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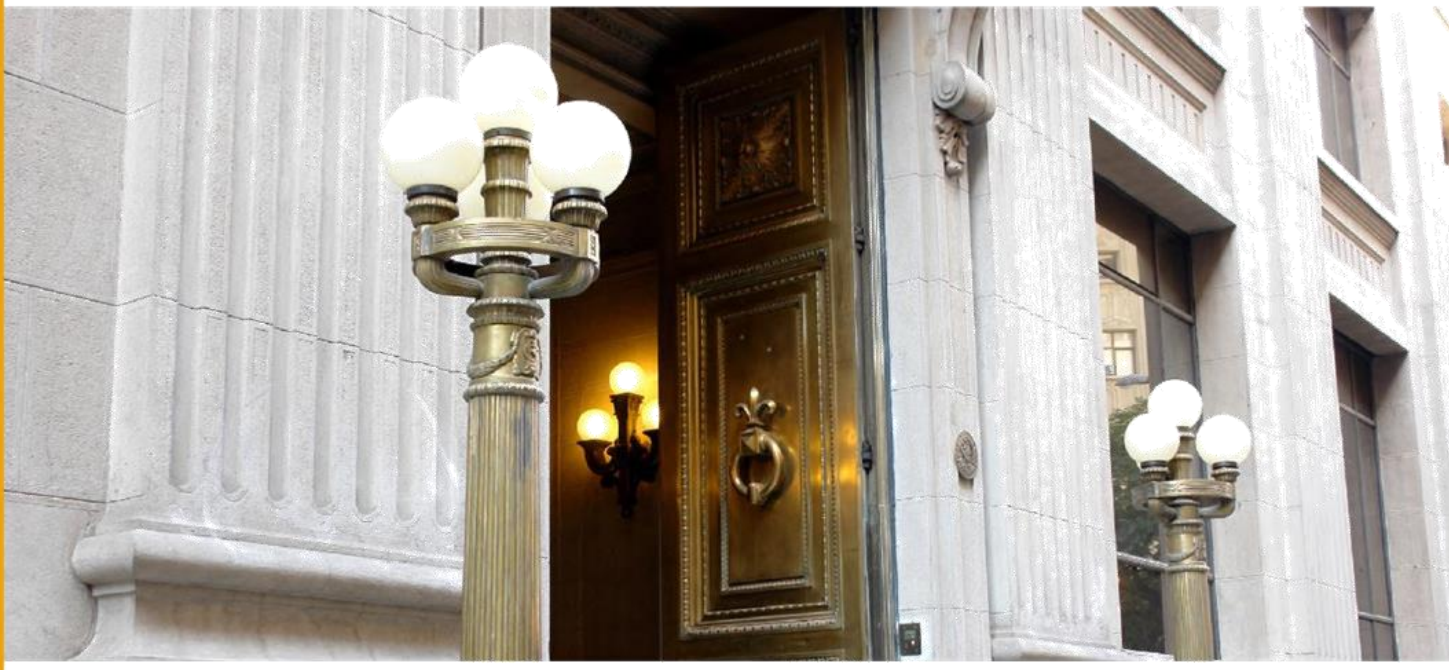

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Sovereign Wealth Funds and Optimal Foreign Reserves*

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Resumen

Este trabajo estudia la interacción entre los fondos soberanos (SWF) y la acumulación de reservas internacionales en economías abiertas vulnerables a crisis financieras. Una autoridad fiscal que gestiona fondos de estabilización y de ahorro de largo plazo interactúa con un banco central que acumula reservas internacionales. Durante las crisis, el fondo de estabilización apoya a los hogares mediante transferencias, mientras que las reservas internacionales proveen liquidez a los bancos, mitigando efectos adversos en sus balances. Ambos instrumentos de política afectan la carga tributaria de los hogares y los costos asociados a la acumulación de reservas, generando trade-offs para cada autoridad. No obstante, estas autoridades no internalizan el impacto de sus decisiones sobre la otra. Como resultado, emerge una relación negativa endógena entre el tamaño óptimo de los SWF y la acumulación de reservas internacionales. El trabajo provee evidencia empírica entre países consistente con estos resultados, utilizando un novedoso panel de datos sobre la existencia y los tipos de SWF.

Abstract

We study the interaction between sovereign wealth funds (SWFs) and foreign reserve accumulation in open economies vulnerable to financial crises. A fiscal authority that manages stabilization and long-term SWFs interacts with a central bank that accumulates foreign reserves. During crises, the stabilization SWF supports households through transfers, while foreign reserves provide liquidity to banks, mitigating adverse balance sheet effects. Both policy tools affect households' tax burden and the costs associated with reserve accumulation, creating trade-offs for each authority. However, these authorities do not internalize the impact of their actions on each other. As a result, an endogenous negative relationship emerges between the optimal size of SWFs and the accumulation of foreign reserves. We provide cross-country empirical evidence supporting these results using a novel panel dataset on the existence and types of SWFs.

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1 Introduction

In this paper, we address the question of how do Sovereign Wealth Funds (SWFs) established by the government and foreign reserves accumulated by the central bank coexist. Specifically, we study the relationship between the institutional arrangement of the SWF and the accumulation of foreign reserves. Both, in the context of an open economy prone through financial crisis through the balance sheet of banks.

To motivate our work, Figure 1a shows the total value of foreign reserves in the world, and the total value of assets under management of SWFs. While the worldwide stock of foreign reserves increased 547% between 2000 and 2024, the stock of assets managed by SWFs increased by about 933% during the same period. By the end of 2024, countries worldwide were managing more assets under SWFs than through central banks in the form of foreign reserves.

Most of the recent literature has studied the strong build-up of foreign reserves since the 2000s by central banks around the world (Aizenman and Lee 2007; Bianchi and Lorenzoni 2022), yet less attention has been paid to the accumulation of assets by SWFs and, more importantly, its relationship with foreign reserves. Figure 1b shows the evolution of reserves and SWFs' assets relative to 2000. From the figure it is worth noticing that the official hoarding of foreign reserves slowed down following the global financial crisis, while the accumulation of assets under management of SWFs kept increasing.[1] On average, are SWFs substitutes to foreign reserves? How do they interact with each other? What are the policy implications of this interaction? We answer these questions in our work.

In spite of the fact that SWFs are not officially considered as foreign reserves when they are not in control of the central bank, they could have similar objectives under economic

[1] It is worth mentioning that Between 2009 and 2012, some central banks deployed reserves through bailout programs, QE-type policies, and other arrangements, whereas between 2015 and 2018, some of them announced reserve accumulation programs.

distress, such as a stabilization purpose. On the one hand, SWFs that have a savings long-term usage simply transfer resources to support structural goals such as pensions or infrastructure. On the other hand, stabilization SWFs may act during a crisis to support the economy. Specifically, this type of funds could be considered as a tool to deploy counter-cyclical fiscal policy in times of distress.[2] Therefore, considering that one crucial motive to accumulate foreign reserves is to insure the economy against a financial crisis (Jeanne and Rancière 2011; Calvo et al. 2013; Céspedes and Chang 2024), SWFs could be either substitutes or complements to foreign reserves, depending on their purpose.

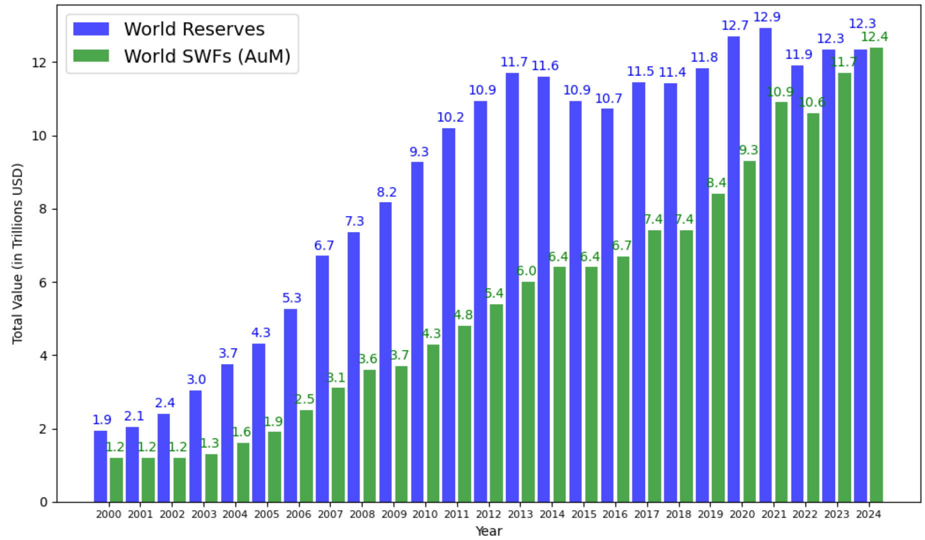
We develop a two-sector (tradable and non-tradable) three-period small open economy model with financial frictions, building upon the framework of Céspedes and Chang (2024). The economy is inhabited by households, firms, banks, a central bank, and the government. The central bank accumulates reserves to mitigate the adverse effects of binding financial constraints, whereas the government may accumulate assets in the form of both short-term and long-term SWFs. The former serves macroeconomic stabilization purposes in case of a crisis, and the latter is commited to provide transfers to households only in the final period.

Banks borrow from abroad at a risk-free interest rate. Households and firms borrow from domestic banks to, respectively, consume and invest in capital to produce in the future. Banks have a balance sheet collateral constraint, which is subject to financial shocks that can make the constraint bind (i.e., a financial crisis). The central banks borrows by issuing long-term bonds abroad to buy reserves and intervene in the case of a crisis. This represents a cost to households that is paid for with taxes. Yet, it also has the potential benefit that, in the case of a crisis, the central bank may provide liquidity to banks, relaxing the constraint and stimulating the provision of credit in the economy.

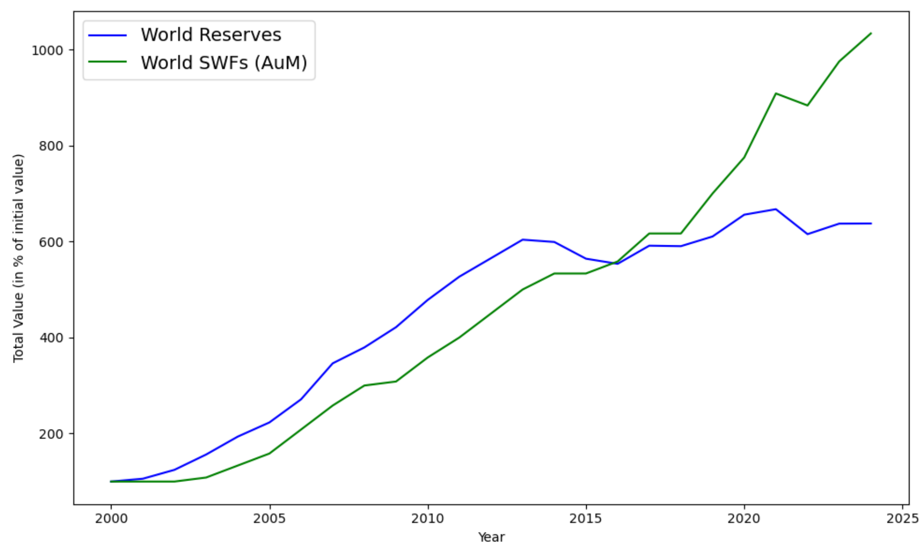
A crucial aspect of our model is the government. We add it by defining a government that spends exogenous quantities funded with taxes. The government may also receive a

[2] The IMF (2010) defines the the role of a SWF as: "...to finance future potential deficits...and, to smooth fiscal spending, or to finance regular extraordinary public debt amortization..."

Figure 1: Sovereign Wealth Funds and Foreign Reserves - Worldwide



(a) SWF and Foreign Reserves (USD trillions)



(b) Evolution of Foreign Reserves Relative to 2000

Note: The upper panel shows the global stock of foreign reserves and the total stock of assets under management by SWFs in USD trillions. The lower panel shows their evolution relative to their level in 2000. Sources: Own calculations based on López (2023), www.globalswf.com, and the IMF's COFER database.

windfall income endowment, akin to a boom in government-owned commodity producers. The government may save resources to allocate them with different purposes. The first one, could be a stabilization purpose in case of a crisis. In this case, the government may transfer resources to households, which alleviates their tax burden. This is no free lunch. Savings from the government imply larger debt taken by households to consume and pay for taxes in the first period. Also, the government can allocate resources to stablish a long-term fund, which yields better returns than the short-term one, at the expense of only being available to households in the final period of the economy. Thus, a short-term fund is a stabilization SWF.

