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Measuring Systemic Risk: A Quantile Factor Analysis*

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Abstract

This paper proposes a novel measure to quantify systemic risk from the information contained in asset returns. In the context of the external habits formation model of Campbell and Cochrane (1999), and under the assumption that stock returns are heteroskedastic, I show that equilibrium risk premium has a factor structure where the factors are a monotonic transformation of the surplus consumption ratio, a state variable that captures the systemic risk in the structural model. The restrictions implied by the model suppose a setup where one of the factors affects the variance of excess returns. Therefore, the factor model is estimated employing an adapted version of the Quantile Principal Components estimation procedure proposed by Sagner (2019). Simulations of the structural model under alternative parameterizations calibrated for the US show a good performance of the proposed systemic risk metric computed at quantiles different than the median. When estimated using quarterly post-war data, the proposed measure displays significant hikes that coincide with both several US recession periods and episodes of substantial financial market turbulences. Finally, the systemic risk estimator can forecast sharp shifts in both economic activity and industrial production up to one quarter ahead.

Resumen

Este artículo propone una nueva medida para cuantificar el riesgo sistémico a partir de la información contenida en el retorno de activos financieros. Así, en el contexto del modelo de formación de hábitos de Campbell y Cochrane (1999), y bajo el supuesto de retornos heterocedásticos, demuestro que el premio por riesgo de equilibrio posee una estructura factorial donde los factores son una transformación monótonica del ratio consumo excedente; una variable de estado que captura el riesgo sistémico dentro del modelo estructural. Las restricciones impuestas por el modelo implican una configuración donde uno de los factores afecta la varianza del exceso de retorno. Por lo tanto, el modelo factorial es estimado utilizando una versión adaptada de la metodología Componentes Principales Cuantiles propuesta por Sagner (2019). Simulaciones del modelo estructural bajo parametrizaciones alternativas calibradas para EE.UU. muestran un buen desempeño de la medida de riesgo sistémico propuesta, calculada en cuantiles distintos de la mediana. La medida estimada utilizando datos trimestrales de post-guerra muestra aumentos significativos que coinciden con varios períodos de recesión en EE.UU. y episodios de turbulencias en los mercados financieros. Finalmente, el estimador de riesgo sistémico puede pronosticar cambios bruscos de la actividad económica y producción industrial en un horizonte de hasta un trimestre.

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1 Introduction

After the Great Recession of 2007-2009, there has been a revived and increasing interest by both the academic community and policymakers on how to model and quantify systemic risk. Perhaps most of this trend, if not all, can be conceived as a response to two issues closely related to each other. First, systemic risk is a concept that lacks a unified definition, although there is agreement that it is related to risks of major dysfunction in financial markets (Hansen, 2013). Second, since systemic risk involves the financial system, it becomes highly desirable to measure and monitor it to support risk management and macro-prudential policies with useful information concerning the current and future state of the economy.

The preceding arguments suggest that systemic risk is rather a multifactorial construct, i.e. more than one notion, and consequently more than one metric, are needed to capture the complex and dynamic nature of financial markets and the economy before, during, and after periods of financial distress. To this extent, the related empirical literature has proposed several measures to quantify systemic risk¹. These measures span different dimensions of the concept, but they typically concentrate around four key aspects that characterize the financial system in a broad sense, namely leverage, liquidity, linkages between financial institutions, and asset prices². Many of these indicators were developed to serve also as an early-warning tool capable of signaling future episodes of financial distress, conferring thus time to policymakers to implement prudential actions towards mitigating the buildup of systemic risk and its potential losses for the overall economy. On the contrary, Giglio *et al.* (2016) find that only a reduced number of the immense variety of systemic risk measures available meets this objective.

In this paper, I propose a novel metric to quantify systemic risk based on asset returns that has a structural interpretation. More precisely, I show that in the context of the consumption-based asset pricing model with external habits of Campbell and Cochrane (1999) and under the assumption that the volatility of stock returns is counter-cyclical, the equilibrium risk premium has a two-factor structure. In this setup, factors are a monotonic transformation of the surplus consumption ratio (i.e., the proportion of consumption above the habit level), a state variable that contains the systemic risk in the structural model. Despite its connection with conventional measures based on asset returns, my approach departs from the traditional empirical literature in two key aspects. First, the equilibrium conditions of the model imply that one of the factors affects the variance of risk premium only. Therefore, popular econometric techniques for extracting unobserved factors from stock returns, such as Principal Components (PC), are no longer suitable because this factor is not identified at the center of the distribution of innovations to excess returns. Second, the unobserved factors

¹See Biais *et al.* (2012) for a recent and comprehensive survey of quantitative measures of systemic risk.

²Some recent studies related to leverage in this context are Geanakoplos and Pedersen (2011) and Frazzini and Pedersen (2012). Systemic risk measures related to liquidity are more abundant, e.g. Chordia *et al.* (2001), Pastor and Stambaugh (2003), Getmansky *et al.* (2004), Chan *et al.* (2007), Brunnermeier *et al.* (2011), Khandani and Lo (2011), among many others. Measures based on linkages between financial institutions generally incorporate new developments of modern network models; some examples are Huang *et al.* (2011), Billio *et al.* (2012), Adrian and Brunnermeier (2016), and Acharya *et al.* (2017). Systemic risk measures based on asset returns are among the oldest ones. Chen *et al.* (1986), Connor and Korajczyk (1988), Fama and French (1993), Chow *et al.* (1999), and recently Kritzman and Li (2010) and Kritzman *et al.* (2011) are some examples of this class of systemic risk indicators.

implied by the model are related in a nonlinear fashion. Consequently, nonlinear estimation methodologies are required to avoid misspecification bias.

To get a systemic risk estimator under this approach, I consider the Quantile Principal Components (QPC) estimation procedure proposed by [Sagner \(2019\)](#). This methodology addresses the issues mentioned before by allowing the estimation of both linear and nonlinear factor models at any quantile of the distribution of excess returns. In addition, the rotation required by the procedure was adapted so that it meets the restrictions implied by the model. In particular, I imposed the nonlinear relation between the factors and the sign restrictions on the factor loadings. In the latter case, one can note that the sign of one loading depends straightforwardly on the quantile of interest, whereas the sign of the remaining loading is a function of structural parameters. I show that if the state variable of the model is persistent enough, then the sign of the corresponding loading is unambiguously negative for all assets. I refer to the overall estimation procedure as Adapted Quantile Principal Components or AQPC for short.

I then solve and simulate the external-habit-based model under distinct parameterizations calibrated for the US economy with the purpose to compute an estimator of systemic risk from artificial data via the AQPC procedure, and to study its performance both individually and relative to the PC methodology. I find that, when computing the AQPC-based measure of systemic risk at a quantile different than the median, the precision of the estimator is high, which indicates that, on average, the surplus consumption ratio estimated under my approach can be effectively regarded as the true one. This good performance tends to decline when the surplus consumption ratio becomes more persistent and the risk-free rate is counter-cyclical. Intuitively, in the first case, because the long-run value of the state variable turns large, the stochastic discount factor tends to a constant, which implies that equilibrium returns become less sensitive to systemic risk and, therefore, more sensitive to idiosyncratic shocks. In the second case, the dynamic behavior of the risk-free rate induces a weak pro-cyclicality of the risk premium. Thus, excess returns are, as in the previous case, relatively more sensitive to idiosyncratic shocks. My simulations also show that when the estimate of systemic risk is computed with the AQPC procedure at the median of the distribution of returns or using the PC methodology, misspecification bias is large and severely affects the precision of the estimators.

Lastly, I compute the AQPC-based systemic risk measure using quarterly US stock data over the period from 1954 to 2018. The proposed estimator displays significant systemic risk spikes that coincide not only with several recession periods in the US, but also with some episodes of financial turbulences that did not trigger a recession in subsequent quarters like the Flash Crash of 1962, the S&P 500 decline of 22% over eight months in 1966, and Black Monday of October 1987. Moreover, the estimate of systemic risk can forecast sharp shifts in macroeconomic activity up to one quarter ahead, with an accuracy that outperforms PC-based measures. The preceding feature highlights the usefulness of the proposed indicator as an additional or complementary early-warning signal that policymakers can incorporate into their monitoring and prudential policy-making process.

This paper is related to the long literature which seeks to identify and estimate one or more systemic risk factors from a set of asset returns. Most studies in this area build on arbitrage arguments, as in the Arbitrage Pricing Theory (APT) developed by [Ross \(1976\)](#), or on equilibrium arguments, as in the Intertemporal Capital Asset Pricing Model (ICAPM)

developed by [Merton \(1973\)](#). In this sense, one strand of the literature relies on the idea that systemic risk, because of its lack of specificity, is a concept that cannot be quantified *ex-ante*. But, because it becomes evident *ex-post*, it can be measured in terms of its effects on other key observable variables. [Chen *et al.* \(1986\)](#) and [Fama and French \(1993\)](#) are good examples of this approach. In particular, the first paper assumes that systemic risk is well characterized by five macroeconomic variables that explain, to some extent, changes in the cash flow of firms and its relevant discount rate: the term spread of US government bonds, expected and unexpected inflation, industrial production growth, and the yield premium between high- and low-grade corporate bonds. The second paper, on the other hand, relies on firm characteristics to quantify systemic risk, namely the return of the market portfolio, firm size, book-to-market equity, bond maturity, and default risk. The rest of the related literature assumes, largely motivated by the APT, that systemic risk can be gauged by portfolios constructed out of traded assets. [Connor and Korajczyk \(1988\)](#) follow this approach and model systemic risk through five portfolios represented by five factors that were obtained using the PC methodology. More recently, [Kritzman *et al.* \(2011\)](#) propose a systemic risk metric, named the absorption ratio, which amounts to the fraction of the total variance of a panel of asset returns explained or “absorbed” by the first factor computed via PC. Intuitively, the absorption ratio measures how coupled is the financial market, i.e., it captures its fragility because negative shocks tend to propagate more easily and broadly in highly correlated markets (see [Ang and Chen, 2002](#); [Ang *et al.*, 2002](#); [Hong *et al.*, 2007](#)). [Chow *et al.* \(1999\)](#) and [Kritzman and Li \(2010\)](#), on its part, employ a rather different methodology to obtain an indicator of financial turbulence. In these articles, the authors use the Mahalanobis distance to determine whether a given asset return is exceptionally away from the cross-section average according to their historical joint distribution³. Accordingly, the indicator signals a turbulent financial market if, in a given period in time, the proportion of unusual returns (i.e., returns that are far away from their historical average) increases.

The rest of the paper is structured as follows. Section 2 briefly reviews the consumption-based asset pricing model with external habits of [Campbell and Cochrane \(1999\)](#), which corresponds to the basis of the proposed systemic risk measure. Simulations results under different parameterizations intended to study the precision of the metric are also reported. Section 3 presents the systemic risk estimate for the US that was computed using post-war data. A discussion regarding its in- and out-of-sample properties to evaluate its coherence with known recession periods and early-warning properties, respectively, is also provided. Finally, Section 4 concludes. Proofs and data descriptions were left in the Appendix.

2 Model

In this section, I revisit the external habits formation model of [Campbell and Cochrane \(1999\)](#) over which my measure of systemic risk builds up. Then, I simulate the model using parameters calibrated for the US economy and compute the systemic risk measure from artificial data using an adapted version of the high dimensional quantile factor analysis (QFA) framework proposed by [Sagner \(2019\)](#). Finally, I evaluate the performance of my

³The Mahalanobis distance is a weighted Euclidean distance, where the weights are given by the inverse of the covariance matrix.

measure within this context in terms of (i) how well it captures the dynamics of the systemic risk of the model, and (ii) whether my measure outperforms similar metrics based on PC.

2.1 An External-Habit-Based Asset Pricing Model

There is a representative investor in the economy who has lifetime utility over consumption C_t relative to a level of habit X_t in the following manner:

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j} - X_{t+j})^{1-\gamma} - 1}{1-\gamma} \right] \quad (1)$$

where $0 < \beta < 1$ represents the subjective discount factor and $\gamma > 0$ denotes the risk-aversion coefficient. The habit formation process X_t is external, also known as “catching up with the Joneses” following [Abel \(1990\)](#), in the sense that past consumption affects the habit formation process, but the latter does not affect current consumption. Because the representative investor derives utility from consumption that is over the level of habit, X_t cannot be below C_t for (1) to be well defined, and so X_t can be interpreted as a consumption subsistence level. It is convenient to capture this relation in terms of the *surplus consumption ratio* $S_t \equiv (C_t - X_t)/C_t$, i.e., the amount of consumption above the subsistence level as a proportion of total consumption. Thus, if $S_t \rightarrow 0$, the level of habit is close to consumption, and the economy enters a very bad state of nature (recession). Conversely, if $S_t \rightarrow 1$, then consumption is very large compared to the level of habit and, consequently, the economy lies in a good state of nature (boom). Moreover, note that in this model the coefficient of relative risk aversion is given by

$$-\frac{u_{CC}(C_t, X_t)}{u_C(C_t, X_t)} C_t = \frac{\gamma}{S_t} \quad (2)$$

where $u(C_t, X_t) = [(C_t - X_t)^{1-\gamma} - 1]/(1-\gamma)$ is the instantaneous utility function, $u_C(\cdot) = \partial u(C_t, X_t)/\partial C_t$ and $u_{CC}(\cdot) = \partial^2 u(C_t, X_t)/\partial C_t^2$. Thus, the representative investor becomes relatively more risk averse during recessions, i.e., when consumption is close to its subsistence level. During booms, on the contrary, risk aversion is relatively low and close to the risk-aversion coefficient γ .

