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### **Optimal Monetary Policy and Incomplete Information: Does the Real Exchange Matter?**\*

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#### Abstract

In a small economy, with complete markets and domestic price stickiness, a monetary policy rule that reacts to domestic inflation implements the efficient allocation, as long as it also reacts to the natural rate of interest. In this case, a policy response to the exchange rate or any other foreign variable is inefficient. We show that, when the central bank is unable to observe the natural rate of interest, a domestic inflation targeting rule that reacts also to the real exchange rate is optimal. This rule is able to fully stabilize domestic inflation and, at the same time, induce efficient movements in relative prices (terms of trade) through nominal devaluations. Indeterminacy can arise, but a stronger policy response to domestic inflation can prevent this from happening.

#### Resumen

En una economía pequeña y abierta, con mercados completos y rigidez en los precios internos, una regla de política monetaria que reacciona a la inflación doméstica implementa la asignación eficiente, siempre que también reaccione a la tasa de interés natural. En este caso, una respuesta de política al tipo de cambio o cualquier otra variable externa es ineficiente. En este trabajo mostramos que, cuando el banco central no es capaz de observar la tasa de interés natural, una regla de metas de inflación doméstica que reacciona también al tipo de cambio real es óptima. Esta regla es capaz de estabilizar completamente la inflación doméstica y, al mismo tiempo, inducir movimientos eficientes en los precios relativos (términos de intercambio) a través de devaluaciones nominales. Esta regla puede generar indeterminación, pero una respuesta de política más agresiva a la inflación doméstica puede evitar que esto suceda.

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#### 1 Introduction

What is the role of foreign variables, and in particular of the exchange rate, in the design of monetary policy? According to Clarida et al. (2001) and Galí and Monacelli (2005), in small open economies with complete markets and domestic price stickiness, the policy objectives are isomorphic to the ones in closed economies. This means that it is optimal to stabilize domestic inflation and the the gap between output and its flexible price level.<sup>1</sup> The exchange rate, or indeed any foreign variable, should not be policy targets. In this setup, stabilizing domestic inflation removes inefficient movements in relative prices, related to sticky prices. Stabilizing the output gap ensures that labour is efficiently allocated.

Galí and Monacelli (2005) showed that, to implement the efficient allocation, the policymaker requires full knowledge of the exogenous shocks that affect the first-best level of the variables. This would imply that TFP shocks are directly observable by the central bank. If this is the case, a simple Taylor-type rule that reacts to the natural rate of interest –which depends on TFP– to domestic inflation and to the output gap implements the efficient allocation. This rule does not require any reaction to the exchange rate or, more generally, to any foreign variable.

In practice, however, the natural rate of interest is not known in real time. This limitation, as stressed by Orphanides and Williams (2002) and Woodford (2003) in the case of closed economies, complicates policymaking in practice and may limit the scope for stabilization policy. This problem is also present in open economy models, in the spirit of Galí and Monacelli (2005) and Corsetti et al. (2010). As a consequence, natural interest rate–based policies cannot be implemented in practice. In this context, implementable rules, which react only to observable variables, could be sub-optimal.

In this paper we assess the role of the real exchange rate in implementable rules. We show that, when the central bank is unable to observe the natural interest rate, reacting to the real exchange rate and to domestic inflation is optimal. We analyti-

<sup>1.</sup> As noted by Catão and Chang (2013), domestic inflation targeting is still nearly optimal in alternative models with realistic calibrations, such as De Paoli (2009) and Faia and Monacelli (2008), even if the conditions stressed by Galí and Monacelli (2005) fail.

cally derive the optimal policy response coefficients, in a simple and implementable rule. These coefficients ensure that domestic inflation is zero and, at the same time, that the relative price of domestic goods moves efficiently. This movement is generated by optimal changes in the nominal exchange rate. Under our optimal rule the devaluation rate increases when the real exchange rate is below its long-run level (or flexible price level) and vice-versa. This mechanism, which in a different context is also present in Uribe (2003), does not require the central bank to observe the long-run level of the real exchange rate. As is the case in Uribe (2003), our rule is prone to indeterminacy if the policy response to the exchange rate is sufficiently large in absolute value. However, indeterminacy can be avoided if the policy response to domestic inflation is increased. In our baseline calibration, we find that such a response is within the range of values we observe in practice.

Using the method of undetermined coefficients, we derive the optimal policy responses in a domestic inflation-based Taylor rule (DITR) augmented with a response to the real exchange rate. As in Galí and Monacelli (2005), we consider productivity shocks which do not induce a policy trade-off. In addition, we consider the implications of our augmented DITR in the face of cost-push shocks. In this case, which induce a trade-off between stabilizing domestic inflation and the output gap, our rule is capable of fully stabilizing domestic inflation. In our baseline calibration this rule has a better performance than a rule which excludes a response to the real exchange rate. Finally, we conclude that our rule can also be expressed in terms of the expected real exchange rate depreciation. In this specification the problem of indeterminacy, related to the size of the policy response to the exchange rate, is absent.

This paper is organized as follows. Section 2 lays out the main features of the Galí and Monacelli (2005) model we use as a benchmark. Besides showing the optimal, but not implementable, rule that emerges in this context, we discuss the implementable –but sub-optimal rules– proposed in that paper. Section 3 presents alternative simple and implementable policy rules. We consider the DITR, proposed by Galí and Monacelli (2005), but extend it to include a response to the real exchange rate. Then, we use the method of undetermined coefficients to analytically derive the optimal policy responses, in the face of productivity shocks. In Section 4, we test

the performance of our proposed rule in the face of cost-push shocks, we also consider the implication of an alternative rule, one that reacts to the expected depreciation rate. Section 5 concludes.

