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Abstract

Copper price is fundamental for the Chilean economy and, thus, for the Central Bank of Chile's forecasts. The goal of this document is to provide a theoretical tool that allows not only to understand the determinants of the evolution of copper price, but also forecast it. We define isoelastic demand and supply functions with both temporary and permanent shocks. We also allow for inventory storage based on a non-arbitrage condition with respect to expected prices. We use a shooting algorithm to solve for equilibrium with rational expectations, with supply, demand, and inventory data as inputs. We also isolate the USD exchange rate component of copper prices. We find that in the short run, temporary shocks play a minor role, whereas the USD Broad index and expectations contribute significantly. Also, in the long run, permanent demand and supply shocks seem to explain the major dynamics. We also present suggestive evidence that imprecise information can explain short-run volatility in expectations. Likewise, the model and methodology used are applicable to any storable commodity, assuming that supply, demand, price and inventories data are always available.

Resumen

El precio del cobre es fundamental para la economía chilena y, por ello, para las predicciones del Banco Central de Chile. El propósito de este documento es proveer de una herramienta teórica que no solo nos permita entender los determinantes del precio del cobre, sino también predecirlo. Utilizamos un modelo con funciones de demanda y oferta isoelásticas con shocks tanto temporales como como permanentes. A su vez, introducimos el almacenamiento de inventarios basado en una condición de no arbitraje con respecto a los precios esperados. Utilizamos un algoritmo de disparo para resolver el equilibrio con expectativas racionales, usando como insumo datos de oferta, demanda e inventarios. Además, aislamos el componente del tipo de cambio del dólar de los precios del cobre. Encontramos que, en el corto plazo, los shocks temporales juegan un rol menor, mientras que el índice del dólar estadounidense amplio y las expectativas contribuyen significativamente. Además, en el largo plazo, encontramos que los shocks permanentes sobre oferta y demanda parecieran explicar la dinámica general. También presentamos evidencia sugerente de que la presencia de información imprecisa puede explicar la volatilidad de las expectativas a corto plazo. Asimismo, el modelo y la metodología utilizados son aplicables a cualquier *commodity* almacenable, asumiendo que los datos de oferta, demanda, precio e inventarios están siempre disponibles.

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*The views and conclusions presented here are exclusively those of the authors and do not necessarily reflect the position of the Central Bank of Chile or its Board members. Emails: farroyo@bcentral.cl; fletelier@bcentral.cl;

1 Introduction

Commodities are a central part of emerging economies. For most of them, they constitute a major part of total exports and imports, as well as a significant part of GDP. This implies that commodity prices will impact directly on the cycle through both economic activity and financial markets. Hence, understanding these price dynamics of prices becomes key to monetary policy.

In this paper we develop a model to study prices for any given storable commodity. The model consists of demand and supply functions with transitory and permanent shocks. We also add expectations in the form of expected demand and supply growth. Agents are allowed to arbitrage between moments of time by accumulating or selling inventories with linear costs. This implies that, in equilibrium, expectations must be consistent with current prices and inventories. Hence, our model solves for commodity prices within a rational expectations framework. As a result, we can decompose and quantify the role of fundamentals in price behaviour. We use our model for copper (2000 to 2020, quarterly data), given its importance for the Chilean economy. We find that permanent shocks play a more relevant role in the medium and long run, whereas expectations explain better short term changes. We also separate the exchange rate component from real fundamentals. We find that the recent increases in copper prices (from $2.4 to $3.7 per pound in 2020) are mostly explained by depreciation in the USD.

We also incorporate our estimates into an error correction model and test for out-of-sample forecasting capacity between 2015 and 2020. We find that there are gains in predictability relative to projecting a random walk, indicating that our estimations add useful information to the last observed price.

Another main finding is that prices are more volatile than what fundamentals suggest. This is similar to Shiller [1980]. A possible explanation is that information flows change expectations, that is, that although fundamentals are stable, the information around them may not be, hence inducing additional volatility. To study this hypothesis, we use our model to simulate moments and contrast them with the observed ones. We introduce different shocks into our model and see how moments change. First, we shock interest rates and future curve slopes according to their data-based volatility. We find that these shocks do not make the model adjust well to the data. We then proceed to shock expectations by adding a difficulty in distinguishing permanent from transitory shocks. We do this by filtering fundamentals in real time (instead of ex post filtering), to simulate the case of an agent that is making decisions on the spot. We find that this last exercise delivers model determined moments that are in better accordance with the data. Finally, to support this hypothesis, we also find evidence of uncertainty in expectations by looking at market reactions to Consensus Forecast announcements. We find that markets react to CF, even though these are themselves not 100% precise predictions. This suggests that the market expectations have error themselves, since otherwise they would not pay attention to these not fully accurate predictions.

The paper is organized as follows. Section 2 presents the model. Section 3 explains how data is incorporated into it. Section 4 provides a forecasting exercise. Section 5 shows how the model historically decomposes changes in copper prices. Section 6 shows moment performance of the model and how it varies according to different innovations. Section 7 presents evidence of inaccuracy in market expectations. Section 8 contrasts our results to those of Chile's expert copper committee. Section 9 concludes.

2 Model

2.1 Supply and Demand

Given that we are analyzing the case of one particular market, we follow a partial equilibrium analysis. Similar to Deaton and Laroque [1992] and Knittel and Pindyck [2016], we assume isoelastic demand and supply functions:

$$D_t = Z_t p_t^{-\rho} \tag{1}$$

$$S_t = A_t p_t^{-\eta} \tag{2}$$

These type of functions simplify the analysis by separating into the component that depends on the price (endogenous) and the one that does not (exogenous). The parameters ρ , η represent demand and supply elasticities respectively.

Exogenous components A_t, Z_t incorporate any factor that determines the supply and demand that are not the price. We assume that there are two types of possible shocks: transitory $\epsilon_t^{A,trans}$, $\epsilon_t^{Z,trans}$ or permanent $\epsilon_t^{A,perm}$, $\epsilon_t^{Z,perm}$. Also, there are long-term expected trends γ_A, γ_Z, which will materialize as a constant sequence of permanent shocks.

$$logA_t = \rho_A logA_{t-1} + (1-\rho_A)[log(A) + \epsilon_t^{A,perm} + \gamma_A t] + \epsilon_t^{A,trans} \tag{3}$$

$$logZ_t = \rho_Z logZ_{t-1} + (1-\rho_Z)[log(Z) + \epsilon_t^{Z,perm} + \gamma_Z t] + \epsilon_t^{Z,trans} \tag{4}$$

2.2 Inventories

Risk-neutral agents maximize profits by arbitraging with inventories over time with a discount factor β and depreciation δ. At each time t, they choose the amount of inventories in the following period, X_{t+1}. The non-arbitrage condition implies that $E_t(p_{t+1})\beta(1-\delta) = p_t$, if inventories are positive.

