DOCUMENTOS DE TRABAJO

Contracts, Firm Dynamics, and Aggregate Productivity

Bernabe Lopez-Martin
David Perez-Reyna

N° 910 Abril 2021

banco central Chile

Documentos de Trabajo del Banco Central de Chile
Working Papers of the Central Bank of Chile
Agustinas 1180, Santiago, Chile
Teléfono: (56-2) 3882475; Fax: (56-2) 3882231

Documento de Trabajo
N° 910

Working Paper
N° 910

Contracts, Firm Dynamics, and Aggregate Productivity*

Bernabe Lopez-Martin
Central Bank of Chile

David Perez-Reyna
Universidad de Los Andes

Abstract

We construct a firm-dynamics framework to evaluate the impact of the enforcement of contracts between final goods producers and intermediate goods suppliers on firm life-cycle growth, technology accumulation, and aggregate productivity. We show that contractual incompleteness implies a wedge on profits, which disincentives technology accumulation and is potentially correlated with technology, in addition to wedges on production decisions. We find that our model accounts for differences in output per worker of up to 33 percent across economies. The impact on firm life-cycle growth, the age and size distribution of firms is quantitatively significant.

Resumen

Construimos un modelo de dinámica de firmas para evaluar el impacto del cumplimiento de contractos entre productores de bienes finales y proveedores de bienes intermedios sobre el crecimiento de las firmas en su ciclo de vida, la acumulación de tecnología, y la productividad agregada. Mostramos que la incompletitud de contratos implica una distorsión en los beneficios de las firmas, lo cual desincentiva la acumulación de tecnología, y es potencialmente correlacionada con la tecnología además de generar distorsiones en las decisiones de producción. Encontramos que nuestro modelo explica diferencias de producto por trabajador de hasta 33 por ciento entre distintas economías. El impacto en el crecimiento de ciclo de vida de las firmas, la distribución de firmas según tamaño y edad es cuantitativamente significativo.

* We would like to thank, for their valuable comments and discussions, Manuel Amador, Pol Antràs, Pedro Bento, Angel Calderón, Marcela Eslava, Timothy Kehoe, Matthias Kehrig, David Lagakos, Diego Restuccia, Esteban Rossi-Hansberg, Immo Schott, Gabriel Ulyssea, Gustavo Ventura, and Fabrizio Zilibotti. We are also grateful to seminar participants at the Inter-American Development Bank, U. Jorge Tadeo Lozano, Banco de la República de Colombia, U. de los Andes, California State University - East Bay, U. de Montréal, PUC Chile, Universidad del Rosario, and conference participants at the SAET Conference on Current Trends in Economics (Rio de Janeiro,2016), the Public Economic Theory Conference (Rio de Janeiro, 2016), the Annual Conference in Economics and Public Policy at U. Iberoamericana (Mexico City, 2017), the Annual Colombian Economics Conference (Bogota, 2018), the Global Young Scholars Conference (Shanghai, 2019), the Society for Computational Economics Conference (Ottawa, 2019) and the Virtual MACS Junior Conference (2020). We would also like to thank Diego Parra and Sergio Flórez for their outstanding research assistance. The views and conclusions presented here are exclusively those of the authors and do not necessarily reflect the position of the Central Bank of Chile or its Board members. Emails: bernabe.lopezmartin@gmail.com, d.perez-reyna@uniandes.edu.co

1 Introduction

A central area of research in macroeconomics and development aims at identifying sources of distortions that account for significant differences in total factor productivity and output per capita across countries. A recent literature has focused on the analysis of these distortions at the firm level and their aggregate consequences.[1] It is understood that idiosyncratic distortions not only affect the allocation of inputs of production across firms, but also the incentives to invest in technology and productivity within the firm. Both channels have, at least in theory, a significant impact on aggregate productivity. Identifying the sources of these distortions is of paramount importance to assist the design of economic policies aiming at promoting economic development. In turn, the development of quantitative frameworks provides an understanding of the mechanisms and the potential impact of different distortions faced by firms on aggregate outcomes.

In line with this literature, we construct a dynamic framework of heterogeneous firms to evaluate the impact of contract enforcement on firm life-cycle growth and aggregate productivity. We build upon the model of Acemoglu et al. (2007), who provide a tractable structure where firms that produce final goods (henceforth, firms) need to procure intermediate goods from suppliers. The first building block of this model is the representation of technology as the range of intermediate inputs used by firms. Recent work has documented the relationship between the size of firms and the number of suppliers (e.g., Bernard et al., 2019a; Bernard et al., 2019b). The second building block is the well established approach to incomplete-contracting models of the firm originated by Grossman and Hart (1986) and Hart and Moore (1990). The producer of final goods decides the range of intermediate goods that it will employ for production. This range represents the technology of the firm: a more advanced technology is more productive, but entails higher costs in terms of direct pecuniary costs as well as those that emerge from contracting with more suppliers. Suppliers undertake relationship-specific activities, some of which are contractible while the rest are nonverifiable and noncontractible.

In our model the quality of the contracting institutions in an economy is represented by the range of contractible activities. Producers of final goods decide the investment levels in contractible activities undertaken by the supplier of each intermediate good. However, suppliers choose investment in noncontractible activities, a decision that anticipates the results of a bargaining game. This results in an allocation of resources that is not efficient: suppliers tend to underinvest in noncontractible activities given that they are not the full residual claimants of the output gains obtained from their investments. Thus, contractual incompleteness has a negative impact on technology adoption and can potentially generate sizable productivity differences across countries.

We expand the analysis of this friction by analyzing its impact in a framework of firm dynamics (Hopenhayn, 1992; Hopenhayn and Rogerson, 1993). This

[1] See Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Hsieh and Klenow (2009). Below, we provide an overview of the literature.

approach allows us to make a contribution that we outline in the following manner. First, we show that the friction under study generates different *wedges* (or distortions equivalent to taxes) on production and profits. We show that the wedge on profits is dependent on the technology of the firm. Second, we show how these wedges affect not only the size of the firm but also the dynamic incentives to invest in technology and productivity within the firm, which will determine the life-cycle growth profile of firms and aggregate productivity. Additionally, we establish its impact on the age and size distribution of firms, which is consistent with a series of studies that have documented the smaller size of firms in developing economies (e.g., Tybout, 2000; Poschke, 2018; Garcia-Santana and Ramos, 2015). Third, our analysis allows us to connect our quantitative results with the literature that studies alternative frictions in similar theoretical frameworks. For example, an extensive literature has studied the role of financial frictions, by examining alternative specifications (incomplete enforcement, collateral constraints, etc.), parameterizations, and margins through which they affect aggregate productivity.[2] A similar comparison can be made with the literature that studies firm registration costs or labor market regulation. To the best of our knowledge, we are the first to explore the role of firm-supplier contract enforcement in a quantitative framework of firm dynamics.

We provide empirical support for our theoretical analysis. At the country level we estimate a robust and economically significant relationship between judicial quality and output per capita and TFP. The econometric results are strengthened with the use of well established instrumental variables. Then, we exploit cross-country firm-level data and document strong correlations of the quality of our institutions of interest with firm size, and revenue per worker. Furthermore, we find support for the dynamic mechanism of our model by documenting the effect of judicial quality on revenue growth, investment in research and development, and growth of revenue per worker.

We use our theoretical model to highlight the mechanisms that make contract enforcement relevant to account for differences in output per worker. For our quantitative analysis we consider the U.S. economy as a benchmark and calibrate our model under the assumption of contract completeness. Some of the parameters are standard and obtained from the literature of firm dynamics, while others are calibrated to replicate key statistics of the U.S. economy, such as firm exit rates, firm life-cycle growth, and the distribution of employment by age of the firm. We then document how the economy performs, in general equilibrium, as the range of contractible activities is reduced. This affects the investment in technology at the firm level, the age and size distribution of firms, and aggregate productivity. The model is able to account for differences in output per worker of up to 33 percent, which is comparable to losses generated by financial frictions in similar quantitative frameworks. Furthermore, we observe considerable differences in firm growth when comparing economies with and without contract incompleteness: average firm size for 26 to 30 year old firms is 2.6 times that of young firms when contracts are complete (replicated by

[2]This literature is extensive, some examples are: Amaral and Quintin (2010), Buera et al. (2011), D'Erasmo and Moscoso-Boedo (2012), Greenwood et al. (2013), Midrigan and Xu (2014), Moll (2014), Lopez-Martin (2019), Lopez-Martin (2017), Hill and Perez-Reyna (2017).

calibration in the baseline reference), while firm growth is negligible when contracts are incomplete. Finally, we assess the role of key parameters of the model. Based on our results, we stress the importance of frictions that distort the ability of firms to enforce contracts with suppliers.

This article is organized as follows. In Section 2, we provide a summary of the related literature; with an overview of quantitative research related to firm dynamics, misallocation and aggregate productivity, as well as empirical work on the relationship between the quality and efficiency of judicial institutions and firm performance. In Section 3 we present our own empirical motivation and results. In Section 4, the quantitative framework is described and the characterization of the equilibrium is provided. The parameterization of the model is outlined and discussed in Section 5. In Section 6, we examine in more detail how contract incompleteness with suppliers implies wedges for the firms, how these affect incentives, and their potential size-dependence. Section 7 presents the quantitative analysis of the model. Lastly, Section 8 concludes with final comments.

2 Relation to the Literature

Our work is related to different strands of the literature on firm dynamics, misallocation, and aggregate productivity. It is connected to the literature that evaluates the effects of idiosyncratic distortions, in models where productivity is endogenous (see Bhattacharya et al., 2013; Gabler and Poschke, 2013; Hsieh and Klenow, 2014; Ranasinghe, 2014; Buera and Fattal-Jaef, 2016; Bento and Restuccia, 2017; Da-Rocha et al., 2017). The analysis of these models has shown that assuming an exogenous distribution of firm productivity can lead to the underestimation of the consequences of distortions that affect the allocation of resources across production units. Distortions can affect incentives to improve productivity, which adds to the effect on the allocation of resources across firms, thus generating an amplification mechanism. This effect can be particularly detrimental when distortions are more severe for the most productive firms, often termed *correlated distortions*, as in Bento and Restuccia (2017), Hsieh and Klenow (2014), and Buera and Fattal-Jaef (2016).[3]

Related to the previous line of research, we contribute to the literature that aims to identify and evaluate the sources of size dependent distortions and distortions faced by firms in general. For example, D'Erasmo and Moscoso-Boedo (2012), Busso et al. (2012), Ulyssea (2018), Lopez (2017), and Lopez-Martin (2019), among others, analyze tax evasion or the informal sector.[4] Lagos (2006), Moscoso-Boedo and Mukoyama (2012), Da-Rocha et al. (2019), Mukoyama and Osotimehin (2019), Lopez and Torres (forthcoming) evaluate the effects of worker firing costs and labor market regulation. Cole et al. (2016) develop a dynamic costly state verification

[3]Hopenhayn (2014) provides theoretical foundations for understanding the quantitative relevance of the correlation between distortions and productivity in an environment with an exogenous productivity distribution.

[4]In some of these studies, the informal sector refers to the *extensive margin*, while the intensive margin refers to firms that are registered but do not fully comply with regulation and tax obligations.

model of venture capital. This friction affects the incentives to invest in different technologies that determine the life cycle growth of firms, the age and size distribution of firms, and aggregate productivity. A series of articles have evaluated the role of crime and extortion (Hill and Perez-Reyna, 2015; Ranasinghe, 2017; Ranasinghe and Restuccia, 2018), and size-dependent policies and tax enforcement (Guner et al., 2008; Garcia-Santana and Pijoan-Mas, 2014; Gourio and Roys, 2014; Garicano et al., 2016; Amirapu and Gechter, 2020; Bachas et al., 2019). Fossati et al. (forthcoming) contribute to the literature by identifying factors that generate firm-level distortions in different countries, their evidence supports the importance of judicial institutions. In line with this general area of research we analyze a particular source of distortions, potentially correlated with firm productivity or technology, which reduces incentives for investment in innovation and firm growth.

Our article is related to several pieces of research in the literature of contractual frictions. Mukoyama and Popov (2020) embed the contract incompleteness structure of Acemoglu et al. (2007) in a dynamic general equilibrium growth model with evolving institutions during the process of industrialization. They show that incompleteness of contracts leads to two types of misallocation that generate production inefficiency: unbalanced use of inputs and unbalanced production of different goods.[5] Boehm and Oberfield (2020) use microdata on Indian manufacturing firms to show that production and sourcing decisions appear systematically distorted in states with weaker enforcement. Boehm (forthcoming) conducts an analysis at the cross country level and computes the losses that result from contracting frictions employing a static multi-industry model, with a focus on input-output relationships across industries. For the reasons described above, our work is complementary to this literature, as well as to their forceful motivation of the study of contract enforcement.

Finally, the empirical results presented in the next section are related to a set of studies that document the impact of the quality and efficiency of judicial institutions on different dimensions of firm performance, such as size and growth, across regions within particular countries, and across different economies. A list of references in this area includes: Beck et al. (2005) and Beck et al. (2006) (cross-country), García-Posada and Mora-Sanguinetti (2015) (Spain), Chakraborty (2016) (India), Giacomelli and Menon (2017) (Italy), Dougherty (2014) and Laeven and Woodruff (2007) (Mexico). The evidence we present is primarily related to Beck et al. (2005) and Beck et al. (2006), who use cross-country firm-level data. We add to their results in several aspects. First, by considering more recent firm-level data (2006-2017), as their databases span the periods 1996-1999 and 1988-2002, respectively. Second, their measures of judicial development are based on survey responses reflecting the perception of managers with respect to legal obstacles, which may be subject to different drawbacks that they discuss, while we rely on the measure of contract enforcement provided by the World Bank in its Doing Business Report. Third, Beck et al. (2006) study the largest publicly traded firms in each country which, as they caution, are relatively more likely to have access to legal resources potentially allowing them to

[5]Additionally, Antràs and Helpman (2006) and Schwarz and Suedekum (2014) introduce the model of Acemoglu et al. (2007) in a context of international trade.

circumvent legal obstacles.[6] We use data from the World Bank Enterprise Surveys, which compile a representative sample of private formal companies in manufacturing and services sectors, with 5 or more employees. Additionally, they centre on firm size as measured by sales and assets. Finally, in addition to size variables and growth in revenues and the number of workers, we also consider revenues per worker and R&D.

3 Empirical Motivation

In this section we provide an empirical motivation to document the role that contract institutions have in determining aggregate productivity, firm size and growth, revenue per worker and its growth, and investment in research and development. We show that higher quality contract enforcement is positively associated with our variables of interest, and that these associations are quantitatively important. We then use our theoretical model to highlight the mechanisms that make contract enforcement relevant to account for differences in output per worker.

In the case of aggregate variables the impact of better contract institutions is sizeable. The results are robust to the introduction of measures of additional frictions and obstacles faced by firms across different economies, including registration costs, tax rates, financial constraints, and the regulatory burden.[7] The country-level results are also robust to exploiting estimates with instrumental variables, which are well established in the literature, to control for potential endogeneity issues. In this regard, our findings are complementary to the empirical results related to other obstacles and distortions faced by firms, among others: Cole et al. (2016), Ranasinghe (2017), Barseghyan and DiCecio (2011), Barseghyan (2008). We describe the data and the methodology, then discuss our main empirical results.