Let $s_t \equiv \log S_t$ be the log surplus consumption ratio. The authors assume that s_t has an autoregressive and heteroskedastic structure, perfectly correlated with innovations to consumption growth, as follows:

$$s_{t+1} = (1 - \phi) \bar{s} + \phi s_t + \lambda(s_t) (\Delta c_{t+1} - E[\Delta c_{t+1}]) \quad (3)$$

where $|\phi| < 1$ is a persistence parameter; $\bar{s} < 0$ is the steady-state value of s_t ; and $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$ is consumption growth, which is assumed to be determined by the following expression:

$$\Delta c_{t+1} = g + v_{t+1} \quad (4)$$

where $g > 0$ is the growth rate of consumption and $v_{t+1} \sim iid \mathcal{N}(0, \sigma^2)$. The term $\lambda(s_t)$ in (3), which governs the heteroskedasticity of the log surplus consumption ratio, corresponds to the *sensitivity function*. It is parameterized by the following expression:

$$\begin{aligned}\lambda(s_t) &= \bar{S}^{-1} \sqrt{1 - 2(s_t - \bar{s})} - 1 \\ \bar{S} &= \sigma \sqrt{\frac{\gamma}{1 - \phi - b/\gamma}}\end{aligned}\tag{5}$$

In equilibrium, the stochastic discount factor M_{t+1} equals the investor's marginal rate of inter-temporal substitution. Therefore

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \cdot \frac{S_{t+1}}{S_t} \right)^{-\gamma}$$

or in logarithmic terms by using (3) and (4)

$$m_{t+1} = \log \beta - \gamma g + \gamma(1 - \phi)(s_t - \bar{s}) - \gamma(1 + \lambda(s_t))v_{t+1}\tag{6}$$

The above equation is the heart of the [Campbell and Cochrane \(1999\)](#) model. To understand its implications, note that in the short run, shocks to c_{t+1} and s_{t+1} move together, as can be seen from equations (3) and (4). Hence, either of these variables accounts for practically the same amount of the resulting variation in the stochastic discount factor. At longer horizons, however, [Cochrane \(2005\)](#) argues that these variables are less and less conditionally correlated, implying that, although s_{t+1} depends on c_{t+1} relative to its recent past, the overall level of consumption can be high or low. Consequently, most of the variation in the stochastic discount factor at longer horizons is driven mainly by shocks to habits. These observations imply that the surplus consumption ratio is a state variable that captures the systemic risk of the financial system. More precisely, assets are risky because they have a bad and volatile performance during occasional deep recessions, and, at the same time, this risk is unrelated to the uncertainty about the long-run average performance of the economy.

The real return on the risk-free asset of this economy is given by the corresponding log-linearized Euler equation of the model⁴, together with equations (5) and (6), as follows

$$r_{t+1}^f = \gamma g - \log \beta - \frac{\gamma(1 - \phi) - b}{2} - b(s_t - \bar{s})\tag{7}$$

The above expression shows that the risk-free rate is a linear function of the log surplus consumption ratio and that this relationship depends on the sign of the parameter b . If $b > 0$, then the risk-free rate is high during recessions and low during booms, suggesting that an inter-temporal substitution effect is predominant: when the economy faces bad (good) times, marginal utility of consumption is high (low), so the investor is willing to borrow (lend) to smooth inter-temporal consumption. Consequently, the equilibrium interest rate is driven up (down). [Wachter \(2006\)](#) exploits this case to study several features of the term structure of nominal interest rates in the US. If on the contrary $b < 0$, then the risk-free rate is procyclical, meaning that in this case a precautionary savings effect dominates: during recessions (booms), uncertainty about the future state of the economy increases (decreases), so investors are more willing to save (spend) and this behavior drives down (up) the equilibrium risk-free rate. [Verdelhan \(2010\)](#) adopts this case in an external-habit-based model and argues

⁴For more details on the derivation of the log-linearized optimality conditions of the model, see the Appendix.

that pro-cyclicality of interest rates is a necessary condition for the model to account for the uncovered interest rate parity puzzle. Lastly, if $b = 0$, then the risk-free rate is constant over time as in the model of [Campbell and Cochrane \(1999\)](#).

Let $r_{i,t}$ be the real return on the i -th risky asset in this economy. Following the enormous empirical literature that started with the seminal paper of [Engle \(1982\)](#), I assume that asset returns are heteroskedastic⁵. Moreover, following [Li \(2001\)](#) and [Li and Zhong \(2005\)](#), I further assume that the heteroskedasticity of $r_{i,t}$ is a function of the sensitivity function $\lambda(s_t)$. This last assumption, although strong, has at least two advantages. First, the sensitivity function given in equation (5) has now an economic interpretation since it corresponds to the price of risk under this assumption. In addition, because $\lambda(s_t)$ is a function of the state variable s_t , the price of risk in the model is, therefore, time-varying and counter-cyclical⁶. Second, and perhaps more importantly, the last assumption is intended to facilitate the estimation and interpretation of the systemic risk measure that will be derived from the model. Otherwise, the aforementioned heteroskedasticity assumption would require the incorporation of a second state variable that (i) would complicate the estimation of the systemic risk measure, as will be explained in the next section; and (ii) would extend the systemic risk concept into a two-dimensional space, hindering in this sense its simplicity in terms of interpretation and empirical application. Thereby, the real return of the i -th risky asset is given by the following expression

$$r_{i,t+1} = E_t[r_{i,t+1}] + (1 + \lambda(s_t)) u_{i,t+1} \quad (8)$$

where the idiosyncratic shocks are such that $u_{i,t+1} \sim \mathcal{N}(0, \zeta_i^2)$; $CORR[u_{i,t+1}, u_{j,t+1}] = \omega_{ij}$, for all $i \neq j$; and $CORR[v_{t+1}, u_{i,t+1}] = \rho_i$. Similar to the case of the risk-free rate, the corresponding log-linearized Euler equation related to risky assets, together with equations (5) to (8), imply that

$$E_t[r_{i,t+1}] - r_{t+1}^f = \zeta_i \left(\frac{2\gamma\sigma\rho_i - \zeta_i}{2} \right) (1 + \lambda(s_t))^2 \quad (9)$$

Thus, equation (9) indicates that the expected excess return or equity risk premium $E_t[r_{i,t+1}] - r_{t+1}^f$ is also a function of the log surplus consumption ratio. In particular, the model predicts a larger risk premium during recession periods, a result that is a direct consequence of the counter-cyclical nature of risk aversion in the model (i.e., it is high in recessions and low in booms).

2.2 Measure of Systemic Risk of Simulated Data

The previous equation describing the expected excess return of the i -th asset corresponds to the basis of my estimations. However, because I do not observe the conditional expectation of excess returns, I use equation (8) to get a similar expression in terms of realized excess returns, $\tilde{r}_{i,t} \equiv r_{i,t} - r_t^f$, as follows

⁵While time-varying volatility of asset returns is a phenomenon that has been known for a long time, the comprehensive survey of [Bollerslev et al. \(1992\)](#) suggests that most formal statistical models addressing this stylized fact started to bloom after the publication of the ARCH framework proposed by [Engle \(1982\)](#).

⁶This is because $\lambda'(s_t) = -(\bar{S}\sqrt{1 - 2(s_t - \bar{s})})^{-1} < 0$ for all s_t .

$$\tilde{r}_{i,t+1} = \zeta_i \left(\frac{2\gamma\sigma\rho_i - \zeta_i}{2} \right) (1 + \lambda(s_t))^2 + (1 + \lambda(s_t)) u_{i,t+1}$$

or alternatively

$$\tilde{r}_{i,t+1} = \eta_i f_t + h_t u_{i,t+1} \quad (10)$$

where $\eta_i = \zeta_i(2\gamma\sigma\rho_i - \zeta_i)/2$, $f_t = (1 + \lambda(s_t))^2$, and $h_t = 1 + \lambda(s_t)$. Two aspects of the above equation are worth highlighting. First, from an estimation point of view, equation (10) has a factor structure in the sense that every element on the right-hand side is unobserved by the researcher: f_t and h_t play the role of the factors, whereas η_i and $u_{i,t+1}$ can be interpreted as the factor loading and the idiosyncratic component of the factor model, respectively. Second, and more importantly, while f_t affects the mean of excess returns, h_t affects its variance. This aspect is the key difference between the multi-factor pricing equation derived from the external-habits formation model and the Arbitrage Pricing Theory developed by [Ross \(1976\)](#) or the Inter-temporal Capital Asset Pricing Model developed by [Merton \(1973\)](#), where all factors affect the mean excess returns. So, any estimation procedure that exploits the information at the center of the distribution of $\tilde{r}_{i,t+1}$, conditional on η_i , f_t , and h_t is unable to identify the latter. To understand this last point, define $u_{i,t+1} \equiv \sigma\Phi^{-1}(z_{i,t+1})$, where $z_{i,t+1} \sim \mathcal{U}[0, 1]$ and $\Phi^{-1}(\cdot)$ is the inverse of the Normal cumulative distribution function. Thus, equation (10) can be rewritten in the following manner

$$\tilde{r}_{i,t+1} = \alpha_i(z_{i,t+1})' \theta_t(z_{i,t+1}), \quad z_{i,t+1} \sim \mathcal{U}[0, 1] \quad (11)$$

where $\alpha_i(z_{i,t+1}) = [\eta_i, \sigma\Phi^{-1}(z_{i,t+1})]'$ is the vector of factor loadings and $\theta_t(z_{i,t+1}) = [f_t, h_t]'$ is the vector of factors. In the jargon of the factor analysis literature, equation (11) corresponds to a location-scale factor model⁷.

Note that although both f_t and h_t do not depend on $z_{i,t+1}$ directly, the dimension of the vector $\theta_t(z_{i,t+1})$ does. In this sense, when the vector of factors is evaluated at the median of $u_{i,t+1}$, then $\Phi^{-1}(0.5) = 0$ and, consequently, $\theta_t(0.5) = f_t$ is a scalar, i.e., we cannot identify h_t by looking at the center of the distribution of $u_{i,t+1}$, and hence of $\tilde{r}_{i,t+1}$. For any $z_{i,t+1} \neq 0.5$, $\theta_t(z_{i,t+1}) = [f_t, h_t]'$ is a 2-dimensional vector, which means that both factors are identified. The previous observation implies that any quantile of the joint conditional distribution of $\tilde{r}_{i,t+1}$, excluding the median, contains additional information about the magnitude and dynamics of the systemic risk variable s_t . This is an important issue because any method aimed to get an estimate of systemic risk from equation (10) based on the information at the center of the aforementioned joint conditional distribution will lead to a loss of information that will ultimately translate into misspecification bias⁸, thus providing a poor description of both the level and dynamics of s_t .

Let τ be a scalar within the $(0, 1)$ interval. Then, the τ -th conditional quantile function of $\tilde{r}_{i,t+1}$, $\mathcal{Q}_{\tilde{r}_{i,t+1}}(\tau) \equiv \inf\{\tilde{r}_{i,t+1} | \Phi(\tilde{r}_{i,t+1}/\sigma | \alpha_i(\tau), \theta_t(\tau)) \geq \tau\}$, is given by

⁷For more details about location-scale factor models, see [Skrondal and Rabe-Hesketh \(2004, pp. 49-93\)](#).

