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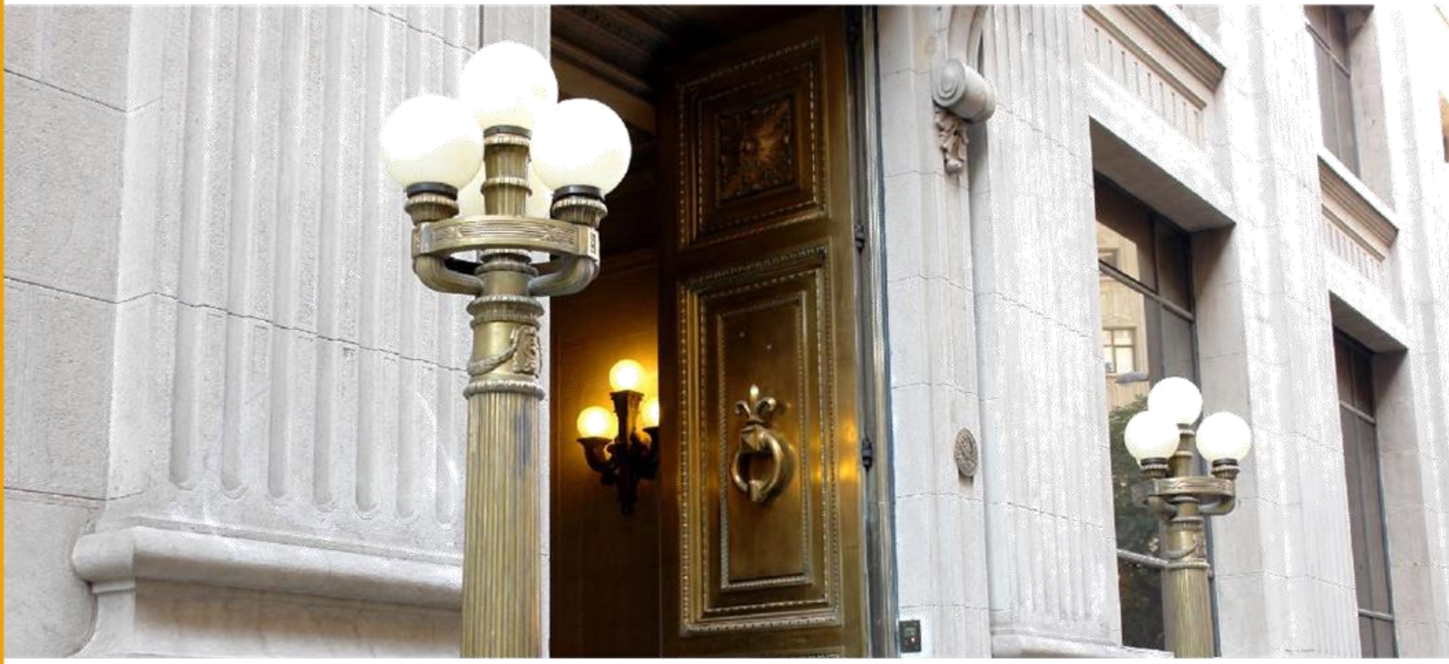

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Risk modeling with option-implied correlations and score-driven dynamics*

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Abstract

In this paper we make use of option-implied volatilities to build a time-varying implied correlation matrix. Then, we use this matrix to estimate jointly both the covariance matrix of the returns and the implied covariance matrix dynamics. Finally, we do a backtest and show that the proposed model can effectively use the risk-neutral information to model the variance of the returns and to forecast the Value-at-Risk. Our results show that the model obtains results comparable to the benchmark while considerably reducing the number of estimated parameters.

Resumen

En este documento utilizamos la volatilidad implícita en opciones financieras para construir una matriz de correlaciones implícita. Luego, usamos esta matriz para estimar conjuntamente la dinámica de la matriz de covarianzas de los retornos y de covarianzas implícitas. Finalmente, el modelo se prueba fuera de la muestra y mostramos que puede, efectivamente, utilizar información neutral al riesgo para modelar la varianza de los retornos y pronosticar Valor-en-Riesgo. Comparando con otros modelos, obtenemos resultados similares, aunque se reducen considerablemente los parámetros a estimar.

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I Introduction

Modeling and forecasting volatility is a crucial activity for financial decision-making. Most of the methodologies harnessing the dynamic of volatilities in financial markets are based on the use of historical returns (backward-looking information). In recent years different studies have shown that option-implied volatilities (forward-looking information) have a higher information content than volatility estimates obtained from historical data and can improve the accuracy of volatility forecasting models (Jiang & Tian, 2005; Wayne, Lui, & Wang, 2010; Schindelhauer & Zhou, 2018; Slim, Dahmene, & Boughrara, 2019) since financial options seem to harness the market expectations about the price of the underlying assets (Poon & Granger, 2003; Vanden, 2006; Christoffersen et al., 2013).

Option-implied volatilities arise from the Black & Scholes (1973) model, which allows the theoretical value of an option to be determined from the historical volatility of the underlying asset. By inverting their formula, we can determine the underlying asset volatility from the financial option market price. This result is known as the implied volatility of the asset underlying the option. Intuitively, one might expect an option's premium to increase as implied volatility increases. This is true for an individual stock, but not necessarily true for an index option. This is because the implied volatility of an index option is made up of the implied volatility of the index plus the covariance of its components. In other words, the value of an option on a set of shares depends on expectations about the volatility of the index and expectations about the covariance matrix of its components. It is for this reason that the implied covariance matrix is used in making financial decisions, such as asset pricing (Bakshi, Cao, & Chen, 1997), volatility forecasting (Bollerslev, Tauchen, & Zhou, 2009), portfolio management (Pan & Poteshman, 2006), financial hedging strategies (Bakshi & Kapadia, 2003) and as an indicator of market risk (Bernales & Valenzuela, 2016). However, these approaches has two common drawbacks. First, estimating multivariate volatilities becomes more and more complex with the increasing number of dimensions. Second, option-implied multivariate volatilities are not directly observable, and therefore, they must be estimated. For this reason, this paper contributes to the recent literature in financial econometric by introducing an econometric approach that can be easily extended to high-dimensional volatility models. In addition, we demonstrate the predictive capacity of the option-implied approach in the context of market risk forecasting.

