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Forecasting Brazilian Inflation with the Hybrid New Keynesian Phillips Curve: Assessing the Predictive Role of Trading Partners*

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Abstract

Despite that the Brazilian economy is a small-open economy to the world's eye, it is still the largest of South America and, thus, it acts as a big source of financial and macroeconomic spillovers to its trading partners abroad in a number of macro-financial variables including inflationary shocks. Consequently, a comprehensive but parsimonious inflation rate modelling yields important advantages. In this line, the aim of this article is threefold. First, to document if the Brazilian inflation follows the Hybrid New Keynesian Phillips Curve (HNKPC) model, tested by econometric means. Second, to extend the scope of the HNKPC from a close- to an open-economy version through a Global Vector Autoregression (GVAR) specification; aiming to quantify the influence of trading partners in forecast accuracy. Third, to compare the multi-horizon predictive ability of the HNKPC in such a way as to identify the predictive gain (or loss) provided by the trading partners and discriminate between them. The HNKPC forecasts are evaluated in a traditional way and compared with several robustness specifications and country-weighting schemes with the GVAR version. The in-sample results do not reject the baseline hypothesis posed by the HNKPC for the Brazilian economy and its main trading partners. However, to a large extent, the evidence favouring the HNKPC at the end of sample plainly weakens. In predictive terms, the results show that the proposed open-economy version of the HNKPC is the best predictive device for Brazilian inflation in a compacted form in the long run. Notably, the euro area and Japan contribute most to forecast accuracy despite the use of a distance-based weighting scheme favouring closer South American trading partners such as Argentina and Chile.

Resumen

A pesar de que la economía brasileña es una economía pequeña y abierta para los ojos del mundo, es la economía más grande de América del Sur y, por lo tanto, actúa como una fuente de derrames financieros y macroeconómicos a sus socios comerciales en el exterior en una serie de variables financieras y macroeconómicas, incluyendo shocks inflacionarios. Así es como un modelo comprehensivo, pero parsimonioso, de la tasa de inflación entrega importantes ventajas. En esta línea, el objetivo de este artículo es triple. Primero, documentar si la inflación brasileña se ajusta econométricamente a una Curva de Phillips Híbrida Neokeynesiana (HNKPC). En segundo lugar, ampliar el alcance de la HNKPC desde una versión de economía cerrada a una de economía abierta a través de una especificación de vector autorregresivo global (GVAR), con el objetivo de cuantificar la influencia de los socios comerciales en la precisión predictiva. En tercer lugar, comparar la capacidad predictiva a múltiples horizontes de la HNKPC de tal manera que identifique la ganancia

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(o pérdida) predictiva facilitada por los socios comerciales y discriminar entre ellos. Los pronósticos de la HNKPC se evalúan de manera tradicional y se comparan con distintas especificaciones de robustez y esquemas de ponderación de países con la versión GVAR. Los resultados dentro de muestra no rechazan la hipótesis base planteada por la HNKPC para la economía brasileña y sus principales socios comerciales. Sin embargo, en gran medida, la evidencia claramente se debilita al final de la muestra. En términos predictivos, los resultados muestran que la versión de economía abierta compacta de la HNKPC es la mejor para predecir la inflación brasileña en el largo plazo. En particular, la Eurozona y Japón son los socios comerciales que más contribuyen a la precisión predictiva, a pesar del uso de un esquema de ponderación basado en la distancia que favorece a socios comerciales más cercanos de América del Sur, como Argentina y Chile.

1 Introduction

Brazil is not only a football giant. The Brazilian economy is the largest of South America[1] and, as such, it acts as a reference for all countries of the region and its main trading partners abroad. Since 1994, particularly after the public-sector re-structuring plan (the *Plano Real*), Brazil concentrates the attention as one of the key emerging markets of the so-called BRIC economies (along with Russia, India, and China). Moreover, in the most recent period, Brazil also has been an important source of economic turmoil and market volatility, and consequently, making more difficult to have a clear appraisal of key developments and prospects of the region. One of these key macroeconomic indicators is the Consumer Price Index (CPI) inflation rate, which is not only relevant for both financial and real-economy decisions, but also, and particularly expected inflation, acts as a latent test on authorities' effectiveness on controlling inflationary pressures and monetary policy priorities.

As the Brazilian inflation is a likely source of spillovers and uncertainty, a comprehensive and parsimonious modelling yields significant advantages. In this sense, the aim of this article is threefold. First, to document if the Brazilian inflation follows the *Hybrid New Keynesian Phillips Curve* (HNKPC) model, introduced by Galí and Gertler (1999), and tested by econometric means in contrast to a calibrated setup.[2] This model result as a practical, convenient representation to understand the driving forces behind a highly indexed inflation such as the Brazilian case. Second, to extend the scope of the HNKPC from a close- to an open-economy version (henceforth, CE-HNKPC and OE-HNKPC) through a Global Vector Autoregression (GVAR) specification (Pesaran, Schuermann, and Weiner, 2004); always having in mind to estimate the influence of Brazil's major trading partners[3] in forecast accuracy. Third, to compare and estimate the multi-horizon predictive ability of the HNKPC in such a way as to identify the predictive gain (or loss) provided by trading partners' information (and the economic structure of the model). The relevance of these exercises is to understand Brazilian inflation dynamics and its predictability using a unified, theoretically-based econometric framework, including up to seven (with the possibility to include even more) countries at the same time aiming to capture most of its foreign-based dynamics.

The use of the GVAR obeys particularly to an open-economy version of the HNKPC, following the same motivation of Medel (2017, 2018). Galí and Monacelli (2005), for instance, develop an open-economy version of the HNKPC which explicitly includes the interaction of a *domestic country* with *the rest of the world* through the foreign exchange rate mechanism, but makes it difficult to distinguish the predictive contribution between different trading partners. The option provided by the OE-HNKPC is to include trading partners as blocs of the same variables used by the domestic country, thus including actual and expected inflation acting as a proxy for the real exchange rate, and a country-specific measure of "global economic slack"; in turn, proxying the hypothesis introduced by Martínez-García and Wynne (2010). Mentioned blocs are aggregated through exogenous weighting schemes, a feature stressed out in this article by using different sets of them, going from an equally-weighted to distance-based weights, and also predicting with a different number of partners to disentangle the most useful country combinations for forecast accuracy.

The results obtained with the CE-HNKPC do not reject the hypothesis proposed by the HNKPC for Brazil and its main trading partners for the *estimation sample* (2000.1-2005.12, in monthly frequency), but weakens in the *evaluation sample* (2006.1-2018.6). This result supports the documented instability of the HNKPC as well as the most recent finding of the "Phillips curve flattening" hypothesis, but still showing an appropriate in-sample goodness of fit and accurate forecasts. In this case, the trading partners contribute just with inflation measures discarding their contribution to the "global economic slack" measure. Coincidentally, after the 2008-9 *Global Financial Crisis*, the inflation rate of the considered countries (with one exception) has been particularly low and hovered over its policy targets. This leaves the discussion open for an active role of the economic slack when expectations are unanchored or when actual inflation is *far* from its target.

[1] According to CIA's *The World Factbook*, in 2017 the Brazilian GDP at PPP reached USD 3,248 trillion (top eight of the world), implying USD 15,600 per capita (roughly 208 million inhabitants). Second largest South American economy (GDP at PPP) is Argentina with USD 922 trillion.

[2] This well-known inflation model comes out as a combination between past and expected values of inflation, obeying to the inertia exhibited by backward-looking price-setter firms and a forward-looking component provided by rational expectations agents' behaviour. It also includes a measure of economic slack as the inflation driving process.

[3] These countries are Argentina (ARG), Chile (CHL), China (CHI), the euro area (EUR), Japan (JPN), and the United States (US), totalising close to 60% of total Brazilian trade on average.

The out-of-sample results show that, at the nowcasting horizon, the best options are the CE-HNKPC and a reduced-form OE-HNKPC; notably, both with a different specification of the economic slack. However, none of them comes out as statistically superior to the traditional autoregression. Beyond the nowcasting horizon, in turn, both HNKPC come out as the best available alternatives. In particular, the simplest CE-HNKPC is slightly better than a two-country parsimonious OE-HNKPC in a horizon within a year, using the unemployment gap as the driving process. For horizons beyond a year, the trading partners' contribution make the OE-HNKPC outperform the remaining options. In particular, the same parsimonious OE-HNKPC mentioned above emerges as the best forecasting model, making use of the unemployment gap, and extracting the information of just a pair of trading partners, *i.e.* the euro area and Japan. These results as robustness exercises, point out that the HNKPC could be considered as a superior forecasting device for Brazilian inflation when including information from just a few but relevant trading partners, and particularly when forecasting at horizons greater than a year.

The rest of the article proceeds as follows. Section 2 reviews the relevant literature concerning the topics covered in this article. These are statistical versus economics-based inflation forecasts plus evidence collected for the Brazilian inflation. Section 3 provides a description of the baseline and robustness HNKPC specifications and the statistical inference carried out for the out-of-sample results. Section 4 presents the results divided into estimation diagnostics and forecast accuracy, including robustness results. Finally, section 5 summarises and concludes.

2 Literature review

In general, there are two ways of thinking in forecasting economic variables: a statistical manner without using any economic theory and an economics-based procedure. This also happens for the case of inflation–see Faust and Wright (2014) for a recent survey. The atheoretical or statistical manner refers to the case when the prediction comes from a model without economic fundamentals, and the appropriate model is obtained purely based on statistical tests' results. Box and Jenkins' (1970) procedures have been the precursors of these reference models due to their fruitful results and efficiency in a broad sense. Some of these models used in this article are the autoregressive moving average (ARMA), the exponential smoothing (ES), and the random walk models (RW). All these models are described in Annex A.

When inflation is forecast with economic models, the task is typically carried out using an NKPC specification. It was introduced by Galí and Gertler (1999) and extended in Galí, Gertler, and López-Salido (2001, 2005). Sachsida (2013) presents a comprehensive, recent literature review on NKPC evidence for the Brazilian case. This revision includes a large variety of NKPC models including linear versus non-linear, time-varying parameters, panel data, Bayesian regressions, and smooth transition regression models. As the nature of these models is diverse, despite that all are referring to the NKPC, they deliver different conclusions. A common practical, useful conclusion, however, is that previously suggested by Blanchard and Galí (2007), that the coefficient of past and expected inflation must be restricted to unity to find appealing and meaningful results. Remarkably, authors encourage other ways to describe the inflation dynamics of Brazil, none of which is similar to the approach adopted in this article. Areosa and Medeiros (2007) derive and estimate a structural NKPC plus a hybrid version; also comparing a close- to an open-economy version. Their results, circumscribed to in-sample estimates, indicates that the Brazilian economy has a fairly important level of nominal indexation, which is also found in this article. When allowing for openness, the indirect impact is significant by changing the weights associated to lagged and expected inflation, and foreign exchange rate and foreign inflation increase its role along with more openness. Mendonça, Sachsida, and Medrano (2012) estimate an HNKPC making use of similar expectation measure to the one used in this article as well as the unemployment rate as the driving process. An important result is that when including the exchange rate, it changes its coefficient's sign from positive to negative when considering data from 2002 onwards. This switching behaviour suggests, in the context of this article, that the relationship with trading partners evolves and, as so, volatile external inflation coefficients are present and adaptive weights could consistently capture better this kind of behaviour.

Machado and Portugal (2014) estimate an HNKPC under the unobserved components framework, also making use of direct measures of inflation expectations and test two economic slack variables for robustness check. The model's adjustment to actual data comes out as appropriate in representing Brazilian inflation dynamics. A crucial factor behind this favourable adjustment is the use of an accommodative, time-varying parameter for the output gap. This also leads to conclude that in the last part of the sample (full sample: 2000.4 to 2011.5) the Phillips curve has flattened, a result shared with several countries succeeding in using the inflation targeting scheme.

Ponzoni and Zilli (2015) estimate an HNKPC for the period 2002-2014 making use of the unemployment rate as a proxy for economic slack. After briefly reviewing Brazil's inflationary history, the authors reveal the difficulties for the unemployment-based HNKPC to accurately describe the inflation rate, despite having found the coefficient sign dictated by the theory. Interestingly, the authors point out that inflation expectations play a key role in inflation dynamics, as do supply shocks proxied by exchange rate shocks. In a different verge, Caetano and Moura (2012), aiming to test the null hypothesis of no sticky information, estimate a Phillips curve for the period 2001-2009. Their results—not surprisingly—, reveal that sticky information does exist, and implies a price update every five quarters.

A deeper econometric analysis of the Phillips curve is presented in Maka and Barbosa (2014) for core inflation (total inflation excluding food and energy prices). Their paper tests several Phillips curve specifications, namely, an autoregressive distributed lag model that encompasses the NKPC, an HNKPC, the Sticky Information Phillips curve, and the Accelerationist Phillips curve (APC)—all of them sharing a baseline NKPC specification but imposing a particular set of restrictions on (to-be-estimated) parameters. The empirical evidence supports the hypothesis posed by the APC only. This finding is important to remark the methodological sensibility that characterises econometric estimations of the Phillips curve, which will be addressed later in this article.

Especially when the aim is forecast accuracy, several recent articles make use of the NKPC for the Brazilian inflation. Altug and Çakmaklı (2016) take advantage of the adoption of the inflation targeting regime by Brazilian authorities in 1999.6 to combine the target with expectation measures, resulting in a specification slightly different to the NKPC. This model, estimated through state-space method, greatly improves it performance when including expectations compared to a version without augmentation. However, when compared to natural benchmarks, the model is outperformed by atheoretical forecasts and the HNKPC, also an issue tackled in this article. A similar result is found in Carlo and Marçal (2016), pointing out that simple univariate models that consider seasonality and a deeper degree of CPI disaggregation outperform those with a more complex structure. At this point, the results of Arruda, Ferreira, and Castelar (2011), show favourable evidence of non-linearities when forecasting with non-linear statistical models and a PC extended with a threshold effect. Ferreira and Palma (2015), in turn, make use of a dynamic model averaging allowing for both model accommodation and time-varying coefficient setup, delivering accurate predictions within a year. These results point out that inflation dynamics could be more complex than what can be captured by simple benchmarks.

Interestingly, Medeiros, Vasconcelos, and de Freitas (2016) also bring more information to the forecasting task, resulting in accuracy improvements. The authors estimate and forecast using models with many candidate regressors, which are reduced through the *Least Absolute Shrinkage and Selection Operator* (LASSO) estimator. As the method tends to keep variables related to government debt and money, the hypothesis of the NKPC is thus less supported for forecasting purposes. All these issues—particularly, CPI disaggregation and a bigger set of covariants–are certainly interesting and not considered in this article; but the disaggregation scheme proposed in Boaretto and Da Silva (2019) offers a concrete starting point for further research.

The results of Medeiros, Vasconcelos, and de Freitas (2016) fit more than well to the findings of the current article. This article investigates the extent to which openness features are correlated with out-of-sample accuracy improvements, and results do not support the hypothesis of *all* foreign sources of inflation predictability as useful, but without rejecting the CE-HNKPC hypothesis. The authors offer an alternative explanation when analysing domestic economy variables as inflation drivers. As their results support government spending and money as the best candidates–certainly more endogenous variables compared to that of trading partners–and that more information spoils out the NKPC proposal, the article documents and confirms that it is very likely that sources of predictability may be found in the domestic economy. In this sense, the CE-HNKPC proposed in this article is suitable because it includes the output or unemployment gap as a measure of economic slack and the OE-HNKPC combines it with the same information coming from trading partners. Moreover, Ponzoni and Zilli (2015) suggest that unemployment does not result in better estimates than the output gap. This overall empirical outcome makes difficult policy making with inflation stabilisation purposes.

The exercise analysed in this article compares the predictive ability of the CE-HNKPC and an OE-HNKPC version in a novel fashion for the Brazilian case. However, a comparison between them should be carefully judged, since an open-economy version typically redounds in the inclusion of more variables in the model. As Hansen (2009) argues, there is no clear relationship between in-sample fit and forecast accuracy, but forecasts tend to be worst with overfitted models. So, if the aim is to forecast a particular variable using the OE-HNKPC, it is preferred to include explanatory variables that capture most of the variance of inflation series. In the OE-HNKPC case, this could be achieved by including the smallest bloc of countries leading to accurate forecasts instead of picking unsorted series from different countries. This is actually explored in this article, suggesting that for forecasting purposes the use of a parsimonious two-country version of the OE-HNKPC is recommended .

3 Econometric approach

In this section, all HNKPC specifications are detailed. To sketch the HNKPC foundations, assume a staggered price-setting scheme *à la* Calvo (Calvo, 1983). Let $1-\theta$ be the fraction of firms that change prices at a given period, and $1-\omega$ the fraction of firms that set prices optimally in a forward-looking manner. Hence, the HNKPC consists of a weighted average between past and future values of inflation plus a driving process $\widetilde{y}_t$, leading to the HNKPC baseline equation:

$$\pi_t - \overline{\pi} = \gamma \widetilde{y}_t + \lambda_b \pi_{t-1} + \lambda_f \mathbb{E}_t[\pi^f_{t,t+h}] + \varepsilon_t, \tag{1}$$

where π_t is actual inflation, $\overline{\pi}$ is an exogenous policy target, $\mathbb{E}_t[\pi^f_{t,t+h}] = \widetilde{\pi}_t$ is the inflation expectation at period f measured with a forecast made h-steps-ahead at period t, and $\widetilde{y}_t$ is a real marginal cost measure whose construction is detailed in Annex B. $\{\gamma, \lambda_b, \lambda_f, \sigma^2_\varepsilon\}$ are parameters to be estimated, and ε_t is a cost-push shock, $\varepsilon_t \sim iid(0, \sigma^2_\varepsilon)$. This specification constitutes a reduced form coming from the optimisation problem of a structural NKPC where:

$$\begin{aligned} \lambda_b &= \frac{\omega}{\phi}, \\ \lambda_f &= \frac{\beta\theta}{\phi}, \\ \gamma &= \frac{[(1-\omega)(1-\theta)(1-\beta\theta)]}{\phi}, \\ \phi &= \theta + \omega\left[1-\theta(1-\beta)\right], \end{aligned} \tag{2}$$

where β is a discount factor. Note that equation (1) results in a convenient specification for forecasting purposes and allowing many price settings.

3.1 Closed-economy HNKPC

The baseline specification is the univariate equation (1). To avoid part of the simultaneity in the variables of the right-hand side, equation (1) is estimated with the *Generalised Method of Moments* (GMM) estimator. However, this method eliminates *methodological* simultaneity only, as the series exhibit high correlation given their underlying similar data generating processes and face similar global shocks, particularly, commodity prices shocks. I make use of lagged observations of the same variables as instrumental variables (IV). Recall that the problem that GMM addresses is the orthogonality condition $\mathbb{E}_t[\mathbf{x}'_t\varepsilon_t]$ that no longer holds. Hence, it is necessary to "instrumentalise" the $\mathbf{x}'_t$ matrix with another one, say $\mathbf{m}_t$, containing ℓ IV ($\ell \geq k$) which fulfils:

$$\mathbb{E}_{t-1}[(\pi_t - \overline{\pi} - \gamma\widetilde{y}_t - \lambda_b\pi_{t-1} - \lambda_f\mathbb{E}_t[\pi^f_{t,t+h}]) \times \mathbf{m}_{t-1}] = 0. \tag{3}$$

In this context, a formal test for IVs' suitability is analysed through the Hansen's J-statistic:

$$J(\widehat{\boldsymbol{\beta}}, \widehat{\mathbf{w}}_T) = \frac{1}{T}(\pi_t - \mathbf{x}'_t\widehat{\boldsymbol{\beta}})'_t\mathbf{m}\widehat{\mathbf{w}}_T^{-1}\mathbf{m}'(\pi_t - \mathbf{x}'_t\widehat{\boldsymbol{\beta}}), \tag{4}$$

where $\widehat{\mathbf{w}}_T$ is a $\ell \times \ell$ symmetric and positive-definite *weighting matrix*, as it weights the moments considered in the estimations. Hence, GMM finds the vector of coefficients:

$$\widehat{\boldsymbol{\beta}} = (\mathbf{x}'\mathbf{m}\widehat{\mathbf{w}}_T^{-1}\mathbf{m}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{m}\widehat{\mathbf{w}}_T^{-1}\mathbf{m}'\pi_t, \tag{5}$$

that minimises equation (4). As $J(\widehat{\boldsymbol{\beta}}, \widehat{\mathbf{w}}_T) \sim \chi^2_{\ell-k}$, along with the estimated coefficients it is also reported the p-value that test the null hypothesis: $\mathbb{E}_T[J(\widehat{\boldsymbol{\beta}}, \widehat{\mathbf{w}}_T)] = 0$. If p-value $> \alpha\%$, the IV are valid at $\alpha\%$-level of significance, and the specification qualifies to be the forecasting model together with coefficients with the expected sign.

The estimation of the weighting matrix is made according to the Hansen (1982) recommendation–the inverse of the covariance matrix, *i.e.* $\widehat{\mathbf{w}}_T = \widehat{\mathbf{s}}^{-1}$, and avoiding potential autocorrelation with the Newey and West (1987) heteroscedasticity and autocorrelation-correction method. The estimation of both covariance matrices–for the two stages, IV and final regression–is set in the same manner. The whitening lag specification is set automatic, to be selected according to the Bayesian Information Criterion (BIC) choosing in a maximum of three lags.

