_U} < 0$, and $C^D_{a_U a_D} < 0$.

(iii) The demand is downward sloping and θ_D represents higher demand: $P_{a_D} < 0$ and $P_{\theta_D} > 0$

(iv) Firm D's problem is concave: $P_{a_D}(a_D, \theta_D) < C^D_{a_D a_D}(a_D, a_U)$.

We consider a planner that regulates the market and can potentially contract with both firms. It values consumer surplus, which, for a level of consumption a_D and for a given type θ_D, is given by $V(a_D, \theta_D) \equiv \int_0^{a_D} P(x, \theta_D)dx - P(a_D, \theta_D)a_D$. The planner also weights U's profit according to $0 \leq \lambda_U \leq 1$ and D's profits according to $0 \leq \lambda_D \leq 1$.

Lastly, consider the following available transfers: from the principal to U, T^U; from D to the principal, T^D; and from D to U, T^{DU}. Then, the principal maximizes the social welfare, which is given by,

$$\begin{aligned} SW(a_U, a_D, \theta_U, \theta_D, T^U, T^D, T^{DU}) =& V(a_D, \theta_D) - T^U + T^D + \lambda_U \left[T^U - C^U(a_U, \theta_U) + T^{DU}\right] \\ &+ \lambda_D \left[P(a_D, \theta_D) \cdot a_D - C^D(a_U, a_D) - T^D - T^{DU}\right] \quad (1) \end{aligned}$$

We first consider a benchmark mechanism corresponding to a principal that deals simultaneously with U and D under asymmetric information.

2.1 Benchmark: the centralized mechanism

In a centralized mechanism, the principal deals directly with U and D, and there is no contracting between them; therefore, $T^{DU} \equiv 0$. As the problem is standard, we briefly summarize the mechanism, its properties, and the additional assumptions required. Figure 1 outlines the structure of this mechanism.

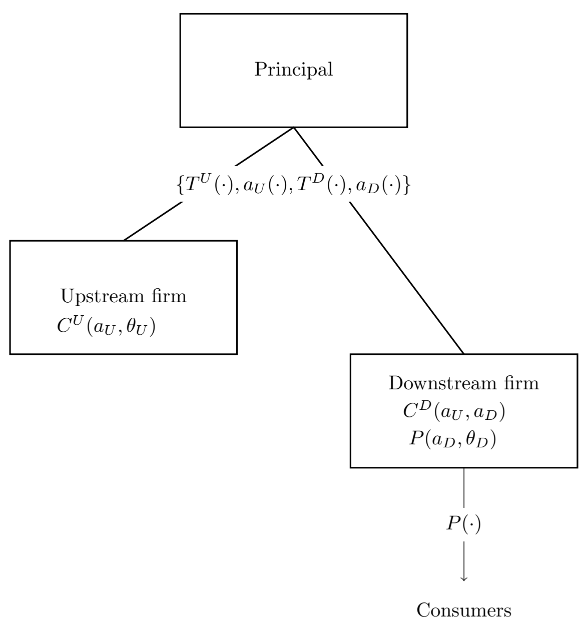

Figure 1: Centralized mechanism.

By the Revelation Principle, we focus on direct and truthful mechanisms.

Definition 1 *A direct mechanism is given by allocations* $a_U(\theta_U, \theta_D)$*,* $a_D(\theta_U, \theta_D)$ *and transfers* $T^U(\theta_U, \theta_D)$*,* $T^D(\theta_U, \theta_D)$*. Therefore, the expected social welfare generated by a mechanism is given by,*

$$\mathbb{E}_\theta[SW(a_U(\cdot), a_D(\cdot), \theta_U, \theta_D, T^U(\cdot), T^D(\cdot), 0)]$$

If D's type is θ_D and it reports $\tilde{\theta}_D$, its profits are given by:

$$\pi^D(\tilde{\theta}_D, \theta_D) = \mathbb{E}_{\theta_U}\left[P(a_D(\theta_U, \tilde{\theta}_D), \theta_D) \cdot a_D(\theta_U, \tilde{\theta}_D) - C^D(a_D(\theta_U, \tilde{\theta}_D), a_U) - T^D(\theta_U, \tilde{\theta}_D)\right],$$

Analogously, if U's type is θ_U and reports $\tilde{\theta}_U$, its profits are given by:

$$\pi^U(\tilde{\theta}_U, \theta_U) = \mathbb{E}_{\theta_D}\left[T^U(\tilde{\theta}_U, \theta_D) - C^U(a_U(\tilde{\theta}_U, \theta_D), \theta_U)\right]$$

Definition 2 *1. The mechanism satisfies incentive compatibility (IC) iff:*

$$\Pi^i(\theta_i) \equiv \underset{\tilde{\theta}_i \in \Theta_i}{max}\ \pi^i(\tilde{\theta}_i, \theta_i) = \pi^i(\theta_i, \theta_i) \quad \forall \theta_i \in \Theta_i, \textit{and } i \in \{U, D\}$$

2. The mechanism satisfies voluntary participation iff $\Pi^i(\theta_i) \geq 0 \quad \forall \theta_i \in \Theta_i,\ i \in \{U, D\}$

As it is standard in adverse selection problems, we need the single crossing property to characterize incentive compatibility with local constraints. For that purpose, we make an additional assumption on the demand function faced by firm D.

Assumption 2 $P_{\theta_D}(a_D, \theta_D)$ *is increasing and convex in* a_D*.*

Lemma 1 ***(Incentive Compatibility)***
A mechanism is incentive-compatible iff:

(i) $\mathbb{E}_{\theta_D}[a_U(\theta_U, \theta_D)]$ *is decreasing in* θ_U *and* $\Pi^{U\prime}(\theta_U) = \mathbb{E}_{\theta_D}\left[-C^U_{\theta_U}(a_U(\theta_U, \theta_D), \theta_U)\right]$

(ii) $\mathbb{E}_{\theta_U}[a_D(\theta_U, \theta_D)]$ *is increasing in* θ_D *and* $\Pi^{D\prime}(\theta_D) = \mathbb{E}_{\theta_U}\left[P_{\theta_D}(a_D(\theta_U, \theta_D), \theta_D) \cdot a_D(\theta_U, \theta_D)\right]$

With this, the planner's maximization problem can be written as follows:

$$\max_{\substack{a_U(\theta_U, \theta_D), T^U(\theta_U, \theta_D), \\ a_D(\theta_U, \theta_D), T^D(\theta_U, \theta_D)}} \mathbb{E}_\theta\left[V(a_D, \theta_D) - T^U(\theta_U, \theta_D) + T^D(\theta_U, \theta_D) + \lambda_U \Pi^U(\theta_U) + \lambda_D \Pi^D(\theta_D)\right] \tag{2}$$

subject to incentive compatibility and voluntary participation as defined above.

Lemma 2 *The expected social welfare generated by a centralized mechanism can be determined by allocations* (a_U, a_D) *and is given by,*

$$\mathbb{E}_\theta[SW(a_U, a_D, \theta_U, \theta_D, T^U, T^D, 0)] = \mathbb{E}_\theta[K(a_U, a_D, \theta_U, \theta_D; \lambda_U, \lambda_D)]$$

where $K(a_U, a_D, \theta_U, \theta_D; \lambda_U, \lambda_D) := V(a_D, \theta_D) - h^{U,\lambda_U}(a_U, \theta_U) + h^{D,\lambda_D}(a_U, a_D, \theta_D)$ *, and*

$$h^{U,\lambda_U}(a_U, \theta_U) = C^U(a_U, \theta_U) + (1-\lambda_U)\frac{F_U(\theta_U)}{f_U(\theta_U)} C^U_{\theta_U}(a_U, \theta_U) \quad (3)$$

$$h^{D,\lambda_D}(a_U, a_D, \theta_D) = P(a_D, \theta_D) \cdot a_D - C^D(a_D, a_U) - (1-\lambda_D)\frac{1-F_D(\theta_D)}{f_D(\theta_D)} P_{\theta_D}(a_D, \theta_D) \cdot a_D \quad (4)$$

The principal's valuation can be written as the sum of the consumer surplus, the "virtual cost" h^{U,λ_U} of U and the "virtual profits" h^{D,λ_D} of D. Firm U's virtual cost exceeds the production cost C^U by an amount equal to the share of the agent's informational rent that the principal does not value $(1-\lambda_U)$. The same holds true for the virtual profits of D, which are lower than the actual profits resulting from informational rents.

The centralized allocation generates a double distortion on the production of the final good a_D when $\lambda_D, \lambda_U < 1$. When informational rents are costly, the principal directly induces a lower a_D (see Eq. 4) and a lower level of investment a_U (see Eq. 3), which in turn indirectly reduces a_D through higher marginal costs.

We can fully characterize the optimal centralized mechanism by using standard tools and imposing additional conditions that guarantee regularity (see Assumption 3).

Assumption 3 *(i)* $F_U(\theta_U)$ *and* $F_D(\theta_D)$ *have densities* $f_U(\theta_U), f_D(\theta_D) > 0$.

(ii) $\frac{F_U(\theta_U)}{f_U(\theta_U)}$ *is increasing and* $\frac{1-F_D(\theta_D)}{f_D(\theta_D)}$ *is decreasing.*

(iii) $a_D \cdot P_{\theta_D}(a_D, \theta_D)$ *is increasing and convex in* a_D *for all* $\theta_D \in \Theta_D$.

Lemma 3 *The welfare-maximizing* ***centralized allocation*** $(a^*_U(\theta_U, \theta_D), a^*_D(\theta_U, \theta_D))$ *satisfies,*

$$(a^*_U(\theta_U, \theta_D), a^*_D(\theta_U, \theta_D)) \in \underset{a_U, a_D}{\operatorname{argmax}}\ \mathbb{E}_\theta\left[K(a_U, a_D, \theta_U, \theta_D; \lambda_U, \lambda_D)\right] \quad (5)$$

We now turn to studying contracts with delegation, where the principal only deals with the upstream firm and delegates the choice of actions (a_U, a_D).

3 Delegation contracts

The principal often cannot or chooses not to deal directly with the upstream and downstream firms. Instead, it delegates to U the infrastructure provision together with the task of dealing with D. This implies that U contracts with D privately and sets the prices for the infrastructure. In facilities such as airports, for example, the government selects a firm that, in addition to carrying out an infrastructure contract, becomes a regulated monopolist and sells infrastructure access to airlines and carriers.

This section studies the principal's problem of delegating to an upstream monopolist. The main challenge for the principal is overcoming the misalignment between its objective function and that of firm U. While the planner values consumer surplus and a fraction λ_D of D's profits, U does not care about either.

To provide the right incentives to U, the principal offers a delegation contract, whose payments depend on the actions chosen by U and D and the report by U about its type. Formally,

Definition 3 *A delegation contract is given by a payment rule $x(a_U, a_D; \tilde{\theta}_U)$ to the upstream firm that depends on its report $\tilde{\theta}_U$, its own action a_U and the downstream firm's action a_D.*

A delegation contract does not specify actions a_D and a_U as in a typical mechanism but indirectly influences them through the payment rule. In fact, it is suboptimal to establish actions at this stage since information about D, summarized by θ_D, is unavailable. Moreover, as we explain below, the payment rule cannot depend on D's type because it would induce the upstream firm to report untruthfully about the downstream type.

Figure 2 summarizes our delegation framework.

Conditional on the payment rule $x(a_U, a_D; \tilde{\theta}_U)$ contracted with the principal, U deals with D through a direct mechanism. This includes a production level a_D, a transfer T^{DU}, and a commitment to an investment a_U, which can depend on D's report.[2] In this delegation framework, D earns profits from selling a_D and, therefore, the transfer T^{DU} flows from D to U.

Definition 4 *A downstream mechanism is given by a production rule $a_D : [\underline{\theta}_D, \overline{\theta}_D] \longrightarrow \mathbb{R}_+$, a transfer rule $T^{DU} : [\underline{\theta}_D, \overline{\theta}_D] \longrightarrow \mathbb{R}$ and an investment rule $a_U : [\underline{\theta}_D, \overline{\theta}_D] \longrightarrow \mathbb{R}_+$.*

U chooses the downstream mechanism, knowing its type θ_U (which affects its costs) and the report made to the principal $\tilde{\theta}_U$, which determines its payments. Therefore, given $(\theta_U, \tilde{\theta}_U)$, its problem becomes

[2] Firm D does not care about θ_U directly, but only about investment a_U. Therefore, this is not an informed principal problem as in Myerson (1983) or Maskin and Tirole (1990).