#### 2 Optimal Monetary Policy in a Canonical SOE Model

Galí and Monacelli (2005) characterize the optimal monetary policy in a New Keynesian small open economy model. In this setup, competitive monopolistic firms set domestic prices infrequently. Households decide how much to consume and how many hours to work. It is assumed that financial markets are complete. These assumptions give rise to a highly tractable framework and to simple and intuitive log-linearized equilibrium conditions. The non-policy block of the model is represented by a two-equation dynamical system for domestic inflation,  $\pi_{H,t}$ , and the output gap,  $x_t$ :

$$x_t = x_{t+1} - \frac{1}{\sigma_{\alpha}} \left( i_t - \pi_{H,t+1} - r_t^n \right)$$
(1)

$$\pi_{H,t} = \beta \pi_{H,t+1} + \kappa_{\alpha} x_t \tag{2}$$

The domestic output gap,  $x_t$ , is defined as the deviation of (log) domestic output  $y_t$  from its natural level  $y_t^n$ , where the latter is in turn defined as the equilibrium level of output in the absence of nominal rigidities and conditional on world output,  $y_t^*$ .

$$x_t = y_t - y_t^n \tag{3}$$

$$y_t^n = \Gamma a_t + \alpha \Psi y_t^* \tag{4}$$

where  $\Gamma \equiv \frac{1+\varphi}{\sigma_{\alpha}+\varphi} > 0$ .

Equation (1) is a dynamic IS-type equation derived from households' optimization process. Equation (2) is a New Keynesian Phillips curve for domestic inflation that emerges from the first order conditions of firms. In the IS equation the policy rate is represented by  $i_t$ , whereas the natural rate of interest is represented by  $r_t^n$ . The parameter  $\sigma_{\alpha}$  determines the response of the output gap to changes in the real ex-ante interest rate. This coefficient is a function of the relative risk aversion parameter,  $\sigma$ , and of the proportion of foreign goods in the consumer basket,  $\alpha$ . This proportion is associated to the degree of openness of the economy. More precisely,  $\sigma_{\alpha} = \frac{\sigma}{1+\alpha(\omega-1)}$ .<sup>2</sup> In general, the degree of openness influences the sensitivity of the output gap to changes in the domestic real interest rate. If  $\omega > 1$ , an increase in openness raises the sensitivity of the output gap to changes in the policy rate.

The slope of the New Keynesian Phillips curve is given by  $k_{\alpha} \equiv \lambda(\sigma_{\alpha} + \varphi)$ , where  $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ . As in the closed economy version, the slope depends (inversely) on the degree of price stickiness,  $\theta$ . It also depends on the degree of openness,  $\alpha$ , the coefficient related to the disutility of labor,  $\varphi$  and the discount factor,  $\beta$ . In the case in which  $\omega > 1$ , an increase in the degree of openness,  $\alpha$ , flattens the slope of this New Keynesian Phillips curve.

The natural rate of interest, defined as the equilibrium real interest rate in the absence of nominal rigidities, is given by:

$$r_t^n = \rho - \sigma_\alpha \Gamma(1 - \rho_a) a_t + \alpha \sigma_\alpha (\Theta + \Psi) E_t \{ \Delta y_{t+1}^* \}$$
(5)

where  $\Theta \equiv (\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1) = \omega - 1$  and  $\Psi \equiv -\frac{\Theta\sigma_{\alpha}}{\sigma_{\alpha} + \varphi}$ . The natural rate depends on two exogenous variables: domestic productivity,  $a_t$  and world output,  $y_t^*$ . Both variables can be expressed as AR(1) processes:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{6}$$

$$y_t^* = \rho_y y_{t-1}^* + \varepsilon_t^* \tag{7}$$

Productivity shocks,  $\varepsilon_t^a$  and foreign output shocks,  $\varepsilon_t^*$ , are i.i.d. structural shocks.

<sup>2.</sup> The  $\omega$  coefficient is a function of the elasticity of substitution between domestic and foreign goods,  $\eta$ , and of the elasticity of substitution across varieties of foreign goods,  $\gamma$ . It also depends on the degree of openness and on the relative risk aversion of households:  $\omega = \sigma \gamma + (1 - \alpha)(\sigma \eta - 1)$ .

Now, in this economy the households' consumer basket is composed of domestic and foreign goods. As a result CPI inflation,  $\pi_t$ , and domestic inflation,  $\pi_{H,t}$ , are linked according to:

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{8}$$

where  $s_t \equiv p_{F,t} - p_{H,t}$  denotes the effective terms of trade,  $p_{F,t}$  is the price index for imported goods (in domestic currency), and  $p_{H,t}$  is the price of domestically produced goods.

It is assumed that the law of one price holds, implying that the (log) price of imported goods, in local currency, depend on the nominal exchange rate,  $e_t$  and on foreign prices,  $p_t^*$ :

$$p_{F,t} = e_t + p_t^* \tag{9}$$

The real exchange rate,  $q_t$ , is defined as the relative price of a consumption basket in the rest of the world and in the domestic economy. Given the CPI definition (equation 8), the real exchange rate,  $q_t$ , is proportional to the terms of trade,  $s_t$ :

$$q_t = (1 - \alpha)s_t \tag{10}$$

In addition, the real exchange rate evolves according to the UIP condition:

$$E_t(q_{t+1}) = r_t - r_t^*$$
(11)

where  $r_t = i_t - E_t(\pi_{t+1})$  and  $r_t^* = i_t^* - E_t(\pi_{t+1}^*)$  represent the real *ex-ante* interest rate in the domestic economy and in the rest of the world, respectively.

#### 2.1 Welfare and Optimal Policy: A Special Case

As shown by Galí and Monacelli (2005), in the particular case in which  $\sigma = \eta = \gamma =$ 1, a second-order approximation to the welfare of the representative consumer can be derived analytically. For this parameter configuration the welfare criterion can be expressed as a function of the variance of domestic inflation and of the output gap:

$$W = \frac{1-\alpha}{2} \left[ \frac{\varepsilon}{\lambda} Var(\pi_{H,t}) + (1+\varphi) Var(x_t) \right]$$
(12)

where  $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ . In this case the coefficients  $\Theta$  in (5) become zero. As a result, both the natural rate of interest and the natural output level depend only on productivity shocks:

$$r_t^n = \rho - \sigma \Gamma (1 - \rho_a) a_t \tag{13}$$

$$y_t^n = \Gamma a_t \tag{14}$$

In this case, the flexible price equilibrium allocation characterized by zero domestic inflation is optimal. This means that, in the face a productivity shock, which are the only ones modifying the natural rate of interest,  $r_t^n$ , it is possible to stabilize domestic inflation and the output gap simultaneously. Hence, for this shock, the *divine coincidence* holds. Therefore, the optimal policy is isomorphic to the one in a closed economy setup: the first best is achieved if both, domestic inflation and the output gap are zero.