Due to the above, the use of implied covariances or correlations (IC) has had less attention. This fact comes from the complexity of identifying the implied covariance matrix is due to its stochastic nature (Driessen, Maenhout, & Vilkov, 2009), the number of variables that must be identified in the correlation matrix and because these grow exponentially with the number of returns. One possible solution to this problem is to impose equi-implied correlations as Skintzi & Refenes (2005) and Driessen, Maenhout, & Vilkov (2013). This assumption is too restrictive though, because this correlations are different for each pair of returns in fact. For this reason, Buss & Vilkov (2012) allow the parameters to be different by estimating a explicit parametric form for the IC, although this method is only valid when the IV are above the current return's volatility, assumption that holds most of the time. Finally, Numpacharoen & Numpacharoen (2013) propose a modification to this method in order to ensure that the IC matrix will always be positive-semidefinite.

In this paper we address two problems. First, we identify the implied covariance matrix based on the methodologies of Buss & Vilkov (2012) and Numpacharoen & Numpacharoen (2013) using daily data of DAX index. Then, we model both the dynamics of the implied convariance matrix and the covariance matrix of the returns based on the Gorgi, Hansen, Janus, & Koopman (2018) methodology. Finally, we evaluate the predictive capacity of the model on the return covariance matrix by backtesting against common alternatives such as the EWMA and DCC models. We evaluate the models in prediction of the Value-at-Risk (VaR) of the returns.

Our results show that the model correctly fits the dynamics of the implied covariance matrix and the covariance matrix of the returns. These results are supported by the fact that the parameters obtained behave according to expectations and their statistical significance. Regarding the out-of-sample results, the backtest shows that the model outperforms to the EWMA model and obtain results at least as good as the DCC model. However, the proposed model considerably reduces the number of estimated parameters, which is why we consider it superior to the benchmark. These results suggest new lines of research in the development of multivariate models that allow the inclusion of information involved in financial options for the development of forecasting models.

We present our method in Section II. In Section III we describe the data, the benchmarks and the backtest exercise. We also show our main results. The Section IV concludes.

II Method

In this section we summarize the method we use to identify each element of the implied covariance matrix (in addition to the diagonal, which comes directly from option implied-volatilities data) and the estimation method. We also describe how we use the implied covariance matrix to model the covariances of the returns.

II.1 Implied Correlation Matrix

In order to identify the IC matrix, we use the methodology[1] employed by Numpacharoen & Numpacharoen (2013) to ensure the positive semidefiniteness of the IC matrix in every period. To do this, we need to establish the following definition.

Definition 1: A valid $n \times n$ correlation matrix must hold the following conditions:

1. The matrix must by symmetric, $\rho_{ij} = \rho_{ji}$ for $i \neq j$.

2. The elements in the diagonal should be equals to 1 (i.e. $\rho_{ii} = 1$).

3. The elements off the diagonal must be real numbers in the closed interval $[-1, 1]$.

4. The matrix must be positive semidefinite (i.e. its eigenvalues must be greater or equal to zero).

We can use definition 1 to fully identify the IC matrix. To show this, we define the positive semidefinites $k \times k$ matrices A and B and define $\kappa \in [0, 1]$. By doing this, we can say that the matrix $C = (1 - \kappa)A + \kappa B$ is also positive definite. In the same way, we define the following relationship between the implied correlation matrix and realized correlation matrix, that we call $R_t^Q := \{\rho_{ij,t}^Q\}$ and $R_t^P := \{\rho_{ij,t}^P\}$ respectively [2].

$$R_t^Q = R_t^P + \kappa_t (I - R_t^P) \tag{1}$$

[1]We recognize this method was first developed by Buss & Vilkov (2012) and also used by Buss et al. (2016), but their method do not ensure the covariance to be positive semidefinite for all periods.

[2]The superscripts Q and P indicates risk-free and physical measures. Meucci (2011) realize a detailed description of this metrics and their use in quantitative finance

Now we need to identify the elements of the matrix R_t^Q. For this purpose we assume that the implied correlations are consistent with the correlation formula such that the implied correlation of a market index (I superscript) is composed by the implied correlations and volatilities of the market as we show below.

$$(\sigma_t^{Q,I})^2 = \sum_{i=1}^{N} w_{i,t}^2 (\sigma_{i,t}^Q)^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,t} w_{j,t} \sigma_{i,t}^Q \sigma_{j,t}^Q \rho_{ij,t}^Q, \quad i \neq j \tag{2}$$

where σ_i^Q and σ_j^Q are the implied volatilities of i and j; w_i y w_j are the weights of i and j in the index I; and ρ_{ij}^Q is the implied correlation between i y j.

Note that all the components in (2) are identified by data except by ρ_{ij}^Q. But by using (1) and the fact that $\rho_{ij,t}^Q = \rho_{ij,t}^P + \kappa_t(1 - \rho_{ij,t}^P)$, we can fully identify R^Q, being κ_t the only missing parameter we need. With the purpose of showing this explicitly we combine (1) and (2) and show the resulting equation for κ_t.

$$\kappa_t = \frac{(\sigma_t^{I,Q})^2 - \sum_{i=1}^{N}\sum_{j=1}^{N} w_{it} w_{jt} \sigma_{i,t}^Q \sigma_{j,t}^Q \rho_{ij,t}^P}{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,t} w_{j,t} \sigma_{i,t}^Q \sigma_{j,t}^Q (1 - \rho_{ij,t}^P)}, \quad i \neq j \tag{3}$$

From which every element can be identified by data. Once we obtain κ_t replacing data, we can identify R_t^Q using (1) again. For the purpose of our empirical application we define $\rho_{i,t}^P$ as a 75 days rolling-window correlation and $w_{i,t}$ as

$$w_{it} = \frac{p_{it} s_{it}}{\sum_{i=1}^{N} p_{it} s_{it}} \tag{4}$$

Where p_{it} and s_{it} are the price and the quantity of the share i in circulation on the day t.

Finally, we can recover the IC matrix, V_t^Q, from the R_t^Q by doing the transformation:

$$V_t^Q = \mathrm{diag}\{V_t^Q\}^{1/2} R_t^Q \mathrm{diag}\{V_t^Q\}^{1/2} \tag{5}$$

Where $\mathrm{diag}\{V_t^Q\}^{1/2}$ are the implied volatilities that we obtain from data.

