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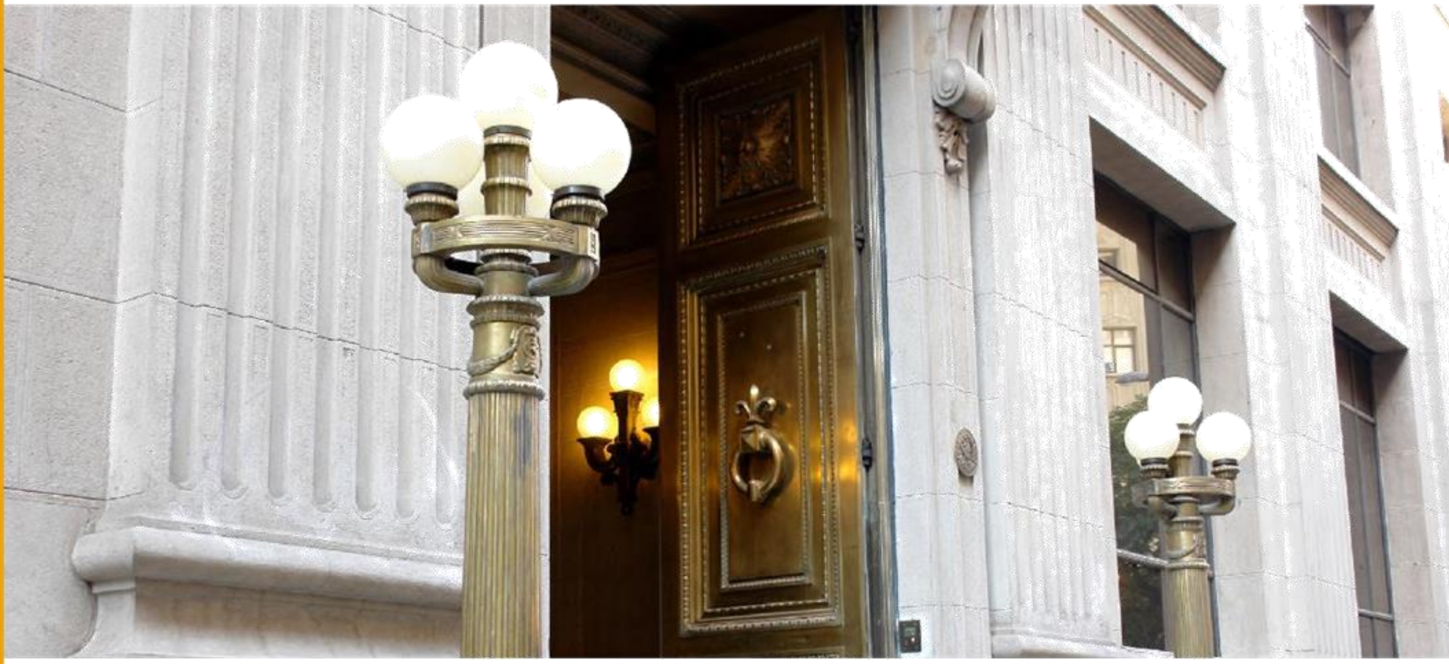

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Inequality, Nominal Rigidities, and Aggregate Demand*

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Abstract

This paper studies wage and price flexibility as a means of absorbing adverse shocks. We focus on economies with unequal access to financial markets and where the monetary authority is constrained by the zero lower bound. We show that the economy becomes more volatile in this setting when wages are more flexible. As our model assumes financial frictions, wage flexibility translates into output volatility via a redistribution channel, which operates through aggregate demand. We find that this volatility depends on the relative wage and price rigidity. Additionally, we show that the redistribution channel gains prominence when the central bank is at the zero lower bound. We conclude that in these kinds of economies, the usual recommendation of making labor markets more flexible to restore high output levels, is mistaken.

Resumen

Este artículo estudia la flexibilidad de precios y salarios como forma de absorber shocks adversos. Nos enfocamos en economías con acceso desigual a los mercados financieros y donde la autoridad monetaria está limitada por la zero lower bound. Mostramos que la economía se vuelve más volátil en este entorno cuando los salarios son más flexibles. Dado que nuestro modelo asume fricciones financieras, la flexibilidad salarial se traduce en volatilidad del producto a través de un canal de redistribución, que opera a través de la demanda agregada. Encontramos que esta volatilidad depende de la rigidez relativa de precios y salarios. Además, mostramos que el canal de redistribución es más importante cuando el banco central se encuentra en la zero lower bound o no puede reaccionar efectivamente a los shocks. Concluimos que en este tipo de economías, la recomendación habitual de flexibilizar los mercados laborales para restaurar el pleno empleo es errónea.

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1 Introduction

Letting wages adjust freely after an economic downturn is one of the main elements of the classical economists' toolkit. According to this argument, if wages fall, labor demand increases and output returns to its potential level. However, as Galí (2013) shows this is not necessarily true in the presence of price and wage rigidities. In this paper, we extend this analysis to an heterogeneous agent economy, at the zero lower bound. We show that in a model where there is heterogeneity in marginal propensities to consume (MPC), there is a distributional channel of nominal rigidities that exacerbates the losses from wage flexibility depending on the degree of price rigidities and the degree of inequality.

Making wages more flexible to restore full employment was first challenged by Keynes (1936). He disagreed with the classical theory by postulating that aggregate demand (AD) matters for the determination of output. He also pointed out that the classical theory is wrong in assuming that the AD does not depend on wages. In his view, the AD depends on wages as long as they affect: (i) the return on assets or (ii) the average marginal propensity to consume (MPC). The first channel operates if wages affect the interest rate in the economy, switching the incentives to consume and invest. This first mechanism was explored by Galí (2013), among others, who show that this channel matters if wages alter the real interest rate; i.e., wages affect the AD only indirectly through the endogenous response of the central bank. The second channel operates if wage adjustments redistribute resources between agents with different MPCs, affecting their levels of consumption and aggregate demand. Recent literature on monetary policy has emphasized that in the presence of market incompleteness the indirect effects of monetary policy dominate, where one important component are fluctuations in labor income. (see Kaplan et al. (2016) and Auclert (2019) among others).

This paper builds on the second channel–the average MPC. We start from the observation that, when there are agents who are unable to save or borrow, the AD depends on the distribution of all income sources (in particular, labor income) and not only on the interest rate, as in the baseline New Keynesian model. In such a case, the shift of resources in the cycle between workers and firm owners affects the extent to which wages determine consumption dynamics. There is a shift in the average aggregate MPC when: (i) income from assets (which includes bonds and firms' profits) and labor is unequally distributed; (ii) the MPCs of workers and other agents differ; and (iii) wages fluctuate differently from other sources of income. That is, specifically, the intuition put forward by Keynes (1936) Chapter 19.

In this paper, we show both analytically and numerically that the relative rigidity of wages and prices can drive this redistribution. The relative rigidities affect the evolution of the real wage, which has an active role determining the AD in the presence of limited access to financial markets. In short, the relative rigidities determine who gets the resources from aggregate fluctuations: workers or firm owners. If prices fall less than wages in a downturn, then there is a redistribution of resources from workers to firm owners. Hence, if workers have more restricted access to financial markets and their MPCs are higher, the average MPC of the economy shifts because of the differential nominal rigidities that the different agents face. In turn, limited access to financial markets activates the channels proposed by Keynes, and the final effect is governed by the relative nominal rigidities. In this context, wage flexibility might not be desirable if it generates countercyclical redistribution from high- to low-MPC consumers. We show that this is especially acute if the degree of price rigidity is high.

The existence of price and wage rigidities has been broadly studied in the literature. Dhyne et al. (2006) provide evidence on price rigidities, finding that the average spell of prices in the Euro Area is about one year, while in the U.S. it is about two quarters. While Bils and Klenow (2004) and Nakamura and Steinsson (2013) show a shorter duration of prices (about four months), they do not rule out the existence of price rigidities. Regarding nominal wage rigidities, there is broad evidence reviewed by Taylor (2016). He highlights the evidence from France studied by Le Bihan et al. (2012) and Iceland collected by Sigurdsson and Sigurdardottir (2011), which shows that wages in both cases stay fixed, on average, for about one year.

We rationalize the concepts above by building a textbook New Keynesian model with limited asset participation and price and wage rigidities as in Colciago (2011) and Furlanetto and Seneca (2012). To capture market incompleteness, we assume there is a share of agents without access to financial markets as in Galí et al. (2007) and Bilbiie (2008)[1], implying different MPCs across the population. We call these constrained agents *Hand to Mouth* (HtM).[2]

The main result of this paper is an analytical characterization of the equilibrium in a simplified economy where prices and wages are set in advance. We derive an aggregate demand that depends on wages, as emphasized by Keynes (1936). We show that with wage rigidities and a share of HtM

[1]See also the Debortoli and Galí (2017) who compare the results of Two-Agent and Heterogenoues-Agent New Keynesian models.

[2]Kaplan et al. (2014) provide evidence of the existence of HtM households in the US and Europe. They find that 30% of households do not hold assets on average. Our calculations find that this share has been barely stable over the 2000-2018 period.

agents, the AD (and output) depends on wage and price inflation and in particular on the wage and price processes, which are given by the respective degrees of stickiness. This dependence arises from the fact that when there are wage rigidities, the price adjustment process is not isomorphic to the wage adjustment. When prices and wages fluctuate separately, the income of firm owners and of workers fluctuates differently. That implies that redistribution between workers and firm owners can arise from nominal rigidities. If workers are financially constrained and firms' owners are not, different price and wage rigidities generate switches in the average MPC of the economy.

In our model the final effect of the different features, conditional on demand shocks, operates through two channels: an *interest rate channel* and a *distributional channel*. The former is the conventional procyclical response of monetary policy to the different shocks when the Taylor rule responds to endogenous variables. The latter corresponds to how aggregate demand is affected by the redistribution of resources in the cycle among different households. On the one hand, we show that through the distributional channel, wage flexibility amplifies the cycle by making income of the high MPC agent more volatile. On the other hand, we show that wage flexibility stabilizes output if the monetary policy response to this excessive volatility is strong enough. The final effect depends on the share of HtM agents, and more interestingly, on the degree of price rigidities. The importance of price rigidities for the gains from wage flexibility was already emphasized by Galí and Monacelli (2016), who showed that more flexible wages translates into a more stable economy (with lower prices and higher demand for its goods) if prices are sufficiently flexible. Our argument is similar, though we highlight an alternative channel, which operates through redistribution of resources between workers and firm owners.

We show that the conduct of monetary policy–and what it reacts to– matters. As prices and wages do not have the same effect on aggregate demand (as in a model without wage rigidities and financial frictions), reacting to price inflation is not enough to stabilize output. If prices are too rigid, the distributional channel gains prominence. Therefore, we show that monetary policy should react to wage inflation to dampen the distributional channel. In our model, monetary policy is more effective when it reacts to wage inflation than when it reacts to price inflation.

Finally, we study the gains from wage flexibility using the full model. We show that the results presented in the simple model still stand. In this setup, we find no gains from wage flexibility with limited access to financial markets, since it stimulates the distributional channel. This result is especially acute if monetary policy cannot react, i.e., is in the zero lower bound. We show that there are losses from wage flexibility caused by an excessive volatility of prices, wages, and output.

This paper contributes to the literature mainly because it helps to clarify the effects of the interaction between incomplete markets and price and wage rigidities. We obtain a closed-form characterization of the economy subject to these three frictions: price rigidities, wage rigidities, and limited asset markets participation. Previous literature has not emphasized the role of the three frictions together, but rather detracted from the role of price rigidities in shaping redistribution. This paper is similar to Broer et al. (2019). However, we show analytically how the gains from wage flexibility may depend on price rigidities and the monetary policy stance, and not only how wage rigidities affect the aggregate outcome. Moreover, we uncover the distributional channel of nominal rigidities which arises in models with incomplete markets and nominal rigidities.[3]

The remainder of the paper is organized as follows. Section 2 describes the model. In Section 3 we solve analytically a simple version of the model where we assume prices and wages are set in advance. In Section 4 we conduct quantitative exercises in the full model. And finally, we present our conclusions in Section 5.

2 Model

Our setup is a New Keynesian model with limited asset market participation and wage rigidities, building on the work of Bilbiie (2008), Furlanetto and Seneca (2012), Debortoli and Galí (2017), among others. In particular, we assume there is a fraction of agents that cannot borrow or lend and do not own firms. Workers supply labor in a monopolistically competitive environment and are subject to staggered wage setting. Firms are also subject to price rigidities and supply their goods in a monopolistically competitive environment. Additionally, we assume that monetary policy follows a Taylor rule which is bounded from below by zero.

Households. The economy is populated by a continuum of households of mass 1, where a fraction λ cannot borrow, lend or own firms, while the remainder $1-\lambda$ has full access to financial markets and owns the firms in the economy. We refer to the former as constrained agents, denoted by c, and to the latter as unconstrained, denoted by u. Each household is composed by a continuum of members that supply differentiated labor varieties denoted by $j \in [0, 1]$. We assume there is perfect insurance within the household, which equalizes members' consumption.

[3] Additionally, Colciago (2011) studies this question while he does not show explicitly how these mechanisms interact.

Households' lifetime utility is given by

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \chi_{t+k} \left(\frac{(C_{t+k}^K)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{N_{t+k}^K(j)^{1+\varphi}}{1+\varphi} dj \right), \tag{1}$$

for $K \in \{u, c\}$, where χ_t represents a shock to preferences, C_t^K is final good consumption and $N_t^K(j)$ denotes hours worked supplied to variety j.