To implement the optimal allocation, the central bank can follow a Taylor-type rule which reacts to both domestic inflation and the output gap. For this rule to be optimal, however, it should also factor in the natural rate of interest. The optimal rule can be expressed as:

$$i_t = r_t^n + \phi_\pi \pi_{H,t} + \phi_x x_t \tag{15}$$

A response to domestic inflation and/or to the output gap in (15) ensures the first-best allocation is implemented, as long as the determinacy condition is satisfied.<sup>3</sup>

<sup>3.</sup> To ensure determinacy, the policy coefficients should satisfy:  $\kappa_{\alpha} (\phi_{\pi} - 1) + (1 - \beta) \phi_x > 1$ .

Galí and Monacelli (2005) acknowledge that rule (15) may be difficult to implement in practice, as this would require that productivity shocks, affecting  $r_t^n$  and  $x_t$ , are directly observable by the central bank. In this context, Galí and Monacelli (2005) assess the performance of three alternative rules that respond to observable variables. These implementable, but inefficient rules are:

$$i_t = \phi_H \pi_t \tag{16}$$

$$i_t = \phi_H \pi_{H,t} \tag{17}$$

$$e_t = 0 \tag{18}$$

where rule (16) responds only to CPI inflation, denoted CITR. Rule (17) reacts only to domestic inflation and is denoted DITR. Finally, a rule that fully stabilizes the nominal exchange rate, in (16), is denoted PEG. These three rules deviate from the efficient equilibrium. They all generate substantial volatility in domestic inflation, which explains most of the welfare loss that Galí and Monacelli (2005) found in each case. This result is robust to alternative values of the steady-state markup and the labor supply elasticity.

Now, implementable rules in (16) and (17) do not consider additional variables, other than inflation. It has been shown, however, that central banks in open economies react not only to inflation but also to foreign observable variables such as the exchange rate or the foreign interest rate.<sup>4</sup> In the next section we will

<sup>4.</sup> Clarida et al. (1998), using data since the mid-1970s to the mid-1990s, provide evidence that Italy, France, Germany and Japan pursued an implicit form of inflation targeting. In this context, however, the monetary authority in those countries reacted not only to inflation, but also to the exchange rate and/or foreign policy rate. Lubik and Schorfheide (2007), on the other hand, find that the central banks of Australia and New Zealand did not explicitly target exchange rates over the last two decades. The Bank of Canada and the Bank of England, on the other hand, did. This finding is robust over different specifications of the monetary policy reaction function. The relevance of the exchange rate and other foreign variables, in the inflation targeting strategies of small open economies, is also discussed in Clarida et al. (2001), Mohanty and Klau (2004), Aizenman et al. (2011), Svensson (2000), and Taylor (2001), among others.

determine whether simple and implementable policy rules, that also react to the exchange rate, can implement the optimal allocation.

#### 3 Optimal and Implementable Simple Rules

In the case of a tractable model, the implementation of the efficient equilibrium, through a Taylor-type rule like (15), requires the central bank to react to domestic inflation and/or the output gap, as well as to  $r_t^n$ . This is only possible if the monetary authority observes the productivity shocks that hit the economy and determine  $r_t^n$  and  $x_t$ . However, as noted by Clarida et al. (1999) and Senay and Sutherland (2019), central banks have incomplete information due to imperfect observability: when setting interest rates, they may not have all the relevant information available about the state of the economy. It takes time to collect and process certain data and, even if it has access to data in real time, some key variables such as the natural level of output are not directly observable and are likely measured with great error.<sup>5</sup> Hence, the rule in (15) is optimal, but not implementable.

In the light of the empirical evidence on the relevance of the real exchange rate, we propose a Taylor rule in an incomplete information scenario: a scenario that assumes the central bank is unable to observe the natural rate of interest, nor the shocks hitting the economy. We do so in the tractable model of Galí and Monacelli (2005) and consider an augmented DITR rule that reacts only to observable variables, namely domestic inflation and the real exchange rate. This augmented rule takes the form:

$$i_t = \phi_\pi \pi_{H,t} + \phi_q q_t \tag{19}$$

To derive a closed form solution for the optimal coefficients in (19) we proceed in two steps. First, we find analytical expressions for the variances of domestic inflation and the output gap. Those expressions will be functions of the structural parameters of the model, including the monetary policy coefficients. In the second step, we find the values of  $\phi_{\pi}$  and  $\phi_q$  that minimize the welfare function in (2.1).

<sup>5.</sup> This is the case in the special calibration considered by Galí and Monacelli (2005).

As in Christiano (2002), we use the method of undetermined coefficients to solve the model with the augmented DIT rule in (19). With this method we obtain a linear approximation to the solution of a dynamic rational expectations model, which can be used to derive the model implications for second moments. Specifically, since we have a system of dynamic equations, we assume that there exists a solution that depends only on each structural shock. In the face of a productivity shock,  $\varepsilon_t^a$ , the solution to the model is:

$$\begin{bmatrix} x_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} \Psi_{xa} \\ \Psi_{\pi a} \\ \Psi_{za} \end{bmatrix} \varepsilon_t^a$$

where  $\mathbb{X} = \{x_t, \pi_t, i_t\}$  are the variables and  $\mathbb{B} = \{\Psi_x, \Psi_\pi, \Psi_i\}$  represent the set of reduced form coefficients. The above solution can be generalized for any other structural shock,  $\varepsilon_t^s$ , in which case the reduced form coefficients of the system will be given by  $\mathbb{B} = \{\Psi_{xs}, \Psi_{\pi s}, \Psi_{is}\}$ .

#### 3.1 Optimal Policy Coefficients

We solve the model using the method of undetermined coefficients, and obtain a closed form solution for the variances of domestic inflation,  $V(\pi_{H,t})$ , and of the output gap,  $V(x_t)$ . As shown in Table 1, these depend on the structural parameters of the economy as well as on the variance of the productivity shock,  $V(\varepsilon_t^a)$ .