II.2 Model assumptions

Intending to use the IC matrix to model the historical variances, we base this part of the methodology in the work of Gorgi et al. (2018) which allows us to estimate jointly both covariance matrices.

Let r_t be a $k \times 1$ vector of daily log-returns. Let $\mathcal{F}_{t-1}$ the σ-algebra generated by the past values of r_t and let V_t^Q the IC matrix. Assume the following conditional densities:

$$r_t | \mathcal{F}_{t-1} \sim N_k(0, \Sigma_t) \tag{6}$$

$$V_t^Q | \mathcal{F}_{t-1} \sim W_k(\Sigma_t^Q, \nu) \tag{7}$$

Where Σ_t is the covariance matrix of the multivariate normal distribution and Σ_t^Q is the mean of the Wishart distribution with $\nu \geq k$ degrees of freedom. The assumptions (6) and (7) implied the following distribution for the squared returns:

$$r_t r_t' | \mathcal{F}_{t-1} \sim W_k(\Sigma_t, 1) \tag{8}$$

The measurement equations are defined as follows:

$$r_t = \Sigma_t^{1/2} \varepsilon_t, \quad \varepsilon_t \sim N_k(0, I_k) \tag{9}$$

$$\Sigma_t^Q = (V_t^Q)^{1/2} \Omega_t (V_t^Q)^{1/2}, \quad \Omega_t \sim W_k(\nu, \frac{I_k}{\nu}) \tag{10}$$

Where ε_t and Ω_t are a random vector and a random matrix respectively. By assumption we establish that the relation between the conditional mean of the squared returns and the conditional mean of the IC is proportional as follows:

$$\Sigma_t = \Lambda^{1/2} \Sigma_t^Q \Lambda^{1/2} \tag{11}$$

Here $\Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_k\}$ is a $k \times k$ diagonal matrix that does not vary in time and shows the proportional relationship between these two matrices.

II.3 Score-driven dynamics

To incorporate the score-driven dynamics, we define the vector f_t, which contains the time-varying parameters of the model. In our case, this vector contain the covariance matrices, so we define f_t as the following:

$$f_t = \text{vech}(V_t^Q) \tag{12}$$

Where the operator vech(A) stack the information in A in a vector. The V_t^Q dynamics is modeled such as the vector f_t is updated by the score function of the conditional density function. In this way, the dynamics of f_t is given by:

$$f_{t+1} = (1-\beta)\omega + \beta f_t + \alpha s_t \tag{13}$$

Where ω is a vector containing the unconditional mean of f_t, β is the autoregressive factor of the covariance dynamics and α is the contribution of the score, s_t, to the dynamics. The vector s_t is a martingale sequence withe zero mean a finite variance defined as:

$$s_t = S_t \nabla_t \tag{14}$$

$$\nabla_t = \frac{\partial log\ p_N(r_t|f_t, \mathcal{F}_{t-1};\theta)}{\partial f_t} + \frac{\partial log\ p_W(\Sigma_t^Q|f_t, \mathcal{F}_{t-1};\theta)}{\partial f_t} \tag{15}$$

$$S_t = \mathcal{I}_t^{-1/2},\ \mathcal{I}_t = E_{t-1}\left[\nabla_t \nabla_t'\right] \tag{16}$$

Here θ is a vector of static parameters —including α, β and ω—. ∇_t is the score vector of the likelihood function, defined as the sum of score vector of both return and covariance distributions. The function S_t accounts for the curvature of the likelihood function with respect to the parameter f_t, and p_N and p_W are the multivariate normal and Wishart distributions respectively. It should be noted that there is no single way to define S_t (Creal et al., 2013). However, here the scale function based on the Fisher information matrix, $\mathcal{I}_t$, is used for its intuitive interpretation based on Fisher's scoring-algorithm.

An important feature of this model is that the recursion presented in (13) is analogous to a GARCH process. The main difference is due to the fact that the dynamics of the covariance matrix in a GARCH process is driven by the square of the returns, while the score dynamics uses all the information of the probability density, instead of the first moments of distribution. On the other hand, the likelihood function is available in closed form and can be decomposed as the sum of the likelihood functions. In this way, the likelihood function $\mathcal{L}_t(\theta)$ can be expressed as:

$$\mathcal{L}_t(\theta) = log\ p_N(r_t|f_t, \mathcal{F}_{t-1}; \theta) + log\ p_W(\Sigma_t^Q|f_t, \mathcal{F}_{t-1}; \theta) \tag{17}$$

This property is due to the assumption of independence between the innovations of both innovations, that is, due to $E[\varepsilon_t \Omega_t] = 0$. Asymptotic properties of the model are reviewed in greater detail in the work of Gorgi et al. (2018).

Finally, we exploit equations (11) and (8) to recover the time-variant covariance matrix of returns, Σ_t, from the process in (13) and model its dynamics. Due to the inclusion of option-implied covariances and the Wishart distribution to model its innovations, this model will hereinafter be referred to as Implied-Covariance Wishart or ICW.

III Empirical application

In this section we describe the data we used and show the results of the estimation. Finally we describe the benchmark and backtesting exercises and show the results of forecasting univariate series and portfolios of different sizes.

III.1 Data

The data is made up of 25 daily series returns and their respective implied volatilities. The series correspond to components of the DAX index plus the index. The volatilities correspond to implied volatilities in 30-day European options, which is why they were transformed to daily volatility considering 25 trading days per month. The implied volatilities in the database correspond to the average implied volatilities in put and call-options for each share in the DAX. The components are shown in Table 1.