2.3 Equilibrium

Equilibrium is defined by the vector of prices such that market clearing and non-arbitrage conditions are satisfied. Formally, p_t^* is an equilibrium price if, given this price, equations 5, 6, and 7 hold.

$$S_t(p_t^*) - D_t(p_t^*) = X_{t+1} - X_t \tag{5}$$

$$E_t(p_{t+1})\beta(1-\delta) = p_t^*, \ if \ X_{t+1} \geq 0 \tag{6}$$

$$E_t(p_{t+1})\beta(1-\delta) < p_t^*, \ if \ X_{t+1} = 0 \tag{7}$$

That is, the current price of the commodity has to be consistent with future price expectations. The intuition is that for each current price, there will be a different expected inventory path. In equilibrium, the expected inventory path must be consistent with the expected price trajectory.

Therefore, at each moment t, equilibrium price is a non-linear function of state variables and shocks:

$$P = p(X_t, \epsilon_t^{Z,perm}, \epsilon_t^{A,perm}, \epsilon_t^{Z,trans}, \epsilon_t^{A,trans}, \gamma_Z, \gamma_A) \tag{8}$$

An important point is the role of trends. At any given moment, trend growth can be summarized as a sequence of permanent shocks that have already taken place. Therefore, the role of trends γ_Z,γ_A is to change expectations, because they determine, given a starting point, by how much will supply and demand grow and how it should be reflected in the current price. Consequently, in practice γ_Z,γ_A summarize market long-run expectations.

The model is solved numerically. Details are available in the appendix, but intuitively, the process consists of testing different paths for expected prices and see whether they are consistent with the non-arbitrage, non-negativity, and transversality conditions (avoid having unsold inventories at infinity). That is, for each initial price it is possible to construct a path of (expected) future prices. Given the market clearing equation in each period, those expected prices will imply an expected path for inventories. The equilibrium expected path must not violate neither the non-negativity constraint (inventories cannot be expected to be zero at any point) nor the transversality condition (in expectations, inventories cannot be left unsold forever). Given that the excess demand function is continuous and decreasing in price, for any initial positive inventory holdings, a solution always exists. Moreover, since in our case the excess demand function is *strictly* decreasing in prices, the solution is unique. Therefore, we can always find a price path that solves the model. The initial price of that path will be the solution for each moment t.

3 Empirical application

With the previous model we can estimate a theoretical price based on fundamentals. In this paper we do it for copper, but this can be done for any other commodity as long as the commodity is storable and the required data is available. We follow a series of steps in order properly calculate this price. First, it is necessary to define the market price that is being modeled. Second, we generate the time-series data for variables that are model-determined (such as exogenous demand and supply variables A_t and Z_t). We then proceed to calibrate the parameters and finally run the model.

3.1 Data

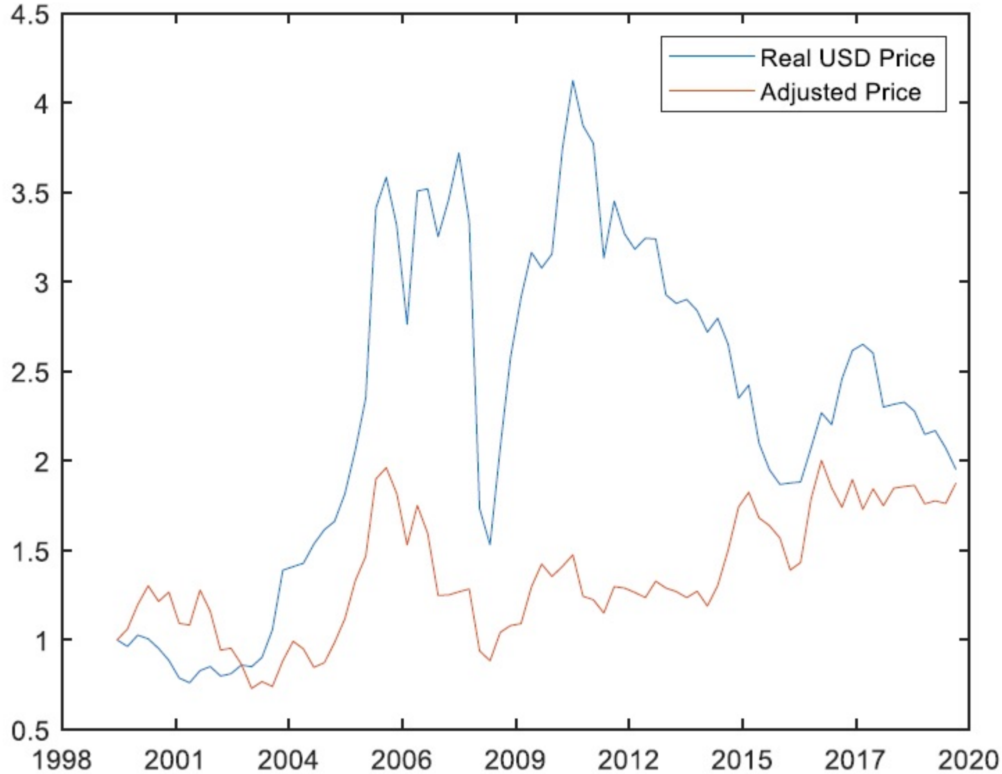
Data used correspond to copper price data, USD Index, inventories, global supply and demand, with a quarterly frequency. The interval under analysis is since 2000Q1 to 2020Q2, and data was obtained from Bloomberg for the case of the copper price and USD Broad Index, and from CRU Group, for inventories, global supply and demand.

3.2 First step: Define what market price is being modeled

The model estimates an equilibrium price based on supply, demand and inventories. This price is defined as the one that affects directly demand and supply. The issue is that the price is only observable in USD, usually in international markets. Given that the US only represents 12% of world production and 10% of world consumption (World Copper Factbook 2019), a big part of the price time series is directly associated with the USD Index. This implies that many changes in the price time series will be the result of international exchange rate markets, which our model is not designed to capture. Intuitively, the problem is that the USD copper price may not be the price that drives *directly* global consumers and producers. For example, it is not clear whether a Chinese construction company that wishes to import copper cares more about the cost in USD or in RMB. Therefore, our theoretical price (the one that influences *directly* supply and demand) is only partially observed in the data.

Hence, we proceed to "clean" the copper price from its USD component and use the model to explain the remaining part. We do this by regressing (in logs) real copper prices in USD against the USD index. We work with the residual from this regression, and call it the "adjusted" price (Figure 1). We are aware that this strategy is possibly subject to some degree of endogeneity. However it is not the goal in this paper to identify the role of exchange rates in commodity prices, we only take this role as given and proceed to filter the data from it. This particular strategy is not central to the model nor to this document, and therefore it can be replaced in the future without major inconvenience. It its only one of many ways in which the econometrician can proceed to construct the relevant price time series. We leave alternative strategies for further research.

Figure 1: Adjusted versus observed price in USD.



Notes: Values in level, CPI adjusted; 2000Q1 standardized equal to 1.