Variable	Closed-Form Solution
$V(x_t)$	$\left(\frac{\Gamma\sigma_{\alpha}(1-\rho_{a})(1-\beta\rho_{a})(\rho_{a}-1-\phi_{q}(1-\alpha))}{(\phi_{\pi}-\rho_{a})\kappa_{\alpha}+\sigma_{\alpha}(1-\beta\rho_{a})((1-\rho_{a})+\phi_{q}(1-\alpha))}\right)^{2}\frac{V(\varepsilon_{t}^{a})}{1-\rho_{a}^{2}}$
$V(\pi_{H,t})$	$\left(\frac{\kappa_{\alpha}\Gamma\sigma_{\alpha}(1-\rho_{a})(\rho_{a}-1-\phi_{q}(1-\alpha))}{(\phi_{\pi}-\rho_{a})\kappa_{\alpha}+\sigma_{\alpha}(1-\beta\rho_{a})((1-\rho_{a})+\phi_{q}(1-\alpha))}\right)^{2}\frac{V(\varepsilon_{t}^{a})}{1-\rho_{a}^{2}}$

Table 1: Analytical Variances under Rule (19)

Based on the above results, we derive the policy coefficients that minimize the welfare criterion in (2.1). More specifically, the variances of the output gap and domestic inflation can be fully stabilized  $V(x_t) = 0$  and  $V(\pi_{H,t}) = 0$ , as long as the policy response to the real exchange rate,  $\phi_q$ , is:

$$\phi_q = -\frac{1-\rho_a}{1-\alpha} \tag{20}$$

In addition to the previous condition, the optimal policy coefficients  $\phi_{\pi}$  and  $\phi_q$  should satisfy the following determinacy condition:<sup>6</sup>

$$\sigma_{\alpha}(1-\beta)\phi_q(1-\alpha) + \kappa_{\alpha}(\phi_{\pi}-1) > 0 \tag{21}$$

The optimal response to the real exchange rate, in (20), is negative and increasing, in absolute value, with the degree of openness. It also increases as the shock becomes less persistent.<sup>7</sup> From (21) it is clear that the policy response to inflation,  $\phi_{\pi}$ , should be greater than one. It will need to increase as  $\phi_q$  becomes larger, in absolute value.

#### 3.2 Optimal Exchange Rate Dynamics

We have shown that the rule in (19) can implement the optimal allocation, as long as conditions (20) and (21) are satisfied. Under this rule the policymaker is not required to observe the natural rate of interest and can react only to observable variables. It is assumed the central bank has information regarding the persistence of the productivity shock,  $\rho_a$ , and hence it can respond in an optimizing manner to

<sup>6.</sup> Two additional determinacy conditions must hold. The first one is:  $\sigma_{\alpha}(1+\beta)(2+\phi_q(1-\alpha))+\kappa_{\alpha}(1+\phi_{\pi}) > 0$ . It is, however, a redundant one. Once (21) is satisfied, the previous condition holds. The second one is  $|a_0| < 1$  where  $a_0 \equiv \frac{\beta\sigma_{\alpha}}{\phi_{\pi}\kappa_{\alpha}+\sigma_{\alpha}+\phi_q(1-\alpha)\sigma_{\alpha}}$ . Again, this condition holds, under the optimal policy response to the real exchange rate, once (21) is satisfied. This requires that  $(1-\beta) + \rho_{\alpha}\beta + \lambda + \frac{\varphi}{\lambda} > \beta$ , which is satisfied under the present calibration. If this inequality is not satisfied, for a different calibration, the condition  $|a_0| < 1$  could be restored if the policy response to domestic inflation,  $\phi_{\pi}$  is increased. This mechanism will also ensure that condition (21) is satisfied.

<sup>7.</sup> We assume the policymaker is unable to observe the structural shocks, but knows  $\alpha$  and  $\rho_a$ . Now, even if  $\rho_a$  is not inferred correctly, there are efficiency gains from responding to the real exchange rate. We discuss this issue in Subsection 3.2.

domestic inflation and the real exchange rate. In addition to be known  $\rho_a$  is assumed to be time invariant. The evidence for emerging and developed economies in García-Cicco et al. (2010) and Aguiar and Gopinath (2007) show that this assumption is valid.

To understand the mechanism through which our rule implements the optimal allocation, we compare the performance of two alternative rules. The first one is a suboptimal DITR rule, in (15), where  $\phi_q=0$ . The second one is the optimal rule, in (19), in which  $\phi_q = -\frac{1-\rho_a}{1-\alpha}$ . In both rules we set  $\phi_{\pi}$  to 1.5, which ensures determinacy.<sup>8</sup>

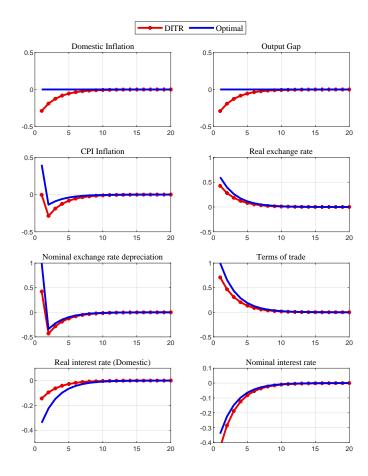
In the face of a productivity shock, the efficient allocation is characterized by zero domestic inflation, zero output gap and a decline in the relative price of domestic goods. This decline is equivalent to a real exchange rate depreciation. Now, the DITR is unable to implement the efficient allocation for any value of  $\phi_{\pi}$ . As is clear, from the analytical variances in Table 1, once  $\phi_q=0$  the DITR can only approximate the efficient allocation if  $\phi_{\pi}$  tends to infinity, which is unfeasible. In short, a fundamental flaw of the DITR is that an equilibrium in which domestic inflation is zero and the policy rate declines cannot be implemented. If domestic inflation is zero, the policy rate is also zero, but in this case, inflation itself cannot be zero. As a result, when  $\phi_{\pi}=1.5$ , the decline in the real rate is not able to induce the optimal allocation. Under this rule, domestic inflation, the output gap, the contemporaneous devaluation and the real exchange rate lie below the efficient equilibrium (see Figure 1).

Now, the optimal and implementable rule, where  $\phi_{\pi}=1.5$  and  $\phi_{=}-\frac{1-\rho_{a}}{1-\alpha}$ , is able to stabilize two of the relevant dimensions that characterize the efficient allocation. On the one hand, it ensures that the relative price of domestic goods declines even if domestic inflation is zero. This movement in relative prices is induced by a larger contemporaneous depreciation, which ensures that both the real exchange rate and the terms of trade move optimally. This depreciation is *validated* in equilibrium by the optimal policy response to the real exchange rate,  $\phi_{q}$ . On the other hand,

<sup>8.</sup> The rest of coefficients in the model are calibrated according to Galí and Monacelli (2005). Under this calibration the optimality and stability conditions (20) and (21) hold. See Appendix A for details on calibrated parameters.

this rule ensures that domestic inflation and the output gap are stabilized, since the real rate declines even as domestic inflation tends to zero. Again, the optimal policy response to the real exchange rate,  $\phi_q$ , induces a policy rate reduction in the efficient equilibrium. In short, the optimal rule can fully stabilize domestic prices and at the same time generate an efficient movement in relative prices thanks to the nominal depreciation. This last element, which is key for implementing the optimal allocation, is absent from the DITR rule.