⁸In this context, [Onatski \(2015\)](#) finds that the estimation of factor models where the number of estimated factors is smaller than the true one (i.e., the estimation model is misspecified), can seriously affect the quality of both the estimated factors and factor loadings.

$$\mathcal{Q}_{\tilde{r}_{i,t+1}}(\tau | \alpha_i(\tau), \theta_t(\tau)) = \alpha_i(\tau)' \theta_t(\tau), \quad \tau \in (0, 1) \quad (12)$$

where now I refer to $\theta_t(\tau)$ and $\alpha_i(\tau)$ as *quantile factors* and *quantile factor loadings*, respectively. Given a panel of N excess returns observed during T periods, equation (12) can be estimated using the QPC procedure proposed by Sagner (2019). Broadly speaking, this methodology is a simple iterative procedure, which in this context is based on the minimization of the average quantile loss $(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T \rho_\tau(\tilde{r}_{i,t+1} - \alpha_i' \theta_t)$, with $\rho_\tau(x) = (\tau - \mathbb{1}\{x < 0\})x$, for a given value of $\tau \in (0, 1)$. The algorithm returns estimators of $\alpha_i(\tau)$ and $\theta_t(\tau)$ by running quantile regressions in two iterative steps. At each step, an estimator of one of these objects is obtained considering a preliminary estimate of the other one. Convexity of the quantile loss function $\rho_\tau(\cdot)$ when either α_i or θ_t is held fixed ensures the convergence of the algorithm to a local minimum.

A key ingredient of the QPC procedure is the choice of the identifying restrictions (also known as *rotation*) to estimate and, perhaps more importantly, to interpret both the quantile factors and quantile loadings. The algorithm considers three alternative restrictions for such purposes: (i) the default rotation of the PC methodology via MLE; (ii) a recursive rotation; and (iii) an errors-in-variables rotation. In the context of the external-habits formation model described previously, the first rotation implies that $\hat{\theta}_t(\tau)$ is an orthogonal vector and closely related to the eigenvectors associated with the two largest eigenvalues of the covariance matrix of $\tilde{r}_{i,t+1}$. Under the second rotation, $\hat{\theta}_t(\tau)$ is also an orthogonal vector, but the assumptions on $\hat{\alpha}_i(\tau)$ imply that there exists one excess return that is affected by f_t only. Finally, the last rotation imposes all restrictions on $\hat{\alpha}_i(\tau)$ by assuming that there are two excess returns, say $\tilde{r}_{i,t+1}$ and $\tilde{r}_{j,t+1}$ with $i \neq j$, that are affected only by f_t and h_t , respectively. However, by looking back at equation (12), one can note that none of these rotations adequately characterize the quantile factor structure implied by the model.

The previous observation highlights the fact that we need to adapt the identifying restrictions of the QPC methodology in order to get estimators of $\alpha_i(\tau)$ and $\theta_t(\tau)$ that are interpretable within the context of the external-habits formation model. Therefore, since the model has two unobservable factors, the factor analysis literature tells us that we need to impose four restrictions to uniquely identify the parameters of the quantile factor model⁹. The first restriction conditions the relationship between the two quantile factors by imposing that $f_t(\tau) = h_t(\tau)^2$ for all t and $\tau \in (0, 1)$, as derived in equation (10). The remaining rotations correspond to sign restrictions. So, the second restriction enforces the condition $h_t(\tau) > 0$ for all t and $\tau \in (0, 1)$, which follows directly from the definition of the sensitivity function $\lambda(s_t)$ in (5). The last two restrictions are related to the sign of the quantile factor loadings $\alpha_i(\tau)$. Note that the sign of the second quantile loading $\alpha_i^{(2)}(\tau) = \sigma \Phi^{-1}(\tau)$ depends on the value of τ , i.e., $\alpha_i^{(2)}(\tau) < 0$ if τ lies within the $(0, 0.5)$ interval; $\alpha_i^{(2)}(\tau) = 0$ if $\tau = 0.5$, in which case $h(\tau)$ is not identified as mentioned previously; and $\alpha_i^{(2)}(\tau) > 0$ if τ belongs to the $(0.5, 1)$ interval. The sign of the first quantile loading $\alpha_i^{(1)}(\tau) = \zeta_i(2\gamma\sigma\rho_i - \zeta_i)/2$, however, deserves more attention. Looking at its definition, it is straightforward to see that the sign of $\alpha_i^{(1)}(\tau)$ depends on whether the difference $2\gamma\sigma\rho_i - \zeta_i$ is positive or negative, which in turn depends on the calibration adopted. But in an estimation context, this identifying restriction cannot

⁹In general, for a given $\tau \in (0, 1)$, if a quantile factor model has $K(\tau)$ factors, then one needs to impose $K(\tau)^2$ restrictions to uniquely identify the quantile factors and quantile loadings.

be implemented directly, because it requires the knowledge of the magnitude of structural, unobservable parameters.

Proposition 1 (Sign of First Quantile Factor Loading). *Let $\alpha_i^{(1)}(\tau)$ be the first quantile factor loading of the excess returns implied by the external-habit formation model given by equation (10). Let ϕ , b , and γ be the persistence of the log surplus consumption ratio, the parameter governing the cyclicalness of the risk-free rate, and the risk-aversion coefficient, respectively. Thus, if $\phi \rightarrow \min\{1, 1 - b/\gamma\}$, then $\alpha_i^{(1)}(\tau) < 0$ for all i and $\tau \in (0, 1)$.*

The above proposition posits that if the state variable of the model is very persistent, then the first quantile loading is unambiguously negative for all risky assets. While it is true that the parameter ϕ is also unobservable, simulations performed by Wachter (2006) show that this parameter determines the first-order autocorrelation of the price-dividend ratio $(P/D)_t$. Thus, the knowledge of the persistence of $(P/D)_t$ is informative about the magnitude of ϕ , and hence of the sign of $\alpha_i^{(1)}(\tau)$. The intuition behind this result is as follows. When s_t is very persistent, the volatility of stock returns is less sensitive to consumption growth shocks. As a consequence, the correlation between $r_{i,t+1}$ and Δc_{t+1} , which is proportional to ρ_i , is small relative to the variance of stock returns, which in turn is proportional to ζ_i . Therefore, the difference $2\gamma\sigma\rho_i - \zeta_i$ is negative for all risky assets.

In summary, the QPC methodology with a rotation (or identifying restrictions) adapted to the quantile factor model (12), or Adapted Quantile Principal Components (AQPC) algorithm hereafter, consists of the following steps. For a given $\tau \in (0, 1)$, start by guessing initial values for the vectors of quantile factor loadings $\hat{\alpha}_i(\tau)$. Then, using the guessed quantile loadings, obtain estimates of the quantile factors $\hat{\theta}_t(\tau) = [\hat{f}_t(\tau), \hat{h}_t(\tau)]'$ using the nonlinear quantile regression procedure proposed by Koenker and Park (1996) across cross-sections for each $t = 1, \dots, T$, subject to the restrictions $\hat{f}_t(\tau) = \hat{h}_t(\tau)^2$ and $\hat{h}_t(\tau) > 0$. In the next step, fix the estimated quantile factors and get estimates of the quantile loadings $\hat{\alpha}_i(\tau) = [\hat{\alpha}_i^{(1)}(\tau), \hat{\alpha}_i^{(2)}(\tau)]'$ using nonlinear quantile regressions across periods for each $i = 1, \dots, N$, subject to the corresponding sign restrictions. In the final step, compute the discrepancy between these estimates and the initial guesses, and if the difference is smaller than a predefined accuracy level, the algorithm ends. Otherwise, repeat the previous steps until convergence is achieved¹⁰.

2.2.1 Calibration

The model described in the previous section was calibrated to the US economy using quarterly data covering the period from 1954 until the end of 2018¹¹. Tables 1 and 2 summarize the calibrated parameters under nine sets of parameterizations given by three alternative values of ϕ and b . Rather than to calibrate these last parameters, I prefer to assign them alternative values to study the performance of the AQPC estimator of $\alpha_i(\tau)$ and $\theta_t(\tau)$ under different

¹⁰As mentioned in Sagner (2019), the convergence of the algorithm is local. To ensure that the AQPC estimators $\hat{\alpha}_i(\tau)$ and $\hat{\theta}_t(\tau)$ are a global optimum, one can consider, for example, different initial guesses of the quantile factors and quantile loadings to start the algorithm, and then keep the corresponding AQPC estimators that deliver the smallest value for the loss function.

¹¹All data sources and constructed variables are detailed in Section A.3.1 of the Appendix.

persistence degrees of the log surplus consumption ratio and different cyclical patterns of the risk-free rate.

The parameters g and σ were chosen to match the consumption data, i.e., they correspond to the mean and standard deviation, respectively, of the real per capita consumption growth rate of non-durables and services. Regarding the preference parameters, I follow standard real business cycle literature and set the value of the risk-aversion coefficient γ to 2. The subjective discount factor β matches the inverse of the average real risk-free gross rate. The last is the difference between the return of the 3-months Treasury Bill and expected inflation, where the latter variable was approximated by a bivariate VAR(1) model using the risk-free rate and inflation. My choice for the value of β is above the one in [Campbell and Cochrane \(1999\)](#) and [Wachter \(2006\)](#) (0.97 and 0.98, respectively), due to the US monetary easing period between 2011 and 2015, not considered in these papers, where the Fed funds rate reached virtually the zero lower bound.

As mentioned in Proposition 1, the parameter that governs the dynamics of the log surplus consumption ratio plays a key role in the estimation of the systemic risk measure. Thus, instead of assigning a particular value to ϕ , I consider three alternative magnitudes. In particular, I consider a case where s_t is less persistent ($\phi = 0.50$), persistent ($\phi = 0.90$), and highly persistent ($\phi = 0.99$). Similarly, I consider that the parameter b adopts the values -0.010, 0, and 0.010, which imply a procyclical, constant, and counter-cyclical risk-free rate, respectively. Both parameters, along with γ and σ , determine the steady-state value of the surplus consumption ratio \bar{S} according to equation (5), and consequently, of the log surplus consumption ratio $\bar{s} \equiv \log \bar{S}$. My results indicate that, depending on the parameterization used, consumption is, on average, between 0.9% to 8.9% above the subsistence level.

I use the [Fama and French \(1993\)](#) portfolios to calibrate the parameters of real stock returns. More precisely, I consider six portfolios formed by the intersection of two categories of size and three categories of the book-to-market ratio. In this setup, the volatility parameters ζ_i are related to the standard deviation of the corresponding portfolio returns σ_{r_i} through equations (5) and (8) by noticing that $V[r_{i,t+1}] = (\zeta_i/\bar{S})^2$, which suggests the calibration $\zeta_i = \bar{S} \cdot \sigma_{r_i}$. Because this parameter depends on \bar{S} , which in turn depends on ϕ and b , its value varies across parameterizations. However, by looking at Table 1, one can see that differences are negligible when ϕ is low. On the contrary, when s_t becomes more persistent, differences across parameterizations become more apparent when the parameter b turns from negative to positive. The parameter ρ_i , on the other hand, is slightly more laborious to calibrate. In Lemma 2 of the Appendix, I show that this parameter is proportional to the correlation between returns of the i -th portfolio and consumption growth. Moreover, because the proportion is a function of σ , and especially of ϕ and \bar{S} , the value of ρ_i depends on the parameterization adopted by showing a decreasing pattern as b goes from negative to positive values. Nevertheless, as in the case of ζ_i , the main differences are only noticeable when ϕ is close to 1. Finally, the correlation between stock returns ω_{ij} were chosen to match the sample correlation between portfolio returns given that $CORR[r_{i,t+1}, r_{j,t+1}] = \omega_{ij}$ for all $i \neq j$, as implied by the model in equation (8).

2.2.2 Results

The model was solved numerically and simulated for each of the nine calibrations described previously to study how different combinations of the structural parameters affect the performance of the systemic risk estimator implied by the model¹². In particular, for each calibration, I simulate the model 1,000 times to obtain a total of 250 quarters of artificial data at each simulation, which is equivalent to have roughly 63 years of data at each simulation.

Next, I standardize the artificial excess returns and extract the measure of systemic risk using three alternative estimation methodologies: (i) the AQPC estimator with $\tau = \{0.4, 0.5, 0.6\}$; (ii) the Adapted PC (APC) estimator, which corresponds to the Principal Components estimator under the identifying restrictions of AQPC; and (iii) the PC estimator under the default rotation. For the first two estimators, the calibrations shown in Table 1 entail that when the log surplus consumption ratio is less persistent or persistent (i.e., $\phi = 0.5$ or $\phi = 0.9$, respectively), the sign of the first quantile loading is positive, whereas it is negative when s_t is highly persistent ($\phi = 0.99$); a result that is in line with the implications of Proposition 1.