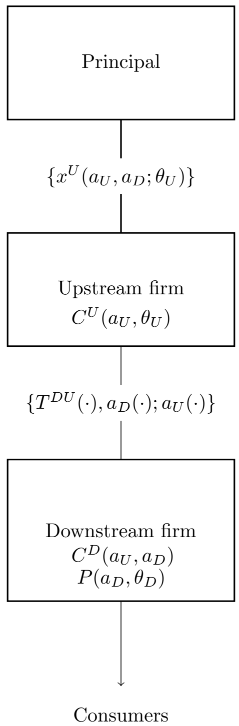

Figure 2: Delegation contract

$$\max_{\substack{a_D(\theta_U,\theta_D),a_U(\theta_U,\theta_D)\\ T^{DU}(\theta_U,\theta_D)}} \quad \mathbb{E}_{\theta_D}\left[x(a_U, a_D; \tilde{\theta}_U) - C^U(a_U, \theta_U) + T^{DU}(\theta_U, \theta_D)\right] \tag{6}$$

s.t.

$$\theta_D \in \underset{\tilde{\theta}_D}{\text{argmax}}\, \pi^D_{del}(\theta_D, \tilde{\theta}_D) := P(a_D(\theta_U, \tilde{\theta}_D), \theta_D) \cdot a_D(\theta_U, \tilde{\theta}_D)$$
$$- C^D(a_D(\theta_U, \tilde{\theta}_D), a_U(\theta_U, \tilde{\theta}_D)) - T^{DU}(\theta_U, \tilde{\theta}_D)$$
$$\Pi^D_{del}(\theta_D) := \pi^D_{del}(\theta_D, \theta_D) \geq 0.$$

We denote the solution of this problem as $L^x(\theta_U, \tilde{\theta}_U)$.

3.1 Implementable allocations

The payment rule $x(\cdot)$ induces a reporting strategy by U that maximizes its profits, which includes the payment $x(\cdot)$, its costs and the rent it can extract from the downstream market. This is, formally, $\tilde{\theta}^x_U(\theta_U) \in arg\max_{\tilde{\theta}_U} L^x(\theta_U, \tilde{\theta}_U)$. Moreover, denoting by $\hat{a}_U(\theta_U, \theta_D)$ and $\hat{a}_D(\theta_U, \theta_D)$ the solution to (6), we can write the allocation induced by x as $a^x_U(\theta_U, \theta_D) = \hat{a}_U(\theta_U, \theta_D|\tilde{\theta}^x_U(\theta_U))$ and $a^x_D(\theta_U, \theta_D) = \hat{a}_D(\theta_U, \theta_D|\tilde{\theta}^x_U(\theta_U))$. We say that an allocation rule

$a_U(\theta_U, \theta_D), a_D(\theta_U, \theta_D)$ is implementable through delegation if there exists a payment x such that $a_D^x(\theta_U, \theta_D) = a_D(\theta_U, \theta_D)$ and $a_U^x(\theta_U, \theta_D) = a_U(\theta_U, \theta_D)$.

As in the mechanism design literature, it is without loss of generality to consider payment rules that induce a truthful report $\tilde{\theta}_U^x(\theta_U) = \theta_U$, which, in turn, allows for a sharp characterization of the implementable allocation rules.

Lemma 4 *For any $x(a_U, a_D, \tilde{\theta}_U)$, there exists a payment rule $\bar{x}(a_U, a_D, \tilde{\theta}_U)$ that induces truthful revelation, i.e., $\tilde{\theta}_U^{\bar{x}}(\theta_U) = \theta_U$ and the same allocation rules: $a_U^x(\theta_U, \theta_D) = a_U^{\bar{x}}(\theta_U, \theta_D)$, $a_D^x(\theta_U, \theta_D) = a_D^{\bar{x}}(\theta_U, \theta_D)$.*

We now provide a characterization of the set of implementable allocations.

Proposition 1 *An allocation $(\hat{a}_U(\theta_U, \theta_D), \hat{a}_D(\theta_U, \theta_D))$ is implementable iff there exist functions $G : \mathbb{R}_+^2 \to \mathbb{R}$ and $\ell : \mathbb{R}_+ \times [\underline{\theta}_U, \bar{\theta}_U] \to \mathbb{R}$ such that*

$$(\hat{a}_U, \hat{a}_D) \in \underset{a_U, a_D}{\operatorname{argmax}} \, \mathbb{E}_{\theta_D} \left[G(a_U, a_D) + \ell(a_U, \theta_U) - C^U(a_U, \theta_U) + h^{D,0}(a_U, a_D, \theta_D) \right]$$

with G increasing in both variables, and $\frac{\partial^2 \ell(a_U, \tilde{\theta}_U)}{\partial a_U \partial \tilde{\theta}_U} \leq 0$.
The delegation contract that implements this allocation can be written as

$$x(a_U, a_D; \tilde{\theta}_U) = G(a_U, a_D) + \ell(a_U, \theta_U) + b(\theta_U)$$

where

$$b(\theta_U) = \mathbb{E}_{\theta_D} \left[\int_{\theta_U}^{\bar{\theta}_U} \frac{\partial C^U}{\partial \theta_U}(\hat{a}_U(s, \theta_D), s) ds - G(\hat{a}_U, \hat{a}_D) - \ell(\hat{a}_U, \theta_U) + C^U(\hat{a}_U, \theta_U) - h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D) \right].$$

The delegation contract must ensure truth-telling and participation. Truth-telling, in particular, implies an additively separable function consisting of three terms. The first term, $G(a_U, a_D)$, depends on observable actions and provides direct incentives for downstream production, allowing the principal to partially counteract the output contraction typical of a monopolist. For example, consumer surplus is a natural candidate for $G(\cdot)$. The second term, $\ell(a_U, \theta_U)$, considers U's report and investment, and it can be used to internalize informational rents, optimally distorting investment (since informational rents must be taken into account). Finally, for U to report truthfully, the term $b(\theta_U)$ must be added. This plays the same role as a transfer rule in the mechanism design literature, and, as in that case, it is entirely determined by $G(\cdot)$ and $l(\cdot)$. However, in contrast with that literature, the allocation rules $\hat{a}_U$ and $\hat{a}_D$ are *induced* by $G(\cdot)$ and $l(\cdot)$, and not directly determined by the principal.[3]

Note that the expected social welfare generated by a delegation contract $x(a_U, a_D, \theta_D)$, is given by

$$\mathbb{E}_\theta[SW(a_U^x(\cdot), a_D^x(\cdot), \theta_U, \theta_D, x(\cdot), 0, T^{DU}(\cdot))]$$

Moreover, in a result reminiscent of revenue equivalence, whenever an allocation is implementable, the social welfare generated by it can be written as a function uniquely of the allocation, with transfers being determined by it. Moreover, social welfare can be written through the same functional form as in a centralized mechanism, but with a crucial difference. Even if the planner values D's profits (at a rate λ_D), the value generated by delegation does not include it. Delegation, then, entails a cost, even for implementable allocations. Proposition 2 formalizes this result.

Proposition 2 *An allocation implementable through delegation can achieve the same expected social welfare as if implemented by a centralized mechanism when the planner does not value downstream profits. In that case, with* $\ell(a_U, \theta_U) = -(1-\lambda_U)\frac{F_U(\theta_U)}{f_U(\theta_U)} C^U_{\theta_U}(a_U, \theta_U)$,

$$\mathbb{E}_\theta[SW(a_U^x(\cdot), a_D^x(\cdot), \theta_U, \theta_D, x(\cdot), 0, T^{DU}(\cdot))] = \mathbb{E}_\theta[K(a_U^x, a_D^x, \theta_U, \theta_D; \lambda_U, 0)].$$

Together, propositions (1) and (2) show the potential costs of delegation. First, the optimal centralized allocation is not necessarily implementable. Second, even if implementable, the social welfare generated is strictly smaller whenever $\lambda_D > 0$.

In the centralized problem, the virtual cost associated with a_U is given by U's actual cost $C^U(\cdot)$ plus informational rents $(1-\lambda_U)\frac{F_U}{f_U}C^U_{\theta_U}$. By choosing $\ell(\cdot) = -(1-\lambda_U)\frac{F_U}{f_U}C^U_{\theta_U}$ the planner makes the upstream firm internalize exactly this informational rent.

With delegation, U, besides its costs, considers the rents it can extract from the downstream relationship, which are given by the virtual profits $h^{D,0}(a_U, a_D, \theta_D)$. Therefore, if the planner can set $G(a_U, a_D)$ to reflect the consumer surplus and $\ell(a_U, \theta_U)$ to reflect the relevant share of informational rents, the allocation can be implemented without giving up welfare.

[3] Writing $G(\cdot) + \ell(\cdot)$ imposes a particular structure on the incentives the principal can give to U. It is not possible to provide any function $\Upsilon(a_U, a_D, \theta_U)$, since implementability requires that $\Upsilon_{a_D,\theta_U} = 0$ and $\Upsilon_{a_U,\theta_U} \leq 0$.

3.2 Implementability of centralized allocations

From the analysis above, it follows that two elements may render the centralized allocation unimplementable. First, firm U never internalizes any fraction of D's profits. That is why h^{D,λ_D} is evaluated at $\lambda_D = 0$ in U's objective function, highlighting the wedge between the planner, who values D's profits at λ_D, and U, who does not. Second, the planner does not observe D's report $\tilde{\theta_D}$; the payment rule only depends on D's observable allocation. The following proposition summarizes the conditions under which the centralized allocation is implementable through delegation.

Proposition 3 *The centralized allocation can be implemented through delegation iff the function* $H(a_D, \theta_D) = V(a_D, \theta_D) - \lambda_D \frac{1-F_{\theta_D}(\theta_D)}{f_{\theta_D}(\theta_D)} P_{\theta_D}(a_D, \theta_D) \cdot a_D$ *satisfies* $\frac{\partial^2 H(a_D,\theta_D)}{\partial a_D \partial \theta_D} = 0$, *and therefore can be written as* $H(a_D, \theta_D) = H_a(a_D) + H_\theta(\theta_D)$. *The payment rule that implements the centralized allocation, in this case, is* $x(a_U, a_D, \theta_U) = H_a(a_D) - (1 - \lambda_U)\frac{F_U}{f_U} C^U_{\theta_U} + b(\theta_U)$, *where* $b(\theta_U)$ *is defined as in Proposition 1.*

The intuition follows from the planner's inability to set a payment rule that depends on θ_D. If the marginal benefit from consumption $\partial V / \partial a_D$ depends on θ_D, the planner cannot perfectly align the upstream firm's incentives with welfare maximization. The same problem occurs when the planner values downstream profits, i.e., $\lambda_D > 0$. These conditions are summarized in the following corollary.

Corollary 1 *The condition* $\frac{\partial^2 H(a_D,\theta_D)}{\partial a_D \partial \theta_D} = 0$ *is not satisfied if either*

(i) $\frac{\partial^2 V(a_D,\theta_D)}{\partial a_D \partial \theta_D} \neq 0$ *or,*

(ii) $\lambda_D \neq 0$.

Nevertheless, there are some cases where the benchmark allocation can be implemented through delegation. For example, if the planner does not value D's profit ($\lambda_D = 0$) and its private information is additively separable in the inverse demand function, the condition in Proposition 3 holds, and the benchmark can be implemented. The latter condition is widely used in the literature, as it is common to model the private information as a shift of the demand function (see, e.g., Basso, Figueroa, & Vásquez, 2017, Vives, 1988).

Corollary 2 *If* θ_D *is a parallel shift in demand, i.e.,* $P(a_D, \theta_D) \equiv \theta_D + \Psi(a_D)$, *and* $\lambda_D = 0$, *the centralized allocation can be implemented through delegation.*

This possibility result is even more general and can be applied to contexts where the planner does not maximize social welfare. If the planner's objective function is of the form

$H(a_D, a_U)$ and she does not care directly about the agents' utilities, the benchmark allocation can be implemented. This comes directly from Proposition 3, since the planner can induce U to internalize any function of the form $G(a_U, a_D)$.

This is in line with the results of N. Melumad et al. (1992) and N. D. Melumad et al. (1995), where the principal is a firm that cares about the actions of agents (a_U, a_D in our context). The designer cares neither about consumer or agents' surplus, which potentially depends on their types (θ_U, θ_D), nor their actual payoffs. When a firm only cares about the actions, the principal can implement the benchmark case through delegation.

Finally, we establish an important result about the usefulness of providing incentives for downstream production in a delegation environment. Under delegation, the introduction of incentives for downstream production always increases social welfare. Formally,

Proposition 4 *Under delegation, any incentive $G(a_U, a_D) > 0$ induces a higher social welfare than $G(a_U, a_D) = 0$.*

This section has established that delegation generally entails a cost in the context of regulation. The principal cannot provide the incentives needed to implement the benchmark allocation. A related concern is the potential complexity of the delegation mechanism, which can make it impractical. In the next section, we study a particular class of delegation contracts, widely used in practice, where the planner provides linear incentives on the quantity produced downstream.

4 Linear Delegation Contracts

In the absence of complex mechanisms, subsidies from the planner to U that are linear in the final good's quantity (a_D) are widely observed in practice. For example, regulated airports' aeronautical revenue typically comes from passenger service fees, security charges, and other charges based on passenger throughput, all commonly set by the planner. Another example can be found in healthcare. The clinical and ancillary service operations of La Ribera Hospital in Valencia (Spain) and the amortized cost of the infrastructure investment were paid for via an annual per capita payment from the Valencia government. By 2011, the Hospital covered 18% of the Valencia population and the per capita fee was \$ 656 euros (Sosa Delgado-Pastor et al., 2016).