3.1 Description of Data

We use the measure of *contract enforcement* provided by the World Bank in its Doing Business Report, for both country and firm-level specifications. The measure summarizes the time and cost for resolving commercial disputes through local courts. The range of this index is 1 to 100, where 100 denotes perfect contract enforcement. These indices have been used in the literature as a proxy of the quality of contract enforcement (e.g., Acemoglu et al., 2009; Boehm, forthcoming).

[6] In fact, they caution that their results are not robust to sensitivity tests.

[7] A measure of employment rigidity (average of a difficulty of hiring index, a rigidity of hours index, and a difficulty of firing index.) was not statistically significant in most of our specifications, and their introduction did not alter our main results.

Table 1 Country-Level Empirical Analysis.

	per worker GDP (OLS)			**per worker GDP (IV)**		
contract enforcement	0.021***	0.007***	0.002	0.116***	0.117***	0.099***
credit-output ratio	0.012***	0.005***	0.004***	-0.002	0.001	0.002**
government corruption	—	-0.019	-0.071*	—	-0.296***	-0.273***
registration costs	—	-0.003***	-0.003***	—	-0.001*	-0.002***
property rights	—	0.018***	0.016***	—	0.005	0.011**
tax rate	—	—	0.000	—	—	0.004*
business freedom	—	—	0.018***	—	—	-0.010
R-squared (IV: F-statistic)	0.393	0.556	0.584	69.30***	13.72***	11.43***
n.observations	1,548	1,233	993	1,248	1,034	826

Statistical significance: *** 1%, ** 5%, * 10%.
All regressions include year fixed effects and intercepts. Per worker GDP is in logs.
Robust standard errors.

As a measure of financial development we rely on the ratio of total private sector credit to output, from the Global Financial Development Indicators, which is standard in this type of analysis and is often used as a target in the calibration of quantitative models. We also include the *total tax rate*, from the Doing Business Report, which computes an approximation of total taxes that a medium-size company must pay in a given year. To control for regulatory efficiency, we use the *business freedom* index from the Doing Business Report, with a range of 0 to 100, where 100 is the business environment with least restrictions. It is based on 10 different factors, including the cost, number of procedures and number of days needed to start a business, to obtain a license, and to close a business. Additionally, we consider firm *registration costs*, expressed in terms of income per capita.

The variables are available for the period 2006-2017 at an annual frequency. From the Penn World Tables we obtain real GDP per worker in PPP terms and TFP (Feenstra et al., 2015). To counteract potential endogeneity issues we employ distance to the Equator as an instrumental variable, following Acemoglu et al. (2014) and Barseghyan (2008).[8] We rely on firm level data from the World Bank Enterprise Surveys (WBES), which provides close to 100,000 observations at the firm level from 2006 to 2017. In the Appendix we provide a description of this database. Our main variables of interest are revenue per worker (as a proxy for productivity), firm size in terms of the number of workers, and investment in research and development.

3.2 Cross-Country Evidence

The estimation results with country-level data for GDP per worker are shown in Table 1 (variable in logs), for both OLS and IV regressions.[9] The index of contract enforcement is highly significant and positive in most specifications. Furthermore, this result is robust to alternative specifications (additional results available upon request). The coefficient for the proxy for financial development resulted positive and statistically significant by OLS, but loses significance and magnitude in our IV specification, making contract enforcement more relevant.

[8]Following Barseghyan (2008) we use distance to the Equator as an instrument that takes into account the fact that "geographical characteristics and the extent to which major European languages have been adopted in a country are correlated with the quality of the country's institutions" (Barseghyan 2008, pg 149). Alternative estimations with European settler mortality and language of origin as instrumental variables provided similar results (Barseghyan, 2008). We show regressions with TFP in the Appendix.

[9]In the case of IV estimations we show the F-statistic test of relevance for instruments in Table 1, it is highly significant in all specifications. Additionally, we conduct the Cragg-Donald test and results show that the instruments are relevant. All results are available upon request.

Table 2 Firm-Level Empirical Analysis.

	size (logs)			**revenue per worker (logs)**			**R&D decision**		
contract enforcement	0.057***	0.036***	0.021**	0.190***	0.152***	0.077***	0.009***	0.001	0.026***
credit-output ratio	0.007**	0.008***	0.009***	0.001	0.005	0.006	0.003	0.004*	0.003***
government corruption	—	0.074	0.057	—	0.324*	0.219	—	0.008	0.125***
registration costs	—	-0.001	0.000	—	-0.002	0.001	—	-0.001	0.000
property rights	—	-0.003	-0.006	—	-0.014	-0.029***	—	-0.001	-0.007***
tax rate	—	—	0.003	—	—	0.010	—	—	-0.002***
business freedom	—	—	0.013**	—	—	0.065***	—	—	0.011***
foreign ownership	0.708***	0.715***	0.736***	0.494***	0.513***	0.527***	0.036***	0.038***	0.040***
exports	0.855***	0.873***	0.840***	0.287***	0.298***	0.275***	0.160***	0.165***	0.167***
firm age	0.018***	0.018***	0.018***	0.005***	0.004***	0.004***	0.001***	0.001***	0.001***
R-squared	0.891	0.892	0.893	0.981	0.982	0.983	0.358	0.367	0.374
n. observations	91,084	82,132	76,089	77,107	69,691	64,886	58,258	54,052	48,948

Statistical significance: *** 1%, ** 5%, * 10%.
All regressions include time, country and sector fixed effects, and exclude intercepts. Errors clustered at the country level.

The coefficient for registration costs is negative and robust across specifications, in line with the results of Barseghyan (2008). Additionally, tax rates, and the indices of business freedom and property rights lose significance with an IV specification. The coefficient for contract enforcement is economically important. Consider the last regression under IV estimation: an increase by one unit in this index increases output per worker by 9.9%. For example, if institutions in Bangladesh (with a level of contract enforcement of 20.8 in 2015) improved to the level registered in Germany (74.9), output per worker would increase by 5.37 times (= 0.099×54.2), when the observed ratio in output per worker is 10.5. In a less extreme example, improving the judicial quality of Argentina (52.1) to the level of Sweden (68), would imply an increment in output per worker of 1.57 times, while the observed ratio in output per worker is 2.38.[10]

3.3 Empirical Evidence at the Firm Level

We use the WBES firm-level data to estimate the impact of *contract enforcement* on size and revenue per worker (variables in logs), and the probability that a firm invests in research and investment.[11] To measure R&D we rely on a dummy variable available in the WBES that takes the value of 1 if the firm states that it spent resources in research and development during the fiscal year previous to the one in which it was interviewed. In the case of R&D we estimate a linear probability model, with similar results found with probit and logit models. At the firm-level, we consider a dummy variable that indicates that a firm exports, and a dummy variable that indicates a firm has foreign ownership. These variables point to the exposition of the firm to international competition and technologies, which have been found to be relevant in the literature.

Table 2 shows the estimation results. We estimate that better contract enforcement (an increase of 1 unit in the index) is associated with an increase in size of 2.1%, an increase in revenue per worker of 7.7% and a higher probability of investing in R&D of 2.6%. For example, if a firm in Angola could benefit from the institutional environment of Chile in terms of contract enforcement quality (26.5 compared to 65 in 2010), its size in terms of workers would increase by 80.9% and its revenue per worker would increase by 296%.

[10] We show in the Appendix that the impact on TFP is approximately half of that on output per worker.

[11] In the Appendix we describe the data in more detail.

Table 3 Firm Level Empirical Analysis - Latin America WBES Panel.

	Δ% revenues			**Δ% revenue per worker**			**Δ% size**		
contract enforcement	0.002***	0.002*	0.003***	0.001	0.001	0.002***	0.001**	0.000	0.001**
credit-output ratio	-0.002	-0.003	-0.003***	-0.003**	-0.003*	-0.003***	0.001*	0.000	-0.001
government corruption	—	0.014	-0.046***	—	0.001	-0.056***	—	0.008	0.006
registration costs	—	0.000	-0.000	—	-0.000	-0.001***	—	0.000*	0.000***
property rights	—	0.000	0.000**	—	-0.000	0.001***	—	0.000	-0.000
tax rate	—	—	0.000*	—	—	0.000***	—	—	-0.000
business freedom	—	—	0.002***	—	—	0.002***	—	—	0.001
employees (log)	0.004	0.004	0.000	0.018***	0.018***	0.020***	-0.015***	-0.015***	-0.020***
age (log)	-0.014*	-0.016*	-0.022**	-0.011*	-0.011**	-0.020***	0.000	-0.001	0.004
exports	-0.016	-0.015	-0.026	-0.022	-0.022*	-0.039**	-0.000	0.001	0.005
foreign	-0.004	-0.004	0.027	-0.020	-0.018	0.007	0.013	0.012	0.013
R-squared	0.079	0.080	0.093	0.061	0.062	0.063	0.049	0.054	0.069
n. observations	1,376	1,376	884	1,373	1,373	882	1,654	1,654	1,076

Statistical significance: *** 1%, ** 5%, * 10%.
All regressions exclude intercepts. Errors are clustered at the country level. Outcomes are expressed as yearly p.p. changes.

In order to analyze the dynamic effect of contract enforcement on firms, we construct a panel using available surveys for Latin American countries, given the availability of surveys for these countries for similar years. In our panel we consider three waves of surveys (2006, 2010 and 2017) for selected Latin American countries included in the WBES database for which we could track firms across at least two waves.[12] We regress changes in three variables (percentage change in revenues, revenue per worker and size) between two waves of surveys, normalized by the number of years (variables are expressed as annual percentage changes), and we use controls of the initial year.

Table 3 shows the results for Latin America. We find that contract enforcement is associated with growth in revenues, size and revenue per worker. For example, if a firm in 2010 in Colombia could benefit from contract enforcement in Argentina (36 compared to 65), its annual growth in revenues would have been 8.7% higher, its revenue per worker would have been 5.8% higher, and the firm would be 2.9% bigger. As a robustness exercise for our results, in the Appendix we provide regressions using a panel available from the WBES for Eastern European and Asian countries.

4 Quantitative Framework

We analyze an economy where a continuum of firms produce an homogeneous final good. We will refer to these production units as *firms*, as opposed to intermediate good suppliers. These firms purchase intermediate goods from suppliers, while suppliers need to invest in a range of activities to deliver the intermediate goods. Firms invest each period to improve their technology level, this level of technology refers to the measure of intermediate goods (a higher level of technology implies a larger range of intermediate goods). We first describe the static problem and the contracting problem faced by firms, which builds upon Acemoglu et al. (2007). In our model the technology level is given in any period. Then we describe the dynamic problem of firms, that decide how much to invest in improving their technology level for the next period. We assume that there is a representative household endowed with a unit of time that is inelastically supplied to firms as labor.

4.1 Technology and Payoffs

We first describe the static problem faced by firms. We denote the technology level of a firm by $n \in \mathbb{R}_+$, which represents the range of intermediate goods the firm can use in production, and is a state variable for the firm. In this sense, a higher n represents a more complex final good. For each $j \in [0, n]$, $\overline{x}(j)$ is the quantity of intermediate input j. We introduce a term with decreasing returns to scale in labor

[12] Descriptive statistics are available in the Appendix.

to the original production function:[13]

$$y = z^{1-\beta}\, n^{\beta\,(\kappa+1-1/\alpha)} \left[\int_0^n \overline{x}(j)^\alpha \, dj\right]^{\beta/\alpha} l^\nu \tag{1}$$

with $\kappa > 0$ and $0 < \alpha < 1$. Parameter α determines the degree of complementarity between inputs, so that the elasticity of substitution is $1/(1-\alpha)$. Parameter κ controls the elasticity of output with respect to the level of the technology, while ν governs the decreasing marginal productivity of labor. The specification for the term $n^{\beta\,(\kappa+1-1/\alpha)}$ allows for the separation of the elasticity of substitution between inputs and the elasticity of output with respect to the level of technology. As is standard in these models, and following the literature, firms are subject to exogenous productivity shocks, which we denote z.

There is a number of profit-maximizing suppliers that produce the intermediate goods, which have an outside option ω. The supplier of an intermediate input makes a relationship-specific investment, with constant marginal cost c_x for each activity necessary for production, which we consider to be in terms of the cost of labor.[14] The production function of intermediate inputs is Cobb-Douglas and symmetric in the activities:

$$\overline{x}(j) = \exp\left[\int_0^1 \ln\, x(i,j)\, di\right], \tag{2}$$

where $x(i,j)$ is the level of investment in activity i performed by the supplier of input j. Payment to supplier j consists of two parts: an ex ante payment $\tau(j) \in \mathbb{R}$ before the investments $x(i,j)$ take place and payment $s(j)$ after these investments are completed. The payoff to supplier j, taking into account her outside option is:

$$\pi_x(j) = \max\left\{\tau(j) + s(j) - \int_0^1 c_x\, x(i,j) di, \omega\right\}.$$

The profits of the firm are:

$$\pi = y - \int_0^n [\tau(j) + s(j)]\, dj - w\, l,$$

where w is the wage rate.

[13]We depart from Acemoglu et al. (2007), who consider a monopolistic competition framework, whereas we consider a technology with decreasing returns to scale where labor enters in a standard fashion. These are important features to take into account when we describe the parameterization of the model. Additionally, as we demonstrate in our quantitative exercises, including physical capital does not modify our main results.

[14]In general equilibrium the wage level will go down as contract institutions worsen, reducing the marginal cost of the activities of suppliers and, to some extent, moderating the negative effects of more adverse conditions (in this sense, the results are conservative). In the stationary equilibrium there is a fixed measure of these agents that do not display any type of dynamics (for example, this similar to the intermediaries in Cole et al., 2016), and represent a small part of the labor force.

4.2 Equilibrium under Complete Contracts

We first consider a benchmark economy where contracts are complete (i.e. the first best). With complete contracts a firm pays each supplier the outside option: it makes a contract offer $[\{x(i,j)\}_{i\in[0,1]}, \{s(j), \tau(j)\}]$ for every input $j \in [0, n]$.

We consider a subgame perfect equilibrium, that can be represented as a solution to the following problem:

$$\max_{\{\{x(i,j)\}_{ij}, \{s(j),\tau(j)\}_j, l\}} y - \int_0^n [\tau(j) + s(j)]\, dj - w\, l$$

subject to (1), (2) and the participation constraint of suppliers:

$$s(j) + \tau(j) - c_x \int_0^1 x(i,j) di \geq \omega \quad \forall j \in [0, n].$$

This last condition is satisfied with equality in equilibrium, so there are no rents for suppliers. Since all activities are symmetric, the firm chooses the same investment level x for all activities in all intermediate inputs. With this condition the problem becomes:

$$\pi^*(n, z) \equiv \max_{\{x,l\}} z^{1-\beta}\, n^{\beta\,(\kappa+1)}\, x^\beta\, l^\nu - n\,(x\, c_x + \omega) - w\, l. \tag{3}$$

Notice that (3) is strictly concave in x and l as long as $1 - \beta - \nu > 0$.