Figure 1: Impulse responses to a productivity shock under alternative policy rules



When compared to the optimal, but not implementable, rule in Galí and Monacelli (2005), equation (15), our rule also needs a policy reaction to domestic inflation as a mechanism to avoid indeterminacy. In the case of Galí and Monacelli (2005) the policy response to  $r_t^n$  ensures that in equilibrium the policy rate declines, inducing an output expansion and an efficient movement in relative prices. In our rule the policy reaction to  $q_t$  has the same effect: it induces a decline in the policy rate that generates an efficient expansion in output and a decline in the relative price of domestic goods. Hence, the response to  $q_t$  is able to mimic the policy response to the unobservable natural rate of interest. The advantage of our rule is that this is possible without the need to observe  $r_t^n$  nor the productivity shocks.

Under our rule, the optimal  $\phi_q$  coefficient ensures that devaluations take place when the real exchange rate is below is flexible price level. This mechanism, which is also present in the PPP rules considered in Uribe (2003), does not require the central bank to observe the real exchange rate equilibrium level.

Our rule requires the central bank to know the persistence of the unobserved shock,  $\rho_a$ . This is assumed to be time invariant in the model, and can eventually be inferred by the central bank. There is the risk, however, that this inference process is imperfect. To test the robustness of our rule to misspecified values of  $\rho_a$ , we perform two exercises. First we assume that the central bank underestimates the value of the persistence assuming  $\rho_a = 0.48$  instead of  $\rho_a = 0.6$ . In the second exercise, we assume the opposite: the central bank overestimates the true coefficient so  $\rho_a = 0.9$ . As shown in Figure 2, the misspecified scenarios generate a larger real interest rate decline, compared to the DITR. As a result, the nominal and the real exchange rate depreciate by more. Domestic inflation and the output gap are not fully stabilized in either of these two alternative scenarios. However, the volatility of both variables is below the one obtained under the DITR, as shown in Table 2. Therefore, responding to the real exchange rate, even if the optimal response cannot be implemented, is better than not reacting at all.

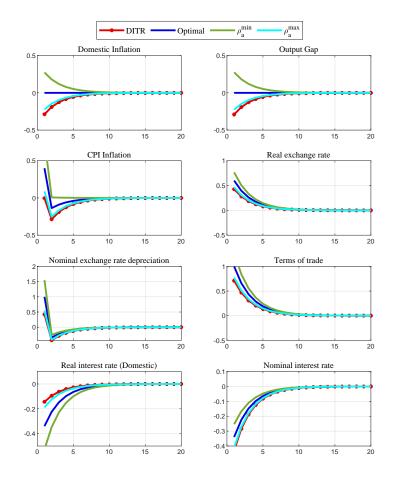


Figure 2: Impulse responses to a productivity shock under alternative policy rules

Variances	Optimal	$\phi_q = 0$	$\rho_a^{min}=0.48$	$\rho_a^{max}=0.9$
Domestic Inflation	0.00	0.15	0.13	0.09
Output Gap	0.00	0.15	0.14	0.09
CPI Inflation	0.19	0.15	0.61	0.12
Expected Depreciation RER	0.07	0.04	0.12	0.04
Depreciation RER	0.43	0.22	0.71	0.26
Loss Function	0.00	3.25	2.94	1.93

 Table 2: Variances and Loss Function (Productivity Shock)

#### 3.3 Indeterminacy trap and policy escape

Our rule induces efficient movements in relative prices even if domestic inflation is zero in equilibrium. This is possible through nominal contemporaneous depreciation which affects the relative price of foreign goods. This mechanism is triggered by the explicit, and optimal, policy response to the real exchange rate. As a result, the nominal exchange rate depreciates when the actual real exchange rate is below its flexible price level and vice-versa.

The previous mechanism is also present in the PPP rules considered in Uribe (2003). In that contribution it is shown that PPP rules might facilitate the economy's adjustment to fundamental shocks. However, Uribe (2003) shows that these rules can also generate indeterminacy of the rational expectations equilibrium and endogenous fluctuations due to arbitrary revisions in expectations. Thus, PPP rules can give rise to situations in which exchange rate instability, both nominal and real, occurs simply because the public expects it. Our rule is also prone to indeterminacy. In contrast with Uribe (2003)'s PPP rule, ours reacts not only to the real exchange rate, but also to domestic inflation. This makes it possible to escape the indeterminacy trap through explicit policy actions and still preserve the advantages of PPP rules (the exchange rate adjustment).

To understand the mechanisms that make our rule prone to indeterminacy, as well as the potential remedies, we evaluate the determinacy condition (21) under the optimal policy response to the real exchange rate,  $\phi_q = -\frac{1-\rho_a}{1-\alpha}$ . In this case determinacy requires that:

$$\frac{\left(1-\beta\theta\right)\left(1-\theta\right)\left(1+\frac{\varphi}{\sigma_{\alpha}}\right)\left(\phi_{\pi}-1\right)}{\theta(1-\beta)} > 1-\alpha$$
(22)

where the right-hand side of (22) is the absolute value of the optimal response to the real exchange rate,  $\phi_q$ . In practice, condition (22) sets an upper bound for the response (in absolute value) to the real exchange rate. We will show that this condition is not binding if the policy response to inflation,  $\phi_{\pi}$ , is adjusted accordingly.

As in Uribe (2003), if the response to exchange rate increases, the equilibrium may become indeterminate. However, in our case if the increase in the optimal response to the real exchange rate is due to a decline in  $\alpha$ , condition (22) does not change. In our case, the source of the increase matters. The equilibrium may become indeterminate if the persistence coefficient,  $\rho_a$ , declines.

Price stickiness affects the probability of indeterminacy. As prices become more flexible, an increase in  $(1-\theta)$ , the probability of indeterminacy declines. On the contrary, if prices become more rigid (a higher  $\theta$ ), indeterminacy is more likely. An increase of the relative risk aversion coefficient,  $\sigma_{\alpha}$ , as well as an increase in the Frisch elasticity  $(\frac{1}{\varphi})$ , increase the risk of indeterminacy.