We now analyze the effectiveness of a linear scheme and compare its performance to that of a centralized mechanism. In short, we show that under mild conditions, the optimal linear contract yields a significant share of the benefits that could be obtained with centralization.

To perform the analysis, we first characterize the optimal subsidy to the planner and, using a linear quadratic setting, we address some economic intuitions. Then, we define the welfare measure we use to compare the outcomes of different mechanisms. We provide bounds for the expected fraction of welfare loss using a linear contract, and finally, we illustrate the mechanism through numerical analysis.

Formally, the linear contract that we consider is composed of $G(a_D, a_U) = s \cdot a_D$, $\ell(a_U, \theta_U) = -(1-\lambda_U)\frac{F_U(\theta_U)}{f_U(\theta_U)} C^U_{\theta_U}(a_U, \theta_U)$, and $b(\theta_U)$ as defined in Proposition 1 to guarantee truthful revelation and minimize informational rents.

Definition 5 *The payment rule of a linear delegation contract is given by*

$$x^s(a_U, a_D, \theta_U) = s \cdot a_D - (1-\lambda_U)\frac{F_U(\theta_U)}{f_U(\theta_U)} C^U_{\theta_U}(a_U, \theta_U) + b(\theta_U) \tag{7}$$

We denote the induced allocations by $a_U^{x^s}(\cdot) := a_U^s(\cdot)$ *and* $a_D^{x^s}(\cdot) := a_D^s(\cdot)$*.*

4.1 Optimal and regret-minimizing linear subsidies

We first consider the forces at play when determining an optimal subsidy. While it is clear that a higher subsidy s increases $a_D^s(\cdot)$, and by complementarity the investment level a_U^S, for any realization of (θ_U, θ_D). However, it does not reach the optimal level that a regulator would prefer, which is caused by the linearity of the tool. The optimal subsidy $\hat{s}$ solves:

$$\max_s \ \mathbb{E}_\theta \left\{ V(a_D^s, \theta_D) - x^U(a_U^s, a_D^s, \theta_U) + \lambda_U \left[x^U(a_U^s, a_D^s, \theta_U) - C^U(a_U^s, \theta_U) + h^{D,0}(a_U^s, a_D^s, \theta_D) \right] \right\}$$

Using proposition (2), the objective function can be rewritten as a function of the allocations $(a_U^s(\cdot), a_D^s(\cdot))$:

$$\max_s \ \mathbb{E}_\theta \left\{ V(a_D^s, \theta_D) - h^{U,\lambda_U}(a_U^s, \theta_U) + h^{D,0}(a_U^s, a_D^s, \theta_D) \right\}$$

Moreover, using an envelope argument, only the effects through a_D^s are relevant in the first-order condition, as can be seen in the next proposition.

Proposition 5 *Consider a linear delegation contract, the first-order condition of the optimal subsidy is given by:*

$$\mathbb{E}_\theta \left\{ \left[P(a_D^s, \theta_D) - \frac{\partial C^D(a_U^s, a_D^s)}{\partial a_D} - \frac{1-F_D(\theta_D)}{f_D(\theta_D)} (P_{\theta_D, a_D}(a_D^s, \theta_D) a_D^s + P_{\theta_D}(a_D^s, \theta_D) \right] \frac{\partial a_D}{\partial s} \right\} = 0$$

The Proposition shows that the effect of an increase in s is a weighted sum of its effect across all realizations of (θ_U, θ_D). The term in square brackets is the derivative of the objective function with respect to a_D, which is weighted by the effect of s on a_D. A subsidy increase unambiguously increases $a_D(\theta_U, \theta_D)$ (since $\frac{\partial a_D}{\partial s} > 0$). However, for some realizations, this is detrimental (the planner would like a lower a_D), and for others, it is beneficial. On average, the effect should be 0 at $\hat{s}$.

Another desirable property of a delegation contract is that it minimizes the regret, that is, its performance loss compared to the optimal centralized mechanism. Therefore, to evaluate the performance of a linear subsidy, we consider the expected fraction of welfare that is lost through the use of a linear subsidy. Proposition 2 establishes that the expected welfare depends only on the allocation rule (a_U, a_D), regardless of the mechanism used. In particular, the social welfare can be written as the expected value of the function $K(a_U, a_D, \theta_U, \theta_D; \lambda_U, \lambda_D)$ (see Eq. 5). We consider, therefore, the s that minimizes, in expectation, the value of the "money left on the table" when using a linear subsidy instead of the centralized one.

Definition 6 *We define the* ***linear delegation regret*** *as*

$$\begin{aligned} R :=& \min_s \mathbb{E}_\theta \left[\frac{K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D) - K(a_U^s, a_D^s, \theta_U, \theta_D; \lambda_U, 0)}{K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)} \right] \\ =& \min_s 1 - \mathbb{E}_\theta \left[\frac{K(a_U^s, a_D^s, \theta_U, \theta_D; \lambda_U, 0)}{K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)} \right], \end{aligned}$$

which captures the (expected) fraction of welfare loss due to delegation through a linear subsidy. We define s^* as the regret-minimizer linear subsidy.

4.2 Linear subsidies in linear-quadratic environments

We specialize the analysis to a linear quadratic setting, which can be seen as a second-order approximation of any environment. This allows us to provide an explicit upper bound on the delegation regret for linear subsidies, which turn out to perform well under many specifications. We consider the following functional forms:

$$\begin{aligned} P(a_D, \theta_D) &= A + \theta_D - B \cdot a_D, \\ C^U(\theta_U, a_U) &= \frac{1}{2}\theta_U a_U^2, \\ C^D(a_D, a_U) &= (c - d \cdot a_U) \cdot a_D, \end{aligned} \qquad \begin{aligned} \theta_D &\sim U[0,1], \\ \theta_U &\sim U[1,2], \end{aligned}$$

with $A - c > 1$ [4] and $B > d^2$. Here, $A - c$ measures market size, while d captures the "coupling" of U and D's payoffs because it measures the reduction of D's marginal costs due to U's investment. It is direct to show that consumer surplus can be written as:

$$V(a_D, \theta_D) \equiv V(a_D) = \frac{B}{2} a_D^2$$

Since the consumer surplus is independent of θ_D, by Proposition 3 (Corollary 2), the centralized mechanism is implementable with delegation when $\lambda_D = 0$. However, in the more general case, there are two sources of welfare loss: (1) the planner's concern for the downstream firm's profits ($\lambda_D > 0$), which impedes the implementation of the centralized allocation, and (2) the linearity of the subsidy.

Moreover, both in the centralized mechanism and a delegation contract, the allocation a_U is completely determined by a_D by

$$\frac{a_D^*(\theta_U, \theta_D)}{a_U^*(\theta_U, \theta_D)} = \frac{a_D^s(\theta_U, \theta_D)}{a_U^s(\theta_U, \theta_D)} = \frac{\phi(\theta_U, \lambda_U)}{d}$$

where $\phi(\theta, \lambda) = \theta(2-\lambda) - (1-\lambda)$. Therefore, we can write $\mathbb{E}_\theta[SW(a_U^x(\cdot), a_D^x(\cdot), \theta_U, \theta_D, x(\cdot), 0, T^{DU}(\cdot))]$ as a function of a_D as:

$$\mathbb{E}_\theta\Big[K\Big(d\frac{a_D^x}{\phi(\theta_U, \lambda_U)}, a_D^x, \theta_U, \theta_D; \lambda_U, \lambda_D\Big)\Big] = \mathbb{E}_\theta\Big[V(a_D, \theta_D) - h^{U,\lambda_U}\left(d\frac{a_D}{\phi(\theta_U, \lambda_U)}, \theta_U\right) + h^{D,\lambda_D}(a_D, \theta_D)\Big]$$

Consequently, he allocation rules a_D^* and a_D^s fully characterize the linear delegation regret. Solving the benchmark problem (see 5) and the allocation induced by a subsidy s (Proposition 1), we obtain:

$$a_D^*(\theta_U, \theta_D) = \frac{\phi(\theta_U, \lambda_U) \cdot [A - c + \phi(\theta_D, \lambda_D)]}{B\phi(\theta_U, \lambda_U) - d^2} \tag{8}$$

$$a_D^s(\theta_U, \theta_D) = \frac{\phi(\theta_U, \lambda_U) \cdot [s + A - c + \phi(\theta_D, 0)]}{2B\phi(\theta_U, \lambda_U) - d^2} \tag{9}$$

This allows for pinpointing the inefficiencies induced by a linear contract.

A first result that sheds light on the sources of inefficiency is that the benchmark allocation rule a_D^* is always steeper, with respect to θ_D, than the one induced by a linear subsidy.

Lemma 5 *For any s,*

$$\frac{\partial a_D^*(\theta_U, \theta_D)}{\partial \theta_D} \geq \frac{\partial a_D^s(\theta_U, \theta_D)}{\partial \theta_D}$$

[4] This condition ensures production even for the lowest value of θ_D, which is $\theta_D = 0$.

This result comes from the interplay between two countervailing forces. First, in any decentralized mechanism, since U does not value D's profits, a quantity distortion is more profitable (to decrease information rents) and therefore a_D^s is more sensitive than a_D^* to private information θ_D. This can be seen in the numerator of both expressions in Eqs. (8)–(9), and noting that $\frac{\partial \phi(\theta_D, \lambda_D)}{\partial \theta_D} = 2 - \lambda_D$. The opposing force is due to the typical monopoly power and decreases the downstream quantity and its sensitivity to information, making a_D^s less sensitive to θ_D. This can be seen in the denominator of the expressions above. Lemma 5 proves that this second force, the market power, dominates.

A second economic insight comes from the result that the planner can affect, through the subsidy s, the *level* of the allocation rule a_D^s, but not its slope with respect to θ_D. By increasing the linear subsidy, the planner does not affect, on the margin, the extra informational rents captured by D in its relationship with U if the allocation a_D changes. Therefore, the contract between U and D is not marginally affected by s.

The parameter λ_D represents the misalignment between the planner and U about the valuation of D's profits. It plays an essential role in shaping the delegation regret; therefore, we explicitly write the dependence $R(\lambda_D)$. We first consider $\lambda_D = 0$, which implies that the benchmark allocation is implementable (by Corollary 2).[5] The regret, in this case, can be directly interpreted as the loss induced by the linearity of the delegation contract (as compared to the optimal non-linear one that replicates the centralized benchmark).

Proposition 6 *The delegation regret when the planner does not value downstream profits ($\lambda_D = 0$), $R|_{\{\lambda_D=0\}}$, is decreasing in market size ($A - c$). Moreover, it can be bounded using the following results:*

- $R|_{\{\lambda_D=0,d=0\}} = \frac{1}{4}\left[1 - \frac{z(A-c)}{4}\right] \leq \frac{1}{4}$
- $R|_{\{\lambda_D=0,d=\sqrt{B}\}} \leq 0.61 - 0.118 \cdot z(A - c)$

where z is an increasing function satisfying $\lim_{x\to 1} z(x) = 0$ and $\lim_{x\to\infty} z(x) = 4$.

As the market size increases, the suboptimal provision of incentives of the linear contract becomes less important in relative terms, leading to a lower regret. If problems are uncoupled ($d = 0$), linear subsidies perform well. Even for a minimal market size, the regret is bounded by 25%, and linear subsidies become nearly optimal as the market size approaches infinity. On the other hand, for highly coupled structures ($d = \sqrt{B}$), the linear incentives' performance decreases. The regret can get close to 60 % for small markets, and the inefficiency does not disappear (close to 14%) for large markets.

[5]Since in this setting, $V(a_D, \theta_D) = \Psi(a_D) = -\frac{B}{2}a_D^2$

In the other case, when $\lambda_D > 0$, the misalignment between the planner and U matters. In a centralized mechanism, as λ_D grows, the principal chooses higher (and more efficient) levels of a_D, even if they involve more informational rents to D. In a delegation mechanism, however, the principal must provide higher subsidies s^* for U to induce higher levels of a_D. We analyze this problem numerically in Appendix C, showing that the regret decreases as λ_D increases. This implies that the bounds in Proposition 6 are global lower bounds, as they hold for the lowest λ_D.

5 Empirical application: airport's operations

This section aims to understand the cost of delegation in a complex market: aviation. Public airports have failed to manage travel delays, affecting air travelers and airlines. As Yan and Winston (2014) discuss, the problem lies in the inefficient pricing guidelines set by the Federal Aviation Administration, which has in turn induced a wave of airport privatization worldwide (Howell, Jang, Kim, & Weisbach, 2022).

We calibrate a model to reflect the characteristics of a US airport as closely as possible. We use functional forms estimated for US airports and parameters based on observed passenger volumes (a_D) and airport size (investment a_U), mostly from Yan and Winston (2014), who estimate a model of airport competition in the San Francisco Bay Area.