Despite the solution offered by the IV, some other problems could arise. A common setback is when IV are *weak instruments* (Stock, Wright, and Yogo, 2002). The problem could be easily explained by comparing the bias of two available estimators (*Ordinary Least Squares*: $\boldsymbol{\beta}^{OLS} = (\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}'\pi_t$ versus GMM, equation (5)). So, the relative asymptotic bias could be expressed as (with $\boldsymbol{\eta} = \mathbf{m}\widehat{\mathbf{w}}_T^{-1}\mathbf{m}'$):

$$\text{Relative Asymptotic Bias} = \frac{\operatorname*{plim}_{T\to\infty}\left[\widehat{\beta} - \beta\right]}{\operatorname*{plim}_{T\to\infty}\left[\beta^{OLS} - \beta\right]} = \frac{\operatorname*{plim}_{T\to\infty}\left[(\mathbf{x}'\boldsymbol{\eta}\mathbf{x})^{-1}\mathbf{x}'\boldsymbol{\eta}\pi_t - \beta\right]}{\operatorname*{plim}_{T\to\infty}\left[(\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}'\pi_t - \beta\right]} = \frac{\mathbb{C}\left[\boldsymbol{\eta}, \varepsilon\right]}{\mathbb{C}\left[\mathbf{x}, \varepsilon\right]} \cdot \mathbb{C}\left[\boldsymbol{\eta}, \mathbf{x}\right]^{-1}. \tag{6}$$

From equation (6) it is easy to notice that the higher $\mathbb{C}\left[\boldsymbol{\eta}, \mathbf{x}\right]$, the smaller the relative asymptotic bias. Note also that:

$$\begin{aligned} \mathbb{V}\left[\widehat{\beta}\right] &= \sigma^2_\varepsilon \left(\mathbf{x}'\boldsymbol{\eta}\right)^{-1} \left(\boldsymbol{\eta}'\boldsymbol{\eta}\right) \left(\boldsymbol{\eta}, \mathbf{x}\right)^{-1} \\ &= \sigma^2_\varepsilon \left(\mathbf{x}'\mathbf{x}\right)^{-1} \left(\mathbf{x}'\boldsymbol{\eta}\right)^{-1} \left(\boldsymbol{\eta}'\boldsymbol{\eta}\right) \left(\boldsymbol{\eta}, \mathbf{x}\right)^{-1} \left(\mathbf{x}'\mathbf{x}\right) \\ &= \mathbb{V}\left[\beta^{OLS}\right] \cdot \rho^{-2}_{\boldsymbol{\eta}\mathbf{x}}. \end{aligned} \tag{7}$$

Hence, the lower the correlation between $\mathbf{x}$ and $\boldsymbol{\eta}$ (termed as $\rho_{\boldsymbol{\eta}\mathbf{x}}$), the higher the variance of the IV estimator relative to that of *Ordinary Least Squares* (OLS). For the set of IV used in each estimation the Stock and Yogo (2010) test is applied to measure the "strength of identification". Note that it is computed through the Cragg-Donald F-statistic. More details on the econometrics of weak instruments can be found in Bound, Jaeger, and Baker (1995), Stock, Wright, and Yogo (2002), and Moreira (2009). A deep overview for the specific case of the NKPC can be found in Nason and Smith (2008).

3.2 Open-economy HNKPC

The GVAR methodology was introduced by Pesaran, Schuermann, and Weiner (2004) in search for a flexible procedure able to include key interactions across a big number of countries. The result is a specific Structural VAR (SVAR) that comes from stack country-level VAR previously defined in two blocs: the domestic and the foreign variables. The foreign variables enter the domestic equation as weighted averages of the same variables of the remaining countries. As the weights are exogenously imposed, it is easy to define first the model in a "compressed" manner, making possible its estimation, to then "decompress" it for further post-estimation handling. The extensive form model eliminates any bloc of variables, treating every variable as part of an ordinary VAR. Nevertheless, given the mechanics of the GVAR, it avoids the *curse of dimensionality* confronted by VAR models with too many coefficients to be estimated (and exponentially arisen when a new variable is included).

The model's flexibility comes from the fact that it is possible to model a country-level VAR including specific variables and different lag length. This is permitted because the key issue of the GVAR is the stacking step. Notice that this also allows for multi-regional analysis (a subset of countries) at the same stage with country level analysis. As a SVAR procedure, it provides the advantage of accommodating non-stationary series, computing cross-country impulse response functions, and forecasting.

For formal description purposes (following closely Pesaran, Schuermann, and Weiner, 2004; and Medel, 2018), assume that there are i=0,1,...,N+1 countries across the time span t=1,...,T, where the country i=0 is the reference country. Now, assume that each country is modelled using k_i domestic and k_i^* foreign variables (hereafter, "*" will refer to foreign variables). In this article, for each country k_i=k_i^*=3, and hence k=6 (accounting: k_i={π_{it}, $\widetilde{\pi}_{it}$, $\widetilde{y}_{it}$} and k_i^*={π_{it}^*, $\widetilde{\pi}_{it}^*$, $\widetilde{y}_{it}^*$}). So, for each country i it is defined the $k_i \times 1$ vector $\mathbf{x}_{it} = [\pi_{it}; \widetilde{\pi}_{it}; \widetilde{y}_{it}]'$ and the vector of order $k_i^* \times 1$ of foreign variables $\mathbf{x}_{it}^* = [\pi_{it}^*; \widetilde{\pi}_{it}^*; \widetilde{y}_{it}^*]'$, and hence the OE-HNKPC is:

$$\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{\Phi}_i \mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{i0} \mathbf{x}_{it}^* + \boldsymbol{\varepsilon}_{it}, \tag{8}$$

where $\mathbf{a}_{i0}$ is a $k_i \times 1$ vector containing constants to be estimated, Φ_i is a $k_i \times k_i$ matrix containing lagged coefficients, Λ_{i0} is a $k_i \times k_i^*$ matrix containing the foreign variables relevant for the country i, and $\boldsymbol{\varepsilon}_{it}$ is $k_i \times 1$ vector of errors. Notice that equation (8) could include more lags of the foreign variables vector, and it nests the VAR(1) if Λ_{i0}=...=Λ_{ip^*}=0. It is assumed that $\boldsymbol{\varepsilon}_{it} \sim iid(\mathbf{0}, \mathbf{\Sigma}_{ii})$; hence, errors are uncorrelated and with mean equal to 0 and finite variance σ_{ii}^2. Note that $\mathbf{\Sigma}_{ii} = \mathbb{C}\left[\varepsilon_{ilt}, \varepsilon_{ist}\right]$ with $l \neq s$, and $\mathbf{\Sigma}_{ii}$ is nonsingular. This assumption could be easily relaxed for a spillover analysis with a long enough sample, since the elements of the diagonal must be estimated now. However, since $\mathbf{x}_{it}^*$ is included in the estimation, $\boldsymbol{\varepsilon}_{it}$ already contain foreign information.

The foreign variables included in $\mathbf{x}_{it}^* = [\pi_{it}^*; \widetilde{\pi}_{it}^*; \widetilde{y}_{it}^*]'$ constitute a weighted average of the same variable defined for the remaining N countries:

$$\pi_{it}^* = \sum_{j=0}^{N} \omega_{ij}^{\pi} \pi_{jt},\ \widetilde{\pi}_{it}^* = \sum_{j=0}^{N} \omega_{ij}^{\widetilde{\pi}} \widetilde{\pi}_{jt},\ \widetilde{y}_{it}^* = \sum_{j=0}^{N} \omega_{ij}^{\widetilde{y}} \widetilde{y}_{jt}, \tag{9}$$

where $\{\{\omega_{ij}^{\pi}\},\{\omega_{ij}^{\widetilde{\pi}}\},\{\omega_{ij}^{\widetilde{y}}\}\}_{j=0}^{N}$ is the set of N weights for each of the k_i^* foreign variables relevant for the country i. The simplest weight scheme is the equally-weighted average with ω_{ij}^{π}=$\omega_{ij}^{\widetilde{\pi}}$=$\omega_{ij}^{\widetilde{y}}$=$1/N$, $\forall i \neq j$. Obviously, as the sequences $\{\omega_{ij}^{x}\}$ are weights, $\Sigma_{j=0}^{N} \omega_{ij}^{x} = 1$. Note, however, that the weighting scheme is key to move away from matrix $\mathbf{x}_t = [\mathbf{x}_{it}, \mathbf{x}_{it}^*]$ non-invertibility, every time that $\mathbf{x}_t$ has a strong common component. So, to alleviate a possible excess of collinearity, a bigger weight could be assigned to the "most exogenous" (independent) country, which, in this case, seems to be Argentina. This actually happens with a distance-based weighting scheme, described later.

By now, equation (8) represents a VARX*(1,1) model, *i.e.* a VAR(1) model including exogenous variables X*. So, the advantage of the GVAR method is that it actually models all the variables contained in the weighted average. Hence, it includes the N+1 variables $\mathbf{x}_{it}$. This is made by stacking all the countries into one equation using the predetermined weights. As the weights are known, it is possible to estimate the equations separately and then continue with the stacking step.

Define the next $(k_i + k_i^*) \times 1$ vector $\mathbf{z}_{it}$:

$$\mathbf{z}_{it} = \begin{bmatrix} \mathbf{x}_{it} \\ \mathbf{x}_{it}^* \end{bmatrix}. \tag{10}$$

Equation (8) could be rewritten as:

$$\mathbf{A}_i \mathbf{z}_{it} = \mathbf{a}_{i0} + \mathbf{B}_i \mathbf{z}_{i,t-1} + \boldsymbol{\varepsilon}_{it}, \tag{11}$$

where $\mathbf{A}_i$ contains contemporaneous restrictions, $\mathbf{A}_i$=$[\mathbf{I}_k, -\mathbf{\Lambda}_{i0}]$, with $\mathsf{rank}(\mathbf{A}_i)$=$k_i$ and $\mathbf{B}_i$=$[\mathbf{\Phi}_i, \mathbf{0}]$. If the foreign variables are included with a lag, then its coefficient matrix $\Lambda_{i,t-1}$, will appear in $\mathbf{B}_i$ as $\mathbf{B}_i$=$[\mathbf{\Phi}_i, \mathbf{\Lambda}_{i,t-1}]$. A global vector $\mathbf{x}_t$ (suppressing the i-index) will be of the shape $\mathbf{x}_t = [\mathbf{x}_{0t}, \mathbf{x}_{1t}, ..., \mathbf{x}_{Nt}]'$, and the order in which the foreign variables enter into $\mathbf{x}_{it}$ and the stacking order is irrelevant. To have a view of the matrices involved, it is suggested to have a look at the $\mathbf{A}_i$ shape for the case considered in this article:

$$\mathbf{A}_i = \begin{bmatrix} 1 & 0 & 0 & -\gamma_{ii}^{\tilde{y}^*} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\lambda_{ii}^{\pi^*} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\lambda_{ii}^{\tilde{\pi}^*} \end{bmatrix}. \tag{12}$$

Now, once that all the $\mathbf{x}_{it}$ vectors are already contained in the $\mathbf{z}_{it}$ vectors, it is easy to notice the following identity:

$$\mathbf{z}_{it} = \mathbf{W}_i \mathbf{x}_t, \tag{13}$$

where $\mathbf{W}_i$ (time-fixed) is a $(k_i + k_i^*) \times k$ matrix containing the known country-level weights. Pesaran, Schuermann, and Weiner (2004) label equation (13) as "the link", as it links the country-specific model ($\mathbf{z}_{it}$) using all the global variables ($\mathbf{x}_t$). The shape of the $\mathbf{W}_i$ matrix when i=0 is the following:

$$\mathbf{W}_{i=0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_{01}^{\tilde{y}} & 0 & 0 & \omega_{02}^{\tilde{y}} & 0 & 0 & \omega_{03}^{\tilde{y}} & 0 & 0 & ... & \omega_{06}^{\tilde{y}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_{01}^{\pi} & 0 & 0 & \omega_{02}^{\pi} & 0 & 0 & \omega_{03}^{\pi} & 0 & ... & 0 & \omega_{06}^{\pi} & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_{01}^{\tilde{\pi}} & 0 & 0 & \omega_{02}^{\tilde{\pi}} & 0 & 0 & \omega_{03}^{\tilde{\pi}} & ... & 0 & 0 & \omega_{06}^{\tilde{\pi}} \end{bmatrix},$$

and the 3×3 submatrix of zeros (below the 3×3 identity submatrix) is moving one block (of 3 columns) to the right when the country is changed across i=1,...,6.

Using the link equation in the country-specific model delivers:

$$\mathbf{A}_i \underbrace{\mathbf{W}_i \mathbf{x}_t}_{\mathbf{z}_{it}} = \mathbf{a}_{i0} + \mathbf{B}_i \underbrace{\mathbf{W}_i \mathbf{x}_{i,t-1}}_{\mathbf{z}_{i,t-1}} + \boldsymbol{\varepsilon}_{it}, \tag{15}$$

and $\mathbf{A}_i\mathbf{W}_i$ and $\mathbf{B}_i\mathbf{W}_i$ are both $k_i \times k$ matrices. Stacking these equations yields:

$$\mathbf{G}\mathbf{x}_t = \mathbf{a}_0 + \mathbf{H}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{16}$$

where:

$$\mathbf{a}_0 = \begin{bmatrix} \mathbf{a}_{00} \\ \mathbf{a}_{10} \\ \vdots \\ \mathbf{a}_{N0} \end{bmatrix}, \mathbf{G} = \begin{bmatrix} \mathbf{A}_0\mathbf{W}_0 \\ \mathbf{A}_1\mathbf{W}_1 \\ \vdots \\ \mathbf{A}_N\mathbf{W}_N \end{bmatrix}, \mathbf{H} = \begin{bmatrix} \mathbf{B}_0\mathbf{W}_0 \\ \mathbf{B}_1\mathbf{W}_1 \\ \vdots \\ \mathbf{B}_N\mathbf{W}_N \end{bmatrix}, \boldsymbol{\varepsilon}_t = \begin{bmatrix} \boldsymbol{\varepsilon}_{0t} \\ \boldsymbol{\varepsilon}_{1t} \\ \vdots \\ \boldsymbol{\varepsilon}_{Nt} \end{bmatrix}. \tag{17}$$

As $\mathbf{G}$ is a non-singular $k \times k$ matrix and of full rank generally, it allows the OE-HNKPC representation:

$$\mathbf{x}_t = \mathbf{G}^{-1}\mathbf{a}_0 + \mathbf{G}^{-1}\mathbf{H}\mathbf{x}_{t-1} + \mathbf{G}^{-1}\boldsymbol{\varepsilon}_t, \tag{18}$$

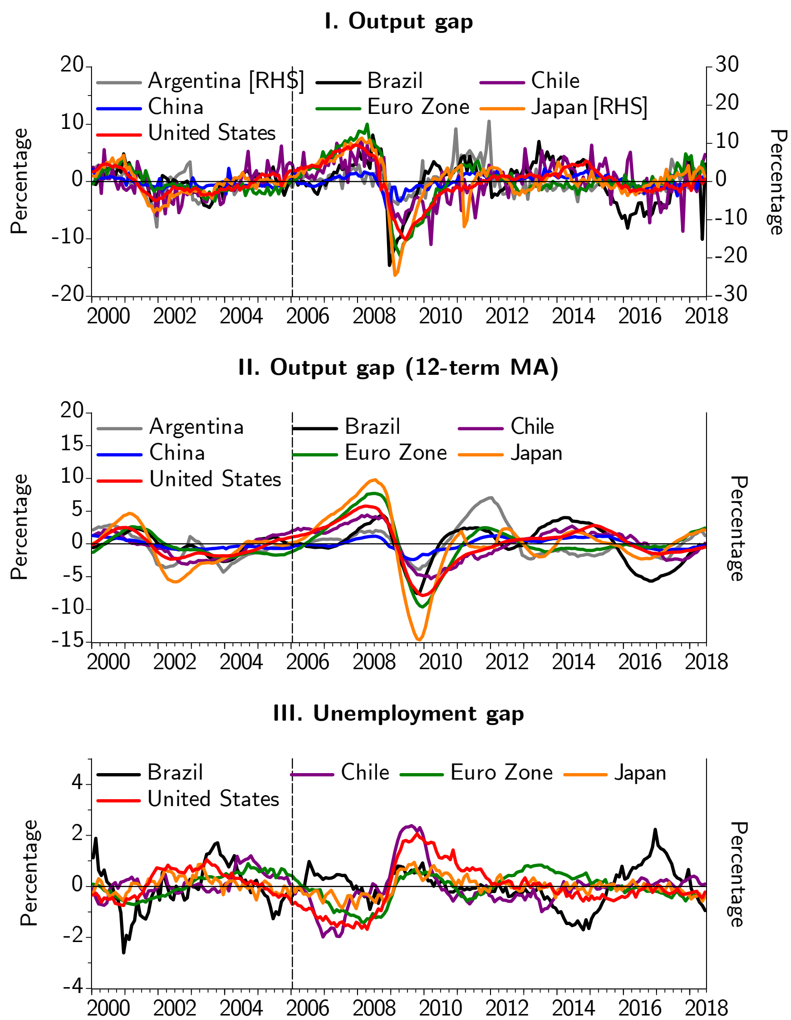
which can be solved recursively as an SVAR(1) model. Note that the structure of the model is commanded by the $\mathbf{G}$ matrix, which contains no row-crossed terms. This allows to estimate each country-level equation separately, to then stack all the $\mathbf{A}_i\mathbf{W}_i$ results (numerically) in $\mathbf{G}$. This method provides the advantage of covering many countries (or regions) and allowing different specifications for each country.[4]

[4]Some technical difficulties could arise when $\mathbf{G}$ is nonsingular. However, as Chudik and Pesaran (2016) suggest, the problem could be alleviated by including more lags of the foreign variables acting as an external unobservable factor, or as suggested above, making use of a different weighting scheme.

3.3 Inflation expectations, economic slack measures, and OE-HNKPC weighting schemes

The HNKPC assumes that inflation expectations are directly measured, as opposed to being completely instrumentalised through lags of actual inflation or other related exogenous variables. The full dataset is described in Annex C. This article makes use of the *Consensus Forecasts* (CF) dataset of inflation expectations which are described in Annex C.2. Two remarkable features are that CF developers ask respondents for two forecasting horizons: December of the current calendar year and December of the next year. Hence, a weighting scheme between the two series is used to come out with just one variable and to reflect that, as the horizon is coming closer, price changes are less uncertain than those at longer horizons, losing the characteristic of an "expectation" variable (see Annex C.2 for details). Also, just for the Brazilian case, another direct measure of inflation expectations is used, namely the "Focus" expectations generated by the *Banco Central do Brasil*. This series has a unique fixed forecasting horizon, defined as "IPCA-inflation accumulated over the next 12 months", where IPCA stands for its Portuguese "Broad National Consumer Price Index". This series is henceforth referred to as "BRA: Focus".

Figure 1: Economic slack measures: output gap and unemployment gap (*)
Full sample: 2000.1-2018.6



(*) Vertical line=start of evaluation sample (2006.1). Source: Author's calculations.

Regarding the use of economic slack measures, as above mentioned, there are two variables constructed for this purpose: the output gap and the unemployment gap. For convenience and forecast feasibility, both series are built following the same methodology described in Annex B and depicted in Figure 1. Notice that the output gap enters the baseline specification contemporaneously, making it difficult to estimate model coefficients, as the output gap depends on lagged inflation and vice versa. This is why GMM comes out as an appropriate estimation method for the CE-HNKPC.

In economic terms, it is also plausible for past values of the output gap to influence current inflation dynamics, and hence, it must be reflected in the second-stage estimation. A way to tackle this issue consists in using a 12-order moving average version of the output gap, depicted in Figure 1, panel II, and re-doing the whole exercise with this transformed variable. The same argument could be applied to the unemployment gap; however, its estimates are not that volatile as those of the output gap and no smoothing transformation is applied. Finally, the unemployment gap measure is constructed for all countries except Argentina and China due to insufficient data availability. In these cases, when the OE-HNKPC is estimated, the output gap measure is used (with the opposite sign).

Table 1: OE-HNKPC weighting schemes (*)

	ARG	*BRA*	*CHL*	*CHI*	*EUR*	*JPN*	*US*
	Fixed first principal component weights						
ARG	0.000	0.105	0.096	0.260	0.135	0.217	0.186
BRA	0.102	0.000	0.096	0.261	0.136	0.218	0.187
CHL	0.101	0.105	0.000	0.259	0.134	0.216	0.185
CHI	0.121	0.125	0.114	0.000	0.161	0.258	0.221
EUR	0.105	0.109	0.099	0.269	0.000	0.225	0.193
JPN	0.115	0.119	0.108	0.294	0.153	0.000	0.211
US	0.111	0.115	0.104	0.284	0.148	0.238	0.000
	Equal weights						
ARG	0.000	0.167	0.167	0.167	0.167	0.167	0.167
BRA	0.167	0.000	0.167	0.167	0.167	0.167	0.167
CHL	0.167	0.167	0.000	0.167	0.167	0.167	0.167
CHI	0.167	0.167	0.167	0.000	0.167	0.167	0.167
EUR	0.167	0.167	0.167	0.167	0.000	0.167	0.167
JPN	0.167	0.167	0.167	0.167	0.167	0.000	0.167
US	0.167	0.167	0.167	0.167	0.167	0.167	0.000
	Trade-based weights						
ARG	0.000	0.308	0.042	0.203	0.266	0.022	0.159
BRA	0.105	0.000	0.034	0.278	0.320	0.040	0.222
CHL	0.036	0.085	0.000	0.359	0.222	0.082	0.217
CHI	0.010	0.038	0.020	0.000	0.404	0.172	0.356
EUR	0.012	0.043	0.012	0.396	0.000	0.104	0.432
JPN	0.003	0.013	0.011	0.415	0.257	0.000	0.302
US	0.008	0.031	0.012	0.366	0.454	0.128	0.000
	Distance-based weights						
ARG	0.000	0.262	0.537	0.032	0.064	0.033	0.073
BRA	0.369	0.000	0.286	0.051	0.118	0.049	0.127
CHL	0.569	0.216	0.000	0.034	0.063	0.038	0.080
CHI	0.062	0.071	0.063	0.000	0.124	0.573	0.108
EUR	0.147	0.194	0.138	0.146	0.000	0.127	0.247
JPN	0.066	0.068	0.070	0.577	0.108	0.000	0.111
US	0.160	0.197	0.166	0.120	0.234	0.123	0.000

(*) Each row adds to unity. Source: Author's calculations.