A key element of our environment is that both the government, which is specifically a fiscal authority in this framework, and the central bank act independently. This is, their strategic interaction has a non-cooperative nature, where each authority takes the actions of the other as given. As a result, the best response function of each authority exhibits a negative relationship between the size of the stabilization SWF and foreign reserves. On one hand, the central bank takes as given the short-term fund, accumulating reserves to provide liquidity in case of a crisis considering as given how much the government will alleviate households' tax burden in this case. Conversely, the government takes as given the liquidity provision in case of a crisis (i.e. foreign reserves), choosing based on this how much of its savings to allocate into stabilization purposes. For each authority, taking as given larger values of the other's instrument, implies a lower usage of its own. In turn, a set of equilibria emerge. In these, optimal foreign reserves are negatively related with both the presence and the size of the stabilization SWF.

The main mechanism behind these results is that the tax burden of the household is a decreasing function of the SWF when it can stabilize the economy. Lower taxation leads households to borrow less, which in turn relaxes the collateral constraint. The central bank still stabilizes through foreign reserves but, given the lower probability of financial crisis due to the stabilizing SWF, it accumulates lower foreign reserves compared to a scenario without a stabilizing SWF. In this case, the SWF and foreign reserves are imperfect substitutes. To our knowledge, we are the first to provide theoretical framework that allows for the derivation of a central bank's optimal reserves policy in tandem with optimal SWF policies by the government. This the first main contribution of this paper.

We complement our theoretical work with cross-country evidence that points out in the direction of the model's predictions. For this purpose, we use a comprehensive novel dataset on the extensive margin of SWFs and their type, across countries and time, developed in Acosta-Henao, Martínez, and Rondón-Moreno (2026). The data cover 115 SWFs in 85 countries between 1970 and 2022. Crucially, the dataset provides a classification for each of the SWFs in the data according to their *de jure* description, such that it can be either a stabilization SWF or not.[3] We complement these data with relevant macroeconomic variables that the literature identifies as key in explaining the accumulation of foreign reserves.

We estimate panel fixed effects specifications and show that, on average, countries with SWFs have less foreign reserves than countries without them. Furthermore, we show that this difference stems from stabilization SWF. Thus, having a stabilization SWF is the mechanism behind a lower accumulation of foreign reserves. This result supports the models' implication for the optimal accumulation of reserves, where optimally having a stabilization SWF implies optimal lower reserves than an economy without this type of fund.

Though suggestive, and not causal, these result underscore the relevance of the possible institutional arrangements of a SWF for the accumulation of foreign reserves. Uncovering that it is the type of fund, specifically a stabilization SWF, rather than its mere presence what is really related with the accumulation of foreign reserves, is the second main contribution of this work.[4]

Finally, we study how the equilibria in the decentralized economy compares to that of a social planner that has access to all policy instruments simultaneously, internalizing the effects of each instrument over the economy. compared to the decentralized economy, the

[3]Non-stabilization SWF could have savings purposes to be transferred to households in the long term, strategic purposes, or other.

[4]Disclaimer: This work does not suggest that sovereign wealth funds are misaligned relative to the level implied by the current stock of international reserves in a particular country. We focus on a positive, rather than normative, analysis an provide cross-country empirical evidence supporting the model's findings.

planner has a unique equilibrium where the optimal size of a stabilization SWF is zero, and all public foreign assets accumulated are allocated to the long-term fund. The optimal level of reserves is positive, and smaller than a decentralized equilibrium with also an optimal size of 0 for the stabilization SWF. Yet, the planner's reserves are larger than those of decentralized equilibria with a positive SWF. This is the result of internalizing the benefits and costs of both policies implemented simultaneously at all possible levels of the economy. This result sheds light on the fact that there might be other set of costs and constraints associated to fiscal and monetary policy that go beyond our model, which justify establishing a stabilization SWF. This, in turn, affects the accumulation of foreign reserves by the central bank as explained before. This analysis is the third and last contribution of our work.

Related literature. The question of why countries accumulate foreign reserves goes back, at least, to Heller (1966) who views reserves as an instrument to cover current account adjustments. Following the financial crises in emerging markets at the end of the nineties, foreign reserves where viewed as an appropriate self-insurance mechanism against the volatility of capital flows (Feldstein, 1999).

From a theoretical perspective, Jeanne and Rancière (2011) estimate the optimal level of reserves to insure an economy against financial dry-ups in international markets while Calvo et al. (2013) do the same in a statistical model. More recently, people have introduced foreign reserves into sovereign default models to study optimal reserves holdings and management when governments can default on their debt (Alfaro and Kanczuk 2009; Bianchi et al. 2018; Barbosa-Alves et al. 2024), while others have highlighted the role of reserves as a potential macroprudential instrument (Bianchi and Lorenzoni, 2022).

Our paper builds on Céspedes and Chang (2024), where reserves can be used to alleviate financial frictions. We explicitly introduce fiscal policy to this framework to model the interaction between the strategic interaction government and the central bank. This interaction leads endogenously to establishing, or not, different SWFs, determining the optimal size of foreign reserves which, in turn, also determine the optimal size of each fund.

On the empirical front, several papers argue that countries build up their stock of reserves due to precautionary motives. Aizenman and Lee (2007) find that countries with greater capital mobility indices tend to have greater levels of reserves. Obstfeld et al. (2010) find a positive relationship between broad money and the level of reserves, which they interpret as evidence of the desire of central banks to protect domestic banks from *internal drains* by having sufficient reserves.[5] Similarly, Ghosh et al. (2017) present evidence that the motives behind reserves accumulation have shifted from current account concerns in the eighties to capital and financial account risks in the 2000s. Closer to our paper, for a sample of twenty two countries, Aizenman and Glick (2009) show that foreign reserves holdings fall over time following the creation of a SWF. That is, if SWF have any impact on reserves, it is only observable in the long-term. Furthermore, Aizenman et al. (2015) show that there is a negative relationship between the presence of a SWF and the level of foreign reserves across countries.

In response to the growing size of SWFs after the global financial crisis, a growing body of literature focusing on the structural characteristics of these funds has emerged (Gintschel and Scherer 2008; Megginson et al. 2013; Bortolotti et al. 2015; López 2023). There has been also great interest in understanding the macro-fiscal and macro-financial linkages of SWFs. For instance, Griffith-Jones and Antonio Ocampo (2012) explore the role of these funds in explaining the broader accumulation of foreign reserves in developing countries during the commodities boom of the late 2000s.

We complement these studies by showing that it is not the mere presence of the fund, but its type what matters for such relationship, and provide a theory behind this observation.

The rest of the paper is organized as follows: The second section develops the theoretical model. The third section characterizes the policy equilibria. The fourth section analyzes

[5] An internal drain is defined as an event where domestic residents withdraw their resources from the economy and buy assets abroad (capital outflows).

the model's implications for SWF policy and the optimal level of reserves, along with its mechanisms. The fifth section provides empirical evidence supporting the model's main results. The sixth section studies welfare through the lens of a social planner. The last section concludes.

2 Model

We propose a theoretical framework for a small open economy subject to liquidity constraints. In this environment, both the central bank and the central government have incentives to independently accumulate external assets. However, the central government follows exogenous expenditure path and may establish a SWF. The government uses the SWF's resources for either for long-term investment (*savings*), economic intervention during a crisis (*stabilization*), or both. As a result, our framework allows us to analyze how the central bank's optimal reserve accumulation policy interacts with different types SWFs optimally chosen by the central government.

Households and firms borrow from domestic banks, which in turn obtain funding from international markets to finance consumption and production. External borrowing by banks is subject to an *occasionally binding* collateral constraint, which also limits the borrowing capacity of domestic agents. When this constraint binds, domestic agents are unable to secure sufficient external funding, forcing them to reduce investment and consumption in equilibrium. We refer to this scenario as a financial crisis.

Foreign reserves in our model serve as a welfare-improving instrument through their role as a liquidity support policy. By relaxing the collateral constraint, these reserves facilitate higher levels of consumption and investment. Thus, the central bank determines the optimal level of reserves by weighing these benefits against the associated costs of reserve accumulation.

The central government finances public transfers and expenditures using a combination of an initial endowment, public foreign borrowing, and lump-sum taxes. The initial endowment–which could be interpreted as windfall income from commodity exports–and public foreign borrowing provide the government with the resources to accumulate external assets in the form of a Sovereign Wealth Fund. This fund is established with specific policy objectives, which are either short-term stabilization or long-term savings.

Importantly, the central government internalizes the effects of its fiscal policy over the borrowing behavior of domestic agents. In particular, it anticipates that transferring resources during periods of financial stress can help alleviate the effects of a binding collateral constraint by reducing their financing needs. Therefore, the government selects its optimal SWF policy by balancing the benefits of short-term fiscal stabilization (less frequent crises) and long-term investment returns (higher future consumption) against the opportunity costs of these resources.

Our model builds on the framework developed by Céspedes and Chang (2024). In absence of the central government, windfall income (i.e. the endowment), and the possibility of setting up and using a SWF, our model collapses to theirs.

2.1 Environment

Consider a three period ($t = 0, 1, 2$) small open economy model with tradable and non-tradable goods, and borrowing from international markets. The economy is inhabited by a representative household that owns firms and banks. Foreign reserves policy is conducted by the central bank, while the government is responsible for fiscal policy. It is convenient to present the timing of our model by separating the analysis of the private and public sectors. The sequence of events for private agents is the following:

In Period 0 households and firms borrow from banks to finance their consumption and tax payments to the government. Banks, in turn, obtain funding from international markets to provide domestic credit.

In Period 1, households roll over their initial debt and borrow to pay for taxes, while firms borrow from banks to acquire new capital. Simultaneously, banks roll over their previous debt and seek additional foreign funding to meet the new financing needs of households and firms. During this period banks' borrowing is subject to an occasionally binding collateral constraint, which depends on their net worth. A binding constraint leads to an increase in domestic borrowing costs and a subsequent reduction in firms' investment.

Finally, in Period 2, firms produce using the capital they bought in period $t = 1$. Households decide their consumption based on their disposable income, which is calculated after all debts are settled and net transfers from banks and firms are received.[6]

The specific timing for the public sector–the central bank and the central government–will depend on the details of the liquidity and stabilization policies they decide to follow. Generally, it proceeds as follows: Both agents recognize that facing a binding collateral constraint in period 1 would be costly in terms of welfare. Thus, in $t = 0$, the central bank borrows from international markets at the risk-free world rate to accumulate reserves. Concurrently, the government receives its initial endowment in the form of an internationally traded good, and decides whether to borrow from international markets. It then chooses the amount of external assets dedicated to a stabilization SWF and to a long-term savings SWF.

In $t = 1$, if the constraint binds, the central bank uses its reserves and the government uses the stabilization SWF to help stabilize the economy. In $t = 2$, the government distributes to households the profits from its long-term investments, net of any fiscal expenditures for this period. The central bank is paid by the private sector for its use of foreign reserves. Both agents then proceed to settle their accounts with the rest of the world.

[6] Notice that, in equilibrium, facing a collateral constraint with positive probability of being binding in period 1 reduces welfare as it yields an inefficiently low level of consumption and borrowing in period 0.

2.1.1 Households

The economy is populated by identical households of mass 1. They consume tradable goods in both the initial and final periods ($t = 0, 2$). Their preferences are given by $U(C_0) + \beta\mathbb{E}(C_2)$, where C_t denotes consumption of tradables in period t, β the discount factor, $U(\cdot)$ is a CRRA function, and $\mathbb{E}(\cdot)$ is the expectations operator.

Households own domestic banks and firms, with profits Π_b and Π_f respectively, being their only source of income. These profits are distributed at $t = 2$. To finance consumption and pay lump-sum taxes at $t = 0$, C_0 and T_0, households must borrow L_0^H (in units of tradables) from domestic banks at a gross interest rate of R_0.

$$L_0^H = C_0 + T_0$$

At $t = 1$, though households do not consume they must take on a new loan, L_1^H, at a gross rate of R_1 from domestic banks. This loan is necessary to repay their initial debt and cover any taxes collected from them in this period.

$$L_1^H = R_0 L_0^H + T_1$$

As a result, households' financing needs at $t = 1$ are partially determined by tax payments in both $t = 0$ and $t = 1$. We define this combined tax burden at $t = 1$ $T^b \equiv R_0 T_0 + T_1$, such that a negative T^b implies a net positive transfer of resources from the government to households.

In the final period ($t = 2$), households receive profits from firms (Π_f) and banks (Π_b), pay off their debts and remaining taxes, and consume whatever they have left. The households' budget constraint in this period is then:

$$C_2 + R_1 L_1^H + T_2 = \Pi_f + \Pi_b$$

Since we assume that there are no financial frictions in the initial period, the domestic lending interest rate at $t = 0$ is equal to the world's risk-free rate, so $R_0 = R_0^*$. The household's intertemporal budget constraint is given by:

$$C_2 + R_1(R_0^* C_0 + T^b) + T_2 = \Pi_f + \Pi_b \tag{1}$$

Households are atomistic. Therefore, when choosing their consumption at $t = 0$, households treat interest rates, taxes, and profits as given. As in Céspedes and Chang (2024), a financial friction can cause the domestic gross rate, R_1, to exceed the international gross rate, R_1^*, at $t = 1$. Consequently, households' decisions at $t = 0$ are based on the expected cost of rolling over their debt at $t = 1$. Denoting this expectation as $\mathbb{E}(R_1)$, the household's optimal initial consumption is determined by the following Euler equation:

$$U'(C_0^*) = \beta R_0^* E(R_1) \tag{2}$$

Considering that taxes are lump-sum, the households' consumption decision is not directly affected by fiscal policy. Thus, regardless of T^b, households consumption will be optimal conditional on their available disposable income.