3.3 Second step: Generate Exogenous Variables Series

With supply and demand data, we can solve for the exogenous time series by residual in equations 9 and 10 :

$$A_t = S_t p_t^{-\eta} \tag{9}$$

$$Z_t = D_t p_t^{-\rho} \tag{10}$$

Given that S_t, D_t y p_t are observable, we need to impute the value of price elasticities η and ρ to obtain A_t and Z_t. López et al. [2009] estimate them to be 0.07 and 0.12 respectively. Other authors place these values between [0.05-0.2] and [0.03-0.24] (see Figure 6 in the appendix for more detail, with values from Takeuchi [1972], Banks [1974], Mardones et al. [1984], Vial [1988], Vial [2004] and Newhouse and Sloan [1966]). In this case, we will take 0.07 and 0.12 as a baseline, in line with what is indicated in Central Bank of Chile [2020].

However, we also make estimations with alternatives elasticities as a robustness check.

3.4 Filtering: permanent shocks vs transitory shocks

Given the elasticities, series of A_t and Z_t are generated. The next step is to differentiate between the permanent and the transitory components.

$$A_t = A_t^{perm} + A_t^{trans} \tag{11}$$

$$A_t = Z_t^{perm} + Z_t^{trans} \tag{12}$$

Series can be filtered with various methodologies. This document uses a Hodrick-Prescott filter for quarterly series, but the model is compatible with any filtering method deemed appropiate.

Once the transitory component has been estimated, the autoregressive parameters ρ_A, ρ_Z are obtained with an AR(1) estimation.

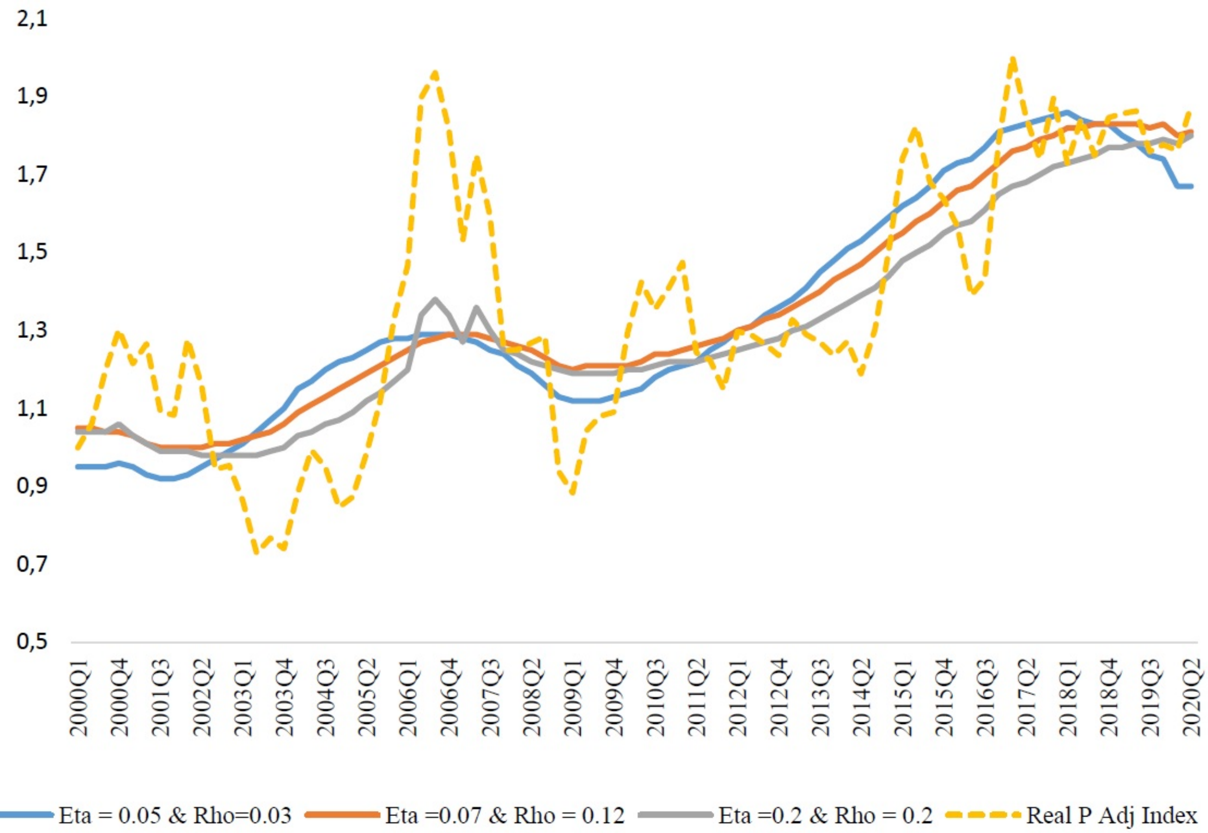
Finally, it is necessary to also determine the expected trends in demand and supply growth γ_Z, γ_A. For the model, the important thing is their relative value, not their levels. Intuitively, since in the very long run inventories don't play a significant role, essentially demand will be equal to supply. That is, it does not matter if they grow at 2%, 4% or 8%, it is their relative performance the only relevant factor when it comes to price determination. Therefore, what matters is the ratio $\frac{\gamma_A}{\gamma_Z}$. We estimate it such that it minimizes the squared error of the model.

$$P^{model} = p(X_t, \epsilon_t^{Z,perm}, \epsilon_t^{A,perm}, \epsilon_t^{Z,trans}, \epsilon_t^{A,trans}, \frac{\gamma_A}{\gamma_Z}) \tag{13}$$

The parameters β and δ are calibrated based on the futures curve and the literature, taking values 0.9975 and 0.001 respectively.

The model estimated prices can be seen in Figure 2. The orange line corresponds to the model's estimated price with the elasticity values of López et al. [2009].

Figure 2: Model price versus Real price



Note: Values in levels, CPI adjusted; 2000Q1 standardized equal to 1. Estimations done ex-post for different elasticities as indicated for each color.

Two conclusions can be drawn from the estimation exercise. The first one is that the model tracks well the real price in the medium and long run. This is because the main inputs of the model are supply and demand shocks, and therefore prices cannot systematically move away from them for a long time, otherwise inventories would either skyrocket or hit the zero lower bound. On the other hand, it is clear that the short-run dynamic is explained more by changes in expectations than by fundamentals. The latter is close to Shiller [1980]'s finding for the equity market, where he finds that the short-run price dynamics differ significantly from dividend dynamics. The second conclusion is that in the long run there are no great differences according to the elasticities used. That is, the model has a certain degree of robustness to the elasticity that one could attribute to it. This robustness is greater as one looks at a longer horizon.

4 Forecast

The next objective is to evaluate the model's usefulness to make short and medium term projections. A synthesis of the aforementioned would be that the model captures well the price based on fundamentals, while the observed price represents expectations. Since both factors are relevant, we propose an econometric model that includes both to optimize the projection throughout the horizon of interest.