A remarkable feature of the GVAR-based OE-HNKPC is the exogenous weighting schemes across countries, as suggested by equation (8), in which the same specification allows to accommodate different economic reasonings. For this case, there are many appealing options available, ranging from trade-based to time-varying weights (Di Mauro and Pesaran, 2013) or even leaving them open to be estimated (Gross, 2013). To fully exploit the potential of the model, this article makes use of five different set of weights: (i) fixed-weights coming from the first principal component obtained with the estimation sample and kept fixed for the evaluation sample (labelled as PC1), (ii) equal weights (EW), (iii) trade-based weights (Trade) using a 10-year average of bidirectional trade statistics, (iv) distance-based weights (Distance) fixed by definition, and (v) unlike those of (i), weights from the first principal component but estimated with a rolling sample (PC2).

All weights (except PC2) are presented in Table 1. Trade-based weights are fixed through the full sample and are obtained as the sum of exports plus imports over the total trade of the countries comprising the GVAR (to ensure unity). The distance-based weights take Brasilia as a centre and count the distance to the other capital cities (Buenos Aires, Santiago, Beijing, Lisbon, Tokyo, and Washington DC, considering Lisbon as the closest capital city to Brasilia within the euro area). Then, to reflect closeness–which is the aim with this set of weights–, I take the inverse of the distance of each capital city and adding to a number acting as a pivot. Finally, each inverse distance is divided by the pivot to obtain a sequence of weights adding it to unity. This implies that these weights are fixed and distance measuring units are irrelevant, and will change only when adding new countries. The aim of these weights is to reflect that proximity could imply intensity of one trading partner as a covariate of the other. In this case, Argentina and Chile take a larger share unlike the remaining weighting schemes (see Table 1).

3.4 Forecast evaluation framework

The statistical measure used to evaluate the accuracy of all point forecasts is the Root Mean Squared Forecast Error (RMSFE) defined as:

$$\mathsf{RMSFE}_h = \left[\frac{1}{T} \sum_{t=1}^{T} (\pi_{t+h|t+h} - \pi_{t+h|t})^2 \right]^{\frac{1}{2}}, \tag{19}$$

where $\pi_{t+h|t}$ is the h-step-ahead forecast of $\pi_{t+h|t+h}$ made at period t. Note that this statistic is computed given a forecasting horizon h=$\{1, 6, 12, 24\}$ months ahead, and hence, the difference $T - t$ is variable depending on h. To make a more appealing comparison between the many models, all RMSFE figures are scaled to the RMSFE obtained with the RW. Thus, the analysed statistic corresponds to the RMSFE ratio defined as:

$$\mathsf{RMSFE\ ratio} = \frac{\mathsf{RMSFE}_h^{\mathcal{M}}}{\mathsf{RMSFE}_h^{\mathsf{RW}}}, \tag{20}$$

where $\mathcal{M}$ is the full set of candidate models including all robustness versions (totalising 26 when comparing with the base setup plus 21 originated with single and pairwise country combinations). Hence, as the RW acts as a pivot, values greater than unity imply a worse performance of the competing model. Figures below unity represent a "predictive gain" of (1-RMSFE ratio)% compared to the RW.

To investigate to what extent the predictive gains are statistically significant, I make use of the unconditional t-type test of Giacomini and White (GW, 2006) providing the advantage of comparing *forecasting methods* instead of *forecasting models*. As the null hypothesis (NH) is defined as *the competing model has a superior predictive ability compared to the* RW, a one-side t-type GW statistic is used accordingly.

Formally, the NH: $\mathbb{E}_t(d_h) \leq 0$ is tested against the alternative AH: $\mathbb{E}_t(d_h) > 0$, where:

$$\mathsf{d}_h = (\pi_{t+h|t+h} - \pi_{t+h|t}^{\mathsf{RW}})^2 - (\pi_{t+h|t+h} - \pi_{t+h|t}^{\mathcal{M}})^2, \tag{21}$$

using the Newey and West (1987) heteroscedasticity- and autocorrelation-corrected estimator of the standard deviation of d_h. The NH is rejected if the subsequent t-statistic is greater than $t_{\alpha\%}$, corresponding to the tabulated value of a normal distribution with probability α%.

4 Results

This section analyses both kinds of results: in- and out-of-sample. The in-sample results are related to estimation diagnostics and stability, whereas the out-of-sample results exclusively to dynamic forecasts precision (RMSFE ratio). Robustness results are analysed separately. Notice that there are several models to be evaluated, and they will be labelled as follows:

1. Benchmark models (3 models): autoregression (AR), exponential smoothing (ES), and the random walk (RW),
2. CE-HNKPC (4 models): the baseline uniequational HNKPC displayed in equation (1) (UEq); to allow a fairer comparison with the VAR-based HNKPC, I also consider a VAR version of equation (1) estimated with *Ordinary Least Squares* (VAR); and for both mentioned versions, the 12-order MA output gap is used; labelled, thus, as CE-HNKPC-UEq-MA and CE-HNKPC-VAR-MA,
3. OE-HNKPC (20 models): the baseline OE-HNKPC specification with CF expectations and output gap using PC1, EW, Trade, Distance, and PC2 weighting schemes, and the same specifications using the MA output gap (PC1-MA, etc.). Also, a compacted version is used in which the OE-HNKPC corresponds to the VARX* specification of equation (8), labelled as "X*". Finally, this compacted "X*" version also makes use of the MA output gap (OE-HNKPC-PC1-X*-MA, etc.).

All these models add up to 27 different forecasts, 24 of them coming from an HNKPC-based model. The models considered later with the Focus and unemployment gap are separately analysed; thus, not deserving special notation. For reference, the baseline specification corresponds to the extended OE-HNKPC making use of the output gap without any transformation, trade-based weights, and CF expectations (thus, OE-HNKPC-Trade).

4.1 In-sample diagnostics: closed-economy HNKPC

Table 2 presents the coefficient estimation results of the CE-HNKPC using GMM with the estimation sample. Although the focus is on the Brazilian economy, the results for the other economies are shown for reference. In particular, all these results are useful to actually corroborate whether or not the HNKPC is rejected, to be used later in the VAR-based OE-HNKPC.

To ease a comparison between countries, the estimates are restricted to both inflation coefficients adding to unity, *i.e.* $\lambda_b + \lambda_f = 1$. Having in mind that the dependent variable is the actual inflation rate minus the inflation target of each country (assuming none for Argentina and China), lagged inflation coefficient dominates price formation, except for the euro area. For the particular case of Brazil, the lagged inflation coefficient is 0.969 and that of expected coefficient is 0.030 using CF (3% of the lagged inflation coefficient) and 0.959 and 0.040 using Focus (4% of the lagged inflation coefficient), corroborating the high indexation of the Brazilian inflation (Areosa and Medeiros, 2007). A remarkable result is that Brazil exhibits the biggest output gap coefficient relative to the other countries when using Focus expectations, despite that the output gap dynamics does not look too different when compared with the remaining countries (see Figure 1). All IV are valid according to the J-statistic p-value. Moreover, all estimated coefficients are statistically significant and are in line with the NKPC theory. The proposed model specification fits well to data, except for the euro area exhibiting an adjusted goodness-of-fit coefficient of 3.1% (compared to, for instance, 47.7% obtained with the AR(2) model).

As coefficient instability is a recognised problem with this kind of estimations, Figure 2 shows the evolution of key model parameters in a rolling-window scheme throughout the evaluation sample. Figure 2, panel I depicts the coefficient of the lagged inflation for all countries. Despite the euro area—as mentioned before—the adjustment of lagged inflation is rather stable especially after the *Global Financial Crisis* of 2008-09 (GFC), with values steady around 0.80-0.90. For the euro area, the "switching" estimates obey to the difficulty for the model to identify between past and expected inflation in a prolonged period of low, stable inflation close to the target, with highly persistent and firmly anchored expectations. Thus, for the euro area the AR model could exhibit a much precise performance due to its parsimony.

Table 2: GMM estimates of the CE-HNKPC using the output gap (*)

Infl. exp.:	*ARG* CF	*BRA* CF	*BRA* Focus	*CHL* CF	*CHI* CF	*EUR* CF	*JPN* CF	*US* CF
	Dependent variable: $\widehat{\pi}_t = \pi_t - \pi^{Target}$							
	Estimation sample: 2000.1-2005.12 (Focus: 2002.8-2005.12)							
π^{Target}	-	4.5%	4.5%	3.0%	-	2.0%	2.0%	2.0%
$\widehat{\pi}_{t-1}$	0.852	0.969	0.959	0.861	0.833	0.373	0.660	0.557
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.023]	[0.000]	[0.000]
$\widetilde{\pi}_t$	0.147	0.030	0.040	0.138	0.166	0.626	0.339	0.442
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
$\widetilde{y}_t$	0.361	0.076	0.595	0.108	0.455	0.055	0.017	0.159
	[0.016]	[0.096]	[0.000]	[0.004]	[0.001]	[0.081]	[0.097]	[0.000]
$\overline{\pi}$	-0.055	-0.071	-0.865	-0.385	0.254	-0.765	-0.695	-0.618
	[0.741]	[0.665]	[0.000]	[0.083]	[0.142]	[0.000]	[0.000]	[0.002]
$\overline{R}^2$	0.982	0.937	0.793	0.850	0.786	0.031	0.666	0.798
S.E. Reg	1.649	0.850	1.780	0.471	0.761	0.261	0.256	0.377
J-Stat.	2.062	2.034	4.259	3.666	4.487	6.809	4.298	2.007
p-value	0.356	0.565	0.372	0.453	0.343	0.146	0.231	0.734
	Instrumental variables list (lags)							
Constant								
π_t	(2)	(2)(6)	(2)(9)	(2)	(2)(5)	(2)(6)	(2)	(4)(5)
$\widetilde{\pi}_t$	(2)	(3)(11)	(2)(9)	(5)(8)	(2)	(7)	(3)	-
$\widetilde{\pi}_t^{Current}$	-	-	-	-	(7)	(11)	-	-
$\widetilde{\pi}_t^{Next}$	-	-	-	-	(9)	-	-	-
$\widetilde{y}_t$	(2)(3)	(7)	(6)(7)	(1)(4)(9)	(1)	(5)	(4)(5)(11)	(1-3)(12)

(*) Equation: $\widehat{\pi}_t = \overline{\pi} + \lambda_b \widehat{\pi}_{t-1} + (1-\lambda_b)\widetilde{\pi}_t + \gamma \widetilde{y}_t + \varepsilon_t$, with $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$.
Coefficient *p*-value in $[\cdot]$. Source: Author's calculations.

The mirror component of the lagged inflation component is the coefficient of expected inflation, depicted recursively in Figure 2, panel II. Accordingly, these estimates fluctuate between 0.10-0.20, becoming steady after the GFC. Note that between 2011 and 2014 Brazilian estimates of λ_b are greater than unity, implying a negative λ_f that turns back to the positive zone until the end of the sample. This result is largely due to the rapid inflation rise occurred between mid-2012 and the beginning of 2016, when inflation grew almost 700 basis points in 3.5 years from 4.9% (2012.6) to 10.7% (2016.1) which is much better captured by the persistence of the series rather than its expectations.

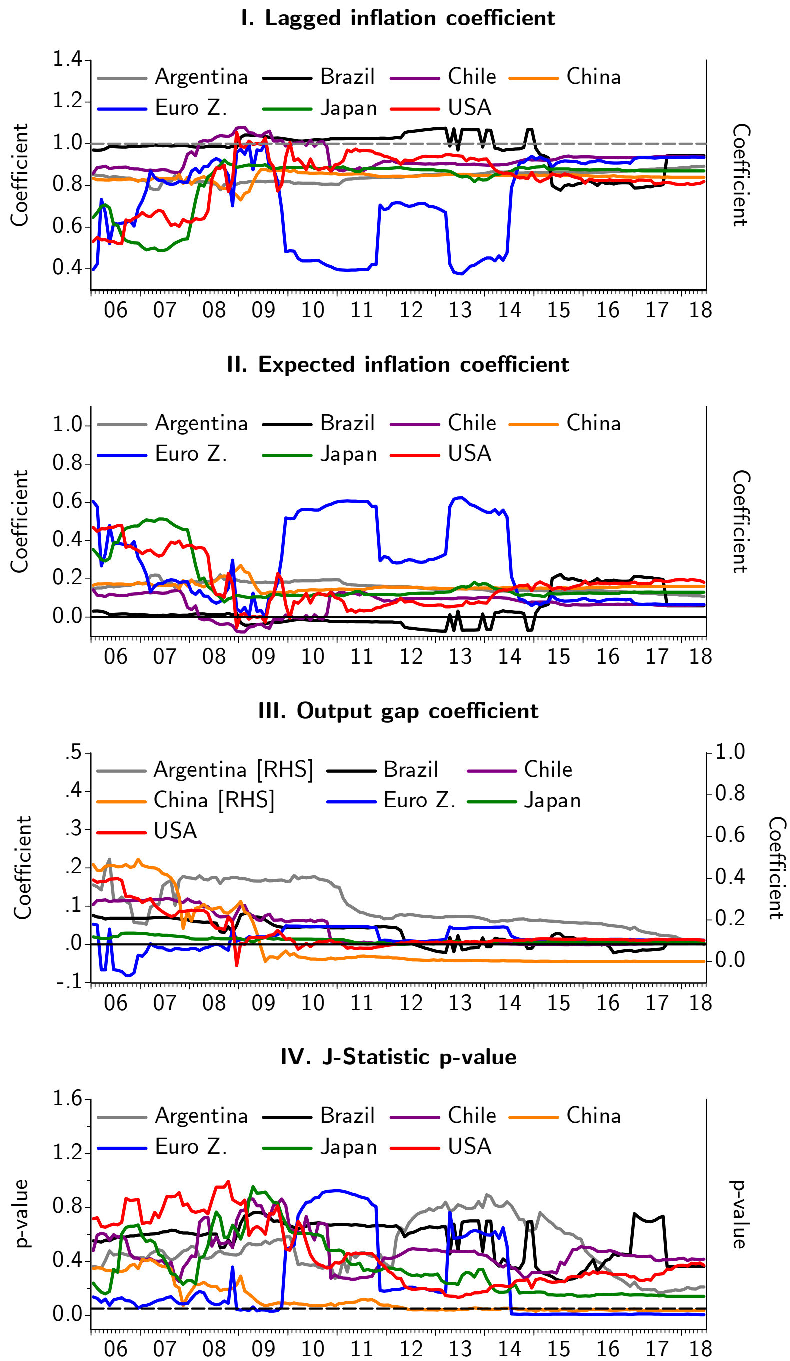
Figure 2, panel III shows the output gap coefficient along the sample, revealing a common behaviour towards flattening, particularly after the GFC. The explanations behind the "Phillips curve flattening" hypothesis have been largely analysed in the literature. The mainstream explanation relies on the weakening relationship between inflation and economic activity when business cycle disturbances on the supply side are much more volatile than those on the demand side (Jacob and van Florenstein Mulder, 2019). So, supply shocks are so influential to fade out the correlation between inflation and domestic activity, a phenomenon attributable to increasing globalisation forces (Bean, 2006; Iakova, 2007). Naturally, extracting global components from inflation (for instance, to obtain core inflation) and estimating economic slack measures with a larger share of non-tradables reduces the scope of the Phillips curve flattening (Constâncio, 2015).

The observed flattening, however, does not reduce the potential importance of the forecasting exercise. In particular, and considering the OE-HNKPC, it precisely makes use of global measures of all variables, including the relevant *ad-hoc* "global economic slack" to explain domestic inflation and thus, capturing global forces much better than domestic economic slack; in turn, playing a more prominent role for inflation dynamics.[5] Also, the estimates with the unemployment gap acting as economic slack (Figure 3) goes against the flattening hypothesis at least

[5]The explanation and forecast exercise with global output gap measures has its foundations in Martínez-García and Wynne (2010) and subsequent papers by the same authors.

for Brazil, the euro area, and Japan. So, there is evidence to not reject the HNKPC as a valid inflation model and forecasting device.

Figure 2: CE-HNKPC: recursive estimates of key coefficients,
restricted estimation, using the output gap and CF (*)



(*) Horizontal line in panel IV=10%. Source: Author's calculations.

Moreover, Figure 2, panel IV suggests that the same IV set chosen with the estimation sample is significant across the evaluation sample, with China and the euro area with a slightly lower level of significance, recalling that the

NH is *IV are valid*; thus, seeking for non-rejection of the NH to conceive the HNKPC as valid. For the case of Brazil, not a single point falls below 20%, having valid IV during the whole evaluation sample.

When analysing the CE-HNKPC-UEq by using the unemployment gap as economic slack measure, the estimates are more well-behaved (*i.e.* smoothed) than in the previous case. Due to data availability, the estimates exclude Argentina and China, but they are considered in the OE-HNKPC using the output gap (entering with the opposite sign).

The point estimates using the unemployment gap are presented in Table 3. For Brazil, the results with Focus expectations are quite similar for the inflation-based coefficients when comparing with the output gap (0.971 versus 0.969 for lagged inflation and 0.029 versus 0.030 for expected inflation). However, with the remaining countries and BRA: CF, the estimates are more balanced between expected and lagged inflation, except with the euro area. However, this region largely increases its goodness of fit from 3.1% to 28.3% compared to using the output gap, and with lagged inflation commanding the dynamic of the series displaying a coefficient of 0.976 (0.024 for expected inflation).

Table 3: GMM estimates of the CE-HNKPC using the unemployment gap (*)

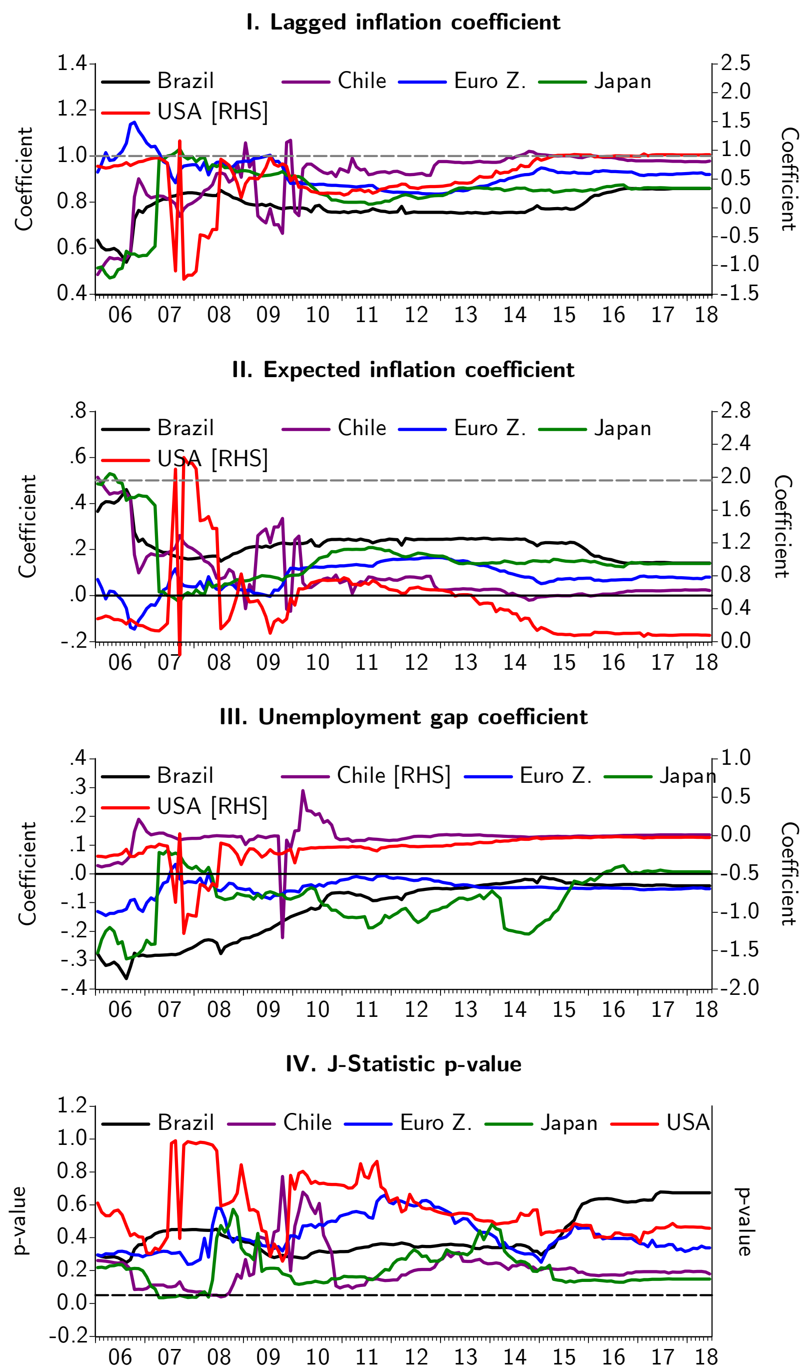
	BRA	*BRA*	*CHL*	*EUR*	*JPN*	*US*
Infl. Exp.:	CF	Focus	CF	CF	CF	CF
	Dependent variable: $\widehat{\pi}_t = \pi_t - \pi^{Target}$					
	Estimation sample: 2000.1-2005.12 (Focus: 2002.8-2005.12)					
π^{Target}	4.5%	4.5%	3.0%	2.0%	2.0%	2.0%
$\widehat{\pi}_{t-1}$	0.642	0.971	0.489	0.976	0.508	0.662
	[0.000]	[0.000]	[0.000]	[0.000]	[0.022]	[0.000]
$\widetilde{\pi}_t$	0.358	0.029	0.511	0.024	0.492	0.338
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
$\widetilde{u}_t$	-0.347	-0.601	-0.473	-0.146	-0.287	-0.328
	[0.045]	[0.069]	[0.082]	[0.037]	[0.064]	[0.100]
$\overline{\pi}$	-1.252	0.082	-1.451	-0.011	-0.988	-0.472
	[0.000]	[0.775]	[0.000]	[0.940]	[0.022]	[0.063]
$\overline{R}^2$	0.901	0.937	0.781	0.283	0.545	0.797
S.E. Reg	1.017	0.967	0.551	0.221	0.304	0.373
J-Stat.	1.576	4.279	6.720	5.104	3.066	2.360
p-value	0.454	0.369	0.242	0.276	0.215	0.669
	Instrumental variables list (lags)					
Constant						
π_t	(2)	(2)(6)	(2)(3)(12)	(3)(5)	(2)(6)	(10)
$\widetilde{\pi}_t$	(2)	(2)(8)	(2)(12)	(2)(5)	(4)	(1)(5)
$\widetilde{y}_t$	(3)(5)	(4)(10)	(4)(10)	(2)(6)	(9)	(2)(3)(6)

(*) Equation: $\widehat{\pi}_t = \overline{\pi} + \lambda_b \widehat{\pi}_{t-1} + (1-\lambda_b)\widetilde{\pi}_t + \gamma \widetilde{u}_t + \varepsilon_t$, with $\varepsilon_t \sim iid(0, \sigma^2_\varepsilon)$.
Coefficient p-value in $[\cdot]$. Source: Author's calculations.