In equilibrium, however, fiscal policy may crowd out household consumption. As we show below, a larger tax burden, T^b, can cause the collateral constraint to bind more frequently under certain conditions. This leads to an expected interest rate, $\mathbb{E}(R_1)$, that is greater than R_1^*. In this scenario, households are underborrowing and consuming at inefficiently low levels.

This externality is a direct result of atomistic households participating in a competitive credit market. In other words, households do not internalize the effects of their consumption and borrowing decisions over interest rates, taxes, and profits in equilibrium, which leads them to borrow less than the case where they do internalize such effects.

2.1.2 Firms

Domestic competitive firms produce Y_2 units of tradable goods at $t = 2$ using capital, K_2. Capital is purchased in period $t = 1$ at a unit price of Q_1, and its is financed by borrowing $L_1^f = Q_1 K_2$ from domestic banks. Firms' profits are given by:

$$\Pi_f = Y_2 - R_1 Q_1 K_2$$

where output is a decreasing returns to scale function of the form $Y_2 = Z K_2^{\alpha}$ (with $0 < \alpha \leq 1$), and R_1 is the loan's financial cost.[7]

A competitive firm takes prices as given and demands capital until its marginal product equals its marginal cost. In this case, the marginal cost of capital is the product of its price, Q_1, and the loan's gross domestic interest rate, R_1. Therefore, the optimal demand for capital is given by:

$$K_2^d = \left(\frac{\alpha Z}{R_1 Q_1} \right)^{\frac{1}{1-\alpha}} \tag{3}$$

Capital is then produced in period 1 using tradables and non-tradables, denoted by I_W and I_H. More specifically, the production of capital follows a Cobb-Douglas aggregator of the form $K_2 = \frac{1}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} I_H^{\gamma} I_W^{1-\gamma}$. With the price of tradables as the numeraire, the price of capital in terms of tradables is given by the following relationship:

$$Q_1 = X_1^{\gamma}$$

[7] Notice that productivity of each firm, Z, is homogeneous across firms. Therefore, firms are identical, and can be thought of as a representative firm, despite having a decreasing returns to scale technology.

where X_1 is the real exchange rate of this model, and the optimal demand curves for tradables and non-tradables are, respectively, given by:

$$I_W = (1-\gamma)Q_1 K_2 \quad \text{and} \quad I_H = \gamma \left(\frac{Q_1}{X_1}\right) K_2 \tag{4}$$

2.1.3 Banks

Domestic banks act as intermediaries between the economy and international markets. All loans between banks and private domestic and foreign agents have a maturity of one period. At $t = 0$, banks lend an amount L_0 of tradables to households, at the domestic interest rate R_0, which they finance by borrowing D_0 from international markets at the world rate R_0^*. Thus, the initial period budget constraint is given by

$$L_0 = D_0$$

At the beginning of $t = 1$, banks collect $R_0 L_0$ from households, borrow again from international markets D_1 at interest rate R_1^*, and receive an endowment comprised of tradables (T) and non-tradables (N). Additionally, we allow for the possibility that banks receive liquidity support from the central bank equal to F, which is a credit from the central bank also at interest rate R_1^*. Then, banks use these resources to pay back debt acquired in the initial period ($R_0^* D_0$), and finance an amount L_1 of new loans to households and domestic firms. Banks' budget constraint at $t = 1$ implies that banks will borrow from foreign investors at $t = 1$ the amount necessary to lend domestically,

$$D_1 = L_1 - (T + X_1 N + F) \tag{5}$$

In the last period, banks recoup $R_1 L_1$ from domestic agents, pay back international markets and the central bank ($R_1^* D_1 + R_1^* F$), and transfer profits to households. Thus, banks' profits are given by

$$\Pi_b = R_1 L_1 - R_1^*(D_1 + F)$$

2.1.4 Fiscal Authority

We model a fiscal authority that finances an *exogenous* expenditure path $\{G_0, G_1, G_2\}$ with an initial endowment, taxes, and sovereign debt. This fiscal authority has access to both long-term and short-term investment technologies in international markets. In our model, the accumulation of external assets by the fiscal authority is equivalent to establishing a sovereign wealth fund. Thus, maturity of these investments abroad determine the type of SWF.

The fiscal authority starts the initial period with a legacy endowment of tradable goods denoted by A, and doesn't receive any other endowment in the following periods.[8] Additional to this endowment, the government can issue long-term debt, B, and collect taxes from households at $t = 0$ to finance its expenditure (G_0), to invest S tradable goods in a long-term project, or save some resources through external safe assets in international markets (s_0). Thus, the government's budget constraint in the initial period is given by

$$A + B + T_0 = G_0 + s_0 + S$$

In the intermediate period, the fiscal authority receives the returns of investing s_0 at the risk-free rate R_0^*, and collects taxes from households (T_1) to finance government expenditures. In principle, the government could also save again in international markets some resources s_1 between $t = 1$ and $t = 2$. Hence, the fiscal authority's budget constraint in the intermediate period is given by

$$R_0^* s_0 + T_1 = G_1 + s_1$$

[8] We do not restrict A to be strictly positive, and allow for the possibility that the government starts with a inherited debt that needs to paid immediately. Yet, intuitively, a positive A can be thought of as windfall income from commodities.

In the final period, this fiscal authority pays for government expenditure G_2, redeems its long-term debt, $R_0^* R_1^* (1 + \tau^{fa}) B$, and back stops any operations of the Central Bank by transferring T_{BC}. To cover these outlays, it collects (T_2) from households, it uses the returns from investing s_1 at the risk-free rate between $t = 1$ and $t = 2$, and it receives $R_0^* R_1^* (1+\tau^{lt}) S$ from investing in the long-term project. Thus, the budget constraint at $t = 2$ is equal to

$$R_1^* s_1 + R_0^* R_1^* (1 + \tau^{lt}) S + T_2 = G_2 + R_0^* R_1^* (1 + \tau^{fa}) B + T^{CB}$$

Without loss of generality, we assume that any resources invested in S cannot be accessed by the fiscal authority until the last period, while resources invested in s_0 or s_1 have immediate access. Thus, S is an illiquid asset that must be held to maturity.[9] Conversely, investing in s_0 is more liquid at the cost of lower return than the long-term asset.

We combine each period's constraint and derive the fiscal authority's intertemporal budget constraint (Equation 6). It shows that the net financial cost of long-term debt and any transfers to the central banks are financed with the path of primary surpluses plus the return of the legacy endowment and the return on of long-term assets.

$$R_0^* R_1^* \left(T_0 - G_0\right) + R_1^* \left(T_1 - G_1\right) + T_2 - G_2 = R_0^* R_1^* \tau^{fa} B + T^{CB} - R_0^* R_1^* (A + \tau^{LT} S) \tag{6}$$

As we show below, a key decision by the fiscal authority is how much to invest in the long-term project versus how much of resources to leave for short-run expenditures. To see this, we define the stock of external liquid assets (ELA) as the amount of resources that a fiscal authority has available to cover short-run expenditures. Note that, given a level of legacy endowment, ELA increases either by issuing more long-term debt in international

[9] Another possibility would be that liquidation of the long-term project before the last period is costly, and the fiscal authority would earn $r < R_0^*$ for each unit of S that is liquidated. The key idea is that, from the perspective of the fiscal authority's investment portfolio, allocating resources to s_0 is different from allocating resources to S.

markets, or by investing less in the long-term project.

$$ELA \equiv A + B - S$$

Combining the definition of ELA with the fiscal authority's budget constraints of the initial and intermediate periods yields Equation 7. This expression shows that resources in ELA are used to cover the primary fiscal deficits at $t = 0$ and $t = 1$, and any investment in international markets from $t = 1$ to $t = 2$. In turn, we interpret ELA as a stabilization sovereign wealth fund that aims to implement counter-cyclical fiscal policy and smooth fiscal spending.

$$ELA = G_0 - T_0 + \frac{G_1 - T_1}{R_0^*} + \frac{s_1}{R_0^*} \tag{7}$$

To understand the cost of allocating resources to ELA, it is illustrative to substitute S in Equation 6, which yields Equation 8. First, this expression shows that the opportunity cost of ELA is equal to the net cost of not investing in the long-term project (τ^{lt}). Second, given that this fiscal authority has access to a long-term project, the value of the legacy endowment augments to $A(1+\tau^{lt})$ in it's intertemporal finances. Lastly, the financial cost of long-term debt depends on how these borrowed resources are used. Borrowing an additional unit from international markets has a net cost equal to the term premium (τ^{fa}), unless this additional unit is allocated to the long-term project. In this case, the net financial cost of borrowing falls to $\tau^{fa} - \tau^{lt}$.

$$R_0^* R_1^* (T_0 - G_0) + R_1^* (T_1 - G_1) + T_2 - G_2 + R_0^* R_1^* (1+\tau^{lt})A = R_0^* R_1^* (\tau^{fa} - \tau^{lt})B + T^{CB} + R_0^* R_1^* \tau^{lt} ELA \tag{8}$$

The fiscal authority policy is characterized by a households tax burden in the first two periods, taxes in the final period, a level of long-term borrowing B, and the size of the stabilization SWF (ELA). We denote a fiscal policy as set Γ that contains elements $\Gamma = \{T^b, T_2, B, ELA, s_1\}$. Given an exogenous path for government expenditure, Γ is an endogenous policy set chosen by the fiscal authority.

2.1.5 Central Bank

We model a central bank that has access to long-term debt at $t = 0$ and can act as a lender of last resort at $t = 1$. Its only policy instrument is to provide liquidity in tradable goods to domestic banks when the economy remains in a crisis after all agents, including the government, have implemented their actions. However, to provide tradables at $t = 1$, it needs to preemptively accumulate them in the initial period.[10]

Specifically, we assume that the central bank can borrow a quantity F_0 of tradables at $t = 0$ from international markets by issuing long-term debt B_{CB}. This debt has to be paid back in the final period with a financial cost equal to $R_0^* R_1^*(1 + \tau^{cb})$. Note that we assume that the fiscal authority and the central bank face the same borrowing conditions. Once the central bank issues its debt at $t = 0$, it immediately lends F_0 back to foreign borrowers in international markets in exchange for a one-period return equal to R_0^*. Thus, the central bank starts $t = 1$ with an foreign reserves stock equal to $R_0^* F_0$ which can be used to lend again at the world rate R_1^*, either to foreign borrowers or, if necessary, to domestic banks. We denote the amount of liquidity support to domestic banks as F. In the final period, the central bank collects $R_1^* F$ from banks and $R_1^*(R_0^* F_0 - F)$ from international borrowers to pay back its outstanding long-term debt. Since the amount that the central bank collects is not sufficient to fully redeem its debt, we assume that the remainder– $\tau^{cb} R_0^* R_1^* F_0$–is supported by the government with the transfer T^{CB}.

[10]Note that we are excluding the possibility of the C.B to issue short-term debt at $t = 1$. This assumption is necessary to generate a positive official demand for reserves. However, this assumption is without lost of generality as issuing debt during the crisis would be subject to a risk-premium that makes it more expensive for the central bank to pay for than than preemptive accumulation in the initial period.

2.1.6 Collateral Constraint

Contracts between domestic banks and foreign investors are subject to an incentive compatible constraint (ICC). Specifically, we assume that bank profits should be greater or equal to a fraction θ of banks final period income, net of payment to the central bank. That is,

$$\Pi_b \geq \theta R_1 L_1 - R_1^* F \qquad (ICC)$$

This type of constraint emerges naturally in environments where incentives between the lender–in this case the foreign lender–and the borrower–the domestic bank–are not aligned. Therefore, in any incentive compatible contract the lender needs to guarantee that the borrower obtains a minimum fraction of profits such that optimal behavior by the latter is aligned with the objective function of the former.[11] As in Holmstrom and Tirole (2011), we assume that any public lending is not subject to the collateral constraint. In other words, the government can always collect the debt it is owed by its citizens.

The ICC in the final period defines a collateral constraint for domestic borrowing at $t = 1$. To see this, replace banks $t = 1$ budget constraint and the definition of Π_b in the ICC, and after some reorganization we obtain Equation 9. This equation underscores that this economy's external private borrowing at $t = 1$ is limited by the share of endowment that is *credibly pledgeable* to foreigners.

$$D_1 \leq \frac{R_1(1-\theta)}{R_1^* - R_1(1-\theta)} (T + X_1 N + F) \tag{9}$$

We assume that fraction θ is a *random variable* whose realization occurs at the start of $t = 1$, and determines the fraction of the domestic interest rate that is pledgeable to international markets, $R_1(1-\theta)$.[12] Everything else equal, a greater value of θ implies tighter

[11] See Holmström and Tirole (2011) for a greater discussion.