As in Uribe (2003), reacting to the real exchange rate makes our rule prone to indeterminacy. In contrast, our rule has a mechanism that can preserve the advantages of reacting to the real exchange rate and induce determinacy: an increase in the policy response to inflation. For instance, in our baseline calibration any value of  $\rho_a$  below 0.48 generates indeterminacy of the rational expectations equilibrium. Hence, if we push the response to the real exchange rate to the upper limit, by setting  $\rho_a$  to 0, the system will be undetermined. In this extreme case, however, we can induce determinacy by increasing the response to inflation,  $\phi_{\pi}$ , from 1.5 to 2.9. This latter value is within the range of estimated policy responses to inflation.<sup>9</sup> It ensures determinacy, of the rational expectation equilibrium, for any possible value of  $\rho_a$ .

<sup>9.</sup> See Clarida et al. (1998), Lubik and Schorfheide (2007), and Aizenman et al. (2011), among others.

#### 4 Robustness Exercises

In this section we assess the robustness of our findings in two dimensions. First, we analyze the implications of considering supply shocks, which are innovations to the New Keynesian Phillips curve in (2). Second, we reformulate our policy reaction function as a forward-looking rule in which the central bank reacts to expected real depreciation and contemporaneous domestic inflation.

#### 4.1 Cost-Push Shocks

In the face of a cost-push shock it is not possible to stabilize domestic inflation and the output gap at the same time. In short, this type of shocks generate a policy trade-off. Hence, setting the variance of domestic inflation and the output gap to zero is not feasible. If domestic inflation is fully stabilized, this will be associated with a positive output gap variance, and vice-versa. In the Galí and Monacelli (2005) model, this type of shock is not considered. However, this shock can be introduced in the model by appending a stochastic element to the NKPC Phillips curve in (2). This modified equation takes the form:

$$\pi_{H,t} = \beta \pi_{H,t+1} + \kappa_{\alpha} x_t + v_t \tag{23}$$

where  $v_t$  is an AR(1) process:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \tag{24}$$

As before, we follow Christiano (2002) and use the method of undetermined coefficients to derive closed form solutions for the variances of domestic inflation and the output gap. Results are presented in Table 3.

Variable	Closed-Form Solution
$V(x_t)$	$\left(\frac{\phi_{\pi}-\rho_{v}}{(1-\beta\rho_{v})\sigma_{\alpha}(1-\rho_{v})+(\phi_{\pi}-\rho_{v})\kappa_{\alpha}+\sigma_{\alpha}(1-\beta\rho_{v})\phi_{q}(1-\alpha)}\right)^{2}\frac{V(\varepsilon_{t}^{v})}{(1-\rho_{v}^{2})}$
$V(\pi_{H,t})$	$\left(\frac{\sigma_{\alpha}(\rho_v - 1 - \phi_q(1 - \alpha))}{(1 - \beta \rho_v)\sigma_{\alpha}(1 - \rho_v) + (\phi_{\pi} - \rho_v)\kappa_{\alpha} + \sigma_{\alpha}(1 - \beta \rho_v)\phi_q(1 - \alpha)}\right)^2 \frac{V(\varepsilon_t^v)}{(1 - \rho_v^2)}$

Table 3: Analytical Variances under Rule (19) and Cost-Push Shock

From the closed-form solutions it is possible to derive the policy response to the real exchange rate that fully stabilizes domestic inflation, making  $V(\pi_{H,t})=0$ . This response is given by:

$$\phi_q = -\frac{1-\rho_v}{1-\alpha} \tag{25}$$

As expected, if the previous policy response fully stabilizes domestic inflation, it will not be able to stabilize the output gap. From Table 3 we conclude that, when  $\phi_q = -\frac{1-\rho_v}{1-\alpha}$ , the variance of the output gap is inversely related to the slope of the New Keynesian Phillips curve,  $\kappa_{\alpha}$ , becoming independent from the rest of the parameters in the model, including the policy coefficients. In this case the variance of the output gap is given by:

$$V(\pi_{H,t}) = \left(\frac{1}{\kappa_{\alpha}}\right)^2 \frac{V(\varepsilon_t^v)}{(1-\rho_v^2)}$$
(26)

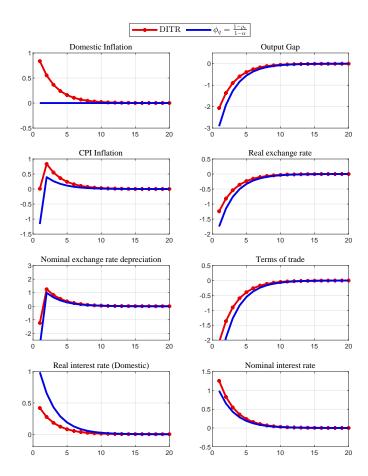
As in the case of productivity shocks, fully stabilizing domestic inflation not only requires that  $\phi_q = -\frac{1-\rho_v}{1-\alpha}$ , but also that the following determinacy condition is satisfied:

$$\frac{\left(1-\beta\theta\right)\left(1-\theta\right)\left(1+\frac{\varphi}{\sigma_{\alpha}}\right)\left(\phi_{\pi}-1\right)}{\theta(1-\beta)} > 1-\rho_{v}$$
(27)

As before, the probability of indeterminacy increases when the shock is more persistent and when prices are stickier. An increase of the relative risk aversion coefficient,  $\sigma_{\alpha}$ , as well as an increase in the Frisch elasticity  $(\frac{1}{\varphi})$ , increase the risk of indeterminacy. In all these cases indeterminacy can be avoided by a more aggressive policy response to inflation,  $\phi_{\pi}$ .

Given the presence of a policy trade-off, an equilibrium with zero domestic inflation and positive output gap variance is not necessarily an efficient one. To determine the advantages, and potential costs, of fully stabilizing domestic inflation, we assess the performance of a DITR where  $\phi_q=0$  and a rule in which  $\phi_q = -\frac{1-\rho_v}{1-\alpha}$ . Figure 3 shows the response of the economy under these two alternative rules.