Table 3 shows the average correlation between the simulated log surplus consumption ratio and the same variable obtained from the three estimators mentioned in the previous paragraph. Several findings are worth highlighting from this table. First, the AQPC estimator with $\tau \neq 0.5$ has a good performance in terms of extracting the systemic risk measure implied by the model of stock returns data. In fact, the average correlation, which can be interpreted as a measure of estimation precision, can be as high as 0.94, suggesting that the log surplus consumption ratio derived from this estimator can be very close to the true s_t . Second, the precision of the AQPC systemic risk measure tends to decrease as s_t becomes more persistent, and as r_{t+1}^f turns counter-cyclical. For instance, when the log surplus consumption ratio is less persistent, and the risk-free rate is pro-cyclical, the average correlation is about 0.93 under $\tau = 0.4$. If the latent variable of the model becomes highly persistent ($\phi = 0.99$), all other things being equal, the correlation decreases to around 0.80. If besides r_{t+1}^f becomes counter-cyclical, then the average correlation falls back to roughly 0.60. Intuitively, when $\phi \rightarrow \min\{1, 1 - b/\gamma\}$, the log surplus consumption ratio is close to a random walk, which implies that the stochastic discount factor tends to a constant. Consequently, the excess returns are less sensitive to s_t and relatively more sensitive to the idiosyncratic shock $u_{i,t+1}$, implying thus a low correlation between the estimated and the true s_t . Similarly, a counter-cyclical risk-free rate induces a risk premium that is small during recessions and large during boom periods. In other words, the co-movement of r_{t+1}^f relative to s_t acts like a buffer that reduces the sensitivity of the excess returns to variations of the state variable of the model, resulting in a low correlation between the estimator \hat{s}_t and the true s_t . A pro-cyclical risk-free rate, on the contrary, amplifies the sensitivity of $\tilde{r}_{i,t+1}$ to s_t , which translates into a high correlation. Third, as expected, the AQPC estimator performs poorly when computed at the median of the joint distribution of excess returns. In particular, my results show that, when $\tau \neq 0.5$, the average correlation ranges from 0.60 to 0.95, whereas it drops to values between 0.05 to 0.50 otherwise. This occurs because, as mentioned previously, when

¹²The model was solved numerically by using Dynare. Alternatively, one can use the numerical algorithm developed by Wachter (2005), which is based on a grid of values for s_t to solve for the price-dividend ratio as the fixed point of the Euler equation of the model.

$\tau = 0.5$ the second quantile factor h_t cannot be identified from the artificial data and, consequently, its estimator is too noisy to be useful: the identifying restrictions $\hat{f}_t(0.5) = \hat{h}_t(0.5)^2$ and $\hat{h}_t(0.5) > 0$ generate an imprecise measure of systemic risk that dramatically reduces the average correlation between \hat{s}_t and s_t . Lastly, my simulations show that, within the context of an external-habit-formation model like the one described, the AQPC estimator with $\tau \neq 0.5$ outperforms PC-based estimators, where the latter display average correlations below 0.55 and typically around 0.23. This is an expected result because the APC and PC estimators exploit the mean of the joint distribution of $\tilde{r}_{i,t+1}$ to extract the factors. However, this is precisely the part of the distribution where h_t cannot be identified. Note that, despite the low average correlation, the APC estimator has a better performance relative to the AQPC estimator with $\tau = 0.5$; a finding that can be explained by remembering that the latter estimator, since it does not have a closed-form solution, has to be approximated through numerical algorithms such as the interior-point algorithm popularized by [Karmankar \(1984\)](#)¹³.

3 Systemic Risk Measure for the US

This section starts by presenting the data that was used to characterize the US stock market during the post-war period and then explaining how the AQPC methodology described in the previous section was applied to this data to compute a habit-based systemic risk measure for the US economy. Next, I discuss the in-sample properties of the proposed indicator compared to a PC-based index, and in terms of its coherence with several past recession periods. Finally, I employ the criterion proposed by [Giglio *et al.* \(2016\)](#) to evaluate the out-of-sample predictive power of my measure, and thus its usefulness as an early-warning indicator.

3.1 Data and Estimation

I obtain the US stock market data from the Annual Update database of the Center for Research in Security Prices (CRSP) available from 1954 until 2018 in daily frequency. Specifically, I consider data on stock prices for all US corporations with 780 trading months record (i.e., with no missing observations) to construct stock returns in excess of the risk-free rate¹⁴. Alternatively, I consider stock prices from 1990 onwards (348 trading months) as a way to control for the potential effects of survival bias over my estimations, which could be significant, especially in those industries that are more subject to firms entry and exit such as the service and financial-based sectors. Then, in both cases, this variable was expressed in quarterly frequency with the aim to eliminate high-frequency fluctuations that would be otherwise difficult to explain by a model that contains macroeconomic variables like consumption growth. These criteria imply a panel of 114 and 977 firms over the period from 1954q1 to 2018q4, and from 1990q1 to 2018q4 respectively¹⁵, which accounts, on average, for almost

¹³For a brief review and recent developments on this topic, see [Potra and Wright \(2000\)](#) and [Wright \(2004\)](#).

¹⁴I did not include dividends in the computation of stock returns because this would require to introduce an additional stochastic process for this variable in the model, thus increasing the number of state variables, and hence the number of quantile factors of the statistical factor model.

¹⁵See Section A.3.2 in the Appendix for more details about the construction of this database.

16% and 35% of the total yearly amount of shares traded by all firms considered in the CRSP database (Figure 1a). In terms of the distribution of industries, Figure 1b shows that the full sample is concentrated mostly in manufacturing (74%), followed distantly by transportation and public utilities (11%), mining (8%), and retail trade (5%). On the other hand, the most recent sample (1990 onwards) has also an important, although lower, participation of the manufacturing sector (49%), but now industries like finance, insurance, and real estate; and services represent a significant share of this sample (17% and 9%, respectively).

As discussed in the previous section, the sign restriction on the first quantile factor loading, besides other restrictions, is key for the identification of the AQPC estimator of the systemic risk measure. In this sense, Proposition 1 indicates that this sign depends crucially on the magnitude of the persistence parameter ϕ , where the latter can, in turn, be inferred from the first-order autocorrelation of the price-dividend ratio of the S&P 500 index, ϕ_{SP} , according to the findings of Wachter (2006). The estimation of this parameter using an AR(1) model reveals that the price-dividend ratio of the S&P 500 is very persistent: the point estimate of ϕ_{SP} is about 0.976 and has a standard error of 0.013¹⁶. Moreover, the data also provides indirect evidence in favor of a counter-cyclical risk-free rate, i.e., in favor of $b > 0$. To support this point, I regressed the realized real risk-free rate ($r_{t+1}^f - \pi_{t+1}^e$) on the weighted consumption growth over the past 10 years $\sum_{j=1}^{40} \phi_{SP}^j \Delta c_{t-j}$ that works as a proxy for the log consumption surplus ratio s_t using the following model

$$r_{t+1}^f - \pi_{t+1}^e = \nu_1 + \nu_2 \sum_{j=1}^{40} \phi_{SP}^j \Delta c_{t-j} + \varepsilon_{t+1}$$

where $\phi_{SP} = 0.976$ in agreement with the previous result. The OLS estimate for ν_2 is -0.035 with a robust standard deviation of 0.007, in line with the hypothesis that $b > 0$. Figure 2 shows the historical evolution of both variables. The inverse relationship between the risk-free rate and past cumulative consumption growth becomes apparent after a visual inspection of the graph, in particular during the period before 2011. From this year onwards, when short term nominal interest rates were close to 0% and the Fed announced new rounds of its large-scale asset purchase program, the relationship becomes rather diffuse, which would explain the estimated magnitude and standard error of ν_2 ¹⁷. In this manner, these results suggest that the persistence parameter ϕ is close to the upper bound of its support ($1 - b/\gamma$). Therefore, by Proposition 1, we can consider that the sign of the first quantile factor loading is negative for all risky assets i . The sign of the second quantile loading, on its part, depends straightforwardly on the value of τ employed for the AQPC estimation. Finally, I consider the non-negativity constraint on the second factor, and the quadratic relationship between the quantile factors implied by equation (10). In the final step of the estimation, I standardize all excess returns and apply the AQPC methodology under the four identification restrictions just discussed, and $\tau = 0.6$ ¹⁸.

¹⁶This result is in line with the findings of Campbell (1991), and more recently, Chevillon and Mavroeidis (2018) and Golinski *et al.* (2018), among many others.

¹⁷The OLS estimate and robust standard error of ν_2 using data until the last quarter of 2010 are -0.542 and 0.101, respectively.

¹⁸In light of the discussion of Section 2.2, the choice of τ is arbitrary, as long as $\tau \neq 0.5$. My results are robust to alternative values of this parameter (not reported).

Thus, to recap, the estimated model is the following

$$\begin{aligned}
\mathcal{Q}_{\tilde{r}_{i,t+1}^*}(0.6 | \alpha_i(0.6), \theta_t(0.6)) &= \alpha_i^{(1)}(0.6) \cdot f_t(0.6) + \alpha_i^{(2)}(0.6) \cdot h_t(0.6) & (13) \\
\text{s.t.} & f_t(0.6) = h_t(0.6)^2, & \forall t \\
& h_t(0.6) > 0, & \forall t \\
& \alpha_i^{(1)}(0.6) < 0, & \forall i \\
& \alpha_i^{(2)}(0.6) > 0, & \forall i
\end{aligned}$$

where $\alpha_i(0.6) = [\alpha_i^{(1)}(0.6), \alpha_i^{(2)}(0.6)]'$, $\theta_t(0.6) = [f_t(0.6), h_t(0.6)]'$, and $\tilde{r}_{i,t+1}^*$ is the standardized realized excess return on the i -th risky asset. Finally, because I do not observe the structural parameters of the model, the estimated quantile factor $\hat{h}_t(0.6)$ is rather a monotonic transformation of s_t that is proportional to $1 + \lambda(s_t)$. So, given that $\partial\lambda(s_t)/\partial s_t < 0$ for all s_t , the proposed measure signals high (low) systemic risk when its magnitude is high (low).

3.2 In-Sample Properties

Figure 3 plots the estimated systemic risk measure for the US under both samples, along with the recession periods identified by the National Bureau of Economic Research (NBER). From this figure, we note that the measures are positively correlated (correlation coefficient around 0.76) and they exhibit significant spikes, i.e., increases that are over 1.96 standard deviations above its mean, that coincide with several well-documented economic recessions in the US: the 1960-1961 recession, the 1973 oil crisis coupled with the 1973-1974 stock market crash that came after the collapse of the Bretton Woods system, the double-dip recession of the early 1980s, the early 1990 recession that started after the oil price shock by August of that year, as well as the Great Recession of 2007-2009. The measure under both samples also displays a dramatic increase during the collapse of the dot-com bubble by the end of the first quarter of 2000, an event that preceded the 2001 recession.

Interestingly, the AQPC-based measure computed under the full sample signals three financial episodes that did not trigger a recession in the following periods: (i) the Flash Crash of 1962, when the stock market dropped 22%, and the recovery came at the end of that year, after the end of the Cuban Missile Crisis; (ii) the S&P 500 decline of 22% over eight months during 1966 that occurred after the Fed increased the interest rate to control inflation; and (iii) the Black Monday of October 1987 that began in Hong Kong and spread shortly to the west hitting Europe and the US, where the Dow Jones plummeted 23% in one day.

Table 4 reports summary statistics of the proposed measure, and an alternative measure obtained via PC under the traditional rotation as in [Connor and Korajczyk \(1988\)](#)¹⁹. Several statistical facts about both estimates of systemic risk stand out in this table. First, the skewness of the AQPC-based measure is positive and almost 7 to 8 times larger than that of the PC-based measure. This feature is a direct consequence of the restriction $\hat{h}_t(\tau) > 0$ imposed by the habits-formation model through the parameterization of the sensitivity function $\lambda(s_t)$. The PC-based measure, on the contrary, is allowed to take both positive and

¹⁹I extracted one factor in order to make both measures comparable.

negative values as seen in Figure 4 for the case of the full sample and has, therefore, a skewness close to 0. Second, while it is true that the AQPC-based indicators have larger kurtosis, this statistic scaled by 1 plus the skewness squared is smaller than the PC-based measure (1.32 and 1.55 versus 3.52)²⁰. This result implies that, there are more extreme values in the latter measure of systemic risk, consistent with the visual inspection of Figure 4. Third, the AQPC-based measure under the full sample and the 1990-onwards sample is counter-cyclical and has a contemporaneous correlation with industrial production and national economic activity of -0.268 and -0.222, and of -0.334 and -0.323, respectively. This statistical fact is in line with the definition of the quantile factor $h_t(\tau)$, in the sense that during recessions, consumption and habits are close together, i.e., s_t is small and hence $h_t(\tau)$ is large. The PC-based indicator, on the contrary, depicts virtually no correlation with industrial production, and a negative correlation with the economic activity index. Finally, all measures show a positive correlation with the national financial conditions index computed by the Chicago Fed. Thus, periods of high (low) systemic risk signaled by these measures coincide with tighter (looser) than average conditions in US money, debt, and equity markets.