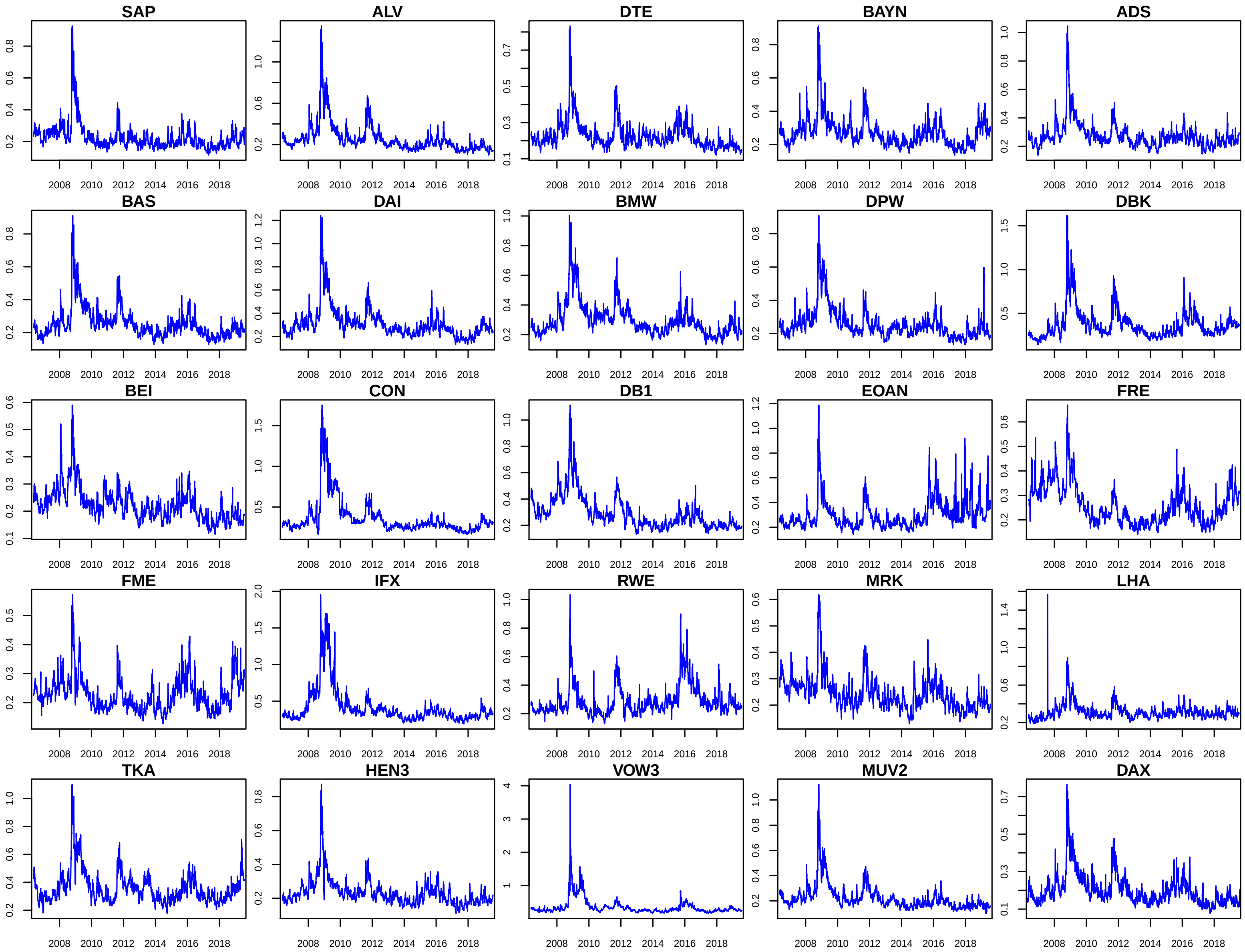
The sample data is between May 19, 2006 and June 23, 2019, which correspond to 3,343 market days. The data was partitioned in such a way that the data was left from January 2, 2017 (645 trading days) onwards to test the fit of the models outside the sample. The series are shown in Figure 1.

Table 1. Returns series in the database

Ticker	Firm	Industry
SAP	SAP	Software
ALV	Allianz	Insurance
DTE	Deutsche Telekom	Communications
BAYN	Bayer	Pharmacy and chemicals
BAS	BASF	Chemestry
DAI	Daimler	Manufacture
BMW	BMW	Manufacture
ADS	Adidas	Textile
DPW	Deutsche Post	Logistic
MUV2	Munich Re	Insurance
CON	Continental	Manufacture
DBK	Deutsche Bank	Banking
BEI	Beiersdorf	Consumer goods and chemicals
IFX	Infineon Technologies	Semiconductors
DB1	Deutsche Borse	Shares
EOAN	E.ON	Energy
FME	Fresenius Medical Care	Medical services
FRE	Fresenius	Medical services
LHA	Deutsche Lufthansa	Air transport
MRK	Merck	Pharmaceutical
RWE	RWE	Energy
TKA	ThyssenKrupp	Industry and manufacture
HEN3	Henkel	Consumer goods and chemicals
VOW3	Volkswagen Group	Manufacture

The data was obtained from the Thomson Reuters datastream Eikon platform.

Figure 1. Option-implied volatilities in the database



III.2 Estimates

Table 2 shows the results of the maximum likelihood (ML) estimate of the ICW model for the 2698 trading days of the in-sample partition. The model was estimated for a selection of $k \in \{6, 12, 24\}$ series randomly selected from the sample. For the case of $k = 24$ the model is estimated for all DAX components. The first three columns of Table 2 show three estimates with $k = 6$. The next three columns show three estimates with $k = 12$, while the last column corresponds to the model estimate with $k = 24$.

Results in Table 2 show that the estimate of the parameter ν increases as k increases. This parameter corresponds to the degrees of freedom of the Wishart distribution, so it must increase as the dimensions of the covariance matrix increase for the matrix to be non-singular. It can be observed that this parameter is significant for all the estimates made with different values of k, indicating that the model maintains its properties at different sample sizes.

Regarding the parameter β, we obtained estimates close to one, which shows the high persistence of the covariance matrix over time for all estimates. On the other hand, the parameter α indicates the relevance of the score function in the dynamics of the covariance matrix. Finally, significant parameters λ_i were obtained and greater than one in all cases, which indicates that the transformation $\Sigma_t = \Lambda^{1/2}\Sigma_t^Q\Lambda^{1/2}$ correctly captures the relationship between the implied covariance matrix and the covariance matrix of the returns.

Finally, the results indicate that the estimates of the ICW model parameters remain relatively constant between estimates at different dimensions. Additionally, all the model parameters were significant for all estimates. This is due to the importance of co-movements between the different components of the DAX in its dynamics. This result suggests that it is possible to set a single autoregressive parameter, β, for the entire set of returns, which we do in order to reduce the parameters to estimate as much as possible.

In the following subsections, the backtesting of the ICW model is performed with the parameters shown in the last column of Table 2, where $k = 24$.