Lemma 1 in Appendix A shows that the values for activities and labor under complete contracts are given by:

$$x^* = \frac{1}{n}\left[\left(\frac{\nu}{w}\right)^\nu \left(\frac{\beta}{c_x}\right)^{1-\nu} z^{1-\beta} n^{\beta\kappa}\right]^{\frac{1}{1-\nu-\beta}},\ l^* = \left[\left(\frac{\nu}{w}\right)^{1-\beta} \left(\frac{\beta}{c_x}\right)^{\beta} z^{1-\beta} n^{\beta\kappa}\right]^{\frac{1}{1-\nu-\beta}} \tag{4}$$

and production is:

$$y^* = \left[\left(\frac{\nu}{w}\right)^\nu \left(\frac{\beta}{c_x}\right)^{\beta} z^{1-\beta} n^{\beta\kappa}\right]^{\frac{1}{1-\nu-\beta}}. \tag{5}$$

4.3 Equilibrium under Incomplete Contracts

We now consider an economy with incomplete contracts. Contract incompleteness is modeled as the fraction of activities that are not contractible. That is, for every intermediate input, we define $\mu \in [0, 1]$ such that investments in activities $0 \leq i \leq \mu$ are observable and contractible, while $\mu < i \leq 1$ are not contractible. The contract stipulates investments for the contractible activities but not for the $1 - \mu$ noncontractible activities: suppliers will decide investment in $1 - \mu$ in anticipation of the ex-post distribution of revenue.

The timing is as follows:

- z and n are fixed at the beginning of the period.
- The firm hires labor l, offers contract $[\{x_c(i,j)\}_{i=0}^{\mu}, \tau(j)]$ for every intermediate input $j \in [0,n]$, where $x_c(i,j)$ is investment level in a contractible activity, and $\tau(j)$ is an upfront payment to supplier j (can be positive or negative).
- Potential suppliers decide whether to apply for the contracts.
- Suppliers $j \in [0,n]$ choose investment levels $x(i,j)$ for all $i \in [\mu,1]$. In contractible activities $i \in [0,\mu]$, investment is $x(i,j) = x_c(i,j)$.
- Suppliers and firm bargain over the division of revenue (suppliers can withhold their services in noncontractible activities).
- Output y is produced and distributed.

We consider a symmetric subgame perfect equilibrium (SSPE) and we denote hired labor, investment in contractible activities, investment in noncontractible activities, and upfront payment to suppliers by $\{\hat{l}, \hat{x}_c, \hat{x}_n, \hat{\tau}\}$. A SSPE is solved by backward induction, at the penultimate stage of the game given l and x_c.

To determine the division of the surplus between the firm and its suppliers, we use the Shapley value following Acemoglu et al. (2007) and Hart and Moore (1990). We are interested in constructing a symmetric equilibrium. Suppose $x_n(-j)$ is investment in noncontractible activities for all suppliers other than j, while investment by supplier j is $x_n(j)$. Denote the Shapley value of supplier j by $\overline{s}_x[l, x_c, x_n(-j), x_n(j)]$, for which an explicit expression is derived below. In equilibrium, symmetry is satisfied $x_n(j) = x_n(-j)$, so x_n is a fixed point given by:

$$x_n = \arg\max_{x_n(j)} \overline{s}_x(l, x_c, x_n, x_n(j)) - (1-\mu)\, c_x\, x_n(j). \tag{6}$$

Let $s_x(l, x_c, x_n) \equiv \overline{s}_x(l, x_c, x_n, x_n)$. In a symmetric equilibrium output of the firm is given by $y = z^{1-\beta}(n^{\kappa+1}\, x_c^{\mu}\, x_n^{1-\mu})^{\beta}\, l^{\nu}$. The Shapley value for the firm is obtained as a residual:

$$s(l, x_c, x_n) = z^{1-\beta}\,(n^{\kappa+1}\, x_c^{\mu}\, x_n^{1-\mu})^{\beta}\, l^{\nu} - n\, s_x(l, x_c, x_n)$$

The contract offered by the final-good firm has to satisfy the participation constraint for suppliers:

$$\overline{s}_x(l, x_c, x_n, x_n) + \tau \geq \mu\, c_x\, x_c + (1-\mu)\, c_x\, x_n + \omega \tag{7}$$

The maximization problem of the (final good) firm is:

$$\max_{\{l, x_c, x_n, \tau\}} s(l, x_c, x_n) - n\,\tau - w\, l \quad \text{s.t. (6) and (7).}$$

We can obtain τ from the participation constraint that will be satisfied with equality in equilibrium, which allows us to rewrite the problem of the firm as:

$$\max_{\{l,x_c,x_n\}} s(\,\cdot\,) + n[\overline{s}_x(\,\cdot\,) - \mu\, c_x\, x_c - (1-\mu)\, c_x\, x_n)] - \omega\, n - w\, l$$

s.t. condition (6), and the upfront payment needs to satisfy:

$$\hat{\tau} = \mu\, c_x\, \hat{x}_c + (1-\mu)\, c_x\, \hat{x} + \omega - \overline{s}_x(\hat{l}, \hat{x}_c, \hat{x}_n, \hat{x}_n)$$

It can be shown that:

$$s_x(l, x_c, x_n) = \frac{\beta}{\alpha+\beta}\, y/n \quad \text{and} \quad s(l, x_c, x_n) = \frac{\alpha}{\alpha+\beta}\, y.$$

$\frac{\alpha}{\alpha+\beta}$ is interpreted as the bargaining power of the firm, increasing in α and decreasing in β. The role of these parameters is discussed with more detail below.

4.3.1 Characterization of Equilibrium

Using the incentive compatibility constraint, the problem of the supplier is given by

$$x_n = \arg\max_{\{x_n(j)\}} \frac{\beta}{\alpha+\beta}\, z^{1-\beta} \left[\frac{x_n(j)}{x_n}\right]^{(1-\mu)\alpha} x_c^{\beta\mu}\, x_n^{\beta(1-\mu)}\, n^{\beta(\kappa+1)-1}\, l^{\nu} - (1-\mu)\, c_x\, x_n(j).$$

In this problem there are two differences with respect to the first best. First, the supplier receives a fraction $\frac{\beta}{\alpha+\beta}$, so the supplier is not a full residual claimant of the return to its investment in noncontractible activities and thus underinvests relative to the optimal level. Second, multilateral bargaining distorts the concavity of the private return, which now depends on α, instead of just depending on β, as in equation (3). The solution is obtained from the first-order condition of the problem and solving for the fixed point $x_n(j) = x_n$. This results in a unique x_n:

$$x_n = \overline{x}_n(x_c, l) = \left[\frac{\alpha\beta}{\alpha+\beta} \times (c_x)^{-1} x_c^{\beta\mu} z^{1-\beta} n^{\beta(\kappa+1)-1}\, l^{\nu}\right]^{1/[1-\beta(1-\mu)]}. \tag{8}$$

Taking this as given the problem of the firm is:

$$\begin{aligned}\pi_i(z, n; \mu) \equiv \max_{\{x_c,l\}} & \, z^{1-\beta}[x_c^{\mu}\, \overline{x}_n(x_c, l)^{1-\mu}]^{\beta} n^{\beta(\kappa+1)}\, l^{\nu} \\ & - c_x n \mu x_c - c_x n(1-\mu)\, \overline{x}_n(l, x_c) - \omega\, n - w\, l\end{aligned} \tag{9}$$

In Appendix A we prove that

$$l_i = h_1(\mu) \cdot l^*, \quad x_c = h_1(\mu) \cdot x^*, \quad x_n = h_2(\mu) \cdot x_c$$

and

$$y_i \equiv z^{1-\beta} n^{\beta(1+\kappa)} x_c^{\beta\mu} x_n^{\beta(1-\mu)} l_i^{\nu} = h_3(\mu) \cdot y^*, \tag{10}$$

where

$$h_1(\mu) \equiv \left[\frac{1}{\alpha+\beta} \left(\frac{\alpha+\beta-\alpha\beta(1-\mu)}{1-\beta(1-\mu)} \right)^{1-\beta(1-\mu)} \alpha^{\beta(1-\mu)} \right]^{\frac{1}{1-\nu-\beta}}$$

$$h_2(\mu) \equiv \alpha \frac{1-\beta(1-\mu)}{\alpha+\beta-\alpha\beta(1-\mu)}, \quad h_3(\mu) \equiv h_1(\mu)^{\beta+\nu} \cdot h_2(\mu)^{(1-\mu)\beta}.$$

The equations above express different wedges $\{h_1(\mu), h_2(\mu), h_3(\mu)\}$ on output and inputs of production that imply distortions relative to the baseline with perfect contracts. Notice that $h_1(1) = 1$ (no distortion when μ equal one) and $h_1'(\mu) > 0$ (the distortion is decreasing in μ), so $x_c \leq x^*$ and $l_i \leq l^*$. With $h_2(1) = \alpha/(\alpha+\beta)$ and $h_2'(\mu) > 0$, we obtain $x_n < x_c$. With these results, it is straightforward to verify that output with complete contracts is higher than under incomplete contracts $y_i < y^*$.

Furthermore, in Appendix A we prove that we can express profits under complete and incomplete contracts as, respectively:

$$\pi^* = (1-\beta-\nu)\, y^* - \omega n \quad \text{and} \quad \pi_i(\mu) = (1-\beta-\nu)\, h_1(\mu)\, y^* - \omega n. \tag{11}$$

The implied wedge on profits under incomplete contracts affects incentives to invest in technology. We discuss below how incomplete contracts generate a distortion that depends on the technology level of the firm.

4.4 Dynamic Problem of the Firm

We now describe the dynamic problem of firms. Technology n, a state variable, is accumulated over time with investment $e \geq 0$ in a stochastic innovation technology. Thus, the level of technology level summarizes the history of investment and success in innovations and governs the size of the firm (Klette and Kortum 2004). Furthermore, it is lost when the firm closes, regardless of whether exit is due to an exogenous exit shock or because it is optimal to close the firm. Finally, technology is assumed to be firm-specific and there is no market for its trade.

The dynamic problem of the firm can be written in recursive form as follows:

$$\begin{aligned} v(n,z) = \max_{\{e \geq 0\}} \; & \pi(n,z) - e - c_f \\ + \quad & \gamma\,(1-\phi) \sum_{\{n',z'\}} \Lambda(z' \,|\, z) \cdot P(n' \mid n, e) \cdot \max\{v(n',z'),\, \underline{v}\} \end{aligned} \tag{12}$$

where $\pi(n,z)$ is the level of profits, whether with complete or incomplete contracts, that depends on the level of technology n and the stochastic productivity shock z, e are expenditures in the innovation technology, γ is the discount parameter and ϕ is the probability of an exogenous exit shock. The per-period fixed cost of production c_f generates endogenous exit of firms, while the exit value $\underline{v}$ when a firm decides to close down is set at zero. Exogenous firm productivity evolves according to a discrete Markov process $\Lambda(z' \mid z)$. This exogenous process is important to generate firm exit but cannot drive long-term firm productivity growth due to its mean-reverting nature.

In every period firms can invest in the innovation good e to increase the stock of technology.[15] Three outcomes are possible every period, depending on the amount of investment in the innovation good in the previous period: technology may increase by a proportion ψ, it may remain constant, or decrease by ψ.

Technology is defined on the grid $\{\underline{n}, \underline{n}\,(1+\psi), \underline{n}\,(1+\psi)^2, \ldots, \overline{n}\}$, where $\underline{n}$ and $\overline{n}$ are the lowest and highest possible levels of technology, respectively. The probability of a successful outcome is given by:

$$P(n' = n\,(1+\psi) \mid n, e) = \frac{(1-\xi)\cdot(e/n)}{1+(e/n)}.$$

There are diminishing returns to innovation investment e. Fixing a probability of success in innovation, $P(n\,(1+\psi) \mid n, e)$, the necessary investment in innovation goods e to increase the productivity of the firm by a fixed percentage is proportional to technology n.[16] Parameter ξ determines the expected return to investment in innovation. The probability of a negative outcome is given by:

$$P(n' = n/(1+\psi) \mid n, e) = \frac{\xi}{1+(e/n)}.$$

with the remaining probability assigned to the current technology level.

4.5 Entry of New Firms

A new firm enters with an initial level of technology $\underline{n}$. The value of a potential entering firm, net of the entry cost, is given by:

$$v_e = \int v(\underline{n}, z)\, dF(z) - c_e$$

where $F(z)$ is the unconditional distribution of idiosyncratic firm productivity z. In equilibrium a break-even condition needs to be satisfied; $v_e = 0$.

4.6 Representative Household and Model Equilibrium

We close the model by assuming there is an infinitely lived representative

[15] The stochastic innovation process builds on Pakes and McGuire (1994) and Farias et al. (2012). For related stochastic specifications see Klette and Kortum (2004) and Atkeson and Burstein (2010).

[16] For a similar adjustment that depends on firm productivity, see Hsieh and Klenow (2014).

household with preferences over consumption sequences given by:

$$\sum_{t=0}^{\infty} \gamma^t \, u(c_t)$$

with c_t denoting aggregate consumption in period t, $\gamma \in (0, 1)$ is the discount factor, $u(c)$ is assumed to satisfy standard conditions. The household has an endowment of labor that is inelastically supplied in the market. Resources for the household are $c = d + w - \overline{en} + \overline{ex}$, where $\overline{en}$ denotes aggregate firm creation costs, $\overline{ex}$ is the aggregate exit value of firms, d denotes aggregate dividends from the firms and suppliers.

We study the stationary equilibrium of this economy, where prices and aggregate variables are constant, as well as the distribution of firms over state variables. The equilibrium wage clears the market for labor, where the firms in the economy use the available amount of labor that is inelastically supplied by the household.

5 Parameters and Calibration

We start our analysis with the baseline model. As is standard in the literature, we set parameter values that jointly contribute to replicate key statistics of the U.S. economy. The critical institutional parameter μ represents the share of activities, of each intermediate input, for which investment is observable and contractible. For the undistorted economy we assume perfect contracts.[17]

5.1 Predetermined Parameters

We first enumerate the set of predetermined parameters in Table 4, assigning standard values in the literature. In the model, the length of a time period represents one year. The discount factor γ of 0.99, jointly with an exogenous death rate of firms of 0.04 (which is a calibrated parameter discussed below), determine an *effective discount value* of 0.95 for the firms, which is within the range of commonly used values.

[17] In the quantitative financial development literature, for example, assuming perfect markets is standard for the U.S.

Table 4. Predetermined Parameters.

description/role of parameter	symbol	value
discount factor	$\gamma \cdot (1-\phi)$	0.95
exponent on technology and intermediate inputs	β	0.45
elasticity of substitution intermediate inputs	α	0.50
elasticity of output w.r.t. technology	κ	0.30
exog. productivity process: autocorrelation	ρ	0.60
exog. productivity process: volatility	σ_ε	0.25

The returns to scale in the production function are jointly determined by ν and β. In Acemoglu et al. (2007), the authors consider a monopolistic competition framework, where β determines the elasticity of demand. Their benchmark value for this parameter is 0.75, in a model without labor or physical capital in the production function. This number is consistent with the generally accepted range of the elasticity of substitution between final-good varieties.