A Johansen cointegration test indicates the existence of a long-term relationship between both variables. Therefore, in this section we formally propose to test forecastability with an Error Correction Model (ECM) as in expressions 14 and 15.

$$\triangle p_t = \alpha_1(\beta_1 p_t - \beta_2 p_t^{model} + \gamma) + \alpha_2 \triangle p_{t-1} + \alpha_3 \triangle p_{t-1}^{model} + \epsilon_t \tag{14}$$

$$\triangle p_t^{model} = \delta_1(\beta_1 p_t - \beta_2 p_t^{model} + \gamma) + \delta_2 \triangle p_{t-1} + \delta_3 \triangle p_{t-1}^{model} + \epsilon_t^{model} \tag{15}$$

We additionally test the ECM with the constraint $\beta_1 = 1$, $\beta_2 = 1$. The complete exercise consists of first estimating the model and then obtaining and evaluating out-of-sample forecasts for up to 8 following quarters.

For this exercise, we use a real-time estimated HP filter. It is important to take into account that if we want to analyze predictive capacity then we must incorporate estimates made with the information available on a real time basis. Hence, the HP filter is estimated in real time (minimum 40 sample quarters) and the model is fed with the most up-to-date estimate possible for each moment in time.

The projections are evaluated for 8 quarters, which coincides with the usual projection horizon of Central Bank of Chile. In turn, it is contrasted with a random walk (RW) projection for a better evaluation. Since we impose at least 40 quarters for the HP filter, that implies that the first date for estimation results is 2010Q1. This is then used as the initial window for 2010Q1-2014Q4 estimation and then modified over time according to both a rolling window and expanding window criteria.

Table 1: RMSE 2015Q1-2020Q2

	HORIZON	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
Rolling Window	VECM	0.19	0.25	0.24	0.28	0.32	0.30	0.28	0.33
	Restricted VECM	0.17	0.21	0.21	0.23	0.26	0.25	0.26	0.26
	RW	0.16	0.23	0.27	0.28	0.29	0.26	0.24	0.24
Expanding Window	VECM	0.16	0.20	0.19	0.21	0.22	0.20	0.18	0.18
	Restricted VECM	0.16	0.19	0.18	0.19	0.21	0.21	0.21	0.22
	RW	0.16	0.23	0.26	0.26	0.26	0.23	0.19	0.18

The projection is evaluated out of sample for the period 2015Q1-2020Q2. That is, the estimation and subsequent projection is carried out without incorporating any future data, in order to obtain results that emulate those of an econometrician who tries to project in real time. Tables 1 and 2 summarize the performance of the models according to the Mean Square Error (RMSE) and Standard Errors (SE) metrics.

Table 2: SE 2015Q1-2020Q2

	HORIZON	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
Rolling Window	VECM	0.19	0.24	0.20	0.23	0.27	0.24	0.22	0.27
	Restricted VECM	0.17	0.21	0.20	0.21	0.24	0.24	0.24	0.24
	RW	0.16	0.23	0.27	0.28	0.29	0.26	0.23	0.23
Expanding Window	VECM	0.16	0.19	0.19	0.20	0.21	0.18	0.16	0.16
	Restricted VECM	0.16	0.19	0.18	0.19	0.20	0.20	0.21	0.21
	RW	0.16	0.23	0.26	0.26	0.26	0.23	0.18	0.18

Note: Tables 1 and 2 show standard errors of forecasts from 1 to 8 quarters ahead for different econometric approaches. Out of sample period was 2015-2020. As a reference for the scale of standard errors, the average index price between 2015 and 2020 was 1.7. Yellow shaded numbers indicate the lowest SE.

The results show that the model provides relevant information when making medium-term projections relative to what is already observed in the current price. In both versions, restricted and unrestricted, the model outperforms the RW with a lower RMSE almost throughout the entire horizon. This is repeated for both the rolling window and the expanding window simulation.

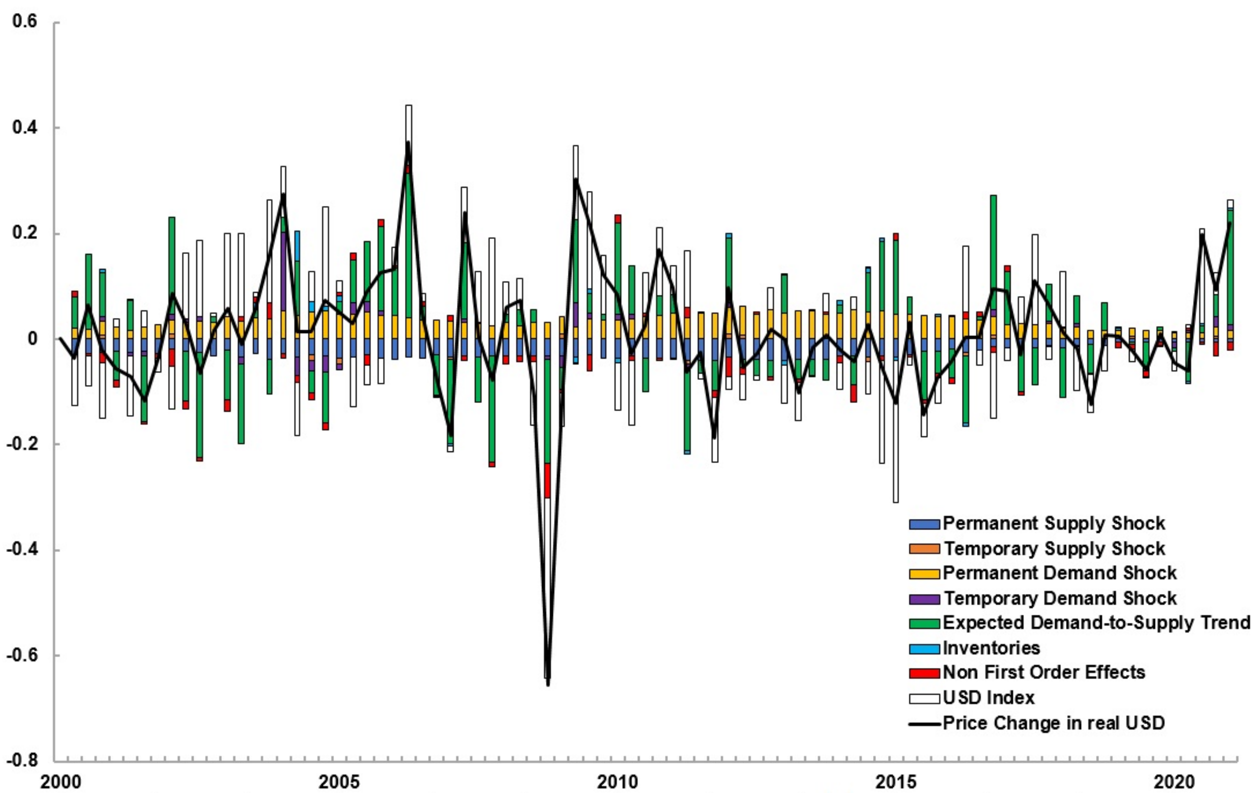
It is also worth noting that the model improves its performance when moving from a rolling window scheme to one with an expansive window. This makes sense since, by being expansive, the estimate incorporates new data without discarding the previous ones. Unless there is a structural change in the market, this implies a gain in estimation efficiency (with a subsequent gain in predictability).