Households face the following period resource constraint

$$P_t C_t^K + Q_t B_t^K = B_{t-1}^K + \int_0^1 W_t(j) N_t^K(j) dj + D_t^K. \tag{2}$$

Earnings are given by labor income $\int_0^1 W_t(j) N_t^K(j) dj$ and profits D_t^K, which proceed from firms ownership. $P_t C_t^K$ is total (nominal) expenditure on the final good and $Q_t B_t^K$ are bond purchases, where P_t is the price of the final good while Q_t is the price of bonds.

We assume the preference shock follows an exogenous AR(1) process, given by

$$\log \chi_t = (1-\rho_\chi) \log \chi + \rho_\chi \log \chi_{t-1} + \sigma_\chi \varepsilon_t^\chi.$$

Intertemporal optimization implies the following Euler equation for unconstrained households

$$1 = R_t \mathbb{E}_t \left\{ \beta \frac{\chi_{t+1}}{\chi_t} \left(\frac{C_t^u}{C_{t+1}^u} \right)^\sigma \frac{1}{\Pi_{p,t+1}} \right\}, \tag{3}$$

where $R_t = 1/Q_t$. Constrained households have no access to financial markets. Hence, their consumption equals current income from labor

$$C_t^c = \frac{W_t}{P_t} N_t. \tag{4}$$

Finally, aggregate consumption is given by

$$C_t = (1-\lambda) C_t^u + \lambda C_t^c. \tag{5}$$

Final Good Producers. Firms producing the final good operate in a perfectly competitive environment and combine a continuum of measure one of intermediate goods $Y_t(i)$ to produce a homogeneous final good Y_t according to

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \tag{6}$$

where ϵ_p is the elasticity of substitution among good varieties.

Solving the optimization problem of the firm, we obtain the following demand function for intermediate inputs

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_p} Y_t, \tag{7}$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon_p} di \right)^{\frac{1}{1-\epsilon_p}}$ is the price of the final good.

Intermediate Goods Producers. There is a continuum of intermediate firms, indexed by $i \in [0, 1]$. These firms operate in a monopolistic competitive environment. Hence, each firm produces a single-differentiated good and operates as a monopoly in its own market. Intermediates production technology is given by

$$Y_t(i) = N_t(i)^{1-\alpha}, \tag{8}$$

where $Y_t(i)$ is firm i output and $N_t(i)$ is labor input. We assume that each firm i demands different kinds of labor provided by the households, with an elasticity of substitution ϵ_w. Thus, we have $N_t(i) = \left(\int_0^1 N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$, where $N_t(i,j)$ is the amount of labor variety j demanded by firm i. Then, a standard cost minimization problem derives the demand for each labor variety

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i), \tag{9}$$

where $W_t(j)$ is the wage of labor variety j. From Equation (9), note that the total demand across firms for variety j is given by $N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t$.

Firms also face price rigidities. In the next sections we consider two different types of rigidities: prices set in advance and Calvo pricing, which we will describe in detail later. Assuming prices are set in advance allows us to study the role of nominal rigidities in aggregate Demand analytically. On the other hand, with Calvo pricing, we take into account the role of expectations and dynamics to describe the impact of nominal rigidities.

Wage Setting. The wage for each labor variety is set by a union operating in a monopolistically competitive market. Unions choose the wage rate that maximizes a weighted average of unconstrained and constrained lifetime utility, which, for example, with Calvo pricing writes

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \chi_{t+k} \Bigg[(1-\lambda) \left(\frac{(C^u_{t+k}-1)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{(N^u_{t+k|t})^{1+\varphi}}{1+\varphi} dj \right) + \lambda \left(\frac{(C^c_{t+k}-1)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{(N^c_{t+k|t})^{1+\varphi}}{1+\varphi} dj \right) \Bigg], \tag{10}$$

where θ_w is the parameter of wage rigidities.

In the next sections we consider two different types of rigidities: wages set in advance and Calvo wage adjustment. We will describe these problems in detail below.

Monetary Policy. We assume that the monetary authority follows a Taylor rule that is subject to the zero lower bound, given by

$$R_t = \max \left\{ \overline{R} \left(\frac{\Pi_{p,t}}{\overline{\Pi}_p} \right)^{\phi_\pi}, 1 \right\}, \tag{11}$$

where parameter ϕ_π determines the response of the central bank to deviations of inflation from its steady state level.

Equilibrium. In this economy all production is consumed

$$Y_t = C_t. \tag{12}$$

The relation between aggregate output and employment can be written as[4]

$$N_t = \Delta_{w,t} \Delta_{p,t} Y_t^{\frac{1}{1-\alpha}}, \tag{13}$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon_p}{1-\alpha}} di$.

Finally, we assume bonds are in zero net supply. Hence, equilibrium in the bonds market requires

$$(1-\lambda) B^u_t + \lambda B^c_t = 0. \tag{14}$$

[4] See Appendix B.1 for the derivation.

Since constrained agents have no access to financial markets, the last expression implies $B_t^u = B_t^c = 0$.

3 Aggregate Demand with Prices and Wages Set in Advance

In this section, we illustrate how the interaction between price and wage rigidities shapes redistribution over the business cycle and how such interaction affects aggregate demand. To clarify the mechanisms of our model, we assume the wage and price setting processes as follows. Workers supply labor in a monopolistically competitive environment. However, we include a staggered wage setting by assuming that a fraction of workers set nominal wages in advance (i.e., before the shocks are realized). The remaining workers are not constrained to set wages. We use the same simplification for firms' pricing problem.

The aim of this section is to solve the model to obtain an aggregate demand equation when there is limited access to financial markets. The resulting equation will be useful to explain how market incompleteness interacts with price and wage rigidities in shaping aggregate demand.

3.1 The Consumption Gap

Combining equations (3) to (5) we can solve for the following Euler equation of aggregate consumption (in log deviations from the steady state)[5]

$$\widehat{c}_t = \mathbb{E}_t\left\{\widehat{c}_{t+1}\right\} - \frac{1}{\sigma}\left(\widehat{r}_t - \mathbb{E}_t\left\{\widehat{\pi}^p_{t+1}\right\} - (1-\rho_\chi)\widehat{\chi}_t\right) + \frac{\lambda}{(1-\lambda)\gamma+\lambda}\mathbb{E}_t\left\{\Delta\widehat{\gamma}_{t+1}\right\}, \tag{15}$$

where $\gamma_t \equiv \frac{C_t^u}{C_t^c}$ is the *consumption gap* between the unconstrained and the constrained households. Notice that Equation (15) is the usual Euler equation with an additional term that depends on the growth rate of consumption inequality. This equation can be solved forward to obtain

$$\widehat{c}_t = -\frac{\lambda}{(1-\lambda)\gamma+\lambda}\widehat{\gamma}_t - \frac{1}{\sigma}\mathbb{E}_t\sum_{k=0}^{\infty}(\widehat{r}_{t+k} - \widehat{\pi}^p_{t+k+1} - (1-\rho_\chi)\widehat{\chi}_{t+k}). \tag{16}$$

Equation (16) shows that consumption, through aggregate demand, is directly affected by inequality. The direction of this dependence is determined by the cyclicality of the consumption gap, which is an endogenous variable. To derive the consumption gap, recall that unconstrained agents work and own the firms; hence their income is given by the sum of labor and profit earnings, i.e.,

[5] In what follows hat ($\hat{x}$) variables correspond to log-deviations with respect to the steady-state.

$C_t^u = \frac{W_t}{P_t} N_t + \frac{1}{1-\lambda} \frac{D_t}{P_t}$. Constrained households, meanwhile, only receive labor income; hence, their consumption is given by $C_t^c = \frac{W_t}{P_t} N_t$. Then, in equilibrium, it must be that the consumption gap is given by

$$\gamma_t = \frac{W_t N_t + \frac{1}{1-\lambda} D_t}{W_t N_t}.$$

As Debortoli and Galí (2017) show, the consumption gap can be written in terms of the economy's price markup

$$\gamma_t = \frac{1 - \alpha + \frac{1}{1-\lambda} \left(\mathcal{M}_t^p - (1-\alpha) \right)}{1-\alpha}, \tag{17}$$

where $\mathcal{M}_t^p$ is the average price markup.[6] Equation (17) in log-deviations from the steady state reads

$$\widehat{\gamma}_t = \Psi \widehat{\mu}_t^p, \tag{18}$$

where $\Psi \equiv \frac{\mathcal{M}^p}{(1-\lambda)\left(1-\alpha+\frac{1}{1-\lambda}(\mathcal{M}^p-(1-\alpha))\right)}$. Equation (18) represents a relation that is at the core of the results that follow: it is only the price markup that determines the consumption gap, as the only source of inequality in the model is the ownership of firms. The coefficient Ψ, which determines the relationship between the consumption gap and markups, depends negatively on the share of unconstrained agents; i.e., of the fraction of firms owners. This occurs because having a lower share of firms' owners implies that any increase in the price markup (and hence in firms' profits) is distributed among a smaller share of agents. Therefore, firm owners experience a greater increase in their income, leading to a larger rise in the consumption gap between firm owners and workers. The next step is to understand how the price markup evolves over the business cycle.

Firms Average Markup. As Equation (18) shows, firms' markups are important in the equilibrium of the model. This is not only because they are a source of fluctuations, like in any New Keynesian model, but because they shape inequality and aggregate demand. Regarding the firms' labor demand, the average markup is given by

$$\mathcal{M}_t^p = (1-\alpha) \frac{P_t Y_t}{W_t N_t},$$

[6] To obtain this expression, we used $\frac{W_t N_t}{P_t Y_t} = (1-\alpha) \frac{Y_t}{\mathcal{M}_t^p N_t} \frac{N_t}{Y_t} = \frac{1-\alpha}{\mathcal{M}_t^p}$ and $\frac{D_t}{P_t Y_t} = \frac{P_t Y_t - W_t N_t}{P_t Y_t} = 1 - \frac{1-\alpha}{\mathcal{M}_t^p}$.

which log-linearized around the steady state yields

$$\widehat{\mu}_t^p = -\frac{\alpha}{1-\alpha}\widehat{y}_t - \widehat{\omega}_t. \tag{19}$$

From (18) and (19) we get

$$\widehat{\gamma}_t = -\Psi\left(\frac{\alpha}{1-\alpha}\widehat{y}_t + \widehat{\omega}_t\right). \tag{20}$$

Equation (20) describes the evolution of the consumption gap and its drivers. We highlight two results from this expression. First, the consumption gap depends negatively on output. This occurs because decreasing returns on labor imply a reduction in the firms' average markup following an increase in production (and hence employment). Second, the gap depends negatively on real wages, $\widehat{\omega}_t$. That occurs as wages raise marginal costs and hence drive firms' markups down. As a result, income is redistributed towards workers; i.e., the consumption gap drops.

3.2 Equilibrium Wages

In this subsection, we derive the equilibrium real wage. To do so, we first obtain wage and price inflation schedules. We derive two Phillips-like equations for prices and wages and show how the real wage depends on relative nominal rigidities between prices and wages. To obtain closed-form solutions for price and wage inflation, we make some simplifying assumptions, summarized in Proposition 1.

Proposition 1 (Price and wage dynamics)**.** *Assume there is a continuum of measure one of firms (unions) in a monopolistic competitive environment, in which a share θ_p (θ_w) of firms (unions) set prices (wages) in advance, while the remainder $1-\theta_p$ ($1-\theta_w$) set prices (wages) considering the value of the shocks in t. Assume also, that firms maximize profits by taking into account their production function, Equation (8); while unions maximize aggregate welfare of the members of the union of each task.*

Under these assumptions, the evolution of price inflation is given by

$$\widehat{\pi}_t^p = \kappa_\pi\left(\widehat{\omega}_t + \frac{\alpha}{1-\alpha}\widehat{y}_t\right) + \mathbb{E}_{t-1}\widehat{x}_t^p, \tag{21}$$

where $\kappa_\pi \equiv \frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$ *and* $\widehat{x}_t^p \equiv \widehat{\omega}_t + \alpha\widehat{n}_t^m(j) + \widehat{\pi}_t^p$.

While the evolution of real wages is given by

$$\widehat{\omega}_t = \kappa_\omega(\varpi_1 \widehat{y}_t + \varphi \widehat{n}_t) - \varsigma \widehat{\pi}^p_t + \mathbb{E}_{t-1}\widehat{x}^w_t, \tag{22}$$

where $\kappa_\omega \equiv \frac{1-\theta_w}{1+\theta_w\varphi\epsilon_w-(1-\theta_\omega)\varpi_2}$, $\varsigma \equiv \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w-(1-\theta_\omega)\varpi_2}$ *and* $\widehat{x}^w_t \equiv \varsigma\left(\varpi_1\widehat{y}_t + \varpi_2\widehat{\omega}_t + \varphi\widehat{n}^m_t(j) + \widehat{\pi}^p_t\right)$.

Proof. See Appendix B.2. □

Proposition 1 describes the evolution of inflation and the real wage. Equation (21) shows that in our setting, price inflation depends on wages and output; while Equation (22) describes the relationship between the real wage with output, labor, and price inflation. Both equations are a Phillips-like relationship for prices and wages as we obtain a positive relationship between the output gap and price inflation on the one hand and the marginal rate of substitution and wages on the other. These relationships depend on the degrees of price and wage stickiness, θ_p and θ_w. If wages or prices are fully flexible, these relationships break. If prices are fully flexible, price inflation is free to evolve (and the aggregate supply becomes infinitely inelastic). Also, if wages are fully flexible, the real wage is given by the labor supply at all times.[7,8] Notice that the real wage depends negatively on the price inflation rate, with this relation given by the parameter ς, which is a function of the degree of wage rigidities. Therefore, the real wage depends on the price inflation rate because of the wage rigidities.