5.1 Set-up and parameters

Following Morrison and Winston (2007) and Yan and Winston (2014), we use a constant-elasticity demand function $Q(p) = A \cdot p^{\xi}$, so that the inverse demand is given by:

$$P(a_D, \theta_D) = \theta_D \cdot \left(\frac{a_D}{A}\right)^{1/\xi} .$$

Under this assumption the condition of Proposition 3 does not hold, and therefore the centralized allocation cannot be implemented through delegation. We use a demand elasticity of -1.54, as estimated by Yan and Winston (2014), and assume for simplicity that $\theta_D \sim U[2, 3]$ so the demand scale parameter A is adjusted to reflect observed passenger volumes at observed prices.[6]

[6] Using this interval prevent us from dealing with zero demand and with negative payoffs that may happen with $\theta_D = 1$

Upstream costs and private information are modeled to reflect increasing marginal costs:

$$C^U(a_U, \theta_U) = \theta_U \cdot \frac{1}{2} \cdot \kappa_U \cdot a_U^2 \ ,$$

where units are such that a_U represents the number of runways, the usual measure of airport capacity and a main driver of downstream costs through landing and takeoff delays. We also assume for simplicity that $\theta_U \sim U[1, 2]$ and adjust the cost scale parameter κ_U to reflect observed airport construction costs.

Critical to our study is the relationship between upstream investment and downstream costs. For the latter, we use the function estimated by Yan and Winston (2014). The airlines' costs are given by:

$$C^D(a_U, a_D) = \frac{a_D \cdot \kappa_D}{L \cdot K} \left(\left[h + \frac{1}{60} \cdot \delta \left(\frac{a_D}{a_U} \right) \right] \cdot Z(K) + \tau \cdot K + SC(dist) \cdot L \cdot K \right) \tag{10}$$

where a_D is the number of passengers (million pax per year), L is the average aircraft's load factor, K is the aircraft capacity (seats) and κ_D is a scale parameter to reflect airlines' costs given the choices of units. h is the (undelayed) operating time (hours per flight), and δ is the average delay (minutes per flight) at the airport, a function of the ratio between passenger volume and airport capacity. $Z(K)$ is the aircraft operating cost function per hour, a function of the aircraft size. The second term in the parentheses on the right-hand side of equation 10, τ, is the landing fee, and the third term is the average per-flight cost that varies with the distance traveled.

Yan and Winston (2014) estimate the functions $\delta(\cdot)$ and $Z(\cdot)$ using traffic delays recorded in the FAA's Aviation System Performance Metrics (ASPM) database:

$$\ln(\delta) = \left(b_0 + b_1 \cdot \ln \left(\frac{a_D}{a_U} \right) + b_2 \left[\ln \left(\frac{a_D}{a_U} \right) \right]^2 \right) \tag{11}$$

$$Z(K) = \exp(4.5715 + 0.6982 \cdot ln(K)) \tag{12}$$

The function $\delta(a_D, a_U)$ is key to our example as it reflects the impact of upstream investment on downstream costs. The summary of parameter values is presented in Table 1, where most are from Yan and Winston (2014).

Table 1: Parameter values

Parameter	Value	Note
A	145,397	Demand scale parameter
ξ	-1.54	Demand elasticity
κ_U	500	Upstream cost scale parameter
κ_D	5	Downstream cost scale parameter
L	80%	Aircraft load factor
K	146	Aircraft capacity (seats)
h	4.93	Average undelayed flight time (hours)
τ	2	US$ per-seat fee
SC	0.8232	US$ per pax
$dist$	2.04	1000 miles
b_0	-2.9886	Delay function parameter
b_1	4.2989	Delay function parameter
b_2	-0.7852	Delay function parameter

Note: SC is computed according to the linear function of distance estimated by Yan and Winston (2014): $SC(dist) = 2.19 - 0.67 \cdot dist$ USD per pax.

5.2 Results

Unlike the linear quadratic setting, our model does not yield closed-form solutions for the firms' actions, a_D and a_U. We therefore numerically analyze the performance of linear contracts. Our primary focus is identifying the optimal linear contract, characterized by the subsidy per passenger (s^*), and evaluating its performance using the associated delegation regret R.

Our numerical procedure is as follows. First, to analyze the centralized mechanism, we establish a grid of possible values for the planner's valuations of upstream and downstream profits λ_U, λ_D. For each point on the grid, we compute the optimal actions (a_U, a_D) and the resulting welfare, as per Lemmas 1 and 2. We then calculate the expected welfare, $\mathbb{E}[K^C]$, by averaging across all grid points, leveraging the uniform distributions of the type parameters.

For the linear contract, on the other hand, we need to take into account the lack of dependence of s^* on θ_D. To analyze the linear delegation contract, since s is fixed ex ante, we iterate over candidate values of s and, for each, integrate welfare over the (θ_U, θ_D) grid. We compute the downstream firm's action induced by the downstream mechanism, a_D, and the corresponding induced action from the upstream firm, a_U. This procedure calculates the expected welfare under delegation, $\mathbb{E}[K^s]$. We then find the regret-minimizing subsidy, s^*, by selecting the s that yields the lowest delegation regret. Lastly, we compute the welfare associated with the delegation contract that does not include subsidies (i.e., $s = 0$).

Table 2 summarizes our results for various planner weights, λ_U and λ_D. Our main finding is that a simple linear delegation contract performs remarkably well, yielding only minimal welfare losses. The regret, $R(s^*)$, is consistently small across all specifications. For instance, the maximum regret is 1.8%, which occurs when the planner's objective is fully aligned with downstream profits while disregarding upstream profits ($\lambda_D = 1, \lambda_U = 0$). This scenario intuitively represents the worst case, as it creates the most significant misalignment between the planner's goal and that of the upstream monopolist, who does not internalize downstream outcomes. For comparison, a laissez-faire policy ($s = 0$) is far inferior, generating regrets between 22% and 27%.

The model's quantitative predictions also align well with real-world observations. The predicted airport size ranges from 2.3 to 3.2 runways under the centralized mechanism and 2.5 to 3.2 runways under the delegation contract. These figures are comparable to those of airports in the San Francisco Bay Area, where San Francisco International (SFO) and Oakland (OAK) have four runways each, while San José (SJC) has two. Similarly, the predicted annual traffic of seven to ten million passengers under both mechanisms is on the same order of magnitude as that of SJC.

The resulting optimal subsidy is approximately \$1,200 per passenger. While substantial, this figure is plausible given that we analyze a monopoly, the market structure where output is most severely restricted. This required subsidy would be significantly smaller in a more competitive environment with weaker incentives to limit quantity.

6 Conclusion

The article studies optimal regulation mechanisms and contracts in vertical markets where different firms produce the input and the final good. The focus is on upstream monopolies, such as airports or clinical laboratory services, which are essential for downstream production, and the interaction with downstream firms may be a critical duty of the monopoly. The article's novelty is that it considers delegation contracts as a regulatory alternative to centralized mechanisms. The optimal delegation contract is a payment rule that depends on the type declared by the upstream firm and observed (ex-post) quantities, including downstream production.

We provide three main contributions, including the characterization of the optimal delegation contract, the complete set of allocations implementable with this regulatory regime, and the analysis of the performance of a linear contract. The study establishes that delegation contracts cannot generally achieve the same result as the direct regulation of both

Table 2: Numerical results

λ_U	s^*	$\mathbb{E}[K^C]$	$\mathbb{E}[K^s]$	$R(s^*)$	$R(s=0)$
Panel A: $\lambda_D = 0$					
0	1,130	30,433	29,937	1.68%	22.06%
0.333	1,190	30,975	30,707	0.89%	22.95%
0.667	1,260	31,661	31,571	0.29%	24.18%
1	1,330	32,605	32,552	0.17%	25.94%
Panel B: $\lambda_D = 1/3$					
0	1,150	31,161	30,649	1.70%	22.47%
0.333	1,210	31,717	31,440	0.90%	23.38%
0.667	1,270	32,421	32,328	0.30%	24.62%
1	1,350	33,389	33,337	0.16%	26.39%
Panel C: $\lambda_D = 2/3$					
0	1,160	31,897	31,366	1.72%	22.88%
0.333	1,220	32,467	32,179	0.92%	23.80%
0.667	1,290	33,188	33,091	0.30%	25.04%
1	1,360	34,181	34,127	0.16%	26.82%
Panel D: $\lambda_D = 1$					
0	1,180	32,640	32,089	1.75%	23.27%
0.333	1,240	33,224	32,923	0.94%	24.20%
0.667	1,300	33,964	33,860	0.32%	25.46%
1	1,380	34,982	34,924	0.17%	27.24%

firms. It also illustrates under which conditions simpler linear contracts can be considered a reasonable monopoly regulation policy.

The analysis suggests several avenues for future research. First, studying the optimal delegation contract when the centralized allocation cannot be implemented would be interesting. Second, we deem it relevant to examine delegation when the downstream mechanism is limited by exogenous factors such as competition law forbidding transfers to prevent price discrimination. Third, the analysis can be extended to include other downstream market structures.

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Appendix A. Proof of Lemma 1 to Proposition 4

Proof of Lemma 1

Upstream firm.(⇒) By Envelope Theorem $\Pi^{U'}(\theta_U) = \pi^U_{\tilde{\theta}_U}(\tilde{\theta}_U, \theta_U)|_{\tilde{\theta}_U=\theta_U} = \mathbb{E}_{\theta_D}\left[-C^U_{\theta_U}(a_U, \theta_U)\right]$. By (IC) $\pi^U_{\theta_U,\tilde{\theta}_U}(\tilde{\theta}_U, \theta_U) = \mathbb{E}_{\theta_D}\left[-C^U_{\theta_U,a_U}(a_U, \theta_U)\partial a_U/\partial\tilde{\theta}_U\right] \geq 0$, as $C^U_{\theta_U,a_U}(a_U, \theta_U) > 0$ then $a_U(\theta_U, \theta_D)$ is decreasing in θ_U. (⇐) The converse is standard.
Downstream firm.(⇒) By Envelope Theorem $\Pi^{D'}(\theta_D) = \pi^U_{\tilde{\theta}_D}(\tilde{\theta}_D, \theta_D)|_{\tilde{\theta}_D=\theta_D} = \mathbb{E}_{\theta_U}[P_{\theta_D}(a_D(\theta), \theta_D)\cdot a_D(\theta)]$. By (IC) $\pi^D_{\theta_D,\tilde{\theta}_D}(\tilde{\theta}_D, \theta_D) = \mathbb{E}_{\theta_U}[\partial a_D/\partial\theta_D(P_{\theta_D a_D}(a_D(\theta), \theta_D)a_D(\theta) + P_{\theta_D}(a_D(\theta), \theta_D)] \geq 0$, by assumption (2) $P_{\theta_D}(a_D(\theta), \theta_D)\cdot a_D(\theta)$ is convex in a_D, therefore $a_D(\theta_U, \theta_D)$ is increasing in θ_D. (⇐) The converse is standard.

