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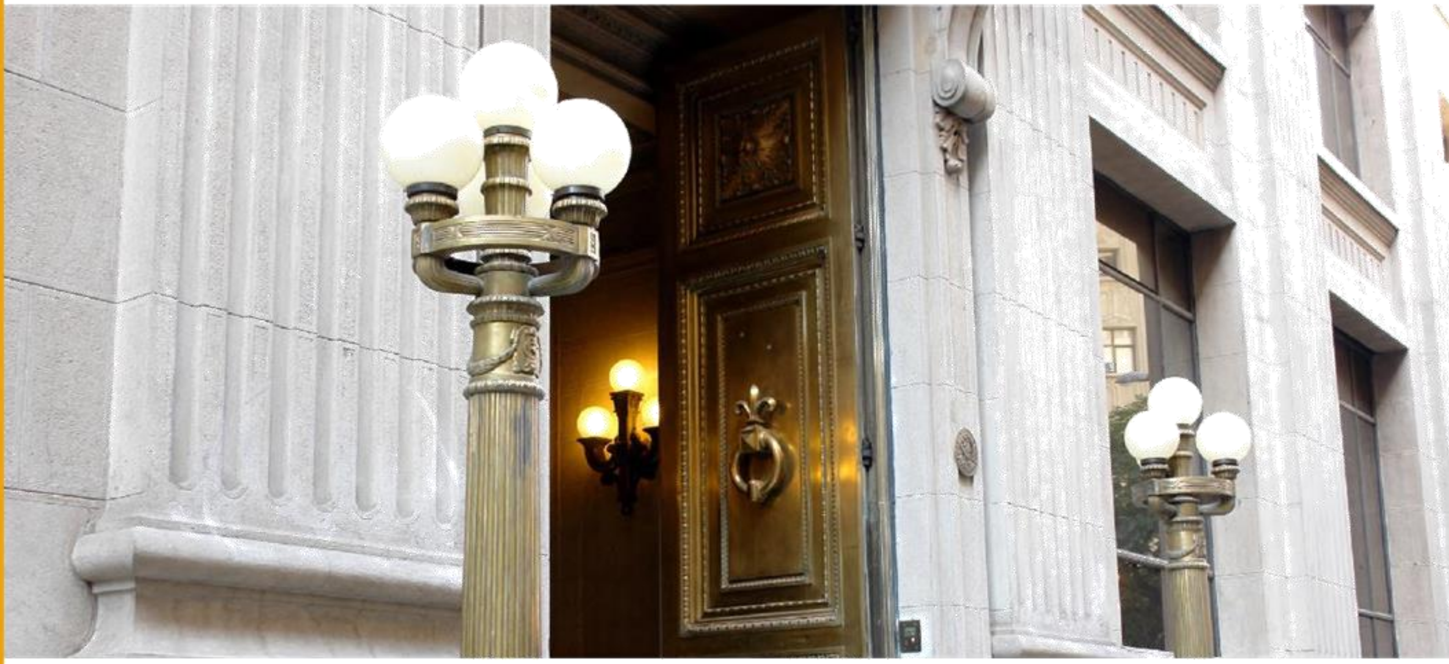

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Estimates of the US Shadow-Rate

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Abstract

This paper provides several estimates for the shadow rate (SR) of the short-term interest rate in US. We assume maximal models with two and three Gaussian factors, and we use forward rates to estimate the parameters of the models. Based on that we conclude that point estimates of SR should be taken with caution as they depend on the characteristics of the dataset, in terms of the sample-size, maturities, and smoothness. The latter is even more crucial than other settings discussed previously in the literature, such as the number of factors.

Resumen

Este documento proporciona varias estimaciones de la tasa sombra de la tasa de interés a corto plazo en Estados Unidos. Usamos modelos máximos con dos y tres factores Gaussianos y utilizamos tasas forward para estimar los parámetros de los modelos. Sobre esta base, llegamos a la conclusión de que las estimaciones puntuales de la tasa sombra deben tomarse con cautela, ya que dependen de las características del conjunto de datos, en términos del tamaño de la muestra, los vencimientos y la uniformidad del rango de tasas forward que se utilicen. Este último es incluso más crucial que otros elementos discutidos previamente en la literatura, como el número de factores.

I Introduction

There is an extensive literature on the estimate of the yield curve based on dynamic factors and non-arbitrage opportunities. Singleton (2009) proposes a framework that has three key ingredients: (i) a relationship between short-term interest rate and the dynamic factors, (ii) a dynamic specification of the factors under the physical measure (P), and (iii) a dynamic specification of the factors under the risk-neutral measure (Q). Given a time-series of a set of yields, several procedures can be apply to recover the parameters of the model. In particular, when all relationships (i), (ii) and (iii) are linear and disturbances are Gaussian, then the standard Kalman Filter can be applied. In that case, observed yields are related with the factors in the measurement equation, using the parameters of the Q-measure; meanwhile the dynamic of the factors is characterized by the transition equation, using the parameters of the P-measure. In the last decade, simple procedures have been proposed, exploiting both the linearity of relationships (i), (ii), and (iii) the normality assumption. For instance, Diebold & Li (2006) adapt the popular Nelson & Siegel (1987) model to a dynamic version (DNS) which allow them to relate yields with factors. In order to solve the model, they propose a two-step approach: first, they extract factors using calibrated parameters of the Q-measure, and second they estimate the parameters of the P-measure by fitting a VAR on the extracted factors. Hamilton & Wu (2012) propose to run both a VAR model and a linear regression, when the latter involves an extra yield that has measurement error, that assumption was introduced in the literature of yield curve estimation by Chen & Scott (1993). Parameters of the model can be obtained from empirical ones (VAR and OLS) by using minimum-distance procedure. Finally, Adrian et al. (2013) propose a 3-step linear regressions approach, starting with a VAR of the main principal components of large set of yields which provide parameters of the P-measure, and then moving to pooled regressions of the excess returns, which impose non-arbitrage conditions. Based on the latter they identify the parameters of the Q-measure. It is important to stress that a limited number of parameters can be identified, this means that some restrictions should be imposed in system of equations (i), (ii), and (iii). Different sets of restrictions identify the same number of parameters; these sets are known as normalizations. In this paper we use the normalization proposed by Giglio & Kelly (2018), which allow us to identify all possible parameters (maximal model) in both 2 and 3 factor models. Further, this normalization affects only equations (i) and (iii), providing simple formulas for pricing (Q-measure). Although previous procedures are simple to implement, they do not impose the non-negative restriction on the short-term interest rate. This restriction was relevant during the global financial crisis, in which several central banks reduced their monetary policy rates significantly, putting those close to zero. In the case of the current Covid crisis, that restriction is again relevant implying that a yield curve model should consider such restriction. In that respect, Krippner (2012) proposes a very simple adjustment to include explicitly this restriction under the normality assumption. This implies a closed-form relationship between forward rate and factors, which is obtained from the truncated normal distribution. Thus, the model preserve the features (i)-(iii) but controls for the fact that short-term interest rate cannot be below of a given threshold, which it is generalized as Effective Lower Bound (ELB). Christensen & Rudebusch (2015) introduce this constraint into the DNS model and use of the Extended Kalman Filter (EKF) for solving the model, because the truncation introduces a non-linearity in the measurement equation. Another use of EKF on this context (ELB constraint) is proposed in Wu & Xia (2016); however, in that case, authors use forward rates instead of yields. Krippner (2015) provides a comprehensive revision of models and methods that include the ELB constraint. In particular, the authors suggests the use of iterated EKF (IEKF), but there is not theoretical support for using IEKF instead of EKF. In this paper, we estimate 2 and 3 Gaussian-factors yield curves models with the ELB constraint using forward rates, and applying both EKF and IEKF. In that sense, we extend the analysis developed in Wu & Xia (2016). Our discussions will be focused on the so-called Shadow-Rate (SR), which is the unconstrained short-term interest rate.

Many empirical papers (Adrian et al., 2013; Wu & Xia, 2016) rely on smoothed yields to estimate yield-curves. Smoothed yields are generated from actual transactions, but coupon and other effects are removed, also they are interpolated across maturities. Two key datasets of smoothed yields for US are GSW (Gürkaynak et al., 2007) and LW (Liu & Wu, 2020). Both datasets (GSW and LW) are based on raw data provided by CRSP.[1] GSW and LW datasets are similar in excluding: (i) securities with special-features (eg., callable, flower bonds), and (ii) instruments recently issued. In both cases, the goal is to avoid any possible liquidity premium. However, GSW and LW datasets have some differences: (i) the GSW uses a parametric method to summarize daily yield curves, meanwhile the LW uses a non-parametric kernel smoothing, and (ii) the GSW excludes Treasury bills (short-term securities), meanwhile the LW includes

[1]This center is associated to the business school of University of Chicago and it collects financial data from the 60s. It started with the need of the vice president of Merry Lynch to illustrate portfolio performance to his clients.

those. Liu & Wu (2020) provide significant evidence that these differences, along with others related with outlier extraction matter. Indeed, they show that conclusions change if LW dataset is used instead of GSW dataset when authors replicated the results reported in Cochrane & Piazzesi (2005) and Giglio & Kelly (2018). In this paper, we use both datasets, considering monthly frequency from January 1990 to December 2019 as a benchmark, and we also use a sample that includes 2020.[2] This allows us to extend the results presented in Liu & Wu (2020) by providing an additional case: the SR derived in Wu & Xia (2016) using the LW dataset.