Our price and wage arrangements assume prices are set in advance. This implies that firms and workers set wages taking an expectation of the future demand for goods and labor before the shocks realize (in $t-1$). That is why the terms $\mathbb{E}_{t-1}\hat{x}^p_t$ and $\mathbb{E}_{t-1}\hat{x}^w_t$ appear in the prices and wage setting schedules. When prices and (or) wages are fully sticky, the evolution of these variables are given by these expectations that are the best the restricted agents can do. Throughout this section we assume shocks are iid with zero mean, so these expectation terms are zero.

As the real wage depends on price inflation, by combining Equations (21) and (22), we obtain the *real wage schedule*, which is presented in the following proposition.

Proposition 2 (The real wage schedule)**.** *From the evolution of the real wage and price inflation,*

[7] With flexible wages $\hat{\omega}_t = \overline{\omega}\hat{y}_t + \overline{\varphi}\hat{n}_t$.

[8] These two Phillips-like equations are very close to the ones derived in the basic New Keynesian model with Calvo rigidities. The difference is the backward looking nature of these ones, while in the New Keynesian they are forward-looking. We assume these simplifications to derive an analytical solution of our problem.

the real wage is given by

$$\widehat{\omega}_t = \Xi_y \widehat{y}_t + \mathbb{E}_{t-1}\widehat{x}_t, \tag{23}$$

where $\Xi_y \equiv \frac{\kappa_\omega\left(\varpi_1 + \frac{\varphi}{1-\alpha}\right) - \varsigma\kappa_\pi\frac{\alpha}{1-\alpha}}{1+\varsigma\kappa_\pi}$ *and* $\widehat{x}_t \equiv \frac{\widehat{x}^w_t - \varsigma\widehat{x}^p_t}{1+\varsigma\kappa_\pi}$.

Proof. This result follows directly from Proposition 1. □

Equation (23) describes the real wage in this economy, which is a function of the output gap. The parameter Ξ_y governs the cyclicality of the real wage, which depends on the relative wage and price rigidities.

3.3 Aggregate Demand

Aggregate demand in our economy, as in any New Keynesian model, corresponds to the aggregate Euler equation in addition to goods market clearing. In our model, as Equation (15) shows, the aggregate Euler equation depends on the consumption gap. Hence, before solving for aggregate demand, we present the equilibrium consumption gap, which is characterized in Proposition 3.

Proposition 3 (The Consumption Gap)**.** *The equilibrium consumption gap is given by*

$$\widehat{\gamma}_t = -\Theta_y \widehat{y}_t - \Psi\mathbb{E}_{t-1}\widehat{x}_t, \tag{24}$$

where

$$\Theta_y \equiv \Psi\left(\underbrace{\frac{\alpha}{1-\alpha}}_{\textit{Employment}} + \underbrace{\frac{\kappa_\omega\left(\varpi_1 + \frac{\varphi}{1-\alpha}\right)}{1+\varsigma\kappa_\pi} - \frac{\varsigma\kappa_\pi\frac{\alpha}{1-\alpha}}{1+\varsigma\kappa_\pi}}_{\textit{Real Wage}} \right).$$

Proof. This result follows directly from replacing Equation (23) in the expression for the consumption gap (20). □

Equation (24) shows that consumption inequality depends on output, where the coefficient Θ_y represents the cyclicality of the consumption gap. The cyclicality depends on two channels: *Employment* and *Real Wage.* The former derives from the switch in the labor quantity required by firms in the presence of decreasing returns on labor. The latter enters due to the dependence of the consumption gap (through markups) on the real wage. More importantly, with wage rigidities, the real wage depends on both price and wage inflation. Hence, the cyclicality of the consumption gap depends on the dynamics of nominal wages and prices, represented by the parameters κ_ω and κ_π.

Price and wage rigidities have different effects on the cyclicality of the consumption gap. While both κ_ω and κ_π fall when wages or prices become more rigid, their impact on inequality is different. When prices are more rigid (given a degree of wage stickiness) consumption inequality becomes more countercyclical, whereas when wages are more sticky inequality becomes less countercyclical. The intuition is that these rigidities generate a distribution of resources between workers and firms' owners. When there is a recession and wages do not fall by much, it is firms' owners who bear the shock. This implies that the consumption gap does not react as much as in the case with flexible wages.

Next, we show how the previous result translates into aggregate demand (and output). We present our main result: an aggregate demand equation with wage and price rigidities and limited access to financial markets. Given (15) and (24), we derive the IS equation, as presented by Proposition 4.

Proposition 4 (The IS equation)**.** *Under iid shocks, the IS equation of this economy with financial frictions and price and wage rigidities is given by*

$$\widehat{y}_t = -\frac{1}{\sigma}\frac{1}{1-\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta_y}\left(\widehat{r}_t - \widehat{\chi}_t\right), \tag{25}$$

Proof. See Appendix B.3. □

Equation (25) presents the Euler equation after deriving the consumption gap, replacing the gap into the original Euler (Equation (15)), and assuming goods market clearing. The main difference between this Euler equation and the one derived from a representative agent without wage rigidities is that the slope (the relationship between output and the interest rate) is significantly affected by other features of the model. In particular, the slope depends on the way resources are distributed in the cycle, which in this case depends on the rigidities as we explain below.

Equilibrium in the Simplified Economy. If the economy is subject to *iid* shocks, the equilibrium in this economy is summarized by the following equations

$$\widehat{y}_t = -\frac{1}{\sigma}\frac{1}{1-\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta_y}\left(\widehat{r}_t - \chi_t\right), \tag{26}$$

$$\widehat{\pi}_t^p = \Upsilon_y \widehat{y}_t, \tag{27}$$

$$\widehat{\omega}_t = \Xi_y \widehat{y}_t, \tag{28}$$

$$\widehat{r}_t = \phi_\pi \widehat{\pi}_t^p + \phi_\omega \widehat{\pi}_t^\omega + \varepsilon_t^{mp}, \tag{29}$$

$$\widehat{\pi}_t^\omega = \widehat{\omega}_t + \widehat{\pi}_t^p. \tag{30}$$

Equations (26)-(30) characterize: (i) the IS equation; (ii) the relation between price inflation and output (obtained by replacing the equation for real wages into the equation for price inflation)[9]; (iii) the cyclicality of real wages; (iv) the evolution of the interest rate; and (v) the definition of wage inflation. Notice that due to the iid shocks assumption we made, the terms with expectations dissapear (both past and future). Therefore, what follows in this section can be interpreted as the *impact* responses of the variables to the different shocks.

3.4 The Distributional Channel of Nominal Rigidities

As we can observe in Proposition 4, output depends directly on price and wage rigidities. This dependence arises because nominal rigidities affect how income is distributed in the cycle, distorting the average marginal propensity to consume. In this way, we obtain the mechanism proposed by Keynes (1936), which is that wages enter aggregate demand if they distort the average MPC. Our approach to obtain this result is through nominal rigidities, and we call this *the distributional channel of nominal rigidities*. Next, we study how the distributional channel affects the variance of the output gap.[10]

To study the role of the distributional channel, let us for now assume that monetary policy fully controls the real interest rate, i.e., $\widehat{r}_t = \varepsilon_t^{mp}$, where ε_t^{mp} is an exogenous monetary policy shock.[11]

[9] With $\Upsilon_y \equiv \frac{1}{1-\varsigma}\left[\kappa_\omega(\varpi_1 + \frac{\varphi}{1-\alpha}) - \frac{\varsigma\alpha}{1-\alpha}\right]$.

[10] We are interested in the second moment as that is the welfare relevant indicator.

[11] Another way of obtaining this type of rule is by having a monetary policy rule that fully targets the expected inflation $r_t = \mathbb{E}_t\pi_{t+1} + \varepsilon_t^{mp}$ as in Bilbiie (2020). Notice that with our assumptions of iid shocks we have $\mathbb{E}_t\pi_{t+1} = 0$, so these two rules are equivalent under the assumptions of Proposition 4.

With these assumptions, the output gap (in the absence of preference shocks) is given by

$$\hat{y}_t = -\frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda} \Psi \left(\frac{\alpha}{1-\alpha} + \frac{\kappa_\omega \left(\varpi_1 + \frac{\overline{\varphi}}{1-\alpha} \right)}{1+\varsigma\kappa_\pi} - \frac{\varsigma\kappa_\pi \frac{\alpha}{1-\alpha}}{1+\varsigma\kappa_\pi} \right)} \varepsilon_t^{mp}, \tag{31}$$

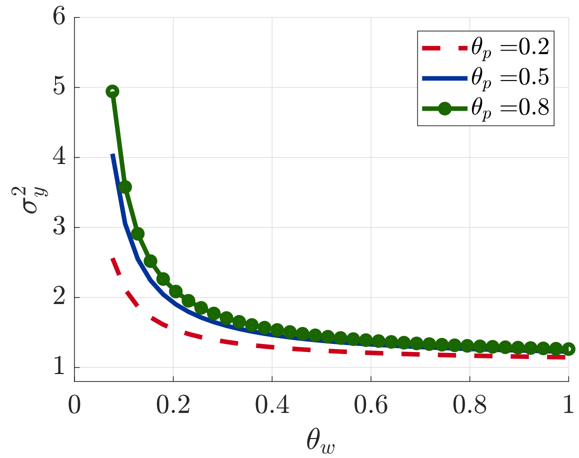
where we use the expression for Θ_y. Hence, through the coefficient Θ_y, output depends explicitly on the relative wage and price rigidities. This can be observed by the dependence of output on the parameters κ_ω and κ_π. Hence, in this model, amplification of the monetary policy transmission is not just obtained from incomplete markets but from a higher degree of price stickiness relative to wage stickiness. That is the distributional channel of nominal rigidities on aggregate demand. Notice that when prices get more sticky (given a degree of nominal wage rigidity), the parameter κ_π falls, and the response of output to the monetary policy shock is amplified.[12] The intuition is simple. The stickier prices are, the more price markups rise in a downturn. That implies that workers with high MPCs (as they are more financially contrained) lose more than firm owners with low MPCs (who are unconstrained). Therefore, the response of consumption (and then output) is amplified by the higher countercyclicality of markups generated by high price stickiness.

However, wage rigidity dampens the effect of monetary policy through redistribution. Having more rigid wages implies that workers are more protected from aggregate shocks as their income fluctuates less in our setup. Therefore, firm owners bear the costs of recessions when wages are more rigid. In that case the distributional channel is weaker and the economy stabilizes. This implies that as a consequence of the distributional channel, there are no gains from wage flexibility, conditional on a real rate shock.

Figure 1 describes how the distributional channel operates depending on price and wage rigidities. It shows that, conditional on monetary policy shocks (i) there are never gains from wage flexibility: the variance of output monotonically increases with wage flexibility ($\theta_\omega \to 0$); and (ii) the variance of output increases with price rigidity ($\theta_p \to 1$).

[12] In Appendix D we show that the derivative of Equation (31) is negative, meaning that the economy becomes more sensitive to monetary policy shocks when price rigidity increases.

Figure 1: Variance of output for different calibrations, conditional on monetary policy shocks.



Notes: This figure shows the variance of the output gap for different levels of price rigidities, θ_π, as a function of wage rigidities, θ_ω. The calibration assumed in this figure is the following: $\lambda = 0.3$, $\alpha = 0.25$, $\epsilon = \epsilon_w = 6$, and $\sigma = \varphi = 1$.

Importantly, we obtain this dependence of output in the relative wage and price rigidity because of wage rigidities. Recall that the parameters ς, κ_ω, and κ_π depend on the degrees of price and wage rigidities (θ_π and θ_ω). Recall also that $\varsigma = \frac{\theta_\omega(1+\varphi\epsilon_w)}{1+\theta_\omega\varphi\epsilon_w-(1-\theta_\omega)\varpi_2}$, which is equal to zero if wages are fully flexible ($\theta_\omega = 0$). This implies that when wages are fully flexible, aggregate demand does not depend on price rigidities. The reason why this is the case is that with wage rigidities the real wage depends explicitly on price inflation. That dependence is given by the parameter ς, which is the pass-through from price inflation to the real wage (see Equation 22). This pass-through is stronger when wages are more rigid. That happens because whenever nominal wages are rigid an increase in price inflation makes the real wage fall. Naturally, the higher the wage rigidity, the stronger the relationship between the real wage and price inflation. Then, if wages are flexible, it can be shown that aggregate demand, conditional only on real rate shocks, does not depend on any rigidity and we get

$$\hat{y}_t = -\frac{1}{\sigma}\frac{1}{1-\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Psi\left(\varpi_1+\frac{\alpha+\varphi}{1-\alpha}\right)}\varepsilon_t^{mp}, \tag{32}$$

which is the expression obtained by Bilbiie (2008) and Debortoli and Galí (2017).