The larger unemployment gap coefficient in absolute terms is obtained for Brazil using Focus expectations (-0.601); then Chile with -0.473 and Brazil again with CF (-0.347). However, as will be shown, for Chile the coefficient hovered to zero after the GFC becoming statistically non-significant, whereas for Brazil it approaches zero but not enough to be negligible. The goodness-of-fit coefficient, similar to the previous case, suggests the best adjustment for Brazil and then for the United States and Chile. All IV come out as valid according to the J-statistic p-value.

When considering the stability of the key coefficients, Figure 3 (panels I and II) reveal steadier and more affine estimates compared to using the output gap, especially after the GFC. The coefficient dynamic after 2009 in Figure 3, panel I is quite revealing when compared to Figure 2, panel I tending to concentrate the dynamics in lagged inflation. For the unemployment gap, Figure 3, panel III shows that for Chile, Japan, and the United States the flattening hypothesis applies from 2011 onwards–*i.e.* the period of low, controlled inflation. However, and unlike the output gap case, for Brazil and the euro area the coefficient is different from zero throughout the whole evaluation sample, ultimately playing a role when forecasting. Finally, Figure 3, panel IV reveals that the IV chosen with the estimation sample remain valid until the last available observation.

Figure 3: CE-HNKPC: recursive estimates of key coefficients, restricted estimation, using the unemployment gap and CF (*)



(*) Horizontal line in panel IV=10%. Source: Author's calculations.

In sum, and particularly for the Brazilian case, there is evidence to not reject the CE-HNKPC-UEq proposal when using, at least, the unemployment gap as a measure of economic slack with all observations available. For its trading partners, the evidence is mixed throughout the evaluation sample depending on the economic slack measure used and inflationary experience.

Table 4 shows the results of weak instruments testing for both output and unemployment gap using CF and Focus expectations. The test is composed by the Cragg-Donald F-statistic which is contrasted with the Stock and Yogo (2010) critical values in search of a statistical threshold of the percentage of bias compared to the OLS case (GMM being equal to OLS at 100%). In this sense, the test does not follow a "critical-value-based" decision rule, but 20% is recommended as a rule of thumb of the "identification strength". From this point of view, only three cases exceed the 20% threshold, which are BRA: Focus and the United States, both with the output gap, and Japan with the unemployment gap. Notice that, as will be reviewed later, the best forecasting option of BRA: Focus is obtained with the OE-HNKPC and the unemployment gap and, hence, no special alleviating treatment is made for this case.

All these results are important because, as will be shown later, in most cases with the CE-HNKPC, the best out-of-sample results are obtained with the VAR estimation. This implies that the simultaneity in coefficient estimation provided by VAR (OLS) is preferable to a better option in controlling bias (GMM). This is also reflected with the OE-HNKPC which is intrinsically estimated simultaneously, but it comes out as the best forecasting option of the whole exercise at h=12 and 24, as will be shown later. Thus, the information contained in the model is as useful as it is difficult to disentangle the precise contribution of each variable. It is important to mention that the exercise carried out in this paper is circumscribed to the NKPC without questioning its economic foundations. Instead, previous to it being used and extended as a forecasting device, it is statistically tested with the most suitable estimator (GMM). The estimates of Tables 2 and 3 are favourable for the CE-HNKPC proposal and, at the same time, make the in-sample statistical inference of the OE-HNKPC more demanding.

Table 4: CE-HNKPC-UEq weak instruments diagnostics (*)

Country	Instruments	C-D F-stat.	S-Y c.v.			Relevant
			10%	15%	20%	MSC
		Output gap				
ARG	$\pi_{t-2}, \widetilde{\pi}_{t-2}, \widetilde{y}_{t-2}, \widetilde{y}_{t-3}$	83.39	24.58	13.96	10.26	-0.807
BRA: CF	$\pi_{t-2}, \pi_{t-6}, \widetilde{\pi}_{t-3}, \widetilde{\pi}_{t-11}, \widetilde{y}_{t-7}$	11.25	26.87	15.09	10.98	-1.233
BRA: Focus	$\pi_{t-2}, \pi_{t-9}, \widetilde{\pi}_{t-2}, \widetilde{\pi}_{t-9}, \widetilde{y}_{t-6}, \widetilde{y}_{t-7}$	7.94	29.18	16.23	11.72	-4.160
CHL	$\pi_{t-2}, \widetilde{\pi}_{t-5}, \widetilde{\pi}_{t-8}, \widetilde{y}_{t-1}, \widetilde{y}_{t-4}, \widetilde{y}_{t-9}$	14.81	29.18	16.23	11.72	3.008
CHI	$\pi_{t-2}, \pi_{t-5}, \widetilde{\pi}_{t-2}, \widetilde{\pi}^{Current}_{t-7}, \widetilde{\pi}^{Next}_{t-9}, \widetilde{y}_{t-1}$	27.61	29.18	16.23	11.72	4.975
EUR	$\pi_{t-2}, \pi_{t-6}, \widetilde{\pi}_{t-7}, \widetilde{\pi}^{Current}_{t-11}, \widetilde{y}_{t-5}$	22.38	29.18	16.23	11.72	-1.300
JPN	$\pi_{t-2}, \widetilde{\pi}_{t-3}, \widetilde{y}_{t-4}, \widetilde{y}_{t-5}, \widetilde{y}_{t-11}$	37.43	26.87	15.09	10.98	1.220
US	$\pi_{t-4}, \pi_{t-5}, \widetilde{y}_{t-1}, \widetilde{y}_{t-2}, \widetilde{y}_{t-3}, \widetilde{y}_{t-11}$	3.67	29.18	16.23	11.72	5.610
		Unemployment gap				
BRA: CF	$\pi_{t-2}, \widetilde{\pi}_{t-2}, \widetilde{y}_{t-3}, \widetilde{y}_{t-5}$	42.61	24.58	13.96	10.26	0.990
BRA: Focus	$\pi_{t-2}, \pi_{t-6}, \widetilde{\pi}_{t-2}, \widetilde{\pi}_{t-8}, \widetilde{y}_{t-4}, \widetilde{y}_{t-10}$	20.85	21.68	12.33	9.19	-0.330
CHL	$\pi_{t-2}, \pi_{t-3}, \pi_{t-12}, \widetilde{\pi}_{t-2}, \widetilde{\pi}_{t-12}, \widetilde{y}_{t-4}, \widetilde{y}_{t-10}$	14.59	31.50	17.38	12.48	2.790
EUR	$\pi_{t-3}, \pi_{t-5}, \widetilde{\pi}_{t-2}, \widetilde{\pi}_{t-5}, \widetilde{y}_{t-2}, \widetilde{y}_{t-6}$	13.98	29.18	16.23	11.72	-1.000
JPN	$\pi_{t-2}, \pi_{t-6}, \widetilde{\pi}_{t-4}, \widetilde{y}_{t-9}$	8.71	26.87	15.09	10.98	0.940
US	$\pi_{t-10}, \widetilde{\pi}_{t-1}, \widetilde{\pi}_{t-5}, \widetilde{y}_{t-2}, \widetilde{y}_{t-3}, \widetilde{y}_{t-6}$	57.58	31.50	17.38	12.48	2.690

(*) "C-D F-stat." stands for Cragg-Donald F-statistic. "S-Y c.v." stands for Stock and Yogo (2004) critical values. "MSC" stands for moment selection criteria. Source: Author's calculations.

4.2 In-sample diagnostics: open-economy HNKPC

In an attempt to make the results of the OE-HNKPC more comparable with those of the CE-HNKPC, in this subsection I display the base rolling coefficients of the OE-HNKPC, despite the caveat that they are obtained from a VAR estimation. Thus, not surprisingly, these estimates exhibit a more volatile dynamic due to a higher (non-controlled) collinearity. This is also the reason why I discard analysing point estimates at the end of the estimation sample, and I do not impose the restriction for inflation coefficients to add to unity. The support for this analysis relies on the Granger causality test for exogeneity.

Starting with the output gap version, CF inflation expectations, and trade-based weights,[6] the upper panel of Table 5 shows the traditionally used information criteria for lag length selection, indicating a lag equal to one for all countries according to the preferred Bayesian Information Criterion (BIC). The remaining IC suggest a bigger number of lags, particularly, the Akaike IC and the Hannan-Quinn IC to a lesser extent. This means that BIC implies an estimation of 294 coefficients, whereas AIC a total of 1116 coefficients. In sum, BIC parsimony is contributing to forecast accuracy as it avoids exaggerating inflation dynamics from both domestic and foreign blocs.

Notice that the foreign variables $\mathbf{x}_{it}^{*}$ of equation (8), later used in equation (18), can be viewed actually as VAR exogenous block, helping to understand the mechanics inside the proposed OE-HNKPC. Hence, testing VAR's block exogeneity is relevant for two reasons: (i) it reveals the extent to which is possible to rely statistical inference on coefficient estimates subject to a possible invalidating share of bias, (ii) it reveals the extent to which a likely common "global inflation factor" exists and affects the estimation of the domestic HNKPC parameters, particularly that of expected inflation.[7]

The Granger causality results of the VAR exogeneity block for the foreign variables are presented in Table 5 (null hypothesis: foreign variables do not cause domestic variables). Focusing on Brazil using CF, the only independent (*i.e.* exogenous) variable is the output gap. Also, the Brazilian foreign output gap measure Granger causes its foreign inflation measure, which also happened with Focus expectations. This means that the foreign information used for predicting Brazilian inflation, while useful, is not clear from which component of trade partners' Phillips curve it is coming from. When using Focus, however, there is no evidence that all $\mathbf{x}_{it}^{*}$ variables are exogenous. A similar situation is obtained with the remaining countries when using either CF or Focus. A remarkable case is China, that could be considered as a full exogenous bloc, except for its foreign inflation with CF; but fully exogenous with Focus. In sum, using neither CF nor Focus expectations, in not a single country does the $\mathbf{x}_{it}^{*}$ vector act as an exogenous bloc in full.

Also, in Annex D, Figure D1, I show the residuals of each of the equations comprising the OE-HNKPC-Trade for all countries, as well as the adjusted goodness-of-fit coefficient. It is possible to visually corroborate that all residuals are well behaved, showing major variations during specific periods of higher inflation volatility. For the case of Brazil, more volatile residuals are found for the output gap equation, not attributable to any specific event nor particularly different to any other country, whereas expected inflation displays greater volatility during the 2001-02 period only.

Figure 4 displays the lagged and expected inflation coefficients across the estimation sample along with those of the output gap. Figure 4, panel I reveals fuzzy dynamics of the lagged inflation coefficient ranging between 0.2 and 1.0. For the particular case of Brazil, the coefficient starts in 0.63 and ends in 0.78, with a swing comparable to that of Figure 2. The more volatile estimates are obtained with Japan and the United States, both with a similar movement but with a more pronounced decline for the United States case since 2014.

Figure 4, panel II, displays the expected inflation coefficients. As the estimates are not restricted to add to unity, these are steadier than those of lagged inflation, except for the case of Argentina. In general, the coefficients range between 0.0 and 0.2, with the euro area displaying a higher coefficient from 2015 onward. In turn, Figure 4, panel III shows the rolling estimates of the output gap coefficient. Roughly speaking, all countries exhibit the same sort of hump-shaped behaviour, except for China and the United States displaying virtually an insensitiveness of inflation to the output gap, *i.e.* the NKPC flattening hypothesis. Notice, however, that a bigger role is found for this driving process during the more volatile period of inflation, *i.e.* the GFC, being similar to the estimates of Figure 2 (including China).

Overall, according to these diagnostics and despite sharp movements in estimates, the coefficients do not threaten the proposal of the HNKPC for the open economy.

[6] The in-sample results of the OE-HNKPC using Focus expectations are available upon request.

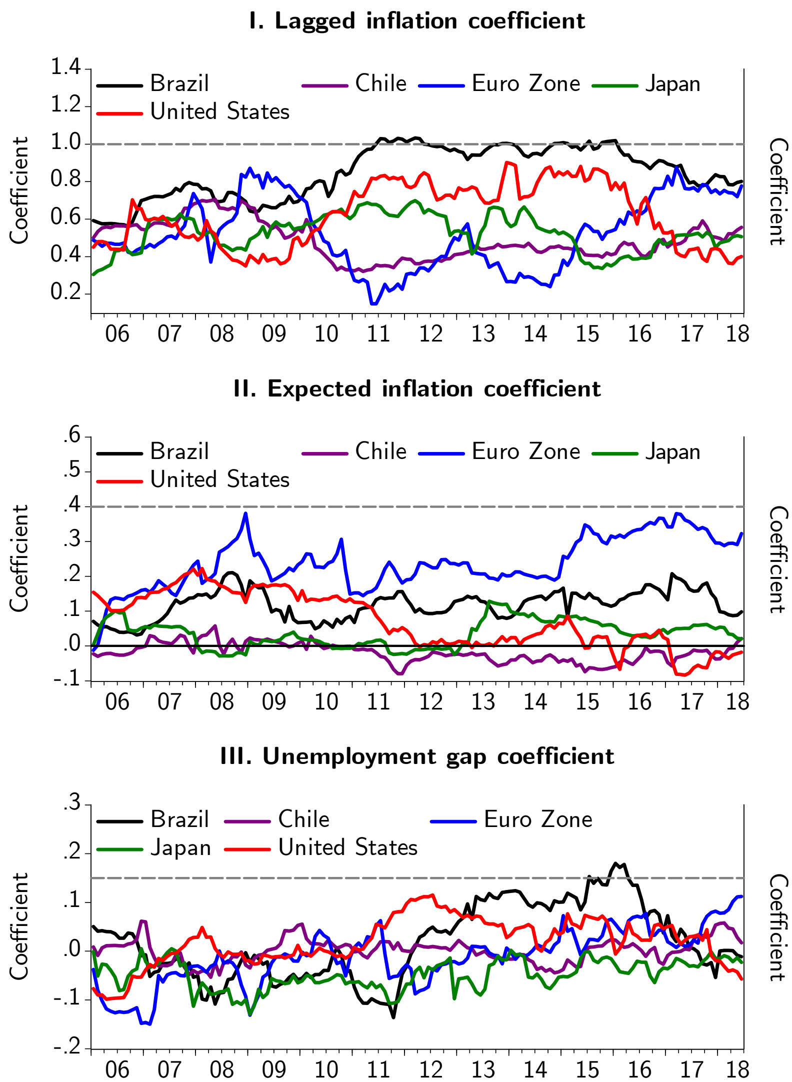
[7] Some empirical evidence on the predictability of *ad-hoc* global inflation factors to domestic inflation rates can be found in Ciccarelli and Mojon (2010), Medel (2016), Medel, Pedersen, and Pincheira (2016) and the papers cited therein.

Table 5: OE-HNKPC-Trade diagnostics; output gap (*)

	ARG	*BRA*	*CHL*	*CHI*	*EUR*	*JAP*	*US*
Output gap + CF							
Lag length							
BIC	1	1	1	1	1	1	1
AIC	6	6	1	6	1	6	5
HQ	2	2	1	1	1	1	1
Block exogeneity: Granger causality test p-value							
Dependent variable: Foreign inflation							
π_t	0.001	0.216	0.320	0.128	0.379	0.704	0.175
$\widetilde{\pi}_t$	0.041	0.667	0.339	0.793	0.747	0.027	0.766
$\widetilde{y}_t$	0.364	0.702	0.311	0.132	0.002	0.655	0.702
$\widetilde{\pi}_t^*$	0.037	0.608	0.015	0.060	0.005	0.126	0.127
$\widetilde{y}_t^*$	0.011	0.000	0.137	0.044	0.017	0.991	0.939
All	0.000	0.000	0.004	0.085	0.005	0.093	0.553
Dependent variable: Foreign output gap							
π_t	0.363	0.607	0.112	0.926	0.058	0.006	0.027
$\widetilde{\pi}_t$	0.012	0.965	0.403	0.526	0.416	0.001	0.001
$\widetilde{y}_t$	0.656	0.205	0.157	0.783	0.038	0.291	0.027
π_t^*	0.876	0.663	0.855	0.579	0.093	0.107	0.307
$\widetilde{y}_t^*$	0.635	0.676	0.825	0.432	0.227	0.977	0.998
All	0.092	0.692	0.686	0.947	0.172	0.001	0.001
Dependent variables: Foreign inflation expectations							
π_t	0.000	0.351	0.008	0.744	0.532	0.247	0.792
$\widetilde{\pi}_t$	0.194	0.751	0.088	0.581	0.247	0.405	0.667
$\widetilde{y}_t$	0.000	0.819	0.398	0.263	0.695	0.836	0.334
π_t^*	0.133	0.076	0.217	0.257	0.065	0.001	0.006
$\widetilde{\pi}_t^*$	0.820	0.003	0.469	0.776	0.223	0.431	0.798
All	0.000	0.000	0.008	0.415	0.209	0.000	0.036
Output gap + Focus plus CF							
Block exogeneity: Granger causality test p-value							
Dependent variable: Foreign inflation							
π_t	0.027	0.210	0.152	0.559	0.380	0.870	0.548
$\widetilde{\pi}_t$	0.973	0.863	0.249	0.867	0.343	0.105	0.471
$\widetilde{y}_t$	0.635	0.592	0.572	0.287	0.109	0.707	0.447
$\widetilde{\pi}_t^*$	0.469	0.377	0.155	0.190	0.041	0.247	0.001
$\widetilde{y}_t^*$	0.048	0.000	0.488	0.383	0.883	0.505	0.244
All	0.000	0.000	0.071	0.446	0.029	0.098	0.031
Dependent variable: Foreign output gap							
π_t	0.216	0.288	0.638	0.855	0.394	0.031	0.334
$\widetilde{\pi}_t$	0.093	0.447	0.188	0.948	0.860	0.015	0.496
$\widetilde{y}_t$	0.687	0.035	0.752	0.082	0.749	0.321	0.384
π_t^*	0.042	0.025	0.669	0.975	0.368	0.028	0.945
$\widetilde{y}_t^*$	0.355	0.030	0.874	0.573	0.267	0.098	0.138
All	0.034	0.011	0.805	0.463	0.226	0.000	0.222
Dependent variables: Foreign inflation expectations							
π_t	0.017	0.002	0.001	0.462	0.601	0.058	0.857
$\widetilde{\pi}_t$	0.010	0.743	0.275	0.995	0.025	0.215	0.873
$\widetilde{y}_t$	0.040	0.136	0.017	0.270	0.234	0.446	0.201
π_t^*	0.526	0.097	0.008	0.362	0.316	0.002	0.079
$\widetilde{\pi}_t^*$	0.544	0.018	0.342	0.609	0.558	0.440	0.599
All	0.000	0.000	0.001	0.524	0.093	0.000	0.076

(*) BIC=Bayesian Information Criterion (IC), AIC=Akaike IC, HQ=Hannan-Quinn IC. Sample: 2000.1-2005.12. Shaded cells: p<10%. Source: Author's calculations.

Figure 4: OE-HNKPC-Trade: recursive estimates of key coefficients
using CF and the output gap (*)



(*) Vertical lines=1.0, 0.40, and 0.15 shown for reference only. Source: Author's calculations.

Similar to the previous case, in Table 6, upper panel, I display the results of lag length criteria using the unemployment gap and CF inflation expectations. The preferred BIC suggest one lag for all countries, implying 294 coefficients to be estimated, followed by the Hannan-Quinn IC with only Brazil increasing to two lags, and the less parsimonious AIC suggesting the estimation of 942 coefficients.