[12] The variable θ is the only source of uncertainty in the model for which we assume that it can take n values, denoted by θ_s, $s = 1, ..., n$, between $\theta_L = \theta_1 > 0$ and $\theta_H = \theta_n < 1$, plus each θ_s with probability

financial conditions in international markets as that foreign investors recognize a lower share of the domestic bank's collateral as pledgeable.

Notice from (9) that tighter financial conditions can be offset either by raising the domestic interest rate, R_1, above the world rate, R_1*, or by the central bank providing more liquidity support to domestic banks (F). However, the costs of these possibilities differ. While a greater R_1 reduces the optimal demand for capital (Equation 3), which implies lower borrowing taken by domestic firms, more liquidity support increases the amount of resources that the central bank itself needs to borrow from abroad. Thus, a spread in interest rates–$R_1 - R_1^*$–simultaneously attracts more external resources and curbs domestic borrowing. Therefore, when a shock to θ is sufficiently large, R_1 endogenously adjusts to guarantee that Equation 9 binds.

2.2 Competitive Equilibrium

We follow the conventional definition of a competitive equilibrium where, given a fiscal and reserves policy, privates agents are optimizing, and markets clear.

Competitive equilibrium. A competitive equilibrium with foreign reserves and fiscal policy is given by $C_0 \geq 0$, $F \geq 0$, a Fiscal Policy set Γ and, for all possible states (of θ and hence of the economy) $s = 1, 2, ..., n$, $R_{1s} \geq R_1^*$, and non-negative K_{2s}, Q_{1s}, X_{1s}, and I_{Ws} that satisfy Equations 2, 3, 4, 8, 9, and

$$C_{2s} = ZK_{2s}^{\alpha} + R_1^* T - R_1^* I_{Ws} - R_0^* R_1^* C_0 - R_1^* T^b - T_2 \geq 0 \tag{10}$$

$$\frac{R_1^*}{R_1^* - (1-\theta_s)R_{1s}}[T + X_1 N + F] - (R_0^* C_0 + Q_{1s} K_{2s} + T^b) \geq 0, = \text{if } R_{1s} > R_1^* \tag{11}$$

To solve for this equilibrium, it is convenient to characterize the continuation equilibrium. Once θ is realized, the state of the economy is determined by set $\{\theta_s, F, \Gamma, C_0\}$. In any state

$\pi_s > 0$ where $\sum_{i=1}^{n} \pi_s = 1$.

s, variables K_{2s}, Q_{1s}, X_{1s}, I_{Hs} and I_{Ws} are pinned down by the domestic interest rate R_{1s}. To see this, consider a feasible set $\{\theta_s, F, \Gamma, C_0\}$ and a level of $R_{1s} \geq R_1^*$. Since markets clear in equilibrium, firms optimal demand for I_{Hs} has to be equal to domestic banks endowment of non-tradables (N) which is the only source of non-tradables in this economy. Combining $I_{Hs} = N$, Equation 4, and firms optimal demand for capital (Equation 3), we derive the level of the exchange rate as a function of parameters and R_{1s}.

$$X_{1s} = \left[\frac{\alpha Z}{R_{1s}} \left(\frac{\gamma}{N} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha(1-\gamma)}} \tag{12}$$

Equation 3 determines the level of K_{2s} in equilibrium while, as discussed before, Q_{1s} is equal to X_1^{γ}. Thus, the optimal demand of firms for tradable goods I_{ws} is equal to $\frac{1-\gamma}{\gamma} N X_{1s}$. As a result, firms demand for tradables and non-tradables, capital, and output depend on R_{1s}.

We turn now to households consumption in the final period, which after replacing bank and firm profits in equilibrium, is given by Equation 10. In principle, the period 2 value of the path of household taxes ($R_1^* T^{bs} + T^{2s}$) is state-dependent since the fiscal authority chooses T^b and T_2 after observing the realization of θ. However, after substituting Equation 8 and T^{CB}, Equation 13 shows that households pay the same period-2 value of taxes regardless of their timing. Moreover, this value of taxes depends on variables that are chosen in the initial period: initial household initial consumption, fiscal authority and central bank long-term debt, and the size of ELA. Therefore, household consumption in the final period only depends on the state of the economy through firms output and their demand for tradables, which we already discussed are a function of R_{1s}.

$$C_{2s} = ZK_{2s}^{\alpha} + R_1^*(T - I_{ws}) - R_0^* R_1^* C_0$$
$$- \Big[R_0^* R_1^* G_0 \; + \; R_1^* G_1 \; + \; G_2 + (\tau^{fa} - \tau^{lt}) R_0^* R_1^* B \; + \; \tau^{cb} R_0^* R_1^* F_0 \tag{13}$$
$$+ R_0^* R_1^* \tau^{lt} ELA \; - \; R_0^* R_1^* (1 + \tau^{lt}) A \Big]$$

The level of R_{1s} is determined in the domestic credit market and whether domestic banks collateral constraint binds in equilibrium or not. Depending on θ, Equation 9 binds or not. First, consider a continuation equilibrium where the collateral constraint is not binding at $t = 1$. In this state, domestic banks lend to the economy at the world rate,[13] and the real exchange rate, the price of capital, and the level of capital are at their frictionless levels. We denote these levels as X_{1f}, Q_{1f}, and K_{2f}, respectively. For a given θ, C_0, F and T^b, the demand for domestic loans and the upper bound on the supply of loans in a frictionless continuation equilibrium, denoted by L_{1f}^d and $\bar{L}_{1f}^s$, are given by

$$L_{1f}^d = R_0^* C_0 + Q_{2f} K_{2f} + T^b$$
$$\bar{L}_{1f}^s = \frac{1}{\theta} \left(T + X_{1f} N + F \right)$$

Thus, in equilibrium, the constraint does not bind for a given θ as long as the demand for domestic loans L_{1f} is less or equal to $\bar{L}_{1f}^s$.

It is possible to determine an endogenous level of θ, denoted as threshold $\hat{\theta}$, such that domestic banks collateral constraint does not bind whenever $\theta \leq \hat{\theta}$, and is binds otherwise. To find $\hat{\theta}$, we equate L_{1f} equal to $\frac{1}{\hat{\theta}} \left(T + X_{1f} N + R_0^* F \right)$, which is $\bar{L}_{1f}^s$ with θ evaluated at $\hat{\theta}$, and we rearrange terms to derive Equation 14. Function $\hat{\theta}(C_0, F, T^b)$ establishes the threshold for θ where stringent financial conditions in international markets turn into a domestic crisis.

$$\hat{\theta}(C_0, F, T^b) \equiv \frac{T + X_{1f} N + F}{R_0^* C_0 + Q_{2f} K_{2f} + T^b} \tag{14}$$

[13] This is the case of the initial period ($R_0 = R_0^*$) since, by assumption, D_0 is not subject to a collateral constraint.

The previous analysis follows closely Céspedes and Chang (2024). Similar to this work, we find that $\hat{\theta}(C_0, F, T^b)$ is endogenous to household debt and liquidity policy. Specifically, greater initial household debt reduces $\hat{\theta}$, whereas greater liquidity to banks increases $\hat{\theta}$. However, our model provides a role for fiscal policy which is absent in Céspedes and Chang (2024). Interestingly, higher taxes make a financial crisis more likely in this economy. The reason is that a greater tax burden increases the leverage of private households which brings the domestic banks closer to the collateral constraint.

Now, consider a feasible θ such that, for a given $\{C_0,\ T^b,\ F\}$, L^d_{1f} is greater than $\bar{L}^s_{1f}$. This is a scenario where if R_{1s} is equal to R^*_1, credit demand is greater than what domestic banks can supply due to tighter international financial conditions. As shown by Equation 9, banks increase their pledgeable endowment by raising the domestic interest rate since they can promise a larger share of their endowment.[14] In turn, a greater R_{1s} depreciates the exchange rate in equilibrium, reducing firms investment and their demand for credit. Therefore, an R_{1s} greater than R^*_1 in equilibrium guarantees that the domestic credit market clears by solving Equation 15.

$$R^*_0 C_0 + Q_{2s} K_{2s} + T^b = \frac{R^*_1}{R^*_1 - (1-\theta) R_{1s}} [T + X_{1s} N + F] \tag{15}$$

Proposition 1 (Characterization of Continuation Equilibria)**.**

For a given set $\{\theta, F, \Gamma, C_0\}$, let $\hat{\theta}(C_0, F, T^b)$ be determined by Equation 14, and let $\rho(C_0, F, T^b)$ solve Equation 15.

- *If $\theta \leq \hat{\theta}(C_0, F, T^b)$, then R_{1s} is equal to R^*_1, K_{2s}, Q_{1s}, X_{1s}, and I_{Ws} are equal to their frictionless values, and C_{2f} is given by Equation 13 where K_{2s} and I_{Ws} are evaluated at their frictionless values.*

- *If $\theta > \hat{\theta}(C_0, F, T^b)$, then R_{1s} is equal to $\rho(C_0, F, T^b)$, X_{1s} is determined by Equation 12*

[14] This is true as long as $R_{1s} < \frac{R^*_1}{1-\theta_L}$ which we assume to hold. Otherwise, the leverage ratio in Equation 9 is not defined for all possible values of θ.

evaluated at $\rho(C_0, F, T^b)$*,* K_{2s} *is determined by Equation 3,* Q_{1s} *is equal to* X_1^γ *and* $I_{Ws} = \frac{1-\gamma}{\gamma} N X_{1s}$*, and* C_{2s} *is given by Equation 13.*

Proposition 1 characterizes the continuation equilibrium for any set $\{\theta, F, \Gamma, C_0\}$. Given this characterization for all possible values of θ, one can calculate the expected value of R_1 which, in turn, determines the optimal level of initial consumption through the Euler Equation (2). A competitive equilibrium is reached when, for a given fiscal and liquidity policy, the level of initial consumption C_0 is consistent with the Euler Equation and Proposition 1, and, simultaneously, Proposition 1 is consistent with C_0. Notice that Γ and F are taken as given by private agents in this economy. We now turn to the optimal choice of F and Γ by the central bank and the government, respectively.

3 Policy Equilibria

In this section, we study the optimal policies by the fiscal authority and the central bank, which choose Γ and F, respectively, to maximize households' lifetime utility in a competitive equilibrium. Since both policy-makers act simultaneously and, by assumption, independent from each other, this optimal policy choice is akin to the solution of a strategic game. We are interested in a perfect foresight equilibrium, which we solve by backward induction.

3.1 Continuation Equilibrium with Policy

As we showed in the previous section, the state of the economy at $t = 1$ is characterized by set $\{\theta, F, \Gamma, C_0\}$. At this point, the fiscal authority can choose the level of tax burden (T^b), T_2 and s_1 while it takes ELA and B as given since these variables were chosen in the initial period. Meanwhile, the central bank starts this period with a stock of reserves equal to $R_0 F_0$ and chooses how much liquidity support to allocate to domestic banks through F. Both policy makers want to maximize the level of consumption of households in the final period and, importantly, take as given the actions of the other.

Equation 8 establishes the level of households consumption at $t = 2$ in any continuation equilibrium. It is straight forward to see that household consumption only depends directly on policy variables that are chosen in the initial period: fiscal authority and central bank long-term debt, and the size of ELA. This implies that the timing of taxes between T^b and T_2, s_1 and the amount of liquidity support F matter for C_{2s} only through their impact on the domestic interest rate. Therefore, the effect of policies depend on how F and Γ affect $\hat{\theta}(C_0, F, T^b)$ and $\rho(C_0, F, T^b)$.

Proposition 2 (Dominant Strategies - Continuation Equilibria)**.**
In any continuation equilibrium, s_1 equal to zero and F equal to $R_0^* F_0$ are dominant strategies for the fiscal authority and the central bank, respectively.

Proposition 2 states that the fiscal authority and the central bank have dominant strategies in the intermediate period. To see why, we start with the central bank. Suppose any feasible Γ. A strategy where F is equal to $R_0^* F_0$ means that the central bank depletes all of its stock of reserves as liquidity support for domestic banks. Could the households consumption increase if the central bank were to choose F by less than $R_0 F_0$? The answer is no for two reasons. The first is that the outside option of the central bank, which is investing international markets, produces the same return R_1^* supporting domestic banks. The second reason is that, for any feasible value of T^b, $\hat{\theta}(C_0, F, T^b)$ is increasing in F, and $\rho(C_0, F, T^b)$ is non-increasing in F. Therefore, choosing F less than $R_0^* F_0$ would at best keep households consumption unchanged and, at worst, increase the domestic interest rate which lowers consumption via a lower output from firms. Hence, $F = R_0^* F_0$ is a dominant strategy for the central bank.