Figure 3: Impulse responses to a cost-push shock under alternative policy rules



In the face of a cost-push shock, stabilizing domestic inflation requires a contraction in the output gap. This generates a decline in marginal costs that reduces, and eventually stabilizes, domestic inflation. As shown in Figure 3, such a policy rule generates a larger increase in the real rate than the DITR. As a result, fully stabilizing domestic inflation generates a larger contraction in output and consumption, as well a larger decline in the relative price of foreign goods (i.e. a greater real exchange rate appreciation).

To determine the performance of the two alternative rules, we compute the second moments and the welfare loss function in each case. As shown in Table 4, the gains from fully stabilizing domestic inflation more than compensate the costs related to increasing output volatility. More specifically, the welfare loss decline from 1.76, under the DITR, to 0.9 under an extended rule in which  $\phi_q = -\frac{1-\rho_v}{1-\alpha}$ .

Variances	$\phi_q = \frac{-1 - \rho_v}{1 - \alpha}$	$\phi_q = 0$
Domestic Inflation	0.00	0.06
Output Gap	0.75	0.38
CPI Inflation	0.08	0.06
Expected Depreciation RER	0.03	0.02
Depreciation RER	0.18	0.09
Loss Function	0.90	1.76

Table 4: Variances and Loss Function (Cost-Push Shock)

As before, if the persistence,  $\rho_v$ , is between 0 and 0.48, the system will be undetermined. Since the determinacy condition is identical to the one under the productivity shocks, we can induce determinacy by increasing the response to inflation,  $\phi_{\pi}$ , from 1.5 to 2.9.

#### 4.2 Forward-Looking Taylor Rule

Under the optimal rule, movements in the contemporaneous real exchange rates induce efficient adjustment in relative prices. In our model these movements, in turn, are linked to the expected real depreciation. This relationship between the contemporaneous real exchange rate and the expected depreciation rate is also present in Uribe (2003) as part of the mechanism inducing changes in relative prices. Given this direct link, there should be an equivalence between a rule that reacts to the contemporaneous exchange rate, like (19), and one that reacts to the expected real depreciation. To test this conjecture, we reformulate the policy reaction function as follows:

$$i_t = \phi_\pi \pi_{H,t} + \phi_q E_t \left( \Delta q_{t+1} \right) \tag{28}$$

The previous rule assumes that the central bank has model consistent expectations about  $E_t(\Delta q_{t+1})$ . It is under this assumption that we derive closed form solutions for the variances of domestic inflation and the output gap,  $V(\pi_{H,t})$  and  $V(x_t)$ , under (28). As before, we follow Christiano (2002) and use the method of undetermined coefficients to do so. The results are presented in Table 5 and Table 6 below.

Variable	Closed-Form Solution
$x_t$	$\left(\frac{\Gamma\sigma_{\alpha}(1-\rho_{a})(1-\beta\rho_{a})(\phi_{q}(1-\alpha)-1)}{(\phi_{\pi}-\rho_{a})\kappa_{\alpha}+\sigma_{\alpha}(1-\rho_{a})(1-\beta\rho_{a})(1-\phi_{q}(1-\alpha))}\right)^{2}\frac{V(\varepsilon_{t}^{a})}{1-\rho_{a}^{2}}$
$\pi_{H,t}$	$\left(\frac{\kappa_{\alpha}\Gamma\sigma_{\alpha}(1-\rho_{a})(\phi_{q}(1-\alpha)-1)}{(\phi_{\pi}-\rho_{a})\kappa_{\alpha}+\sigma_{\alpha}(1-\rho_{a})(1-\beta\rho_{a})(1-\phi_{q}(1-\alpha))}\right)^{2}\frac{V(\varepsilon_{t}^{a})}{1-\rho_{a}^{2}}$

**Table 5:** Analytical Variances under Rule (28) and Productivity Shock

Variable	Closed-Form Solution
$V(x_t)$	$\left(\frac{\phi_{\pi}-\rho_{v}}{\sigma_{\alpha}(1-\beta\rho_{v})\phi_{q}(1-\alpha)(1-\rho_{v})-\sigma_{\alpha}(1-\beta\rho_{v})(1-\rho_{v})-(\phi_{\pi}-\rho_{v})\kappa_{\alpha}}\right)^{2}\frac{V(\varepsilon_{t}^{v})}{(1-\rho_{v}^{2})}$
$V(\pi_{H,t})$	$\left(\frac{\sigma_{\alpha}(1-\phi_q(1-\alpha))}{\sigma_{\alpha}(1-\beta\rho_v)\phi_q(1-\alpha)(1-\rho_v)-\sigma_{\alpha}(1-\beta\rho_v)(1-\rho_v)-(\phi_{\pi}-\rho_v)\kappa_{\alpha}}\right)^2 \frac{V(\varepsilon_t^v)}{(1-\rho_v^2)}$

Table 6: Analytical Variances under Rule (28) and Cost-Push Shock

As shown in Table 5, in the face of productivity shocks, it is possible to simultaneously stabilize domestic inflation and the output gap by setting  $\phi_q$  to:

$$\phi_q = \frac{1}{1 - \alpha} \tag{29}$$

In contrast to the contemporaneous rule, the optimal policy response to the real exchange rate in (29) is independent from the persistence of the shock, as it depends only on the degree of openness,  $\alpha$ . As the economy becomes more open, the optimal policy response to the real exchange rate increases.

As shown in Table 6, in the face of cost-push shocks it is not possible to simultaneously stabilize domestic inflation and the output gap. However, it is possible to fully stabilize domestic inflation. In this case, the response to the real exchange rate is identical to the one in (29), and the variance of the output gap is given by:

$$V(\pi_{H,t}) = \left(\frac{1}{\kappa_{\alpha}}\right)^2 \frac{V(\varepsilon_t^v)}{(1-\rho_v^2)}$$
(30)

As with the contemporaneous rule, if domestic inflation is fully stabilized in the face of cost-push shocks, the variance of the output gap will depend on the slope of the New Keynesian Phillips curve,  $\kappa_{\alpha}$ , as shown in (30).