In summary, the AQPC-based indicators depict interesting features when evaluated in-sample. In particular, the level and dynamics of systemic risk are coherent with economic activity altogether with financial conditions, since several recessions and episodes of financial market turmoils coincide with sharp increases of the proposed measure. Moreover, given that the in-sample properties of the proposed measure computed under both samples are very similar, the analysis in the subsequent sections will consider the systemic risk measure obtained using the full sample only.

3.3 Early-Warning Indicator Properties

To be useful to policymakers, systemic risk measures should also be able to signal, to some extent, future periods of macroeconomic distress. This additional requirement is aimed to give policymakers enough time to implement corrective actions towards mitigating the build-up of downside risks that would otherwise result in broad losses for the overall economy.

In this section, I evaluate the ability of the proposed systemic risk indicator, both standalone and relative to the PC-based measure, to forecast future adverse macroeconomic shocks. To this extent, I employ a procedure based on [Giglio *et al.* \(2016\)](#), which consists of four basic steps.

In the first step, shocks to macroeconomic variables are proxied by innovations to the Industrial Production Index (IPI); and the Chicago Fed National Activity Index (CFNAI) together with its subcomponents personal consumption and housing (PCH); production and income (PI); sales, orders and inventories (SOI); and employment, unemployment and hours (EUH). These innovations are merely the residuals of AR(p) models, where the lag order p is chosen according to the Akaike information criterion²¹.

Next, in the second step, I forecast future macroeconomic shocks using quantile regressions of the form

²⁰[Rohatgi and Szekely \(1989\)](#) show that the scaled kurtosis $K/(S^2 + 1)$ is bounded below by 1, where K is the kurtosis and S is the skewness. Thus, a distribution with a relative high scaled kurtosis has fatter tails.

²¹In my results, most values for the autoregressive order are concentrated around 6 and 4 quarters for the IPI and CFNAI indices, respectively.

$$\mathcal{Q}_{\tilde{y}_{t+h}^{(n)}}(q|\xi_t^{(l)}) = a_1(q) + a_2(q)\xi_t^{(l)}, \quad q \in (0, 1)$$

where $\tilde{y}_{t+h}^{(n)}$ is an innovation to the n -th macroeconomic variable, $n \in \{\text{IPI, CFNAI, PCH, PI, SOI, EUH}\}$, h quarters ahead, $\xi_t^{(l)}$ is a measure of systemic risk obtained via either the Adapted Quantile Principal Components or the Principal Components methodology, i.e., $l \in \{\text{AQPC, PC}\}$, and $a_1(q)$ and $a_2(q)$ are quantile-specific parameters to be estimated. Regarding the values of q , I consider the 5th and 20th quantiles of $\tilde{y}_{t+h}^{(n)}$ to characterize future adverse shocks to macroeconomic variables, and for the sake of completeness, I also consider the median ($q = 0.5$), and the 80th and 95th quantiles to represent benign innovations. These choices highlight the potential nonlinear relationship between systemic risk and future crisis and boom periods. Regarding the forecasting horizon h , I focus the attention on out-of-sample forecasts within a year, i.e., $h = \{1, 2, 4\}$ quarters. This forecasting exercise using quantile regressions resembles the vulnerable growth methodology proposed by [Adrian *et al.* \(2019\)](#), where future quantiles of GDP growth are forecasted using variables that capture the actual macroeconomic and financial conditions or, in other words, the systemic risk of the overall economy.

The third step, on its part, repeats the previous ones in a real-time fashion, starting with 10 years (40 quarters) of data and then adding one new quarter of data at each repetition until reaching the end of the sample. Note that because the estimations are conducted using information up to time t at each repetition, the AR(p) model of the first step used to generate the innovations $\tilde{y}_t^{(n)}$ may change when incorporating new observations. Analogously, the measures of systemic risk are entirely re-computed when new data becomes available at each repetition.

Lastly, in the fourth step, I evaluate the predictive accuracy of the systemic risk indicators based on AQPC and PC, and of unconditional quantiles (UQ) using the test of [Diebold and Mariano \(1995\)](#). Let $\hat{\mathcal{Q}}_{\tilde{y}_{t+h}^{(n)}}(q|\xi_t^{(l)})$ be the h -quarters ahead forecast of the q -th quantile function of the innovation to the n -th macroeconomic variable $\tilde{y}_{t+h}^{(n)}$, conditional on the l -th systemic risk measure $\xi_t^{(l)}$, and let $\hat{e}_{t+h}^{(n)}(q|\xi_t^{(l)}) \equiv \tilde{y}_{t+h}^{(n)} - \hat{\mathcal{Q}}_{\tilde{y}_{t+h}^{(n)}}(q|\xi_t^{(l)})$ be the corresponding forecast error. The predictive accuracy is measured using the following loss function

$$\mathcal{L}_q^{(n)} \left(\left\{ \xi_t^{(l)} \right\}_{t=40}^{T-h} \right) = \frac{1}{T - (h + 40)} \sum_{t=40}^{T-h} \left(q - \{1\} \left\{ \hat{e}_{t+h}^{(n)}(q|\xi_t^{(l)}) < 0 \right\} \right) \hat{e}_{t+h}^{(n)}(q|\xi_t^{(l)})$$

for all n , l , h , and q .

Tables 5 and 6 display statistics about the out-of-sample predictive accuracy of the AQPC systemic risk indicator versus the unconditional quantile estimate, and versus the PC-based measure, respectively. In both tables, bold values denote loss functions that are statistically lower at the 10% significance level compared to those of the competing estimator. Several findings emerge from this exercise. In particular, when looking at the center of the distribution of macroeconomic shocks, the proposed measure generates out-of-sample forecasts that significantly outperform those of PC at various horizons (e.g., the loss function improves almost 50% on average), although they are in general not better than UQ-based predictions. A possible explanation for this last result could be the fact that my indicator contains

measurement errors that veil its informational content relative to the unconditional quantile estimate.

Note, furthermore, that the success of the AQPC-based measure is found at the tails of the distribution of shocks. More precisely, at short forecast horizons, this index offers better out-of-sample predictions for extremely adverse or extremely positive shocks compared to the other measures under evaluation. In fact, under the AQPC-based index, the quantile loss function is reduced, on average, by around 17% when forecasting periods of economic expansions and by roughly 5.5% when trying to anticipate future periods of economic distress. Moreover, this good forecasting performance can also be understood from a historical perspective by looking at Figure 5, where the estimate of systemic risk predicts, a quarter in advance, sharp declines in industrial production during four episodes of major market distress: the double-dip recession of the 1980s, Black Monday in 1987, the burst of the dot-com bubble at the beginning of the 1990s, and the Great Recession of 2007-2009. Out-of-sample forecasts on economic activity during these episodes, on its part, arrive somewhat late except in the case of the recession that followed the collapse of the dot-com bubble.

At longer horizons or less extreme shocks, however, the forecasting power of the AQPC-based systemic risk tends to dilute, particularly in the upper tail of the distribution of the macroeconomic shocks, where improvements relative to PC or UQ predictions are now lower and around 6% on average. Figure 5 also tells us that signals based on out-of-sample forecasts, although predicting sharp decreases of industrial production and economic activity, arrive 2 to 3 quarters belated.

Thus, to sum up, I find that the proposed systemic risk measure contains useful information regarding the future state of the economy, and it can be exploited as an early-warning indicator. In this sense, the AQPC-based indicator is capable of predicting sharp contractions and expansions of economic activity and industrial production but only up to one quarter ahead.

4 Conclusions

In this paper, I propose a novel measure to quantify systemic risk from a set of asset prices. In particular, I show that in the context of the external-habits-formation model of [Campbell and Cochrane \(1999\)](#), and under the assumption that stock returns are heteroskedastic, equilibrium excess returns have a factor structure. The restrictions implied by the model entail the existence of two factors that are simply a monotonic transformation of the log surplus consumption ratio, a state variable that characterizes the systemic risk in the structural model. However, unlike the traditional asset pricing literature, one of the unobserved factors affect the variance of excess returns, and both factors are related in a nonlinear fashion. Because of the preceding restrictions, classical tools for extracting unobserved factors from asset returns such as Principal Components (PC) are not suitable in this case because one of the factors cannot be identified at the center of the conditional distribution of the idiosyncratic component of excess returns. Instead, I use the Quantile Principal Components procedure proposed by [Sagner \(2019\)](#) for such purpose, where the rotation considered is governed by the restrictions mentioned before, plus sign restrictions on the loadings that are determined by the magnitude of a subset of structural parameters.

Then, I solve and simulate the model using different sets of parameterizations calibrated from US macroeconomic and financial data to compute the AQPC-based measure of systemic risk from artificial data. My results show that, when computing the measure at a quantile different than the median, the precision of the estimator is high, suggesting that, on average, estimated systemic risk via the AQPC procedure can be regarded as the true one. The good performance of the AQPC estimator tends to decrease as the log surplus consumption ratio becomes more persistent or when the risk-free rate is counter-cyclical, because, in both cases, the risk premium is less sensitive to the underlying state variable and, therefore, more responsive to idiosyncratic shocks. When systemic risk is computed using the AQPC estimator at the median, or the PC methodology, misspecification bias can be very large.

Finally, I compute the AQPC-based indicator using actual US post-war data. The proposed systemic risk estimator depicts significant hikes that coincide not only with several well-individualized US recession periods but also with financial episodes that did not trigger a recession in subsequent quarters. As expected from the structural model, the estimator of systemic risk can forecast sharp macroeconomic contractions up to one quarter ahead more accurately than PC-based indices. This feature highlights the usefulness of the proposed measure as an additional early-warning indicator that policymakers can incorporate into their monitoring toolkit.

A Appendix

A.1 Optimality Conditions of the Model

The representative investor in this economy takes care of consumption and saving by choosing the sequences $\{C_t, B_{t+1}, A_{i,t+1}\}_{t=0}^{\infty}$, to solve the following optimization problem

$$\begin{aligned} \max_{\{C_t, B_{t+1}, A_{i,t+1}\}_{t=0}^{\infty}} \quad & E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j} - X_{t+j})^{1-\gamma} - 1}{1-\gamma} \right] \\ \text{s.t.} \quad & C_{t+1} + B_{t+1} + \sum_{i=1}^N A_{i,t+1} = (1 + r_t^f) B_t + \sum_{i=1}^N (1 + r_{i,t}) A_{i,t} \end{aligned} \quad (\text{A.1})$$

where B_{t+1} denotes the quantity of a one-period, real risk-free discount bond purchased in period t and maturing in period $t + 1$; and, similarly, $A_{i,t+1}$ represents the quantity of the i -th risky asset held by the representative investor that was purchased in period t and that pays off a capital gain plus a risk premium in period $t + 1$.

The first-order conditions are the following

$$C_t : (C_t - X_t)^{-\gamma} - \delta_t = 0 \quad (\text{A.2})$$

$$B_{t+1} : -\delta_t + E_t \left[\beta \delta_{t+1} (1 + r_{t+1}^f) \right] = 0 \quad (\text{A.3})$$

$$A_{i,t+1} : -\delta_t + E_t \left[\beta \delta_{t+1} (1 + r_{i,t+1}) \right] = 0 \quad (\text{A.4})$$

where δ_t is the Lagrange multiplier of the optimization problem (A.1). Let M_{t+1} be the stochastic discount factor defined in Section 2.1. After combining equations (A.2) and (A.3), together with the definition of the surplus consumption ratio S_t , we get the following Euler equation for the real risk-free return

$$(1 + r_{t+1}^f) E_t [M_{t+1}] = 1 \quad (\text{A.5})$$

Using the first-order Taylor approximation $\log(1 + r_{t+1}^f) \approx r_{t+1}^f$, and the property of log-Normal random variables $\log(E_t[M_{t+1}]) = E_t[m_{t+1}] + V_t[m_{t+1}]/2$, the log-linearized version of (A.5) is given by

$$\begin{aligned} r_{t+1}^f &= -E_t [m_{t+1}] - \frac{V_t [m_{t+1}]}{2} \\ &= \gamma g - \log \beta - \frac{\gamma(1 - \phi) - b}{2} - b(s_t - \bar{s}) \end{aligned}$$

where in the last equality, I used the log stochastic discount factor given in equation (6). This result corresponds to the risk-free rate that appears in expression (7).