5 Historical decomposition

Another interesting exercise is to use the model to decompose historical price changes. According to the model, the difference between the estimated results with a constant long-term expectation and the observed data implies that expectations change from period to period. The model can then be re-estimated by adapting the necessary changes in expectations to "match" the observed price and quantify its role (that is, by residual). This can be seen in Figure 3. Figure 3 decomposes the effects of shocks and expectations on the quarterly price changes according to the model. Additionally, the effect of the USD Index is added to reconstruct the price in dollars (see section "First step"), which is the price observed by the policy maker. Recall that it is not the goal of this document to explain the role of the USD Index on prices, we only take it as given and explain the remaining behavior.

Figure 3 shows that, in the short term, the main relevant factors of variations from one quarter to another are changes in expectations and the USD Index. Instead, the permanent components play a more constant and stable role over time. That is, long run changes are mostly determined by permanent supply and demand shocks. Another interesting result is the scarce role of transitory shocks. They play a very small role in terms of quarterly price changes. This is because agents arbitrate shocks between different moments in time through inventories, distributing any temporary shock along the entire expected price curve, and therefore diminishing its initial impact.

Figure 3: Historical decomposition



Notes: QoQ variation. Units in decimals, i.e. 0.1 equals 10%.

6 Moments

We now proceed to test additional statistics from the model. In this section we analyze the model's moments and test what kind of shocks and possible innovations can help explain what is not captured. The historical decomposition exercise assigns a relevant role to expectations in the short term. However, since these are built by residue, it is possible that there are other factors incorporated there.

To answer this, we simulate the model with different types of additional shocks (that have not yet been modeled) and compare the moments against those of the observed prices (adjusted according to step 1). Our base simulation is the one with the model as it has been presented so far: with temporary and permanent supply and demand shocks and a given long-term expectation (and therefore fixed). We then simulate additional results by adding different types of shocks: shocks to the futures curve (Basis), shocks to the interest rate, and filtering errors. Each exercise was done with 1000 simulations. The results can be seen in table 3.

The base simulation shows something that can also be seen in Figure I: supply and demand shocks generate very little quarterly volatility. The volatility of the copper price (adjusted) quarter over quarter is 10.7%, while the base model only generates 2.2%. Thus there are other non-modeled factors that affect short-term dynamics.

The second simulation adds shocks to the expectations equation 6. These were calibrated by looking at the volatility of the 3-month copper futures curve. This volatility was applied to the base model. The standard deviation rises to 18%, above the 10% in the data. In turn, the kurtosis rises to 11.3, above the 3.2 in the data. Obliquity also rises to 2.6, a long way from its empirical value of -0.28.

The third simulation adds shocks to the interest rate. In this case, volatility rises to 3.5%, without exceeding the empirical 10%. The shocks were calibrated with the volatility of the one year UST. However, again kurtosis and obliquity grow significantly to 4.8 and 0.72. This should not come as a surprise since in the model the interest rate operates via the futures curve, and therefore the effects should be similar to those of the second simulation, although of lesser amount given that it is less volatile.

The fourth simulation feeds the volatility of permanent shocks calibrated with the filtered data in real time, not ex post. That is, we incorporate the standard deviation of permanent shocks, when these shocks are estimated with a real time HP filter. Volatility rises to 4.3% compared to 2.2% in the baseline. Interestingly, in this case both Kurtosis and obliquity are much more closer to their empirical values. This suggests that information could play an important part in short-run volatility.

Finally, a Jarque-Bera normality test is carried out for all cases. The empirical data do not reject the null hypothesis that the data (quarterly adjusted price difference) is normally distributed. This result is only repeated for the simulation fed with an HP filter in real time. Both the distribution generated by the base exercise, and with shocks to the curve and the rate reject normality.

Table 3: Moment Analysis

	Base	Basis shocks	Interest Rate Shocks	Real Time HP	Data (adjusted prices)
Standard Deviation	0.022	0.181	0.035	0.043	0.107
Skewness	0.372	2.607	0.716	0.023	-0.275
Kurtosis	3.498	11.377	4.814	2.957	3.282
Rejects normality?	Y	Y	Y	N	N

From the exercises carried out, the main conclusion is that what could mostly explain quarterly volatility is the difficulty of agents to perceive in real time which components of demand and supply are permanent and which transitory. This misperception implies that the market validates prices that are later corrected as new information emerges. Those corrections imply greater volatility, but without generating Kurtosis or Obliquity. This hypothesis is also proposed by Shiller [1980] to explain short-term volatility in the stock market.

7 Signal extraction

The results indicate that information flow could potentially play a relevant role in short-term dynamics. In this section we provide an estimate of how the market values its own uncertainty.

The methodology to be used is that of the literature that models agents with imprecise information. This literature states that agents are constantly exposed to informational signals, each with a certain degree of precision. Agents then combine the information optimally by weighting the signals based on their relative precision, where a greater precision implies greater weighting.

In this case, we analyze the effect of a Consensus Forecast (CF) announcement on future copper prices. Every month, CF announces the average forecast of various market operators for the price of copper in the next quarter (as well as several other quarters thereafter). The market reacts to this announcement by adjusting the price of copper to some degree. In this paper, we only consider announcements that forecast the upcoming quarter, i.e, we avoid announcement that forecast the current quarter. Our goal is to estimate how much the market weights its own beliefs and from there infer the implicit precision in it.

It is worth mentioning that using announcements to study expectations in commodity markets is not new. For example, in Känzig [2021], the author proposes a novel identification strategy to shed light on the role of oil supply expectations. He uses variation in futures prices in a tight window around OPEC announcements in order to identify an oil supply news shock. In our case, we use CF announcement to identify information shocks and model how these innovations should impact prices within a rational inattention framework.

7.1 Estimation

Formally, before CF, expectations are given by:

$$E(p_{t+1}) = E(p_{t+1}|m) \tag{16}$$

Where "m" represents market information. When CF is announced, agents receive the following signal:

$$E(p_{t+1}|CF) = E(p_{t+1}|m) + \epsilon_t \tag{17}$$

CF already contains market information and adds an innovation ϵ_t. Given this signal, the market makes the following bayesian update of expectations:

$$E(p^{'}_{t+1}) = E(p_{t+1}|m) + K[E(p_{t+1}|CF) - E(p_{t+1}|m)] \tag{18}$$

$$K = \frac{\sigma_m^2}{\sigma_m^2 + \sigma_\epsilon^2} \tag{19}$$

Where K is the weight that the agent gives to information innovation, also known as Kalman Gain.