To determine whether the response in (29) is feasible, we need to check whether the determinacy conditions hold. In the case of the forward-looking rule in (28) the following three determinacy conditions should be satisfied:

$$\kappa_{\alpha}(\phi_{\pi} - 1) > 0 \tag{31}$$

$$2\sigma_{\alpha}(1+\beta)(1-\phi_q(1-\alpha))+\kappa_{\alpha}(1+\phi_{\pi})>0$$
(32)

$$1 > |a_0| \tag{33}$$

where

$$a_0 \equiv \frac{\beta \sigma_\alpha (1 - \phi_q (1 - \alpha))}{\phi_\pi \kappa_\alpha + \sigma_\alpha (1 - \phi_q (1 - \alpha) \sigma_\alpha)}$$

If the response to the real exchange rate is  $\phi_q = \frac{1}{1-\alpha}$ , conditions (31) to (33) imply that the system is determined if  $\phi_{\pi} > 1$ . Hence, it is possible to fully stabilize inflation (and the output gap in the case of productivity shocks) with a forward-looking rule like (28), irrespective of the persistence and source of the shock, as long as the *Taylor principle* holds.

Now, the system may become undetermined if  $\phi_{\pi} > 1$  and  $\phi_q$  is sufficiently large so condition (31) or (32) is not satisfied. In this case the  $\phi_q$  is set to a different value than the one implied by  $\phi_q = \frac{1}{1-\alpha}$ . For our calibration, the maximum value that this coefficient can attain is  $\phi_q=2$ , whereas the value that fully stabilizes inflation, which is coherent with (29), is given by  $\phi_q=1.7$ . In Table 7 we present the second moments under three alternative rules, one in which  $\phi_q = \frac{1}{1-\alpha}=1.7$ , another in which  $\phi_q = 0$ and finally one that contains the maximum possible response to the real exchange rate,  $\phi_q=2$ . As we already proved, in the face of productivity shocks it is possible to stabilize both domestic inflation and the output gap if  $\phi_q = \frac{1}{1-\alpha}$ . Here, it is also possible to stabilize domestic inflation in the face of cost-push shocks. This will generate costs in terms of output gap volatility.

If a DITR with zero response to the real exchange rate is implemented, the volatility of the output gap will decline in the face of cost-push shocks. In the case of productivity shocks, as we already now, the DITR rule is suboptimal: it increases the volatility of both, domestic inflation and the output gap. As shown in Table 8, in terms of welfare the performance of the DITR rule is worst for productivity shocks, cost-push shocks and for both of them. Furthermore, if the response to the real exchange rate is pushed to the limit,  $\phi_q=2$ , the performance of this rule is better than DITR. In this context, we conclude that even if the policy response to

the real exchange rate is not intended to stabilize domestic inflation, it is better to incorporate a response which is different from zero.

Variable	$\phi_q = \frac{1}{1-\alpha}$	$\phi_q = 0$	Max. $\phi_q = 2$	
Productivity Shock				
Domestic Inflation	0.00	0.15	0.01	
Output Gap	0.00	0.15	0.01	
CPI Inflation	0.19	0.15	0.29	
Expected Depreciation RER	0.07	0.04	0.09	
Depreciation RER	0.43	0.22	0.51	
NKPC Shock				
Domestic Inflation	0.00	0.06	0.01	
Output Gap	0.75	0.38	0.89	
CPI Inflation	0.08	0.06	0.12	
Expected Depreciation RER	0.03	0.02	0.04	
Depreciation RER	0.18	0.09	0.22	
Both Shocks				
Domestic Inflation	0.00	0.21	0.02	
Output Gap	0.75	0.53	0.91	
CPI Inflation	0.27	0.21	0.41	
Expected Depreciation RER	0.11	0.05	0.12	
Depreciation RER	0.62	0.31	0.73	

 Table 7: Model Variances under Rule (28)

**Table 8:** Loss Function under Rule (28)

Shock	$\phi_q = \tfrac{1}{1-\alpha}$	$\phi_q = 0$	Max. $\phi_q = 2$
Productivity Shock	0.00	3.25	0.31
NKPC Shock	0.90	1.76	1.19
Both Shocks	0.90	5.00	1.50

#### 5 Conclusion

In this paper we derive an optimal and implementable policy rule in the context of a small open economy with complete markets and domestic price stickiness. We show that, when the central bank is unable to observe the natural interest rate, reacting to domestic inflation and to the exchange rate (or terms of trade) could fully stabilize domestic inflation and the output gap in the face of productivity shocks.

Using the method of undetermined coefficients we find a closed form solution for such an optimal and implementable policy rule. The optimal response to domestic inflation should satisfy the Taylor principle. The optimal response to the real exchange rate is positively related to the degree of openness in the economy and inversely related to the persistence of the productivity shock. This response ensures that the policy interest rate moves to induce the optimal path for domestic prices relative to foreign ones. Given that –in the optimal equilibrium– domestic prices are fixed, this rule ensures that the nominal exchange rate moves so as to induce optimal relative price movements.

In our proposed rule the policy reaction to the real exchange rate induces a decline in the policy rate that generates an efficient expansion in output and a decline in the relative price of domestic goods. Hence, this response is able to mimic the policy response to the unobservable natural rate of interest. The advantage of our proposed rule is that this is possible without the need to observe neither the natural rate nor the productivity shock.

Our proposed rule induces efficient movements in relative prices even if domestic inflation is zero in equilibrium. This is possible through nominal contemporaneous depreciation which affects the relative price of foreign goods. This mechanism is triggered by the explicit response to the real exchange rate. This reaction, however, may generate indeterminacy of the rational expectations equilibrium, as in Uribe (2003). This situation, however, can be prevented if the policy response to inflation is increased. Hence, our proposed rule has the advantage of inducing efficient relative price movements and can eliminate the risk of indeterminacy, which is present in standard PPP rules.

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## A Appendix: Calibration

### Model Parameters

Parameter	Description	Value
σ	Household's relative aversion coefficient	1.0
$\eta$	Degree of substitution between domestic and foreign goods	1.0
$\gamma$	Degree of substitutability between goods produced in the rest of the world	1.0
$\epsilon$	Degree of substitutability between different varieties	6.0
$\varphi$	Inverse of Frisch elasticity	3.0
$\theta$	Index of price stickiness: probability of non resetting prices	0.75
$\beta$	Discount factor	0.99
$\alpha$	Degree of openness	0.4
$\phi_{\pi}$	Policy response to inflation	1.5
$\phi_q$	Policy response to the real exchange rate	$\frac{-1-\rho_a}{1-\alpha}$ or $\frac{1}{1-\alpha}$
$ ho_a, ho_v, ho_z$	Shock persistence for $a_t$ , $v_t$ and $z_t$	0.66

#### Table 9: Deep parameter values

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