Proof of Lemma 2

The expected payoffs of the firms, when they report their true type, are,

$$\Pi^U(\theta_U) = \mathbb{E}_{\theta_D}[T^U(\theta_U, \theta_D) - C^U(a_U(\theta_U, \theta_D), \theta_U)]$$
$$\Pi^D(\theta_D) = \mathbb{E}_{\theta_U}[P(a_D(\theta_U, \theta_D), \theta_D) \cdot a_D(\theta_U, \theta_D) - C^D(a_U(\theta_U, \theta_D), a_D(\theta_U, \theta_D)) - T^D(\theta_U, \theta_D)]$$

By Lemma 1,

$$\Pi^{U'}(\theta_U) = -\mathbb{E}_{\theta_D}\left[\frac{\partial}{\partial\theta_U}C^U(a_U(\theta_U, \theta_D), \theta_U)\right],$$
$$\Pi^{D'}(\theta_D) = \mathbb{E}_{\theta_U}\left[\frac{\partial}{\partial\theta_D}P(a_D(\theta_U, \theta_D), \theta_D) \cdot a_D(\theta_U, \theta_D)\right],$$

therefore,

$$\Pi^U(\theta_U) - \Pi^U(\bar{\theta}_U) = \int_{\theta_U}^{\bar{\theta}_U} \mathbb{E}_{\theta_D}\left[\frac{\partial}{\partial\theta_U}C^U(a_U(t, \theta_D), t)\right]dt$$
$$\Pi^D(\theta_D) - \Pi^D(\underline{\theta}_D) = \int_{\underline{\theta}_D}^{\theta_D} \mathbb{E}_{\theta_U}\left[\frac{\partial}{\partial\theta_D}P(a_D(\theta_U, t), t) \cdot a_D(\theta_U, t)\right]dt$$

Binding the agent's individual-rationality constraints for extreme types, $\Pi^U(\bar{\theta}_U) = 0$ and

$\Pi^D(\underline{\theta}_D) = 0$. Integration by parts shows that,

$$\mathbb{E}_\theta[\Pi^U(\theta_U)] = \int_{\underline{\theta}_U}^{\bar{\theta}_U} \Pi^U(\theta_U) dF_U(\theta_U) = \mathbb{E}_\theta\left[\frac{\partial}{\partial\theta_U} C^U(a_U(\theta_U,\theta_D),\theta_U)\cdot\frac{F_U(\theta_U)}{f_U(\theta_U)}\right]$$
$$\mathbb{E}_\theta[\Pi^D(\theta_D)] = \int_{\underline{\theta}_D}^{\bar{\theta}_D} \Pi^D(\theta_D) dF_D(\theta_D) = \mathbb{E}_\theta\left[\frac{\partial}{\partial\theta_D} P(a_D(\theta_U,\theta_D),\theta_D)\cdot a_D(\theta_U,\theta_D)\cdot\frac{1-F_D(\theta_D)}{f_D(\theta_D)}\right]$$

Replacing this expression on $\mathbb{E}_\theta[\Pi^U(\theta_U)]$ and $\mathbb{E}_\theta[\Pi^D(\theta_D)]$ we have that,

$$\mathbb{E}_\theta[T^U(\theta_U,\theta_D)] = \mathbb{E}_\theta\left[C^U(a_U,\theta_U) + \frac{F_U(\theta_U)}{f_U(\theta_U)}\frac{\partial}{\partial\theta_U}C^U(a_U,\theta_U)\right]$$
$$\mathbb{E}_\theta[T^D(\theta_U,\theta_D)] = \mathbb{E}_\theta\left[P(a_D,\theta_D)\cdot a_D - C^D(a_D,a_U) - \frac{1-F_D(\theta_D)}{f_D(\theta_D)}\cdot\frac{\partial}{\partial\theta_D}P(a_D,\theta_D)\cdot a_D\right]$$

Therefore, replacing on the expected social welfare,

$$\begin{aligned}\mathbb{E}_\theta[SW(a_U,a_D,\theta_U,\theta_D,T^U,T^D,0)] &= \mathbb{E}_\theta\Big[V(a_D,\theta_D) - \Big(C^U(a_U,\theta_U) + (1-\lambda_U)\frac{F_U(\theta_U)}{f_U(\theta_U)}C^U_{\theta_U}(a_U,\theta_U)\Big) + \\ &\Big(P(a_D,\theta_D)\cdot a_D - C^D(a_D,a_U) - (1-\lambda_D)\frac{1-F_D(\theta_D)}{f_D(\theta_D)}P_{\theta_D}(a_D,\theta_D)\cdot a_D\Big)\Big] \\ &= \mathbb{E}_\theta\left[V(a_D,\theta_D) - h^{U,\lambda_U}(a_U,\theta_U) + h^{D,\lambda_D}(a_U,a_D,\theta_D)\right]\end{aligned}$$

Proof of Lemma 3

Standard proof apply.

Proof of Lemma 4

Standard proof apply.

Proof of Proposition 1

($\Rightarrow$)Let $(\hat{a}_U,\hat{a}_D)$ be an allocation implementable by the principal; this means that exists a payment rule x that induces truthful revelation and solves (6). Then

$$\pi^U(\theta_U,\theta_U) = \max_{\tilde{\theta}_U}\pi^U(\theta_U,\tilde{\theta}_U) = \max_{\tilde{\theta}_U}\mathbb{E}_{\theta_D}\left[x(\hat{a}_U,\hat{a}_D,\tilde{\theta}_U) - C^U(\hat{a}_U,\theta_U) + h^{D,0}(\hat{a}_U,\hat{a}_D,\theta_D)\right],$$

by envelope theorem,

$$\frac{\partial \pi^U(\theta_U, \tilde{\theta}_U)}{\partial \theta_U} = -\mathbb{E}_{\theta_D}\left[\frac{\partial C^U}{\partial \theta_U}(\hat{a}_U, \theta_U)\right].$$

Then,

$$\begin{aligned}\mathbb{E}_{\theta_D}\left[x(\hat{a}_U, \hat{a}_D, \theta_U) - C^U(\hat{a}_U, \theta_U) + h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D)\right] =& \mathbb{E}_{\theta_D}[\pi(\bar{\theta}_U, \bar{\theta}_U)] \\ &+ \mathbb{E}_{\theta_D}\left[\int_{\theta_U}^{\bar{\theta}_U} \frac{\partial C^U}{\partial \theta_U}(\hat{a}_U(s, \theta_D), s) ds\right]\end{aligned}$$

setting $\pi(\bar{\theta}_U, \bar{\theta}_U) = 0$, we have that

$$\mathbb{E}_{\theta_D}\left[x(\hat{a}_U, \hat{a}_D, \theta_U)\right] = \mathbb{E}_{\theta_D}\left[\int_{\theta_U}^{\bar{\theta}_U} \frac{\partial C^U}{\partial \theta_U}(\hat{a}_U(s, \theta_D), s) ds + C^U(\hat{a}_U, \theta_U) - h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D)\right] \tag{13}$$

Then any delegation contract $x(\hat{a}_U, \hat{a}_D, \tilde{\theta}_U)$ is incentive compatible if (13) holds. In particular, given that (13) holds point-wise, therefore $\partial^2 x(\cdot)/\partial a_D \partial \theta_U = 0$. Then $x(\hat{a}_U, \hat{a}_D, \theta_U)$ can be written as $x(\hat{a}_U, \hat{a}_D, \theta_U) = G(a_U, a_D) + H(a_U, \theta_U)$. Let's write $H(a_U, \theta_U) = \ell(a_U, \theta_U) + b(\theta_U)$, replacing in (13), this function $b(\theta_U)$ is,

$$\begin{aligned}b(\theta_U) =& \mathbb{E}_{\theta_D}\left[\int_{\theta_U}^{\bar{\theta}_U} \frac{\partial C^U}{\partial \theta_U}(\hat{a}_U(s, \theta_D), s) ds + C^U(\hat{a}_U, \theta_U) - h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D)\right] \\ &- \mathbb{E}_{\theta_D}\left[G(\hat{a}_U, \hat{a}_D) + \ell(\hat{a}_U, \theta_U)\right].\end{aligned}$$

($\Leftarrow$) Let's define $\Gamma\left(\theta_U, \tilde{\theta}_U\right)$ as the optimal payoff of the upstream firm type θ_U when it reports $\tilde{\theta}_U$,

$$\begin{aligned}\Gamma\left(\theta_U, \tilde{\theta}_U\right) &= \max_{a_U, a_D} \mathbb{E}_{\theta_D}\left[x(a_U, a_D, \tilde{\theta}_U) - C^U(a_U, \theta_U) + h_D^0(a_U, a_D, \theta_D)\right] \\ &\Leftrightarrow \max_{a_U, a_D} \mathbb{E}_{\theta_D}\left[G(a_U, a_D) + \ell(a_U, \tilde{\theta}_U) - C^U(a_U, \theta_U) + h_D^0(a_U, a_D, \theta_D)\right]\end{aligned} \tag{14}$$

$b(\theta_U)$ is independent of a_U and a_D. By Mirrlees (1986), to establish incentive compatibility, it suffices to show that $\forall \theta_U$

$$\frac{\partial \Gamma\left(\theta_U, \tilde{\theta}_U\right)}{\partial \theta_U} \text{ is (weakly) increasing in } \tilde{\theta}_U. \tag{15}$$

By envelope theorem,

$$\frac{\partial \Gamma\left(\theta_U, \tilde{\theta}_U\right)}{\partial \theta_U} = \mathbb{E}_{\theta_D}\left[-\frac{\partial C^U}{\partial \theta_U}(a_U(\tilde{\theta}_U, \theta_D), \theta_U)\right]$$

Note that, as a single-crossing property hold, i.e., $\partial^2 C^U / \partial \theta_U \partial a_U > 0$, it suffices to show that $a_U(\theta_U, \theta_D)$ is decreasing in $\tilde{\theta}_U$. As $\partial^2 \Gamma / \partial \tilde{\theta}_U \partial a_U = \mathbb{E}_{\theta_D}[\partial^2 \ell / \partial \tilde{\theta}_U \partial a_U] \leq 0$, by Topkis it holds.

To complete the proof,

$$\mathbb{E}_{\theta_D}\left[G(\hat{a}_U, \hat{a}_D) + \ell(\hat{a}_U, \tilde{\theta}_U) - C^U(\hat{a}_U, \theta_U) + h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D)\right] + b(\theta_U) = \mathbb{E}_{\theta_D}\left[\int_{\theta_U}^{\bar{\theta}_U} \frac{\partial C^U}{\partial \theta_U}(a_U(s, \theta_D), s) ds\right]$$

$$b(\theta_U) = \mathbb{E}_{\theta_D}\left[\int_{\theta_U}^{\bar{\theta}_U} \frac{\partial C^U}{\partial \theta_U}(a_U(s, \theta_D), s) ds - G(\hat{a}_U, \hat{a}_D) - \ell(\hat{a}_U, \tilde{\theta}_U) + C^U(\hat{a}_U, \theta_U) - h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D)\right]$$

Proof of Proposition 2

Let $(\hat{a}_U(\theta_U, \theta_D), \hat{a}_D(\theta_U, \theta_D))$ be an allocation implementable by delegation through $\ell(a_U, \theta_U) = -(1-\lambda_U)\frac{F_U(\theta_U)}{f_U(\theta_U)} C^U_{\theta_U}(a_U, \theta_U)$.. When the planner implements this allocation in a centralized mechanism, following definition 3, the expected social welfare that generates is

$$\mathbb{E}_\theta\left[V(\hat{a}_D, \theta_D) - h^{U,\lambda_U}(\hat{a}_U, \theta_U) + h^{D,\lambda_D}(\hat{a}_U, \hat{a}_D, \theta_D)\right].$$

To prove Proposition 2, we show that the expected social welfare when $(\hat{a}_U(\theta_U, \theta_D), \hat{a}_D(\theta_U, \theta_D))$ is implemented through delegation is the same. To implement the allocation by delegating, following Proposition 1, the payment rule must be $x(a_U, a_D; \tilde{\theta}_D) = G(a_U, a_D) + \ell(a_U, \theta_U) + b(\theta_U)$, where $b(\theta_U)$ is defined in Proposition 1 and $\ell(a_U, \theta_U)$ as

$$\ell(a_U, \theta_U) = -(1-\lambda_U)\frac{F_U(\theta_U)}{f_U(\theta_U)} C^U_{\theta_U}(a_U, \theta_U).$$

Then,

$$\begin{aligned}\mathbb{E}_\theta[x(\hat{a}_U, \hat{a}_D; \theta_U)] =& \mathbb{E}_\theta \Bigg[G(\hat{a}_U, \hat{a}_D) + \ell(\hat{a}_U, \theta_U) + \int_{\theta_U}^{\bar{\theta}_U} C^U_{\theta_U}(a_U(s, \theta_D), s) ds + C^U(\hat{a}_U, \theta_U) \\ & - G(\hat{a}_U, \hat{a}_D) - \ell(\hat{a}_U, \tilde{\theta}_U) - h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D) \Bigg] \\ =& \mathbb{E}_\theta \left[\int_{\theta_U}^{\bar{\theta}_U} C^U_{\theta_U}(\hat{a}_U(s, \theta_D), s) ds + C^U(\hat{a}_U, \theta_U) - h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D) \right] \\ =& \mathbb{E}_\theta \left[h^{U,0}(\hat{a}_U, \theta_U) - h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D) \right].\end{aligned}$$