The ultimate goal is to obtain an estimate for σ_m^2, which would give us the precision that the market assigns to its own pre-CF expectation. For this we must have an estimate of K and σ_ϵ^2.

To do this, it is useful to remember that in equilibrium the non-arbitrage condition holds and hence:

$$E(p_{t+1})\beta(1-\delta) = p_t \qquad \rightarrow \qquad \%\triangle E(p_{t+1}) = \%\triangle p_t \tag{20}$$

Combining with previous equations, we obtain:

$$\%\triangle E(p_{t+1}) = \%\triangle p_t = K[\frac{E(p_{t+1}|CF)}{p_t[\beta(1-\delta)]^{-1}} - 1] \tag{21}$$

$$\%\triangle p_t = \alpha[\frac{E(p_{t+1}|CF)}{p_t} - 1] + \epsilon_t \tag{22}$$

Therefore, a very close approximate value for K can be estimated by regressing the changes in the market price and the distance between the predicted by CF and the current price. This estimate gives 0.3, that is, if CF announces a forecast 10% higher than the current price, the market reacts by raising the price by 3%.

$$\alpha \cong K = 0.3 = \frac{\sigma_m^2}{\sigma_m^2 + \sigma_\epsilon^2} \tag{23}$$

This gives us a close estimate of K. The next step is to obatin an estimate of σ_ϵ^2. By using CF's projections, we can know CF's precision. From a theoretical point of view, the precision of CF is given by the sum of the precisions of its components (market information plus innovation). Data from 2010 to 2020 indicate that CF hits on average its predictions with a margin of error of 8%. This implies that:

$$\sigma_{CF}^{-1} = (0.08)^{-1} = \sigma_m^{-1} + \sigma_\epsilon^{-1} \tag{24}$$

Hence, we are left with two equations (23 and 24) and two unknowns (σ_m and σ_ϵ), and can therefore solve for the value for market precision and innovation precision:

$$\sigma_m = 0.13 \qquad \sigma_\epsilon = 0.2 \tag{25}$$

What this result indicates is that markets react to the CF announcement as if, for them, the accuracy of their prior information was 13%. This number is close to the dollar price volatility, which is what CF predicts (table 4).

Table 4: Signal Precision

	Δp_t^{USD}	$\sigma^{marketprecision}$	$\Delta USDIndex$	$\Delta p_t^{adjusted}$
Standard Deviation	0.126	0.13	0.03	0.107

Note: Table 4 shows standard deviations of indicated variables. Variable frequency is quarterly for all cases and in first log-differences.

8 Expert's Projections

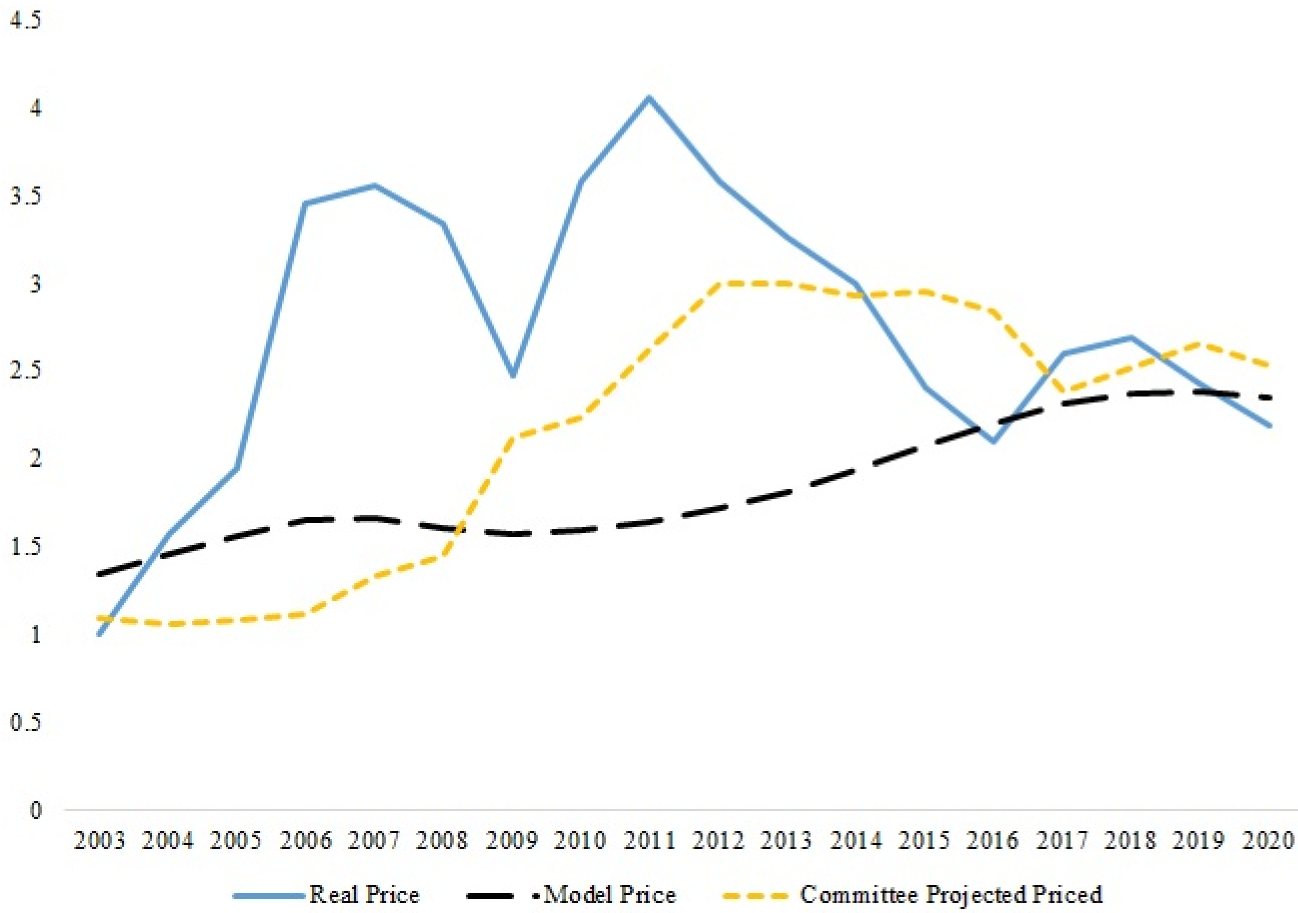
In Chile, given the importance of copper, a committee of experts is tasked annually with projecting its price. Their report consists of a 10 year average forecast of the US price of copper. In this section, we contrast their projections against our model output.

First, we compare their forecast against the value of copper at the moment of the projection and the model's estimation of the fundamental price. We normalize all of them by setting the real price in 2003 to 1 and the others to their relative initial position in 2003. All indexes are in real USD (corrected by CPI). The three price series can be seen in figure 4. Clearly, until 2015, observed prices were significantly above the forecast (at the moment of each forecast) and closer to the model's estimation of fundamental price. Recall that the model focuses on the non-exchange rate component of the price. Additionally, it filters short term volatility by stabilizing expectations (which are volatile in the short run) and filtering transitory from permanent shocks. Therefore the main difference between model and actual price is twofold: USD broad index effect and non-fundamental behaviour.