3.5 Wage Flexibility and the Role of Monetary Policy with Inequality

As Galí (2013) uncovered, the effect of wage flexibility in New Keynesian models hinges crucially on how monetary policy is conducted. In this section, we show that in models with heterogeneity, monetary policy rules that only respond to price inflation are not sufficient to counteract the effect of highly volatile wages. In RANK, targeting price inflation is isomorphic to targeting wage inflation (unless we are interested in welfare). That is because prices are the only channel through which wage fluctuations affect output. Then, the role of higher–or lower– wage flexibility depends primarily on the response of price inflation to it. Hence, having a rule that reacts to wages or prices has similar qualitative effects on the economy.

Studying monetary policy design in models with heterogeneity and market incompleteness is relevant. Most of the models with household heterogeneity assume simple monetary policy rules. They do so, because the focus is on the impact of heterogeneity and not the conduct of monetary policy. However, as we explained above, with income heterogeneity, price and wage inflation affect aggregate demand directly (through the real wage). Moreover, aggregate demand depends differently on prices and wages, which means that the responses to price and wage inflation might no longer be equivalent.

Let us assume that the Taylor Rule is given by

$$\widehat{r}_t = \phi_\omega \widehat{\pi}_t^\omega + \phi_\pi \widehat{\pi}_t^p,$$

where monetary policy reacts to deviations of the nominal wage and the price inflation rates from their steady states (assumed at zero). Substituting the Taylor rule into the Euler equation delivers

$$\widehat{y}_t = -\frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta_y} \left[\phi_\omega \widehat{\pi}_t^\omega + \phi_\pi \widehat{\pi}_t^p - \widehat{\chi}_t\right].$$

Recalling that $\widehat{\pi}_t^\omega = \widehat{\omega}_t + \widehat{\pi}_t^p$, $\widehat{\omega}_t = \Xi_y \widehat{y}_t$, and $\widehat{\pi}_t^p = \Upsilon_y \widehat{y}_t$, the previous expression implies

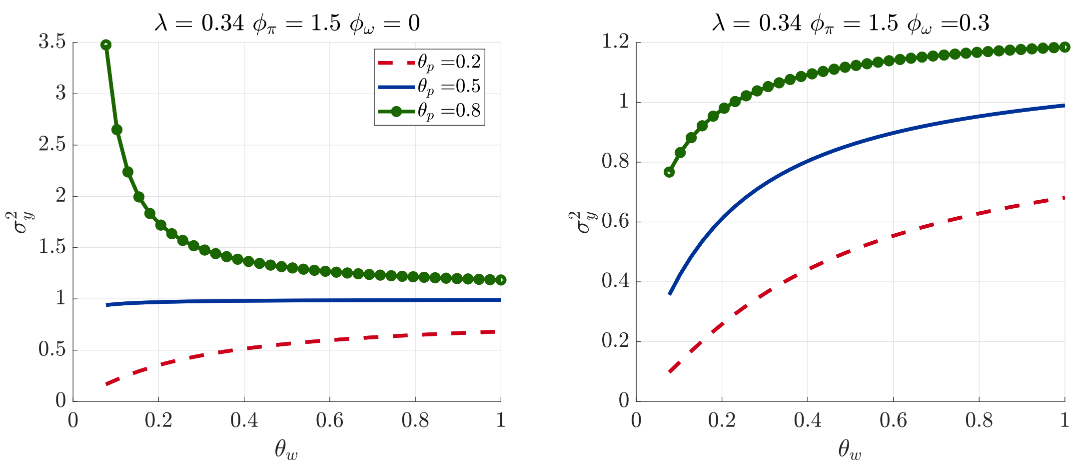
$$\widehat{y}_t = -\frac{1}{\sigma}\left(1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta_y + \frac{1}{\sigma}\left[\phi_\omega\left(\Xi_y + \Upsilon_y\right) + \phi_\pi \Upsilon_y\right]\right)^{-1} \widehat{\chi}_t. \tag{33}$$

Hence, output depends on the cyclicality of prices Υ_y and the cyclicality of wages Ξ_y through the interest rate response, in addition to the distributional channel represented by the expression $\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta_y$.

With this type of policy rule, monetary policy has the ability to directly counteract the excessive volatility of wages if the response to them is sufficiently strong. Figure (2) shows the variance of the output for two alternative Taylor rules. The left-hand panel shows the variance of output with a Taylor rule that does not respond to wages, while the right-hand panel shows the variance if monetary policy reacts to nominal wage inflation too. When monetary policy does not react to wages, it does not offset the amplifying effects of redistribution since the real wage is still too volatile (if prices are too sticky). That translates to output through aggregate demand. However, if monetary policy reacts (strongly) to wage inflation, it activates an additional countercyclical response. In this case, monetary policy reacts to the high volatility of wages and counteracts the distributional effect of high wage flexibility.

Therefore, we have two opposing effects from wage flexibility in our model. One depends on the share of HtM agents and the other on the ability of monetary policy to react to aggregate outcomes, whether prices or wages. The former controls the degree of redistribution, and the latter acts as a countercyclical device. The left-hand panel in Figure (2) shows that the strength of these effects depends on the degree of price rigidities. If prices are flexible, wage flexibility is stabilizing while if prices are sticky wage flexibility is distabilizing. An additional result from this is that there is a threshold for θ_π which turns wage flexibility from distabilizing to stabilizing. These results depend on the response of monetary policy to wages or prices, as the right-hand panel shows. For that specific calibration, monetary policy can restore the ability of wage flexibility to stabilize output.

Figure 2: Variance with alternative Taylor rules.



Notes: This figure shows the variance of the output gap for different levels of price rigidities, θ_π as a function of wage rigidities, θ_ω. The calibration assumed in this figure is the following: $\lambda = 0.3$, $\alpha = 0.25$, $\epsilon = \epsilon_w = 6$, and $\sigma = \varphi = 1$.

However, the previous result does not hold for every $\phi_\omega > 0$. To show this, we compute the value of ϕ_ω that turns wage rigidities from amplifying to stabilizing, which is given by [13]

$$\overline{\phi}_\omega = \frac{\sigma\lambda(\epsilon_p - 1)(1-\alpha)(1-\alpha+\alpha\epsilon_p)\theta_\pi}{(\epsilon_p - \lambda(\epsilon_p-1)(1-\alpha))(1-\alpha+\theta_\pi\alpha\epsilon_p)} - \frac{(1-\theta_\pi)(1-\alpha)}{1-\alpha+\theta_\pi\alpha\epsilon_p}\phi_\pi, \tag{34}$$

which we refer to as the threshold ϕ_ω. Equation (34) shows that the threshold depends on the share of HtM and the degree of wage rigidities. Figure 3 displays $\overline{\phi}_\omega$.

As the share of HtM and price rigidities increase, so too does the threshold. These two are the main drivers of the distributional channel. Hence, to offset the distributional channel, monetary policy must react to wage inflation sufficiently strongly.

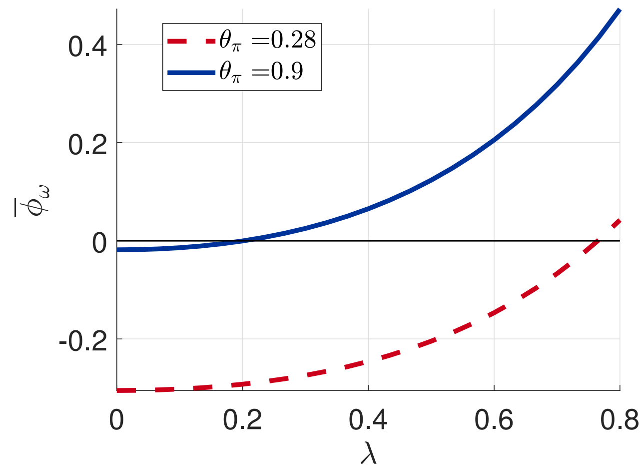
In the case of Figure (3), for some combinations of parameters (i.e., low λ and low θ_p) the threshold is negative. We interpret that as combinations of parameters in which wage fluctuations are not a constraint for monetary policy when stabilizing output. The negative values are a consequence of having $\phi_\pi > 0$, which helps in stabilizing output when responding to prices. However, with extremely sticky prices, the role of ϕ_π disappears. Equation (34) also shows that when prices are fully sticky, the required response to wages is bounded. This means that monetary policy counteracts the distributional channel more effectively when it responds to wage inflation; i.e., monetary policy does not need to set $\phi_\omega \to \infty$ to stabilize output. This is more evident if we compare the threshold for ϕ_ω with the one for ϕ_π, which reads

$$\overline{\phi}_\pi = \frac{\sigma\lambda(\epsilon_p - 1)(1-\alpha+\alpha\epsilon_p)}{\epsilon_p - \lambda(\epsilon_p-1)(1-\alpha)}\frac{\theta_\pi}{(1-\theta_\pi)} - \frac{1-\alpha+\alpha\epsilon_p\theta_\pi}{(1-\theta_\pi)(1-\alpha)}\phi_\omega. \tag{35}$$

Notice that if prices are fully sticky ($\theta_p = 1$), $\overline{\phi}_\pi \to \infty$. Whereas, in the case of ϕ_ω that is not the case. This means that monetary policy is not sufficiently effective to counteract the impact of wage flexibility if prices are sticky. Therefore, policy makers should consider responding to wages.

[13]In Appendix E we get this threshold by computing the derivative of the variance of output to θ_w, equalizing it to zero, and solving for the minimum parameter required to turn wage flexibility from amplifying to stabilizing.

Figure 3: Threshold for ϕ_ω.



Notes: This figure shows the variance of the output gap for different levels of price rigidities, θ_π as a function of wage rigidities, θ_ω. The calibration assumed in this figure is the following: $\lambda = 0.3$, $\alpha = 0.25$, $\epsilon = \epsilon_w = 6$, and $\sigma = \varphi = 1$.

The main takeaway of this exercise is that monetary policy plays a vital role in offsetting the effects of redistribution. When wages are too volatile with respect to prices, monetary policy should react to wages to offset the distributional effects of shocks and stabilize aggregate demand. Therefore, in economies with income inequality and incomplete markets, in which wage inflation is more volatile than price inflation, the monetary authority should target wages in addition to prices to stabilize output effectively.

4 Gains from Wage Flexibility: Calvo Price and Wage Adjustment

In this section, we use our model to quantitatively investigate the gains from wage flexibility and how such gains depend on: the relative nominal rigidities; the degree of market incompleteness; and the zero lower bound on the nominal interest rate (ZLB). We consider the latter because it helps us study the effects of the distributional channel of nominal rigidities in a model without the simplifications we made in Section 3. Now we switch from a setup where prices are set in advance to one in which prices and wages are subject to Calvo pricing. This allows us to take into account agents' expectations and the dynamics of the economy, unlike in Section 3, and analyze if the main results we showed previously still hold.

4.1 Price and Wage Setting à la Calvo

Now firms face price stickiness à la Calvo. Hence, in every period, they reset prices with probability $(1-\theta_p)$. A firm that is able to reset prices in period t, chooses the price P_t^* that maximizes the following sum of discounted profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - TC_{t+k}(Y_{t+k|t}) \right) \right\}, \tag{36}$$

subject to the demand constraint given by

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} Y_{t+k}, \tag{37}$$

which is the demand faced in $t+k$ by a firm that sets prices optimally in t. Total cost of producing $Y_{t+k|t}$ units is $TC_{t+k}(Y_{t+k|t}) \equiv W_{t+k} \left(\frac{Y_{t+k|t}}{A_{t+k}} \right)^{\frac{1}{1-\alpha}}$. $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}^u}{C_{t+k}^u} \right)^{-\sigma}$ is the stochastic discount factor, which depends only on the consumption of the unconstrained agent. The first-order condition for profits maximization reads

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M}^p MC_{t+k|t} \right) \right\} = 0, \tag{38}$$

where $\mathcal{M}^p \equiv \frac{\varepsilon_p}{\varepsilon_p - 1}$ is the desired markup and $MC_{t+k|t}$ is the nominal marginal cost.

The wage for each labor variety is set by a union operating in a monopolistically competitive market. Unions choose the wage rate that maximizes a weighted average of unconstrained and constrained lifetime utility, given by

$$\begin{aligned} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \chi_{t+k} \Bigg[(1-\lambda) \Bigg(\frac{(C_{t+k}^u - 1)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{(N_{t+k|t}^u)^{1+\varphi}}{1+\varphi} dj \Bigg) \\ + \lambda \Bigg(\frac{(C_{t+k}^c - 1)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{(N_{t+k|t}^c)^{1+\varphi}}{1+\varphi} dj \Bigg) \Bigg], \end{aligned} \tag{39}$$

subject to households' resource constraint and the sequence of demands for the labor variety they represent, given by

$$N_{t+k|t}^K = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\epsilon_w} N_{t+k}^K, \tag{40}$$

where W^* is the optimal wage chosen by a union that last resets its wage at t, $N^K_{t+k|t}$ is labor supply for the household's members whose wage was last reoptimized in period t and ϵ_w is the elasticity of substitution among labor varieties.

Assuming firms demand for constrained and unconstrained workers labor is the same, i.e., $N_t(j)^u = N_t(j)^c = N_t(j)$, the first-order condition of the union is

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \chi_{t+k} N^{1+\varphi}_{t+k|t} \left[\left((1-\lambda) \frac{1}{(C^u_{t+k})^\sigma N^\varphi_{t+k|t}} + \lambda \frac{1}{(C^c_{t+k})^\sigma N^\varphi_{t+k|t}} \right) \frac{W^*_t}{P_{t+k}} - \mathcal{M}^w \right] = 0, \tag{41}$$

where $\mathcal{M}^w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$ is the desired markup and $\Pi_{w,t} \equiv \frac{W_t}{W_{t-1}}$ is the gross inflation rate of wages.