Similarly, the definitions of S_t and M_t , together with equations (A.2) and (A.4), result in the following Euler equations related to risky asset returns

$$E_t [M_{t+1} (1 + r_{i,t+1})] = 1 \quad (\text{A.6})$$

for all i , or in its log-linear version

$$\begin{aligned}
E_t[r_{i,t+1}] &= -E_t[m_{t+1}] - \frac{V_t[m_{t+1}]}{2} - \frac{V_t[r_{i,t+1}]}{2} - COV_t[m_{t+1}, r_{i,t+1}] \\
&= \gamma g - \log \beta - \gamma(1 - \phi)(s_t - \bar{s}) \\
&\quad + \left(\gamma \sigma \rho_i \zeta_i - \frac{\zeta_i^2 + \gamma^2 \sigma^2}{2} \right) (1 + \lambda(s_t))^2 \\
E_t[r_{i,t+1}] - r_{t+1}^f &= \zeta_i \left(\frac{2\gamma \sigma \rho_i - \zeta_i}{2} \right) (1 + \lambda(s_t))^2
\end{aligned}$$

where in the second equality, I used the expression for m_{t+1} and $r_{i,t+1}$ given by equations (6) and (8), respectively, and in the last equality, I replaced the expression of the risk-free rate derived previously. This result corresponds to the expected excess return shown in (9).

To finalize, the log-linearized versions of equations (A.5) and (A.6), together with equations (3) to (5) and (8), conform the optimality conditions of the model.

A.2 Proof of Proposition 1

We start by providing two Lemmas that will be useful for the proof. The following Lemma characterizes the unconditional variance of the log surplus consumption ratio s_t .

Lemma 1. *Let s_t be the log surplus consumption ratio described by equations (3) and (4), and let $\lambda(s_t)$ be the sensitivity function given by equation (5). Then,*

$$V[s_{t+1}] = \frac{(\bar{S}^{-1} - 1)^2 \sigma^2}{1 - \phi^2 - \bar{S}^{-1} \sigma^2} \quad (\text{A.7})$$

Proof. Let $\tilde{s}_t \equiv s_t - \bar{s}$ be the log surplus consumption ratio expressed as deviations from its steady-state value \bar{s} . From equations (3) and (4) we have that $\tilde{s}_{t+1} = \phi \tilde{s}_t + \lambda(\tilde{s}_t) v_{t+1}$, where $\lambda(\tilde{s}_t) = \bar{S}^{-1} \sqrt{1 - 2\tilde{s}_t} - 1$. Thus, $E[\tilde{s}_{t+1}] = 0$ and

$$V[\tilde{s}_{t+1}] = \left(\frac{\sigma^2}{1 - \phi^2} \right) E[\lambda^2(\tilde{s}_t)]$$

where $E[\lambda^2(\tilde{s}_t)] = \bar{S}^{-2} - 2\bar{S}^{-1} E[\sqrt{1 - 2\tilde{s}_t}] + 1$. To get an expression for the second term, we take a second-order Taylor expansion of $F(\tilde{s}_t) = \sqrt{1 - 2\tilde{s}_t}$ around $\tilde{s}_t = E[\tilde{s}_t] = 0$ as follows

$$\begin{aligned}
F(\tilde{s}_t) &= F(0) + F'(0)\tilde{s}_t + \frac{F''(0)}{2}\tilde{s}_t^2 + o_p(\tilde{s}_t^2) \\
&= 1 - \tilde{s}_t - \frac{\tilde{s}_t^2}{2} + o_p(\tilde{s}_t^2)
\end{aligned}$$

which implies that $E[F(\tilde{s}_t)] = 1 - V[\tilde{s}_t]/2$, and therefore $E[\lambda^2(\tilde{s}_t)] = (\bar{S}^{-1} - 1)^2 + \bar{S}^{-1} V[\tilde{s}_t]$. Thus, the unconditional variance of \tilde{s}_{t+1} is finally

$$V[\tilde{s}_{t+1}] = \frac{(\bar{S}^{-1} - 1)^2 \sigma^2}{1 - \phi^2 - \bar{S}^{-1} \sigma^2}$$

This last result completes the proof. ■

The second Lemma characterizes the unconditional correlation between each asset return and consumption growth.

Lemma 2. *Let $r_{i,t+1}$ be the real return on the i -th risky asset described by equation (8), and Δc_{t+1} be consumption growth given by equation (4). Then, for all i*

$$CORR[r_{i,t+1}, \Delta c_{t+1}] = \frac{1}{2} \left[\frac{2(1 - \phi^2) - (\bar{S}^{-2} + 1)\sigma^2}{1 - \phi^2 - \bar{S}^{-1}\sigma^2} \right] \rho_i \quad (\text{A.8})$$

Proof. First, I characterize the unconditional moments of $r_{i,t+1}$ and Δc_{t+1} . In the case of the former, from equation (8) we have that $E[r_{i,t+1}] = E[E_t[r_{i,t+1}]]$ and $V[r_{i,t+1}] = (\zeta_i/\bar{S})^2$, for all i . In the latter case, from equation (4) we have that $E[\Delta c_{t+1}] = g$ and $V[\Delta c_{t+1}] = \sigma^2$. Therefore, the unconditional covariance between these two variables is as follows

$$\begin{aligned} COV[r_{i,t+1}, \Delta c_{t+1}] &= E[r_{i,t+1}\Delta c_{t+1}] - E[r_{i,t+1}]E[\Delta c_{t+1}] \\ &= E[\sqrt{1 - 2\tilde{s}_t}\bar{S}^{-1}\rho_i\zeta_i\sigma] \end{aligned}$$

where $E[\sqrt{1 - 2\tilde{s}_t}] = 1 - V[\tilde{s}_t]/2$ and $V[\tilde{s}_t]$ is given by Lemma 1. Hence

$$COV[r_{i,t+1}, \Delta c_{t+1}] = \frac{\bar{S}^{-1}}{2} \left[\frac{2(1 - \phi^2) - (\bar{S}^{-2} + 1)\sigma^2}{1 - \phi^2 - \bar{S}^{-1}\sigma^2} \right] \rho_i\zeta_i\sigma$$

These results imply that the correlation between $r_{i,t+1}$ and Δc_{t+1} is given by

$$CORR[r_{i,t+1}, \Delta c_{t+1}] = \frac{1}{2} \left[\frac{2(1 - \phi^2) - (\bar{S}^{-2} + 1)\sigma^2}{1 - \phi^2 - \bar{S}^{-1}\sigma^2} \right] \rho_i$$

for all i . This last result completes the proof. ■

Proof of Proposition 1. As mentioned in the text, the sign of the first quantile factor loading $\alpha_i^{(1)}(\tau)$ depends on the sign of the difference $d_i \equiv 2\gamma\sigma\rho_i - \zeta_i$. Hence, from Lemma 2 we have that

$$\rho_i = 2 \cdot CORR[r_{i,t+1}, \Delta c_{t+1}] \left[\frac{1 - \phi^2 - \bar{S}^{-1}\sigma^2}{2(1 - \phi^2) - (\bar{S}^{-2} + 1)\sigma^2} \right] \quad (\text{A.9})$$

and

$$\zeta_i = \bar{S}\sqrt{V[r_{i,t+1}]} \quad (\text{A.10})$$

for all i , where $\bar{S} = \sigma\sqrt{\gamma/(1 - \phi - b/\gamma)}$ as given in equation (5). So, we consider three cases depending on the sign of the parameter b .

First, if $b > 0$, then $-1 < \phi < 1 - b/\gamma$ so that the log surplus consumption ratio s_t satisfies the stationary condition. Thus, as ϕ approaches to the upper bound of its support, both \bar{S}^{-1} and \bar{S}^{-2} becomes very close to 0, and \bar{S} is a very large positive number. Moreover, because $b \ll \gamma$ in general, $1 - \phi^2$ becomes close to 0 as well. Therefore, as $\phi \rightarrow 1 - b/\gamma$, then $\rho_i \rightarrow 0$ and $\zeta_i \rightarrow \infty$, which implies that $d_i < 0$, for all i .

Second, if $b = 0$, then $|\phi| < 1$ and as this parameter approaches 1, both \bar{S}^{-1} and \bar{S}^{-2} tend to 0, and $\bar{S} \rightarrow \infty$, consequently. Hence, as $\phi \rightarrow 1$, then $\rho_i \rightarrow 0$ and $\zeta_i \rightarrow \infty$, implying that $d_i < 0$, for all i .

Finally, if $b < 0$, then $|\phi| < 1$ and an argument similar to the previous one applies.

The proof is complete. ■

A.3 Data Description

In this section, I provide the sources of all data used in the paper and describe how the variables of the model were created. In the first part, I describe the data and variables used in the calibration of the external habit-formation model described in Section 2, whereas in the second part, I describe the data related to the estimation of the systemic risk measure for the US of Section 3.

A.3.1 Calibration

Consumption data is from the Bureau of Economic Analysis. The series considered are the real per capita consumption of non-durables (label A796RX0Q048SBEA), and the real per capita consumption of services (label A797RX0Q048SBEA). Both time series are seasonally adjusted and expressed in chained 2012 US dollars. The variable Δc_t in the text is the quarterly log growth of the sum of these two series.

The 3-month Treasury Bill secondary market rate is from the Board of Governors of the Federal Reserve System (label DTB3). The original data is expressed in percentage points on an annual basis and is available in daily frequency. Hence, the quarterly series are averages of the observations within each quarter. I also transform the units so that the data reads as quarterly percentage points.

The consumer price index (CPI) is from the Center for Research on Security Prices (label CPIIND). Quarterly series are averages of monthly observations. Inflation π_t is thus the quarterly growth of the CPI.

The real risk-free rate r_t^f was computed as the difference between the 3-month Treasury Bill rate y_t^{TB} and expected inflation π_t^e . The latter variable was constructed using the following VAR(1) model

$$\begin{bmatrix} y_{t+1}^{TB} \\ \pi_{t+1} \end{bmatrix} = \mu + A \begin{bmatrix} y_t^{TB} \\ \pi_t \end{bmatrix} + W_{t+1} \quad (\text{A.11})$$

where W_{t+1} is a 2-dimensional vector of innovations. Therefore, $\pi_t^e = E_t[\pi_{t+1}] = \hat{\pi}_{t+1}$ is the one-period-ahead quarterly inflation rate predicted by model (A.11).

The [Fama and French \(1993\)](#) portfolio returns based on size and book-to-market ratios are available at Kenneth French's website²². These portfolios include all NYSE, AMEX, and NASDAQ stocks for which data on market equity and (positive) book equity is available in June and December of each year. Two size categories are defined based on the median of the distribution of market equity: small (below the median) and big (above the median). Similarly, three book-to-market categories are defined based on the 30th and 70th percentile

²²<http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html>.

of the corresponding distribution: growth (below the 30th percentile), neutral (between 30th and 70th percentile), and value (above 70th percentile). Because the data consists of daily returns, all observations within a quarter were summed to express them into this frequency. Real portfolio returns are constructed by subtracting π_t^e to each return series.

A.3.2 Measure of Systemic Risk for the US

Stock market data is from the Annual Update daily database of the Center for Research in Security Prices (CRSP). Stock price (label `prc`) is the last non-missing closing price or the bid/ask average in US dollars of a security for a given day. The variable was transformed to quarterly frequency by considering the average price within each quarter. So, excess returns $\tilde{r}_{i,t}$ are constructed as

$$\tilde{r}_{i,t} = 100 \cdot \left(\frac{\text{prc}_{i,t}}{\text{prc}_{i,t-1}} - 1 \right) - y_t^{TB} \quad (\text{A.12})$$

where $\text{prc}_{i,t}$ is the price of the i -th stock in quarter t , and y_t^{TB} is the 3-month Treasury Bill rate.

Data on the S&P 500 index was obtained from Bloomberg. Both the last available quote price (label `PX_LAST`) and net dividend plus tax credit (label `LAST_DPS_GROSS`) of the index were expressed in quarterly frequency by considering the average value of these two variables within a quarter. The price-dividend ratio of the index $(P/D)_t$ was computed as follows

$$(P/D)_t = \frac{\text{PX_LAST}_t}{\text{LAST_DPS_GROSS}_t^*} \quad (\text{A.13})$$

where $\text{LAST_DPS_GROSS}_t^* = \sum_{j=0}^3 \text{LAST_DPS_GROSS}_{t-j}$ are the dividends per share paid during the last year.

The Industrial Production Index (label `INDPRO`), which corresponds to an indicator that measures real output for all facilities located in the US manufacturing, mining, and electric and gas utilities, was obtained from the Board of Governors of the Federal Reserve System and is available in monthly frequency from 1919 onwards. I consider the annual growth rate of the variable. Quarterly series corresponds to the average within a quarter.

The National Activity Index (label `CFNAI`) and its subcategories personal consumption and housing (label `CANDH`); production and income (label `PANDI`), sales, orders and inventories (label `SOANDI`); and employment, unemployment and hours (label `EUANDH`), is from the Federal Reserve Bank of Chicago. The index, which is achievable in monthly frequency since March 1967, has a zero value when the US economy is growing at its historical trend rate. Thus, negative (positive) values indicate below-average (above-average) expansions of the economy. I take monthly averages to transform the data into quarterly frequency.