In our setup, we need to take into account several issues. First, the returns to scale are determined by ν and β, so that their sum should be in line with *span-of-control* values in the literature or its equivalent curvature in monopolistic competition models (e.g. Restuccia and Rogerson, 2008).[18] Second, the weight given to intermediate inputs is larger than the weight on labor and capital (e.g., Jones, 2011). Third, as we will show below, we require $\nu + \beta(\kappa + 1) < 1$ in order to have a wedge that is increasing in the level of technology n. This is the relevant case, although we show below that we are conservative in this aspect. Fourth, ν takes a value of 0.40, as part of the model calibration. Under these considerations, we set β equal to 0.45. We later discuss how our main results change with different parameter values.

The value of α determines the degree of complementarity between intermediate inputs. This parameter is not relevant for calibration, since it does not enter the problem of the firm under complete contracts. However, it does affect the impact of worse judicial institutions given its role in the bargaining process: as α increases, intermediate inputs become more substitutable, and the magnitude of the distortionary effects diminishes. We follow Acemoglu et al. (2007) in fixing its central value at 0.50, and provide a discussion of how quantitative results change for alternative values. Boehm (forthcoming), for example, employs a parameterization that is equivalent to an α of 0.71 in a multi-sector model. Parameter κ controls the elasticity of output with respect to the level of technology. We set a baseline value of 0.30, in the range considered by Acemoglu et al. (2007).

The exogenous productivity component of the production function z follows an AR(1) process, with an autocorrelation parameter of 0.60 and a volatility parameter of 0.25. These values are approximately at the mid-range of estimates for the U.S. (Lee and Mukoyama, 2015), and the numbers used in quantitative firm dynamics

[18] A span-of-control model (our approach) is isomorphic to the monopolistic competition framework.

models. Due to their mean-reverting nature, the exogenous productivity shocks do not drive the life-cycle growth of firms, by neither size nor productivity. Nevertheless they do, as is standard in models of firm dynamics and jointly with other parameters, contribute to determine exit rates by age and size of the firm, and thus the size and age-distribution of firms.

5.2 Calibration

We now turn to the calibration of the model, which we summarize in Table 5. The per-period fixed cost of production c_f, jointly with the exogenous probability of firm exit, denoted by ϕ, determine firm-exit rates in our model. In a stationary equilibrium, total exit and entry rates of firms are equal, we target a level of 0.10, consistent with the literature (e.g., Gabler and Poschke, 2013), see Table 6. Large and productive firms are less likely to exit endogenously in this type of models, and thus their exit rates are mainly generated by exogenous shocks. The range for this moment is approximately 0.04-0.05 (Hsieh and Klenow, 2014; D'Erasmo and Moscoso-Boedo, 2012, and Ranasinghe, 2014); our value of 0.04 is at the lower bound of this range, in line with D'Erasmo and Moscoso-Boedo (2012).

Table 5. Calibrated Parameters.

description/role of parameter	symbol	value
per-period fixed cost of production	c_f	3.761
exogenous firm death rate	ϕ	0.040
innovation technology: size innovation steps	ψ	0.500
innovation technology: *success* probability	ξ	0.673
outside value of suppliers	ω	0.020
prod. function exponent on labor	ν	0.400

The remaining calibrated parameters mainly govern the growth dynamics and employment of firms. The proportional size of each technology step is given by ψ, while the probability of an increase in technology, for a given level of investment, is determined by ξ. We target the growth pattern of firms documented by Hsieh and Klenow (2014) for the U.S., at two points of their life-cycle: the size of survivors of age 6-10 relative to age 1-5, and the size of survivors of age 31-35 relative to age 1-5 (see Table 6). Surviving firms grow faster when they are young, which requires a larger ψ; their growth moderates afterwards. Parameter ω, which represents the outside option for suppliers, affects the growth dynamics of larger and more productive firms as it implies a cost that is increasing in the level of technology (see equation 3). Parameter ν influences the employment of firms;[19] in the U.S. the upper tail of

[19]The elasticity of labor with respect to the level of technology is given by $\beta \cdot \kappa/(1 - \nu - \beta)$, see equation (4). The calibration centers on the final goods firms since, as previously described, intermediate goods firms do not display any type of dynamics by construction, and represent a small measure of the labor force.

the size distribution accounts for a significant part of employment, firms with more than 500 workers account for 0.496 of total employment, compared to 0.467 in our model.[20] The last target we consider is the share of employment in firms of age 41 or older (Hsieh and Klenow, 2014).

We document the fit of the model along several non-target dimensions (see Table 6). Although we do not target the entire distribution of employment by age of the firm, the model replicates this properly. Additionally, in the baseline calibration the ratio of investment in technology to the production of final goods is 0.076. This figure is comparable to the estimate of the ratio of investment in business intangible capital to domestic business value added of 0.064 by McGrattan and Prescott (2010) (see their Table A3). In the Appendix we provide an outline, with some clarifications, of the solution and numerical implementation of the model.

Table 6. Baseline Model: Calibration Moments.

target statistics	data	model
total exit (equal to entry) rate of firms	0.100	0.100
exit rate firms 500+ workers	0.042	0.040
relative size firms 6-10/1-5 years (survivors)	1.597	1.580
relative size firms 31-35/1-5 years (survivors)	2.890	2.964
share of employment at firms with 41+ years	0.280	0.304
employment at firms w/500+ workers	0.496	0.467
non-target statistics	data	model
share of employment at firms with 0-10 years	0.247	0.227
share of employment at firms with 11-20 years	0.207	0.188
share of employment at firms with 21-30 years	0.146	0.148
share of employment at firms with 31-40 years	0.121	0.132
investment in technology/final goods production	0.064	0.076

6 Model Mechanics

In this section we briefly discuss how contract incompleteness implies a distortion, similar to a tax or *wedge*, that affects incentives to invest in technology and, therefore, firm productivity growth and aggregate productivity of the economy. We analyze the mechanism by distinguishing between two effects: one static and one dynamic. First, we can show that, ceteris paribus (and in partial equilibrium, for the purposes of this section), a lower μ curtails firm size. Second, the distortion reduces incentives for the firm to invest in improving technology, this is the dynamic effect.

[20] This moment is obtained from the Longitudinal Business Database of the U.S. Census Bureau, we take the average for the years 2000-2011.

Notice from (10) that y_i is increasing in μ, with better contracts firms will be bigger. The result is rather straightforward if we focus on inputs of production: when $\mu < 1$ there is a wedge, $1 - h_3(\mu)$, that is decreasing in μ. A higher μ results in more input demand and, therefore, increased production.

Given that n is a dynamic decision, to analyze the second effect we focus on the wedge on profits, since profits determine the incentives for the firm to invest in improving their technology level (see equation (12)). In our model this wedge is increasing in n; recall from (11) that $\pi = A\,y^* - \omega\,n$ and $\pi_i = h_1(\mu)\,A\,y^* - \omega\,n$, where $A = 1 - \beta - \nu$, $h_1'(\mu) > 0$ and $h_1(1) = 1$. Consider

$$\frac{\pi_i}{\pi^*} = \frac{h_1(\mu) \cdot A \cdot y^* - \omega \cdot n}{A \cdot y^* - \omega \cdot n} = \frac{h_1(\mu) \cdot A \cdot (y^*/n) - \omega}{A \cdot (y^*/n) - \omega}.$$

As long as $\beta(\kappa + 1) + \nu < 1$, which is true in our benchmark parametrization, $g(n) \equiv (y^*/n)$ is strictly decreasing in n. Then

$$\frac{\partial\,(\pi_i/\pi^*)}{\partial n} = \frac{(1 - h_1(\mu)) \cdot A \cdot g'(n) \cdot \omega}{(A \cdot g(n) - \omega)^2} < 0.$$

The wedge on profits is equal to $1 - (\pi_i/\pi^*)$, so the inequality above implies that this wedge is increasing in n. In other words, firms with a higher technology level are relatively more affected by the friction than firms with lower technology. As μ increases, it is less costly to have a higher n. In our model ω, which is the outside option available to suppliers, plays a crucial role. If $\omega = 0$, the wedge for firms would be equal to $h_1(\mu)$, which does not depend on n. We stress, however, that the wedge need not be increasing in n to affect investment in technology, a constant distortion is sufficient to generate a dynamic disincentive to invest in technology (we evaluate alternative calibrations).[21]

7 Quantitative Analysis

In this section we discuss the main quantitative results. First, we document how contract completeness affects technology accumulation and growth at the firm level, with consequences for the age and size distribution of firms in general equilibrium, as well as aggregate productivity. Second, we analyze the role of different key parameters. Third, we examine the nature of the different wedges that are present in the model. Finally, we evaluate a version of the model with capital in the production function.

7.1 Baseline Quantitative Results

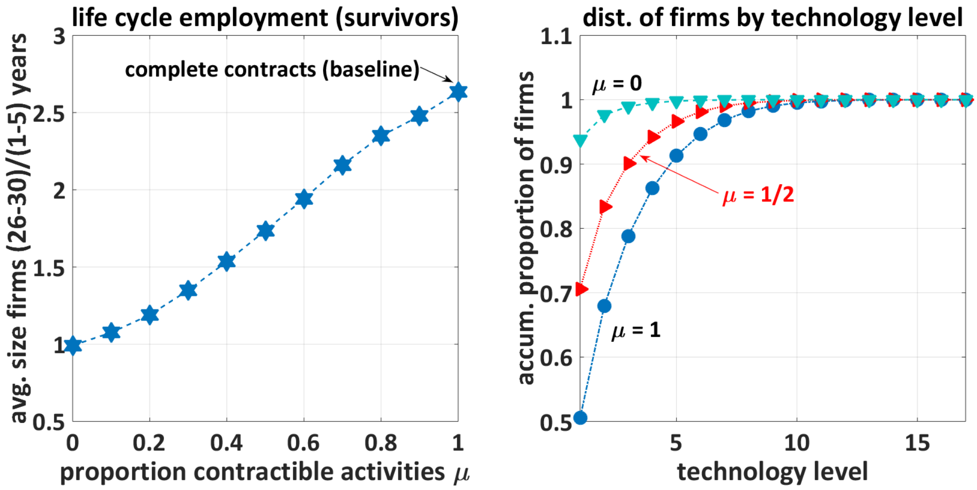
The central exercise consists of reducing μ, the parameter that represents completeness of contracts, starting from the baseline calibration and recomputing the

[21] Interestingly, an early version of Hsieh and Klenow (2014) highlighted the relationship between distortions and profits (rather than output) in the determination of the incentives to invest in higher productivity (Hsieh and Klenow, 2012).

equilibrium of the economy.[22] As contracts become relatively *more incomplete* (i.e., we reduce μ and compute the new equilibrium), the distortion worsens reducing the incentives to invest in technology. In the extreme case of contract incompleteness, firm growth is negligible even after 26 years (see Figure 1, left panel). This directly affects the distribution of technology in the economy (Figure 1, right panel).[23]

The impact on the relevance of older and bigger firms, and the distribution of employment by age and size of firm in general, is quantitatively important: as μ decreases the share of employment in these firms decreases (Figure 2). The consequences of contract incompleteness are economically significant: in the extreme case of contract incompleteness output per worker falls by approximately 33 percent relative to the baseline scenario (Figure 3). These losses are comparable to those found in quantitative models of financial frictions.[24]

Figure 1. Impact of Contract Institutions:
Firm Growth and Accumulation of Technology



7.2 Sensitivity Analysis

We discuss the role of different key parameters for our quantitative results.

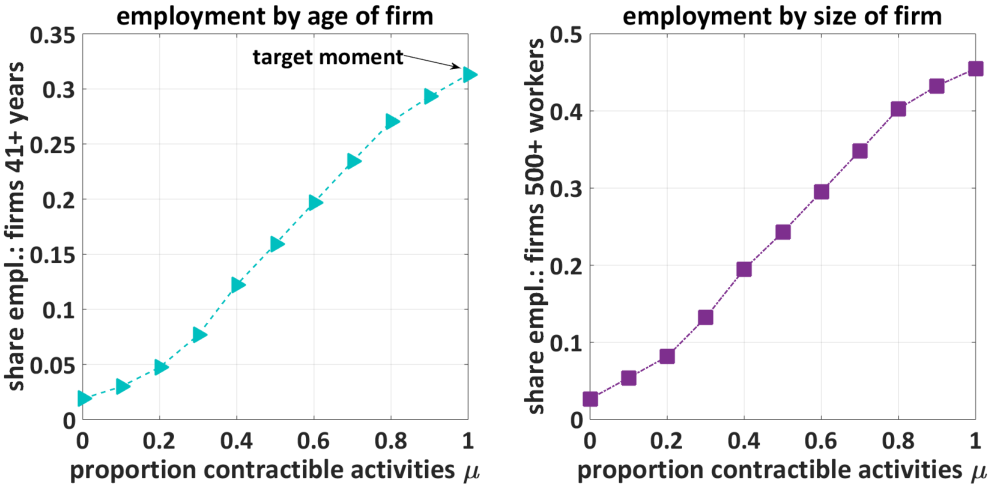
[22]It is only possible to map indirectly this parameter to measures of institutional quality across economies (financial development is typically calibrated using the ratio of credit to GDP, which is measurable and available for a large set of economies). Caselli and Gennaioli (2013) argue that it is reasonable to consider that the judicial system can be inefficient to the extent that contract enforcement is non-existent. Examples of countries in this situation are Cambodia, Malawi and Mozambique, where the monetary cost of enforcing a standardized supplier contract equals the value of the claim (World Bank Doing Business Survey, and Boehm, forthcoming). We discuss some calculations below.

[23]The average level of exogenous productivity does not vary with contract completeness in equilibrium and, therefore, it does not represent a driver of aggregate productivity.

[24]In the baseline parameterization exit rates vary with μ from 0.10 (a target for our calibration), to 0.15. To the best of our knowledge, there is no systematic pattern for exit and entry rates across economies with different levels of development, and the results remain within documented ranges, see Bartelsman et al. (2009).

Parameter α determines the degree of complementarity between inputs. A higher α implies a higher elasticity of substitution between different intermediate inputs, thus every individual supplier becomes less essential in production, increasing the implicit bargaining power of the firm producing final goods. Therefore, the distortion faced by the firm is decreasing in α. In our model, this effect influences the incentives to invest in technology and therefore the life-cycle growth of firms (Figure 3). We show additional results with values of 0.30 and 0.70, the latter is similar to the value of an equivalent parameter in Boehm (forthcoming), employed in a multi-sector model.

Figure 2. Impact of Contract Institutions:
Employment by Age and Size of Firm



Parameter α does not affect the allocation of resources with complete contracts, thus we do not have to recalibrate other parameters to analyze its role. This is not the case for parameter β. This will make the comparison of the different calibrations less transparent as we need to modify other parameters to replicate the target moments discussed for the calibration. We keep the number of modified parameters to a minimum as described next.