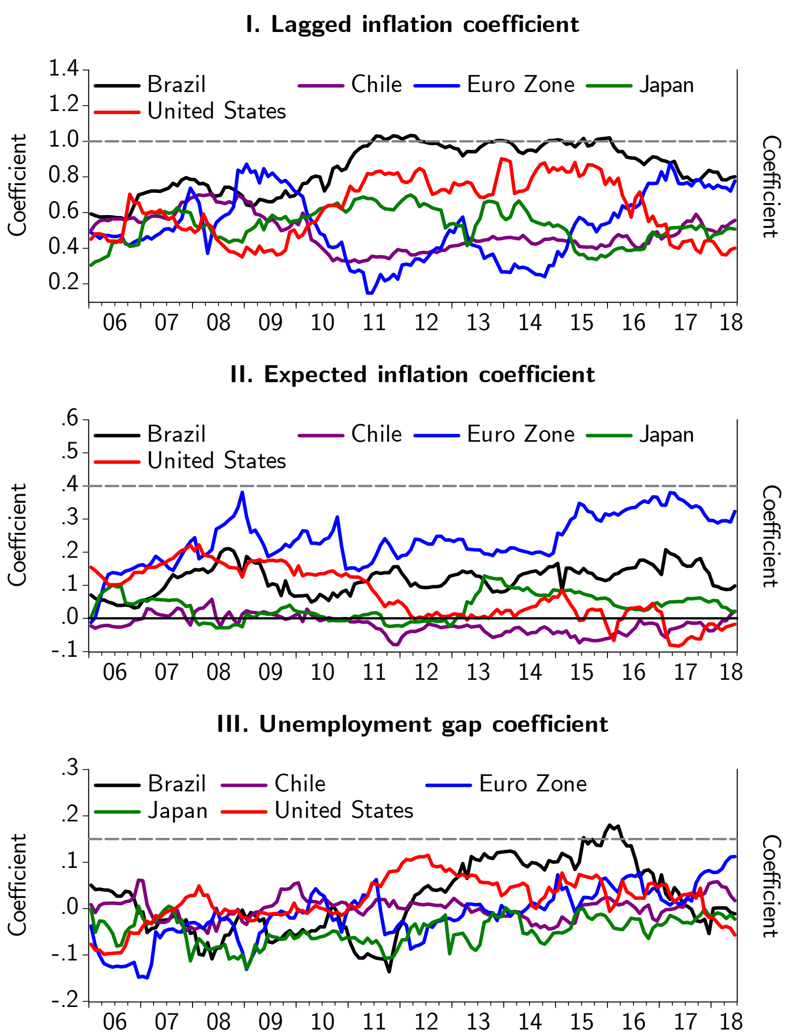
Regarding the results of VAR exogenous block, similar qualitative results are found for Brazil, *i.e.* the output gap independently determined when using CF but endogenous when using Focus. Also, China displays all its domestic variables independently determined, except the foreign output gap with Focus. For the remaining countries, in not a single country do the foreign variables act as an exogenous bloc in full using either CF or Focus expectations.

In terms of rolling estimates, Figure 5, panel I, shows that, in general, and similar to the CE-HNKPC case, lagged inflation coefficients are more stable than when using the output gap. For the case of Brazil, it starts at 0.6 and ends around 0.8, quite like to the CE-HNKPC version, but in this case, as coefficients are not restricted to add to unity, the coefficients stand barely above 1.0 for some observations between 2011 and 2015.

Figure 5, panel II, shows the expected inflation coefficient estimates. For Brazil, the coefficient is relatively stable around 0.15 and complements well the unrestricted mirror component of lagged inflation. Two other cases deserve mention, namely Chile and the United States. In these cases, the coefficient crosses to the negative zone in the second half of the sample. However, this could be due to the fact that are unrestricted estimates, as the same dynamic is noticed with the restricted CE-HNKPC case of Figure 3, panel II.

Figure 5, panel III, displays the rolling coefficients of the unemployment gap. In general, for all cases it shows an increasing pattern throughout the evaluation sample. However, except for Japan, the coefficients move to the positive region in the second half of the sample, which is not noticed with the CE-HNKPC. This means that the driving process is evolving and, thus, the weighting scheme, especially PC2, plays a key role in accommodating the driving process that better describes domestic inflation. For Brazil, this effect is a bit less dramatic noticing that happens during 2012 to 2017 only.

Figure 5: OE-HNKPC-Trade: recursive estimates of key coefficients
using CF and the unemployment gap (*)



(*) Vertical lines=1.0, 0.40, and 0.15 shown for reference only. Source: Author's calculations.

Table 6: OE-HNKPC-Trade diagnostics; unemployment gap (*)

	ARG	*BRA*	*CHL*	*CHI*	*EUR*	*JAP*	*US*
Unemployment gap + CF							
Lag length							
BIC	1	1	1	1	1	1	1
AIC	6	6	1	3	6	1	2
HQ	1	2	1	1	1	1	1
Block exogeneity: Granger causality test p-value							
Dependent variable: Foreign inflation							
π_t	0.001	0.149	0.931	0.055	0.710	0.751	0.341
$\widetilde{\pi}_t$	0.323	0.081	0.130	0.392	0.975	0.655	0.924
$\widetilde{y}_t$	0.057	0.729	0.186	0.174	0.057	0.437	0.702
π_t^*	0.570	0.094	0.001	0.404	0.003	0.036	0.008
$\widetilde{\pi}_t^*$	0.047	0.000	0.174	0.016	0.022	0.894	0.759
All	0.000	0.000	0.000	0.243	0.022	0.098	0.151
Dependent variable: Foreign output gap							
π_t	0.824	0.720	0.092	0.560	0.044	0.891	0.026
$\widetilde{\pi}_t$	0.263	0.018	0.083	0.922	0.190	0.788	0.500
$\widetilde{y}_t$	0.600	0.194	0.154	0.212	0.006	0.052	0.162
π_t^*	0.681	0.123	0.619	0.285	0.590	0.895	0.557
$\widetilde{y}_t^*$	0.394	0.587	0.528	0.714	0.000	0.002	0.668
All	0.291	0.137	0.178	0.635	0.000	0.037	0.157
Dependent variables: Foreign inflation expectations							
π_t	0.000	0.179	0.019	0.905	0.822	0.202	0.189
$\widetilde{\pi}_t$	0.050	0.272	0.493	0.384	0.541	0.071	0.035
$\widetilde{y}_t$	0.000	0.674	0.730	0.548	0.961	0.276	0.357
π_t^*	0.059	0.048	0.021	0.175	0.140	0.000	0.001
$\widetilde{\pi}_t^*$	0.130	0.001	0.031	0.289	0.614	0.614	0.998
All	0.000	0.000	0.003	0.299	0.316	0.000	0.006
Unemployment gap + Focus plus CF							
Block exogeneity: Granger causality test p-value							
Dependent variable: Foreign inflation							
π_t	0.007	0.019	0.086	0.212	0.574	0.920	0.793
$\widetilde{\pi}_t$	0.758	0.098	0.964	0.979	0.753	0.150	0.929
$\widetilde{y}_t$	0.524	0.784	0.241	0.334	0.120	0.552	0.679
$\widetilde{\pi}_t^*$	0.227	0.095	0.005	0.442	0.058	0.007	0.010
$\widetilde{y}_t^*$	0.052	0.000	0.065	0.211	0.049	0.382	0.955
All	0.000	0.000	0.003	0.615	0.138	0.049	0.133
Dependent variable: Foreign output gap							
π_t	0.689	0.456	0.894	0.398	0.427	0.640	0.177
$\widetilde{\pi}_t$	0.730	0.245	0.275	0.031	0.780	0.996	0.969
$\widetilde{y}_t$	0.750	0.066	0.249	0.100	0.227	0.989	0.159
π_t^*	0.141	0.017	0.952	0.910	0.819	0.528	0.287
$\widetilde{y}_t^*$	0.404	0.556	0.215	0.980	0.000	0.045	0.474
All	0.561	0.063	0.300	0.078	0.000	0.086	0.227
Dependent variables: Foreign inflation expectations							
π_t	0.004	0.006	0.068	0.808	0.648	0.094	0.631
$\widetilde{\pi}_t$	0.003	0.701	0.291	0.989	0.411	0.253	0.001
$\widetilde{y}_t$	0.024	0.023	0.274	0.914	0.507	0.390	0.014
π_t^*	0.337	0.003	0.002	0.493	0.202	0.000	0.022
$\widetilde{\pi}_t^*$	0.160	0.001	0.378	0.062	0.953	0.791	0.257
All	0.000	0.000	0.001	0.173	0.465	0.001	0.000

(*) BIC=Bayesian Information Criterion (IC), AIC=Akaike IC, HQ=Hannan-Quinn IC. Sample: 2000.1-2005.12. Shaded cells: p<10%. Source: Author's calculations.

Moreover, these results are in line with previous studies for Brazil such as that of Ponzoni and Zilli (2015) questioning the ability of unemployment gap to describe inflation because of its episodic evidence, in line with other authors applying a flexible, accommodative setup to deal with instability (see Section 2). In this article, despite the adaptive weighting scheme (PC2), I left the coefficients unrestricted to be estimated following the residuals minimisation principle, making use of the information coming from a variety of inflationary experiences and, thus, the methodology will find the most appropriate set of coefficients for a given sample. Any intervention at this stage, although desirable from a theoretical point of view, could be detrimental to keep a fair comparison between the candidate models. Finally, in Annex D, Figure D2, I show the residuals of all equations of the OE-HNKPC-Trade for all countries as well as the adjusted goodness-of-fit coefficients. There are no major differences when compared to Figure D1 and, thus, the residuals are well behaved and reacting in episodes of major inflation volatility. The volatility of unemployment gap residuals is diminished with respect to the output gap, particularly for Brazil, and slightly noticed for the remaining countries.

4.3 Out-of-sample results

The baseline out-of-sample results make use of the CF expectations for all countries as well as the output gap as economic slack measure, and trade-based weights with the OE-HNKPC. As mentioned above, the results are presented in relative terms compared to the RMSFE obtained with the RW to ease a comparison between the different models (and always using the same evaluation sample for a given forecast horizon).

Table 7: Brazil: RMSFE ratio estimates using output gap and CF expectations (*)

		h=1	h=6	h=12	h=24
Statistical	AR	0.941	1.089	1.043	0.803*
	ES	1.519***	1.089***	1.039**	0.983
CE-HNKPC	UEq	0.990	0.856**	0.858**	0.809*
	VAR	0.984	0.834*	0.798**	0.685**
	UEq-MA	1.017	0.833**	0.878	1.182
	VAR-MA	1.046	0.932	0.863	0.739*
OE-HNKPC	PC1	1.442***	2.093***	2.799**	2.465**
	EW	1.445***	2.098***	2.801**	2.463**
	Trade	1.444***	2.100***	2.805**	2.468**
	PC2	1.440***	2.100***	2.802**	2.466**
	Distance	1.441***	2.138***	2.940**	5.018**
OE-HNKPC	PC1-MA	1.451***	2.143***	2.937***	5.025***
	EW-MA	1.445***	2.139***	2.944***	5.021***
	Trade-MA	1.448***	2.140***	2.939***	5.017***
	PC2-MA	1.434***	2.095***	2.801***	2.465***
	Distance-MA	1.448***	2.143***	2.941***	5.021***
OE-HNKPC	PC1-X*	1.082*	1.395**	1.350**	0.860
	EW-X*	1.182**	1.597***	1.437**	0.793**
	Trade-X*	1.112*	1.471**	1.441**	0.921
	PC2-X*	1.173**	1.569***	1.451**	0.817*
	Distance-X*	1.169***	1.636***	1.646**	1.384***
OE-HNKPC	PC1-X*-MA	1.162**	1.643***	1.571***	1.436
	EW-X*-MA	1.162**	1.602***	1.661***	1.501*
	Trade-X*-MA	1.161**	1.624***	1.630***	1.463*
	PC2-X*-MA	1.188**	1.528***	1.332***	0.770*
	Distance-X*-MA	1.153	1.587	1.492	1.462

(*) Orange-shaded cells=RMSFE ratio below unity. Green-shaded cells=best result for a given horizon. GW test results: (***) p<1%, (**) p<5%, (*) p<10%.
Source: Author's calculations.

The baseline results are presented in Table 7 (for Brazil only; the results for the remaining countries are available upon request). The best forecasting model at the nowcast horizon is the AR—while not statistically significant—showing a predictive gain of 5.9% with respect to the RW. For the remaining horizons, the best models are the

CE-HNKPC-UEq-MA (with a predictive gain of 16.7%) and the VAR version of the CE-HNKPC using the output gap with no transformation (20.2% and 31.5% for 12 and 24 steps ahead, respectively); all of them statistically significant. Recall that the increasing predictive gains obey to an RMSFE ratio numerator growing at a faster pace than the denominator; this is so, due to a worse RW performance at longer horizons, despite that the RMSFE of the candidate model is also growing (following the profile suggested by the optimal forecast).

Regarding the OE-HNKPC, three facts are worth mentioning which are present throughout all these comparisons. First, the extended version of the OE-HNKPC persistently shows a lower performance compared to the compacted version ("X*") for all horizons. This fact, however, is expected as the overfitting is said to be detrimental for forecast accuracy. Remarkably, the information contained in the OE-HNKPC-X* is the same as that used in the OE-HNKPC version and thus the results reveal no usefulness in estimating a larger set of coefficients for forecast accuracy. Second, the results with the MA version of the output gap are barely less precise than its comparable versions. Consequently, they will be analysed with Focus expectations and dropped in subsequent robustness results. Third, the predictive gains of the OE-HNKPC are noticed for horizons equal to or beyond a year, being overperformed by the CE-HNKPC in the short-term (except in a particular setup that will be shown below).

4.4 Robustness results I: Focus expectations

In this subsection, I analyse the same baseline models but making use of Focus inflation expectations just for the case of Brazil, whereas for the remaining countries the CF series are still in use. Obviously, the RMSFE ratios of the statistical models does not change but are reported for reference.

Table 8: Brazil: RMSFE ratio estimates using output gap and Focus plus CF expectations (*)

		h=1	h=6	h=12	h=24
Statistical	AR	0.941	1.089	1.043	0.803*
	ES	1.519***	1.089***	1.039**	0.983
CE-HNKPC	UEq	1.030	1.069	1.140	1.232**
	VAR	0.995	0.902	0.916	0.815
	UEq-MA	1.044	1.024	1.118	1.737**
	VAR-MA	1.053	0.995	0.931	0.704**
OE-HNKPC	PC1	1.584***	2.310***	3.326*	3.336***
	EW	1.578***	2.307***	3.330*	3.346***
	Trade	1.584***	2.315***	3.327*	3.346***
	PC2	1.586***	2.313***	3.329*	3.348***
	Distance	1.651***	2.945***	3.698*	6.159***
OE-HNKPC	PC1-MA	1.650***	2.947***	3.700***	6.160
	EW-MA	1.651***	2.946***	3.703***	6.154
	Trade-MA	1.647***	2.946***	3.699***	6.154
	PC2-MA	1.588***	2.315***	3.329***	3.352
	Distance-MA	1.647***	2.951***	3.703***	6.162
OE-HNKPC	PC1-X*	1.320***	1.416**	1.263*	0.806*
	EW-X*	1.297***	1.404**	1.169	0.748**
	Trade-X*	1.386***	1.520**	1.336	0.819
	PC2-X*	1.310***	1.422**	1.198	0.773**
	Distance-X*	1.288***	1.587**	1.917	4.678**
OE-HNKPC	PC1-X*-MA	1.246***	1.552***	1.921***	3.551*
	EW-X*-MA	1.278***	1.702***	2.070***	3.017**
	Trade-X*-MA	1.263***	1.644***	1.936***	2.908**
	PC2-X*-MA	1.194***	1.277***	1.074***	0.719**
	Distance-X*-MA	1.206***	1.429***	1.588***	2.892***

(*) Orange-shaded cells=RMSFE ratio below unity. Green-shaded cells=best result for a given horizon. GW test results: (***) $p<1\%$, (**) $p<5\%$, (*) $p<10\%$.
Source: Author's calculations.

The results are presented in Table 8. They show that the use of Focus expectations systematically worsens baseline results, despite being of a negligible size for certain cases. Table 7 shows that the path of the best forecasting models by horizon is: AR (RMSFE ratio: 0.941), CE-HNKPC-UEq-MA (0.833), CE-HNKPC-VAR (0.798 and 0.685), with the last three being statistically significant. This same path when using Focus shifts from 6 months ahead (because at the nowcasting horizon the best model is still the AR) to 1.024, 0.916, and 0.815, and still the path of the best model only achieves 0.902, 0.916, 0.704 by horizon.

Also, despite that baseline results with the OE-HNKPC are not superior to the RW, the use of Focus expectations exacerbates the out-of-sample inaccuracy. In contrast, the results with OE-HNKPC-X* improves with all weighting schemes except Distance, shedding light on the detrimental effect of overfitting and the usefulness of the GVAR-version of the OE-HNKPC in a compacted version. In sum, next robustness results will favour comparisons using the OE-HNKPC-X*; not considering the MA version of the output gap.

4.5 Robustness results II: Unemployment gap

In this subsection, I analyse the out-of-sample results using the unemployment gap as the driving process for all economic models. Unlike previous analyses, this set of results does not make use of an MA version of the driving process because, as mentioned above, this variable already exhibits a smoothed dynamic.

The results are presented in Table 9 making use of both CF and Focus inflation expectations. The results of statistical models are shown for reference. For the nowcast horizon, there is no material predictive gain with any type of inflation expectation and, even more, the promising CE-HNKPC is spoiled out with respect to the case with output gap. Although at six months ahead both CE-HNKPC (UEq and VAR) using CF are more accurate than the RW, they have an RMSFE ratio still higher than the 0.833 exhibited by the CE-HNKPC-UEq-MA using the output gap (Table 7). At one year ahead, the first predictive gain using the unemployment gap and Focus expectations is noticed, obtained with the CE-HNKPC-UEq. However, it is overperformed by the two CE-HNKPC versions using CF expectations (a predictive gain of 5.7% with Focus compared to 7.5% and 14.6% with UEq and VAR using CF). Still, compared to the output gap case, all these RMSFE ratios are above the 0.798 obtained with the CE-HNKPC-VAR using the output gap (Table 7).

The best forecasting results using the unemployment gap are obtained at 24 months ahead. In particular, all four CE-HNKPC (UEq and VAR with CF and Focus) come out as competitive, but especially the VAR version displaying material predictive gains close to 25%. More remarkably, the OE-HNKPC-X* with Trade and PC2 weights comes out as the best forecasting models for this horizon, using CF as well as Focus expectations. Among them, the best result is obtained with the OE-HNKPC-PC2-X* displaying a statistically significant predictive gain of 37%.

In sum, the path of the best forecasting strategy across horizons is AR (0.941), CE-HNKPC-UEq-MA with CF (0.833), CE-HNKPC-VAR with CF (0.798), and OE-HNKPC-PC2-X* with unemployment gap and Focus (0.630)—the last three cases statistically significant at 5% level of significance.

4.6 Robustness results III: Single-country and pairwise trading-partners analysis

From the previous results, it is possible to draw some conclusions aiming to further analyse the predictive ability of the proposed OE-HNKPC. First, the OE-HNKPC works better at longer horizons and in a compacted version. Second, it is difficult to discriminate between the output or unemployment gap as the best predictive driving process. Third, a similar situation occurs between CF and Focus expectations and, thus, further comparisons are needed to disentangle which features deliver the most accurate forecasts.

To better discriminate between mentioned features, in this subsection I proceed by analysing the predictive ability of the OE-HNKPC in its baseline and compacted versions but going one step deeper in terms of parsimony. For this purpose, I perform a brute-force exercise considering Brazil as the base country and adding just one or a pair of countries in the GVAR estimation, considering all possible combinations (totalising 21 cases). When using a pair, each country is weighted by a half (fixed weights). All these estimations are made for the four horizons analysed in this article and taking CF and Focus inflation expectations as well as output and unemployment gap.

Table 9: Brazil: RMSFE ratio estimates using unemployment gap
and Focus plus CF expectations (*)

		h=1	h=6	h=12	h=24
Statistical	AR	0.941	1.089	1.043	0.803*
	ES	1.519***	1.089***	1.039**	0.983
			Consensus Forecast		
CE-HNKPC	UEq	1.044	0.912	0.925	0.990
	VAR	1.120	0.979	0.854	0.719**
OE-HNKPC	PC1	1.444***	2.123***	3.133*	3.017***
	EW	1.446***	2.126***	3.135*	3.004***
	Trade	1.451***	2.124***	3.136*	3.013***
	PC2	1.441***	2.123***	3.129*	3.016***
	Distance	1.448***	2.126***	3.125*	3.005***
OE-HNKPC	PC1-X*	1.160*	1.277**	1.442**	2.006***
	EW-X*	1.135	1.264*	1.398	1.890
	Trade-X*	1.105*	1.324**	1.135**	0.701***
	PC2-X*	1.105	1.326**	1.132	0.698***
	Distance-X*	1.070	1.182**	1.121	1.078***
			Focus		
CE-HNKPC	UEq	1.091	1.018	0.943	0.943
	VAR	1.176**	1.201*	1.081	0.756
OE-HNKPC	PC1	1.541***	2.295***	2.972**	3.236***
	EW	1.545***	2.297***	2.974**	3.224***
	Trade	1.542***	2.297***	2.969**	3.232***
	PC2	1.540***	2.292***	2.971**	3.233***
	Distance	1.545***	2.299***	2.971**	3.226***
OE-HNKPC	PC1-X*	1.182**	1.480***	1.848***	2.569***
	EW-X*	1.183**	1.489***	1.795**	2.304
	Trade-X*	1.157**	1.309***	1.010***	0.634**
	PC2-X*	1.160**	1.304***	1.005	0.630**
	Distance-X*	1.132**	1.346***	1.277	1.225***

(*) Orange-shaded cells=RMSFE ratio below unity. Green-shaded cells=best result for a given horizon. GW test results: (***) p<1%, (**) p<5%, (*) p<10%.
Source: Author's calculations.