Several papers have documented in the literature that the SR is sensitive to the specification of the model. For instance, Christensen & Rudebusch (2015, 2016), Bauer & Rudebusch (2016), and Krippner (2013) show that the SR can be very sensitive to the number of factors included in the estimation. In that sense, Krippner (2015) shows that, in general, models with two factors used to produce more negative SR than models with three factors. Furthermore, Bauer & Rudebusch (2016) and Christensen & Rudebusch (2015), show the sensitivity of the SR to the ELB. However, Wu & Xia (2016) estimated the SR with the lower bound as a parameter to be estimated and found a small difference in relation to the case in which the lower limit is chosen. In addition, Krippner (2013) shows the sensitivity of the SR to the estimation method, comparing the EKF with the IEKF. Suggests that results obtained using the EKF may depend on whether the initialization parameters in the algorithm estimation are close to the global optimal parameters, for what he proposes to use the IEKF instead. Also he shows that SR is sensitive to the maturity span and observed that using longer maturities can deliver more negative estimates than using shorter maturities. Thus, with this paper we provide some insights on this by showing the sensitivity of the SR to different sets of maturities, set of forward rates, yields datasets, normalization, number of factors and estimation procedure (EKF and IEKF). Based on these exercises, that we described below our main conclusion is that input data matters. In particular, the estimation of the SR using the LW dataset seems to be more robust, to a number of changes in the specification of the model, than the ones obtained by using GSW dataset.

Our first exercise, an update of Wu & Xia's (2016) paper, implies some changes in the parameters of the model. In order to assess the relevance of these changes, we discuss how the SR is affected. In particular, the update provides a different assessment of the monetary policy stance at the end of the original sample. A second exercise provides evidence about

[2] Special thanks to Cynthia Wu for updating LW dataset, that allow us to compare with GSW one during 2020.

relevance of the input dataset for assessing the dynamic of the SR. Indeed, using GSW dataset –used in the original paper— the SR fallen up -1.5% at the end of 2011 and remained around there for two years, then a new reduction is observed getting a second minimum (-4%) in May 2014. Using the LW dataset the SR fallen very rapidly up to -4.5% at the beginning of 2014, exhibiting a V-shape dynamic. This difference is explained by the dynamics of the longest forward rates. We provide robustness exercises of this finding by dropping some forward rates from the estimation procedure. As it is expected, the SR is highly correlated with the longest forward rates when the ELB is active. Thus, to get a precise estimate of the SR, we need proper information about long-term yields. In that respect, we compare the weights of the EKF obtained from both datasets. We conclude that there is no a substantial change in the weights between datasets. Although it seems that long-term forward rates in LW dataset contribute more to the dynamic of the first risk factor during the ELB period relative to the one in the GSW dataset. Further, estimates based on small samples tend to exhibit SR's closer to the ELB. A third exercise consider the sensitivity to the number of factors. We found that previous discussion about discrepancies between 2 and 3 factor models are well-supported when the GSW dataset is used. Indeed, the SR's obtained in these cases provide difference on the monetary policy instance in both: size and timing. However, discrepancies reduce significantly when the LW dataset is used instead. A fourth exercise shows the sensitivity of the SR to the estimation method. We estimate the model with 2 and 3 factors using the IEKF method and we found that the parameters obtained are very similiar to those obtained with EKF procedure while the results are more sensitive to the estimation procedure when GSW dataset is used and 3 factors are considered, while the SR obtained using LW dataset shows robustness to the estimation method using 2 or 3 factors. Finally, an update of the SR during pandemic period (2020) is provided. In that particular case, both datasets tend to agree about the size and timing of SR, although the estimates with the LW data tend to fall faster towards the end of 2020, similarly to what was observed during 2009.

The paper is organized as follow. Section II provides a brief description of the models and the estimation procedures. Section III presents empirical results, and Section IV concludes.

II Theoretical Setting

In this section, we summarize the key components of the Dynamic Term Structure Models (DTSM) covering notation and relevant results. For simplicity interest rates are defined in continuously compounded terms (r_t), thus having the gross return $(1 + R_t)$, then we have: $r_t = \log(1 + R_t)$.

1. Key Components of DTSM

As it is standard in the literature, we assume that the short-term interest rate (r_t) is linear in some dynamic factors (X_t): $r_t = \delta_0 + \delta_1' X_t$. Dynamics of factors are characterized under the physical measure (P) by a vector autoregressive model (VAR) of order one: $X_t = \mu + \Phi X_{t-1} + \Sigma U_t$. It is important to note that P-measure is the one obtained from the time-series of yields, without imposing any non-arbitrage restriction. Further, we extract factors from observed yields, without making any assumption regarding the economic interpretation. Usually, three factors are enough to explain the variance of yields, and they are commonly named as level, slope, and curvature. Diebold & Rudebusch (2013) discuss the economic interpretation of these factors relating inflation with the level, the stage of the business cycle with the slope, and third factor with a statistical element relevant for improving the fit of the data. Finally, to prevent from arbitrage opportunities the P-measure should be adjusted to build a new measure: risk-neutral one or Q-measure. That implies that under this new measure, the dynamic of the factors is: $X_t = \mu_* + \Phi_* \, X_{t-1} + \Sigma V_t$. It is important to note that under P-measure each component of U_t is distributed Gaussian with zero-mean and unit-variance; meanwhile under Q-measure each component of V_t has these properties.

2. Normalization of Gaussian DTSM

The model above has more parameters that the ones that can be identified with the data. Indeed, for the case of three factors, there are 34 parameters but only 22 can be identified (Dai & Singleton, 2000).[3] Table 1 summarizes the possible number of parameters of each object (column 2), and the size of constrained objects (columns 3 to 10). Dai & Singleton

[3] Normalization is not relevant from the results of the shadow-rate, but it is important for comparing results across different studies. The choice of a given normalization is beyond the purpose of this paper and it can be justified from a computational point of view.

(2000), Singleton (2009) and Hamilton & Wu (2012) offer different normalization for these 22 parameters; reported in columns DS, S, and HW, respectively. It should be noted that some authors prefer to constrain the matrix Φ to be lower triangular rather than matrix Φ_* or to set vector μ to be zero rather than vector μ_*. In these settings, vector δ_1 should be estimated which implies that matrix Σ does not have any free parameter, meaning it is the identity matrix. Other authors use alternative normalizations of the model, as Giglio & Kelly (2018), Wu & Xia (2016), Diebold & Rudebusch (2013), Christensen et al. (2010) and Christensen & Rudebusch (2012); reported in columns GK, DR, WX, CLR, and CR, respectively. In these cases, the vector δ_1 is fixed to (1,1,0)' (or (1,1,1)' for GK) at the cost of having free-parameters in the matrix Σ. Giglio & Kelly (2018) impose that matrix Φ_* is diagonal, meanwhile Wu & Xia (2016) impose real eigenvalues with a pair repeated in that matrix. Authors claim that repeated eigenvalues are consistent with the empirical data (1999-2013). A similar constraint is imposed in Nelson & Siegel (1987), when authors consider that deterministic forward interest rate is the solution of a second order differential equation with equal roots.[4] Diebold & Rudebusch (2013) consider a Dynamic version of Nelson-Siegel (DNS), which imply to add more restrictions to the model: (i) unit root in one of the factor under Q-measure, affecting matrix Φ_*, and (ii) $\delta_0 = 0$ for the short-term interest rate. Christensen et al. (2010) and Christensen & Rudebusch (2012) impose a diagonal Σ matrix in addition to the DNS structure, leaving some free parameter in vector μ_*. Christensen & Rudebusch (2012) also add restrictions on the P-measure to improve forecasting-ability: (i) unit root in one of the factors, affecting matrix Φ (ii) zero mean of the same factor, affecting vector μ, and (iii) further zeros in off-diagonal elements of matrix Φ.

In this paper, we consider the GK normalization, but allowing that $\delta_0 \neq 0$. Thus, estimates of the parameters can be used directly into the pricing formula (Q-measure).[5]

3. Short-term and forward rates

Given that the DTSM is linear in the factor, the conditional mean of the m–periods ahead short-term interest rate is $\alpha_{m,t} := E_t^*(r_{t+m})$. For the case of GK normalization, we have

[4] Nelson & Siegel (1987) propose a deterministic forward interest rate and they use the log pure expectation hypothesis to derive zero coupon interest rates. Thus, the model does not have a Jensen term, which is obtained when an affine model is applied. Alfaro (2011) discusses this element in the context of a discrete-time version of the model.

[5] Diebold & Rudebusch (2013) propose a two-step procedure as an alternative to Kalman filter. In that case, the parameters of the measurement equation, which are under Q-measure, are calibrated and estimates of dynamic factors are obtained from that.