Data on the National Financial Conditions Index (label `NFCI`) was obtained from the Federal Reserve Bank of Chicago database in weekly frequency starting the first week of 1971. This indicator provides a comprehensive outlook of US financial conditions in money, debt, and equity markets, as well as in traditional and shadow banking systems. Positive (negative) values indicate tighter (looser) financial conditions relative to its historical mean. Quarterly observations are averages of weekly data within each quarter.

The NBER-based US recession index, available from the Federal Reserve Bank of Saint Louis database at a quarterly frequency (label **USREC**), corresponds to a dummy variable that represents periods of expansions (0) and recessions (1), where the latter begins the first day of the period following a peak and ends on the last day of the period of the trough.

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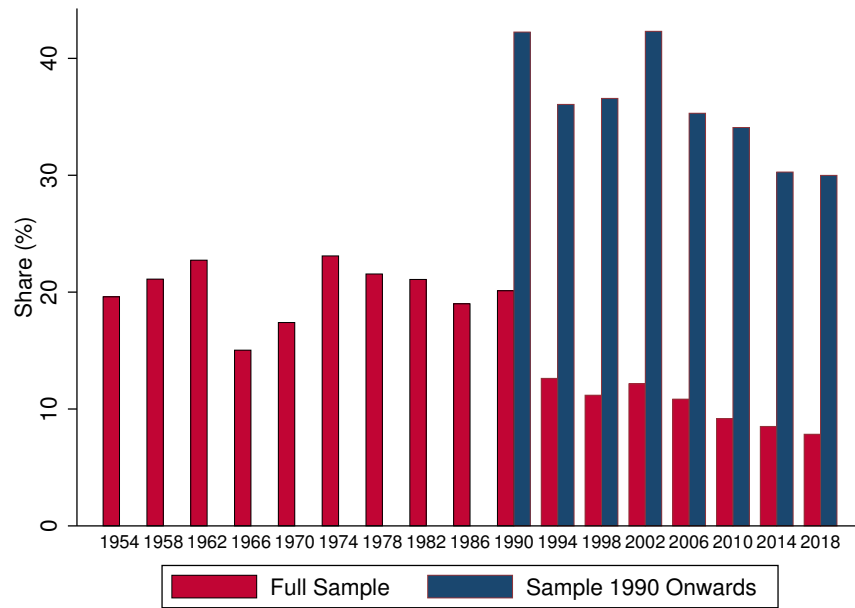
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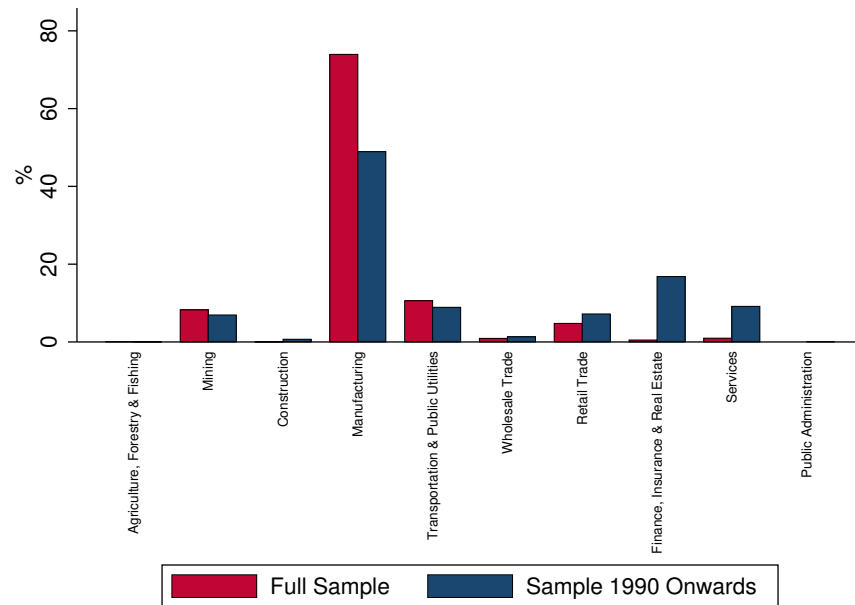
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Figure 1: Sample Characterization

(a) Share of Total Transactions

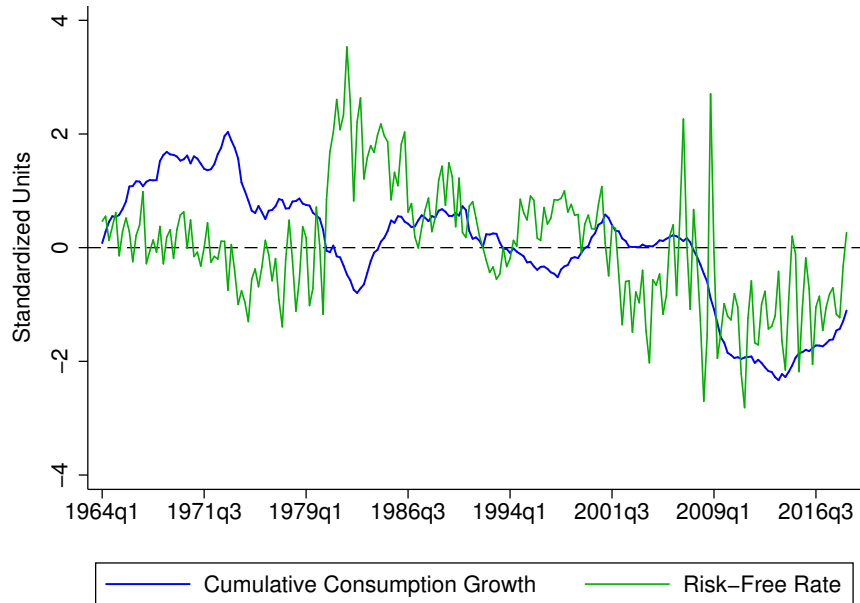


(b) Industries Distribution



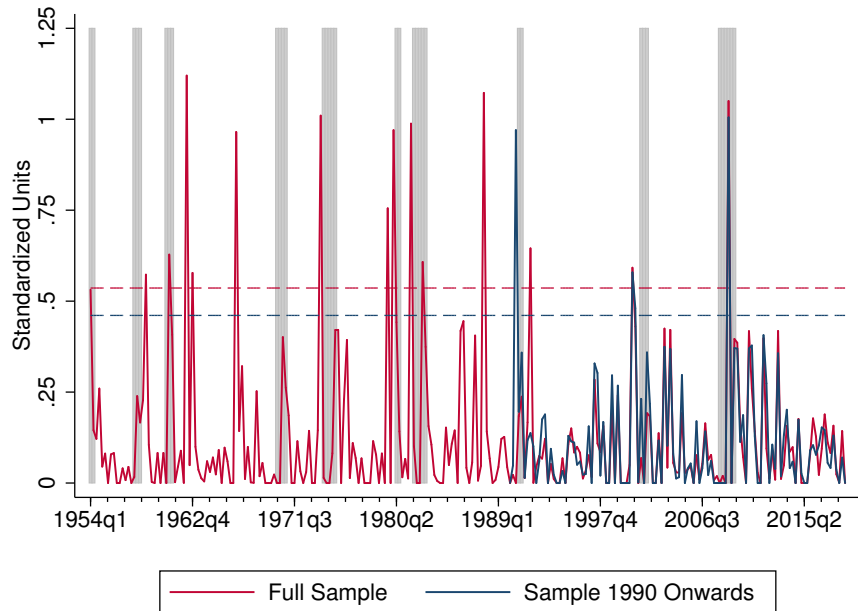
Shares are calculated over the total value of transactions, in US dollars, reported in the CRSP database. Industries distribution is calculated over the total value of transactions, in US dollars, of the corresponding sample.

Figure 2: Risk-Free Rate Cyclicity



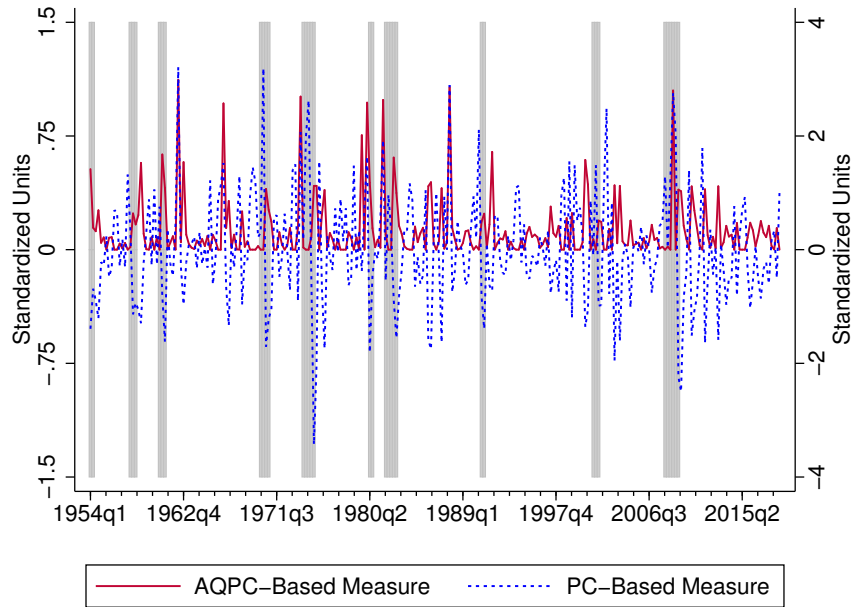
Cumulative consumption growth corresponds to a weighted sum of this variable over the past 10 years and was computed as $\sum_{j=1}^{40} \phi_{SP}^j \Delta c_{t-j}$, where $\phi_{SP} = 0.976$ is the estimated persistence of the price-dividend ratio of the S&P 500 index over the period from 1954q1 to 2018q4. The realized real risk-free rate corresponds to the difference between the yield on the 3-months Treasury Bill and the quarterly expected inflation rate, where the latter was computed using a VAR(1) model. Both variables were standardized.

Figure 3: Measure of Systemic Risk for the US



Horizontal dashed line indicates 1.96 standard deviations above the corresponding sample mean. Shaded areas represent recession periods, as defined by the NBER. Data are quarterly and spans the period from 1954q1 to 2018q4 (full sample), and from 1990q1 to 2018q4 (sample 1990 onwards).

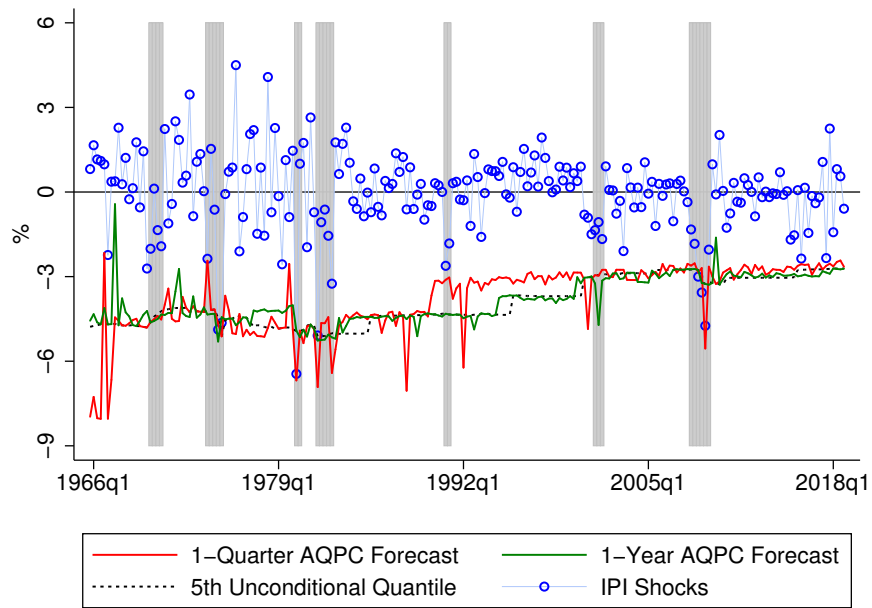
Figure 4: Comparison of Systemic Risk Measures for the US



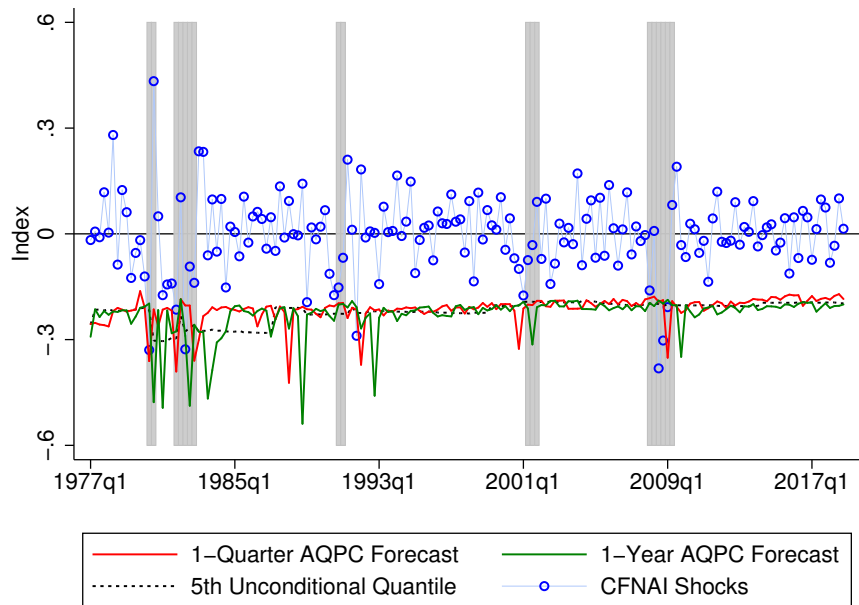
The AQPC-based measure was estimated using an adapted version of the Quantile Principal Components methodology proposed by [Sagner \(2019\)](#), whereas the PC-based measure was estimated via PC under the default rotation, i.e., $T^{-1} \sum_{t=1}^T \hat{\theta}_t \hat{\theta}_t' = 1$ and $\sum_{i=1}^N \hat{\alpha}_i \hat{\alpha}_i' > 0$. Data are quarterly and spans the period from 1954q1 to 2018q4. Shaded areas represent recession periods, as defined by the NBER.