The results using the output gap and CF are presented in Table 10. The first salient feature is that it confirms the damage due to overfitting of extended OE-HNKPC versions for forecast accuracy. The results also support the accuracy of the OE-HNKPC-X* at longer horizons. In terms of the RMSFE ratio, Argentina and the euro area are the single countries exhibiting the most remarkable predictive gains of 20.8% and 21.4%, respectively, and both statistically significant at h=24. By pairwise, Argentina and Chile, Chile and Japan, and the euro area and the United States show the best results with gains of 22%, 21.2%, and 21.7% at h=24, respectively, all figures statistically significant. For the remaining horizons, no statistically significant results are found. Thus, this setup opens a new, promising avenue by exploiting parsimonious versions of the OE-HNKPC-X*.

When analysing the results of Table 11 using Focus expectations, similar general qualitative conclusions are obtained. However, there are a few cases less with a pair of countries displaying unsatisfactory results while others improving with respect to the case of CF: all pairs including Argentina, Chile and the United States, and the euro area and the United States. In turn, the best result is obtained with Argentina and Chile showing a predictive gain of 27.3% at h=24 (statistically significant), which improves with respect to the previous case with CF (22%), but still below the OE-HNKPC-PC2-X* with unemployment gap and Focus (37%).

Table 10: Single-country and pairwise RMSFE ratio (output gap + CF) (*)

		OE-HNKPC	OE-HNKPC-X*			OE-HNKPC	OE-HNKPC-X*
h=1	*ARG*	1.444***	1.115**	h=1	*CHL+CHI*	1.443***	1.135**
h=6		2.099***	1.429***	h=6		2.099***	1.176
h=12		2.801**	1.267**	h=12		2.799**	1.279
h=24		2.464**	0.792***	h=24		2.462**	1.659
h=1	*CHL*	1.443***	1.139*	h=1	*CHL+EUR*	1.446***	1.187**
h=6		2.098***	0.989	h=6		2.096***	1.080
h=12		2.800**	0.973	h=12		2.800**	1.034
h=24		2.464**	0.814*	h=24		2.464**	0.883
h=1	*CHI*	1.445***	1.280***	h=1	*CHL+JPN*	1.444***	1.253**
h=6		2.098***	1.281*	h=6		2.098***	1.076
h=12		2.803**	1.264	h=12		2.799**	0.980
h=24		2.465**	1.254	h=24		2.463**	0.788*
h=1	*EUR*	1.442***	1.157**	h=1	*CHL+US*	1.440***	1.215**
h=6		2.095***	1.027	h=6		2.094***	1.152
h=12		2.798**	0.967	h=12		2.794**	1.090
h=24		2.462**	0.786*	h=24		2.466**	0.838
h=1	*JPN*	1.442***	1.106	h=1	*CHI+EUR*	1.446***	1.130*
h=6		2.098***	1.190	h=6		2.097***	1.199
h=12		2.799**	1.269*	h=12		2.801**	1.224
h=24		2.459**	1.417*	h=24		2.465**	1.221
h=1	*US*	1.436***	1.033	h=1	*CHI+JPN*	1.447***	1.229**
h=6		2.096***	0.994	h=6		2.097***	1.199
h=12		2.796**	1.052	h=12		2.797**	1.164
h=24		2.466**	0.912	h=24		2.461**	1.087
h=1	*ARG+CHL*	1.440***	1.175***	h=1	*CHI+US*	1.441***	1.141*
h=6		2.094***	1.478***	h=6		2.096***	1.100
h=12		2.796**	1.289**	h=12		2.798**	1.039
h=24		2.463**	0.760***	h=24		2.469**	0.922
h=1	*ARG+CHI*	1.442***	1.112**	h=1	*EUR+JPN*	1.447***	1.046
h=6		2.096***	1.347***	h=6		2.096***	1.005
h=12		2.796**	1.218*	h=12		2.797**	1.006
h=24		2.464**	0.780***	h=24		2.462**	1.021
h=1	*ARG+EUR*	1.439***	1.125**	h=1	*EUR+US*	1.441***	1.038
h=6		2.092***	1.493***	h=6		2.094***	0.992
h=12		2.798**	1.308**	h=12		2.797**	1.000
h=24		2.457**	0.788**	h=24		2.461**	0.783*
h=1	*ARG+JPN*	1.446***	1.175**	h=1	*JPN+US*	1.443***	1.073
h=6		2.101***	1.588***	h=6		2.097***	0.971
h=12		2.800**	1.429**	h=12		2.792**	0.989
h=24		2.464**	0.885	h=24		2.461**	0.932
h=1	*ARG+US*	1.443***	1.156**				
h=6		2.093***	1.511***				
h=12		2.796**	1.355**				
h=24		2.467**	0.841**				

(*) Orange-shaded cells=RMSFE ratio below unity. Green-shaded cells=best result for a given horizon. GW test results: (***) p<1%, (**) p<5%, (*) p<10%.
Source: Author's calculations.

Table 11: Single-country and pairwise RMSFE ratio (output gap + Focus plus CF) (*)

		OE-HNKPC	OE-HNKPC-X*			OE-HNKPC	OE-HNKPC-X*
h=1	*ARG*	1.584***	1.150**	h=1	*CHL+CHI*	1.585***	1.138*
h=6		2.316***	1.217**	h=6		2.312***	1.367*
h=12		3.322*	1.042	h=12		3.312*	1.683*
h=24		3.339***	0.761**	h=24		3.343***	2.550*
h=1	*CHL*	1.583***	1.198**	h=1	*CHL+EUR*	1.585***	1.191**
h=6		2.313***	1.134	h=6		2.314***	1.201
h=12		3.325*	1.081	h=12		3.330*	1.138
h=24		3.349***	0.858	h=24		3.348***	0.934
h=1	*CHI*	1.584***	1.258**	h=1	*CHL+JPN*	1.587***	1.157*
h=6		2.315***	1.437**	h=6		2.310***	1.129
h=12		3.322*	1.572**	h=12		3.327*	1.084
h=24		3.346***	1.473*	h=24		3.341***	0.901
h=1	*EUR*	1.586***	1.173**	h=1	*CHL+US*	1.579***	1.140*
h=6		2.314***	1.210*	h=6		2.312***	1.074
h=12		3.329*	1.120	h=12		3.329*	1.012
h=24		3.332***	0.823	h=24		3.335***	0.816*
h=1	*JPN*	1.582***	1.115	h=1	*CHI+EUR*	1.587***	1.185**
h=6		2.312***	1.354**	h=6		2.316***	1.384*
h=12		3.329*	1.556***	h=12		3.326*	1.521*
h=24		3.332***	1.674**	h=24		3.342***	1.466
h=1	*US*	1.582***	1.033	h=1	*CHI+JPN*	1.588***	1.188*
h=6		2.311***	0.944	h=6		2.313***	1.328**
h=12		3.323*	0.960	h=12		3.328*	1.397**
h=24		3.340***	0.793	h=24		3.341***	1.151
h=1	*ARG+CHL*	1.584***	1.165***	h=1	*CHI+US*	1.584***	1.129*
h=6		2.314***	1.257**	h=6		2.314***	1.221
h=12		3.325*	1.076	h=12		3.329*	1.260
h=24		3.336***	0.727**	h=24		3.348***	1.081
h=1	*ARG+CHI*	1.588***	1.138**	h=1	*EUR+JPN*	1.586***	1.107
h=6		2.316***	1.194**	h=6		2.312***	1.110
h=12		3.324*	1.024	h=12		3.324*	1.144
h=24		3.343***	0.747**	h=24		3.338***	1.199
h=1	*ARG+EUR*	1.579***	1.200***	h=1	*EUR+US*	1.581***	1.039
h=6		2.311***	1.269**	h=6		2.313***	0.942
h=12		3.327*	1.060	h=12		3.330*	0.924
h=24		3.341***	0.741**	h=24		3.337***	0.751*
h=1	*ARG+JPN*	1.586***	1.256**	h=1	*JPN+US*	1.590***	1.078
h=6		2.313***	1.312**	h=6		2.312***	1.091
h=12		3.332*	1.112	h=12		3.324*	1.150
h=24		3.342***	0.778**	h=24		3.343***	1.089
h=1	*ARG+US*	1.578***	1.189***				
h=6		2.311***	1.256**				
h=12		3.326*	1.082				
h=24		3.342***	0.752**				

(*) Orange-shaded cells=RMSFE ratio below unity. Green-shaded cells=best result for a given horizon. GW test results: (***) p<1%, (**) p<5%, (*) p<10%.
Source: Author's calculations.

Table 12 displays the results using the unemployment gap as the driving process and CF inflation expectations. Unlike previous cases, the bloc of the euro area and Japan display statistically significant predictive gains for intermediate horizons, h=6 and 12, of 15.5% and 20.6%, respectively. The latter comes out as the best result for h=12 of the whole exercise. Moreover, at h=24, the results with Argentina and Chile show a predictive gain of 31.3%, comparable to the 31.5% obtained with the CE-HNKPC-VAR using the output gap and CF (Table 7), and becoming the top three best specifications of the whole exercise. Overall, multiple enhancements are noticed when comparing these results with the previous ones (Table 8), reclaiming the use of the unemployment gap as the driving process for forecasting purposes.

Table 12: Single-country and pairwise RMSFE ratio (unemployment gap + CF) (*)

		OE-HNKPC	OE-HNKPC-X*			OE-HNKPC	OE-HNKPC-X*
h=1	*ARG*	1.447***	1.131*	h=1	*CHL+CHI*	1.442***	1.175**
h=6		2.125***	1.343***	h=6		2.123***	1.412**
h=12		3.128*	1.136	h=12		3.129*	1.717*
h=24		3.016***	0.708***	h=24		3.006***	2.631*
h=1	*CHL*	1.446***	1.080	h=1	*CHL+EUR*	1.444***	1.129*
h=6		2.127***	1.077	h=6		2.127***	1.172
h=12		3.128*	1.072	h=12		3.140*	1.145
h=24		3.011***	0.812	h=24		3.013***	0.853
h=1	*CHI*	1.441***	1.209**	h=1	*CHL+JPN*	1.446***	1.117
h=6		2.124***	1.439**	h=6		2.125***	0.997
h=12		3.137*	1.715**	h=12		3.129*	0.954
h=24		3.018***	2.343**	h=24		3.006***	0.919
h=1	*EUR*	1.442***	1.072	h=1	*CHL+US*	1.447***	1.111
h=6		2.125***	1.014	h=6		2.124***	1.116
h=12		3.132*	1.022	h=12		3.134*	1.251
h=24		3.018***	1.001	h=24		3.015***	1.267
h=1	*JPN*	1.451***	1.178*	h=1	*CHI+EUR*	1.445***	1.243**
h=6		2.123***	1.172	h=6		2.126***	1.402**
h=12		3.142*	1.120	h=12		3.136*	1.587**
h=24		3.014***	1.432	h=24		3.014***	2.097**
h=1	*US*	1.441***	1.199**	h=1	*CHI+JPN*	1.448***	1.182**
h=6		2.121***	1.233	h=6		2.124***	1.380**
h=12		3.133*	1.353	h=12		3.132*	1.624**
h=24		3.007***	1.362	h=24		3.009***	2.375**
h=1	*ARG+CHL*	1.450***	1.137*	h=1	*CHI+US*	1.449***	1.190**
h=6		2.121***	1.396**	h=6		2.124***	1.425**
h=12		3.125*	1.189	h=12		3.132*	1.641**
h=24		3.012***	0.687***	h=24		3.014***	2.233**
h=1	*ARG+CHI*	1.445***	1.052	h=1	*EUR+JPN*	1.442***	0.965
h=6		2.124***	1.023	h=6		2.122***	0.845*
h=12		3.129*	0.892	h=12		3.131*	0.794**
h=24		3.012***	0.988	h=24		3.012***	0.905
h=1	*ARG+EUR*	1.439***	1.100	h=1	*EUR+US*	1.455***	1.100
h=6		2.124***	1.331**	h=6		2.124***	1.075
h=12		3.136*	1.147	h=12		3.131*	1.051
h=24		3.016***	0.693***	h=24		3.022***	0.996
h=1	*ARG+JPN*	1.440***	1.127	h=1	*JPN+US*	1.448***	1.169*
h=6		2.122***	1.358***	h=6		2.121***	1.056
h=12		3.126*	1.154	h=12		3.141*	1.017
h=24		3.014***	0.705***	h=24		3.012***	1.231
h=1	*ARG+US*	1.443***	1.134*				
h=6		2.127***	1.372***				
h=12		3.127*	1.153				
h=24		3.011***	0.724***				

(*) Orange-shaded cells=RMSFE ratio below unity. Green-shaded cells=best result for a given horizon. GW test results: (***) p<1%, (**) p<5%, (*) p<10%.
Source: Author's calculations.

Table 13: Single-country and pairwise RMSFE ratio (unemployment gap + Focus plus CF) (*)

		OE-HNKPC	OE-HNKPC-X*			OE-HNKPC	OE-HNKPC-X*
h=1	*ARG*	1.546***	1.157**	h=1	*CHL+CHI*	1.539***	1.274***
h=6		2.291***	1.254**	h=6		2.298***	1.789**
h=12		2.967**	0.962	h=12		2.973**	2.487**
h=24		3.223***	0.638***	h=24		3.229***	5.217**
h=1	*CHL*	1.542***	1.216**	h=1	*CHL+EUR*	1.546***	1.255***
h=6		2.291***	1.317**	h=6		2.298***	1.393***
h=12		2.975**	1.293*	h=12		2.970**	1.352**
h=24		3.238***	0.991	h=24		3.234***	0.969
h=1	*CHI*	1.540***	1.284***	h=1	*CHL+JPN*	1.554***	1.251**
h=6		2.298***	1.736**	h=6		2.293***	1.214*
h=12		2.970**	2.187**	h=12		2.965**	1.188
h=24		3.233***	3.063**	h=24		3.232***	1.286
h=1	*EUR*	1.542***	1.246**	h=1	*CHL+US*	1.538***	1.259**
h=6		2.291***	1.411**	h=6		2.296***	1.371**
h=12		2.969**	1.422**	h=12		2.969**	1.445*
h=24		3.229***	1.334	h=24		3.234***	1.825**
h=1	*JPN*	1.548***	1.291**	h=1	*CHI+EUR*	1.545***	1.272**
h=6		2.295***	1.382**	h=6		2.295***	1.657***
h=12		2.966**	1.259*	h=12		2.966**	2.015**
h=24		3.229***	1.470*	h=24		3.230***	2.727**
h=1	*US*	1.546***	1.354***	h=1	*CHI+JPN*	1.547***	1.245***
h=6		2.295***	1.482*	h=6		2.298***	1.641***
h=12		2.971**	1.652*	h=12		2.974**	1.999**
h=24		3.230***	1.842*	h=24		3.237***	2.796***
h=1	*ARG+CHL*	1.547***	1.162**	h=1	*CHI+US*	1.542***	1.294***
h=6		2.297***	1.297**	h=6		2.293***	1.787***
h=12		2.975**	0.998	h=12		2.971**	2.240**
h=24		3.232***	0.621***	h=24		3.231***	3.351***
h=1	*ARG+CHI*	1.544***	1.092*	h=1	*EUR+JPN*	1.545***	1.145*
h=6		2.298***	1.142*	h=6		2.295***	1.052
h=12		2.969**	1.061	h=12		2.696**	0.921
h=24		3.232***	1.057	h=24		3.234***	1.032
h=1	*ARG+EUR*	1.542***	1.145**	h=1	*EUR+US*	1.549***	1.197**
h=6		2.293***	1.260**	h=6		2.293***	1.177
h=12		2.966**	0.979	h=12		2.972**	1.182
h=24		3.231***	0.621***	h=24		3.236***	1.505
h=1	*ARG+JPN*	1.541***	1.158**	h=1	*JPN+US*	1.540***	1.295***
h=6		2.298***	1.269**	h=6		2.292***	1.328*
h=12		2.972**	0.992	h=12		2.968**	1.299
h=24		3.231***	0.636***	h=24		3.237***	1.797
h=1	*ARG+US*	1.544***	1.156**				
h=6		2.295***	1.297***				
h=12		2.970**	1.074				
h=24		3.233***	0.806				

(*) Orange-shaded cells=RMSFE ratio below unity. Green-shaded cells=best result for a given horizon. GW test results: (***) p<1%, (**) p<5%, (*) p<10%.
Source: Author's calculations.

Finally, Table 13 shows the RMSFE ratio using the remaining combination between the driving process and inflation expectation—unemployment gap and Focus expectations. These results are easier to analyse because there are just three cases with statistically significant predictive gains: Argentina alone, and combined with Chile and the euro area; all at h=24. Notably, Argentina with Chile and with the euro area are in a tie delivering the same RMSFE ratio of 0.621 (predictive gain: 37.9%) and becoming the best results of the whole exercise for this horizon.

The second and third best results for this horizon also comes from the OE-HNKPC-X*: with Argentina and Chile (unemployment gap + CF; Table 12) and PC2 (unemployment gap + Focus; Table 9), making it clear that the GVAR-based specification of the OE-HNKPC works well as a long-term, compacted forecasting device. It also claims the use of information from trading partners of a small-scale bloc.[8]

After reviewing a number of specifications and results under different setups, it is difficult to arrive to a concise conclusion. Aiming to alleviate these difficulties, in Table 14 I provide a summary of the best forecasting results by model for a given horizon. The path of best results across horizons is composed by the AR, CE-HNKPC-UEq-MA (with the output gap and CF expectations), OE-HNKPC-X* combining the euro area and Japan (with the unemployment gap and CF expectations), and OE-HNKPC-X* in a tie combining Argentina with Chile and the euro area (with the unemployment gap and Focus expectations).

Some observations in the light of these results are: (i) there are important differences between the composition of the best forecasting setup in the short and the long run. In particular, at longer horizons it is preferable a parsimonious OE-HNKPC version with a pairwise bloc of countries, which is unfavourable for the adaptive weighting scheme and the use of a larger scale of trading partners (noticing that PC2 weights comes out as a competitive option at h=24), (ii) the trading partners that contribute most to forecast accuracy are the euro area and Japan, and Argentina, and Chile in a lesser extent, which is plainly against the use of weighting schemes based in distance such as that used in this article, (iii) Focus inflation expectations turns out as the best option with robustness specifications at longer horizons, working better in the OE-HNKPC rather than the CE-HNKPC specification, and (iv) the unemployment gap delivers the most accurate results at longer horizons, and outperforming virtually all mirror specifications using the output gap. In particular, the unemployment gap works better with the compacted version of the OE-HNKPC(-X*) and Focus inflation expectations.

Table 14: Summary of the best forecasting models by horizons (*)

Model	RMSFE ratio	Infl. exp.	Driving process	CE type	OE weights
			h=1		
AR	0.941	-	-	-	-
CE-HNKPC	0.984	CF	output gap	VAR	-
OE-HNKPC	1.436***	CF	output gap	-	US
OE-HNKPC-MA	1.153	CF	output gap	-	Distance
OE-HNKPC-X*	0.965	CF	unemp. gap	-	EUR+JPN
			h=6		
AR	1.089	-	-	-	-
CE-HNKPC	0.833**	CF	output gap MA	UEq	-
OE-HNKPC	2.092***	CF	output gap	-	ARG+EUR
OE-HNKPC-MA	1.277***	Focus	output gap	-	PC2
OE-HNKPC-X*	0.845*	CF	unemp. gap	-	EUR+JPN
			h=12		
AR	1.039***	-	-	-	-
CE-HNKPC	0.798**	CF	output gap	VAR	-
OE-HNKPC	2.696**	Focus	unemp. gap	-	EUR+JPN
OE-HNKPC-MA	1.074***	Focus	output gap	-	PC2
OE-HNKPC-X*	0.794**	CF	unemp. gap	-	EUR+JPN
			h=24		
AR	0.803*	-	-	-	-
CE-HNKPC	0.685**	CF	output gap	VAR	-
OE-HNKPC	2.463**	CF	output gap	-	EW
OE-HNKPC-MA	0.719**	Focus	output gap	-	PC2
OE-HNKPC-X*	0.621***	Focus	unemp. gap	-	a. ARG+CHL b. ARG+EUR

(*) Orange-shaded cells=best model for given horizon. GW test results: (***) p<1%, (**) p<5%, (*) p<10%. Source: Author's calculations.

[8] Similar conclusions are obtained in De Waal et al. (2015) for the case of South Africa.

5 Summary and concluding remarks

After the public-sector re-structuring *Plano Real* of 1994 Brazil concentrates the attention as one of the key emerging markets of the so-called BRIC economies. However, in the most recent period, Brazil has also been an important source of economic turmoil and market volatility. Consequently, it becomes more difficult to have a clear appraisal on key developments and prospects of the region, including the CPI inflation rate. This is not only relevant for both financial and real-economy decisions, but also it acts as a latent test on authorities' effectiveness on controlling inflationary pressures. As the Brazilian inflation is a likely source of spillovers and uncertainty across the region, a comprehensive and parsimonious modelling yields important advantages.

In this sense, the aim of this article is threefold. First, to document whether the Brazilian inflation follows the *Hybrid New Keynesian Phillips Curve* model, testing it by econometric means–in contrast to a calibrated setup–under different econometric setups. Second, to extend the scope of the HNKPC from a close- to open-economy version through a Global Vector Autoregression specification; always having in mind to estimate the influence of Brazil's major trading partners in forecast accuracy. Third, to estimate and compare the multi-horizon predictive ability of the HNKPC in such a way to identify the predictive gain (or loss) provided by trading partners' information, and the econometric setup behind the best results.

The results obtained with the CE-HNKPC do not reject the hypothesis proposed by the HNKPC for Brazil and its main trading partners for the 2000.1-2005.12 sample but weakens in the 2006.1-2018.6 sample. This result supports the documented instability of the HNKPC as well as the most recent findings of the literature regarding the "Phillips curve flattening" hypothesis. Still, it shows an appropriate in-sample goodness-of-fit and accurate forecasts.