We turn now to the fiscal authority. Consider a feasible F. A strategy of s_1 equal to zero is equivalent to a strategy where the fiscal authority uses the totality of ELA to finance the present value of the stream of fiscal deficits in $t = 0$ and $t = 1$, and not save any resources from $t = 1$ to $t = 2$.[15] For a given choice of ELA, would the fiscal authority

[15]We assume that the fiscal authority cannot borrow from international markets at $t = 1$. Thus, s_1 cannot be negative.

benefit households by choosing an s_1 different from zero? Again, the answer is no. First, from Equation 7 and the definition of tax burden, as s_1 increases, T^b also increases. Thus, the cost of a positive s_1 is, at best, zero or, at worst, a higher domestic interest rate due to $\hat{\theta}(C_0, F, T^b)$ and $\rho(C_0, F, T^b)$. Second, a positive s_1 is an amount of resources that were saved through ELA and not used to finance fiscal deficits. These resources ultimately survived until $t = 2$ for a total return of $R_0^* R_1^*$ whereas if these resources had been saved through the fiscal authority's long-term investment, they would have earned a premium of $R_0^* R_1^*(1 + \tau^{lp})$.

3.2 Optimal Policies at $t = 0$

Proposition 2 implies that the optimal strategies in any continuation equilibrium are determined by variables chosen at $t = 0$. Thus, maximization problems are reduced to variables ELA and B for the fiscal authority, and F_0 for the central bank. For simplicity, let P denote the set of control variables $\{F_0, B, ELA\}$. Since any competitive equilibrium is conditional on the fiscal and liquidity policies and, by Proposition 2, optimal policies in any continuation equilibrium are determined by set P, then households expected lifetime utility of a competitive equilibrium, denoted by W, can be written as a function of P.

$$W(P) = U(C_0(P)) + \beta E[C_2|P]$$

Moreover, as Equation 13 establishes, household consumption in the last period depends on three types of terms: state contingent variables, variables that are chosen at $t = 0$, and terms that are exogenous to policy (t.i.p). Equation 16 expresses the policy objective function in terms of set P, where initial consumption is a function of P by the Euler equation (Equation 2), and $\boldsymbol{\tau} = [\tau^{cb}, (\tau^{fa} - \tau^{lt}), \tau^{lt}]'$ is a column vector whose elements are the financial costs of the different policies.

$$
\begin{aligned}
W(P) = U(C_0(P)) + \beta E[ZK^{\alpha} - R_1^* I_w | P] \\
- \beta \Big[R_0^* R_1^* C_0(P) + R_0^* R_1^* P \cdot \boldsymbol{\tau} \Big] \\
+ t.i.p
\end{aligned}
\tag{16}
$$

As long as K_2, I_w, and C_0 are differentiable with respect to the elements in P, then the optimal policies (strategies) by the central bank and the fiscal authority can be characterized by the respective first order conditions. We start with the optimal choice of the central bank. The first order condition of F_0 is given by

$$
W_{F_0} = \Big[U'(C_0(P)) - \beta R_0^* R_1^* \Big] \frac{dC_0}{dF_0} + \beta \frac{dE[ZK^{\alpha} - R_1^* I_w | P]}{dF_0} - \beta R_0^* R_1^* \tau^{cb} \tag{17}
$$

where the first two terms reflect the marginal benefit of additional liquidity support, while the last term captures its marginal cost. The marginal benefit captures the effect that an additional unit allocated to F does. This relaxes the banks collateral constraint, in turn reducing the expected domestic interest rate. A lower $E(R_1)$, leads households to consume more in the first period, as shown by the first term of the equation, and firms are expected to produce more, as shown by the second term. Meanwhile, the marginal cost of additional liquidity support is equal to the financial cost since it reduces final consumption by $R_0^* R_1^* \tau^{cb}$ in every state.

Importantly, the marginal benefit of F is conditional on P while the marginal cost is not. A greater ELA lowers the marginal benefit of reserves as depicted in W_{F_0}. To see this, denote $\bar{F}_0(ELA)$ as the necessary amount of reserves for the central bank to eliminate crises in this economy. Note that $\bar{F}_0$ falls as the fiscal authority provides households with a larger support through ELA. Céspedes and Chang (2024) argue that central banks optimal choice of reserves lies strictly between 0 and $\bar{F}_0(ELA)$ when $\tau^{cb} > 0$. Hence, as the size of ELA

increases, $\bar{F_0}(ELA)$ gets closer to zero. Eventually, by continuity it must be true that the optimal choice of the central bank also starts to fall with greater ELA. This is,

$$\bar{F_0}(ELA) = \theta_H[R_0^*C_{0f} + Q_{1f}K_{2f} + R_0^*G_0 + G_1] - (T + X_{1f}N) - \theta_H ELA$$

The optimal policy (strategy) of the central bank, is then a function of what that of the fiscal authority. Let $\hat{F_0}(B, ELA)$ denote the central bank's best response to (B, ELA). Function $\hat{F_0}(B, ELA)$ responds to ELA through W_{F_0}, and responds to B through households limited liability.[16]

Let us now turn to the optimal policy (strategy) of the fiscal authority. This policy maker has two instruments, long-term debt and ELA, to maximize households lifetime expected utility. Like in the case F_0, the marginal benefit of ELA is through its effect on the expected domestic interest rate, as the marginal cost is equal to the opportunity cost of not investing that unit of resources in the long term project. This is shown by the first order condition of W with respect to ELA in Equation 18 below. Likewise, the first order condition with respect to issuing long term debt, W_B, is depicted by Equation 19 below.

$$W_{ELA} = \left[U'(C_0(P)) - \beta R_0^* R_1^*\right] \frac{dC_0}{dELA} + \beta \frac{dE[ZK^\alpha - R_1^* I_w|P]}{dELA} - \beta R_0^* R_1^* \tau^{lt} \tag{18}$$

$$\begin{aligned} W_B = &\left[U'(C_0(P)) - \beta R_0^* R_1^*\right] \frac{dC_0}{dELA}\frac{dELA}{dB} + \beta \frac{dE[ZK^\alpha - R_1^* I_w|P]}{dELA}\frac{dELA}{dB} \\ &- \beta R_0^* R_1^* \tau^{lt} \frac{dELA}{dB} - \beta R_0^* R_1^* (\tau^{fa} - \tau^{lt}) \end{aligned} \tag{19}$$

With respect to long-term debt, as long as it is costly ($\tau^{fa} > \tau^{lt}$), the only benefit in

[16] C_2 cannot be negative in any state of the economy. Thus, a greater B through limited liability limits the amount of F_0.

our model from issuing B is that it allows the fiscal authority to allocate more resources to ELA. To see this, assume that B is zero, leading ELA to take values between zero and A. First, consider the case that W_{ELA} is equal to zero for an ELA strictly less than A. In this scenario, the upper bound on all feasible values of ELA is not binding and, therefore, the derivative $\frac{dELA}{dB}$ in W_B is equal to zero. As a result, in this case W_B would be equal to $-\beta R_0^* R_1^* (\tau^{fa} - \tau^{lt})$ which is negative when $\tau^{fa} > \tau^{lt}$. Optimality, then implies that B is equal to zero. Now, consider the case that W_{ELA} evaluated at ELA equal to A is positive. Thus, the fiscal authority would like to allocate more resources to the short-run but cannot unless it issues more long-term debt. This means that the derivative $\frac{dELA}{dB}$ in W_B is equal to one, and the fiscal authority would compare the marginal benefit of relaxing the bound on ELA with the marginal cost which is equal to the financial cost of issuing more debt $(R_0^* R_1^* \tau^{fa})$.

A similar reasoning as with the central bank can be used to show that the fiscal authority's optimal policy is decreasing in F–i.e. its best response to F is decreasing. Thus, as the liquidity support of a central bank increases, the optimal behavior of the fiscal authority is to reduce ELA and, when binding, also B. This is a key result since it indicates that the optimal behavior of the fiscal authority is to increase households tax burden when it expects the central bank to provide a stronger liquidity program. Therefore, indirectly, by implementing a greater F, the central bank is inducing a cost on households that it is not incorporating in its decision. This result is formalized by Proposition 3 as follows:

Proposition 3 (Hidden Cost of Reserves)**.** *As the central bank provides more liquidity support to domestic banks, the optimal response of the fiscal authority is to increase households tax burden.*

The proof of Proposition 3 is straight forward. Let $(\hat{B}, E\hat{L}A)(F_0)$ denote the fiscal authority optimal response to F_0. Replacing this best response in Equation 7 and rearranging terms, we obtain that

$$T^b = R_0 G_0 + G_1 - E\hat{L}A(F_0)$$

which implies that, to the extent that increases in F_0 reduce ELA, then optimally more liquidity support increases households tax burden.

We can now define the policy equilibria as the Nash equilibrium that stems from the strategic interaction of between the central bank and the fiscal authority. This is:

Definition 3.1 (Nash Equilibrium)**.** Let Proposition 2 be satisfied, and let $W(P)$ reflect the household lifetime expected utility in a competitive equilibrium for any feasible P. Let $\hat{F}_0(B, ELA)$ and $(\hat{B}, E\hat{L}A)(F_0)$ denote the central bank's and the fiscal authority optimal strategies at $t = 0$. Then any triplet $\{F_0^*, (B^*, ELA^*)\}$ is an equilibrium if $F_0^* = \hat{F}_0((B^*, ELA^*))$ and $(B^*, ELA^*) = (\hat{B}, E\hat{L}A)(F_0^*)$.

This definition now allows us to analyze the optimal reserves, F_0^* and SWF policies. Where ELA^*, due to its complete usage during a crisis $(s_1 0)$ is isomorphic to a stabilization sovereign wealth fund. Therefore, we can now study the relationship between these instruments in an economy where private agents act competitively and policy makers strategically interact with each other. That is the purpose of the next section.

4 SWF Policy and the Optimal Level of Reserves

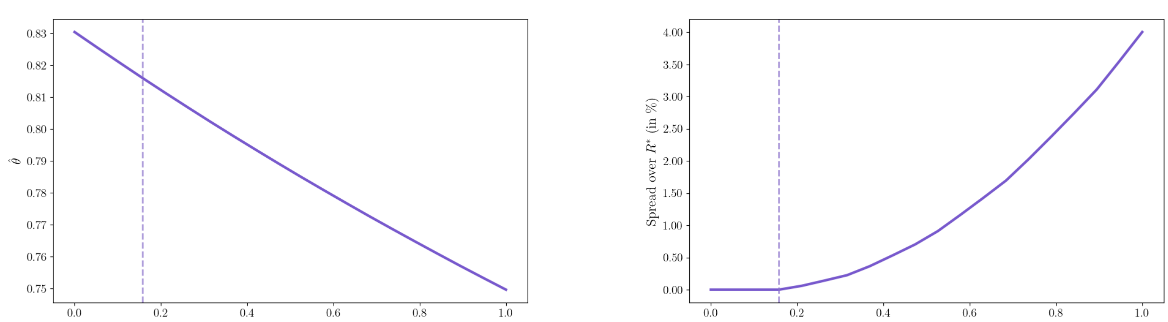
In this section, we implement the characterization of the model's equilibria developed in the previous section to explore its the qualitative outcomes via simulation analysis. Our goal is to explore the model's outcomes and mechanisms that yield empirically testable results.

In our benchmark, we set most of the parameters equal to those in Céspedes and Chang (2024). The world real interest rate, R_t^*, is set to one in both periods 0 and 1. The share of the non-tradable input in the capital production function, γ, is 0.5. We assume the weight of capital in the production function for tradable goods, α, to be 0.8. Regarding the household's preferences, we set the intertemporal discount factor, β, to 0.95, and the elasticity of substitution in consumption, σ, to 2. For the parameter θ, we assume a uniformly distributed grid with 150 points ranging from 0.3 to 0.8. Similarly, we define equally spaced grids for foreign reserves, debt and ELA. Each with 150 points between 0 and 2. The Bank's non-tradable endowment is set to 1, and the tradable endowment is set to 0.2. We assign a value of 2 to the productivity of capital, Z, to ensure that the frictionless marginal productivity of capital is greater than 1.

For the parameters related to fiscal policy, we set government expenditures, G equal to 0.3 in each period, and the size of the SWF, A, to 0.2. In our benchmark simulations, we let the term premium of holding reserves, τ^{cb}, and foreign debt, τ^{FA} to be equal and set to 0.02. The long-term return of the productive investment, τ_{lt} is equal to 0.01. We perform extensive sensitivity analysis about the value of these parameters to study their role in determining the interaction between the presence of the SWF, its policy, and the optimal level of foreign reserves. Table 1 summarizes our parametrization of the model.