During the early 2000s, the USD weakened significantly, hence explaining part of the increase in copper prices. However, the committee forecast does not seem to be too much driven from this and instead it stays closer to model's reading. This switches post 2009 until 2015, from which both observed and model prices get much closer to each other, and so does the committee forecast. This suggests that part of the expert committee also focuses at least partially on the non-ER fundamental determinants of copper prices.

To further analyze this last point, we test the accuracy of the expert's forecast with respect to the average price of the next 10 years. We test their forecast against both average real prices and average model prices. We do this for 2003 to 2020. For the forecasts done in years 2011 to 2020, we test them against the average of the next 9 years, 8 years, and so

Figure 4: Expert Committee Projections vs Actual vs Model



Note: Dollar per pound.

Table 5: Committee's Forecast Precision

	Model	Real Price
Mean Absolute Error	0.10	0.18

Note: Table 5 shows the mean absolute (log) error of the expert committee's forecast relative to the next 10 year average of both the model's estimated fundamental price and the actual price.

on. Table 5 shows the mean absolute error in logs. We find that the committee's forecast matches better the model's estimation than the observed prices by 8% (10% average model error vs 18% average real price error). This reinforces the previous hypothesis that the expert committee's forecast does a better job at focusing more on price long run fundamentals than on the short determinants or ER effects.

9 Conclusion

The model provides a theoretical analysis of commodity dynamics, which complements the empirical approach used in current models. Empirically estimated supply and demand shocks are fed into a theoretical framework, obtaining an estimated price. The main result is that this model estimated price does a good job at capturing long-run price dynamics. This is not the case of the short run, where it seems that expectations play a more relevant role. Additionally, according to the model's reading, these expectations are more volatile than fundamentals themselves, similar to what happens with the stock market according to Shiller [1980]. This implies that the estimated price has much less volatility than its empirical pair.

With this in mind, we tested the projection capacity of an error correction model that incorporates both current prices and those estimated by the model. We tested these projections up to two years ahead. The results show that ECM predicts better than a random walk within the two year horizon from 2015 to 2020. This suggests that the model adds predictive information to that is already implicit in the current price.

We then performed a moments analysis in which we added different shocks to the model (shocks to interest rates, shocks to the future's curve, and shocks to perception of what is a transitory versus permanent shock), to see which type of shocks adapt better to the data. Results suggest that a good part of the short-run dynamic is explained by the difficulty of agents to perceive in real time which shocks are transitory and which are permanent. We also find that when adding shocks to interest rates and to the futures curve, although they generate some volatility, they also generate a kurtosis and obliquity that are incompatible with quarterly data. Shocks to the precision of the available information, on the other hand, present moments similar to those observed.

We contrasted our estimations to those of the expert committee that forecasts the price of copper for the Chilean government. This committee forecasts every year the next ten year average expected price. We find that their forecast, when done, is closer to the model's price than the real one. We also find that the forecast is 8% closer to the next 10 year average of the model than of the real price. This suggests that the expert committee focuses more on fundamentals than short run determinants like expectations or ERs.

Finally, it is worth mentioning that the methodology used here allows a lot of space for improvement. Cleaning the exchange rate component on prices and filtering permanent from transitory shocks are two key steps. In the first case, we used a reduced form regression of copper prices in USD against the USD index and worked with the residual. The second step was done by using a HP filter. We believe that both strategies have their limitations and present a lot of space for improvement. The literature provides with many other alternatives that could enhance our findings. We do no see this as challenge to our findings but more as an opportunity, since our model does not depend on the filtering method of choice.

As a final mention, it is worth clarifying that the model is completely extensible to other commodities, it is not restricted only to copper. Commodities of interest to the monetary authority, such as oil or other minerals in addition to copper, can be perfectly modeled by the equations and methodologies described here. We believe that replicating results shown here would be an important topic for further research.

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A Model solution

The model is solved numerically. The process consists of testing several expected price paths: the "solution" path must be consistent with transversality conditions (we cannot have positive inventories at infinity), non-arbitrage (the expected prices must be such that there are no incentives to arbitrate infinitely), and non-negativity (inventories can never be less than zero at any point). The starting point of the price path that satisfies these conditions is the model's estimated current price.

Formally, given initial values of positive inventories and shocks, the (expected) price path must be such that there are incentives to hold non-negative inventories for at least some time (0,J]. During this time, the (expected) price path evolves such that it compensates opportunity cost (interests) and depreciation.

For t+k<J,

$$E_t(p_{t+k})\beta(1-\delta) = E_t(p_{t+k-1}) \tag{26}$$

During this interval t+k<J, inventories will evolve according to the market clearing equation:

$$S_{t+k}(E_t(p_{t+k})) - D_{t+k}(E_t(p_{t+k})) = X_{t+k+1} - X_{t+k} \tag{27}$$

Another point is that prices cannot grow forever, since this would imply that inventories accumulate infinitely, violating the transversality condition. Therefore, there has to be a moment J upon which prices do not increase. Once $t = J$ has been reached, the (expected) inventories will be zero and therefore the (expected) equilibrium price will be:

For t+k≥J,

$$E(p_{t+k}) = [\frac{Z_{t+k}}{A_{t+k}}]^{\frac{1}{\rho+\eta}} \tag{28}$$

The challenge of the algorithm is to find an expected price path that satisfies (26), (27) and (28), given the initial values of inventories and shocks, and does not violate the non-negativity of inventories. The dynamics of the algorithm can best be seen in Figure 5 and Figure 6. Figure 5 shows different expected price paths tested by the algorithm. Figure 6 shows the expected inventory path that corresponds to each expected price path. For an initial value, initially the expected price path is in line with the depreciation and discount rate, so to ensure the non-arbitrage condition. For every initial price $p_t^* < p^{LR}$, the expected price path will imply that demand will be above supply for some time, and inventories will go down. This guarantees that the expected price path is such that it is expected to deplete inventories at some point and not sustain them forever, in line with the transversality condition. At the same time, inventories can not fall below zero. Therefore, at some point, demand will have to equal supply, or what is equivalent, $p_t^* = p^{LR}$. That moment is $t = J$. The algorithm searches for J such that the described conditions are satisfied. More formally, the algorithm searches for the value of p_t^* such that $X_{t+k} = 0$ for $t + k \geq J$, or equivalently, such that $\sum_{k=1}^{\infty} \triangle X_{t+k} = -X_t$. Once the solution path has been found, the price at t = 0 is the one that corresponds to today, and therefore it is the price delivered by the model.