Given that $T^{DU}(\theta_U, \theta_D) = P(\hat{a}_D, \theta_D) - C^D(\hat{a}_U, \hat{a}_D) - \frac{1-F_D(\theta_D)}{f_D(\theta_D)} P_{\theta_D}(\hat{a}_D, \theta_D) \cdot \hat{a}_D$, the surplus that generates this payment rule is given by:

$$\begin{aligned}&\mathbb{E}_\theta \Bigg[V(\hat{a}_D, \theta_D) - x(\hat{a}_U, \hat{a}_D, \theta_D) + \lambda_U \Big(x(\hat{a}_U, \hat{a}_D, \theta_D) - C^U(\hat{a}_U, \theta_U) + T^{DU}(\theta_U, \theta_D) \Big) \\ &\quad + \lambda_D \Big(P(\hat{a}_D, \theta_D) - C^D(\hat{a}_U, \hat{a}_D) - T^{DU}(\theta_U, \theta_D) \Big) \Bigg] \\ &\quad = \mathbb{E}_\theta \Bigg[V(\hat{a}_D, \theta_D) - h^{U,0}(\hat{a}_U, \theta_U) + h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D) + \lambda_U \Big(h^{U,0}(\hat{a}_U, \theta_U) - h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D) \\ &\quad - C^U(\hat{a}_U, \theta_U) + P(\hat{a}_D, \theta_D) - C^D(\hat{a}_U, \hat{a}_D) - \frac{1 - F_D(\theta_D)}{f_D(\theta_D)} P_{\theta_D}(\hat{a}_D, \theta_D) \cdot \hat{a}_D \Big) \\ &\quad + \lambda_D \Big(P(\hat{a}_D, \theta_D) - C^D(\hat{a}_U, \hat{a}_D) - P(\hat{a}_D, \theta_D) + C^D(\hat{a}_U, \hat{a}_D) + \frac{1 - F_D(\theta_D)}{f_D(\theta_D)} P_{\theta_D}(\hat{a}_D, \theta_D) \cdot \hat{a}_D \Big) \Bigg] \\ &\quad = \mathbb{E}_\theta \Bigg[V(\hat{a}_D, \theta_D) - h^{U,0}(\hat{a}_U, \theta_U) + h^{D,0}(\hat{a}_U, \hat{a}_D, \theta_D) + \lambda_U \cdot \frac{F_U(\theta_U)}{f_U(\theta_U)} C^U_{\theta_U}(\hat{a}_U, \theta_U) \\ &\quad + \lambda_D \frac{1 - F_D(\theta_D)}{f_D(\theta_D)} P_{\theta_D}(\hat{a}_D, \theta_D) \cdot \hat{a}_D \Bigg] \\ &\quad = \mathbb{E}_\theta \Bigg[V(\hat{a}_D, \theta_D) - h^{U,\lambda_U}(\hat{a}_U, \theta_U) + h^{D,\lambda_D}(\hat{a}_U, \hat{a}_D, \theta_D) \Bigg]\end{aligned}$$

Proof of Proposition 3

In the benchmark case, the principal solves the problem (5); therefore, the optimal production assignment solves

$$\begin{aligned} \frac{\partial V(a_D,\theta_D)}{\partial a_D} + \frac{\partial h^{D,\lambda_D}(a_U,a_D,\theta_D)}{\partial a_D} &= 0 \\ -\frac{\partial h^{U,\lambda_U}(a_U,a_D,\theta_D)}{\partial a_U} + \frac{\partial h^{D,\lambda_D}(a_U,a_D,\theta_D)}{\partial a_U} &= 0. \end{aligned} \tag{16}$$

Note that,

$$\begin{aligned} \frac{\partial h^{D,\lambda_D}(a_U,a_D,\theta_D)}{\partial a_D} =& \frac{\partial h^{D,0}(a_U,a_D,\theta_D)}{\partial a_D} + \lambda_D \frac{1-F_D(\theta_D)}{f_D(\theta_D)} \frac{\partial}{\partial\theta_D \partial a_D}(P(a_D,\theta_D)\cdot a_D), \\ \frac{\partial h^{D,\lambda_D}(a_U,a_D,\theta_D)}{\partial a_U} =& \frac{\partial h^{D,0}(a_U,a_D,\theta_D)}{\partial a_U}. \end{aligned}$$

As the upstream firm uses a downstream mechanism with the downstream firm, therefore the problem (6) is equivalent to $\max_{a_U,a_D} \mathbb{E}_{\theta_D}[x(a_U,a_D,\tilde{\theta}_U) - C^U(a_U,\theta_U) + h^{D,0}(a_U,a_D,\theta_D)]$, and the optimal actions (a_U,a_D) solves

$$\begin{aligned} \frac{\partial x(a_U,a_D,\tilde{\theta}_U)}{\partial a_D} + \frac{\partial h^{D,0}(a_U,a_D,\theta_D)}{\partial a_D} &= 0, \\ \frac{\partial x(a_U,a_D,\tilde{\theta}_U)}{\partial a_U} - \frac{\partial C^U(a_U,\theta_U)}{\partial a_U} + \frac{\partial h^{D,0}(a_U,a_D,\theta_D)}{\partial a_U} &= 0. \end{aligned} \tag{17}$$

Therefore, if the delegation contract $x(a_U,a_D,\tilde{\theta}_U)$ induces the optimal actions of the benchmark case, it should be true that $\forall(\theta_U,\theta_D)$

$$\begin{aligned} \frac{\partial x(a_U,a_D,\tilde{\theta}_U)}{\partial a_D} =& \frac{\partial V(a_D,\theta_D)}{\partial a_D} + \lambda_D \frac{1-F_D(\theta_D)}{f_D(\theta_D)} \frac{\partial}{\partial\theta_D \partial a_D}(P(a_D,\theta_D)\cdot a_D), \\ \frac{\partial x(a_U,a_D,\tilde{\theta}_U)}{\partial a_U} =& -\frac{\partial h^{U,\lambda_U}(a_U,a_D,\theta_D)}{\partial a_U} + \frac{\partial C^U(a_U,\theta_U)}{\partial a_U} \end{aligned} \tag{18}$$

For simplicity, we call $H(a_D, \theta_D, \lambda_D) = V(a_D, \theta_D) - \lambda_D \frac{1-F_{\theta_D}(\theta_D)}{f_{\theta_D}(\theta_D)}(P_{\theta_D}(a_D, \theta_D) \cdot a_D)$, then, deriving (18) with respect to θ_D we have that,

$$\frac{\partial^2 x(a_U, a_D, \tilde{\theta}_U)}{\partial a_D^2}\frac{\partial a_D}{\partial \theta_D} = \frac{\partial^2 H(a_D, \theta_D, \lambda_D)}{\partial a_D^2}\frac{\partial a_D}{\partial \theta_D} + \frac{\partial^2 H(a_D, \theta_D, \lambda_D)}{\partial a_D \partial \theta_D}$$

$$\frac{\partial a_D}{\partial \theta_D}\left(\frac{\partial^2 x(a_U, a_D, \tilde{\theta}_U)}{\partial a_D^2} - \frac{\partial^2 H(a_D, \theta_D, \lambda_D)}{\partial a_D^2}\right) = \frac{\partial^2 H(a_D, \theta_D, \lambda_D)}{\partial a_D \partial \theta_D} \tag{19}$$

the expression in the parenthesis in equation (19) is equal to zero by (18). In addition, $\partial a_D/\partial \theta_D \neq 0$, in order to induce different action given different θ_D reports, equation (19) is true iff $\partial^2 H(a_D, \theta_D, \lambda_D)/\partial a_D \partial \theta_D = 0$.

Proof of Proposition 4

Consider a simple linear incentive $G(a_U, a_D) = s \cdot a_D$, with $s \geq 0$, and let $a_U^s(\theta_U, \theta_D)$ and $a_D^s(\theta_U, \theta_D)$ be the allocations that induce the upstream firm with this payment scheme. From the FOC of problem (6), we have that $\partial h^{D,0}/\partial a_D^s = 0$ when we evaluate in $s = 0$. Deriving the previous equation with respect to s, we have that $\partial a_D^s/\partial s > 0$. From proposition 2,

$$\mathbb{E}_\theta[SW(a_U^s(\cdot), a_D^s(\cdot), \theta_U, \theta_D, s(\cdot), 0, T^{DU}(\cdot))] = \mathbb{E}_\theta\left[V(a_D^s, \theta_D) - h^{U,\lambda_U}(a_U^s, \theta_U) + h^{D,0}(a_U^s, a_D^s, \theta_D)\right].$$

Therefore,

$$\begin{aligned}\frac{\partial}{\partial s}\mathbb{E}_\theta\left[V(a_D^s, \theta_D) - h^{U,\lambda_U}(a_U^s, \theta_U) + h^{D,0}(a_U^s, a_D^s, \theta_D)\right] &= \mathbb{E}_\theta\left[\left(\frac{\partial V(a_D^s, \theta_D)}{\partial a_D^s} + \frac{\partial h^{D,0}(a_U^s, a_D^s, \theta_D)}{\partial a_D^s}\right)\frac{\partial a_D^s}{\partial s}\right] \\ &= \mathbb{E}_\theta\left[\left(\frac{\partial V(a_D^s, \theta_D)}{\partial a_D^s}\right)\frac{\partial a_D^s}{\partial s}\right] > 0\end{aligned}$$

given that $V(\cdot)$ is increasing in a_D^s.

Proof of Proposition 5

By proposition 2, the social welfare given by the payment rule $x^s(a_U, a_D, \theta_U) = s \cdot a_D - (1 - \lambda_U)\frac{F_U(\theta_U)}{f_U(\theta_U)}C_{\theta_U}^U(a_U, \theta_U) + b(\theta)$ is:

$$\mathbb{E}_\theta\Bigg[V(a_D^s, \theta_D) - h^{U,\lambda_U}(a_U^s, \theta_U) + h^{D,0}(a_U^s, a_D^s, \theta_D)\Bigg]$$

Then, the optimal subsidy in given by:

$$\begin{aligned}
0 =&\frac{\partial}{\partial s}\mathbb{E}_\theta\left[V(a_D^s,\theta_D) - h^{U,\lambda_U}(a_U^s,\theta_U) + h^{D,0}(a_U^s,a_D^s,\theta_D)\right]\\
=&\mathbb{E}_\theta\left[\frac{\partial}{\partial s}V(a_D^s,\theta_D) - \frac{\partial}{\partial s}h^{U,\lambda_U}(a_U^s,\theta_U) + \frac{\partial}{\partial s}h^{D,0}(a_U^s,a_D^s,\theta_D)\right]\\
=&\mathbb{E}_\theta\left[\left(\frac{\partial}{\partial a_D}V(a_D^s,\theta_D) + \frac{\partial}{\partial a_D}h^{D,0}(a_U^s,a_D^s,\theta_D)\right)\frac{\partial a_D^s}{\partial s} - \underbrace{\left(\frac{\partial}{\partial a_U}h^{U,\lambda_U}(a_U^s,\theta_U) - \frac{\partial}{\partial a_U}h^{D,0}(a_U^s,a_D^s,\theta_D)\right)\frac{\partial a_U^s}{\partial s}}_{=0 \text{ by } (6)}\right]\\
=&\mathbb{E}_\theta\left[\left(\frac{\partial}{\partial a_D}V(a_D^s,\theta_D) + \frac{\partial}{\partial a_D}h^{D,0}(a_U^s,a_D^s,\theta_D)\right)\frac{\partial a_D^s}{\partial s}\right]\\
=&\mathbb{E}_\theta\left[\left(P(a_D^s,\theta_D) - \frac{\partial C^D(a_U^s,a_D^s)}{\partial a_D} - \frac{1-F_D(\theta_D)}{f_D(\theta_D)}(P_{\theta_D,a_D}(a_D^s,\theta_D)a_D^s + P_{\theta_D}(a_D^s,\theta_D)\right)\frac{\partial a_D}{\partial s}\right]
\end{aligned}$$

6.0.1 Proof of lemma 5

For any s

$$\begin{aligned}
\frac{\partial a_D^*(\theta_U,\theta_D)}{\partial\theta_D} &\geq \frac{\partial a_D^s(\theta_U,\theta_D)}{\partial\theta_D}\\
\frac{\phi(\theta_U,\lambda_U)\cdot[A-c+\phi(\theta_D,\lambda_D)]}{B\phi(\theta_U,\lambda_U)-d^2} &\geq \frac{\phi(\theta_U,\lambda_U)\cdot[s+A-c+\phi(\theta_D,0)]}{2B\phi(\theta_U,\lambda_U)-d^2}\\
\frac{[A-c+\phi(\theta_D,\lambda_D)]}{B\phi(\theta_U,\lambda_U)-d^2} &\geq \frac{[s+A-c+\phi(\theta_D,0)]}{2B\phi(\theta_U,\lambda_U)-d^2}\\
[A-c+\phi(\theta_D,\lambda_D)]\cdot(2B\phi(\theta_U,\lambda_U)-d^2) &\geq [s+A-c+\phi(\theta_D,0)]\cdot(B\phi(\theta_U,\lambda_U)-d^2)\\
2B\phi(\theta_U,\lambda_U)(1-\lambda_D)+\lambda_D d^2 &\geq 0
\end{aligned}$$

which is true for any λ_D.