The out-of-sample results show that the path of best results across horizons is composed by the AR, the CE-HNKPC-UEq-MA (using the output gap and CF expectations), the OE-HNKPC-X* combining the euro area and Japan (with the unemployment gap and CF expectations), and the OE-HNKPC-X* in a tie combining Argentina with Chile and the euro area (with the unemployment gap and Focus expectations). This means that, at the nowcasting horizon, the best HNKPC options are not superior to the traditional autoregression. However, beyond the nowcasting horizon, in turn, both CE- and OE-HNKPC come out as the best available alternatives. In particular, the simplest CE-HNKPC is slightly better than a two-country parsimonious OE-HNKPC in a horizon within a year, the latter using the unemployment gap as the driving process. Finally, for horizons beyond a year, the trading partners' contribution makes the OE-HNKPC-X* outperform the remaining options, making use of the unemployment gap, and extracting the information of just a pair of trading partners, *i.e.* the euro area and Japan.

Regarding the proposed OE-HNKPC model, some remarks are: (i) there are important differences between the composition of the best forecasting setup in the short and the long run: at longer horizons it is preferable a parsimonious OE-HNKPC version with just a pairwise bloc of countries, (ii) the trading partners that contribute most to forecast accuracy are the euro area and Japan, and Argentina, and Chile to a lesser extent, (iii) Focus inflation expectations turns out as the best option with the OE-HNKPC-X* specifications at longer horizons, and (iv) the unemployment gap delivers the most accurate results at longer horizons, and outperforming virtually all mirror specifications using the output gap. In particular, the unemployment gap works better with the compacted version of the OE-HNKPC(-X*) and Focus inflation expectations.

The relevance of these exercises is to understand Brazilian inflation dynamics and its predictability using a unified, theoretically based econometric framework, including the most valuable trading partners aiming to capture most of it foreign-based dynamics. The baseline results and robustness exercises, point out that the proposed OE-HNKPC could be considered as a superior forecasting device for the Brazilian inflation when including information from just a few but relevant trading partners, and particularly when forecasting at horizons greater than a year.

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A Statistical benchmarks

A.1 Univariate stationary autoregression

Alongside the RW, stationary AR models complements the most traditional benchmarks used for forecasting inflation as well as many other macroeconomic time-series (Ghysels, Osborn, and Rodrígues, 2006). The fitted models often include an MA component (following the Box and Jenkins, 1970, model selection view); and so, I refer to the ARIMA(p,1,0) particular case for simplicity. This also is due to the high persistence exhibited by inflation series, whose dynamics is well described by an AR(1) with a near-unity coefficient (see Pincheira and Medel, 2016, for details).

The strategy used in this article simply consists of the estimation of equation (A1) across the different p integers using the available sample. In this case, using $p^{\max}$=s=12 (s=annual frequency of the series):

$$\pi_t = \overline{\pi} + \sum_{i=1}^{p \in P} \phi_i \pi_{t-i} + \varepsilon_t, \tag{A1}$$

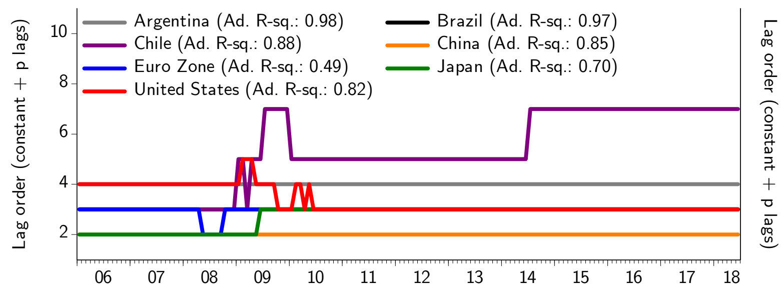
where $\{\overline{\pi},\{\phi_i\}_{i=1}^{p\in P},\sigma_{\varepsilon}^2\}$ are parameters to be estimated, $\varepsilon_t \sim iid\mathcal{N}(0,\sigma_{\varepsilon}^2)$, and P={1,...,12}. For each "p"-model, I compute the Bayesian Information Criterion (BIC) whereas the forecasting model is that with the lower BIC score (reflecting the better adjustment to the true model given the sample size). The BIC is defined as BIC=$-2\mathcal{L} + (1 + p)\log(T)$, where $\mathcal{L}$ is the log-likelihood function, T the sample size, and $(1 + p)$ is the number of coefficients of the model (accounting: one constant plus p AR coefficients).

Many articles analyse the appropriateness of information criteria for forecasting purposes. Among the most used are the BIC, Akaike Information Criterion (AIC), the Hannan-Quinn, and the Mallows Cp Criterion. However, at least these four are derived under the same Kullback and Leibler (1951) principle of *cross entropy*, delivering the same asymptotic results. The BIC produce more parsimonious (in-sample) results with intermediate sample size compared to the AIC. But, this is still not sufficient to ensure a higher out-of-sample accuracy. Hence, for the sake of parsimony, the AR with BIC is preferred. The ϕ_i-coefficient(s) is estimated with the *Ordinary Least Squares* method.

A.2 Diagnostics results of the stationary AR models

Figure A1 presents the chosen lag profile across the evaluation sample of the AR models (showing $p + 1$ lags). For Brazil, there are always two lags plus the constant term. In line with the rolling CE-HNKPC estimates, for the 2008-9 *Global Financial Crisis* more coefficients are required to capture the volatile behaviour of the series. During this period, Chile achieves a maximum of six lags, suggesting that the documented scarce goodness of fit of the CE- and OE-HNKPC may not be due to the HNKPC specification, but rather to a specific high-volatility episode hardly noticeable in the remaining part of the sample. Table A1 presents the first point estimation of all AR models with the *estimation sample* showing significant coefficients and well specified according to the Durbin-Watson statistic.

Figure A1: AR chosen lag length profile across the evaluation sample (*)
Evaluation sample: 2006.1-2018.6



(*) Source: Author's calculations.

Table A1: AR model estimates and diagnostics (*)

	ARG	*BRA*	*CHL*	*CHI*	*EUR*	*JPN*	*US*
	Dependent variable: π_t						
	Estimation sample: 2000.1-2005.12						
ϕ_1	1.505	1.684	1.235	0.922	0.858	0.841	1.188
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
ϕ_2	-0.263	-0.737	-0.321	-	-0.286	-	-0.664
	[0.178]	[0.000]	[0.003]	-	[0.034]	-	[0.000]
ϕ_3	-0.293	-	-	-	-	-	0.401
	[0.021]	-	-	-	-	-	[0.000]
$\overline{\pi}$	9.724	8.399	2.832	1.473	2.192	-0.456	2.717
	[0.017]	[0.000]	[0.000]	[0.170]	[0.000]	[0.006]	[0.000]
$\overline{R}^2$	0.983	0.971	0.881	0.855	0.477	0.704	0.827
S.E. Reg.	1.625	0.548	0.408	0.619	0.199	0.237	0.341
DW Stat.	1.987	1.849	1.885	1.864	2.041	1.843	1.817

(*) Equation: $\pi_t = \overline{\pi} + \phi_1\pi_{t-1} + \phi_2\pi_{t-2} + \phi_3\pi_{t-3} + \varepsilon_t$, with $\varepsilon_t \sim iid\mathcal{N}(0, \sigma_\varepsilon^2)$. Coefficient p-value in $[\cdot]$. "DW Stat." stand for the Durbin-Watson statistic. Source: Author's calculations.

A.3 The exponential smoothing forecast

The ES corresponds *per se* to a forecasting model. The version used in this article corresponds to the *single* ES, but there are available more specifications such as the *double* ES and the Holt-Winters model (see Hyndman *et al.*, 2008). The prediction for h-steps ahead is the same independently of the horizon:

$$\pi_{t+h|t} = \alpha\pi_{t-1} + (1-\alpha)\pi_{t-1+h|t-1}, \tag{A2}$$

with $0 < \alpha \leq 1$. Note that if α=1, the ES coincide with the RW model. The model has also been used for forecasting purposes in Corberán-Vallet, Bermúdez, and Vercher (2011), Kolassa (2011), He, Shen, Tong (2012), and Pincheira and Medel (2015) with relative success for the same reasons of the RW.

A.4 The random walk model

The RW consists of the special AR(1) case where ϕ_1 in equation (A1) is not estimated and it is restricted to ϕ_1=1 instead (with ϕ_j=0 for j>1). This restriction, although simple, entails several methodological as well as economic consequences. The most significant impact is that it turns inflation into a non-stationary variable, theoretically without available statistical inference and divergent predictions in the forecasting horizons. Due to this non-stationarity, it sounds unlikely–at least theoretically–to have room for stabilisation policymaking, since past unpredictable shocks do not vanish in time. Note that this argument is raised assuming that inflation exhibits a unit root; hence, CPI$\sim$I(2). For forecasting purposes, however, over-differentiate series to reach stationarity does not comprise a major setback because it does not necessarily jeopardise forecast accuracy (Dickey and Pantula, 1987).

The empirical evidence has been overwhelmingly in favour of the RW. This is due to the benefit of misspecification that more than offsets the parameter uncertainty arisen from finite sample estimation of a close-to-unity parameter. In this article, I use a *driftless* RW forecast, following the argument given in Medel and Pincheira (2015) and Pincheira and Medel (2016) that driftless RW-based forecasts are unbiased. Iterating forward the AR(1) model, I have:

$$\pi_{t+h} = \overline{\pi}\left[\frac{1-\phi^h}{1-\phi}\right] + \phi^h \pi_t + \sum_{i=0}^{h-1} \phi^i \varepsilon_{t+h-1}. \tag{A3}$$

If π_t is modelled with a driftless RW, *i.e.* ϕ=1 and $\overline{\pi}$=0, the optimal forecast becomes π_{t+h}=π_t at any horizon. Hence, the h-step-ahead forecast error $\mathbb{E}\left[\varepsilon^{RW}_{t+h|t}\right]$=$\mathbb{E}\left[\pi_{t+h} - \pi^{RW}_{t+h|t}\right]$ satisfies:

$$\begin{aligned} \mathsf{Bias}_h &\equiv \mathbb{E}\left[\varepsilon^{RW}_{t+h|t}\right], \\ &= \mathbb{E}\left[\overline{\pi}\left[\frac{1-\phi^h}{1-\phi}\right] - (1-\phi^h)\pi_t + \sum_{i=0}^{h-1}\phi^i \varepsilon_{t+h-i}\right], \\ &= \overline{\pi}\left[\frac{1-\phi^h}{1-\phi}\right] - (1-\phi^h)\mathbb{E}\left[\pi_t\right], \\ &= 0, \end{aligned} \tag{A4}$$

as $\mathbb{E}\left[\pi_t\right]$=$\overline{\pi}/(1-\phi)$. More details can be found in Medel and Pincheira (2015).

B Economic slack measures

This annex follows the output gap construction used in Medel (2015, 2017, 2018), but also including in this article the unemployment gap built with the same methodology. One of the biggest drawbacks when estimating the NKPC is the impossibility to accurately measure the excess of demand. As both HNKPC make use of this measure, it is desirable to have a stable series as new observations are added. The typical alternatives to the marginal cost variable are the output gap ($\widetilde{y}_t$), defined as the difference between current and potential output, and the unemployment gap ($\widetilde{u}_t$) defined as the difference between current unemployment and a *non-accelerating inflation rate of unemployment* (NAIRU) measure. As the estimations are made with monthly data, I use the Industrial Production (IP) index as a proxy of the quarterly GDP for the output gap, whereas the unemployment rate is originally published in monthly frequency. Table B1 presents the descriptive statistics of these series for all countries, using the annual percentage change (Δ^{12}) of the IP level series. Note that the transformation achieves stationarity according to the Augmented Dickey-Fuller (ADF) test and Zivot-Andrews (ZA) test.

Instability in the potential output and the NAIRU arises with the "end-of-sample" problem of filtering, especially when the Hodrick-Prescott (HP) procedure is used; generating an unobservable component.[9] To alleviate this setback, I follow the proposal of Mise, Kim, and Newbold (2005), consisting in adding up to 60 forecast observations to level series prior to performing any filtering procedure. Note that the seasonal adjustment is made with X13-ARIMA-SEATS in its default mode, and the filtering method is HP (λ=129,600).

[9]See Orphanides (2001), Orphanides and van Norden (2002, 2005) and Garratt *et al*. (2008) for a discussion on this matter.

Table B1: Descriptive statistics of industrial production (IP) and unemployment (U) time series (*)

	ARG	*BRA*	*CHL*	*CHI*	*EUR*	*JPN*	*US*	*BRA*	*CHL*	*EUR*	*JPN*	*US*
	Industrial production (Δy_t)							Unemployment (u_t)				
	Estimation sample: 2000.1-2005.12											
Mean	2.74	3.78	4.54	13.79	1.48	1.39	1.39	11.58	11.22	8.87	4.92	5.18
Median	2.81	4.12	4.62	14.45	1.24	2.74	2.12	11.59	11.26	8.80	5.00	5.40
Max.	34.93	11.28	14.60	23.20	7.30	7.79	5.30	14.58	13.11	9.70	5.80	6.30
Min.	-21.83	-6.48	-4.37	2.30	-3.83	-12.76	-5.27	8.41	9.17	8.00	4.00	3.80
Std. dev.	9.07	3.85	3.98	3.96	2.45	5.27	2.78	1.33	0.97	0.46	0.42	0.73
JB-Stat.	6.67	0.77	0.01	3.38	0.65	20.37	10.03	0.30	3.05	2.28	0.84	6.90
p-value	0.04	0.68	0.99	0.18	0.72	0.00	0.01	0.86	0.22	0.32	0.66	0.03
ADF-Stat.	-4.04	-3.36	-5.16	-4.01	-4.54	-3.59	-3.86	-4.01	-5.50	-3.79	-2.51	-3.29
p-value	0.00	0.06	0.00	0.00	0.00	0.03	0.02	0.00	0.00	0.02	0.02	0.00
	Evaluation sample: 2006.1-2018.6											
Mean	2.18	0.17	1.12	11.08	0.70	0.20	0.55	7.70	7.53	9.83	3.93	6.48
Median	1.89	1.21	1.99	9.95	2.05	1.52	2.00	7.56	6.96	9.90	4.00	5.80
Max.	13.25	19.69	14.02	20.10	9.30	27.32	8.55	11.46	11.63	12.70	5.50	10.00
Min.	-16.44	-16.45	-11.46	5.40	-21.62	-33.33	-15.43	4.33	5.70	7.20	2.30	3.80
Std. dev.	5.76	6.86	4.31	4.25	5.80	9.25	4.61	2.03	1.51	1.50	0.76	1.96
JB-Stat.	2.54	0.63	11.47	11.43	186.7	93.05	170.7	10.92	23.61	6.48	2.85	15.56
p-value	0.28	0.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.24	0.00

(*) "JB-Stat." stands for Jarque-Bera test statistic (NH: Data are random). "ADF-Stat." stands for Augmented Dickey-Fuller test statistic (NH: Series has a unit root). ADF equation for *IP* includes a constant plus a trend, and 1 lag for all cases, except ARG (only a constant), EUR (4 lags), and US (3 lags) using the full sample. ADF equation for *U* includes a constant plus a trend and 1 lag for CHL and EUR, whereas for BRA, JPN, and US the Zivot-Andrews test is used due to breakpoints at 2013.12, 2011.4, and 2011.5, respectively.
Source: Author's calculations.

The ARMA forecasting model for IP corresponds to $\Delta^{12}y_t = \overline{y} + \phi\Delta^{12}y_{t-p} + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_{12}\varepsilon_{t-12} + \theta_1\theta_{12}\varepsilon_{t-13}$, with $\varepsilon_t \sim iid\mathcal{N}(0, \sigma_\varepsilon^2)$ and $\{\overline{y}, \phi, \theta_i, \sigma_\varepsilon^2\}$ are parameters to be estimated, whereas for the unemployment it is $u_t = \overline{u} + \phi u_{t-p} + \nu_t + \theta_1\nu_{t-1} + \theta_{12}\nu_{t-12}$, with $\nu_t \sim iid\mathcal{N}(0, \sigma_\nu^2)$ and $\{\overline{u}, \phi, \theta_i, \sigma_\nu^2\}$ are parameters to be estimated. This is a version of the so-called *airline model* (Box and Jenkins, 1970) which has proved to be a model that fits macroeconomic data with substantial success (Ghysels, Osborn, and Rodrígues, 2006). The in-sample estimations are presented in Table B2 (*IP*) and Table B3 (*U*), which also reveal robust results across countries, and a correct specification according to the Durbin-Watson statistic.

Stock and Watson (1999) suggest that especially when the aim is to forecast, the output gap measure provides a convenient alternative since it relies basically on a univariate ensemble, a characteristic shared with the unemployment gap. Also, some of the greater problems associated with the output gap–instead of using the marginal cost–are rather an empirical issue. The forecasts provided by the models of Tables B2 and B3 tackle part of the "end-of-sample" problem.

Table B2: In-sample diagnostics of IP forecasting models (*)

	ARG	*BRA*	*CHL*	*CHI*	*EUR*	*JPN*	*US*
	Dependent variable: $\Delta^{12}y_t$						
	Estimation sample: 2000.1-2005.12						
ϕ	-0.193	-0.276	-0.790	-0.671	-0.336	-0.381	-0.445
	[0.096]	[0.031]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
θ_1	-	-0.896	-	-	-0.898	-0.900	-
	-	[0.000]	-	-	[0.000]	[0.000]	-
θ_2	-0.219	-	-0.752	-0.633	-0.898	-0.900	-
	[0.039]	-	[0.000]	[0.000]	[0.000]	[0.000]	-
θ_3	-	-	-	-	-0.898	-0.900	0.221
	-	-	-	-	[0.000]	[0.000]	[0.007]
θ_{12}	0.314	0.527	0.882	0.833	0.619	0.455	0.889
	[0.065]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
$\overline{y}$	0.144	0.204	0.301	0.400	0.137	0.101	0.149
	[0.675]	[0.001]	[0.000]	[0.000]	[0.000]	[0.066]	[0.490]
$\overline{R}^2$	0.101	0.229	0.778	0.759	0.216	0.286	0.701
S.E. Reg.	3.497	0.962	2.711	1.721	0.759	1.075	1.179
DW Stat.	1.899	2.012	2.098	2.181	2.208	1.731	1.994

(*) Equation: $\Delta^{12}y_t = \overline{y} + \phi\Delta^{12}y_{t-p} + \varepsilon_t + \theta_p\varepsilon_{t-p} + \theta_{12}\varepsilon_{t-12}$ $+\theta_p\theta_{12}\varepsilon_{t-12-p}$ with $\varepsilon_t \sim iid(0, \sigma^2_\varepsilon)$. Coefficient p-value in $[\cdot]$. "DW Stat." stands for the Durbin-Watson statistic.
Source: Author's calculations.

Table B3: In-sample diagnostics of U forecasting models (*)

	BRA	*CHL*	*EUR*	*JPN*	*US*
	Dependent variable: u_t				
	Estimation sample: 2000.1-2005.12				
ϕ	0.871	0.931	0.910	0.852	0.949
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
θ_{12}	0.912	0.910	0.869	0.524	-0.896
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
$\overline{u}$	10.872	10.412	8.731	4.835	5.650
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
$\overline{R}^2$	0.829	0.891	0.951	0.811	0.979
S.E. Reg.	0.589	0.361	0.120	0.175	0.109
DW Stat.	1.652	1.028	1.432	1.936	1.948

(*) Equation: $u_t = \overline{u} + \phi u_{t-p} + \nu_t + \theta_{12}\nu_{t-12}$ with $\nu_t \sim iid(0, \sigma^2_\nu)$. Coefficient p-value in $[\cdot]$. "DW Stat." stands for the Durbin-Watson statistic.
Source: Author's calculations.

C Data

This annex statistically describes the dataset used in this article. There are two kinds of data: inflation time series (actual and expected) and the economic slack measures constructed with the industrial production index and the unemployment rate (see Annex B). Table C1 shows the source and descriptions of all the series. Actual headline inflation is obtained from the OECD Database, except Argentina which is obtained from Cavallo and Bertolotto (2016). The Brazilian inflation rate corresponds to the annual percentage change of the "Índice Nacional de Preços ao Consumidor Amplo, IPCA" (Broad National Consumer Price Index). Inflation expectations are obtained from the *Consensus Forecasts* (CF) monthly report prepared by *Consensus Economics*, whereas the alternative measure for the Brazilian case only (Focus) is obtained from *Banco Central do Brasil*, being the well-known, widely used "Focus" inflation expectations series. The industrial production indexes are obtained from the *OECD Database*, except for Argentina which comes from the *IMF's International Financial Statistics*. Finally, the unemployment rate is obtained from the *FRED Database*, except Chile which corresponds to a backward-based linked-chain series coming from two vintages generated by the *National Statistics Institute*.