Table 1: Parameter Normalizations

		DS	S	HW	GK	WX	DR	CLR	CR
δ_0	1	1	1	1	0	1	0	0	0
δ_1	3	3	3	3	0	0	0	0	0
μ	3	0	3	0	3	3	3	3	2
Φ	9	6	9	9	9	9	9	9	4
μ_*	3	3	0	3	0	0	0	0	3
Φ_*	9	9	6	6	3	2	1	1	1
Σ	6	0	0	0	6	6	6	3	3
	34	22	22	22	21	21	19	16	13

$\mu_* = 0$ then $\alpha_{m,t} = \delta_0 + (1,1,1)\Phi_*^m X_t$. In this paper we consider the forward rate between periods $t+m$ and $t+m+1$, which it can be expressed as a weighted difference of yields: $f_{m,t} = (m+1)y_{m+1,t} - my_{m,t}$. Without any boundary restriction we have: $f_{m,t} = \alpha_{m,t} - 0.5J_m$, where J_m is the Jensen term. In the GK normalization that term is $\delta_1' K_m \Omega K_m' \delta_1$ where $\delta_1 = (1,1,1)'$, $\Omega = \Sigma\Sigma'$, and $K_m = (I - \Phi_*)^{-1}(I - \Phi_*^m)$ a diagonal matrix. Further, the conditional variance of the m-periods ahead short-term interest rate is time-invariant due to Gaussian errors: $\sigma_m^2 = Var_t^*(r_{t+m}) = \delta_1' Var_t^*(X_{t+m})\delta_1$, then ee have: $vec[Var_t^*(X_{t+m})] = (I-\Gamma)^{-1}(I-\Gamma^m)vec[\Omega]$, where $\Gamma = \Phi_* \otimes \Phi_*$.

As Wu & Xia (2016) we consider a restriction on the short-term interest rate: $\tilde{r}_t = \max\{r_t, \underline{r}\}$, with $\underline{r}$ be the Effectively Lower Bound (ELB). Based on the properties of truncated normal distribution, then the conditional mean of the m–periods ahead constrained short-term interest rate is (Krippner, 2012):

$$E_t^*(\tilde{r}_{t+m}) = \underline{r} + \sigma_m \left[\left(\frac{\alpha_{m,t} - \underline{r}}{\sigma_m} \right) \mathcal{N}\left(\frac{\alpha_{m,t} - \underline{r}}{\sigma_m} \right) + n\left(\frac{\alpha_{m,t} - \underline{r}}{\sigma_m} \right) \right] \tag{1}$$

where $\mathcal{N}$ and n are –respectively— cumulative distribution function and density function of the standard normal distribution. Wu & Xia (2016) show that the expression in brackets is almost zero for values below -2, and similar to identity for values above 2. For simplicity they define $g(x) = x\mathcal{N}(x) + n(x)$, with that

$$E_t^*(\tilde{r}_{t+m}) = \underline{r} + \sigma_m g\left(\frac{\alpha_{m,t} - \underline{r}}{\sigma_m} \right). \tag{2}$$

Thus, the constrained short-term interest rate (1) is completely determined by the first two moments of the unconstrained short-term interest rate.[6] In practice, when the short-term interest rate hit the ELB, we observe $\tilde{r}_t = \underline{r}$ which is implicit in the yields and forward rates, thus r_t is the shadow rate (SR). As it is expected, forward rates will be non-linear expressions of the dynamic factors and the yields will be also complex expressions. Under this setting, the forward rate is proportional to the expected m–periods ahead constrained short-term interest rate; however, Jensen terms should be considered as well. Wu & Xia (2016) propose a Taylor approximation of these terms, deriving the following relationship:[7]

$$f_{m,t} = \underline{r} + \sigma_m g\left(\frac{\alpha_{m,t} - J_m - \underline{r}}{\sigma_m}\right) \tag{3}$$

4. Estimation Procedures

We have: (i) a linear transition equation, for which we assume that factors are characterized by a VAR(1) model, and (ii) a non-linear measurement equation (3); that motivates the use of the Extended Kalman Filter (EKF) and the iterated EKF (IEKF)[8]. The EKF is an extension of the Kalman filter that linearizes the non-linear function $g(\cdot)$ around the estimate of the state variable whenever the Kalman filter is updated. While the IEKF maintains the same idea, but instead iterates over the measurement equation until it finds a fixed point before the time update step. Bell & Cathey (1993) show that both the EKF and IEKF are applications of the Gauss-Newton algorithm on the measurement update step, but EKF includes only one iteration.

We use both filters, for a set of one-month forward rates and we add a measurement error with equal variance for all forward rates. In general we use 7 rates, but we analyze the cases where the longest forward rate is excluded from the sample. We consider 2 and 3 factors, in both cases using the maximal models; however, in some of the estimations we fix one of the element of the diagonal of Φ_* to be one given that these elements tend to be very close to that number. Finally, we fix the ELB to be 0.25%, following previous papers.

[6]It is important to stress that under Gaussian disturbances, the constrained short-term interest rate is normally distributed, but truncated at the ELB. Thus, the first two moments are sufficient to characterize the entire distribution.

[7]In the empirical exercises, an extra measurement error term is considered to account for the errors due to the linear approximation. The size of the variance of measurement error depends on the data that we use.

[8]Christensen & Rudebusch (2015) provide evidence on the adequacy of EKF for estimating this model, meanwhile Krippner (2015) suggests the use of IEKF.

III Empirical Application

In this section, we report the results of estimating a 2 and 3 Gaussian-factors yield-curve with the ELB constraint using forward rates. We concentrate our discussion on the GK normalization, and estimating the model under different: (i) datasets of yields (GSW and LW), (ii) set of forward rates, and (iii) time-periods.

1. Descriptive Statistics

Table 2 and 3 show descriptive statistics of forward rates obtained from GSW and LW datasets.[9] We can observe that the shorter forward rates (3m, 6m, 12m and 24m) present fewer differences in their mean between databases, while the longer rates present greater differences. This is line with the fact that LW dataset includes a broader set of instruments in terms of maturities. Additionally, we can note that the longer forward rates from the LW database (84m and 120m) have higher volatilities than those obtained with the GSW data. This fact is important, since as we will see later, long forward rates have more impact on the SR than short/medium forward rates during the ELB.

Table 2: GSW forward rates descriptives

Forward Rate	Mean	Std	q10	q50	q90	Max	Min
3m	2.9725	2.3244	0.1748	2.5491	5.8657	8.4396	0.0155
6m	3.0884	2.3727	0.1828	2.7457	6.1890	8.6220	-0.0044
12m	3.3404	2.3791	0.3052	3.1715	6.3861	8.9010	0.0997
24m	3.8209	2.2622	1.0547	3.8008	6.7702	9.0372	0.3424
60m	4.9084	1.9594	2.2606	4.7171	7.8318	9.0644	1.4466
84m	5.3469	1.8906	2.7526	5.3464	8.1088	9.3244	1.6095
120m	5.6771	1.8442	2.9934	5.7959	8.2848	9.4602	1.8380

Table 3: LW forward rates descriptives

Forward Rate	Mean	Std	q10	q50	q90	Max	Min
3m	2.9217	2.2992	0.1315	2.4957	5.7506	8.3958	0.0504
6m	3.1099	2.3292	0.2281	2.7261	6.1479	8.6717	0.1165
12m	3.3706	2.3813	0.3910	3.0329	6.4141	9.0701	0.2231
24m	3.7919	2.2660	0.9238	3.7988	6.7714	8.8308	0.2908
60m	4.8445	1.9400	2.2970	4.7393	7.7948	9.0956	1.4632
84m	5.2252	1.9090	2.6142	5.3254	7.9230	9.1987	1.4789
120m	5.7847	1.9029	2.9470	6.1196	8.4352	10.0284	1.8272

Data in percentage points.

[9] Forward rates are shown in Figure 8 in Appendix A.

2. Benchmark Results with EKF and IEKF

Table 4 shows the results of replication of Wu & Xia's (2016) paper using the same dataset (GSW). In particular, the first 3-columns (Panel A: 'Original Sample') are results for considering the same normalization and the same sample period, which is January 1990 to December 2013. The next 3-columns (Panel B: 'Update to 2019') show results based on extended sample period to December 2019. In order to compare both results, we focus on the eigenvalues of matrices Φ_* and Φ, having the following result: adding 6 years to the estimation implies small changes in matrix Φ_*, which implies almost no change in the pricing of bonds; however, there are significant changes in matrix Φ, which involves an increase in the persistence of yields. Further, the last 3-columns (Panel C: 'GK normalization') use the GK normalization, which does not impose equal-roots on Φ_*. Table 5 is similar to Table 4 but using the LW dataset instead of the GSW. Again, extending the sample period does not affect the eigenvalues of matrix Φ_*, but the ones of matrix Φ. In contrast to previous table, it is unclear if there is an increase in the persistence of yields. As in the case of the GSW dataset, the use of GK normalization increases the log likelihood and improves information criteria. Similar results are obtained when the estimation is carried out through IEKF (Appendix B). The parameters obtained by IEKF show small changes in relation to those obtained by EKF, however, the eigenvalues tend to be slightly higher under IEKF (Table 7). As we discussed early, yields in the GSW dataset are very smooth due to a parametric estimation of the entire yield curve[10]. In contrast, yields in the LW dataset are generated by local approximation using kernels; therefore, yields are less smooth. That is reflected in the estimation of volatility of the measurement error ($\sqrt{\omega}$), which also accounts for the approximation error of forward rates. In the case of the GSW dataset, this volatility is around 10% meanwhile for the LW dataset is 20%. Also variances of the factors are also higher in LW than GSW dataset. Finally, a two-factor model is estimated. Table 6 shows the results of the estimation of that model using the GK normalization and also considering the case of unit root in Φ_* matrix. The four left columns of the table show the results using the dataset GSW, meanwhile in the four right columns the results with the LW dataset are reported. The parameters are very similar, with the variance of the error ($\sqrt{\omega}$) being the exception. Now, these number suggest a measurement error of 20% and 30%, respectively. These results are quite similar obtained under the IEKF estimate (Appendix B).