Proof of Proposition 6

First, note that using FOC conditions, the social welfare can be written as a function of a_D. The principal chooses s such that

$$s \in \arg\max \mathbb{E}_\theta\left[\frac{K(a_U^s,a_D^s,\theta_U,\theta_D;\lambda_U,0)}{K(a_U^*,a_D^*,\theta_U,\theta_D;\lambda_U,0)}\right]$$

Using Taylor's approximation,

$$
\begin{aligned}
K(a_U^s, a_D^s, \theta_U, \theta_D; \lambda_U, 0) =& K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D) + \frac{1}{2}\frac{\partial^2 K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)}{\partial a_D^2}(\Delta a_D(\theta_U, \theta_D))^2 \\
\frac{K(a_U^s, a_D^s, \theta_U, \theta_D; \lambda_U, 0)}{K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)} =& 1 + \frac{1}{2}\frac{1}{K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)}\frac{\partial^2 K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)}{\partial a_D^2}(\Delta a_D(\theta_U, \theta_D))^2 \\
1 - \frac{K(a_U^s, a_D^s, \theta_U, \theta_D; \lambda_U, 0)}{K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)} =& \frac{1}{2}\frac{1}{K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)}\left|\frac{\partial^2 K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)}{\partial a_D^2}\right|(\Delta a_D(\theta_U, \theta_D))^2
\end{aligned}
$$

where $\Delta a_D(\theta_U, \theta_D) = a_D^*(\theta_U, \theta_D) - a_D^s(\theta_U, \theta_D)$, and

$$
\begin{aligned}
\frac{\partial^2 K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, \lambda_D)}{\partial a_D^2} &= -\left(\frac{B\phi(\theta_U, \lambda_U) - d^2}{\phi(\theta_U, \lambda_U)}\right) \\
(a_D^*(\theta_U, \theta_D) - a_D^s(\theta_U, \theta_D)) &= \frac{\phi(\theta_U, \lambda_U) \cdot [A - c + \phi(\theta_D, 0)]}{B\phi(\theta_U, \lambda_U) - d^2} - \frac{\phi(\theta_U, \lambda_U) \cdot [s + A - c + \phi(\theta_D, 0)]}{2B\phi(\theta_U, \lambda_U) - d^2}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
s &\in \arg\min \mathbb{E}_\theta \left[1 - \frac{K(a_U^s, a_D^s, \theta_U, \theta_D; \lambda_U, 0)}{K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, 0)}\right] \\
s &\in \arg\min \mathbb{E}_\theta \left[\left(\frac{B\phi(\theta_U, \lambda_U)}{2B\phi(\theta_U, \lambda_U) - d^2} - \frac{1}{A - c + \phi(\theta_D, 0)} \cdot \frac{B\phi(\theta_U, \lambda_U) - d^2}{2B\phi(\theta_U, \lambda_U) - d^2}s\right)^2\right]
\end{aligned}
$$

that can be rewritten as, $\min_s \quad \mathbb{E}_\theta\left[(f(\theta) - g(\theta)s)^2\right]$. Let f and g two functions of $\theta = (\theta_U, \theta_D)$ such that,

$$
\begin{aligned}
f(\theta) =& \frac{B\phi(\theta_U, \lambda_U)}{2B\phi(\theta_U, \lambda_U) - d^2} \\
g(\theta) =& \frac{1}{A - c + \phi(\theta_D, 0)} \cdot \frac{B\phi(\theta_U, \lambda_U) - d^2}{2B\phi(\theta_U, \lambda_U) - d^2}
\end{aligned}
$$

Then the optimal price is $s^* = \mathbb{E}_\theta[f(\theta)g(\theta)]/\mathbb{E}_\theta[g(\theta)^2]$. Rearranging the expression, we have that,

$$
\begin{aligned}
R|_{\lambda_D = 0} = 1 - \mathbb{E}_\theta\left[\frac{K(a_U^s, a_D^s, \theta_U, \theta_D; \lambda_U, 0)}{K(a_U^*, a_D^*, \theta_U, \theta_D; \lambda_U, 0)}\right] &= \mathbb{E}_\theta\left[f(\theta)^2\right] - \frac{(\mathbb{E}_\theta[f(\theta)g(\theta)])^2}{\mathbb{E}_\theta\left[(g(\theta))^2\right]} \\
&= \frac{1}{2}\left(r(B, d, \lambda_U) + h(B, d, \lambda_U)\right) - \frac{\left(r(B, d, \lambda_U) + \frac{d^4}{(2B - d^2)(-2(3 - \lambda_U)B + d^2}\right)^2 z(A - c)}{8(r(B, d, \lambda_U) - h(B, d, \lambda_U))}
\end{aligned}
$$

where,

$$r(B,d,\lambda_U) = \frac{2(3-\lambda_U)B^2 - (4-\lambda_U)Bd^2 + d^4}{(2B-d^2)\left(2(3-\lambda_U)B - d^2\right)} = \frac{2(3-\lambda_U)B^2 - (4-\lambda_U)Bd^2 + d^4}{4(3-\lambda_U)B^2 - 2(4-\lambda_U)Bd^2 + d^4}$$
$$h(B,d,\lambda_U) = \frac{1}{2} \cdot \frac{d^2}{(2-\lambda_U)B} \ln\left(\frac{2(3-\lambda_U)B - d^2}{2B-d^2}\right)$$
$$z(A-c) = (A-c+1)(A-c-1)\ln\left(\frac{A-c+1}{A-c-1}\right)^2$$

Note that $z(\cdot)$ is a increasing function, therefore $R|_{\lambda_D=0}$ is a decreasing function of $A-c$ if, $(r(B,d,\lambda_U) - h(B,d,\lambda_U)) > 0$, or equivalent $r(B,d,\lambda_U) > h(B,d,\lambda_U)$. On one side,

$$\begin{aligned} h(B,d,\lambda_U) &= \frac{1}{2} \cdot \frac{d^2}{(2-\lambda_U)B} \ln\left(\frac{2(3-\lambda_U)B - d^2}{2B-d^2}\right) \\ &< \frac{d^2}{2(2-\lambda_U)B}\left(\frac{2(3-\lambda_U)B - d^2}{2B-d^2} - 1\right) \\ &= \frac{d^2}{2(2-\lambda_U)B}\left(\frac{2(2-\lambda_U)B}{2B-d^2}\right) \\ &= \frac{d^2}{2B-d^2} \\ &< \frac{1}{2}. \end{aligned}$$

On the other hand, $r(B,d,\lambda_U)$ is increasing in d, $\frac{\partial r}{\partial d} = \frac{2BD^3\left(4(3-\lambda_U)B-(4-\lambda_U)d^2\right)}{(2B-d^2)^2(2(3-\lambda_U)B-d^2)^2} > 0$. Then,

$$r(B,d,\lambda_U) \geq r(B,0,\lambda_U) = \frac{1}{2}$$

Hence, $\frac{\left(r(B,d,\lambda_U) - \frac{d^4}{(2B-d^2)(2(3-\lambda_U)B-d^2)}\right)^2}{8(r(B,d,\lambda_U)-h(B,d,\lambda_U))} > 0$, then $R|_{\lambda_D=0}$ is decreasing in z. Then

$$\begin{aligned} R|_{\lambda_D=0,d=0} &= \frac{1}{2}\left(r(B,0,\lambda_U) + h(B,0,\lambda_U)\right) - \frac{\left(r(B,0,\lambda_U)\right)^2 z(A-c)}{8(r(B,0,\lambda_U) - h(B,0,\lambda_U))} \\ &= \frac{1}{2}\left(\frac{1}{2}\right) - \frac{(1/2)^2 \cdot z(A-c)}{8 \cdot \left(\frac{1}{2}\right)} = \frac{1}{4}\left[1 - \frac{z(A-c)}{4}\right], \end{aligned}$$

and

$$
\begin{aligned}
R|_{\lambda_D=0,d=\sqrt{B}} &= \frac{1}{2}\left(r(B,\sqrt{B},\lambda_U)+h(B,\sqrt{B},\lambda_U)\right) - \frac{\left(r(B,\sqrt{B},\lambda_U)-\frac{1}{5-2\lambda_U}\right)^2 z(A-c)}{8(r(B,\sqrt{B},\lambda_U)-h(B,\sqrt{B},\lambda_U))} \\
&= \frac{1}{2}\left(\frac{3-\lambda_U}{5-2\lambda_U}+\frac{1}{2(2-\lambda_U)}\ln(5-2\lambda_U)\right) - \frac{\left(\frac{3-\lambda_U}{5-2\lambda_U}-\frac{1}{5-2\lambda_U}\right)^2 z(A-c)}{8\left(\frac{3-\lambda_U}{5-2\lambda_U}-\frac{1}{2(2-\lambda_U)}\ln(5-2\lambda_U)\right)} \\
&\leq \frac{1}{2}\left(\frac{2}{3}+\frac{1}{4}\ln(3)\right) - \frac{1}{72\left(\frac{1}{3}+\frac{1}{4}\ln(3)\right)} z(A-c) \\
&= 0.61 - 0.118 \cdot z(A-c)
\end{aligned}
$$

Appendix B. Extension to many downstream firms

In this Section, we extend the model to consider that the upstream firm deals with n downstream firms that choose a quantity a_D^i to be sold in the market. We denote the vector of quantities as $\hat{a}_D = (a_D^1, ..., a_D^n)$. The cost of firm i is given by $C^{Di}(a_U, a_D^i)$ with $i = 1, ..., n$. Also, each firm i faces an inverse demand $P^i(a_D^i, \theta_D^i)$, where $\theta_D^i \in \Theta_D^i \equiv [\underline{\theta}_D^i, \bar{\theta}_D^i]$ is private information of firm i and is distributed according to F_D^i. We denote the profile of private information of downstream firms as $\hat{\theta}_D = (\theta_D^1, ..., \theta_D^n)$.

We make standard cost and demand function assumptions that are analogous to assumption 1 (ii)–(iv).

Assumption 4 *The following are true:*

(i) Firm i's marginal costs are positive and upstream investment decreases downstream costs and marginal costs: $C^{Di}_{a_D^i} > 0$, $C^{Di}_{a_U} < 0$, *and* $C^{Di}_{a_U a_D^i} < 0$.

(ii) Each demand is downward sloping and θ_D^i *represents higher demand:* $P^i_{a_D^i} < 0$ *and* $P^i_{\theta_D^i} > 0$.

(iii) Firm i's problem is concave: $P^i_{a_D^i}(a_D^i, \theta_D^i) < C^{Di}_{a_D^i a_D^i}(a_D^i, a_U)$.

The planner values the sum of consumer surplus generated in each market; given a_D^i and θ_D^i, we denote this consumer surplus as $\bar{V}(\hat{a}_D, \hat{\theta}_D) \equiv \sum_{i=1}^n \int_0^{a_D^i} P^i(x, \theta_D^i)dx - P^i(a_D^i, \theta_D^i)a_D^i$. Also, the planner weights i's profits according to $0 \leq \lambda_D^i \leq 1$. Therefore, the social welfare generated by investment and productions $(a_U, \hat{a}_D)$, private information $(\theta_U, \hat{\theta}_D)$ and transfers (T^U, T^{Di}, T^{DUi}) is given by,

$$
\begin{aligned}
SW(a_U, \hat{a}_D, \theta_U, \hat{\theta}_D, T^U, T^{Di}, T^{DUi}) =& \bar{V}(\hat{a}_D, \hat{\theta}_D) - T^U + \sum_{i=1}^n T^{Di} + \lambda_U \left[T^U - C^U(a_U, \theta_U) + T^{DU}\right] \\
&+ \sum_{i=1}^n \lambda_D^i \left[P^i(a_D^i, \theta_D^i) \cdot a_D^i - C^{Di}(a_U, a_D^i) - T^{Di} - T^{DUi}\right]
\end{aligned}
$$

Analogously to the monopoly case, in the centralized problem, there is no transfer from downstream firms to upstream firms, i.e., $T^{DUi} = 0 \; \forall i = 1, ..., n$.