4.2 Quantitative Analysis

Calibration. For the baseline calibration we set the parameter α to 0.25 and the discount factor β to 0.994. We initially set the Calvo price and wage parameters to 0.75, which implies an average contract duration of four quarters. We set the parameters ϵ_p and ϵ_w to 6, which implies a steady state markup of 20%. We assume the inverse of the intertemporal elasticity of substitution, σ, and the inverse of the Frisch elasticity, φ, equal to one. Additionally, we fix the coefficient for price inflation of the Taylor rule, ϕ_π, to 1.5 and the one of wage inflation, ϕ_ω, to 0.[14] The interest rate smoothing parameter is set to 0.8.[15] We assume ρ_χ, the autoregressive coefficient of the exogenous preference shock, is 0.8. Regarding the fraction of constrained agents, we assume two scenarios: a Representative Agent (RANK) economy where all households are unconstrained (i.e., $\lambda = 0$) and an economy with a positive fraction of constrained agents, where $\lambda = 0.3$. We solve the model with the extended path method to implement the zero lower bound, and solve all the versions with this method for comparability.

The Gains from Wage Flexibility without the ZLB. We simulate the response of the economy to a contractionary preference shock on different scenarios, depending on the degree of wage flexibility and the access to financial markets. We solve the model for combinations of $\theta_\omega = \{0.3, 0.75\}$, a flexible and a rigid wage case; and $\lambda = \{0, 0.3\}$, without and with inequality, and keep the remaining parameters as described in the calibration. We report the results for the four combinations of these parameters in the plots that follow.

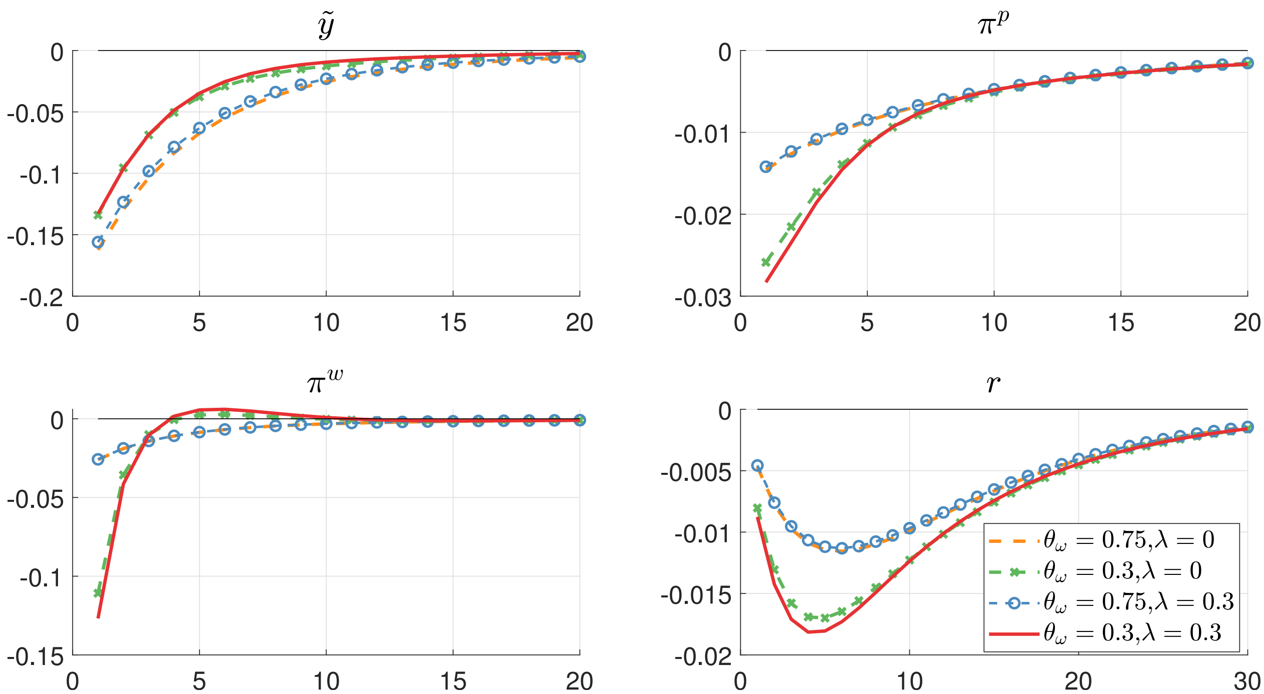
[14] We set these parameters to describe how the distributional channel affects the dynamics of the economy for a monetary policy that does not consider wages and prices as variables with different effects.

[15] For the following simulations we modify rule (11) to incorporate interest rate smoothing.

Figure 4 shows the responses of output, price inflation, wage inflation, and the nominal interest rate to a demand shock in the four afore-mentioned cases. The differences in the responses of output are mostly driven by differences in wage rigidity. The more rigid wages are, the stronger the response of output. Naturally, in the case where wages are flexible, wage inflation falls considerably more than in the case in which wages are rigid. More volatile wages are transmitted to price inflation. Since prices are relatively sticky in this example, the response of inflation is not as strong as the wage inflation rate. However, it is strong enough to trigger a substantial response in the interest rate compared to the case of having rigid wages.

Although the responses of the interest rate with and without inequality are different, they are not quantitatively important. Therefore, in this calibration, monetary policy is successful in counteracting the distributional channel of nominal rigidities. Then, wage flexibility reduces output volatility.

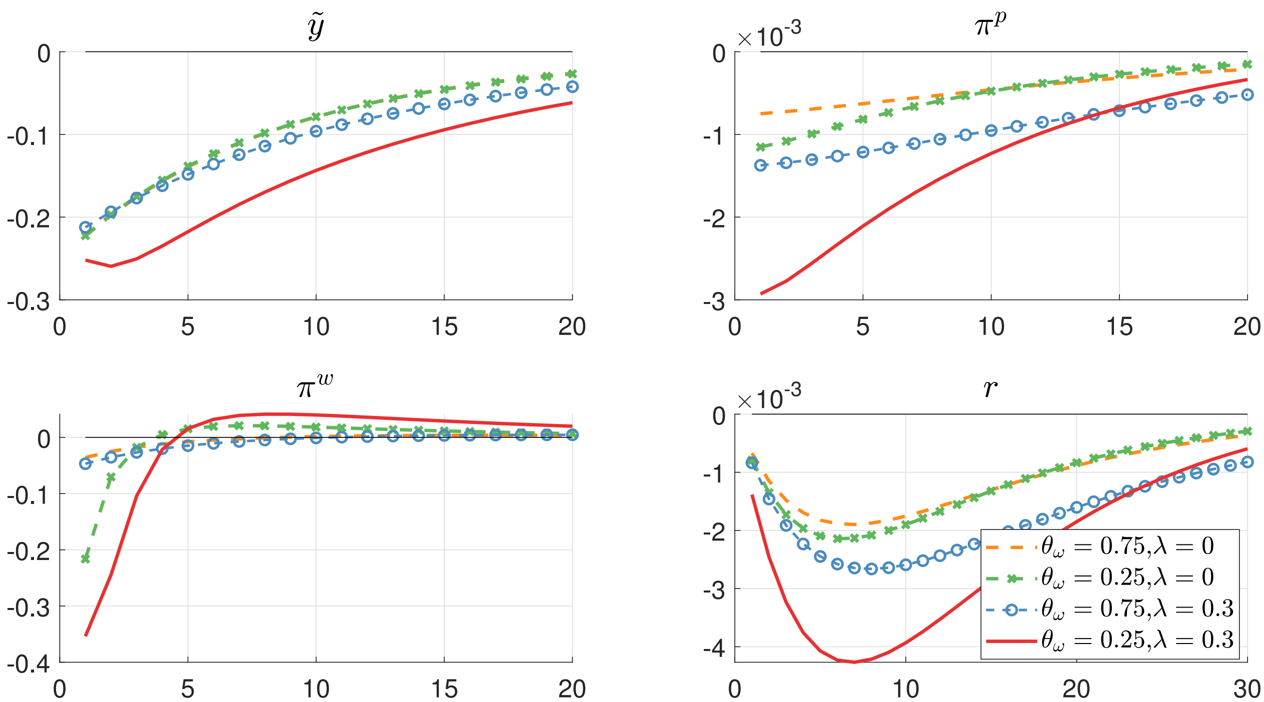
Figure 4: Response of output, prices and wage inflation, and the nominal interest rate to a contractionary preference shock. Baseline calibration.



Notes: This figure shows the responses of output, price and wage inflation, and the nominal interest rate to a contractionary preference shock. We show four calibrations for combinations of $\theta_\omega = \{0.3, 0.75\}$ *and* $\lambda = \{0, 0.3\}$. *This plot assumes the baseline calibration.*

In Figure 5 we set out the role of the distributional channel by assuming prices are almost fully sticky (θ_p=0.95). When prices are sticky, and monetary policy only responds to price inflation, the distributional channel comes into play. In the case of Figure 5, output with inequality and flexible wages falls persistently more that in the other three cases (in which we observe no differences). This is because now monetary policy does not stimulate output, and the distributional effect operates very strongly. Moreover, the case in which there is inequality and wages are sticky behaves like the representative agent case. This suggests that wage rigidity acts as an insurance device for workers even if prices are highly sticky.

Figure 5: Response of output, prices and wage inflation, and the nominal interest rate to a contractionary preference shock. High price rigidity, $\theta_p = 0.95$.

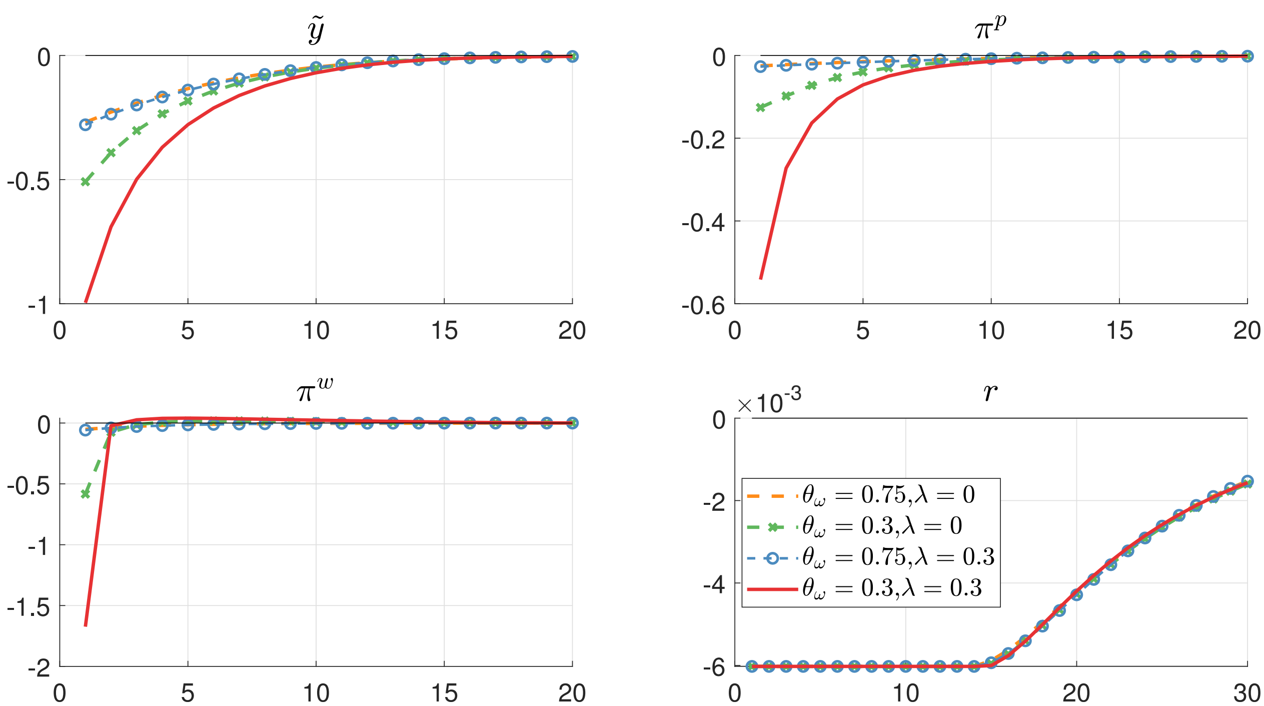


Notes: This figure shows the responses of output, price and wage inflation, and the nominal interest rate to a contractionary preference shock. We show four calibrations for combinations of $\theta_\omega = \{0.3, 0.75\}$ and $\lambda = \{0, 0.3\}$. This plot considers $\theta_p = 0.95$.

The Gains from Wage Flexibility with the ZLB. Another way of analyzing the distributional channel is by imposing the ZLB. We show this in Figure 6. When the ZLB is present, wage flexibility is undesirable. This observation was first made by Billi and Galí (2019), who showed that in the presence of the ZLB, there are always losses from wage flexibility. In such a case, more flexible wages are associated with a more severe deflation. Since the policy rate cannot be further reduced, deflationary expectations induce a drop in demand (due to the associated rise in the real rate), which exacerbates output contraction. For our calibration, in a RANK economy, the impact of having highly flexible wages makes output fall by twice as much as in the case of rigid wages.