[10]For each day, a Svensson (1994)'s model is fitted (Gürkaynak et al., 2007).

Table 4: Replication of Wu and Xia (2016) using GSW dataset

Parameters	A. Original Sample			B. Update to 2019			C. GK normalization		
μ^P	-0.3035	-0.2381	0.0253	0.0561	-0.7021	0.0002	-0.3787	0.3506	-1.0152
	(0.1889)	(0.1814)	(0.016)	(0.1953)	(0.2759)	(0.0202)	(0.3453)	(4.4084)	(4.4411)
Φ	0.9638	-0.0026	0.3445	1.0022	0.0205	-0.3926	0.9884	0.0132	0.0125
	(0.0199)	(0.0183)	(0.4821)	(0.0076)	(0.0065)	(0.21)	(0.0085)	(0.0065)	(0.0062)
	-0.0226	0.942	1.0152	-0.0271	0.9605	0.9413	-0.0122	1.0816	0.1296
	(0.0202)	(0.0212)	(0.5112)	(0.0098)	(0.0087)	(0.2461)	(0.1363)	(0.3306)	(0.3228)
	0.0033	0.0028	0.8868	0.0002	-0.0003	0.9532	-0.0074	-0.1202	0.8368
	(0.0018)	(0.0019)	(0.0384)	(0.0008)	(0.0009)	(0.019)	(0.137)	(0.3272)	(0.3262)
Φ_*	0.9977			0.9995			0.9997		
	(0.0003)			(0.0001)			(0.0001)		
		0.9502	1		0.9566	1		0.9535	
		(0.0012)			(0.0013)			(0.0051)	
			0.9502			0.9566			0.9594
			(0.0012)			(0.0013)			(0.0045)
1200Σ	0.416			0.2856			0.2796		
	(0.0393)			(0.0236)			(0.0221)		
	-0.3999	0.2445		-0.2858	0.261		0.2719	6.2412	
	(0.037)	(0.0233)		(0.0258)	(0.0192)		(2.6132)	(9.8386)	
	-0.011	0.0033	0.039	0.0025	0.0013	0.0376	-0.5482	-6.2438	0.26
	(0.0069)	(0.0034)	(0.003)	(0.0042)	(0.0029)	(0.0032)	(2.6077)	(9.8467)	(0.0193)
$1200\sqrt{\omega}$	0.0893			0.0968			0.0967		
	(0.0027)			(0.0027)			(0.0027)		
$1200\delta_0$	13.3745			29.9475			37.585		
	(1.0445)			(3.9461)			(10.3775)		
$Eigen^P$	0.8452	0.9642	0.9832	0.9662	0.9662	0.9835	0.9632	0.9632	0.9806
$Eigen^Q$	0.9502	0.9502	0.9977	0.9566	0.9566	0.9995	0.9535	0.9594	0.9997
$Llike$	853.0904			978.8732			983.9316		
AIC	-1662			-1914			-1922		
BIC	-1663			-1915			-1923		

Based on the following forward rates: 3 and 6 months, 1, 2, 5, 7, and 10 years from GSW dataset.

Table 5: Replication of Wu and Xia (2016) using LW dataset

Parameters	A. Original Sample			B. Update to 2019			C. GK normalization		
μ^P	-0.3558	0.2119	0.0049	0.0284	-0.4125	-0.0055	-0.035	0.9573	-1.3201
	(0.2346)	(0.2358)	(0.0071)	(0.2138)	(0.2846)	(0.0121)	(0.2097)	(3.8174)	(3.9329)
Φ	0.9799	0.0287	0.0283	1.002	0.0171	0.039	0.9997	0.0156	0.0155
	(0.0147)	(0.0184)	(0.8891)	(0.0078)	(0.0074)	(0.5279)	(0.0073)	(0.0068)	(0.0075)
	0.0186	0.9807	0.6786	-0.0123	0.9849	0.881	0.0704	1.1585	0.2284
	(0.0153)	(0.0174)	(0.941)	(0.0092)	(0.0077)	(0.6896)	(0.1333)	(0.1221)	(0.1301)
	-0.0002	-0.0012	0.9429	-0.0003	-0.0008	0.9308	-0.0807	-0.1741	0.7591
	(0.0006)	(0.0004)	(0.0344)	(0.0004)	(0.0004)	(0.0259)	(0.1369)	(0.1228)	(0.1311)
Φ_*	0.9954			0.9989			0.9989		
	(0.0013)			(0.0004)			(0.0004)		
		0.9773	1		0.9782	1		0.9763	
		(0.0044)			(0.0031)			(0.0024)	
			0.9773			0.9782			0.9796
			(0.0044)			(0.0031)			(0.004)
1200Σ	0.661			0.3731			0.3671		
	(0.3373)			(0.1008)			(0.0992)		
	-0.6527	0.2868		-0.3818	0.2678		4.2576	6.2528	
	(0.3514)	(0.0202)		(0.1124)	(0.0161)		(2.0298)	(4.8094)	
	-0.0193	0.0035	0.0197	-0.0149	0.0031	0.0204	-4.6338	-6.2858	0.2652
	(0.0074)	(0.0017)	(0.0042)	(0.0052)	(0.0015)	(0.0033)	(1.9941)	(4.8126)	(0.0164)
$1200\sqrt{\omega}$	0.2035			0.193			0.193		
	(0.0079)			(0.0067)			(0.0067)		
$1200\delta_0$	13.7203			31.723			32.2547		
	(1.5091)			(8.4217)			(9.1268)		
$Eigen^P$	0.9543	0.9543	0.9949	0.9449	0.9864	0.9864	0.9445	0.9864	0.9864
$Eigen^Q$	0.9773	0.9773	0.9954	0.9782	0.9782	0.9989	0.9763	0.9796	0.9989
$Llike$	-281.9286			-212.2064			-207.2112		
AIC	608			468			460		
BIC	607			467			459		

Based on the following forward rates: 3 and 6 months, 1, 2, 5, 7, and 10 years from LW dataset.

Table 6: GK normalization considering 2 factors

Parameters	GK-2F-GSW		GK-2FR-GSW		GK-2F-LW		GK-2FR-LW	
μ^P	-0.0396	-0.0949	-0.0078	-0.0693	-0.025	-0.095	-0.0026	-0.0707
	(0.0577)	(0.0691)	(0.0278)	(0.0388)	(0.0551)	(0.0655)	(0.028)	(0.0356)
Φ	0.9961	0.0053	0.9966	0.0044	0.9986	0.0043	0.9994	0.0042
	(0.0071)	(0.0068)	(0.0066)	(0.0055)	(0.0064)	(0.0062)	(0.0061)	(0.0052)
	-0.0096	0.9932	-0.0121	0.9913	-0.0094	0.9939	-0.0122	0.9918
	(0.0081)	(0.0064)	(0.0085)	(0.0061)	(0.0066)	(0.0055)	(0.0073)	(0.0056)
Φ_*	0.9984		1.0		0.9984		1.0	
	(0.0003)		(1.0)		(0.0005)		(1.0)	
		0.9744		0.9799		0.9788		0.9841
		(0.0012)		(0.0008)		(0.0021)		(0.0012)
1200Σ	0.3226		0.2438		0.2921		0.248	
	(0.0273)		(0.021)		(0.0272)		(0.0203)	
	-0.1986	0.3453	-0.2323	0.3109	-0.2114	0.3426	-0.2559	0.31
	(0.0456)	(0.0218)	(0.0399)	(0.0169)	(0.0465)	(0.024)	(0.0378)	(0.0195)
$1200\sqrt{\omega}$	0.1927		0.2099		0.2887		0.2992	
	(0.0075)		(0.0072)		(0.013)		(0.012)	
$1200\delta_0$	11.8769		8.2798		12.3599		8.6481	
	(1.2135)		(0.2896)		(1.9021)		(0.2739)	
$Eigen^P$	0.9946	0.9946	0.9939	0.9939	0.9962	0.9962	0.9956	0.9956
$Eigen^Q$	0.9744	0.9984	0.9799	1.0	0.9788	0.9984	0.9841	1.0
$Llike$	-92.8448		-257.4132		-910.2568		-981.0488	
AIC	212		539		1847		1845	
BIC	211		538		1846		1985	