In our model, parameter β determines the weight of the production function given to technology and intermediate inputs. Relative to the baseline calibration we reduce β to 0.40, and decrease the per-period fixed cost of production c_f and innovation parameter ξ, to keep exit rates and firm life-cycle growth on targets. In particular, note that a significant reduction in ξ is required, to 0.12 from the baseline value of 0.673. With a lower β, less weight is given to technology n and intermediate inputs, thus the negative effect of contract incompleteness is reduced relative to the baseline calibration (Figure 4). It has also been shown that the bargaining power of the firm is decreasing in β.

Figure 3. Impact of Contract Institutions:
The Role of Parameter α

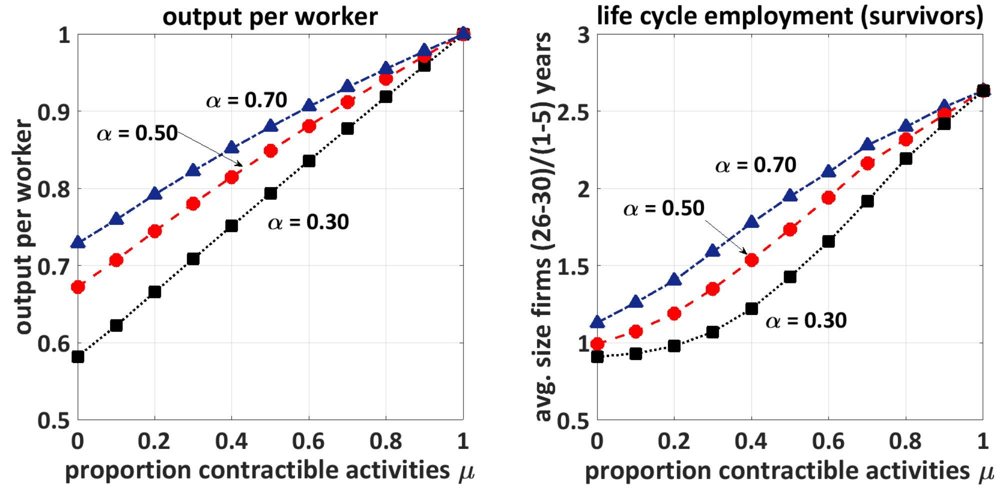
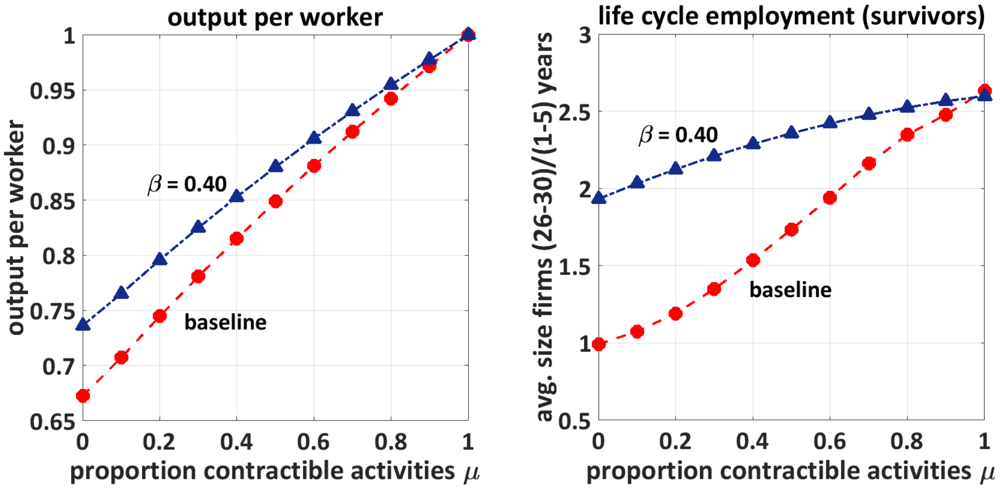


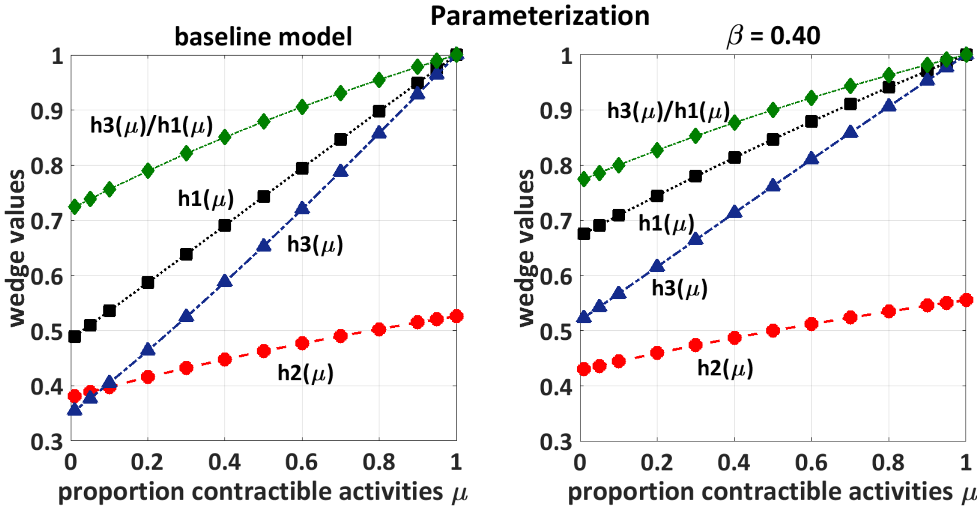
Figure 4. Impact of Contract Institutions:
The Role of Parameter β



7.3 Static and Size-Dependent Distortions

The different wedges of the model were derived and interpreted in Section 4.3.1. We present the values of the different wedges in the model under two alternative calibrations, as a function of contract completeness (Figure 5). With incomplete contracts (see Section 4.3.1 for additional details), relative to an environment with complete contracts we have defined the following distortions: $h_1(\mu)$ is the wedge on labor and contractible activities, $h_3(\mu)$ is the wedge on firm output, $h_2(\mu)$ is the wedge on non-contractible activities relative to contractible activities. Additionally, with incomplete contracts, firm output per worker is proportional to $h_3(\mu)/h_1(\mu)$. As an illustrative example, we can link the firm-level empirical estimations with the theoretical results, specifically on firm size. The empirical results suggest that, due to differences in contract enforcement, on average firm size should be approximately 34 percent smaller for a group of 4 Latin American economies (Argentina, Brazil, Chile, Mexico), relative to either the U.S. or a group of 4 high-income European economies (Austria, Germany, France, Netherlands).[25] In the theoretical model relative average firm size is determined by the wedge $h_1(\mu)$ (see Section 4.3.1), and these differences in size would be associated with a difference in μ of 0.60 (Figure 5).

Figure 5. Wedge Values: Baseline and Alternative Parameterization



Distortions are functions only of parameters α, β, ν and judicial institutions μ. In an environment with incomplete contracts a lower β increases the bargaining power of the firm and reduces the distortions, since the different $h(\mu)$ and $h_3(\mu)/h_1(\mu)$ are higher for all values of μ (Figure 5).

We compute the wedge $1-\tau(\mu) = \pi_i(\mu)/\pi^*$, where π^* are profits in the baseline model with perfect contracts, and $\pi_i(\mu)$ are profits that result from reducing the contract parameter μ, while maintaining the baseline parameterization and prices (Figure 6). This is a partial equilibrium exercise, our goal is to isolate the impact of μ on the relationship between the wedge and the level of technology.[26] In our baseline parameterization (left panel, Figure 6), the wedge on profits is increasing in technology n (as demonstrated in Section 6), but this positive relationship is negligible. Thus, the baseline parameterization is conservative in the sense that higher technology levels are not disproportionately disincentivized. We have shown (Section 6), that for this wedge to be increasing in technology n, it is required that $\nu + \beta(\kappa+1) < 1$, which implies y^*/n is strictly decreasing in n. The condition holds almost with equality under our baseline calibration, but the left hand side is lower

[25] The average of the *enforcing contracts index* for the year 2010 is 61 for the Latin American economies, and 77 for the U.S. as well as for the 4 European economies, then $(77-61)\times 0.021 = 0.34$ (the regression coefficient is documented in Table 2).

[26] In a previous exercise we have discussed that modifying β required altering several additional parameters in order to replicate the moments of the baseline calibration. Comparing the wedges across such different calibrations would make the analysis less transparent.

with a smaller beta, generating a wedge on profits that is more *size-dependent* (right panel, Figure 6).[27]

Figure 6. Size-Dependent Wedge in Profits

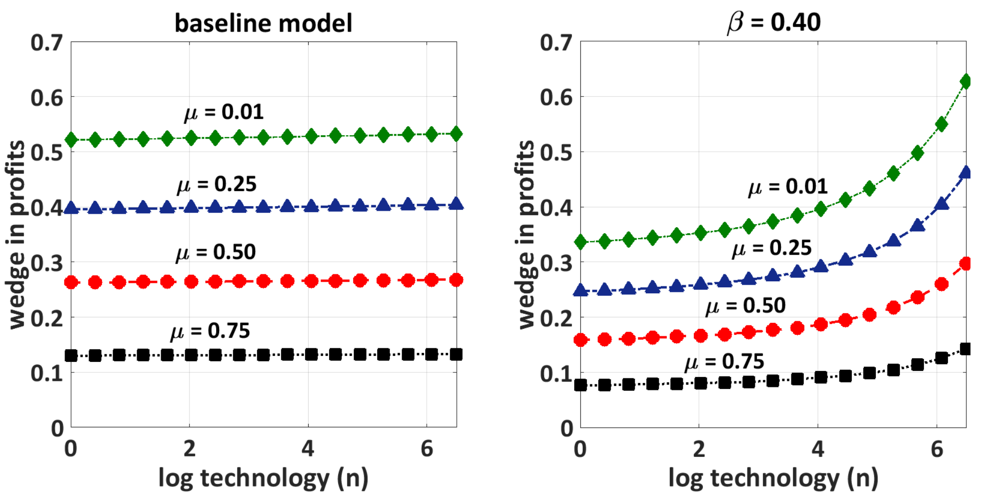
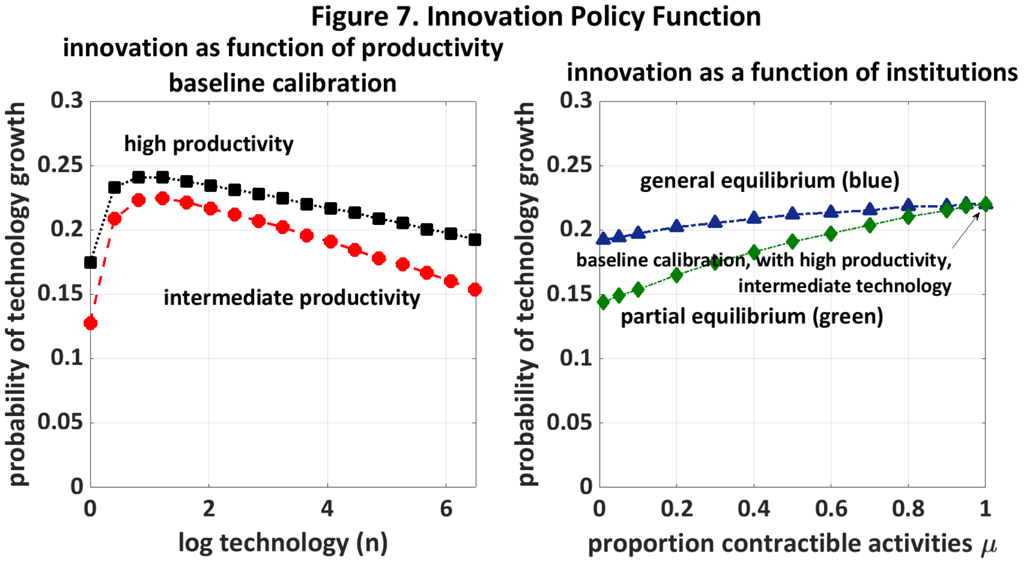


Figure 7. Innovation Policy Function

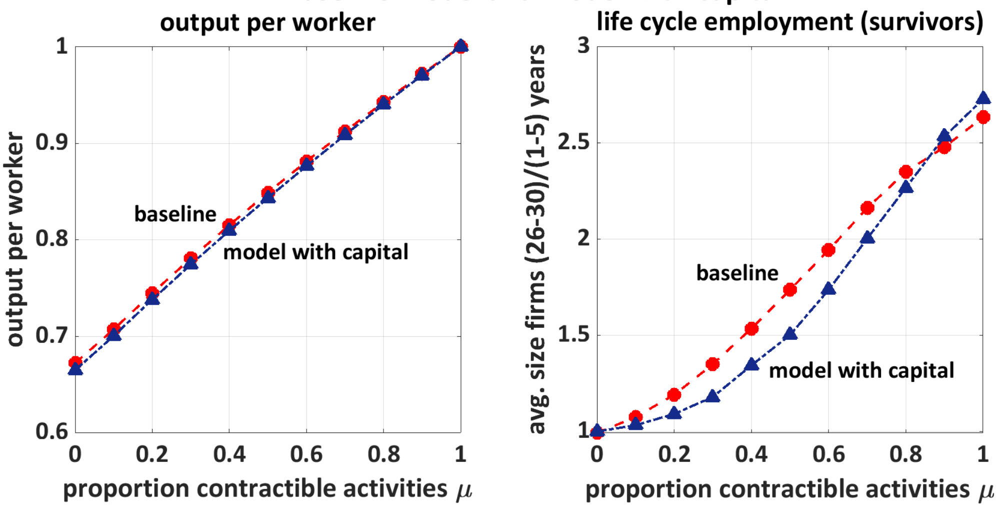


Firm innovation is increasing in exogenous productivity z, but non-monotonic in technology n, which is due to the lower bound in this variable (left panel, Figure

[27]Consistent with the role of this condition we have found negligible effects for α and ω on the size dependent distortion, while ν and κ have a significant influence (additional results are available upon request).

7). Through the wedges already discussed, institutions distort innovation of the firm, although this is in part compensated by general equilibrium effects of lower wages (right panel, Figure 7).[28]

Figure 8. Impact of Contract Institutions: Baseline Model and Model with Capital



7.4 Model with Production Capital

We modify the model by introducing capital in the production function, considering $f(k,l)^{\nu}$ with $f(k,l) = k^{\theta} l^{1-\theta}$, using a standard parameter of θ equal to 1/3 (the main parameters are not modified). The cost of the use of capital is 0.10, equal to the sum of the capital depreciation rate and the interest rate (under a small-open-economy assumption). The quantitative conclusions are practically unchanged (Figure 8).

8 Conclusions

We have constructed a dynamic framework of heterogeneous firms to evaluate the impact of the enforcement of contracts between final goods producers and suppliers on technology accumulation, firm life-cycle growth, and aggregate productivity. We have shown this friction implies a *wedge* on profits that is dependent on the technology level of the firm, in addition to the presence of wedges that affect the level of production. The wedge on profits affects the dynamic incentives to invest in technology and productivity within the firm. This determines the life-cycle growth profile of firms and aggregate productivity, as well as the age and size distribution of

[28]The effect of institutions on innovation depends on the level of productivity z and technology n, the right panel plots the effect for a high productivity level and an intermediate level of technology, where the impact is relatively moderate, as an example to illustrate the role of general equilibrium effects.

firms. We find an economically significant impact of contract enforcement by exploiting a framework similar to those employed in the literature to study firm registration costs, financial and labor market frictions, among other obstacles faced by firms.