In the economy with inequality, the effect is considerably larger. We observe that the output gap falls twice as much as in the RANK case. In this scenario, the effects from greater wage flexibility via deflationary expectations and redistribution reinforce each other due to a feedback loop between the two channels. In fact, redistribution puts downward pressure on output and thus prices, which translates into a rise in the real rate that depresses activity. Given a countercyclical income gap, the latter implies redistribution against the constrained households, which further depresses demand and output. A stronger cut in demand by the unconstrained agents, who respond to more severe deflationary expectations, together with the contractive effects of redistribution, explain the larger drop in activity as compared to the RANK economy. Therefore, in our model, the distributional and monetary policy channels interact to give rise to a sizable drop in activity, largely beyond that observed in a RANK. All the previous analysis implies that nominal rigidities determines the distributional channel, and monetary policy is a key determinant in the transmission of shocks.

Figure 6: Response of output, prices and wage inflation, and the nominal interest rate to a contractionary preference shock in the ZLB.



Notes: This figure shows the responses of output, price and wage inflation, and the nominal interest rate to a contractionary preference shock. We show four calibrations for combinations of $\theta_\omega = \{0.3, 0.75\}$ *and* $\lambda = \{0, 0.3\}$*. This plot assumes the baseline calibration and that monetary policy is subject to the ZLB.*

Output Volatility and Welfare. Now, we generate artificial time series for several variables subject to demand shocks. We consider the two alternative calibrations of the Calvo wage parameter and generate these series using the extended path method to explore the effects of greater wage flexibility. We set the volatility of the innovation so that the ZLB binds 5% of the time. Figure A.1 in Appendix A shows that in the RANK economy, having flexible wages is associated with lower output variability in periods when monetary policy is active, while volatility increases in periods when the ZLB binds. The reduction in rigidities however has only a modest effect on output dynamics. When there are financial frictions (Figures A.2 and A.3 in Appendix A), higher wage flexibility greatly exacerbates the contraction of output in periods when monetary policy is constrained by the ZLB. In line with our previous discussion, the more severe contraction is explained by a larger drop in unconstrained agents' consumption, who respond to increased deflationary expectations, and, to a larger extend, redistribution, which depress constrained agents' consumption. Notice that higher flexibility particularly affects constrained households, whose income is severely reduced due to a large cut in wages, triggering a sizable cut in spending. The present exercise clearly illustrates the relevance of considering the interest rate and redistributional channels jointly, as they do not operate independently. Instead, they interact to enhance each other´s effect, giving rise to a sizable drop in production.

How does wage flexibility affect volatility and welfare? Table 1 presents the results for the volatility of output, the rates of inflation, and the consumption gap for different calibrations of θ_w. In the RANK economy, higher flexibility is associated with a more stable output (while price and wage inflation volatility greatly increases). With inequality we observe that: (i) more flexible wages destabilize output, and (ii) price and wage inflation volatility rise. In terms of welfare losses, Table 2 shows that in an economy with limited asset markets participation, greater wage flexibility increases losses related to all welfare-relevant variables.[16]

	$\tilde{y}$	π^p	π^w
$\theta_w = 0.75$	0.030	0.002	0.002
$\theta_w = 0.30$	0.029	0.004	0.015
Ratio	0.96	2.7	6.4

(a) $\lambda = 0$

	$\tilde{y}$	γ	π^p	π^w
$\theta_w = 0.75$	0.027	0.011	0.001	0.002
$\theta_w = 0.30$	0.036	0.055	0.006	0.019
Ratio	1.3	5	3.8	8.9

(b) $\lambda = 0.3$

Table 1: Standard deviation.

[16]See Appendix C for a derivation of the welfare loss function.

	$\tilde{y}$	π^p	π^w	Total Loss
$\theta_w = 0.75$	0.0012	0.0003	0.0010	0.0025
$\theta_w = 0.30$	0.0011	0.0019	0.0022	0.0052
Ratio	0.9	7.1	2.1	2.1

(a) $\lambda = 0$

	$\tilde{y}$	γ	π^p	π^w	Total Loss
$\theta_w = 0.75$	0.0010	0.0000	0.0002	0.0008	0.0021
$\theta_w = 0.30$	0.0017	0.0003	0.0032	0.0034	0.0087
Ratio	1.7	24.9	14.7	4.1	4.2

(b) $\lambda = 0.3$

Table 2: Consumption equivalent welfare losses.

5 Conclusion

In this paper, we analyze the consequences of nominal rigidities in the presence of incomplete markets and the zero lower bound. We show both analytically and numerically that in the presence of limited asset market participation, the relative price and wage rigidities enter aggregate demand; i.e., both nominal rigidities play a crucial role in determining the response of the economy to shocks. That is due to a distributional effect: if the distribution of income is unequal and there are agents with limited access to financial markets, when prices fluctuate less strongly than wages, the distributional channel gains prominence, and the effects of shocks amplify. That is a consequence of the larger fluctuation of wages relative to prices. When that is the case, workers (who are usually more financially constrained than the owners of firms) suffer more from fluctuations. That implies that wage flexibility may amplify the cycle if prices are rigid relative to wages.

As Billi and Galí (2019), we show that the conduct of monetary policy is important for all the results exposed above. We find that with inequality, responding to price inflation is no longer isomorphic to responding to wage inflation. If monetary policy only reacts to price inflation, it misses the effects that higher wage volatility has on aggregate demand. We show in our model that monetary policy is more effective if it responds to wage inflation rather to price inflation.

Understanding the interaction of these three features (price and wage rigidities and limited access to financial markets) is important for several reasons. First, there is a growing literature that uses these features to study diverse macroeconomic questions, like the effects of fiscal and monetary policy in the presence of incomplete markets. Second, it is important to understand the effects of labor market policies, particularly the policies that pretend to stabilize the economy through wage deflation. We show that these kinds of policies are not desirable under some circumstances since they generate significant aggregate demand effects that could further depress the economy. Then, with high inequality, the economy may experience a sharp contraction from making wages more flexible.

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A Figures

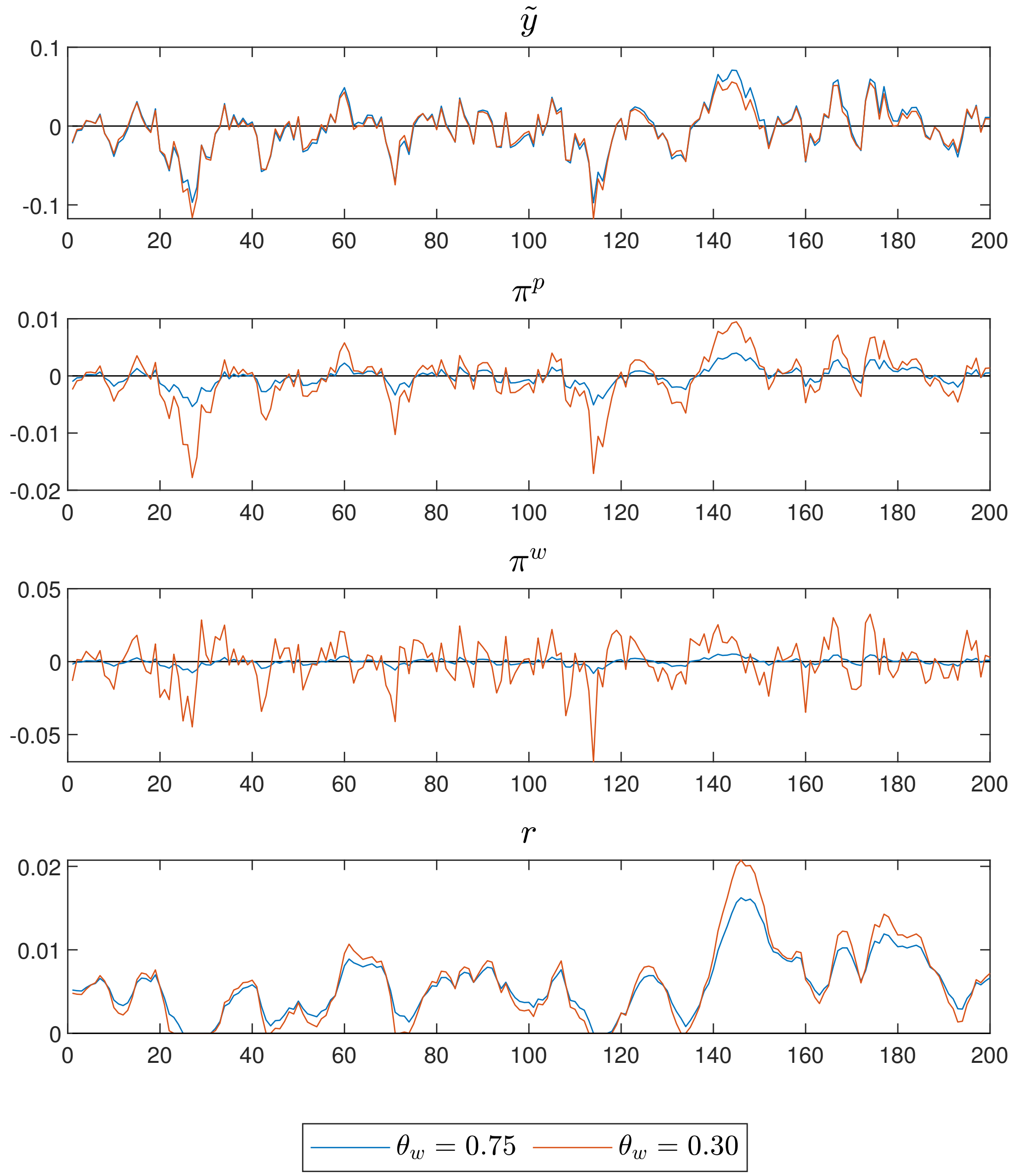

Figure A.1: Fluctuations under preference shocks ($\lambda = 0$).

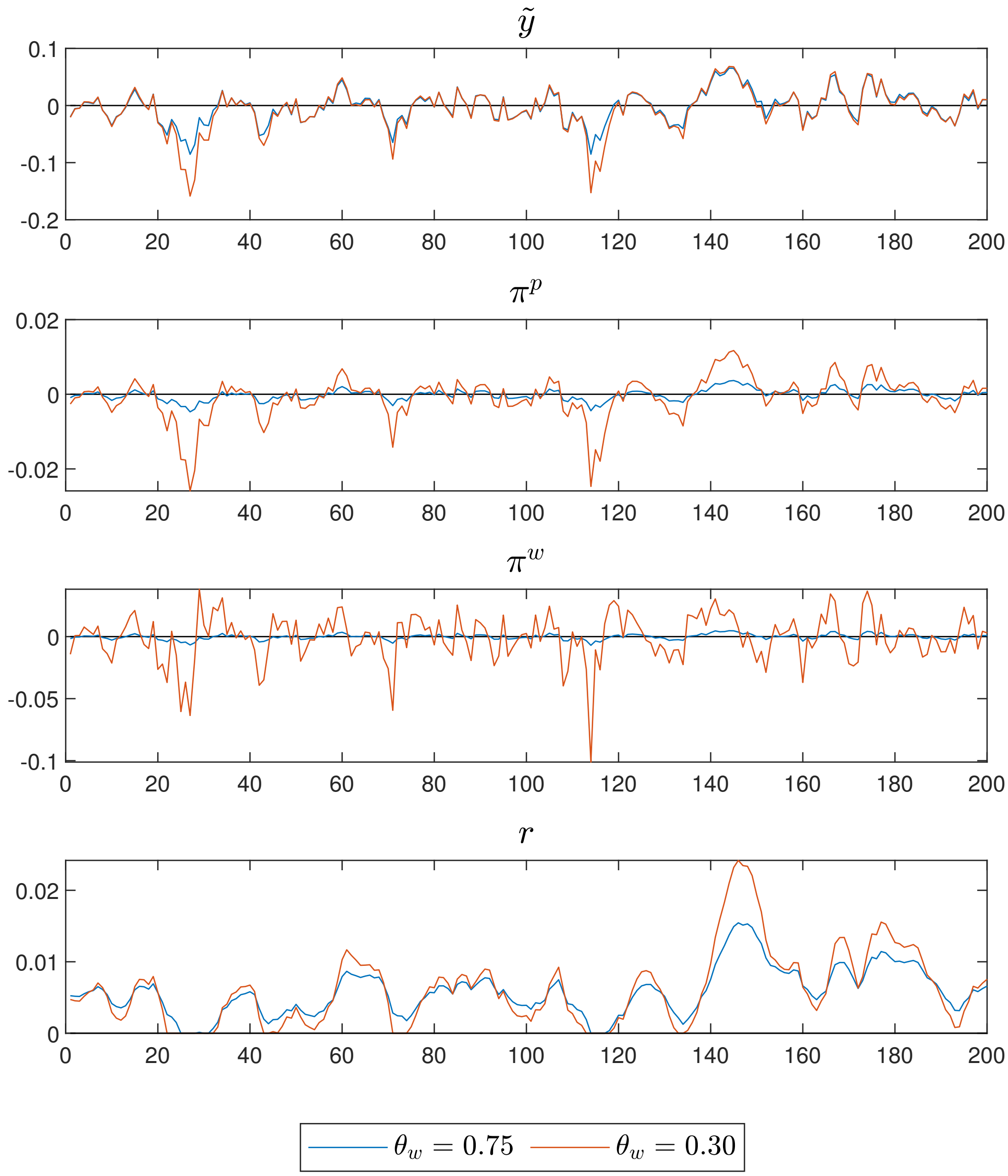

Figure A.2: Fluctuations under preference shocks ($\lambda = 0.3$).