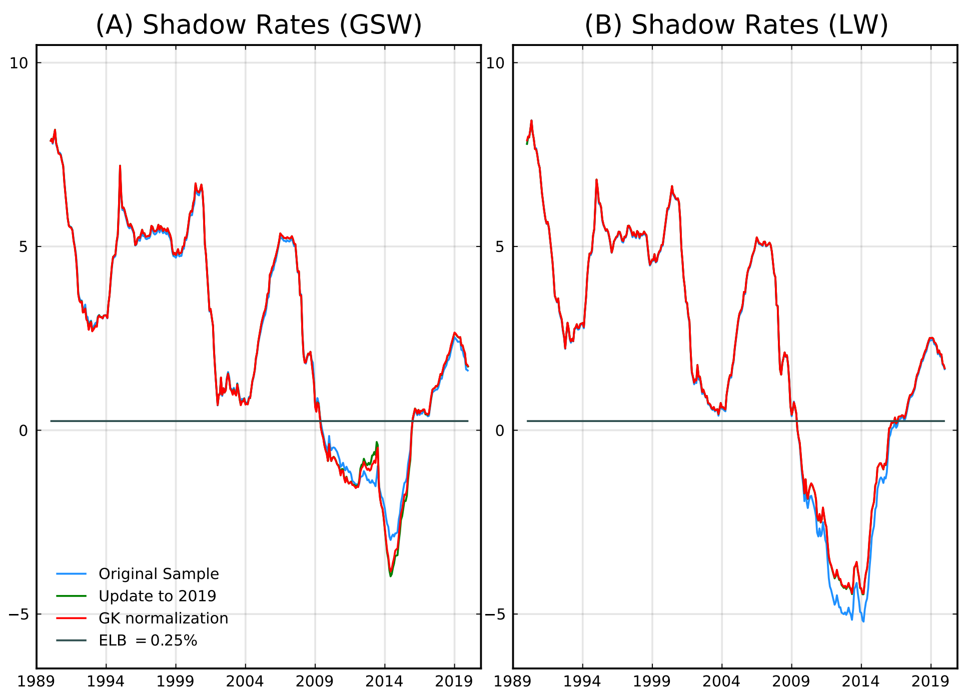
Based on the following forward rates: 3 and 6 months, 1, 2, 5, 7, and 10 years from GSW and LW dataset.

3. Shadow Rates

Figure 1, panel A, shows the SRs using values of Table 4 (GSW dataset). We note that the update in the estimation implies a further reduction of the SR in about 90 basis points in May 2014, from -3% to -3.9%; however, the use of GK normalization does not produce any further change with respect to WX normalization. Figure 1, panel B, shows the SRs derived from Table 5 (LW dataset). In that case, changes regarding the update also imply a similar revision of the SR in about 90 bp in May 2016, from -4.6% to -3.7%.

Clearly, the use of GK normalization does not affect the results. It is important to stress that the SR is used to assess the Monetary Policy Instance (MPI). Thus, the update of dataset (GSW or LW) implies revisions of the estimate of the SR, and magnitudes are relevant for the purpose of assessing the MPI.

Figure 1: Estimates of Shadow Rates



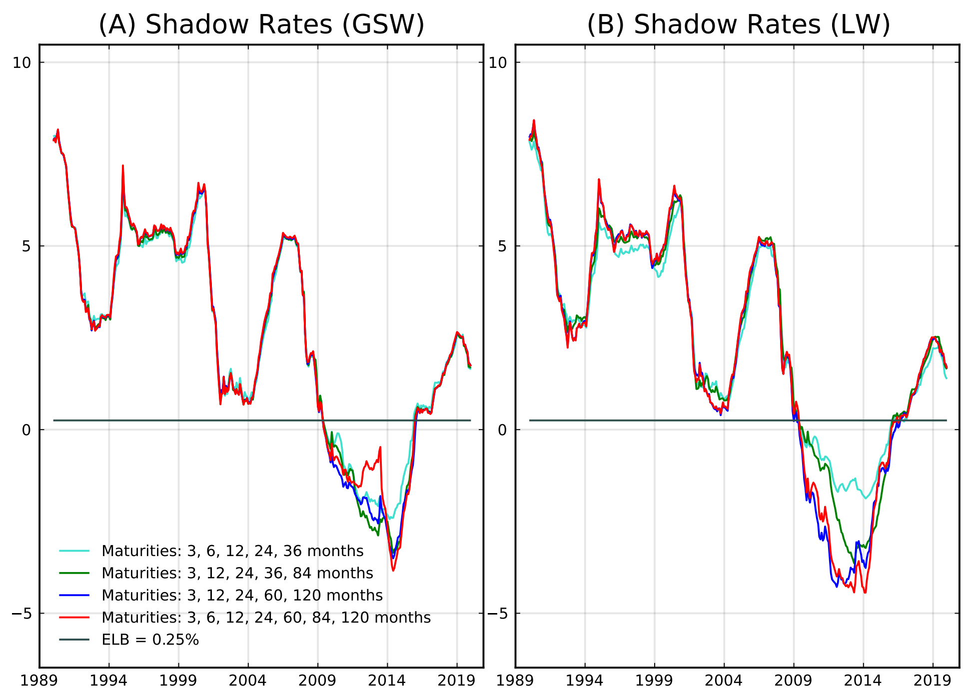
Note: Estimates of shadow rate using original sample used in Wu & Xia (2016), an updated version until December 2019, and the normalization used in Giglio & Kelly (2018). Panel (A) uses GSW database while panel (B) uses LW dataset.

Now we introduce two further changes: (i) different sets of forward rates, and (ii) different sample size. The benchmark model uses the following forward rates: 3 and 6 months, 1, 2, 5, 7, and 10 years. In Figure 2, we compare the SR of the benchmark model (red line) with an alternative that considers only five forward rates (blue line): 3 months, 1, 2, 5, and 10 years.

In the case of the LW dataset, changes are very small; but for the GSW dataset, it shows some differences regarding the recovery of the SR around 2014. Further, reducing the longest forward rate (green lines) implies a different dynamic of SR being closer to the ELB. That is true for both datasets, meaning that the longest forward rate is relevant for assessing the dynamic of the SR during ELB periods. This is clearly observed in the Kalman filter gains, which reduce the weight for short-term forward rate (3 months) and increase that for longest forward rates (5 and 10 years) during the ELB episode.[11].

In Figure 3, we compare the SR of the benchmark model (red line), that use 30 years of data, with samples of 20 years (green line) and 10 years (blue line). All lines include the crisis period and therefore the ELB episode. In both datasets (GSW and LW) the use of the last 10 years implies SRs that are close to the ELB. This suggests that the bias of small-sample reduces the sensitivity of the SR to the movements of the long-term interest rates; however the medium-size sample (20 years) provides a reasonable performance.

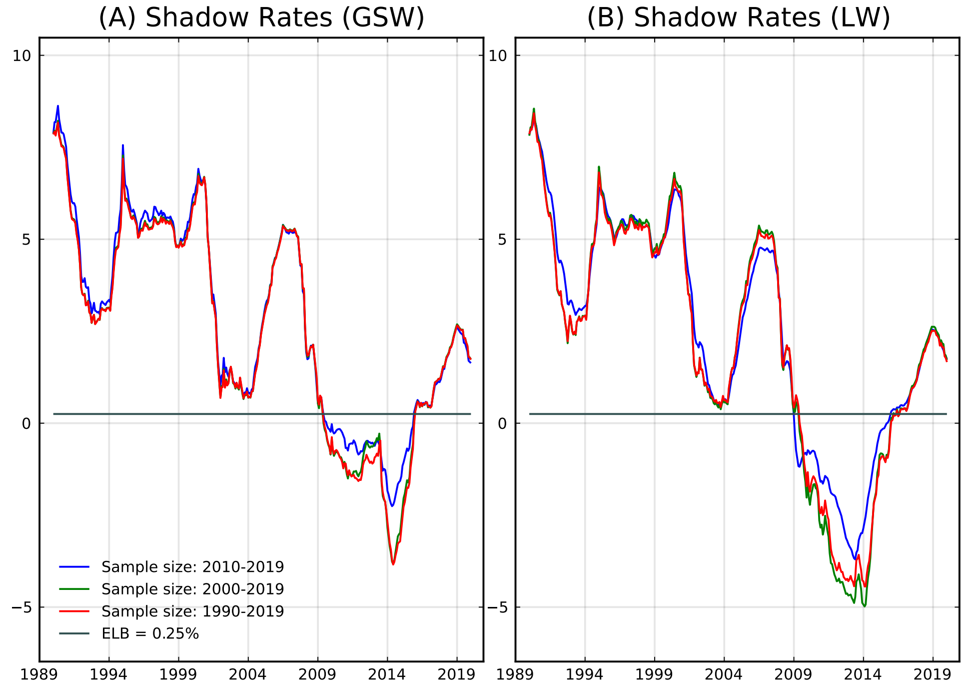
Figure 2: Estimates of Shadow Rates with different sets of forward rates



Note: Estimates of shadow rate using different sets of forward rates. Panel (A) uses GSW database while panel (B) uses LW dataset.

[11]See Appendix C for selected weights obtained from EKF.