4.1 The Laissez-Faire Equilibrium

Figure 2: Threshold $\hat{\theta}$ and Interest Rate as a Function of the *Tax Burden* (T^b)



(a) Threshold $\hat{\theta}(\tau_b)$ (b) Spread $R_1(\tau_b)$

Note: In this plot, we solved the *Laissez-Faire* equilibrium for a grid of values of $G_t \in [0, 0.5]\ \forall\ t = 0, 1, 2$. For each case, we assume that $G_0 = G_1 = G_2$. The *tax burden*, T^b, is defined as in

We define the Laissez-Faire (LF) economy as the equilibrium without any type of policy

Table 1: Summary of Model Parameters

Parameter Name	**Symbol**	**Value/Range**
Structural Parameters		
World real interest rate	R_t^*	1
Share of non-tradable input in capital production	γ	0.5
Weight of capital in tradable goods production	α	0.8
Intertemporal discount factor	β	0.95
Elasticity of substitution in consumption	σ	2
Bank's non-tradable endowment	N	1
Bank's tradable endowment	T	0.2
Productivity of capital	Z	2
Fiscal Parameters		
Government expenditures	G_t	0.3
Initial Endowment	A	0.2
Term premium of holding reserves	τ^{cb}	0.04
Term premium of foreign debt	τ^{FA}	0.04
Long-term return of savings SWF	τ^{lt}	0.2
Grid Parameters		
Grid points for θ	θ	150 points, range [0.4, 0.7]
Grid points for foreign reserves	F	150 points, range [0, 2]
Grid points for foreign debt	B	150 points, range [0, 2]
Grid points for foreign ELA	ELA	150 points, range [0, 2]

intervention during a financial crisis. Because the economy is distorted by both the financial friction and the effects of government spending, initial consumption C_0 depends on the realization of θ_s as well as on the tax burden implied by the government's expenditure path, which is given by

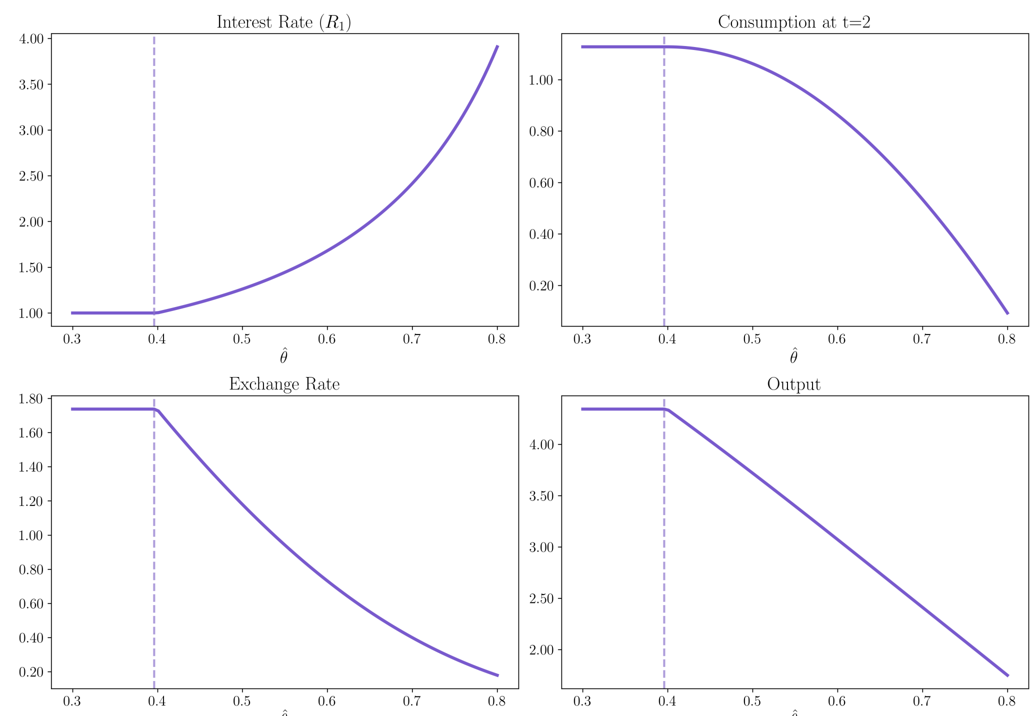
$$T^b = R_0^* G_0 + G_1.$$

As households face higher taxes, their optimal consumption in period 0 falls, but their borrowing needs rise because they must finance the tax burden. Figure 2a shows that

$\hat{\theta}$ is a decreasing function of T^b. Since higher aggregate debt pushes the economy closer to a binding collateral constraint—as illustrated in Figure 2b—the interest rate spread on borrowing increases as the fiscal burden rises.

Under our benchmark parametrization, the threshold $\hat{\theta}$ in this equilibrium is 0.45 and the economy faces a 69 percent probability of facing a financial crisis. As shown in Figure 3, whenever the constraint binds ($\theta > \hat{\theta}$), the spread on the interest rate increases rapidly. As the credit market deteriorates, the exchange rate depreciates and capital becomes expensive. This forces firms to reduce investment which, in turn, lowers output. As a result, expected consumption in $t = 2$ falls. The presence of both the financial friction and the government spending turns out to be very costly in terms of welfare for the agents. In particular, a representative household in this economy would experience a welfare gain equivalent to 18.5 percent of lifetime consumption from switching to a frictionless economy.

Figure 3: Laissez-Faire: Financial Crises



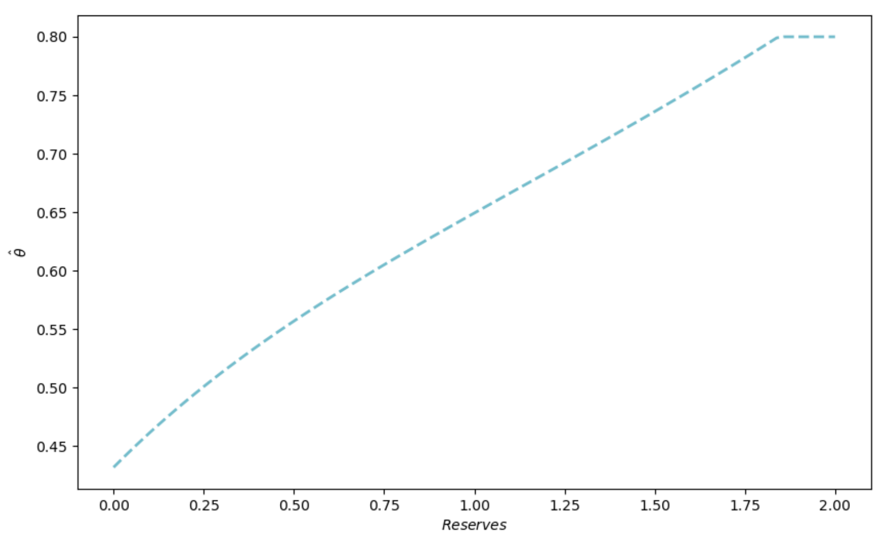
Note: Laissez-Faire equilibrium for different values of θ. Benchmark parameterization.

4.2 Foreign Reserves and Fiscal Stabilization Policies

We now study the equilibrium in an economy where the central bank provides stabilization through liquidity provision (i.e., reserves), and the fiscal authority enacts countercyclical policy by using the resources of the stabilization SWF–ELA–to subsidize its fiscal expenditures.

As in explained in the previous section, contingent on the cost of accumulating reserves, τ^{cb}, the central bank can mitigate the effects of the financial friction by borrowing in international markets to accumulate reserves and subsequently lending these resources to domestic banks in the event of a crisis. Figure 4 shows that, as the central bank accumulates more foreign reserves, the threshold $\hat{\theta}_{RR}$ increases, eventually reaching a level at which the probability of facing a binding collateral constraint becomes zero. Indeed, if the cost of accumulating reserves were zero, the central bank's optimal solution would be trivial.

Figure 4: Threshold $\hat{\theta}$ as a Function of Foreign Reserves



Note: In this plot, the central bank implements foreign reserves policy to provide liquidity to banks in periods where the collateral constraint would bind otherwise. For numbers above the upper-bound of the distribution of θ, the probability of a financial crises is zero.

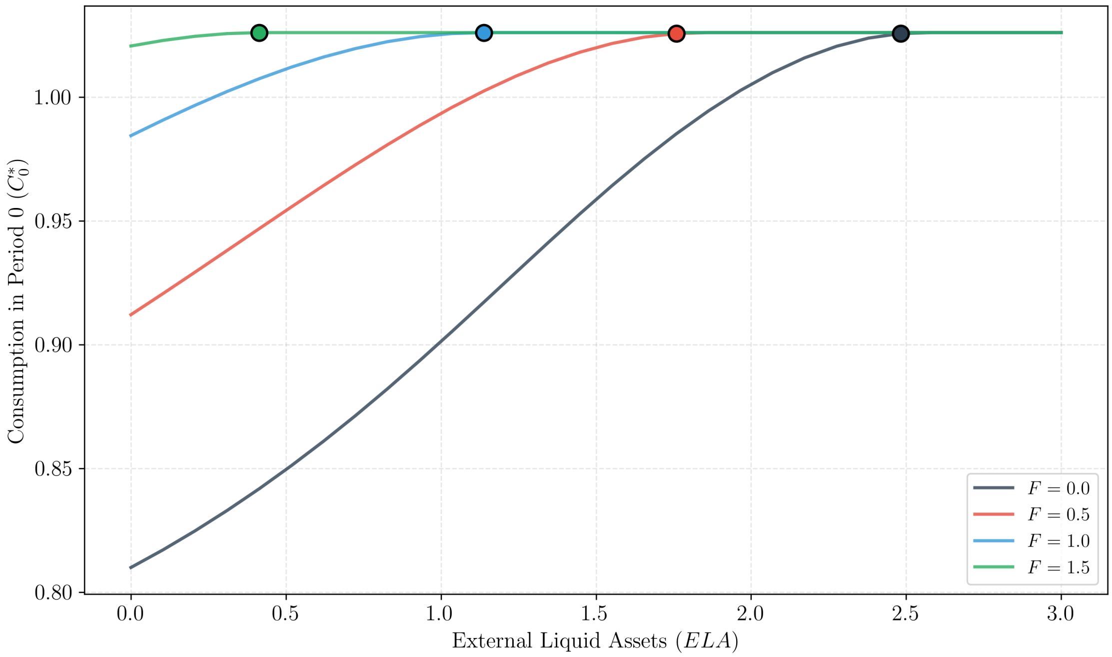
In turn, recall from the previous section that the fiscal authority can use its initial endowment (A) and borrowing capacity (B) to implement countercyclical policy—a stabilization SWF—and (or) to invest in a long-term project with a risk-free return premium τ^{lt}. If it chooses to accumulate resources in the stabilization SWF–ELA–it can be used to provide a fiscal transfer that reduces the household's tax liabilities in period 1 in the event of a financial crisis. If the collateral constraint never binds, the stabilization fund remains inactive and the resources are saved for redistribution in the final period.

In expectation, the presence of the stabilization SWF lowers the household's financing needs and thereby reduces T^b, easing the collateral constraint. Indeed, Figure 5 shows that for any fixed level of reserves–F–, increasing the assets allocated to the stabilization fund raises period-0 consumption. Across reserve levels, the implied consumption profiles converge to essentially the same optimal value of C_0 (1.026) at their respective choices of ELA by the fiscal authority, indicating that the two instruments operate through similar, yet not equal, liquidity-provision channels. In particular, notice that for higher level of reserves, the optimal size of the stabilization SWF is lower to maintain the maximum level of consumption.

A central result of this mechanism is that higher international reserves reduce the optimal size of the stabilization SWF, revealing a clear substitution margin between reserves and fiscal liquid assets. This is, even though both instruments provide liquidity in times of financial distress, there is arguably a role for both policies to be applied simultaneously, where the larger the size of one instrument (e.g. reserves) the lower the optimal size of the other (e.g. stabilization SWF).