Figure 5

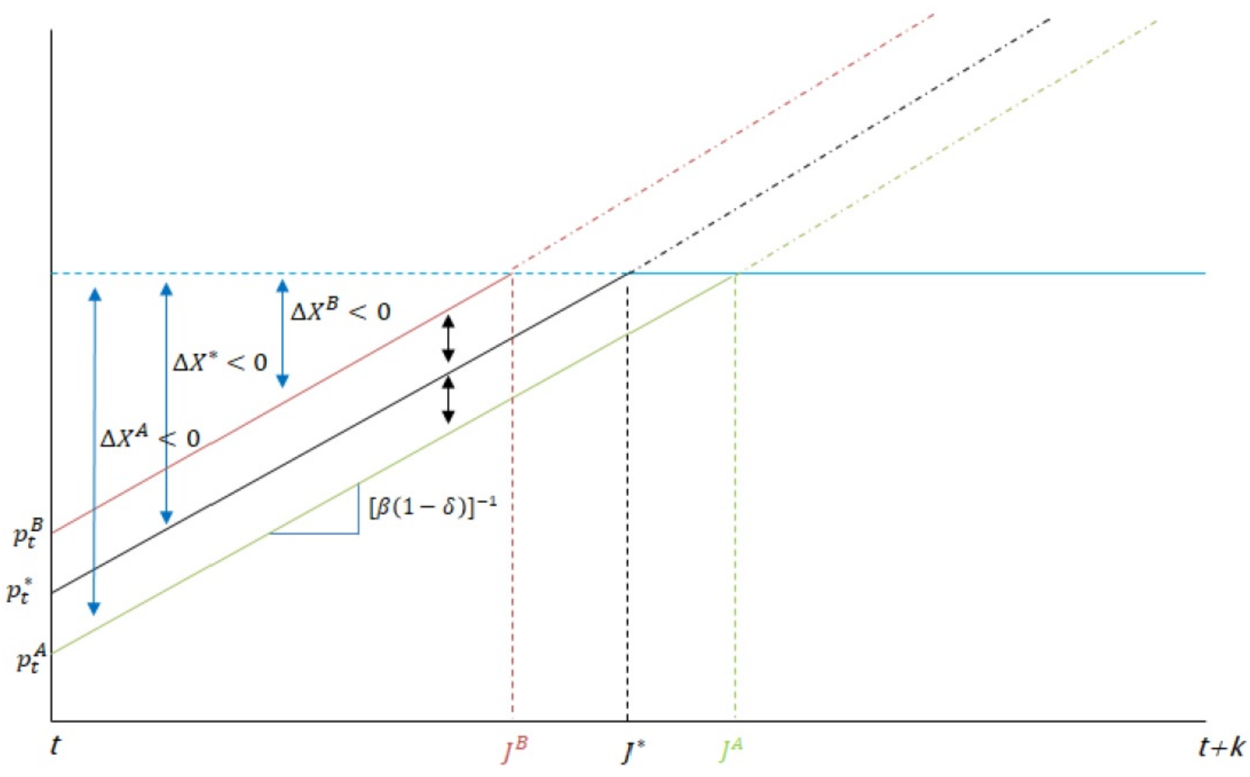

Figure 6

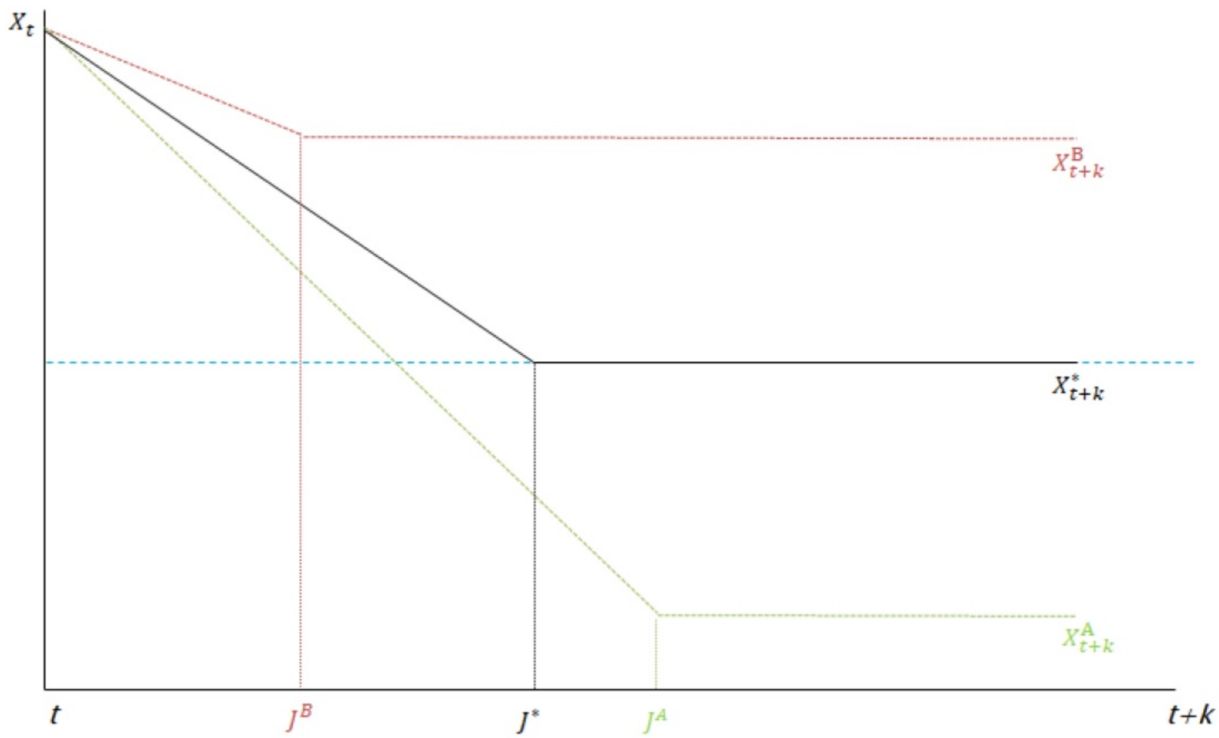

Table 6: Estimation of copper price elasticities: Previous research (percentage change for an increase of 1% in the price).

		Price	
Paper	*Characteristic*	*Short term*	*Long term*
Takeuchi (1974)	Copper demand		[-0.1;-0.3]
Banks (1974)	Copper demand	-0.24	-0.36
Mardones (1984)	Copper demand	[-0.09;-0.47]	[-0.19;-0.92]
Vial (1988)	Copper demand	-0.03	-0.01
Vial (2003)	Copper demand	-0.11	-0.85
Vial (2004)	Copper demand	-0.03	-0.2
Newhouse & Sloan (1966)	Copper supply	0.2	2.47
Banks (1969)	Copper supply	0.2	0.701
Banks (1974)	Copper supply	0.55	0.75
Mardones (1984)	Copper supply	[0.07;0.54]	[0.40;3.30]
Vial (1988)	Copper supply	0.11	0.22
Vial (2003)	Copper supply	0.08	0.25
Vial (2004)	Copper supply	0.05	0.13

Source: López et al. [2009]

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