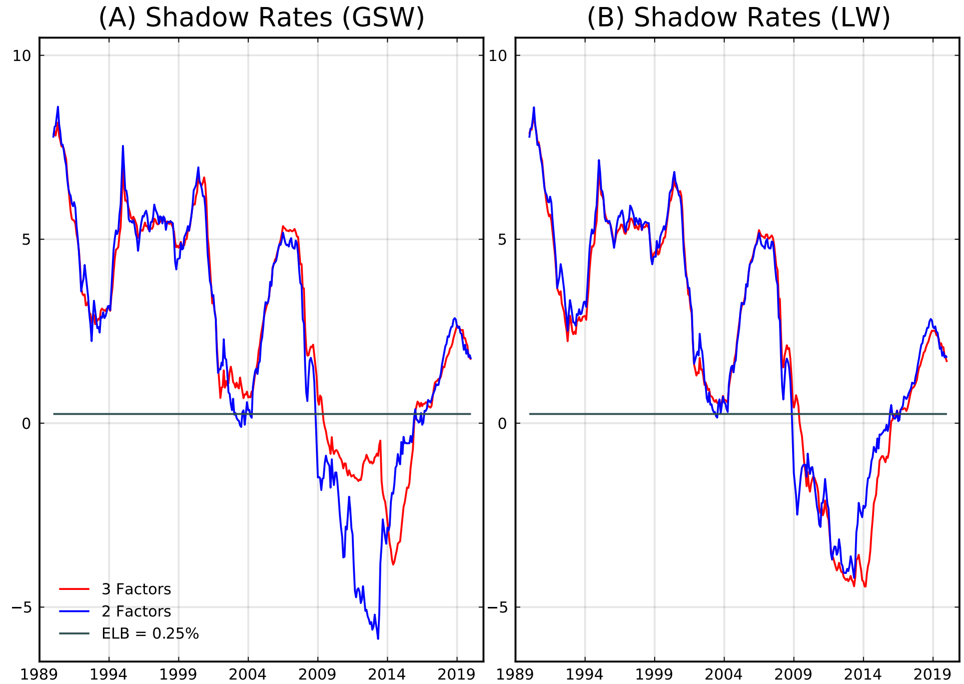
Figure 3: Estimates of Shadow Rates with different sample sizes



Note: Estimates of shadow rate using different sample sizes. Panel (A) uses GSW database while panel (B) uses LW dataset.

Figure 4 shows the estimates of the SR using 3 and 2 factors. As before, the red line represents the 3 factors case, and the blue line represents the 2 factors case. We observe that the 2 factor SR is largely negative relative to the 3 factor SR when we use GSW dataset. In contrast, the use of LW dataset provides small differences, although in both cases the drop is sharper in the estimate with 2 factors. It should be noted that literature suggests relevant differences in SR between the 3 and 2 factor models (Krippner, 2013; Christensen & Rudebusch, 2015). That is consistent with the GSW dataset, but not with LW.

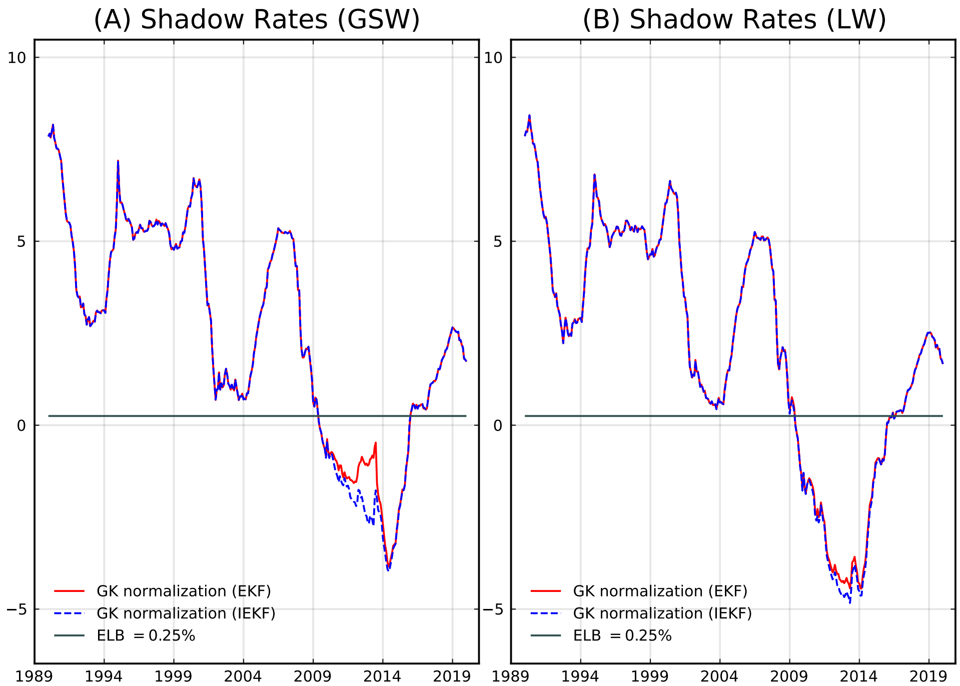
Figure 4: Estimates of Shadow Rates with 3 and 2 factors



Note: Estimates of shadow rate considering 3 and 2 factors. Panel (A) uses GSW database while panel (B) uses LW dataset.

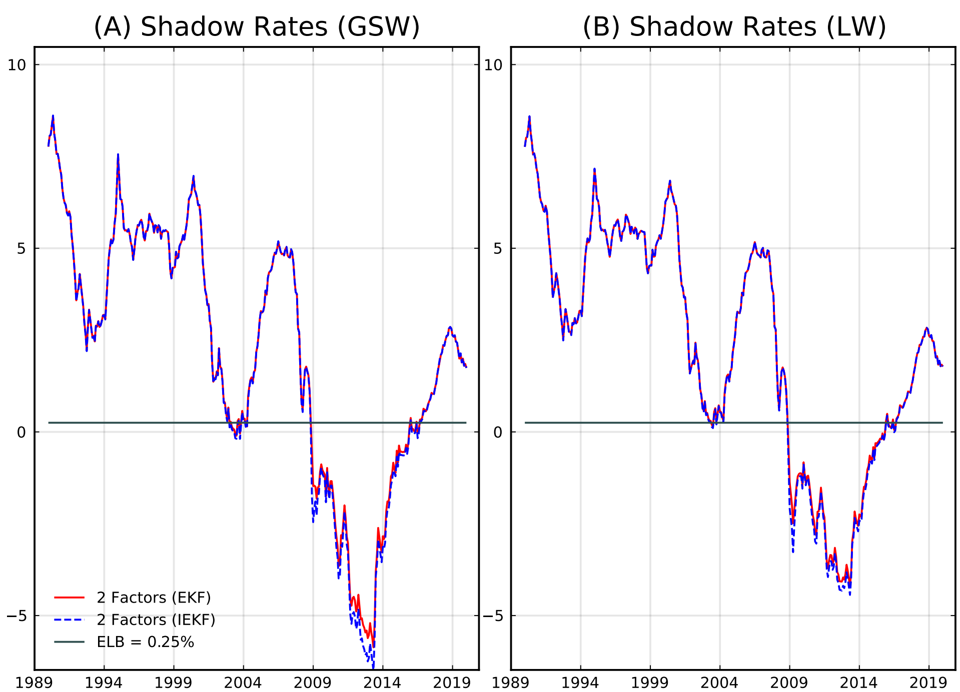
Now we consider the results for the SR obtained by IEKF method. It can be seen in Figure 5 that some difference between EKF and IEKF appears when the GSW dataset; however, these differences are not obtained in the case of the LW database. It should be noted that the differences between GSW and LW are reduced under the estimation with IEKF. On the other hand, estimating the 2-factor model with IEKF, we can observe in Figure 6 that the SR results are similar to those obtained under EKF. In this sense, a lower sensitivity of the model with 2 factors to the estimation method is observed in relation to the case in which we consider 3 factors; however there the difference between datasets remains.

Figure 5: Estimates of Shadow Rates with IEKF



Note: GK normalization shadow rate estimates with IEKF and EKF. Panel (A) uses a GSW data and panel (B) uses LW data.

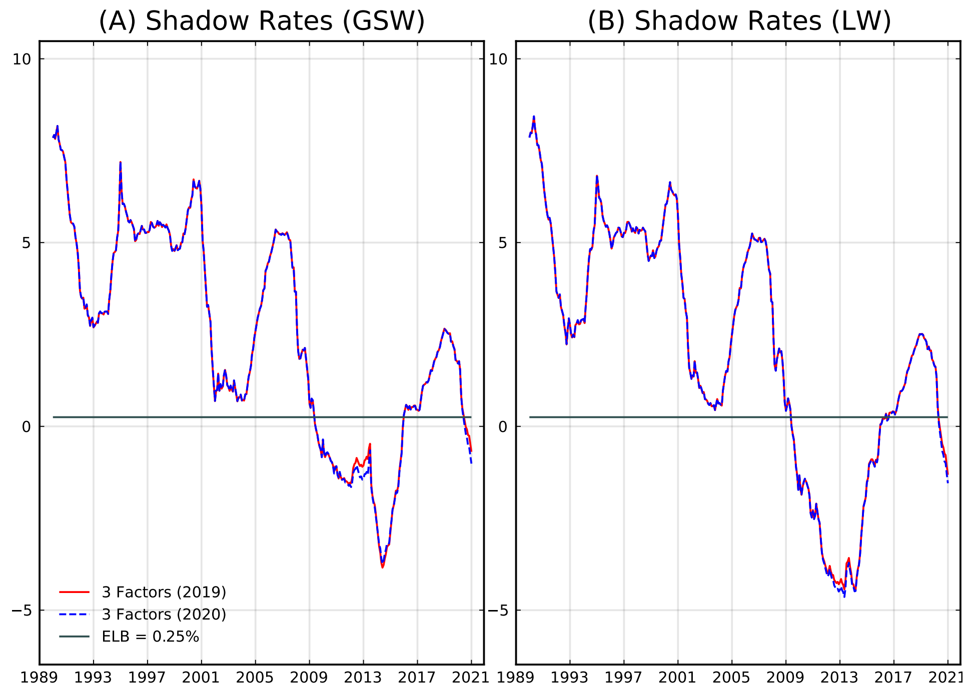
Figure 6: Estimates of Shadow Rates considering 2 factors and IEKF



Note: SR estimates correspond to those shown in Figure 4, but using the IEKF method.