As discussed above, policy equilibria in the decentralized economy emerges from the strategic interaction between the fiscal authority and the central bank, each choosing Γ and F, respectively, to maximize households' lifetime utility. Figure 6 displays the government's best-response schedule for external liquid assets, $ELA^*(F)$, alongside the central bank's best-response schedule for reserve accumulation, $F^*(ELA)$. The two downward-sloping loci reveal the strategic substitutability between these liquidity instruments: increases in ELA

Figure 5: Consumption and External Liquid Assets (Stabilization SWF)

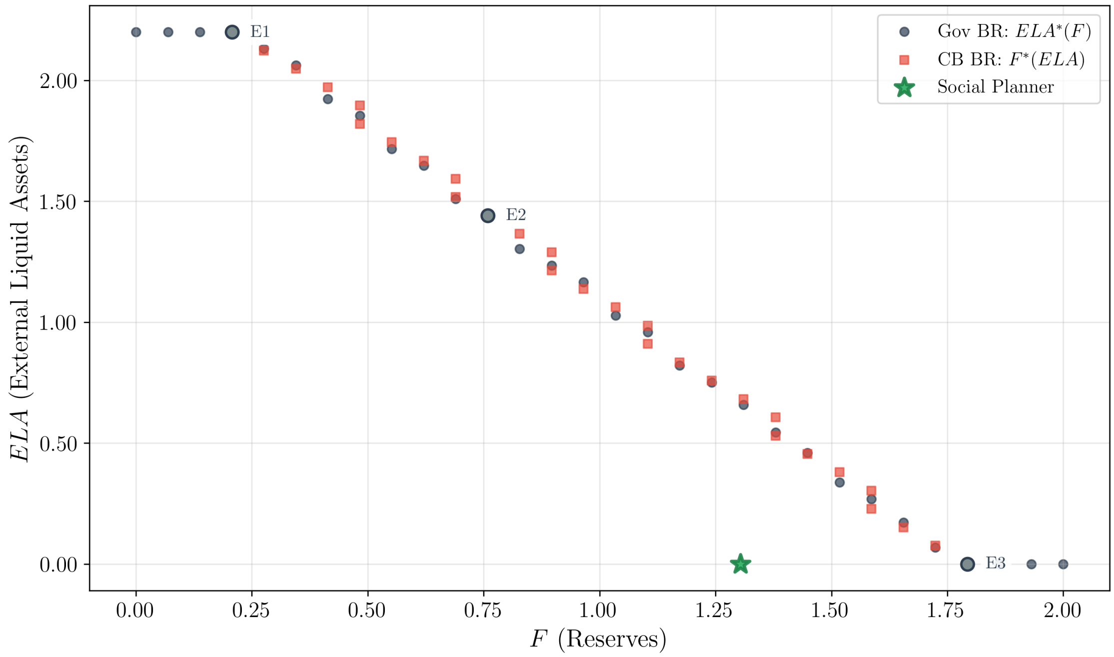


Note: For a given level of foreign reserves, increasing the resources available to the stabilization SWF raises household consumption in period 0. Across reserve levels, all trajectories converge to approximately the same optimal value of C_0 (1.026) at their respective optimal ELA choices, which are indicated by the large circular markers in the figure. All values are computed under the benchmark parameterization.

lower the marginal value of reserves, while higher reserve holdings reduce the optimal size of ELA–the stabilization SWF. Even though both policies supply liquidity in crisis states, they generate distinct fiscal and opportunity costs, producing a non-cooperative allocation that embeds both a fiscal externality (via the tax burden) and a liquidity externality (via the collateral constraint). The intersection of these best responses yields multiple (Nash) equilibria.

Table 2 summarizes the different equilibrium allocations in the decentralized economy, depicted by E1, E2, and E3 in Figure 6. The three Nash equilibria illustrate the substitutability between reserves and the stabilization SWF: low reserve positions (E1) are paired with a large stabilization fund and high debt issuance, whereas high reserve positions (E3) eliminate the need for a stabilization fund and are supported by zero fiscal debt. The intermediate equilibrium allocation (E2) is in between the E1 and E3, exhibiting the possibility

Figure 6: Best-Response Functions of the Government and the Central Bank



Note: The figure plots the government's best response for external liquid assets, $DLA^*(F)$, and the central bank's best response for reserve accumulation, $F^*(DLA)$, as functions of one another. Equilibrium outcomes under the decentralized Nash game are labeled E1–E3. The green star marks the allocation chosen by the social planner. All quantities are computed under the benchmark parameterization.

Table 2: Equilibrium Outcomes: Nash vs. Social Planner

Equilibrium	F (Reserves)	SWF (Stabilization)	B^* (Debt)	U (Utility)	$\hat{\theta}$ (Threshold $\hat{\theta}$)	Crisis Prob. $P(\theta > \hat{\theta})$
E1	0.2069	2.2000	2.0000	-0.3177	0.6916	2.80%
E2	0.7586	1.4414	1.2414	-0.3098	0.6986	0.40%
E3	1.7931	0.0000	0.0000	-0.2943	0.7038	0.00%
Social Planner	1.3034	0.0000	1.1034	-0.2806	0.6132	28.93%

Notes: All values are computed under the benchmark parameterization. Under E3, the central government invests the full endowment A in the long-term project (savings SWF). For the Social Planner, the external liquid assets (ELA) correspond to $F + stab\ SWF$. $\hat{\theta}$ is the equilibrium collateral threshold and $P(\theta > \hat{\theta})$ the implied crisis probability.

of a mixed case where nor the reserves or the size of the stabilization SWF are at their limit levels.

Taking stock of the model's main results. Overall, the results show a clear pattern in how reserves and the stabilization SWF interact. Both instruments ease the collateral constraint by providing liquidity in crisis states, but they do so with different fiscal and opportunity costs. This generates a systematic substitution margin: when reserves are low, the fiscal authority relies more heavily on the stabilization fund, while higher reserve positions reduce the need for external liquid assets and lead to lower debt issuance. Furthermore, in equilibrium the optimal level of reserves is always smaller when the optimal size of the stabilization SWF is greater than zero. Thus, while there is an equilibrium with a sufficiently large level of reserves that eliminates the need to create a SWF, there are other equilibria where this is not the case and both instrument coexist. In the next section, we test the empirical plausibility of these results.

5 Empirical Evidence

In this section we use a panel fixed-effects identification to provide cross-country empirical evidence that supports the model's implications for the relationship between SWFs, their type, and the accumulation of foreign reserves.

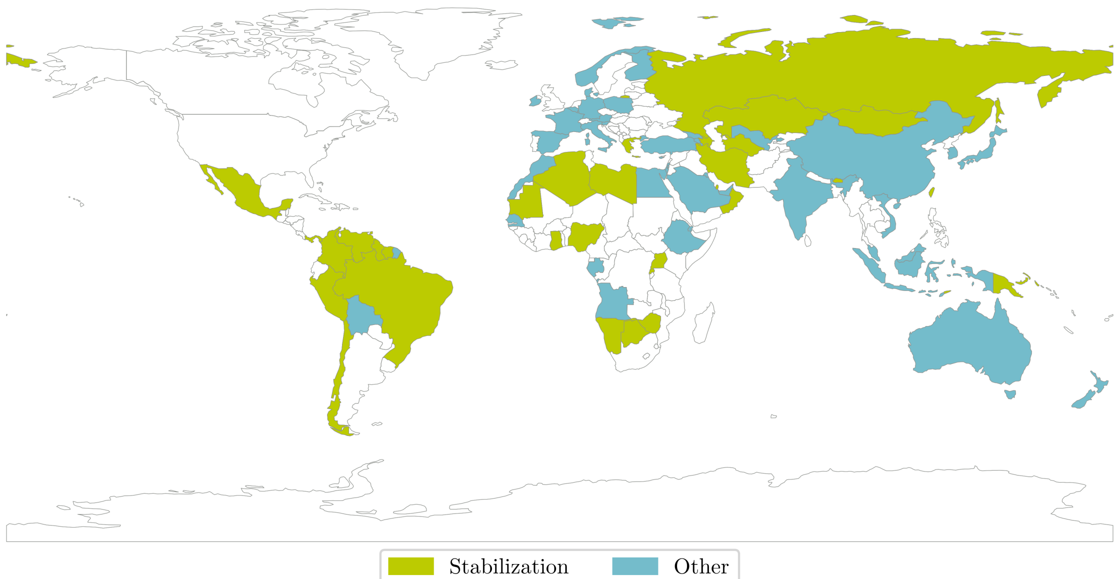
We show that, on average, countries with SWF accumulate less reserves than those without it. Importantly, this result holds only through the presence of a stabilization SWF. These empirical result point out in the direction of the main results of the model, albeit with caution as they do not pose any causal claim.

5.1 Data

The main data source is Acosta-Henao, Martínez, and Rondón-Moreno (2026). This companion paper constructs a novel comprehensive dataset documenting the presence of a national SWF in 86 countries at a yearly frequency between 1970 and 2022. This is, a dummy variable–namely SWF_t^i–that takes the value of 1 if there is at least one SWF operating in country i during year t. In total, there are 114 national SWFs in the sample. More importantly, this dataset provides a classification of each SWF's purpose. In the data, a SWF can be either a stabilization fund–a dummy variable, $StabSWF_t^i$, that is equal to 1 if the fund can be used by the government for macroeconomic stabilization purposes–, a savings or other purpose fund–a dummy variable, $NonStabSWF_t^i$, that is equal to 1 if the fund has a long-term savings purpose, other specified objective (e.g. strategic purposes)–, or other non-specified objective.[17]

Figure 7 provides the most resent snapshot of the presence and type of SWFs around the world. The figure shows the countries with the presence of at least one SWF in 2022. Those in green are categorized as stabilization SWFs, and those in blue have other purposes. The rest are countries for which there is no SWF reported in that year.

Figure 7: Countries with SWF by type in 2022



We complement the data on SWFs and their type with a comprehensive set of macroeconomic variables. When studying the relationship between SWFs, their type, and the

[17]The primary data on sovereign wealth funds (SWFs) by country originates from the International Forum of Sovereign Wealth Funds (IFSWF) Data Project, supplemented with official sources such as the IMF, World Bank, OECD, and national Ministries of Finance. The classification results from a thorough text analysis of the statutes governing each SWF in the sample. See Acosta-Henao, Martínez, and Rondón-Moreno (2026) for further details on these data.

accumulation of foreign reserves, these macroeconomic variables allow us to construct the dependent variable of interest in our study–the foreign reserves to GDP ratio–and to control for other determinants of foreign reserves holdings across countries that are well known in the literature.

For all 86 countries from 1970 to 2022 we construct series of: the foreign reserves to GDP ratio from the 2023 updated dataset from Lane and Milesi-Ferretti (2001); the monetary base, M2, as a fraction of GDP; the Chinn-Ito index, which indicates the degree of mobility in capital flows; the (log of) population; the real per capita GDP in dollars; the yearly nominal exchange rate depreciation rate with respect to the U.S. dollar; and the exports of commodities as a fraction of GDP. The source of these data, namely our baseline variables, is the World Bank´s Development Indicators (WDI).

To complement the baseline variables, we include other variables that allow us to study different motives of reserves accumulation other than the influence of a SWF. Those motives, with their respective additional variables, are: 1) Trade: Imports as share of GDP; and exports volume, from the WDI. 2) Financial Stabilization: Dummies that categories countries as either hard peggers, soft peggers, or none; the short-term debt as share of GDP, all from the International Monetary Fund (IMF); and the average of foreign reserves to GDP ratio from all other countries (i.e. the keeping up with the Joneses effect) which we construct for each country. 3) Mercantilist: Terms of trade index; and GDP growth rate, from the WDI. 4) Financial development: Domestic, private foreign, and public foreign, financial liabilities as share of GDP, all from the IMF. 5) Fiscal: Government gross debt as a share of GDP, and government net lending as a share of GDP, all from the IMF's World Economic Outlook.

5.2 Cross-Country Estimates

As a starting point, we first document how the presence of any type of SWF relates to the accumulation of foreign reserves. For this purpose, we estimate

$$r_{it} = \alpha_i + \alpha_t + \beta SWF^i_{t-1} + \Gamma X_{it-1} + \epsilon_{it} \tag{20}$$

Where r_{it} is the foreign reserves to GDP ratio of country i in year t, α_i and α_t are country and time fixed effects, SWF^i_{t-1} is a dummy variable that takes the value of one if there is a SWF in country i during year $t-1$, X_{it-1} is the set of lagged country-level macroeconomic controls described in the previous section with their corresponding vector of coefficients, Γ, and ϵ_{it} is the mean-zero *i.i.d.* disturbance. Though this regression aims to document the correlation between the presence of a SWF and foreign reserves, controlling for lagged explanatory variables together with country and time fixed effects contributes to deal with reverse causality concerns.[18]

The coefficient of interest in Equation 20 is β, which captures the average difference in reserves to GDP ratio between countries with the presence of a SWF and countries without it that is not explained by either time-invariant country characteristics, a common shock across countries in a given year, or different country-level macroeconomic characteristics. Column 1 in Table 3 shows the result of this estimate, which is significant at a 90% level of confidence. On average countries with a SWF have a foreign reserves to GDP ratio 2.5 pp lower than countries without any type of SWF.

These results indicate that, once a SWF is set, a country is expected to accumulate on average a lower level of reserves. Yet, this does not imply that creating a SWF causes a country to accumulate less foreign reserves. Documenting the reasons by which SWFs

[18]For instance, the possibility of a SWF emerging within a country as a response to its level of reserves. Yet, it is hard to consider this a plausible scenario if the fund was set in period $t-1$ and we analyze how its presence relates to foreign reserves in t, as described by Equation 20.

Table 3: Foreign Reserves to GDP, SWFs, and Central Bank Independence

	(1) Res/GDP	(2) Res/GDP	(3) Res/GDP	(4) Res/GDP	(5) Res/GDP	(6) Res/GDP
$SWF\_i,\, t-1$	0.028	-0.010**	0.034	0.003		
	(0.020)	(0.005)	(0.028)	(0.007)		
$StabSWF\_i,\, t-1$			-0.011	-0.023***	0.018	-0.020***
			(0.035)	(0.008)	(0.026)	(0.005)
$NonStabSWF\_i,\, t-1$					0.021	-0.008
					(0.023)	(0.006)
Observations	1972	1972	1972	1972	1972	1972
R^2	0.696	0.888	0.696	0.889	0.694	0.889
Controls	✓	✓	✓	✓	✓	✓
TimeFE	✓	✓	✓	✓	✓	✓
CountryFE	✗	✓	✗	✓	✗	✓
Sample	Full	Full	Full	Full	Full	Full

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

endogenously emerge goes beyond the scope of this work. However, our data allow us to dig deeper into analyzing the role of heterogeneity across SWFs.