Now, we redefine $\theta = (\theta_U, \hat{\theta}_D)$ and $\theta_{-i} = (\theta_U, \theta_D^1, ..., \theta_D^{i-1}, \theta_D^{i+1}, \theta_D^n)$. If i's type is θ_D^i and it reports $\tilde{\theta}_D^i$, its profits are given by:

$$
\pi^{Di}(\tilde{\theta}_D^i, \theta_D^i) = \mathbb{E}_{\theta_{-i}} \left[P^i(a_D^i(\theta_{-i}, \tilde{\theta}_D^i), \theta_D^i) \cdot a_D^i(\theta_{-i}, \tilde{\theta}_D^i) - C^{Di}(a_D^i(\theta_{-i}, \tilde{\theta}_D^i), a_U(\theta_{-i}, \tilde{\theta}_D^i)) - T^{Di}(\theta_{-i}, \tilde{\theta}_D^i)\right],
$$

If U's type is θ_U and reports $\tilde{\theta}_U$, its profits are given by:

$$
\pi^U(\tilde{\theta}_U, \theta_U) = \mathbb{E}_{\hat{\theta}_D} \left[T^U(\tilde{\theta}_U, \hat{\theta}_D) - C^U(a_U(\tilde{\theta}_U, \hat{\theta}_D), \theta_U)\right]
$$

In this context, a mechanism is incentive-compatible iff:

(i) $\mathbb{E}_{\hat{\theta}_D}[a_U(\theta_U, \hat{\theta}_D)]$ is decreasing in θ_U and $\Pi^{U\prime}(\theta_U) = \mathbb{E}_{\hat{\theta}_D}\left[-C^U_{\theta_U}(a_U(\theta_U, \hat{\theta}_D), \theta_U)\right]$

(ii) $\mathbb{E}_{\theta_{-i}}[a^i_D(\theta_U, \hat{\theta}_D)]$ is increasing in θ^i_D and $\Pi^{D\prime}(\theta_D) = \mathbb{E}_{\theta_{-i}}\left[P^i_{\theta^i_D}(a^i_D(\theta_U, \hat{\theta}_D), \theta^i_D) \cdot a^i_D(\theta_U, \hat{\theta}_D)\right]$

Then the planner's problem can be written as:

$$\max_{\substack{a_U(\theta_U,\hat{\theta}_D), T^U(\theta_U,\hat{\theta}_D), \\ a^i_D(\theta_U,\hat{\theta}_D), T^{Di}(\theta_U,\hat{\theta}_D)}} \mathbb{E}_\theta\left[\bar{V}(a_D, \hat{\theta}_D) - T^U(\theta_U, \hat{\theta}_D) + \sum_{i=1}^n T^{Di}(\theta_U, \hat{\theta}_D) + \lambda_U \Pi^U(\theta_U) + \sum_{i=1}^n \lambda^i_D \Pi^{Di}(\theta^i_D)\right]$$

subject to:

$\mathbb{E}_{\hat{\theta}_D}[a_U(\theta_U, \hat{\theta}_D)]$ is decreasing in θ_U and $\Pi^{U\prime}(\theta_U) = \mathbb{E}_{\hat{\theta}_D}\left[-C^U_{\theta_U}(a_U(\theta_U, \hat{\theta}_D), \theta_U)\right]$
$\mathbb{E}_{\theta_{-i}}[a^i_D(\theta_U, \hat{\theta}_D)]$ is increasing in θ^i_D and $\Pi^{D\prime}(\theta_D) = \mathbb{E}_{\theta_{-i}}\left[P^i_{\theta^i_D}(a^i_D(\theta_U, \hat{\theta}_D), \theta^i_D) \cdot a^i_D(\theta_U, \hat{\theta}_D)\right]$
$\Pi^i(\theta_i) \geq 0, i \in \{U, D_1, ..., D_n\}$

So, the welfare-maximizing centralized allocation $(a^*_U(\theta_U, \hat{\theta}_D), \hat{a}^*_D(\theta_U, \hat{\theta}_D))$ satisfies,

$$(a^*_U(\theta_U, \hat{\theta}_D), \hat{a}^*_D(\theta_U, \hat{\theta}_D)) \in \underset{a_U, a_D}{\operatorname{argmax}}\, \mathbb{E}_\theta\left[\bar{V}(\hat{a}_D, \hat{\theta}_D) - h^{U,\lambda_U}(a_U, \theta_U) + \sum_{i=1}^n h^{Di,\lambda^i_D}(a_U, \hat{a}_D, \hat{\theta}_D)\right]$$

where,

$$h^{U,\lambda_U}(a_U, \theta_U) = C^U(a_U, \theta_U) + (1 - \lambda_U)\frac{F_U(\theta_U)}{f_U(\theta_U)} C^U_{\theta_U}(a_U, \theta_U)$$
$$h^{Di,\lambda^i_D}(a_U, a^i_D, \theta^i_D) = P^i(a^i_D, \theta^i_D) \cdot a^i_D - C^{Di}(a^i_D, a_U) - (1 - \lambda^i_D)\frac{1 - F^i_D(\theta^i_D)}{f^i_D(\theta^i_D)} P^i_{\theta^i_D}(a^i_D, \theta^i_D) \cdot a^i_D$$

In this setting, a delegation contract is given a payment rule $x(a_U, \hat{a}_D; \tilde{\theta}_U)$, where $\tilde{\theta}_U$ is the report of the upstream firm. Therefore, the delegation contract can be written as $x(a_U, \hat{a}_D; \tilde{\theta}_U) = G(a_U, \hat{a}_D) + \ell(a_U, \theta_U) + b(\theta_U)$.

Proposition 7 *The centralized allocation can be implemented through delegation iff the function* $\bar{H}(\hat{a}_D, \hat{\theta}_D) = \bar{V}(\hat{a}_D, \hat{\theta}_D) - \sum_{i=1}^n \lambda^i_D \frac{1-F^i_{\theta^i_D}(\theta^i_D)}{f^i_{\theta^i_D}(\theta_D)} P^i_{\theta^i_D}(a^i_D, \theta^i_D) \cdot a^i_D$ *satisfies* $\frac{\partial^2 \bar{H}(\hat{a}_D, \hat{\theta}_D)}{\partial a^i_D \partial \theta^i_D} = 0$.

Proof. Analogous to the case with one downstream firm, the delegation mechanism can implement the centralized allocation iff:

$$\frac{\partial x(a_U, \hat{a}_D; \theta_U)}{\partial a^i_D} = \frac{\partial \bar{H}(\hat{a}_D, \hat{\theta}_D)}{\partial a^i_D} \quad \forall i. \tag{20}$$

Taking the derivative of (20) with respect to θ^i_D,

$$\frac{\partial^2 x(a_U, \hat{a}_D; \theta_U)}{\partial a_D^i \partial \theta_D^i} = \frac{\partial}{\partial \theta_D^i} \left(\frac{\partial \bar{H}(\hat{a}_D, \hat{\theta}_D)}{\partial a_D^i} \right)$$

$$\frac{\partial a_D^i}{\partial \theta_D^i} \left(\frac{\partial^2 x(a_U, \hat{a}_D; \theta_U)}{\partial^2 a_D^i} - \frac{\partial^2 \bar{H}(\hat{a}_D, \hat{\theta}_D)}{a_D^{i2}} \right) = \frac{\partial^2 \bar{H}(\hat{a}_D, \hat{\theta}_D)}{\partial a_D^i \partial \theta_D^i}$$

the expression in the parentheses is equal to zero due to (20), therefore, the above equality holds only if $\frac{\partial^2 \bar{H}(\hat{a}_D, \hat{\theta}_D)}{\partial a_D^i \partial \theta_D^i} = 0$. ■

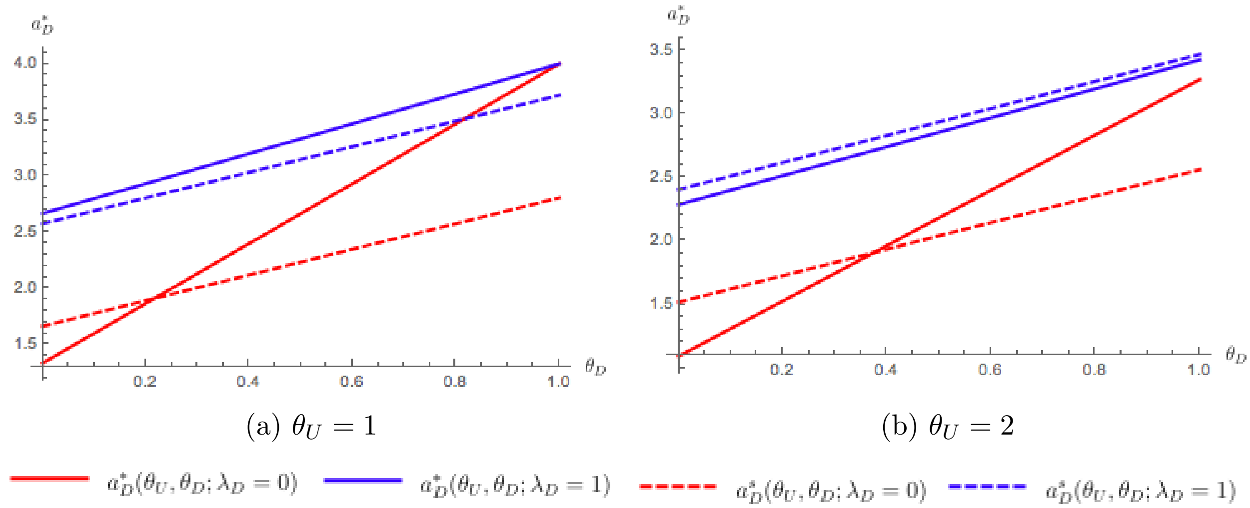
Appendix C. Numerical Analysis

Result 1 *Delegation regret is decreasing in λ_D.*

The intuition of this result (which we derive numerically) is as follows. It is easy to verify that $\partial a_D^*(\theta_U, \theta_D)/\partial \theta_D > 0$, more importantly, $\partial^2 a_D^*(\theta_U, \theta_D)/\partial \lambda_D \partial \theta_D < 0$. Therefore the sensitivity of $a_D^*(\theta_U, \theta_D)$ to θ_D increases as the principal puts more value on the downstream firm's profits. As the principal cares more, informational rents become less important, and he resorts less to distortions. On the other hand, $\partial a_D^s/\partial \theta_D$ does not depend on s. This is so because linear incentives leave the informational rents provided by U to D unaltered. Moreover, $a_D(s)$ does not depend on λ_D. Finally, it is easy to see that $\partial a_D^s/\partial \theta_D < \partial a_D^*/\partial \theta_D$. Therefore as the principal cares more about the downstream firm, the allocations of both mechanisms become more similar and then, the regret decreases in λ_D.

To illustrate the performance of the linear contract, we use numerical analysis with the following parameters as a baseline: $A = 4$, $B = 1$, $c = 2$ and $d = 0.5$. Then, $P(a_D, \theta_D) = 4 + \theta_D - a_D$, $C^U(\theta_U, a_U) = \frac{1}{2}\theta_U a_U^2$, $C^D(a_D, a_U) = (2 - 0.5a_U) \cdot a_D$ and the distributions are $\theta_D \sim U[0, 1]$ and $\theta_U \sim U[1, 2]$.

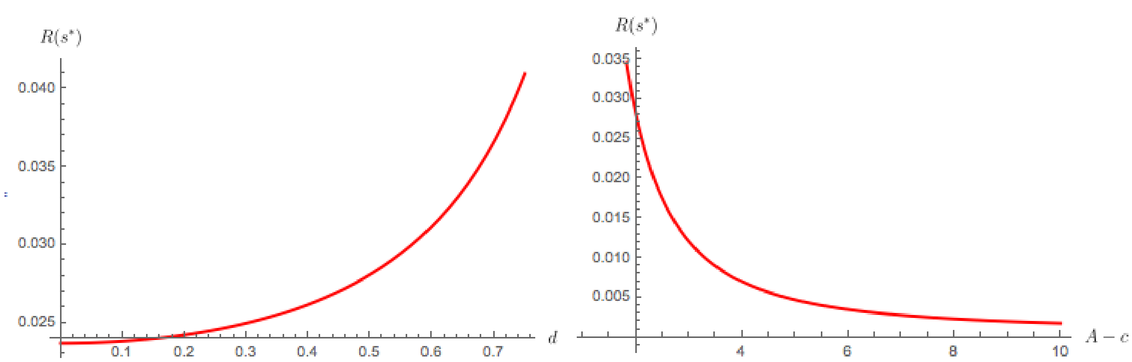
First, we compare the allocations a_D induced by the centralized and delegation mechanisms. In Figure (3), red lines represent the allocation when $\lambda_D = 0$ and blue lines when $\lambda_D = 1$. The dashed lines correspond to the allocation implemented by the linear contract and the solid line by the centralized mechanism. Panels (a) and (b) display the allocations when $\theta_U = 1$ and $\theta_U = 2$, respectively.



(a) $\theta_U = 1$ (b) $\theta_U = 2$

Figure 3: Downstream allocations.

Figure 4 illustrates the size of the delegation regret when $\lambda_D = 0$. Panel (a) shows how the linear delegation regret increases as the upstream and downstream firm problems are more connected via d, while panel (b) shows how it decreases with the market size $(A - c)$.



(a) Different values of D. (b) Different values of $A - C$.

Figure 4: Comparative statics of the linear delegation regret

Finally, we numerically study the behavior of the linear regret as the misalignment between the planner and U regarding d's profit increases, i.e., as λ_D increases. Figure 5 shows that the delegation regret decreases significantly when the planner values the profits of the downstream firm more.

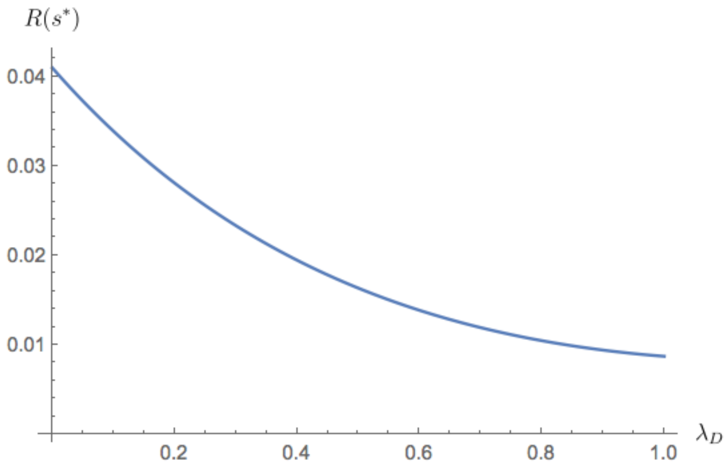


Figure 5: Delegation regret for the baseline case and $\lambda_D > 0$

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