Below we provide an update for SR during 2020. Figure 7 shows the update for 2020 of the estimates with 3 factors and GK normalization. It shows us that the SR does not suffer important changes with the re-estimation of the model with the additional observations and that the SRs tend to agree about the size and timing between estimates considering different datasets. However, the SR obtained with the LW dataset shows a faster drop than that obtained with GSW.

Figure 7: Estimates of Shadow Rates 2020



Note: Estimates of shadow rate considering 3 and GK normalization. Panel (A) uses GSW database while panel (B) uses LW dataset.

IV Conclusions

There is an increasing interest for assessing the monetary policy stance when the short-term interest rate is at the effective lower bound. Under a standard Gaussian model, the shadow rate (SR) could be obtained as a closed-form expression. That allows us to relate the yield curve model to the forward rates and with that to estimate the model by implementing the Extended Kalman filter (EKF) or the Iterated EKF.

In this paper we extend the results in Wu & Xia (2016) by considering a different dataset and by changing the coverage of these datasets, in terms of sample size and maturities. Based on that we offer several estimates of the SR, which suggests that figures should be taken with caution. In particular, having a proper coverage of maturities it seems key for identifying the SR from the long-term interest rates. That is confirmed by the use of the Kalman filter weights, which are zero for short-term forward rate (3 months) during the ELB episode, but significantly higher for longest forward rates (5 and 10 years).

The results emphasize the importance of dataset in the estimation of the SR. In this sense, the SR estimates using 2 and 3 factors show that the number of factors could significantly change the SR estimates as we observed in the case of the GSW database as opposed to the LW database. Additionally, when we change the estimation method, we obtain that the SRs show a certain sensitivity, particularly in the case in which 3 factors and the GSW data are considered. Finally, we also show that updating the parameters towards 2020 barely modified the results obtained initially, although the SR estimated with LW dataset falls faster towards the end of 2020 compared to GSW, similar to what was observed in 2009. These results suggest that the LW dataset seems to be more appropiate for the estimation of the SR in US.

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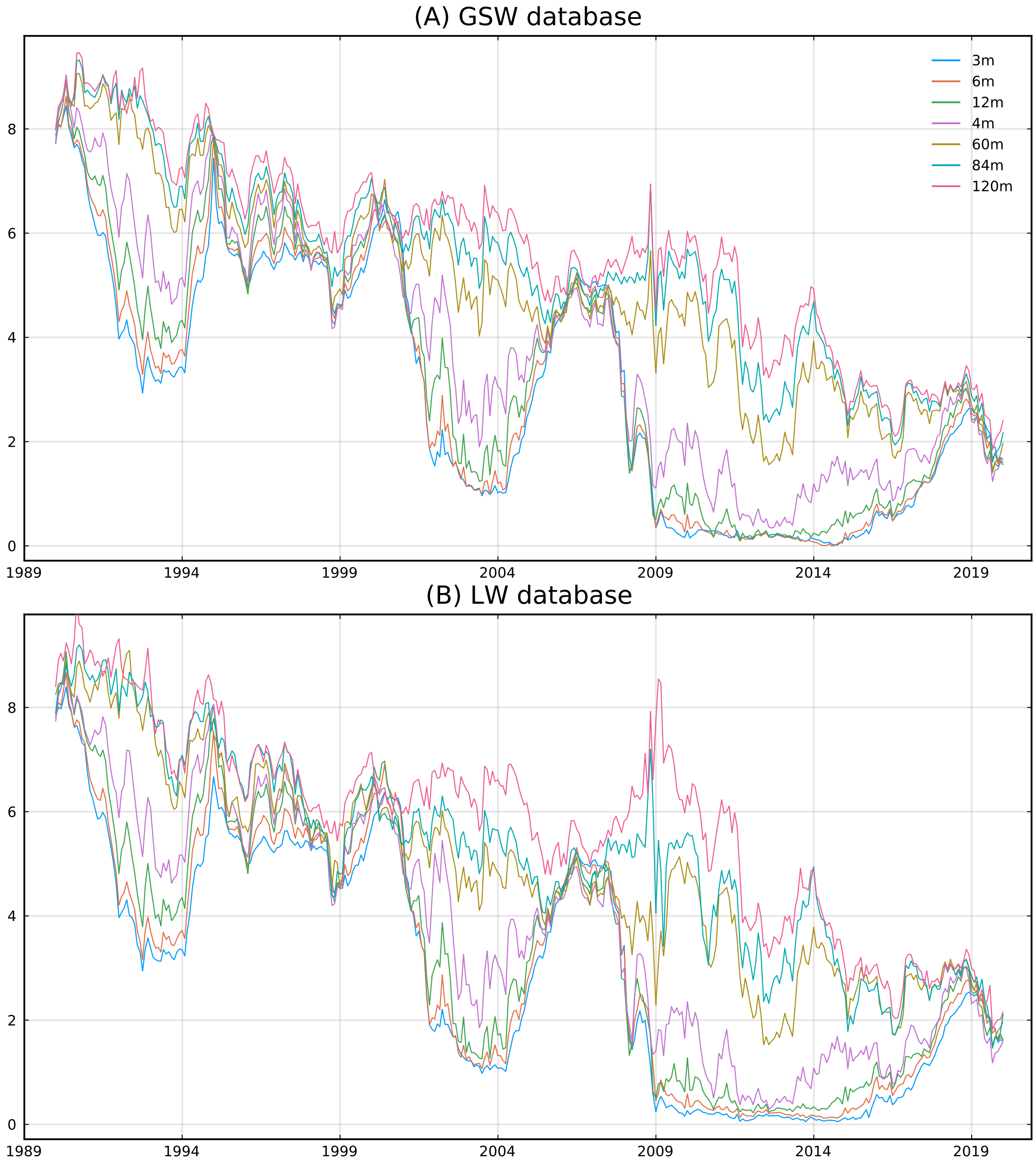
A Forward Rates

Figure 8: US Forward Rates



Note: Panel (A) uses Gürkaynak et al. (2007) (GSW) dataset while panel (B) uses Liu & Wu (2020) (LW) dataset.

B Iterated EKF

Table 7: Replication of Wu and Xia (2016) using GSW dataset and IEKF method

Parameters	A. Original Sample			B. Update to 2019			C. GK normalization		
μ^P	-0.2343	-0.1729	0.0205	0.1229	-0.6272	-0.0058	-0.3685	0.3567	-1.0092
	(0.1872)	(0.1949)	(0.0152)	(0.2037)	(0.282)	(0.0197)	(0.5704)	(7.0202)	(6.7876)
Φ	0.9724	0.0031	0.4096	1.0048	0.0175	-0.1076	0.9889	0.0127	0.0125
	(0.0201)	(0.0197)	(0.5294)	(0.0078)	(0.0067)	(0.2279)	(0.0144)	(0.0072)	(0.0068)
	-0.0147	0.9432	1.0558	-0.024	0.9629	1.0932	-0.012	1.0986	0.1123
	(0.0218)	(0.0224)	(0.5613)	(0.0091)	(0.0084)	(0.2659)	(0.3613)	(1.786)	(1.7228)
	0.0029	0.002	0.9417	-0.0001	0.0	0.9829	-0.0072	-0.1349	0.8574
	(0.0017)	(0.0019)	(0.0424)	(0.0008)	(0.0008)	(0.0213)	(0.3604)	(1.7627)	(1.7429)
Φ_*	0.9977			0.9995			0.9997		
	(0.0003)			(0.0001)			(0.0002)		
		0.9497	1		0.9561	1		0.9529	
		(0.0013)			(0.0014)			(0.0248)	
			0.9497			0.9561			0.9589
			(0.0013)			(0.0014)			(0.0191)
1200Σ	0.4122			0.2804			0.2763		
	(0.0373)			(0.0223)			(0.0201)		
	-0.402	0.2386		-0.2833	0.2459		0.2744	6.2355	
	(0.0347)	(0.0224)		(0.0237)	(0.0199)		(10.0792)	(46.0496)	
	-0.0093	0.0037	0.04	0.0031	0.0033	0.0375	-0.5418	-6.2502	0.2466
	(0.0069)	(0.0035)	(0.0034)	(0.0042)	(0.003)	(0.0034)	(10.0437)	(46.0649)	(0.0216)
$1200\sqrt{\omega}$	0.0893			0.097			0.0969		
	(0.0027)			(0.0028)			(0.0026)		
$1200\delta_0$	13.3824			29.9529			37.5851		
	(0.9725)			(5.3694)			(15.4969)		
$Eigen^P$	0.8891	0.9618	1.0064	0.9701	0.9903	0.9903	0.978	0.978	0.9888
$Eigen^Q$	0.9497	0.9497	0.9977	0.9561	0.9561	0.9995	0.9529	0.9589	0.9997
$Llike$	859.8567			988.6244			988.314		
AIC	-1676			-1933			-1931		
BIC	-1677			-1934			-1932		

Based on the following forward rates: 3 and 6 months, 1, 2, 5, 7, and 10 years from GSW dataset.