In principle, firms could potentially mitigate the distortion caused by contractual incompleteness through vertical integration. This has received attention in the international trade literature (e.g., Antràs and Chor, 2013; Antràs and Helpman, 2006; Schwarz and Suedekum, 2014). However, the possibility of vertical integration confronts the firm with a myriad of other obstacles, particularly in developing economies, that will limit its growth and increase the complexity of the problem.

First, contractual imperfections and monitoring technologies are important in explaining the lack of managerial delegation in developing economies (Laeven and Woodruff, 2007; Caselli and Gennaioli, 2013; Cole et al., 2016; Akcigit et al., 2016, Grobovšek, 2020). As Grossman and Hart (1986) argue, integration shifts the incentives for opportunistic and distortionary behavior, but it does not remove these incentives. Under this view employees, similar to suppliers, require incentives. Thus, in the Property Rights Theories of Grossman and Hart (1986) and Hart and Moore (1990), the effect of contract institutions on vertical integration is not clear.[29] In this regard, Acemoglu et al. (2009) find that contract costs by themselves have no effect on vertical integration. In general, there is no evidence that firms become larger to compensate for inefficiencies of the legal system, and empirical evidence points in the opposite direction (see Section 2). Second, vertical integration may be costly and inefficient (Antràs and Helpman, 2006; Boehm and Oberfield, 2020; Boehm, forthcoming), and implies the firm is forced to develop products for which it has not accumulated know-how and human capital. Furthermore, producing an additional intermediate good will be associated itself with the necessity of additional services and products, subject to the same contracting frictions. Third, financial frictions restrict firm growth while size-dependent distortions, in general, become more severe as the firm becomes larger. Based on these arguments and the findings of the empirical literature already discussed, it seems unlikely that the low quality of judicial institutions is a factor increasing the size of firms in developing economies.

A series of articles in the literature of misallocation consider the interaction of different frictions (e.g., Antunes and Cavalcanti, 2007; Moscoso-Boedo and Mukoyama, 2012; Asturias et al., 2016; Ranasinghe and Restuccia, 2018). This direction of research could offer interesting results in the case of contractual frictions. Additionally, we have abstracted from the possibility that the ability to enforce contracts can alter the industrial structure and comparative advantage across economies (Nunn, 2007; Levchenko, 2007).

To the best of our knowledge, we are the first to explore the role of firm-supplier contract enforcement in a quantitative framework of firm dynamics. We believe there is broad room for further research, specially considering the importance of intermediate goods in economic development (see, for example, Jones, 2011; Grobovšek,

[29] Even in the case of the U.S., shipments of physical goods between vertically integrated plants is modest (Boehm, forthcoming).

2018). The literature has studied different justifications for contract incompleteness including: high costs of enforcement and state verification, the difficulty in describing the terms of a contract, problems of interpretation and the ambiguity of contracts, etc. (see Gennaioli, 2013; Boehm, forthcoming). In addition to issues already enumerated, the influence of contract enforcement on the quality of intermediate goods, or different specifications of multilateral repeated bargaining protocols could be explored.[30] Based on our quantitative results, we have argued that frictions that distort the ability of firms to contract with suppliers are important to understand differences in development across economies.

[30]Repeated bargaining does not eliminate inefficiencies (we will not attempt to present an exhaustive set of references on these issues). Cai (2003), for example, studies a complete-information alternating-offer bargaining game where some of the Markov Perfect Equilibria exhibit wasteful delays. Furthermore, the maximum number of delay periods that can be supported in this type of equilibria increases in the order of the square of the number of players. Cai (2003) provides additional references and an enumeration of potential sources of inefficiencies in these models. Wolinsky (2000) analyzes a model of contracting and recontracting between a firm and its workers, where the unique stationary equilibrium is inefficient. Ray and Vohra (2015) provide a thorough discussion of the possibility (and problems) of achieving efficiency in the context of coalition formation.

References

Acemoglu, Daron, Francisco A. Gallego, and James A. Robinson, "Institutions, Human Capital, and Development ," *Annual Review of Economics*, August 2014, *6* (1), 875–912.

_ , Pol Antràs, and Elhanan Helpman, "Contracts and Technology Adoption," *American Economic Review*, June 2007, *97* (3), 916–943.

_ , Simon Johnson, and Todd Mitton, "Determinants of Vertical Integration: Financial Development and Contracting Costs," *Journal of Finance*, June 2009, *64* (3), 1251–1290.

Akcigit, Ufuk, Harun Alp, and Michael Peters, "Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries," NBER Working Papers 21905, National Bureau of Economic Research, Inc January 2016.

Amaral, Pedro S. and Erwan Quintin, "Limited Enforcement, Financial Intermediation, and Economic Development: A Quantitative Assessment," *International Economic Review*, August 2010, *51* (3), 785–811.

Amirapu, Amrit and Michael Gechter, "Labor Regulations and the Cost of Corruption: Evidence from the Indian Firm Size Distribution," *The Review of Economics and Statistics*, 2020, *102* (1), 34–48.

Antràs, Pol and Davin Chor, "Organizing the Global Value Chain," *Econometrica*, November 2013, *81* (6), 2127–2204.

_ and Elhanan Helpman, "Contractual Frictions and Global Sourcing," NBER Working Papers 12747, National Bureau of Economic Research, Inc December 2006.

Antunes, Antonio R. and Tiago V. de V. Cavalcanti, "Start up costs, limited enforcement, and the hidden economy," *European Economic Review*, January 2007, *51* (1), 203–224.

Asturias, Jose, Sewon Hur, Timothy J. Kehoe, and Kim J. Ruhl, "The interaction and sequencing of policy reforms," *Journal of Economic Dynamics and Control*, 2016, *72* (C), 45–66.

Atkeson, Andrew and Ariel Tomás Burstein, "Innovation, Firm Dynamics, and International Trade," *Journal of Political Economy*, June 2010, *118* (3), 433–484.

Bachas, Pierre, Roberto N. Fattal Jaef, and Anders Jensen, "Size-dependent tax enforcement and compliance: Global evidence and aggregate implications," *Journal of Development Economics*, 2019, *140*, 203 – 222.

Banerjee, Abhijit V. and Esther Duflo, "Growth Theory through the Lens of Development Economics," in Philippe Aghion and Steven Durlauf, eds., *Handbook of Economic Growth*, Vol. 1 of *Handbook of Economic Growth*, Elsevier, August 2005, chapter 7, pp. 473–552.

Barseghyan, Levon, "Entry costs and cross-country differences in productivity and output," *Journal of Economic Growth*, June 2008, *13* (2), 145–167.

_ and Riccardo DiCecio, "Entry costs, industry structure, and cross-country income and TFP differences," *Journal of Economic Theory*, September 2011, *146* (5), 1828–1851.

Bartelsman, Eric, John Haltiwanger, and Stefano Scarpetta, "Measuring and Analyzing Cross-Country Differences in Firm Dynamics," in "Producer Dynamics: New Evidence from Micro Data" NBER Chapters, National Bureau of Economic Research, Inc, June 2009, pp. 15–76.

Beck, Thorsten, Asli Demirguc-Kunt, and Vojislav Maksimovic, "Financial and Legal Constraints to Growth: Does Firm Size Matter?," *Journal of Finance*, February 2005, *60* (1), 137–177.

_ , _ , and _ , "The influence of financial and legal institutions on firm size," *Journal of Banking & Finance*, November 2006, *30* (11), 2995–3015.

Bento, Pedro and Diego Restuccia, "Misallocation, Establishment Size, and Productivity," *American Economic Journal: Macroeconomics*, July 2017, *9* (3), 267–303.

_ and _ , "On Average Establishment Size across Sectors and Countries," Working Papers, University of Toronto, Department of Economics 2018.

Bernard, Andrew B., Andreas Moxnes, and Yukiko U. Saito, "Production Networks, Geography, and Firm Performance," *Journal of Political Economy*, 2019, *127* (2), 639–688.

Bernard, Andrew B, Emmanuel Dhyne, Glenn Magerman, Kalina Manova, and Andreas Moxnes, "The Origins of Firm Heterogeneity: A Production Network Approach," Working Paper 25441, National Bureau of Economic Research January 2019.

Bhattacharya, Dhritman, Nezih Guner, and Gustavo Ventura, "Distortions, Endogenous Managerial Skills and Productivity Differences," *Review of Economic Dynamics*, January 2013, *16* (1), 11–25.

Boehm, Johannes, "The Impact of Contract Enforcement Costs on Value Chains and Aggregate Productivity," *Review of Economics and Statistics*, forthcoming.

_ and Ezra Oberfield, "Misallocation in the Market for Inputs: Enforcement and the Organization of Production," *Quarterly Journal of Economics*, November 2020, *135* (4), 2007–2058.

Buera, Francisco J. and Roberto N. Fattal-Jaef, "The Dynamics of Development: Innovation and Reallocation," February 2016.

_ , Joseph P. Kaboski, and Yongseok Shin, "Finance and Development: A Tale of Two Sectors," *American Economic Review*, August 2011, *101* (5), 1964–2002.

Busso, Matias, Maria Victoria Fazio, and Santiago Levy Algazi, "(In)Formal and (Un)Productive: The Productivity Costs of Excessive Informality in Mexico," Research Department Publications 4789, Inter-American Development Bank, Research Department August 2012.

Cai, Hongbin, "Inefficient Markov perfect equilibria in multilateral bargaining," *Economic Theory*, October 2003, *22* (3), 583–606.

Caselli, Francesco and Nicola Gennaioli, "Dynastic Management," *Economic Inquiry*, January 2013, *51* (1), 971–996.

Chakraborty, Pavel, "Judicial quality and regional firm performance: The case of Indian states," *Journal of Comparative Economics*, 2016, *44* (4), 902–918.

Cole, Harold L., Jeremy Greenwood, and Juan M. Sanchez, "Why Doesn't Technology Flow From Rich to Poor Countries?," *Econometrica*, July 2016, *84*, 1477–1521.

Da-Rocha, José-María, Diego Restuccia, and Marina Mendes Tavares, "Firing costs, misallocation, and aggregate productivity," *Journal of Economic Dynamics and Control*, 2019, *98*, 60 – 81.

Da-Rocha, José-María, Marina Mendes Tavares, and Diego Restuccia, "Policy Distortions and Aggregate Productivity with Endogenous Establishment-Level Productivity," NBER Working Papers 23339, National Bureau of Economic Research, Inc April 2017.

D'Erasmo, Pablo N. and Hernan J. Moscoso-Boedo, "Financial structure, informality and development," *Journal of Monetary Economics*, 2012, *59* (3), 286–302.

Dougherty, Sean M., "Legal Reform, Contract Enforcement and Firm Size in Mexico," *Review of International Economics*, September 2014, *22* (4), 825–844.

Farias, Vivek, Denis Saure, and Gabriel Y. Weintraub, "An approximate dynamic programming approach to solving dynamic oligopoly models," *RAND Journal of Economics*, June 2012, *43* (2), 253–282.

Feenstra, Robert C., Robert Inklaar, and Marcel P. Timmer, "The Next Generation of the Penn World Table," *American Economic Review*, 2015, *105* (10), 3150–3182.

Fossati, Roman, Heiko Rachinger, and Matheus Stivali, "Extent and potential determinants of resource misallocation: A cross-sectional study for developing countries," *The World Economy*, forthcoming.

Gabler, Alain and Markus Poschke, "Experimentation by Firms, Distortions, and Aggregate Productivity," *Review of Economic Dynamics*, January 2013, *16* (1), 26–38.

García-Posada, Miguel and Juan Mora-Sanguinetti, "Does (average) size matter? Court enforcement, business demography and firm growth," *Small Business Economics*, March 2015, *44* (3), 639–669.

Garcia-Santana, Manuel and Josep Pijoan-Mas, "The reservation laws in India and the misallocation of production factors," *Journal of Monetary Economics*, 2014, *66* (C), 193–209.

_ **and Roberto Ramos**, "Distortions and the size distribution of plants: evidence from cross-country data," *SERIEs: Journal of the Spanish Economic Association*, August 2015, *6* (3), 279–312.

Garicano, Luis, Claire Lelarge, and John Van Reenen, "Firm Size Distortions and the Productivity Distribution: Evidence from France," *American Economic Review*, November 2016, *106* (11), 3439–3479.

Gennaioli, Nicola, "Optimal Contracts With Enforcement Risk," *Journal of the European Economic Association*, February 2013, *11* (1), 59–82.

Giacomelli, Silvia and Carlo Menon, "Does weak contract enforcement affect firm size? Evidence from the neighbour's court," *Journal of Economic Geography*, 2017, *17* (6), 1251–1282.

Gourio, Francois and Nicolas Roys, "Size-dependent regulations, firm size distribution, and reallocation," *Quantitative Economics*, July 2014, *5*, 377–416.

Greenwood, Jeremy, Juan Sanchez, and Cheng Wang, "Quantifying the Impact of Financial Development on Economic Development," *Review of Economic Dynamics*, January 2013, *16* (1), 194–215.

Grobovšek, Jan, "Development accounting with intermediate goods," *The B.E. Journal of Macroeconomics*, January 2018, *18* (1), 1–27.

_ , "Managerial Delegation, Law Enforcement, and Aggregate Productivity," *The Review of Economic Studies*, 05 2020, *87* (5), 2256–2289.

Grossman, Sanford and Oliver Hart, "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 1986, *94* (4), 691–719.

Guner, Nezih, Gustavo Ventura, and Xu Yi, "Macroeconomic Implications of Size-Dependent Policies," *Review of Economic Dynamics*, October 2008, *11* (4), 721–744.

Hart, Oliver D. and John Moore, "Property Rights and the Nature of the Firm," Technical Report 1990.

Hill, Enoch and David Perez-Reyna, "Public Law Enforcement: More is not Always Better," December 2015.

_ **and** _ , "Financial development and occupational choice," *Journal of Macroeconomics*, 2017, *54* (PB), 393–409.

Hopenhayn, Hugo A., "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, September 1992, *60* (5), 1127–50.

— , "On the Measure of Distortions," NBER Working Papers 20404, National Bureau of Economic Research, Inc August 2014.

Hopenhayn, Hugo and Richard Rogerson, "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, October 1993, *101* (5), 915–938.

Hsieh, Chang-Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 2009, *124* (4), 1403–1448.

— **and** — , "The Life Cycle of Plants in India and Mexico," May 2012.

— **and** — , "The Life Cycle of Plants in India and Mexico," *The Quarterly Journal of Economics*, 2014, *129* (3), 1035–1084.

Jones, Charles I., "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, April 2011, *3* (2), 1–28.

Klette, Tor Jakob and Samuel Kortum, "Innovating Firms and Aggregate Innovation," *Journal of Political Economy*, October 2004, *112* (5), 986–1018.