Table 2. ML Estimates [a]

Parameter	ML Estimator						
	k=6	**k=6**	**k=6**	**k=12**	**k=12**	**k=12**	**k=24**
ν	90.155	91.472	98733	145.492	143.222	149.021	248.497
	(0.529)	(0.536)	(0.580)	(0.441)	(0.434)	(0.452)	(0.382)
β	0.967	0.971	0.986	0.987	0.991	0.979	0.991
	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
α	0.540	0.518	0.521	0.521	0.507	0.525	0.543
	(0.005)	(0.004)	(0.004)	(0.002)	(0.002)	(0.002)	(0.001)
λ_1	0.347	0.354	0.351	0.364	0.406	0.373	0.395
	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.004)	(0.004)
λ_2	0.348	0.347	0.346	0.715	0.378	0.397	0.396
	(0.004)	(0.004)	(0.004)	(0.014)	(0.004)	(0.004)	(0.003)
λ_3	0.362	0.364	0.360	0.419	0.400	0.387	0.396
	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)	(0.004)
λ_4	0.345	0.343	0.355	0.380	0.378	0.374	0.382
	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.004)	(0.003)
λ_5	0.327	0.343	0.344	0.451	0.368	0.353	0.384
	(0.004)	(0.004)	(0.004)	(0.007)	(0.004)	(0.004)	(0.004)
λ_6	0.345	0.352	0.351	0.418	0.390	0.384	0.420
	(0.004)	(0.004)	(0.004)	(0.005)	(0.004)	(0.004)	(0.004)
λ_7	-	-	-	0.433	0.528	0.407	0.423
				(0.005)	(0.008)	(0.005)	(0.004)
λ_8	-	-	-	0.475	0.374	0.389	0.454
				(0.007)	(0.004)	(0.004)	(0.005)
λ_9	-	-	-	0.393	0.385	0.352	0.396
				(0.005)	(0.004)	(0.004)	(0.004)
λ_{10}	-	-	-	0.548	0.375	0.393	0.398
				(0.008)	(0.004)	(0.004)	(0.004)
λ_{11}	-	-	-	0.373	0.377	0.677	0.381
				(0.004)	(0.005)	(0.001)	(0.004)
λ_{12}	-	-	-	0.371	0.391	0.983	0.467
				(0.004)	(0.004)	(0.015)	(0.006)
λ_{13}	-	-	-	-	-	-	0.403
							(0.004)
λ_{14}	-	-	-	-	-	-	0.368
							(0.003)
λ_{15}	-	-	-	-	-	-	0.383
							(0.004)
λ_{16}	-	-	-	-	-	-	0.388
							(0.005)
λ_{17}	-	-	-	-	-	-	0.403
							(0.005)
λ_{18}	-	-	-	-	-	-	0.423
							(0.005)
λ_{19}	-	-	-	-	-	-	0.423
							(0.005)
λ_{20}	-	-	-	-	-	-	0.412
							(0.005)
λ_{21}	-	-	-	-	-	-	0.386
							(0.004)
λ_{22}	-	-	-	-	-	-	0.402
							(0.005)
λ_{23}	-	-	-	-	-	-	0.481
							(0.006)
λ_{24}	-	-	-	-	-	-	0.396
							(0.004)
$\log L$	-141194	-147853	-148416	-425460	-433402	-375929	-1001419

[a] Standar errors are shown in parenthesis.

III.3 Benchmarking

Out-of-sample model fit is assessed by making a Value-at-Risk 1-step-ahead o forecast of univariate returns and bivariate portfolios of randomly selected series. A comparison is made between the proposed model and the models commonly used to model the dynamics of multivariate volatility series. These models are the EWMA and DCC (Engle, 2002) under the detailed specifications below.

The first benchmark is the EWMA model, which assumes that the dynamics of the covariance matrix follows the following process:

$$\Sigma_{t+1} = c\Sigma_t + (1-c)r_t r_t'$$

Where the dynamics of the conditional variance, Σ_t, is driven by the square of the returns, $r_t r_t'$, and c is a parameter typically set at $c = 0.96$. The second benchmark is the DCC model, which is built from the covariance matrix of the returns adjusted for their standard deviations. The specification of this benchmark is as follows:

$$\Sigma_t = D_t R_t D_t$$

$$\Sigma_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2}$$

$$Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 \epsilon_{t-1}\epsilon_{t-1}' + \theta_2 Q_{t-1}$$

Where R_t is the correlation matrix of the returns, D_t is a diagonal matrix of standard deviations, $\epsilon_{it} = r_{it}/\sqrt{\sigma_{it}}$ y Q_t follows the dynamics described by Engle (2002). To estimate this model, it is necessary to estimate k univariate GARCH models for the k series of returns, since it is necessary to estimate the matrix D_t in the first instance. This is done by estimating a GARCH (1,1) for each series.

Finally, it should be noted that both benchmarks use backward-looking information, while the model with option-implied covariances uses forward-looking information. For this reason, it is more appropriate to make the comparison of the models in their performance to predict volatility of the returns outside the sample, since it could be expected that the implied volatilities contain additional information to the historical volatilities that may be useful to perform forecasting. We evaluate this capacity by estimating the

VaR, which is defined below.

Definition 2: For a confidence level $\alpha \in (0,1)$ and a given time horizon, the VaR of a portfolio is defined as the minimum loss l such that the probability that the loss L exceeds the value l is not greater than $(1-\alpha)$. Formally this is:

$$VaR_\alpha = \inf\{l \in \mathbb{R} : P(L > l) \leq 1-\alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \geq\}$$

That is, the VaR is the quantile of the loss distribution, L. Additionally, the VaR series for a portfolio composed of two shares is defined as follows:

$$VaR_{\alpha t} = \sqrt{w_{it}^2 VaR_{\alpha it}^2 + w_{jt}^2 VaR_{\alpha jt}^2 + 2\rho_{ijt} w_{it} w_{jt} VaR_{\alpha it} VaR_{\alpha jt}}$$

Where the parameters w_{it} and w_{jt} are weights that indicate the weight of the stock in the portfolio. Here we use $w_{it} = w_{jt}$. In this way we can compare the proposed model against the benchmark considering all the elements of the correlation matrix and not only the elements of the main diagonal.