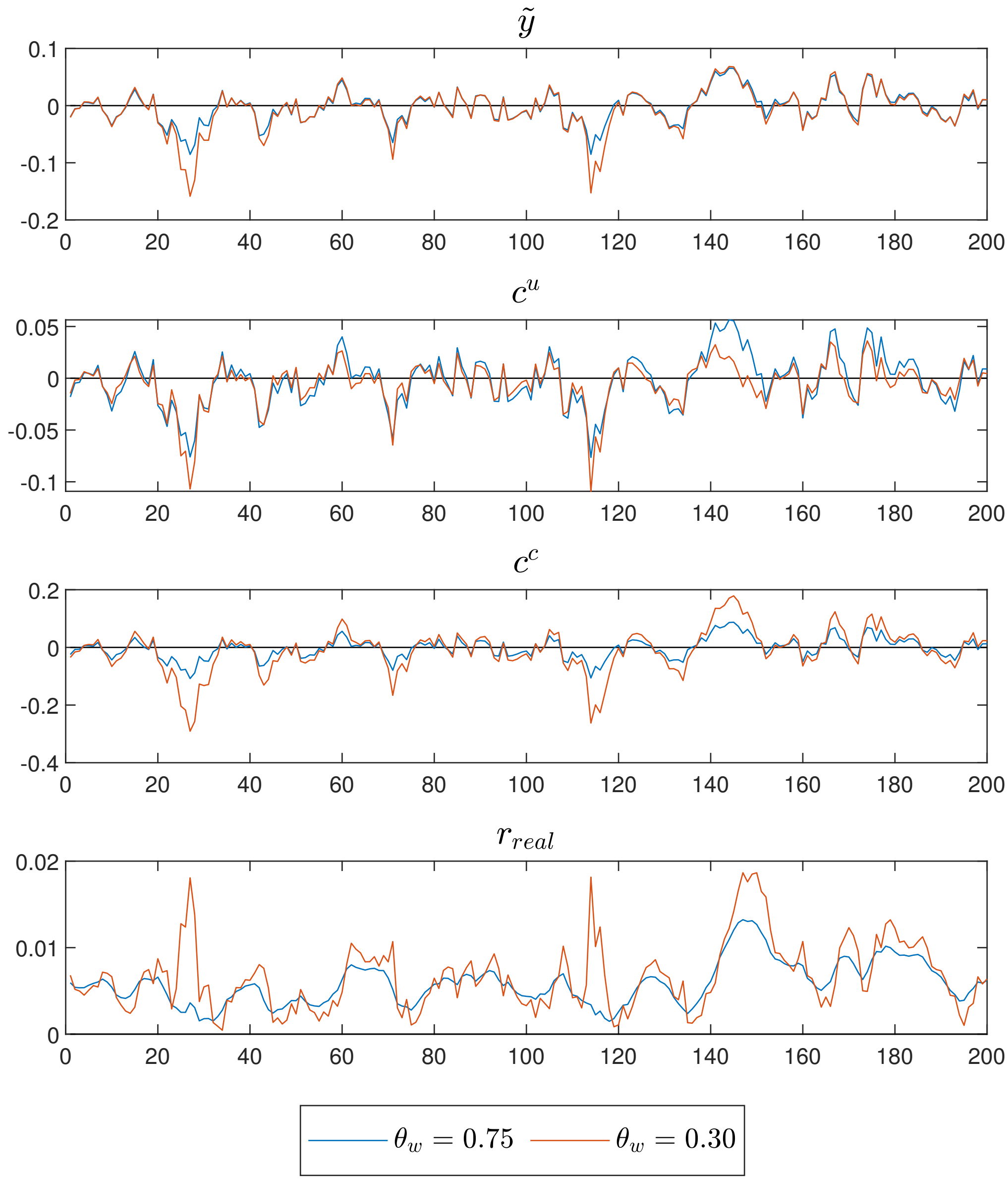

Figure A.3: Fluctuations under preference shocks ($\lambda = 0.3$).

B Proofs and derivations

B.1 Aggregation

Total labor supply must be equal to total demand. This is, $N_t = \int_0^1 \int_0^1 N_t(i,j) di dj$, where i denotes firms and j labor varieties. From the demand of each firm i for variety j we have

$$N_t = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} \left(\int_0^1 N_t(i) di\right) dj.$$

Recalling the demand for each firm i's variety $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} Y_t$ and from the production function of each firm i $N_t(i) = \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}}$, we have

$$\begin{aligned} N_t &= \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} \left(\int_0^1 \left(\left(\frac{Y_t}{A_t}\right)\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p}\right)^{\frac{1}{1-\alpha}} di\right) dj \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \underbrace{\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} di\right)}_{\equiv \Delta_{p,t}} \underbrace{\left(\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj\right)}_{\equiv \Delta_{w,t}}, \end{aligned}$$

which is the same expression as in the main text.

B.2 Proof of Proposition 1

We separate the proof in three parts. First, we describe the evolution of price inflation. Second, we describe the labor supply in the economy with heterogeneity and wage rigidities. Finally, we describe the process for real wages.

Part 1: Price inflation

There is a mass $1 - \theta_p$ of optimizing firms (denoted by superindex o) while the remainder θ_p set prices before the shock is realized (denoted by superindex m).

We start by describing the price setting problem optimizers face. Given the (log-linearized) demand function for a given variety i

$$y_t^o(i) = y_t - \epsilon_p (p_t^o(i) - p_t), \tag{B.1}$$

firms maximize profits setting their price as a markup μ^p over the marginal cost. Optimal pricing implies

$$p_t^o(i) = \mu^p + w_t - \log(1-\alpha) + \frac{\alpha}{1-\alpha} y_t^o(i), \tag{B.2}$$

where $\mu^p \equiv \log(\mathcal{M}^p)$. Substituting the demand (B.1) into the firm optimality condition (B.2) and rearranging yields

$$p_t^o(i) = \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\left(\mu^p + \omega_t - \log(1-\alpha) + \frac{\alpha}{1-\alpha} y_t\right) + p_t, \tag{B.3}$$

where $\omega_t \equiv w_t - p_t$ is the real wage.

Consider next the price setting problem that non-optimizers face. These firms set prices at the end of period $t-1$, and hence, they make their pricing decisions for period t based on the information set available at $t-1$. Optimization implies

$$p_t^m(i) = \mathbb{E}_{t-1}\left(\mu^p + w_t - \log(1-\alpha) + \alpha n_t^m(i)\right). \tag{B.4}$$

On the other hand, the aggregate price index is given by

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon_p} dj\right)^{\frac{1}{1-\epsilon_p}},$$

which implies

$$P_t = \left((1-\theta_p)(P_t^o)^{1-\epsilon_p} + \theta_p(P_t^m)^{1-\epsilon_p}\right)^{\frac{1}{1-\epsilon_p}}.$$

Accordingly, the following relation holds around steady state

$$p_t = (1-\theta_p)p_t^o + \theta_p p_t^m.$$

Substituting the pricing rules from optimizers (B.3) and non-optimizers (B.4) into the aggregate price index yields the following price inflation equation

$$\widehat{\pi}_t^p = \kappa_\pi \left(\widehat{\omega}_t + \frac{\alpha}{1-\alpha} \widehat{y}_t \right) + \mathbb{E}_{t-1} \widehat{x}_t^p, \tag{B.5}$$

where hat variables correspond to log-deviations with respect to steady-state, $\widehat{x}_t^p \equiv \widehat{\omega}_t + \alpha \widehat{n}_t^m(j) + \widehat{\pi}_t^p$ and $\kappa_\pi \equiv \frac{1-\theta_p}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$.

Part 2: Labor supply

Before solving for the wage schedule, we first solve for the *actual* labor supply. This is needed since the labor supply in our model with inequality depends on the consumption gap as it depends on the average marginal utility (thus, depends on both constrained and unconstrained consumption). In our setting, unions maximize households utility by setting the wage (in real terms) to be a markup μ^w over the marginal rate of substitution. However, in a heterogeneous economy like ours, the average marginal utility depends on the fraction of each group of consumers, in particular, it depends on the share of constrained consumers λ. That is why it does not directly depend on the output gap as usual.

To start, we compute three intermediate results. First we show how the average marginal utility of consumption depends on inequality (λ) and the level of consumption of each group. Then we show the relation between individual constrained and unconstrained consumption with GDP and consumption gap. Finally, we show how average marginal utility of consumption depends on GDP and real wages.

Lemma 1. *The average marginal utility of consumption, U, as a function of individual consumptions can be approximated as*

$$\widehat{u}_t = -\sigma(u_c \widehat{c}_t^c + u_u \widehat{c}_t^u), \tag{B.6}$$

with $u_c = \frac{\lambda}{\lambda+(1-\lambda)\gamma^{-\sigma}}$ *and* $u_u = 1 - u_c$, *and* $\gamma \equiv \frac{C^U}{C^C}$ *being the steady-state consumption gap.*

Proof. The marginal utility of consumption of agent $K \in \{c, u\}$ is $(C_t^K)^{-\sigma}$. therefore, the average marginal utility of consumption, U, can be written as $U = \lambda(C_t^c)^{-\sigma} + (1-\lambda)(C_t^u)^{-\sigma}$. Taking a first order approximation around steady-state, we get $\widehat{u}_t = -\sigma(u_c\widehat{c}_t^c + u_u\widehat{c}_t^u)$, with $u_c = \frac{\lambda C_c^{-\sigma}}{\lambda C_c^{-\sigma}+(1-\lambda)C_u^{-\sigma}}$. Replacing $C_u = \gamma C_c$ we get $u_c = \frac{\lambda}{\lambda+(1-\lambda)\gamma^{-\sigma}}$ and $u_u = 1 - u_c$. $\square$

Lemma 1 shows that the average marginal utility of consumption, which is a relevant piece of information for unions to set nominal wages and total hours, depends on the fraction of constrained agents, the income effect on labor supply (given by σ) and the steady-state consumption gap, γ. Clearly, whenever $\lambda = 0$ ($\lambda = 1$), $\widehat{c}_t^c = \widehat{c}_t^u = \widehat{c}_t$, the consumption gap is zero and $u_u = 1$ ($u_u = 0$), so $\widehat{u}_t = -\sigma\widehat{c}_t$, which is the same marginal utility of consumption of a representative agent model.

Lemma 2 shows the aggregate relation between consumption on each segment of the population, GDP and the consumption gap.

Lemma 2. *Consumption of constrained and unconstrained agents can be approximated as*

$$\widehat{c}_t^c = \widehat{y}_t - \frac{(1-\lambda)\gamma}{\lambda + (1-\lambda)\gamma}\widehat{\gamma}_t,$$
$$\widehat{c}_t^u = \widehat{y}_t + \frac{\lambda}{\lambda + (1-\lambda)\gamma}\widehat{\gamma}_t.$$

Proof. From market clearing in the market of final goods and the definition of aggregate consumption we have $Y_t = C_t = \lambda C_t^c + (1-\lambda)C_t^u$. Using the definition of consumption gap, the previous expression is $Y_t = C_t^U\left(\frac{\lambda}{\gamma_t} + 1 - \lambda\right)$. This can be approximated as $\widehat{y}_t = \widehat{c}_t^u - \frac{\lambda}{(1-\lambda)\gamma+\lambda}\widehat{\gamma}_t$. On the other hand, the consumption gap is $\widehat{\gamma}_t = \widehat{c}_t^u - \widehat{c}_t^c$. Using these two equations to solve for constrained and unconstrained consumption as a function of GDP and consumption gap gives the desired equations. $\square$

Finally, Lemma 3 fully characterizes the behavior of the average marginal utility of consumption as a function of GDP and real wages.

Lemma 3. *The average marginal utility of consumption can be written in terms of GDP and real wage as*

$$\widehat{u}_t = -\varpi_1 \widehat{y}_t - \varpi_2 \widehat{\omega}_t, \tag{B.7}$$

with $\varpi_1 \equiv \sigma + \overline{u}\Psi\frac{\alpha}{1-\alpha}$ *and* $\varpi_2 \equiv \overline{u}\Psi$, *and where* $\overline{u} \equiv -\sigma\frac{\lambda(1-\lambda)\gamma^{-\sigma}-\lambda(1-\lambda)\gamma}{[(1-\lambda)\gamma+\lambda][\lambda+(1-\lambda)\gamma^{-\sigma}]}$ *and* $\Psi \equiv \frac{\mathcal{M}^p}{(1-\lambda)\left(1-\alpha+\frac{1}{1-\lambda}(\mathcal{M}^p-(1-\alpha))\right)}$ *comes from the relation between consumption inequality and price markup in equation* (18).