We exploit this novel dataset in a dimension that had not been documented before, the purpose of the SWF. By having categories of SWFs by their type, we can tests directly the implication of the model on the relationship between the type of fund and the accumulation of reserves. This is, we are able to test whether there is cross-country evidence that an economy with a stabilization SWF has lower foreign reserves to GDP than an economy without it. For this, we estimate

$$r_{it} = \alpha_i + \alpha_t + \delta SWF^i_{t-1} + \eta StabSWF^i_{t-1} + \Gamma X_{it-1} + u_{it} \tag{21}$$

Compared to Equation 20, Equation 21 now includes the dummy $StabSWF^i_{t-1}$ which takes the value of 1 if the country has a SWF with a stabilization purpose.

Column 2 in Table 3 shows the result of estimating Equation 21. On average, countries with a stabilization SWF have 2.1 pp less foreign reserves to GDP tan countries without that type of fund. This estimate of coefficient η in Equation 21 is significant at the 95% level of confidence. Importantly, the coefficient of SWF^i_{t-1}, δ, is not significant anymore. Therefore, the results of column 1 in the table, are driven by stabilization SWFs. Once we take into account the type of fund, as shown in column 2 of the table, it is having a stabilization SWF what generates the negative relationship between SWF and reserves.

Altogether, these empirical results provide support for the model's main mechanism. Presuming that policy-makers across countries optimize as we assume in our model, there are countries that locate themselves in the equilibrium E3 shown in Figure 6. This is, an equilibrium with no SWF and the largest level of reserves in the decentralized economies. However, other countries optimally set a SWF, which results into an optimal level of reserves that is lower (equilibria E1 and E2 in Figure 6) than for those without a SWF. This is, our estimates point out toward the substitutability between SWFs and foreign reserves which arguably occurs as long as the SWF has a stabilization purpose.

6 Social Planner and Welfare

So far, we have been silent about welfare implications of the optimal policies in the decentralized equilibrium. On one hand, the emergence of multiple equilibria does not imply that all three equilibria in the decentralized economy have the same welfare. It just shows that, even with the same structural parameters, a small open economy can achieve an equilibrium with different levels of reserves and stabilization SWF. Those equilibria share one characteristic in the extensive margin. It is, the presence of SWF optimally chosen by the fiscal authority, impply the central bank. This is supported by the data, as shown in the previous section.

Nothwithstanding this, it is of relevance to test how the equilibria in the descentralized economy with policy making interacting non-cooperatively compares to a welfare-dominant world. This is a world of a social planner. It chooses B, ELA, and F. That is, the social planner integrates the policy choices of both the fiscal authority and the central bank. Therefore, the planner has access to a long-term investment opportunity and it can reduce households tax burden at $t = 1$ as the fiscal authority, but it can also provide liquidity support to domestic banks as the central bank.

In the model, the social planner faces the same budget constraints as the fiscal authority with two exceptions: at $t = 1$ it invests s_{1sp} in international markets and F_{sp} liquidity support to domestic banks, both investments earning $R_1 * (s_{1sp} + F_{sp})$ at $t = 2$; and, second, there is no need for a transfer to the central bank. As a result, any resources saved through ELA_{sp} are used to finance the stream of fiscal deficits and the liquidity support is provided to domestic banks.

$$ELA_{sp} = R_0^*(G_0 - T_0) + \frac{G_1 - T_1}{R_0^*} + \frac{F_{sp} + s_{1sp}}{R_0^*}$$

With a social planner, households consumption in the final period remains as in Equation 8 with $F_0 = 0$ since there is no issue of long-term debt by the planner that is earmarked to use as reserves. What differentiates the decentralized game form the social planner problem is the impact on the collateral constraint. The key idea is that any resources that are used to support domestic banks come directly or indirectly from households. If ELA_{sp} is positive, then a positive F_{sp} requires that households pay a greater tax burden. Moreover, if ELA_{sp} is equal to zero, then any positive amount of liquidity support is financed through a stream of fiscal surpluses at $t = 0$ and $t = 1$. To see this, we derive function $\hat{\theta}_{sp}(C_0, F_{sp}, ELA_{sp})$ by using the definition of tax burden, the result of Proposition 2 that continues to hold, and the social planner's constraint at $t = 0$ and $t = 1$. Recall that $\hat{\theta}_{sp}(C_0, F_{sp}, ELA_{sp})$ determines the endogenous threshold upon which tighter international conditions become a domestic crisis.

$$\hat{\theta}_{sp}(C_0, F_{sp}, ELA_{sp}) \equiv \frac{T + X_{1f}N + F_{sp}}{R_0^*C_0 + Q_{2f}K_{2f} + R_0^*G_0 + G_1 - R_0^*ELA_{sp} + F_{sp}} \tag{22}$$

Since feasible values of $\hat{\theta}_{sp}(C_0, F_{sp}, ELA_{sp})$ are less than one, a simple derivative of Equation 22 with respect to reserves indicates that increases in F_{sp} reduce the probability of a crisis (a greater $\hat{\theta}_{sp}$). This is true despite of households facing a greater tax burden as F_{sp} gets larger. Additionally, when the constraint binds, R_{1s} must solve Equation 23 to clear the domestic credit market under the presence of a social planner.

In comparison to Equation 15, reserves are less efficient to reduce R_{1s}. While the marginal increase in F increased domestic credit supply by $\frac{R_1^*}{R_1^*-(1-\theta)R_{1s}}$, a social planner's marginal increment of F_{sp} increases domestic credit supply by $\frac{(1-\theta)R_{1s}}{R_1^*-(1-\theta)R_{1s}}$ which is strictly less. The reason is that the social planner takes into account the cost that reserves have on households tax burden, while the central bank does not, as shown by Proposition 3.

$$R_0^*C_0 + Q_{2s}K_{2s} + R_0^*G_0 + G_1 - R_0^*ELA_{sp} + F_{sp} = \frac{R_1^*}{R_1^* - (1-\theta)R_{1s}}[T + X_{1s}N + F_{sp}] \tag{23}$$

The problem of the social planner is then to choose B_{sp}, ELA_{sp}, and F_{sp} to maximize Equation 16 subject to the same constraints faced by the fiscal authority and the central bank with Equation 22 instead of Equation 14, and Equation 23 instead of Equation 15.

Relative to the allocations from the decentralized economy determined by the Nash equilibria, the social planner internalizes the externalities and chooses a configuration in which stabilization is provided solely through reserves while the fiscal authority allocates all endowed resources to a savings SWF.

In contrast to the Nash equilibria, the social planner chooses an intermediate reserve position and dispenses entirely with the stabilization fund, issuing the smallest amount of total debt ($F + B^*$). By internalizing both the fiscal and liquidity externalities embedded in the decentralized game, the planner selects a lower threshold $\hat{\theta}$–hence a higher crisis probability–while still generating higher expected utility than any of the decentralized Nash outcomes. This comparison highlights how strategic interactions shape the mix of liquidity instruments and how the planner reallocates across them to improve welfare, even when doing so alters the implied crisis risk.

Once the fiscal and liquidity externalities are internalized, the planner prefers to rely on reserves as the sole stabilization tool and to allocate all endowed resources to a savings SWF. In this context, even though the planner is a sole entity with access to both instruments–reserves and the stabilization SWF–, the distinction among them stems from their usage. Reserves are used as liquidity provisions to alleviate the balance sheet of banks during a crisis.

The equilibrium represented by a star in Figure 6 shows this last result. The welfare-dominant equilibrium implies using reserves as the only stabilization tool. Yet, this has a lower level of reserves than a decentralized equilibrium with no stabilization SWF (E3), but larger than those equilibria with the presence of the fund (E1 and E2). The former result is due to the fact that the planner internalizes the cost of issuing debt to accumulate reserves over the household's tax burden. The latter result is due to the fact that the planner internalizes that the benefit of establishing the SWF as opposed to a committed savings fund with larger long-term returns–which is alleviating the household's tax burden–can be completely alleviated by choosing more long-term savings and more reserves. Yet, this optimal policy might be desirable but not achievable due to other constraints that are not in our model. For instance, political economy issues, commitment problems (e.g. incentives to deviate resources of a long-term fund into stabilization), or conflicts with other policies from the central bank and the fiscal authority are possible reasons that justify remaining in a decentralized world where each authority takes the actions of the other as given.

7 Conclusion

In this study, we explore the role of external public savings, such as Sovereign Wealth Funds, as potential stabilizing tools and their interaction with central banks' reserve accumulation policies through the lens of a new model that considers both the central bank and the fiscal authority as independent entities. Our results show that assets under management by a stabilization SWF can partially substitute foreign reserves.

We complement our theoretical analysis by using a novel dataset to provide empirical evidence showing a negative relationship between the presence of a SWF and a lower reserves to GDP ratio across countries. The key driver behind this difference is the presence of a stabilization SWF, as in the model.

Potential extensions of this work include investigating potential commitment issues related to the fund's long-term objectives, and expanding our theoretical model to an infinite-period framework to provide deeper insights into the normative implications of SWF design and its interaction with central bank policy.

References

Acosta-Henao, M., H. Martínez, and C. Rondón-Moreno (2026). A global sovereign wealth funds database.

Aizenman, J., Y.-W. Cheung, and H. Ito (2015). International reserves before and after the global crisis: Is there no end to hoarding? *Journal of International Money and Finance 52*, 102–126.

Aizenman, J. and R. Glick (2009). Sovereign wealth funds: Stylized facts about their determinants and governance. *International Finance 12*(3), 351–386.

Aizenman, J. and J. Lee (2007, May). International Reserves: Precautionary Versus Mercantilist Views, Theory and Evidence. *Open Economies Review 18*(2), 191–214.

Alfaro, L. and F. Kanczuk (2009, February). Optimal reserve management and sovereign debt. *Journal of International Economics 77*(1), 23–36.

Barbosa-Alves, M., J. Bianchi, and C. Sosa-Padilla (2024). International reserve management under rollover crises. Technical report, National Bureau of Economic Research.

Bianchi, J., J. C. Hatchondo, and L. Martinez (2018, September). International Reserves and Rollover Risk. *American Economic Review 108*(9), 2629–2670.

Bianchi, J. and G. Lorenzoni (2022). The prudential use of capital controls and foreign currency reserves. In *Handbook of International Economics*, Volume 6, pp. 237–289. Elsevier.

Bortolotti, B., V. Fotak, and W. L. Megginson (2015, 07). The Sovereign Wealth Fund Discount: Evidence from Public Equity Investments. *The Review of Financial Studies 28*(11), 2993–3035.

Calvo, G., A. Izquierdo, and R. Loo-Kung (2013, January-j). Optimal Holdings of International Reserves: Self-insurance against Sudden Stops. *Monetaria 0*(1), 1–35.

Céspedes, L. F. and R. Chang (2024, July). Optimal foreign reserves and central bank policy under financial stress. *American Economic Journal: Macroeconomics 16*(3), 230–67.

Feldstein, M. (1999). A self-help guide for emerging markets. *Foreign Affairs 78*(2), 93–109.

Ghosh, A. R., J. D. Ostry, and C. G. Tsangarides (2017). Shifting motives: Explaining the buildup in official reserves in emerging markets since the 1980s. *IMF Economic Review 65*(2), 308–364.

Gintschel, A. and B. Scherer (2008). Optimal asset allocation for sovereign wealth funds. *Journal of Asset Management 9*(3), 215–238.

Griffith-Jones, S. and J. Antonio Ocampo (2012, 12). Sovereign Wealth Funds: A Developing Country Perspective. In *Sovereign Investment: Concerns and Policy Reactions*. Oxford University Press.

Heller, H. R. (1966, June). Optimal International Reserves. *The Economic Journal 76*(302), 296.

Holmstrom, B. and J. Tirole (2011). *Inside and outside liquidity*. MIT press.

Holmström, B. and J. Tirole (2011). *Inside and Outside Liquidity* (1 ed.), Volume 1. The MIT Press.

Jeanne, O. and R. Rancière (2011, September). The Optimal Level of International Reserves For Emerging Market Countries: A New Formula and Some Applications. *Economic Journal 121*(555), 905–930.

Lane, P. R. and G. M. Milesi-Ferretti (2001). The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries. *Journal of international Economics 55*(2), 263–294.

López, D. (2023, March). SWF 3.0: How sovereign wealth funds navigated COVID-19 and changed forever. *Journal of International Business Policy*.

Megginson, W. L., M. You, and L. Han (2013). Determinants of sovereign wealth fund cross-border investments. *Financial Review 48*(4), 539–572.

Obstfeld, M., J. C. Shambaugh, and A. M. Taylor (2010, April). Financial Stability, the Trilemma, and International Reserves. *American Economic Journal: Macroeconomics 2*(2), 57–94.

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