To perform the backtesting, the DQ_{hit} and DQ_{VaR} tests of Engle & Manganelli (2004) are presented, which are based on a quantile regression to test the independence of the excess series, $Hit_t := 1_{\{VaR_t < r_t\}}$, with its own lags, and the independence between Hit_t and the VaR_t series respectively. On the other hand, the MC tests (Ziggel, Berens, Weiß, & Wied, 2014), based on Monte Carlo simulations, are used to test unconditional coverage, MC_{uc}; error independence, MC_{iid}; and conditional coverage, MC_{cc}. Finally a ratio defined as the quotient between excess obtained and excess expected is shown.

The following subsections present the results for $\alpha \in \{95\%, 99\%, 99.9\%\}$ for a set of randomly composed individual return series and bivariate portfolios.

III.4 VaR forecast: Univariate return series

Table 3 shows the backtest of the VaR forecast for randomly selected individual returns from the data during the 3-year out-of-sample period. Table 3 shows the p-values obtained by the models for each series — BAYN, BMW, SAP and ALV — and for each test. Panels A, B and C show the results obtained for the VaR series with $\alpha = 95\%$, $\alpha = 99\%$ and $\alpha = 99.9\%$ respectively.

The results obtained for these series show that the EWMA model is the one with the worst performance in global terms, with the highest number of failures. On the other hand, the test results worsen in the 3 models while the most extreme quantile is analyzed, although the ICW model shows better results than the benchmark in the most extreme quantiles (Panel B and Panel C).

Table 3. VaR forecasting: Univariate return series[a] [b] [c]

	EWMA				DCC				ICW			
Backtest	BAYN	BMW	SAP	ALV	BAYN	BMW	SAP	ALV	BAYN	BMW	SAP	ALV
	Panel A: α=95%											
DQ_{hit}	0.2599	0.0012	0.3009	0.7463	0.9612	0.8458	0.8458	0.7176	0.9612	0.3793	0.0956	0.2467
DQ_{VaR}	0.1662	0.0031	0.3241	0.5167	0.4880	0.9658	0.8978	0.7114	0.7728	0.3611	0.2490	0.4933
MC_{uc}	0.8310	0.0732	0.9416	0.9524	0.2178	0.0888	0.0942	0.0392	0.2244	0.0012	0.7064	0.0422
MC_{iid}	0.5105	0.1782	0.1073	0.1533	0.9399	0.2619	0.1303	0.3494	0.8844	0.5953	0.4285	0.4736
MC_{cc}	0.9804	0.3322	0.2218	0.3122	0.1232	0.4972	0.2616	0.6856	0.2350	0.7986	0.8456	0.9534
Ratio	0.0481	0.0651	0.0496	0.0496	0.0403	0.0357	0.0357	0.0326	0.0403	0.0248	0.0465	0.0326
	Panel B: α=99%											
DQ_{hit}	0.1411	0.3051	0.3693	0.3693	0.0831	0.6966	0.4668	0.4668	0.6556	0.7809	0.5387	0.5387
DQ_{VaR}	0.0491	0.5793	0.6444	0.6444	0.0250	0.1970	0.7665	0.7665	0.8587	0.7314	0.8204	0.8204
MC_{uc}	0.1924	0.0046	0.0020	0.0000	0.5090	0.7254	0.0224	0.0240	0.5624	0.5326	0.1044	0.1028
MC_{iid}	0.9853	0.7445	0.4446	0.4463	0.8950	0.3673	0.4454	0.4491	0.9152	0.6260	0.7687	0.7724
MC_{cc}	0.0332	0.5220	0.9098	0.9304	0.2148	0.7220	0.8906	0.9008	0.1728	0.7606	0.4530	0.4710
Ratio	0.0155	0.0217	0.0248	0.0202	0.0124	0.0109	0.0202	0.0124	0.0124	0.0078	0.0171	0.0109
	Panel C: α=99.9%											
DQ_{hit}	0.7799	0.6952	0.6139	0.6139	0.8671	0.8233	0.8233	0.8233	0.8233	0.8671	0.7799	0.7799
DQ_{VaR}	0.8025	0.8021	0.6394	0.6394	0.9136	0.7222	0.8833	0.8833	0.8357	0.7658	0.7863	0.7863
MC_{uc}	0.0000	0.0000	0.0000	0.0000	0.0328	0.0016	0.0058	0.0026	0.0064	0.0506	0.0000	0.0000
MC_{iid}	0.9673	0.8487	0.5292	0.5321	0.2509	0.5321	0.0993	0.1031	0.7828	0.5976	0.2634	0.2683
MC_{cc}	0.07080	0.2966	0.9466	0.9436	0.5010	0.9336	0.1894	0.1970	0.4570	0.8150	0.5478	0.5292
Ratio	0.0078	0.0109	0.0140	0.0078	0.0047	0.0062	0.0062	0.0031	0.0062	0.0047	0.0078	0.0047

[a] The out-of-sample period corresponds to the period between January 2, 2017 and June 23, 2019.

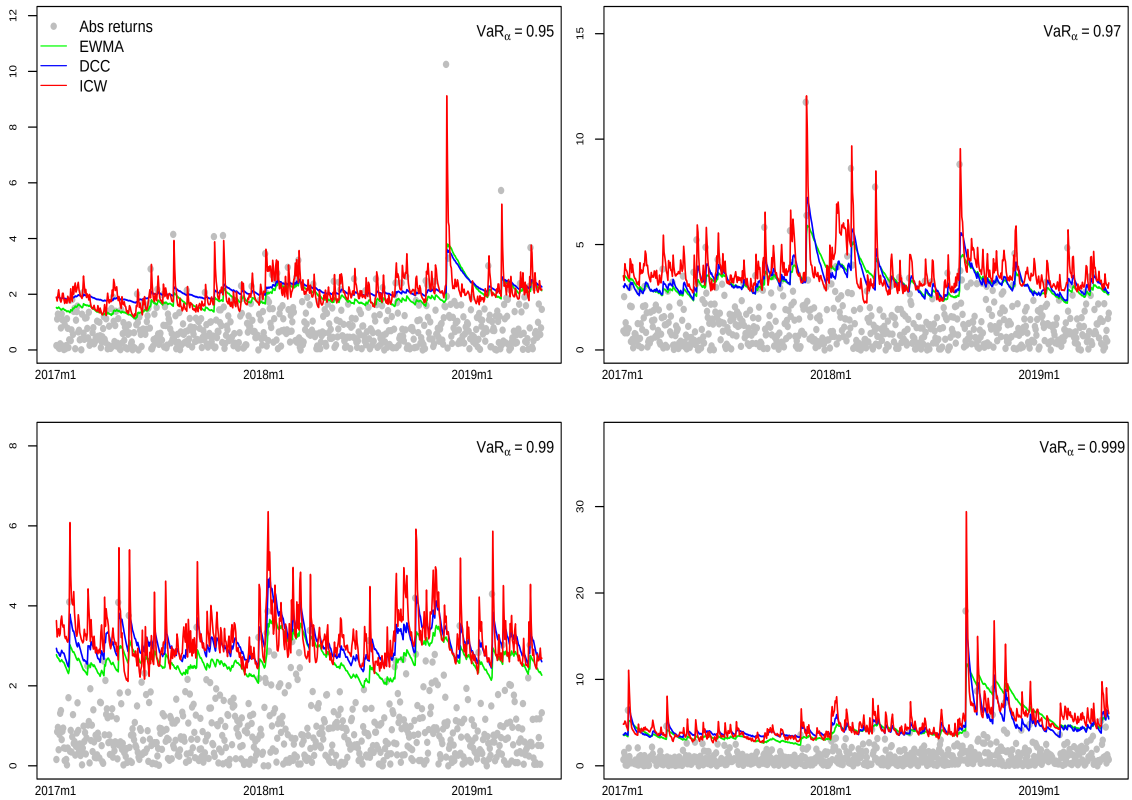
[b] For each test the corresponding p-value of the test statistic is presented.