Table C1: Variable description (*)

Variable	Country	Unity	Scale	Descriptor	Source
Consumer Price Index (transformed to inflation series)	ARG	Index	Oct-07=100	Consumer Price Index	Cavallo and Bertolotto (2016)
	BRA	Index	2010=100	Consumer Prices - All Items	OECD Database
	CHL	Index	2010=100	Consumer Prices - All Items	OECD Database
	CHI	Index	2010=100	Consumer Prices - All Items	OECD Database
	EUR	Index	2010=100	Harmonised CP (19 countries)	OECD Database
	JPN	Index	2010=100	Consumer Prices - All Items	OECD Database
	US	Index	2010=100	Consumer Prices - All Items	OECD Database
Inflation Expectations	ARG	Basis points	None	Ave. % chg. on prev. yr	Consensus Economics
	BRA: CF	Basis points	None	Ave. % chg. on prev. yr	Consensus Economics
	BRA: Focus	Basis points	None	Year-on-year change	Banco Central do Brasil
	CHL	Basis points	None	Ave. % chg. on prev. yr	Consensus Economics
	CHI	Basis points	None	Ave. % chg. on prev. yr	Consensus Economics
	EUR	Basis points	None	Ave. % chg. on prev. yr	Consensus Economics
	JPN	Basis points	None	Ave. % chg. on prev. yr	Consensus Economics
	US	Basis points	None	Ave. % chg. on prev. yr	Consensus Economics
Industrial Production (used for the output gap variable)	ARG	Index	2010=100	Industrial Production	IMF IFS
	BRA	Index	2010=100	Production of total industry sa	OECD Database
	CHL	Index	2010=100	Production of total industry sa	OECD Database
	CHI	Index	2010=100	Production of total industry sa	OECD Database
	EUR	Index	2010=100	Production of total industry sa	OECD Database
	JPN	Index	2010=100	Total retail trade (volume)	OECD Database
	US	Index	2010=100	Production of total industry sa	OECD Database
Unemployment rate (used for the unemployment gap variable)	BRA	Rate	None	Aged 15 and Over	FRED Database
	CHL	Rate	None	National coverage	National Statistics Institute
	EUR	Rate	None	All Persons for the Euro Area	FRED Database
	JPN	Rate	None	All Persons for Japan	FRED Database
	US	Rate	None	All Persons for the US	FRED Database

(*) "sa" stands for seasonally adjusted. Source: Author's calculations.

C.1 Actual inflation data

The descriptive statistics of the inflation series considering the seven countries are presented in Table C2 for three samples. Actual inflation is transformed using the annual percentage change of the CPI. This is made to fit the specification used by the expectation series and to avoid treating additive seasonality if present. CF survey is entirely reported for the same transformation even if CPI-basket re-definitions will be undertaken. The same occurs with the Focus series. The CF expectations series are also the limiting variable for the sample span, starting in 2000. Inflation data are available in a useful quality since the 1960s for the majority of the countries (assuming a backward reconstruction for the Euro Area). Notice that for the full sample, the ADF test for stationarity is presented. According to the ADF test, the inflation series are stationary at 5% of confidence, except Japan CF which is stationary at 10% of confidence.

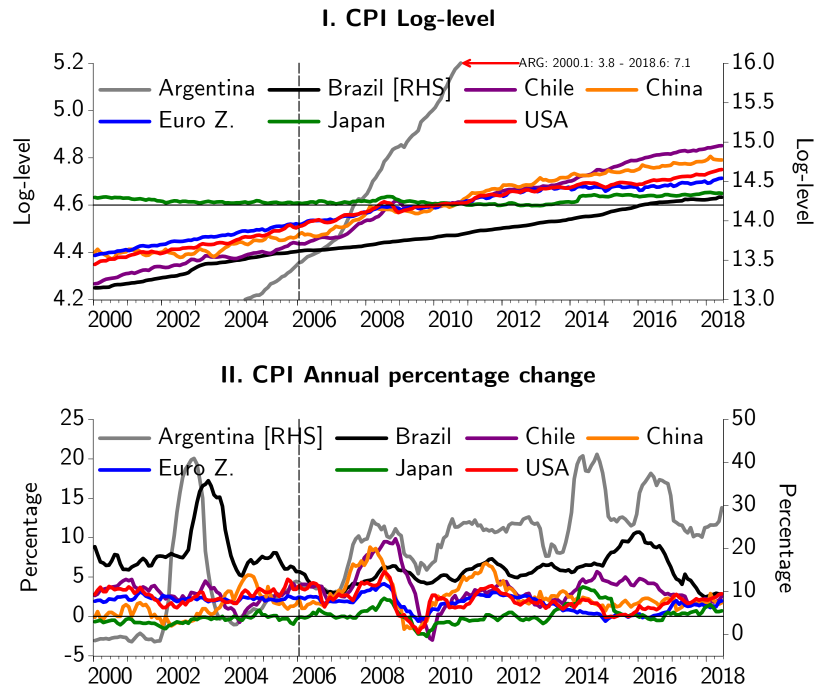
Table C2: Descriptive statistics of actual inflation series (*)

	ARG	*BRA*	*CHL*	*CHI*	*EUR*	*JPN*	*US*
	Actual Inflation (π_t)						
Full sample: 2000.1-2018.6 (232 observations)							
Mean	19.19	6.59	3.25	2.20	1.76	0.06	2.19
Median	22.67	6.28	3.03	1.80	1.98	-0.10	2.12
Max.	41.87	17.23	9.85	8.80	4.13	3.74	5.65
Min.	-1.74	2.46	-3.01	-1.84	-0.62	-2.52	-2.08
Std. dev.	11.86	2.69	1.97	2.07	0.97	1.04	1.29
JB-Stat.	7.72	272.4	79.31	30.28	8.20	93.94	7.42
p-value	0.02	0.00	0.00	0.00	0.02	0.00	0.02
ADF-Stat.	-3.93	-3.01	-3.07	-3.42	-2.73	-2.92	-4.27
p-value	0.01	0.04	0.03	0.01	0.07	0.05	0.00
Estimation sample: 2000.1-2005.12 (72 observations)							
Mean	8.80	8.43	2.79	1.18	2.23	-0.48	2.70
Median	4.90	7.40	2.95	0.98	2.18	-0.44	2.84
Max.	40.95	17.23	4.69	5.25	3.13	0.80	4.71
Min.	-1.74	5.16	-0.75	-1.22	1.68	-1.57	1.10
Std. dev.	12.48	3.20	1.17	1.63	0.27	0.44	0.83
JB-Stat.	29.82	39.88	8.91	8.72	8.81	0.32	1.52
p-value	0.00	0.00	0.01	0.00	0.01	0.85	0.47
Evaluation sample: 2006.1-2018.6 (160 observations)							
Mean	24.18	5.70	3.47	2.68	1.54	0.32	1.95
Median	24.80	5.60	3.19	2.18	1.61	0.20	1.95
Max.	41.87	10.71	9.85	8.80	4.13	3.74	5.65
Min.	9.66	2.46	-3.01	-1.84	-0.62	-2.52	-2.08
Std. dev.	7.56	1.85	2.23	2.09	1.10	1.15	1.40
JB-Stat.	0.66	10.46	23.88	18.08	3.59	22.62	0.72
p-value	0.72	0.01	0.00	0.00	0.17	0.00	0.70

(*) "JB-Stat." stands for Jarque-Bera test statistic (NH: Data are random). "ADF-Stat." stands for Augmented Dickey-Fuller test statistic (NH: Series has a unit root). ADF equations includes a constant with 1 lags (BRA, CHL, EUR, JPN, US), or 4 lags (CHI), or a trend with 1 lag (ARG). Source: Author's calculations.

Figure C1 plots both the log-level of the CPI and its annual percentage change. The most noticeable feature is the high volatility exhibited by the Argentinian inflation through the whole available sample span. This fact, despite being an undesirable, noisy situation for the Argentinian economic authorities, provides a rich source of information–*i.e.* a very different volatility compared to the other countries–in terms of the econometric estimates, and certainly avoiding the collinearity with the Brazilian inflation when considered in the estimation. The predictive results obtained with the robustness OE-HNKPC in a single-country and pairwise country versions support this view of Argentina as a valuable Brazilian inflation covariate.

Figure C1: Consumer price indexes. Log-level and annual percentage change (*)
Full sample: 2000.1-2018.6



(*) Vertical line=start of evaluation sample (2006.1). Source: Author's calculations based on *OECD Database* and Cavallo and Bertolotto (2016).

C.2 Inflation expectation data

The CF expectations are reported monthly, providing the point forecast of 15 to 20 agencies and private consultants for several variables at two fixed horizons: December of the current and the next year. The names of the respondents are explicitly revealed along with their forecasts, making possible a one-by-one accuracy and forecast revision analysis. Given this specific richness of the survey, several articles make use of CF for testing economic/statistic hypotheses. What is important for this article is that these series are produced under the same controlled standards imposed to their respondents. A minor twist consists in the use of the Focus expectations which, as shown, are virtually equal to CF statistically speaking, but built in a framework different to that of CF.

Notice that, as the estimation is made with a constant frequency using recursive estimation, it is necessary to adjust the CF series to have a unique rolling-event forecast. The BRA: Focus series does not require any treatment as it originally targets one horizon only ("IPCA-inflation accumulated over the next 12 months"). The approach used in this article for CF series–similar to that used in Medel (2015, 2017, 2018)–is to create one series with a weighting scheme of the two forecasts in order to better accommodate the information to the targeted rolling-horizon. Hence, the CF forecast series for each month are weighted according to:

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Dec this year	92%	83%	75%	67%	58%	50%	42%	33%	25%	17%	8%	0%
Dec next year	8%	17%	25%	33%	42%	50%	58%	67%	75%	83%	92%	100%

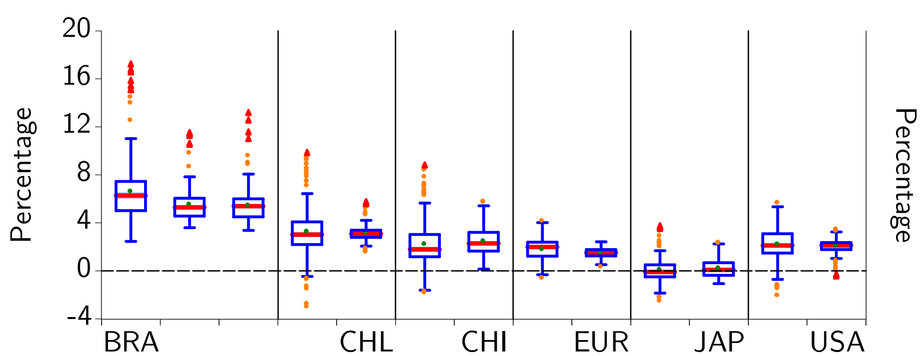
Table C3 shows the descriptive statistics of the weighted CF series plus Focus–with the latter displaying very similar characteristics to the former. Judging by point estimations (mean and median), the accuracy is notably improved across the sample. Figure C2 displays five pairs of boxplots for inflation series, starting with a triplet for Brazil (actual, CF, Focus), and excluding Argentina due to a large number of outliers for a higher level of inflation. A salient feature is the reduced number of outliers and a lower level of inflation of the remaining Brazilian trading partners, challenging the idea of being robust predictors of a more volatile series. Thus, as above mentioned, a key role is left to the Argentinian inflation.

Table C3: Descriptive statistics of expected inflation series (*)

	ARG	*BRA*	*BRA*	*CHL*	*CHI*	*EUR*	*JPN*	*US*
Type:	CF	CF	Focus	CF	CF	CF	CF	CF
Full sample: 2000.1-2018.6 (232 observations)								
Mean	14.48	5.51	5.42	3.15	2.44	1.51	0.20	2.04
Median	10.79	5.30	5.40	3.08	2.29	1.52	0.08	2.12
Max.	63.71	11.50	13.18	5.73	5.75	2.42	2.33	3.43
Min.	-0.30	3.60	3.37	1.55	0.15	0.31	-1.07	-0.45
Std. dev.	11.54	1.42	1.40	0.60	1.15	0.41	0.78	0.61
JB-Stat.	285.9	498.5	944.4	197.4	3.44	2.55	-24.75	142.2
p-value	0.00	0.00	0.00	0.00	0.18	0.28	0.00	0.00
ADF-Stat.	-4.28	-3.62	-3.58	-3.86	-3.19	-3.63	-3.54	-5.35
p-value	0.00	0.00	0.00	0.00	0.02	0.00	0.04	0.00
Estimation sample: 2000.1-2005.12 (72 observations)								
Mean	13.22	6.34	6.35	3.08	1.76	1.40	-0.35	2.21
Median	7.57	5.89	5.80	3.02	1.51	1.45	-0.29	2.19
Max.	63.71	11.50	13.18	4.05	3.75	1.82	0.23	3.06
Min.	-0.30	4.22	4.41	2.08	0.15	0.31	-1.07	1.57
Std. dev.	17.46	1.86	2.04	0.48	1.03	0.24	0.32	0.32
JB-Stat.	41.54	46.95	54.12	1.54	4.60	87.78	3.49	3.00
p-value	0.00	0.00	0.00	0.46	0.10	0.00	0.17	0.22
Evaluation sample: 2006.1-2018.6 (160 observations)								
Mean	15.09	5.11	5.11	3.19	2.76	1.56	0.46	1.96
Median	11.33	5.18	5.16	3.10	2.70	1.68	0.41	2.00
Max.	32.15	7.13	7.29	5.73	5.75	2.42	2.33	3.43
Min.	4.80	3.60	3.37	1.55	0.47	0.52	-1.02	-0.45
Std. dev.	7.16	0.92	0.94	0.65	1.05	0.46	0.80	0.69
JB-Stat.	22.91	7.03	5.39	138.4	5.91	6.74	3.65	41.09
p-value	0.00	0.03	0.07	0.00	0.05	0.03	0.16	0.00

(*) "JB-Stat." stands for Jarque-Bera test statistic (NH: Data are random). "ADF-Stat." stands for Augmented Dickey-Fuller test statistic (NH: Series has a unit root). ADF equations includes a constant with 4 lags (ARG, BRA: CF, BRA: Focus, CHI), or 1 lag (CHL, EUR, US), or a trend and 9 lags (JPN). Source: Author's calculations.

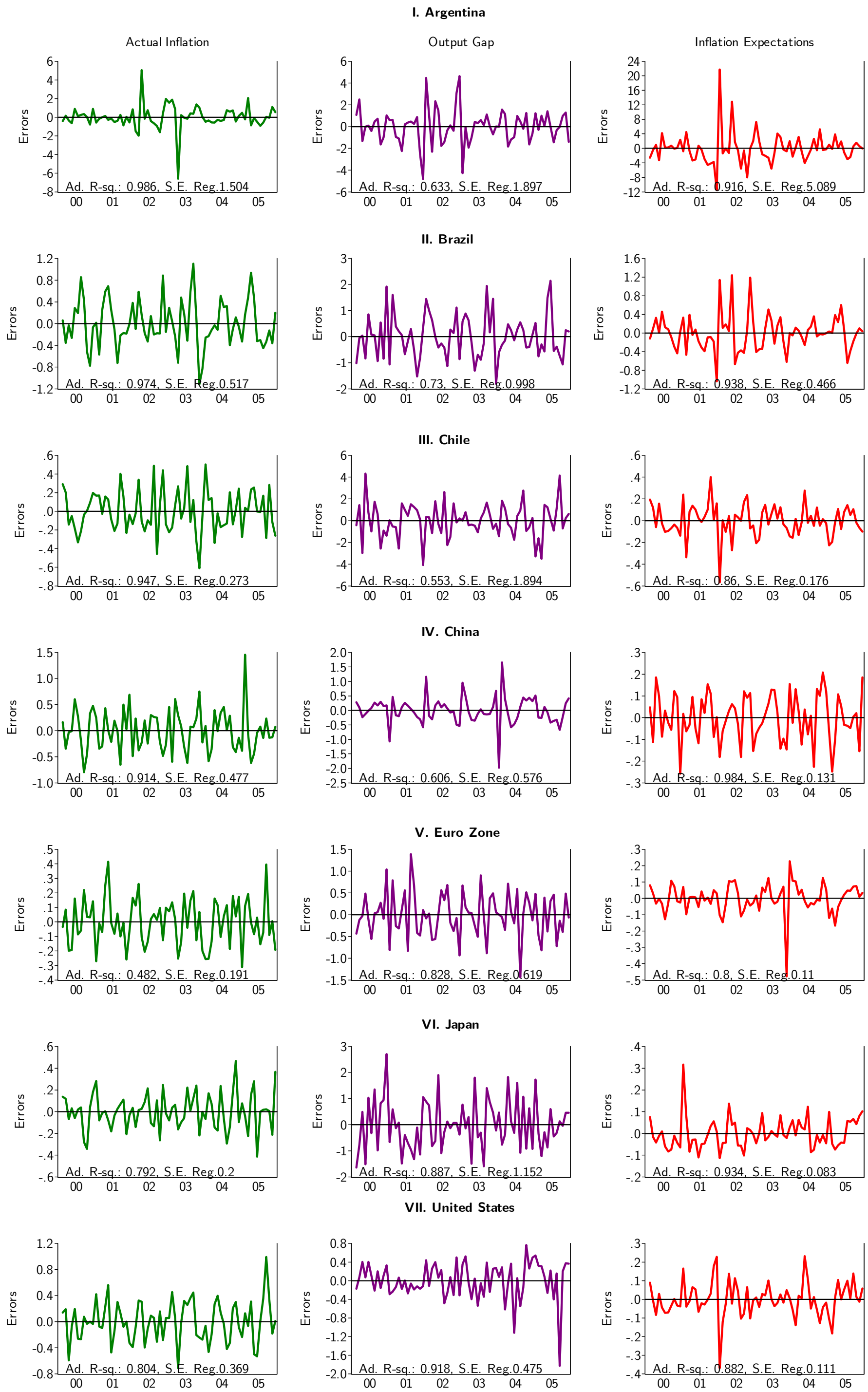
Figure C2: Boxplot of actual and expected inflation series (*)
Evaluation sample: 2006.1-2018.6



(*) BRA: [actual, CF, Focus]. Trading partners: [actual, CF]. Source: Author's calculations.

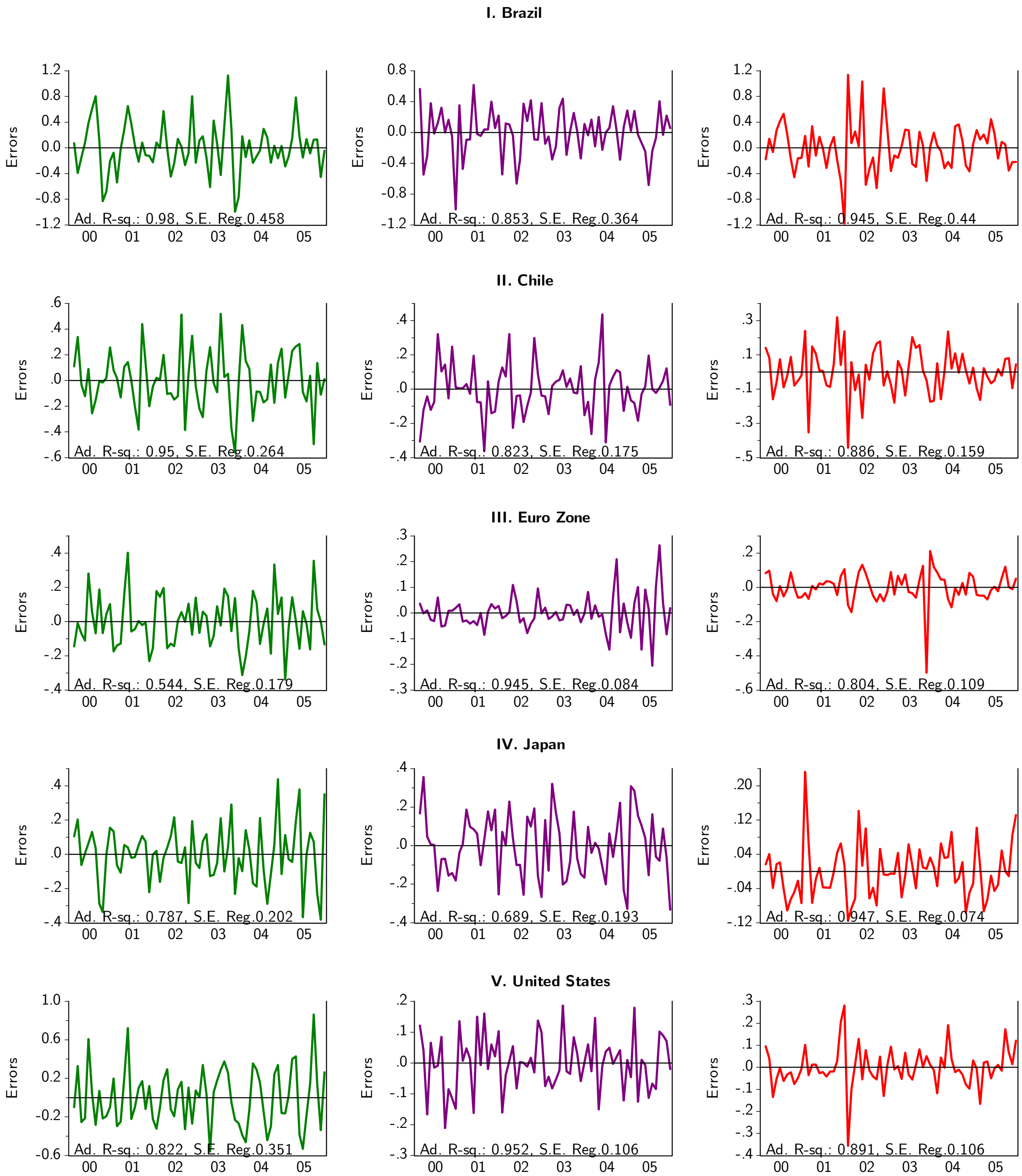
D OE-HNKPC (output gap and trade-based weights) diagnostics: residuals

Figure D1: OE-HNKPC (output gap and trade-based weights) residuals time series and goodness of fit coefficients (*)
Estimation sample: 2000.1-2005.12



(*) Source: Author's calculations.

Figure D2: OE-HNKPC (unemployment gap and trade-based weights) residuals time series and goodness of fit coefficients (*)
Estimation sample: 2000.1-2005.12



(*) Source: Author's calculations.

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