Laeven, Luc and Christopher Woodruff, "The Quality of the Legal System, Firm Ownership, and Firm Size," *The Review of Economics and Statistics*, November 2007, *89* (4), 601–614.

Lagos, Ricardo, "A Model of TFP," *Review of Economic Studies*, 2006, *73* (4), 983–1007.

Lee, Yoonsoo and Toshihiko Mukoyama, "Productivity and employment dynamics of US manufacturing plants," *Economics Letters*, 2015, *136* (C), 190–193.

Levchenko, Andrei A., "Institutional Quality and International Trade," *Review of Economic Studies*, 2007, *74* (3), 791–819.

Lopez, Jose Joaquin, "A quantitative theory of tax evasion," *Journal of Macroeconomics*, 2017, *53* (C), 107–126.

— **and Jesica Torres**, "Size-dependent policies, talent misallocation, and the return to skill," *Review of Economic Dynamics*, forthcoming.

Lopez-Martin, Bernabe, "From Firm Productivity Dynamics to Aggregate Efficiency," *World Bank Economic Review*, 2017, *30* (Supplement), 57–66.

— , "Informal Sector Misallocation," *Macroeconomic Dynamics*, 2019, *23* (8), 3065–3098.

McGrattan, Ellen R. and Edward C. Prescott, "Unmeasured Investment and the Puzzling US Boom in the 1990s," *American Economic Journal: Macroeconomics*, October 2010, *2* (4), 88–123.

Midrigan, Virgiliu and Daniel Yi Xu, "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, February 2014, *104* (2), 422–58.

Moll, Benjamin, "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," *American Economic Review*, October 2014, *104* (10), 3186–3221.

Moscoso-Boedo, Hernan J. and Toshihiko Mukoyama, "Evaluating the effects of entry regulations and firing costs on international income differences," *Journal of Economic Growth*, 2012, *17* (2), 143–170.

Mukoyama, Toshihiko and Latchezar Popov, "Industrialization and the evolution of enforcement institutions," *Economic Theory*, April 2020, *69* (3), 745–788.

_ **and Sophie Osotimehin**, "Barriers to Reallocation and Economic Growth: The Effects of Firing Costs," *American Economic Journal: Macroeconomics*, October 2019, *11* (4), 235–270.

Nunn, Nathan, "Relationship-Specificity, Incomplete Contracts, and the Pattern of Trade," *The Quarterly Journal of Economics*, 2007, *122* (2), 569–600.

Pakes, Ariel and Paul McGuire, "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model," *RAND Journal of Economics*, Winter 1994, *25* (4), 555–589.

Poschke, Markus, "The Firm Size Distribution across Countries and Skill-Biased Change in Entrepreneurial Technology," *American Economic Journal: Macroeconomics*, July 2018, *10* (3), 1–41.

Ranasinghe, Ashantha, "Impact of policy distortions on firm-level innovation, productivity dynamics and TFP," *Journal of Economic Dynamics and Control*, 2014, *46* (C), 114–129.

_ , "Property rights, extortion and the misallocation of talent," *European Economic Review*, 2017, *98* (C), 86–110.

_ **and Diego Restuccia**, "Financial frictions and the rule of law," *Journal of Development Economics*, 2018, *134* (C), 248–271.

Ray, Debraj and Rajiv Vohra, *Coalition Formation*, Vol. 4 of *Handbook of Game Theory with Economics Applications*, Elsevier, 2015.

Restuccia, Diego and Richard Rogerson, "Policy Distortions and Aggregate Productivity with Heterogeneous Plants," *Review of Economic Dynamics*, October 2008, *11* (4), 707–720.

Schwarz, Christian and Jens Suedekum, "Global sourcing of complex production processes," *Journal of International Economics*, 2014, *93* (1), 123–139.

Tauchen, George, "Finite state Markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, 1986, *20* (2), 177–181.

Tybout, James R., "Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?," *Journal of Economic Literature*, March 2000, *38* (1), 11–44.

Ulyssea, Gabriel, "Firms, Informality, and Development: Theory and Evidence from Brazil," *American Economic Review*, August 2018, *108* (8), 2015–47.

Wolinsky, Asher, "A Theory of the Firm with Non-Binding Employment Contracts," *Econometrica*, July 2000, *68* (4), 875–910.

Appendix A Mathematical Appendix

Lemma 1 derives the benchmark values for activities, labor and production.

Lemma 1. *The equilibrium values for activities, labor and production are given by*

$$x^* = \frac{1}{n}\left[\left(\frac{\nu}{w}\right)^{\nu}\left(\frac{\beta}{c_x}\right)^{1-\nu} z^{1-\beta} n^{\beta\kappa}\right]^{\frac{1}{1-\nu-\beta}}, \quad l^* = \left[\left(\frac{\nu}{w}\right)^{1-\beta}\left(\frac{\beta}{c_x}\right)^{\beta} z^{1-\beta} n^{\beta\kappa}\right]^{\frac{1}{1-\nu-\beta}},$$

$$\text{and} \quad y^* = \left[\left(\frac{\nu}{w}\right)^{\nu}\left(\frac{\beta}{c_x}\right)^{\beta} z^{1-\beta} n^{\beta\kappa}\right]^{\frac{1}{1-\nu-\beta}}.$$

Proof. The first order condition of (3) with respect to x is:

$$\beta\, z^{1-\beta}\, n^{\beta\,(\kappa+1)-1}\, x^{\beta-1}\, l^{\nu} = c_x \tag{A1}$$

while the the first order condition with respect to l is:

$$\nu\, z^{1-\beta}\, n^{\beta\,(\kappa+1)}\, x^{\beta}\, l^{\nu-1} = w \tag{A2}$$

Take the ratio of (A1) and (A2):

$$l = \frac{c_x}{w}\frac{\nu}{\beta}\, nx; \tag{A3}$$

replace in (A2):

$$\nu\, z^{1-\beta}\, n^{\beta\,(\kappa+1)}\, x^{\beta}\left[\frac{c_x}{w}\frac{\nu}{\beta}\, n\, x\right]^{\nu-1} = w$$

then:

$$x^{1-\nu-\beta} = \frac{\nu\, z^{1-\beta}\, n^{\beta\,(\kappa+1)}}{w}\left[\frac{c_x}{w}\frac{\nu}{\beta}\, n\right]^{\nu-1}. \tag{A4}$$

(A3) and (A4) yield the result. □

Lemma 2 shows sufficient conditions to guarantee that the objective function in (9) is strictly concave.

Lemma 2. $1 > \beta + \nu$ *is a sufficient condition for the objective function in* (9) *to be strictly concave.*

Proof. If we replace (8) into (9), we can write the objective function as

$$Bx_c^{\frac{\beta\mu}{1-\beta(1-\mu)}}\, l^{\frac{\nu}{1-\beta(1-\mu)}} - c_x n\mu x_c - \omega n - wl, \tag{A5}$$

where

$$B \equiv \frac{\alpha+\beta-\alpha\beta\,(1-\mu)}{1-\beta(1-\mu)}\left[\frac{1}{\alpha+\beta}\left(\frac{\alpha\beta}{c_x}\right)^{\beta(1-\mu)} z^{1-\beta} n^{\beta(\kappa+\mu)}\right]^{\frac{1}{1-\beta(1-\mu)}}.$$

(A5) is a Cobb-Douglas function having x_c and l as inputs, so it is strictly concave in x_c and l as long as

$$\frac{\beta\mu}{1-\beta(1-\mu)} + \frac{\nu}{1-\beta(1-\mu)} < 1,$$

which is equivalent to $1 > \beta + \nu$. □

Proposition 1 shows that μ governs the wedge between input demand, labor and profits under complete contracts and under incomplete contracts. This wedge is decreasing in μ and disappears when $\mu = 1$. One consequence of this proposition is that input demand, labor and profits are increasing in μ.

Proposition 1. *Let*

$$h_1(\mu) \equiv \left[\frac{1}{\alpha+\beta}\left(\frac{\alpha+\beta-\alpha\beta(1-\mu)}{1-\beta(1-\mu)}\right)^{1-\beta(1-\mu)} \alpha^{\beta(1-\mu)}\right]^{\frac{1}{1-\nu-\beta}}$$
$$h_2(\mu) \equiv \alpha\frac{1-\beta(1-\mu)}{\alpha+\beta-\alpha\beta(1-\mu)}$$

and denote by $x_c(n,z;\mu)$ the demand for contractible inputs, $x_n(n,z;\mu)$ the demand for noncontractible inputs, $l_i(n,z;\mu)$ the demand for labor, $y_i(n,z;\mu)$ production and $\pi_i(n,z;\mu)$ profits under incomplete contracts. Similarly, let $x^*(n,z)$ be the demand for inputs, $l^*(n,z)$ the demand for labor, $y^*(n,z)$ production and $\pi^*(n,z)$ profits under complete contracts. Then

$$\begin{aligned}
x_c(n,z;\mu) &= h_1(\mu)\, x^*(n,z)\\
x_n(n,z;\mu) &= h_2(\mu)\, x_c(n,z;\mu)\\
l_i(n,z;\mu) &= h_1(\mu)\, l^*(n,z)\\
y_i(n,z;\mu) &= h_1(\mu)^{\beta+\nu} h_2(\mu)^{(1-\mu)\beta} y^*(n,z)\\
\pi^*(n,z) &= (1-\beta-\nu)y^*(n,z) - \omega n\\
\pi_i(n,z;\mu) &= (1-\beta-\nu)h_1(\mu)y^*(n,z) - \omega n.
\end{aligned}$$

Furthermore, $h_1'(\mu) > 0$, $h_1(1) = 1$ and $h_2'(\mu) > 0$, $h_2(1) = \frac{\alpha}{\alpha+\beta}$.

Proof. First we will prove the properties of $h_i(\mu)$. Noting that $h_1(1) = 1$ and $h_2(1) = \frac{\alpha}{\alpha+\beta}$ is straightforward. To prove that $h_1'(\mu) > 0$ consider

$$f_1(\mu) \equiv (1-\beta(1-\mu))\left[\ln(\alpha+\beta-\alpha\beta(1-\mu)) - \ln(1-\beta(1-\mu))\right] + \beta(1-\mu)\ln\alpha.$$

Proving that $f_1'(\mu) > 0$ is equivalent to proving that $h_1'(\mu) > 0$. Notice that

$$f_1'(\mu) = \beta\left[\ln\left(\frac{\alpha+\beta-\alpha\beta(1-\mu)}{\alpha(1-\beta(1-\mu))}\right) - \frac{\beta}{\alpha+\beta-\alpha\beta(1-\mu)}\right].$$

Let

$$a \equiv \frac{\alpha+\beta-\alpha\beta(1-\mu)}{\alpha(1-\beta(1-\mu))} = 1 + \frac{\beta}{\alpha(1-\beta(1-\mu))}.$$

Since $\beta \in (0,1)$, then $a > 1$. Additionally,

$$\frac{\beta}{\alpha+\beta-\alpha\beta(1-\mu)} = 1 - \frac{1}{a},$$

so proving that $f_1'(\mu) > 0$ is equivalent to proving that $g(a) = \ln a - 1 + \frac{1}{a} > 0$ for $a > 1$. Notice that $g(1) = 0$ and $g'(a) = (a-1)/a^2 > 0$ for $a > 1$.

To prove that $h_2'(\mu) > 0$ consider $f_2(\mu) \equiv \ln(1-\beta(1-\mu)) - \ln(\alpha+\beta-\alpha\beta(1-\mu))$. $f_2'(\mu) > 0$ is equivalent to proving that $h_2'(\mu) > 0$. Notice that

$$f_2'(\mu) = \frac{\beta}{1-\beta(1-\mu)} - \frac{\alpha\beta}{\alpha+\beta-\alpha\beta(1-\mu)}.$$

$f_2'(\mu) > 0$ if and only if $\beta > 0$, which holds by assumption.

To complete the proof we replace (8) into (9).[31] Taking first order conditions with respect to x_c and l yields:

$$\beta\mu\frac{\Psi}{x_c} = c_x n\mu \tag{A6}$$

$$\nu\frac{\Psi}{l} = w, \tag{A7}$$

where

$$\Psi \equiv \frac{\alpha+\beta-\alpha\beta(1-\mu)}{1-\beta(1-\mu)}\left[\frac{1}{\alpha+\beta}\left(\frac{\alpha\beta}{c_x}\right)^{\beta(1-\mu)} z^{1-\beta} x_c^{\beta\mu} n^{\beta(\kappa+\mu)} l^{\nu}\right]^{\frac{1}{1-\beta(1-\mu)}}.$$

If we divide (A6) over (A7) we get:

$$l = \frac{c_x}{w}\frac{\nu}{\beta} n\, x_c. \tag{A8}$$

Replacing (A8) into (A6) and solving for x_c yields:

$$x_c^{1-\nu-\beta} = h_1(\mu)^{1-\nu-\beta} \times \frac{\nu\, z^{1-\beta}\, n^{\beta(\kappa+1)}}{w}\left[\frac{c_x}{w}\frac{\nu}{\beta} n\right]^{\nu-1} \tag{A9}$$

We can then use (A8) and (A9) to get an expression for l_i.

From replacing (A3) and (A4) into (A8) and (A9) we get expressions for x_c and l_i in terms of x^* and l^*. Then, replacing into (8) we get an expression for x_n. Plugging into

$$y_i = z^{1-\beta} n^{\beta(\kappa+1)} x_c^{\beta\mu} x_n^{\beta(1-\mu)} l_i^{\nu}$$

yields the expression for y_i in terms of y^*.

[31] $1 > \beta + \nu$ is a sufficient condition to guarantee that the objective function in (9) is strictly concave. This result is stated and proven in Lemma 2.

To finish the proof, recall that

$$\pi^* = y^* - nx^*c_x - wl^* - \omega n.$$

Since $n\, c_x\, x^* = \beta\, y^*$ and $w\, l^* = \nu\, y^*$ we get the result for π^*. On the other hand

$$\pi_i = y_i - c_x n(\mu x_c + (1-\mu)x_n) - wl_i - \omega n.$$

Plugging the expressions for y_i, x_c, x_n and l_i and the fact that $n\, c_x\, x^* = \beta\, y^*$ and $w\, l^* = \nu\, y^*$ yields

$$\pi_i = \left(\frac{h_2(\mu)^{(1-\mu)\beta}}{h_1(\mu)^{1-\nu-\beta}} - (\mu + (1-\mu)h_2(\mu))\, \beta - \nu \right) h_1(\mu) y^* - \omega n.$$

We will now prove that

$$\frac{h_2(\mu)^{(1-\mu)\beta}}{h_1(\mu)^{1-\nu-\beta}} - (\mu + (1-\mu)h_2(\mu))\, \beta - \nu = 1 - \beta - \nu.$$

First,

$$(\mu + (1-\mu)h_2(\mu))\, \beta = \frac{\alpha + \beta\mu - \alpha\beta(1-\mu)}{\alpha + \beta - \alpha\beta(1-\mu)} \beta.$$

Also,

$$\begin{aligned} \frac{h_2(\mu)^{(1-\mu)\beta}}{h_1(\mu)^{1-\nu-\beta}} &= \left[\alpha \frac{1-\beta(1-\mu)}{\alpha + \beta - \alpha\beta(1-\mu)} \right]^{(1-\mu)\beta} \frac{\alpha + \beta}{\alpha^{\beta(1-\mu)}} \left(\frac{1-\beta(1-\mu)}{\alpha + \beta - \alpha\beta(1-\mu)} \right)^{1-\beta(1-\mu)} \\ &= (\alpha + \beta) \frac{1-\beta(1-\mu)}{\alpha + \beta - \alpha\beta(1-\mu)}, \end{aligned}$$

which implies

$$\frac{h_2(\mu)^{(1-\mu)\beta}}{h_1(\mu)^{1-\nu-\beta}} - (\mu + (1-\mu)h_2(\mu))\, \beta = 1 - \beta.$$

□

Appendix B Additional Country-Level Regressions

In this Appendix we regress (log) TFP from the Penn World Tables and (log) average firm size from Bento and Restuccia (2017) on the various controls considered previously.[32] We find additional evidence that the mechanism that we highlight in this article plays a statistically and economically significant role. In the case of country TFP, contract enforcement resulted statistically significant across the IV regressions (Table B1).