[c] Failed tests are shown in lightgray highlighted text.

Figure 2 and Figure 3 graphically show the results for the VaR series for the univariate returns and for bivariate portfolios. The topleft panel of Figure 2 shows the results obtained by the models with $\alpha = 95\%$ for the TKA series, while the topright, bottomleft and bottom-right sections show the results with α equal to 97% for FRE series, 99% for BEI series and 99.9% for EOAN series respectively.

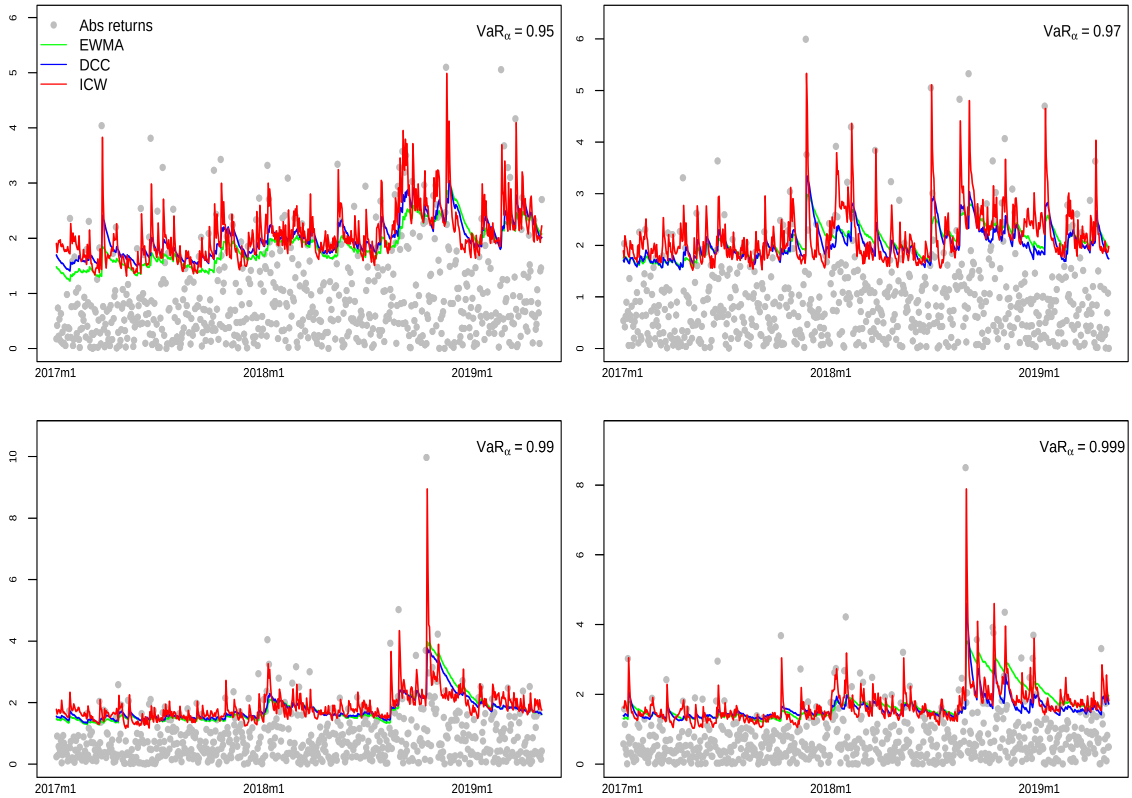
It is observed that the ICW model is much more flexible than the benchmark, returning to its unconditional mean significantly faster than the DCC and EWMA models. Additionally, we can observe that the ICW model adapts better to the volatility of the returns, unlike the benchmark.

Figure 2. VaR forecast: Univariate returns



Note: The VaR series correspond to TKA (upper-left), FRE (upper-right), BEI (bottom-left) and EOAN (bottom-right)

Figure 3. VaR forecast: Bivariate portfolios



Note: The VaR series correspond to TKA/FME (upper-left), FRE/BAYN (upper-right), BEI/DB1 (bottom-left) and EOAN/CON (bottom-right).

III.5 Portfolio simulations

To study the robustness of the results to the size of the portfolios, simulations of portfolios are carried out at different sizes $n = \{6, 12, 18, 24\}$ and at different weights, w_{it} , which determine the weight of each share within the portfolio. In each simulation, the size of the portfolio is previously determined and then the portfolio components with their respective weights are randomly obtained. 5000 simulations are run for each portfolio size, obtaining 20,000 series of portfolio returns with different amounts of components. Once these series of returns have been obtained, the tests estimated in the previous sections are carried out at the confidence levels $\alpha = \{95\%, 99\%, 99.9\%\}$. Table 4 shows the percentage of occasions in which a certain model passes a certain backtest.