Table 8: Replication of Wu and Xia (2016) using LW dataset and IEKF method

Parameters	A. Original Sample			B. Update to 2019			C. GK normalization		
μ^P	-0.2926	0.118	0.0037	0.0329	-0.4084	-0.0041	-0.0009	0.9754	-1.3029
	(0.1926)	(0.2079)	(0.0086)	(0.2041)	(0.2804)	(0.0126)	(0.2075)	(4.5599)	(4.6786)
Φ	0.9796	0.022	-0.2851	1.002	0.0147	0.0429	1.0005	0.016	0.0157
	(0.0118)	(0.0112)	(0.7438)	(0.0074)	(0.0065)	(0.5209)	(0.0073)	(0.0068)	(0.0075)
	0.0171	0.9904	1.2547	-0.0119	0.9906	0.8888	0.0516	1.1562	0.2089
	(0.0141)	(0.0108)	(0.8602)	(0.0087)	(0.0073)	(0.7698)	(0.1755)	(0.3472)	(0.3512)
	0.0001	-0.0008	0.9677	-0.0002	-0.0005	0.9426	-0.0602	-0.1689	0.7823
	(0.0006)	(0.0005)	(0.0361)	(0.0004)	(0.0004)	(0.0283)	(0.18)	(0.3475)	(0.3569)
Φ_*	0.9966			0.999			0.999		
	(0.001)			(0.0004)			(0.0005)		
		0.976	1		0.9774	1		0.9757	
		(0.004)			(0.0031)			(0.0046)	
			0.976			0.9774			0.9791
			(0.004)			(0.0031)			(0.0028)
1200Σ	0.5033			0.3573			0.3598		
	(0.1663)			(0.0895)			(0.0936)		
	-0.4997	0.2809		-0.3678	0.2625		4.2567	6.256	
	(0.1809)	(0.0186)		(0.1024)	(0.0151)		(6.7329)	(7.2653)	
	-0.0162	0.004	0.0216	-0.0147	0.0033	0.0216	-4.629	-6.2882	0.2601
	(0.0061)	(0.0019)	(0.0039)	(0.0055)	(0.0017)	(0.0035)	(6.7953)	(7.2683)	(0.0151)
$1200\sqrt{\omega}$	0.2016			0.1915			0.1916		
	(0.0077)			(0.0065)			(0.0065)		
$1200\delta_0$	15.4037			31.7228			32.2534		
	(2.5805)			(8.8908)			(11.0656)		
$Eigen^P$	0.9717	0.9717	0.9943	0.9518	0.9917	0.9917	0.9766	0.9766	0.9859
$Eigen^Q$	0.976	0.976	0.9966	0.9774	0.9774	0.999	0.9757	0.9791	0.999
$Llike$	-273.6257			-204.7656			-200.1389		
AIC	591			454			446		
BIC	590			452			445		

Based on the following forward rates: 3 and 6 months, 1, 2, 5, 7, and 10 years from LW dataset.

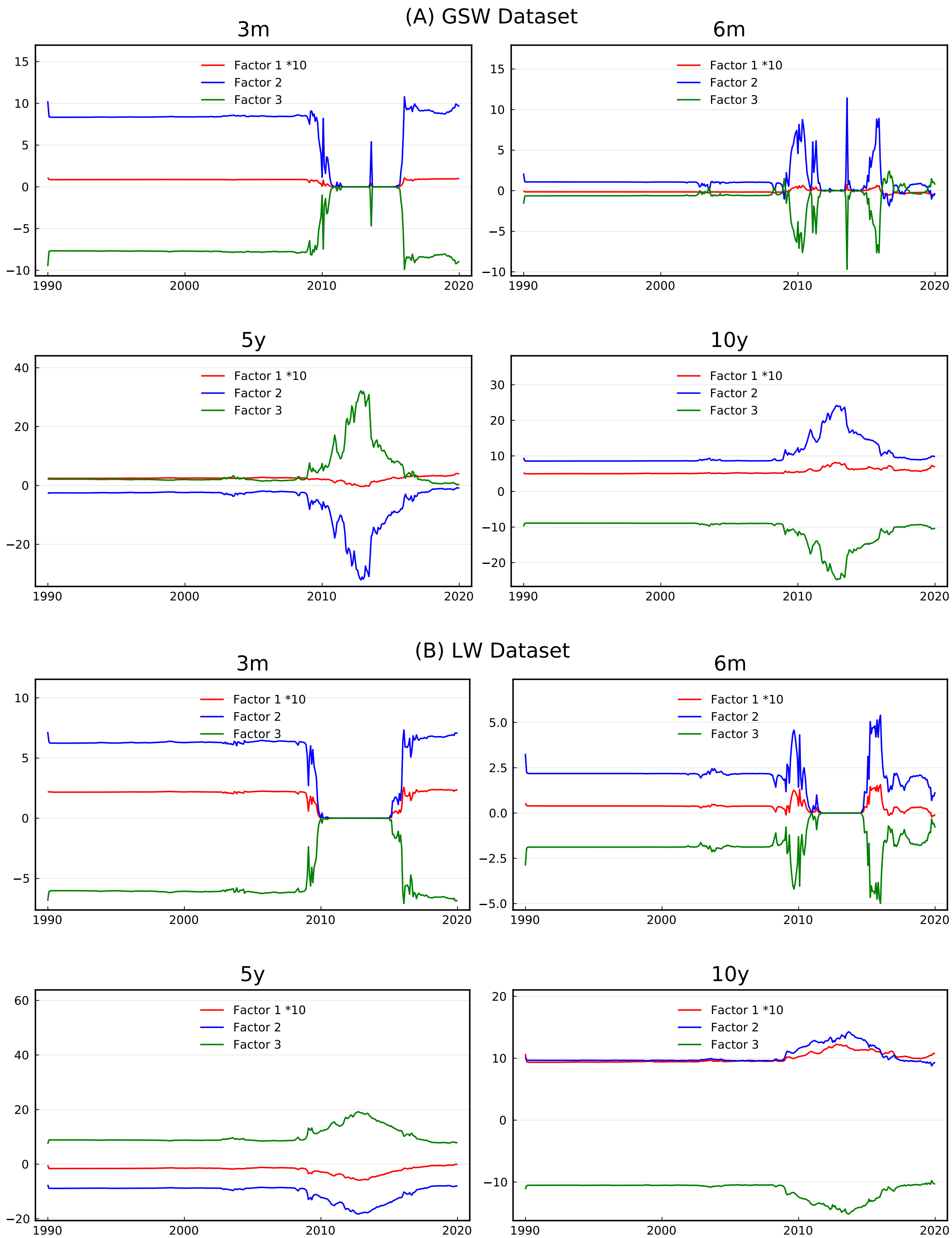
Table 9: GK normalization considering 2 factors and IEKF method

Parameters	GK-2F-GSW		GK-2FR-GSW		GK-2F-LW		GK-2FR-LW	
μ^P	-0.0313	-0.0894	-0.0064	-0.0562	-0.0243	-0.0957	-0.0016	-0.0695
	(0.0605)	(0.0725)	(0.0276)	(0.036)	(0.0547)	(0.0645)	(0.0278)	(0.0366)
Φ	0.9975	0.0058	0.9974	0.0048	0.9989	0.004	1.0002	0.0043
	(0.0071)	(0.0064)	(0.0066)	(0.0053)	(0.0064)	(0.0061)	(0.0061)	(0.0051)
	-0.0093	0.9964	-0.0108	0.9941	-0.0103	0.995	-0.0135	0.9925
	(0.0077)	(0.0061)	(0.0084)	(0.0058)	(0.0064)	(0.0056)	(0.0072)	(0.0056)
Φ_*	0.9985		1.0		0.9984		1.0	
	(0.0003)		(1.0)		(0.0005)		(1.0)	
		0.9735		0.9794		0.9784		0.9838
		(0.0012)		(0.0007)		(0.002)		(0.0011)
1200Σ	0.3195		0.2408		0.2915		0.2475	
	(0.0279)		(0.0218)		(0.0269)		(0.0206)	
	-0.1882	0.3642	-0.231	0.3159	-0.2094	0.3633	-0.2606	0.3236
	(0.0518)	(0.0324)	(0.0473)	(0.0205)	(0.0477)	(0.0308)	(0.0398)	(0.0242)
$1200\sqrt{\omega}$	0.1889		0.2071		0.2857		0.2967	
	(0.0067)		(0.0068)		(0.0123)		(0.0114)	
$1200\delta_0$	12.5671		8.28		12.36		8.6481	
	(1.4242)		(0.1415)		(1.7467)		(0.3174)	
$Eigen^P$	0.9969	0.9969	0.9958	0.9958	0.997	0.997	0.9963	0.9963
$Eigen^Q$	0.9735	0.9985	0.9794	1.0	0.9784	0.9984	0.9838	1.0
$Llike$	-66.3909		-239.1603		-899.2345		-972.0595	
AIC	159		502		1824		1822	
BIC	158		502		1824		1967	

Based on the following forward rates: 3 and 6 months, 1, 2, 5, 7, and 10 years from GSW and LW dataset.

C Kalman Filter Weights

Figure 9: Weights of forward rates with 3 factors



Note: Kalman weights for each factor and maturity indicate the relevance of each component of each forward rate over time. Panel (A) uses GSW database while panel (B) uses LW dataset.

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