Table B1 Country-Level Empirical Analysis: TFP (IV).

contract enforcement	0.059***	0.052**	0.033**	0.035**
credit-output ratio	-0.001	-0.001	-0.000	0.000
government corruption	-0.162	-0.097	-0.067	-0.066
registration costs	0.000	-0.000	-0.001**	-0.001***
property rights	—	-0.001	0.003	0.003*
tax rate	—	—	0.003***	0.003***
business freedom	—	—	—	-0.004
F-statistic (IV)	10.12***	7.83***	9.42***	8.88***
n. observations	849	825	679	679

Statistical significance: *** 1%, ** 5%, * 10%.
All regressions include year fixed effects and intercepts.

In the case of average firm size (in logs, Table B2), the coefficients of contract enforcement are highly significant. To provide an example, according to this estimate if the quality of contract enforcement in Colombia (with a level of 38.45 in 2015) improved to a level equivalent of that in Germany (74.96), average firm size would increase by 62%, compared to the 95% difference observed in the data.

Table B2 Country-Level Empirical Analysis: Avg. Firm Size (OLS).

contract enforcement	0.018***	0.016***	0.017***	0.017***
credit-output ratio	0.002***	0.001*	0.000	0.000
government corruption	0.055**	-0.032	-0.003	-0.002
registration costs	-0.001***	-0.001***	-0.001**	-0.001**
property rights	—	0.007***	0.006***	0.007***
tax rate	—	—	-0.002**	-0.002**
business freedom	—	—	—	-0.002
R-squared	0.341	0.353	0.368	0.368
n. observations	852	838	688	687

Statistical significance: *** 1%, ** 5%, * 10%.
All regressions include year fixed effects and intercepts.

[32] See Bento and Restuccia (2017) and Bento and Restuccia (2018) for alternative specifications and empirical determinants, with similar regressions, of average firm size across countries.

Appendix C Description of WBES Database

The WBES includes data from 2006 to 2017.[33] Firms belong to a representative sample of private formal companies in manufacturing and services sectors (non-agricultural), with 5 or more employees. In total we use 99,223 firm-level observations. Observations for each year vary, Table C1 shows the number of observations available per year.

Table C1. Sample Size

Year	Freq.
2006	12,451
2007	5,062
2008	3,910
2009	10,037
2010	11,866
2011	2,010
2012	270
2013	21,527
2014	17,010
2015	6,094
2016	8,162
2017	1,738
Total	100,137

Table C2 shows the averages, across countries in each region, of control variables we include in the firm level regressions. Surveys for Central Asia and Latin America have on average more observations per country than other regions. The average size of firms is 89.6 workers, and is approximately 18 years old. Additionally, close to a quarter export and 11.4% are foreign owned, according to the definition where more than 50% of the firm is owned by foreign individuals. There is dispersion in the variables across regions. On average, firms in the surveys from Caribbean countries are smaller with 58.7 employees, and older with an average firm age of 27 years, while firms from Central Asian countries tend to export less, with a share of exporters of 11.9%, and are less prone to be foreign owned with a share of 3.6%.

[33] Documentation is available at https://www.enterprisesurveys.org/en/methodology

Table C2. WBES Surveys by Region (averages)

region	number of firms	workers per firm	firm age	share exporters	share foreign
Africa	843.1	65.1	15.4	19.7	15.7
Caribbean	283.2	58.7	27.2	22.4	13.6
Central Asia	2011.7	85.3	16.2	11.9	3.6
East Asia	936.3	186.5	17.3	23.6	13.9
Eastern Europe	772.0	80.0	16.7	29.6	9.1
Latin America	1157.3	105.6	24.0	25.7	9.7
Middle East	816.0	69.4	17.8	27.6	4.7
Average	935.9	89.6	18.2	23.3	11.4

Appendix D Firm-Level Databases

In this section of the Appendix we describe the panel we constructed for Latin American countries, based on WBES. The panel includes firms from Argentina, Bolivia, Colombia, Ecuador, Guatemala, Paraguay, Peru and Uruguay for which we can track data for at least two waves of WBES, which occurred in 2006, 2010 and 2017. Table D1 shows the number of firms for each year and country.

Table D1. Observations Lat. America.

	2006 - 2010	2010 - 2017
country	**freq.**	**freq.**
Argentina	180	176
Bolivia	82	82
Colombia	110	110
Ecuador	75	49
Guatemala	169	82
Paraguay	62	63
Peru	148	148
Uruguay	61	60
Total	887	770

Across the different countries, the distribution of firms is similar to Latin American firms that we consider for the cross-section regressions. The average number of employees per firm for our panel in the initial year is 101.9 compared to 105.6 in the cross section, and firm age in the initial year is 27.5 years compared to 24.0 in the cross section. The proportion of firms that exports is higher, 31.4% compared to 25.7%, and a smaller share of firms are foreign, 8.5% compared to 9.7%. Table D2 shows the average across countries for these variables.

Table D2. Panel of Firms Latin America Summary Statistics.					
country	number of firms	workers per firm	firm age	share exporters	share foreign
Argentina	358	93.8	32.1	40.2	6.7
Bolivia	164	88.7	25.8	17.1	9.8
Colombia	220	94.9	20.6	29.5	3.2
Ecuador	159	105.5	29.6	38.4	10.1
Guatemala	376	112.3	23.9	40.2	4.3
Paraguay	126	74.0	23.8	20.6	14.3
Peru	296	172.1	24.2	35.1	7.4
Uruguay	121	74.0	40.1	29.8	12.4
Average	227.5	101.9	27.5	31.4	8.5

Additionally, we use a panel of firms available from WBES for Eastern Europe and a small number of Asian countries, for the years 2009-2013. We include available data for Central Asia. Table D3 shows the number of firms included for each country. Table D4 shows the average for the variables that we use as controls. Comparing to Table B2, in this panel firms are slightly bigger, 80 workers compared to 96.1 workers per firm. Average age is similar at close to 16 years, with Serbian firms having a larger difference. The proportion of firms that export is close to 30 percent in both databases, while the proportion of firms that are foreign is 10.8% compared to 9.1%.

Table D3. Observations Eastern Europe - Asia, 2009-2013.			
country	**freq.**	**country**	**freq.**
Albania	116	Kazakhstan	64
Armenia	167	Latvia	71
Azerbaijan	61	Lithuania	38
Bulgaria	63	Moldova	168
Croatia	37	Mongolia	103
Czechia	16	Poland	12
Estonia	73	Serbia	101
Hungary	57	Slovenia	79
Total		1728	

Table D4. Eastern Europe - Asia WBES Panel Summary Statistics.					
country	number of firms	workers per firm	firm age	proportion exporters	proportion foreign
Albania	116	27.1	10.6	19.8	10.3
Armenia	167	55.5	10.7	13.8	8.4
Azerbaijan	61	67.7	15.9	8.2	9.8
Bulgaria	63	72.9	15.1	25.4	7.9
Croatia	37	164.1	23.2	45.9	10.8
Czechia	16	290.0	17.7	56.2	31.2
Estonia	73	56.6	17.2	37.0	11.0
Hungary	57	107.5	13.4	45.6	12.3
Kazakhstan	64	61.9	9.5	3.1	3.1
Latvia	71	74.9	13.9	32.4	15.5
Lithuania	38	51.7	13.0	31.6	7.9
Moldova	168	70.5	12.3	13.7	6.0
Mongolia	103	74.9	16.7	7.8	1.9
Poland	12	80.2	19.8	50.0	8.3
Serbia	101	174.1	26.1	45.5	9.9
Slovenia	79	108.2	23.0	64.6	17.7
Average	76.6	96.1	16.1	31.3	10.8

Appendix E Regressions Eastern Europe-Asia Panel

Table E1 shows the results of the regressions for the panel of Eastern Europe and Asia. We lose statistical significance for our variable of interest in some specifications, although the magnitude and sign of the coefficients are similar to those estimated using our constructed panel for Latin American firms.

Table E1. Firm-Level Empirical Analysis - Eastern Europe - Asia WBES Panel.

	Δ% revenues			**Δ% revenue per worker**			**Δ% size**		
contract enforcement	0.002	0.004**	0.002	0.001	0.003**	0.002	0.001*	0.001*	-0.000
credit-output ratio	-0.003	-0.002	-0.001	-0.002	-0.001	-0.001	-0.001**	-0.001	-0.000
government corruption	0.012	0.202**	0.169**	-0.041	0.134	0.116	0.042*	0.059***	0.035*
registration costs	—	-0.004***	-0.005	—	-0.005***	-0.004*	—	0.001	-0.001
property rights	—	-0.011**	-0.012***	—	-0.010***	-0.010**	—	-0.001	-0.002*
tax rate	—	—	0.002	—	—	-0.001	—	—	0.003
business freedom	—	—	0.002	—	—	0.001	—	—	0.001*
employees (log)	-0.007	-0.016	-0.016	0.039***	0.030**	0.031**	-0.043**	-0.044**	-0.045**
age (log)	-0.026	-0.046**	-0.046**	-0.020	-0.036*	-0.039*	-0.009	-0.012	-0.011
exports	-0.058	-0.038	-0.033	-0.031	-0.012	-0.012	-0.016	-0.015	-0.010
foreign	-0.016	0.005	0.003	-0.054	-0.030	-0.032	0.025	0.025	0.023
R-squared	0.130	0.205	0.208	0.106	0.192	0.192	0.119	0.122	0.134
n. observations	952	952	952	947	947	947	1,221	1,221	1,221

Statistical significance: *** 1%, ** 5%, * 10%.
All regressions exclude intercepts. Errors are clustered at the country level. Outcomes are expressed as yearly p.p. changes.

Appendix F Computational Algorithm

We provide an outline of the computational algorithm and the solution method for the model, together with additional details.

The algorithm for solving this type of models consists in normalizing the wage rate in the baseline, then c_e is computed as the value that, in equilibrium under this baseline parameterization, satisfies the break-even condition with equality (Hopenhayn and Rogerson, 1993; D'Erasmo and Moscoso-Boedo, 2012). Entry and fixed costs for firms, and outside values for suppliers are expressed in terms of goods and are proportional to output per worker, following Bento and Restuccia (2017) and others. The lower bound on the endogenous level of technology $\underline{n}$ is normalized. The upper bound $\overline{n}$ is set equal to a sufficiently large number so that it is not binding: we consider 35 levels of technology, while in our simulations the maximum step reached by firms is 18. The exogenous productivity component of the production function follows an AR(1) process, which is discretized following Tauchen (1986) to construct the Markov matrix $\Lambda(z' \mid z)$. The dynamic problem is solved by value function iteration. To compute the stationary equilibrium for the baseline model we generate 50 simulations, each with 50 thousand firms (firms that exit are replaced by new firms), for 300 periods.

When a new equilibrium is computed (e.g., when a new value of μ is set), two equilibrium conditions need to be verified: the break-even condition of firms, the market-clearing condition in the labor market. The two equilibrium variables linked to these conditions are the equilibrium wage and the number of firms.

Documentos de Trabajo **Banco Central de Chile**	**Working Papers** **Central Bank of Chile**
NÚMEROS ANTERIORES	PAST ISSUES
La serie de Documentos de Trabajo en versión PDF puede obtenerse gratis en la dirección electrónica: www.bcentral.cl/esp/estpub/estudios/dtbc. Existe la posibilidad de solicitar una copia impresa con un costo de Ch$500 si es dentro de Chile y US$12 si es fuera de Chile. Las solicitudes se pueden hacer por fax: +56 2 26702231 o a través del correo electrónico: bcch@bcentral.cl.	Working Papers in PDF format can be downloaded free of charge from: www.bcentral.cl/eng/stdpub/studies/workingpaper. Printed versions can be ordered individually for US$12 per copy (for order inside Chile the charge is Ch$500.) Orders can be placed by fax: +56 2 26702231 or by email: bcch@bcentral.cl.

DTBC – 909
Optimal Spending and Saving Strategies for Commodity-Rich Countries
Alvaro Aguirre

DTBC – 908
Uncertainty, Risk, and Price-Setting: Evidence from CPI Microdata
Mario Canales, Bernabe Lopez-Martin

DTBC – 907
Earnings Inequality in Production Networks
Federico Huneeus, Kory Kroft, Kevin Lim

DTBC – 906
Price setting in Chile: Micro evidence from consumer on-line prices during the social outbreak and Covid-19
Jennifer Peña, Elvira Prades

DTBC – 905
Economic Growth at Risk: An Application to Chile
Nicolás Álvarez, Antonio Fernandois, Andrés Sagner

DTBC – 904
Production, Investment and Wealth Dynamics under Financial Frictions: An Empirical Investigation of the Self-financing Channel
Alvaro Aguirre, Matias Tapia, Lucciano Villacorta

DTBC – 903
Earnings Cyclicality of New and Continuing Jobs: The Role of Tenure and Transition Length
Elías Albagli, Gabriela Contreras, Matías Tapia, Juan M. Wlasiuk

DTBC – 902
The Internal Labor Markets of Business Groups
Cristobal Huneeus, Federico Huneeus, Borja Larrain, Mauricio Larrain, Mounu Prem

DTBC – 901
A strategic analysis of "Expectations and the neutrality of money"
Gent Bajraj, Neil Wallace

DTBC – 900
Forecasting Brazilian Inflation with the Hybrid New Keynesian Phillips Curve: Assessing the Predictive Role of Trading Partners
Carlos Medel

DTBC – 899
Searching for the Best Inflation Forecasters within a Consumer Perceptions Survey: Microdata Evidence from Chile
Carlos Medel

DTBC – 898
Capital Flows and Emerging Markets Fluctuations
Jorge Lorca

DTBC – 897
Financial Constraints: a Propagation Mechanism of Foreign Shocks
Rosario Aldunate

DTBC – 896
Sovereign Default Risk, Macroeconomic Fluctuations and Monetary-Fiscal Stabilization
Markus Kirchner, Malte Rieth

banco central Chile

DOCUMENTOS DE TRABAJO ABRIL 2021