Table 4 shows the results of the backtesting. It is observed that the 3 models obtain a similar amount of tests passed in the DQ hit and DQ VaR tests, close to or greater than 90% of successes, although the ICW model obtains a number slightly higher than successful tests in the DQ hit test, and slightly lower in the DQ VaR test. This indicates that the surplus series, Hit_t, exhibits autocorrelation less often in the ICW model than in the benchmark, while the textit Hit_t series tends to show more correlation with the VaR series in the ICW model. These results are consistent across different portfolio sizes and hold at different percentiles analyzed.

On the other hand, in the Monte Carlo tests, the ICW model shows a greater number of successes in at least some of the benchmarks (with the exception of the MC_{cc} test in Panel B). Additionally, at lower quantiles (Panel A), the unconditional coverage tests, MC_{uc}, show the worst results in the 3 models, with the EWMA model obtaining the worst results, while the DCC and ICW models reach about a 55% of successes. On the other hand, at more extreme percentiles (Panel C), the test in which the models perform the worst is the error independence test, MC_{iid}. In this test, the DCC model performs better, followed by the ICW model and finally the EWMA model. Finally, in the conditional coverage test, the models obtain similar results and higher than 90% of successes in all cases, with the ICW model obtaining slightly higher results in the percentiles 95% and 99.9%, and slightly worst in the 99% percentile.

Table 4. Backtesting: Portfolio simulations[a] [b]

	n=6			n=12			n=18			n=24		
Backtest	ICW	EWMA	DCC	ICW	EWMA	DCC	ICW	EWMA	DCC	ICW	EWMA	DCC
	Panel A: α=95%											
DQ_{hit}	**0.9990**	0.9402	0.9704	**0.9992**	0.9382	0.9632	**0.9992**	0.9382	0.9710	**0.9990**	0.9450	0.9628
DQ_{VaR}	0.8964	0.9286	0.9566	0.9180	0.9502	0.9662	0.9322	0.9596	0.9716	0.9334	0.9724	0.9630
MC_{uc}	0.5524	0.1648	0.7062	0.5634	0.0922	0.7348	0.5936	0.0800	0.7652	0.5952	0.0672	0.7684
MC_{iid}	0.9818	0.9862	0.9022	0.9846	0.9906	0.8808	0.9874	0.9928	0.8634	0.9878	0.9940	0.8618
MC_{cc}	0.9564	0.9706	0.9498	0.9638	0.9690	0.9424	**0.9718**	0.9686	0.9416	**0.9670**	0.9618	0.9432
	Panel B: α=99%											
DQ_{hit}	**0.9902**	0.9852	0.9772	**0.9910**	0.9896	0.9648	0.9940	0.9958	0.9664	**0.9938**	0.9934	0.9628
DQ_{VaR}	0.9600	0.9796	0.9788	0.9508	0.9858	0.9822	0.9290	0.9896	0.9862	0.9276	0.9888	0.9904
MC_{uc}	**0.9166**	0.7630	0.9062	0.9178	0.6916	0.9364	0.9232	0.6932	0.9468	0.9342	0.7138	0.9574
MC_{iid}	0.9876	0.9226	0.9906	0.9906	0.7892	0.9934	0.9904	0.6842	0.9962	0.9910	0.6178	0.9956
MC_{cc}	0.9350	0.9420	0.9498	0.9396	0.9522	0.9554	0.9412	0.9600	0.9472	0.9482	0.9666	0.9544
	Panel C: α=99.9%											
DQ_{hit}	**0.9980**	0.9970	0.9692	0.9980	0.9988	0.9746	**0.9996**	0.9990	0.9750	**0.9994**	0.9992	0.9804
DQ_{VaR}	0.9790	0.9902	0.9898	0.9818	0.9958	0.9874	0.9838	0.9954	0.9880	0.9858	0.9984	0.9884
MC_{uc}	0.7852	0.8266	0.6868	0.8466	0.8898	0.7644	0.8752	0.9090	0.8030	0.8978	0.9268	0.8394
MC_{iid}	0.6450	0.5134	0.7230	0.5216	0.3424	0.6306	0.4562	0.2804	0.5634	0.3906	0.2442	0.5184
MC_{cc}	0.9594	0.9716	0.9400	0.9694	0.9818	0.9558	0.9774	0.9844	0.9558	0.9802	0.9874	0.9600

[a] The table shows the percentage of successful tests for each model.

[b] The tests in which the ICW model obtained results equal to or better than any of the benchmarks are shown in lightgray highlighted text. In bold and highlighted the tests that the ICW model outperforms both benchmarks.

It should be noted that the ICW model performs better than the EWMA and at least as good as the DCC model. These results are not penalizing for the number of parameters estimated in each model, which in the case of $k = 24$ reach 74 parameters –3 for each GARCH(1,1) and plus 2 for the DCC– in the DCC model, unlike the 27 parameters estimated in the ICW model.

IV Conclusions

The objective of this paper is to document the predictive capacity of the option-implied volatilities and option-implied correlations on the returns volatility. Our results show that the model correctly captures the relationship between the implied covariances and the covariances of the returns. It is also observed that the factor associated with the score-driven dynamics is significant at different dimensions, indicating that the model manages to capture the dynamics of the covariance matrix. On the other hand, the backtest shows that the model outperforms the EWMA model and obtains results at least as good to the DCC model, although the DCC model needs a greater number of parameters to estimate and we do not penalize for this fact.

The ICW model obtains results comparable to the benchmark in all the tests carried out. However, its advantage is that it considerably reduces the number of parameters that must be estimated, particularly compared to standard models such as DCC and BEKK. On the other hand, the novelty of this methodology is that it allows the inclusion of the option-implied covariance matrix in the estimation of the covariances of the returns. The results obtained are related to results previously observed in the literature on the predictive capacity of implied volatilities on the volatility of returns (Driessen et al., 2013) and on its ability to estimate Value-at-Risk (Slim et al., 2019; Schindelhauer & Zhou, 2018).

We conclude that our results justify a line of research in order to adapt multivariate volatility models with option-implied correlations according to different requirements, such as heavy-tailed distributions, similar to the work of Opschoor et al. (2018), increase the amount of autoregressive parameters, try to capture long memory effects, as in the work of Vassallo et al. (2018), test the robustness at different option expiration periods, test the performance of the model in portfolio optimization and build other benchmarks more suitable than allow comparing models with forward-looking information. These issues are raised as future research.

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