Proof. First note that by combining the results from Lemma 1 and Lemma 2, we get $\widehat{u}_t = -\sigma\widehat{y}_t + \overline{u}\widehat{\gamma}_t$, with $\overline{u}$ previously defined. Then, replacing equation (19) into (18) (average price markup into consumption gap), we get $\widehat{\gamma}_t = \Psi\Big(-\frac{\alpha}{1-\alpha}\widehat{y}_t - \widehat{\omega}_t\Big)$. Combining these results and re-arranging, we get equation (B.7). □

Now we are ready to solve for the actual labor supply schedule. The optimality condition for the union is $\frac{W_t}{P_t}\Big(\frac{\lambda}{(C_t^c)^{-\sigma}} + \frac{1-\lambda}{(C_t^u)^{-\sigma}}\Big) = \mathcal{M}^w N_t^{\varphi}$, where $\mathcal{M}^w$ is the wage markup and we impose the condition that all workers have the same wage and the same number of hours. Note that the term in parentheses is the average marginal utility of consumption across workers. Taking a log-linear approximation, we get $\widehat{\omega}_t + \widehat{u}_t = \varphi\widehat{n}_t$. Replacing the average marginal utility of consumption (B.7), we obtain

$$\widehat{\omega}_t - \varpi_1\widehat{y}_t - \varpi_2\widehat{\omega}_t = \varphi\widehat{n}_t.$$

Re-ordering and defining $\varpi = \frac{\varpi_1}{1-\varpi_2}$ and $\overline{\varphi} = \frac{\varphi}{1-\varpi_2}$, we get the average labor supply

$$\widehat{\omega}_t = \overline{\varphi}n_t + \varpi\widehat{y}_t, \tag{B.8}$$

where $\overline{\varphi}$ and ϖ are parameters that depend on inequality, λ.

Part 3: Real wage

Finally, we can describe the evolution of real wages. As in the case of price inflation, we can separate the problem between optimizers and non-optimizers. Consider first the optimizers problem. Unions optimaly set the wage according to

$$\widehat{\omega}_t^o(j) - \varpi_1 \widehat{y}_t - \varpi_2 \widehat{\omega}_t = \varphi \widehat{n}_t^o(j). \tag{B.9}$$

Given the demand function

$$n_t^o(j) = n_t - \epsilon_w(\omega_t^o(j) - \omega_t), \tag{B.10}$$

we obtain

$$\widehat{\omega}_t^o(j) = \frac{1}{1+\varphi\epsilon_w}(\varpi_1 \widehat{y}_t + \varphi \widehat{n}_t + (\varphi\epsilon_w + \varpi_2)\widehat{\omega}_t). \tag{B.11}$$

Consider next the wage setting problem non-optimizers face. Identical to the firms problem, non-optimizing unions decide wages for period t based on the information set available at $t-1$, implying

$$\mathbb{E}_{t-1}\left(\hat{\omega}_t^m(j) - \varpi_1 \hat{y}_t - \varpi_2 \hat{\omega}_t - \varphi \hat{n}_t^m(j)\right) = 0,$$

which can be rewritten as

$$\widehat{\omega}_t^m(j) = -\widehat{\pi}_t^p + \mathbb{E}_{t-1}\left(\varpi_1 \hat{y}_t + \varpi_2 \hat{\omega}_t + \varphi \widehat{n}_t^m(j) + \widehat{\pi}_t^p\right). \tag{B.12}$$

On the other hand, the aggregate wage is given by

$$W_t \equiv \left(\int_0^1 W_t(i)^{1-\epsilon_w} di\right)^{\frac{1}{1-\epsilon_w}},$$

implying

$$W_t = \left(\theta_w (W_t^m)^{1-\epsilon_w} + (1-\theta_w)(W_t^o)^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}.$$

The previous expression can be written in log-deviation from the steady state as

$$\widehat{\omega}_t = \theta_w \widehat{\omega}_t^m + (1-\theta_w)\widehat{\omega}_t^o.$$

Finally, substituting optimizers and non-optimizers wage setting rules (B.11) and (B.12) into the aggregate wage equation and imposing market clearing yields

$$\widehat{\omega}_t = \kappa_\omega(\varpi_1 \widehat{y}_t + \varphi \widehat{n}_t) - \varsigma \widehat{\pi}_t^p + \mathbb{E}_{t-1}\widehat{x}_t^w, \tag{B.13}$$

where $\kappa_\omega \equiv \frac{1-\theta_w}{1+\theta_w\varphi\epsilon_w-(1-\theta_\omega)\varpi_2}$, $\varsigma \equiv \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w-(1-\theta_\omega)\varpi_2}$ and $\widehat{x}_t^w \equiv \varsigma\left(\varpi_1\widehat{y}_t + \varpi_2\widehat{\omega}_t + \varphi\widehat{n}_t^m(j) + \widehat{\pi}_t^p\right)$.

B.3 Derivation of the IS equation

From (5) we can get

$$C_t = C_t^u\left((1-\lambda)+\lambda\frac{1}{\gamma_t}\right),$$

which can be rewritten in log-deviation from steady state as

$$\widehat{c}_t = \widehat{c}_t^u - \frac{\lambda}{(1-\lambda)\gamma+\lambda}\widehat{\gamma}_t. \tag{B.14}$$

On the other hand, the Euler equation of unconstrained agents is $\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}\mathbb{E}_t[\widehat{r}_t - \widehat{\pi}_{t+1}^p - (1-\rho_\chi)\chi_t]$. Replacing (24) and (B.14) into the previous expression and imposing market clearing we obtain

$$\begin{aligned}
&\widehat{y}_t + \frac{\lambda}{(1-\lambda)\gamma+\lambda}(-\Theta_y\widehat{y}_t - \Psi\mathbb{E}_{t-1}\widehat{x}_t) \\
&\quad = \mathbb{E}_t\left[\widehat{y}_{t+1} + \frac{\lambda}{(1-\lambda)\gamma+\lambda}(-\Theta_y\widehat{y}_{t+1} - \Psi\mathbb{E}_t\widehat{x}_{t+1})\right] - \frac{1}{\sigma}\mathbb{E}_t(\widehat{r}_t - \widehat{\pi}_{t+1}^p - (1-\rho_\chi)\widehat{\chi}_t).
\end{aligned}$$

Assume next that the economy starts at steady state in period $t-1$ and that shocks are iid.

Since shocks are unexpected, then at $t-1$ agents forecast that the economy at t will remain at steady state, i.e., $\mathbb{E}_{t-1}\widehat{x}_t = 0$. Additionally, since shocks have no persistence, all real variables return to steady state at $t+1$.[17] The nominal variables at $t+1$ on the other hand will be determined by monetary policy. We assume the central bank implements a policy such that $\widehat{\pi}^p_{t+1} = 0$. Accordingly, we have $\mathbb{E}_t\widehat{y}_{t+1} = \mathbb{E}_t\widehat{\pi}^p_{t+1} = \mathbb{E}_t\widehat{x}_{t+1} = 0$, and hence the aggregate Euler equation can be written as

$$\widehat{y}_t = -\frac{1}{\sigma}\frac{1}{1-\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta_y}\mathbb{E}_t\left(\widehat{r}_t - \widehat{\chi}_t\right). \tag{B.15}$$

Expression (B.15) is the Euler equation under our simplifying assumptions. Notice that the response of output to the interest rate not only depends on the intertemporal elasticity of substitution, σ, but also on another term, which involves the market incompleteness parameter λ. If $\lambda = 0$ the economy is a RANK and the slope of the Euler equation is given by $-1/\sigma$. However, when market incompleteness is present (with $\lambda > 0$) the elasticity to the real rate depends on another parameter, Θ_y, which governs the cyclicality of the consumption gap.

C Welfare losses

We derive a general welfare loss function for the economy, taking into account limited asset market participation. We assume that the central bank seeks to minimize the weighted utility of constrained and unconstrained agents (with weights given by their relative sizes).[18] Taking a second order approximation of utility around the efficient steady state with no inequality, average welfare losses can be expressed as

$$L = \frac{1}{2}\left[\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)var(\tilde{y}_t) + \sigma\lambda(1-\lambda)var(\widehat{\gamma}_t) + \frac{\epsilon_p}{\lambda_p}var(\pi^p_t) + \frac{(1-\alpha)\epsilon_w}{\lambda_w}var(\pi^w_t)\right], \tag{C.1}$$

where $\lambda_p \equiv \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$ and $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\varphi\epsilon_w)}$. Welfare losses are a function of the output gap, price and wage inflation volatility, and the consumption gap. The latter term captures

[17] At t agents correctly anticipate no shocks at $t+1$, hence it is as if the economy was not affected by any friction at $t+1$ and thus the allocation must coincide with that of the flexible price economy. The latter ensures all real variables return to steady state at $t+1$.

[18] For simplicity, we further assume the existence of a labor subsidy that corrects for the inefficiencies generated by monopolistic competition, and transfers that equate the steady state consumption of constrained and unconstrained households.

inequality and arises from the existence of limited asset markets participation. This is the metric we use to analyze gains from wage flexibility, as well as the degree of output volatility.

D The slope of the IS conditional on monetary policy shocks

A way to write the slope of the IS (denoted by S) is

$$S = -\frac{(1-\alpha)(1+\varsigma\kappa_\pi)}{\sigma(1-\alpha)(1+\varsigma\kappa_\pi)+(\alpha+\kappa_\omega x)\Omega^*}, \tag{D.1}$$

with $\Omega^* = -\sigma\Gamma\Psi = -\sigma\frac{\lambda(\epsilon_p-1)(1-\alpha)}{\epsilon_p-\lambda(\epsilon_p-1)(1-\alpha)}$ and $x = \varpi(1-\alpha)+\overline{\varphi}$. Then, we compute the derivative of this slope, $\frac{\partial S}{\partial\theta_p}$ as

$$\frac{\partial S}{\partial \theta_p} = -(1-\alpha)\varsigma(\alpha+\kappa_w x)\sigma\frac{\lambda(\epsilon_p-1)(1-\alpha)}{\epsilon_p-\lambda(\epsilon_p-1)(1-\alpha)}\frac{(1-\alpha)}{1-\alpha+\alpha\epsilon_w}. \tag{D.2}$$

Notice that as $x > 0$, and all the remaining terms are positive, the slope of the IS increases with the price rigidity in absolute value. This is, output through the IS equation is more volatile conditional on monetary policy shocks.

E Computing the threshold $\overline{\phi}_\omega$

We have

$$S = -\frac{(1-\alpha)(1+\varsigma\kappa_\pi)}{\sigma(1-\alpha)(1+\varsigma\kappa_\pi)+(\alpha+\kappa_\omega x)\tilde{\Omega}+\phi_\omega(\kappa_\omega x-\varsigma\kappa_\pi\alpha)}, \tag{E.1}$$

with $\tilde{\Omega} = \kappa_\pi(\phi_\omega+\phi_\pi)-\sigma\Gamma\Psi$ and $\Gamma\Psi = \frac{\lambda(\epsilon_p-1)(1-\alpha)}{\epsilon_p-\lambda(\epsilon_p-1)(1-\alpha)}$.

$$\frac{\partial S}{\partial \theta_\omega} = \{\kappa_\pi(\alpha+\kappa_\omega x)+(1+\varsigma\kappa_\pi)x\}\left(\tilde{\Omega}+\phi_\omega\right)\frac{1-\alpha}{D^2}\frac{\partial\kappa_\omega}{\partial\theta_\omega}. \tag{E.2}$$

Notice that $\varsigma = 1-\kappa_\omega$

$$\frac{\partial S}{\partial \theta_\omega} = \{\kappa_\pi(\alpha+x)+x)\left(\tilde{\Omega}+\phi_\omega\right)\frac{1-\alpha}{D^2}\frac{\partial\kappa_\omega}{\partial\theta_\omega}. \tag{E.3}$$

All the elements of this derivative are positive for any κ_π, except for the element $\left(\tilde{\Omega}+\phi_\omega\right)$, which sign depends on the degree of price stickiness through κ_π. Recall that $\Gamma\Psi = \frac{\lambda(\epsilon_p-1)(1-\alpha)}{\epsilon_p-\lambda(\epsilon_p-1)(1-\alpha)}$, and

$\kappa_\pi = \frac{1-\theta_\pi}{\theta_\pi}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$, then

$$\tilde{\Omega} + \phi_\omega = \frac{1-\theta_\pi}{\theta_\pi}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}(\phi_\pi + \phi_\omega) - \sigma\frac{\lambda(\epsilon_p - 1)(1-\alpha)}{\epsilon_p - \lambda(\epsilon_p - 1)(1-\alpha)} + \phi_\omega. \tag{E.4}$$

Hence, the threshold is given by the following expression

$$\frac{1-\theta_\pi}{\theta_\pi}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\phi_\pi + \frac{1}{\theta_\pi}\frac{1-\alpha+\theta_\pi\alpha\epsilon_p}{1-\alpha+\alpha\epsilon_p}\phi_\omega = \sigma\frac{\lambda(\epsilon_p - 1)(1-\alpha)}{\epsilon_p - \lambda(\epsilon_p - 1)(1-\alpha)}, \tag{E.5}$$

$$\overline{\phi}_\omega = \frac{\sigma\lambda(\epsilon_p - 1)(1-\alpha)(1-\alpha+\alpha\epsilon_p)\theta_\pi}{(\epsilon_p - \lambda(\epsilon_p - 1)(1-\alpha))(1-\alpha+\theta_\pi\alpha\epsilon_p)} - \frac{(1-\theta_\pi)(1-\alpha)}{1-\alpha+\theta_\pi\alpha\epsilon_p}\phi_\pi. \tag{E.6}$$

Now we can obtain for $\overline{\phi}_\pi$ which is given by

$$\overline{\phi}_\pi = \frac{\sigma\lambda(\epsilon_p - 1)(1-\alpha+\alpha\epsilon_p)}{\epsilon_p - \lambda(\epsilon_p - 1)(1-\alpha)}\frac{\theta_\pi}{(1-\theta_\pi)} - \frac{1-\alpha+\alpha\epsilon_p\theta_\pi}{(1-\theta_\pi)(1-\alpha)}\phi_\omega. \tag{E.7}$$

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