Figure 5: Adverse Shocks Forecast

(a) Industrial Production



(b) National Economic Activity



Forecasts for the 5th quantile of the Industrial Production Index and the Chicago Fed National Activity Index (Figures (a) and (b), respectively). The timing was aligned so that forecasts coincide with realized shocks. Shaded areas represent recessions periods, as defined by the NBER.

Table 1: Calibrated Parameters

Parameter	Value								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Consumption									
$g(\%)$	0.485	0.485	0.485	0.485	0.485	0.485	0.485	0.485	0.485
$\sigma(\%)$	0.444	0.444	0.444	0.444	0.444	0.444	0.444	0.444	0.444
Preferences									
γ	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
β	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
Habits									
ϕ	0.500	0.500	0.500	0.900	0.900	0.900	0.990	0.990	0.990
b	-0.010	0.000	0.010	-0.010	0.000	0.010	-0.010	0.000	0.010
\bar{s}	-4.730	-4.725	-4.720	-3.944	-3.920	-3.894	-2.971	-2.769	-2.422
Stock Returns									
$\zeta_1(\%)$	0.112	0.112	0.113	0.245	0.251	0.257	0.648	0.793	1.122
$\zeta_2(\%)$	0.089	0.090	0.090	0.196	0.201	0.206	0.519	0.635	0.898
$\zeta_3(\%)$	0.093	0.093	0.094	0.204	0.209	0.214	0.539	0.661	0.934
$\zeta_4(\%)$	0.075	0.075	0.075	0.164	0.168	0.172	0.433	0.530	0.750
$\zeta_5(\%)$	0.065	0.065	0.066	0.143	0.146	0.150	0.377	0.462	0.653
$\zeta_6(\%)$	0.078	0.078	0.079	0.171	0.175	0.180	0.452	0.554	0.783
ρ_1	0.159	0.159	0.159	0.154	0.152	0.151	0.161	0.150	0.140
ρ_2	0.173	0.172	0.172	0.166	0.165	0.164	0.174	0.162	0.152
ρ_3	0.175	0.175	0.174	0.169	0.167	0.166	0.177	0.164	0.154
ρ_4	0.179	0.179	0.178	0.173	0.171	0.170	0.181	0.168	0.158
ρ_5	0.153	0.153	0.153	0.147	0.146	0.145	0.154	0.144	0.135
ρ_6	0.171	0.171	0.171	0.165	0.164	0.162	0.173	0.161	0.151

All values are expressed quarterly. Consumption, preferences, and stock return parameters were estimated using data starting 1954q1 and ending 2018q4.

Table 2: Correlation Between Stock Returns ω_{ij}

i	j				
	1	2	3	4	5
2	0.943				
3	0.875	0.968			
4	0.860	0.816	0.739		
5	0.814	0.878	0.851	0.865	
6	0.774	0.867	0.891	0.782	0.893

All values are expressed quarterly. Correlations were estimated using stock return data starting 1954q1 and ending 2018q4.

Table 3: Average Correlation Between Simulated and Estimated s_t

Estimator	Parameterization								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
AQPC ($\tau = 0.4$)	0.928	0.896	0.875	0.816	0.659	0.616	0.795	0.632	0.587
AQPC ($\tau = 0.5$)	0.492	0.453	0.440	0.171	0.099	0.079	0.135	0.101	0.064
AQPC ($\tau = 0.6$)	0.944	0.905	0.867	0.809	0.667	0.606	0.731	0.610	0.573
APC	0.536	0.503	0.473	0.193	0.103	0.089	0.141	0.134	0.071
PC	0.436	0.406	0.385	0.153	0.085	0.072	0.123	0.093	0.056

APC corresponds to the Principal Components estimator with the same identifying restrictions as the AQPC estimator. PC corresponds to the Principal Components estimator under the default rotation, i.e., $T^{-1} \sum_{t=1}^T \hat{\theta}_t \hat{\theta}'_t = I_2$ and $\sum_{i=1}^N \hat{\alpha}_i \hat{\alpha}'_i$ is a diagonal matrix, where I_2 is the identity matrix of size 2. Averages were computed from 1,000 simulations.

Table 4: Systemic Risk Measures Summary Statistics

	AQPC-Based		PC-Based
	Full Sample	1990 Onwards	
Skewness	2.708	2.618	0.353
Kurtosis	10.96	12.17	3.958
Corr. Industrial Production	-0.268	-0.222	0.040
Corr. National Activity	-0.334	-0.323	-0.159
Corr. Financial Conditions	0.386	0.392	0.195

The AQPC-based measure was estimated using an adapted version of the Quantile Principal Components methodology proposed by [Sagner \(2019\)](#). The PC-based measure was estimated via PC under the default rotation, i.e., $T^{-1} \sum_{t=1}^T \hat{\theta}_t \hat{\theta}_t' = 1$ and $\sum_{i=1}^N \hat{\alpha}_i \hat{\alpha}_i' > 0$. Correlation with the Industrial Production Index (IPI) annual growth spans the period from 1955q1 to 2018q4. Correlation with the Chicago Fed National Activity Index (CFNAI) spans the period from 1967q2 to 2018q4. Correlation with the Chicago Fed National Financial Conditions Index spans the period from 1971q1 to 2018q4.

Table 5: Forecast Performance - Adapted Quantile Principal Components v/s Unconditional Quantiles

Variable	Adverse Shocks				Median Shocks				Benign Shocks			
	$q = 0.05$		$q = 0.20$		$q = 0.50$		$q = 0.80$		$q = 0.95$			
	AQPC	UQ	AQPC	UQ	AQPC	UQ	AQPC	UQ	AQPC	UQ	AQPC	UQ
IPI	3.962	4.081	1.416	1.379	0.356	0.256	1.616	1.610	3.386	3.531		
CFNAI	0.969	0.986	0.450	0.423	0.042	0.024	0.473	0.454	1.001	1.063		
PCH	0.095	0.097	0.044	0.042	0.006	0.005	0.046	0.046	0.089	0.088		
PI	0.423	0.425	0.194	0.192	0.028	0.015	0.216	0.214	0.442	0.454		
SOI	0.216	0.223	0.097	0.096	0.009	0.003	0.095	0.093	0.220	0.224		
EUH	0.387	0.374	0.195	0.171	0.031	0.024	0.181	0.164	0.378	0.397		
IPI	4.162	4.081	1.443	1.379	0.272	0.256	1.611	1.610	3.398	3.531		
CFNAI	0.903	0.986	0.432	0.423	0.047	0.024	0.500	0.454	0.928	1.063		
PCH	0.097	0.097	0.043	0.042	0.006	0.005	0.046	0.046	0.088	0.088		
PI	0.427	0.425	0.190	0.192	0.024	0.015	0.229	0.214	0.434	0.454		
SOI	0.217	0.223	0.096	0.096	0.010	0.003	0.099	0.093	0.215	0.224		
EUH	0.367	0.374	0.175	0.171	0.030	0.024	0.174	0.164	0.391	0.397		
IPI	3.982	4.081	1.462	1.379	0.276	0.256	1.637	1.610	3.397	3.531		
CFNAI	0.983	0.986	0.439	0.423	0.054	0.024	0.462	0.454	0.985	1.063		
PCH	0.098	0.097	0.042	0.042	0.006	0.005	0.047	0.046	0.084	0.088		
PI	0.452	0.425	0.196	0.192	0.025	0.015	0.213	0.214	0.435	0.454		
SOI	0.223	0.223	0.102	0.096	0.008	0.003	0.096	0.093	0.219	0.224		
EUH	0.391	0.374	0.171	0.171	0.031	0.024	0.171	0.164	0.383	0.397		

This table reports the loss related to the forecasts of the Adapted Quantile Principal Components (AQPC) and the unconditional quantile (UQ) estimators. Bold values denote losses that are significantly lower at the 10% level compared to those of the competing estimator. Out-of-sample period starts in 1965q1 for the Industrial Production Index (IPI), and 1977q1 for the Chicago Fed National Activity Index (CFNAI) and its subcomponents personal consumption and housing (PCH), production and income (PI), sales, orders and inventories (SOI), and employment, unemployment and hours (EUH).

Table 6: Forecast Performance - Adapted Quantile Principal Components v/s Principal Components

Variable	Adverse Shocks			Median Shocks			Benign Shocks					
	$q = 0.05$			$q = 0.50$			$q = 0.80$			$q = 0.95$		
	AQPC	PC	AQPC	AQPC	PC	PC	AQPC	PC	PC	AQPC	PC	PC
IPI	3.962	4.074	1.416	0.356	1.477	1.711	1.616	1.711	3.386	3.526		
CFNAI	0.969	0.998	0.450	0.042	0.419	0.157	0.473	0.493	1.001	0.943		
PCH	0.095	0.085	0.044	0.006	0.044	0.010	0.046	0.047	0.089	0.088		
PI	0.423	0.464	0.194	0.028	0.166	0.068	0.216	0.250	0.442	0.422		
SOI	0.216	0.211	0.097	0.009	0.097	0.024	0.095	0.107	0.220	0.211		
EUH	0.387	0.397	0.195	0.031	0.150	0.064	0.181	0.178	0.378	0.363		
				$h = 1$								
IPI	4.162	4.134	1.443	0.272	1.333	0.293	1.611	1.680	3.398	3.404		
CFNAI	0.903	1.124	0.432	0.047	0.408	0.065	0.500	0.467	0.928	0.995		
PCH	0.097	0.097	0.043	0.006	0.040	0.008	0.046	0.046	0.088	0.086		
PI	0.427	0.506	0.190	0.024	0.188	0.028	0.229	0.225	0.434	0.445		
SOI	0.217	0.210	0.096	0.010	0.097	0.015	0.099	0.094	0.215	0.230		
EUH	0.367	0.414	0.175	0.030	0.165	0.038	0.174	0.161	0.391	0.391		
				$h = 4$								
IPI	3.982	4.186	1.462	0.276	1.336	0.445	1.637	1.701	3.397	3.715		
CFNAI	0.983	1.152	0.439	0.054	0.433	0.029	0.462	0.465	0.985	1.045		
PCH	0.098	0.099	0.042	0.006	0.043	0.005	0.047	0.046	0.084	0.088		
PI	0.452	0.522	0.196	0.025	0.185	0.018	0.213	0.213	0.435	0.447		
SOI	0.223	0.227	0.102	0.008	0.094	0.007	0.096	0.099	0.219	0.220		
EUH	0.391	0.392	0.171	0.031	0.169	0.028	0.171	0.164	0.383	0.404		

This table reports the loss related to the forecasts of the Adapted Quantile Principal Components (AQPC) and the Principal Components (PC) estimators. Bold values denote losses that are significantly lower at the 10% level compared to those of the competing estimator. Out-of-sample period starts in 1965q1 for the Industrial Production Index (IPI), and 1977q1 for the Chicago Fed National Activity Index (CFNAI) and its subcomponents personal consumption and housing (PCH), production and income (PI), sales, orders and inventories (SOI), and employment, unemployment and hours (EUH).

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