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Price Rigidity and the Granular Origins of Aggregate Fluctuations*

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Abstract

We study the potency of sectoral productivity shocks to drive aggregate fluctuations in the presence of three empirically relevant heterogeneities across sectors: sector size, intermediate input consumption, and pricing frictions in a multi-sector New Keynesian model. We derive conditions under which sectoral shocks matter for aggregate volatility in a simplified model and find the distribution of sector size or input-output linkages are neither necessary nor sufficient to generate aggregate fluctuations. Quantitatively, we calibrate our full model to 341 sectors using U.S. data and find (1) fully heterogeneous price rigidity across sectors doubles the aggregate volatility from sectoral shocks relative to a calibration with homogeneous price rigidity; 2) realistically calibrated sectoral productivity shocks are key to generating sizable aggregate fluctuations of both GDP and prices; 3) heterogeneity of price rigidity matters because it changes the effective distribution of sector size and network centrality. Our quantitative exercise generates large aggregate fluctuations under different empirically plausible monetary policy rules.

Resumen

Este artículo estudia el efecto sobre el PIB de shocks de productividad sectoriales cuando los sectores económicos son heterogéneos en tres dimensiones empíricamente relevantes: tamaño,

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relaciones insumo-producto, y rigidez de precios. Para esto extendemos el modelo neo-Keynesiano para acomodar múltiples sectores económicos. Usando una versión simplificada para dar intuición, se obtiene que, en contraste con la literatura, la distribución del tamaño de sectores o la forma de la matriz insumo-producto no son condiciones ni necesarias ni suficientes para que shocks sectoriales tengan un efecto agregado. Luego, usando el modelo completo calibrado para 341 sectores con datos para Estados Unidos, se muestra que (1) la heterogeneidad sectorial de rigideces de precios duplica el efecto agregado de shocks sectoriales, (2) shocks de productividad empíricamente plausibles son capaces de generar volatilidad del PIB similar a la observada empíricamente, y (3) la heterogeneidad de rigideces de precios cambia la identidad de los sectores clave detrás de fluctuaciones agregadas.

I Introduction

The role of price rigidity for macroeconomics has been at the center of heated discussions for decades. Are prices sticky, and if so, does it matter? The literature typically focuses on the response of real aggregates to nominal shocks, such as the response of GDP to monetary policy shocks. When it comes to the propagation of idiosyncratic, real shocks to the aggregate economy, the conventional wisdom is price rigidity mutes the propagation of productivity shocks. Resulting aggregate fluctuations tend to be unrealistically low.

We argue the literature has disregarded a distinct, important role price rigidity can play for fluctuations at the business cycle frequency: Heterogeneity in price rigidity can interact with other heterogeneous, real features of the economy such that it amplifies the propagation of idiosyncratic productivity shocks. In a realistic calibration using micro data, we show that this interaction can give rise to realistically sized GDP fluctuations that originate from purely sectoral shocks.

We document this role of heterogeneity in price rigidity through the lens of a multi-sector model featuring heterogeneous sector size, input-output linkages across sectors, and sectoral productivity shocks as the source of business cycles. When we calibrate the model to the U.S. for 341 sectors, we generate a monthly standard deviation for GDP of 0.41% nearly matching the standard deviation of 0.52% in the data. Two ingredients are key in generating this result: First, heterogeneous price rigidity across sectors increases the aggregate fluctuations due to sectoral shocks by a factor of two relative to a calibration with homogeneous price rigidity across sectors despite an identical average frequency of price adjustments across calibrations. Second, large idiosyncratic sectoral shocks are necessary to match aggregate fluctuations which echoes results in the literature on micro pricing (see, e.g., Midrigan (2011)). We discipline the size and distribution of sectoral shocks using micro price data from the Bureau of Labor Statistics (BLS).

Heterogeneous price rigidity can interact with sectoral technology shocks and change the effective distributions of sectors' characteristics that previous work associates with idiosyncratic origins from aggregate fluctuations and hence change the propagation mechanism of sectoral shocks.[1] For intuition, consider an economy with two equally-sized sectors, no input-output linkages and both sectors being hit with productivity shocks of equal size but different sign. If both sectors have flexible prices or equal degrees of price stickiness, the economy will experience

[1] Gabaix (2011) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) show in models with flexible prices that fat tails of sectoral size or network centrality may lead to idiosyncratic shocks generating aggregate fluctuations.

no change in the price level and real activity.[2] However, if the sector with the productivity increase had more flexible prices than the other sector, then the economy would experience a boom. If the sector with the negative productivity shock could pass through more of the shock into prices, then instead the cycle would flip sign.

We provide detailed intuition in a simplified model, assuming firms set their prices before observing shocks, according to a sector-specific Poisson process, and facing exogenous nominal demand. These assumptions allow us to analytically characterize the response of aggregate output and prices up to first order. In the model, the cross-sectional dispersion of multipliers mapping sectoral shocks into aggregate volatility, sectoral multipliers for short, determines how much aggregate volatility sectoral shocks can generate. The interaction of the heterogeneous nominal and real features of the economy determine this dispersion: with price rigidity, a physically large or central sector's multiplier becomes *effectively* larger if (i) its prices are more flexible than the average sector's prices; (ii) sectors that buy intermediate inputs from it have more flexible prices than the average sector's prices; and (iii) sectors that buy from intermediate input-demanding sectors, have more flexible prices than the average sector's prices.

This intuition of *effective* importance implies sectoral shocks may generate large aggregate volatility even when all sectors have equal physical GDP shares and input-output linkages – as long as high cross-sectional dispersion of price rigidity exists. We provide general conditions for price rigidity to amplify or reduce the potency of heterogeneous real features of the economy – embodied in our setup by heterogeneous GDP shares and I/O linkages – to generate aggregate fluctuations.

Hence, even in the absence of any such quantitative importance, it is important for policymakers to note that heterogeneity in price rigidity matters in effective, real terms. It may no longer be sufficient to identify the importance of sectors for aggregate fluctuations according to GDP shares or centrality in the production network. The interest of the recent granularity literature such as di Giovanni, Levchenko, and Mejean (2018) or Gaubert and Itskhoki (2018) to identify where aggregate fluctuations originate adds further emphasis to this observation.

The interaction between heterogeneous price rigidity and real features of the economy also directly speaks to the tension between the diversification argument of Lucas (1977) and the granular origin of aggregate fluctuations (see Gabaix (2011) and Acemoglu et al. (2012)). As the number of sectors, K, increases, aggregate fluctuations may die out at a rate of $\sqrt{K}$ – the diversification argument of Lucas (1977) – or the rate of decay can be smaller if fat tails exists

[2]Models with similar ingredients that study the propagation of *aggregate* monetary shocks such as Carvalho (2006) or Nakamura and Steinsson (2010) always generate aggregate fluctuations due to the focus on aggregate shocks.

in the relevant real features of the economy, such as sector size or network centrality.

We provide three novel results in this context. First, fat tails in sectoral price rigidity by itself cannot generate a rate of decay slower than the central limit theorem implies, because price rigidity is bounded: prices cannot get more flexible than fully flexible. Second, heterogeneity in price rigidity is also irrelevant for the rate of decay when it is independent of heterogeneous sectoral features such as size or network centrality – the joint distribution inherits the Pareto properties of the more fat-tailed distribution. Third, however, the distributions of friction-adjusted size or network centrality may be more or less fat-tailed than their frictionless counterparts when we allow for correlations between price rigidities and real features. In this case, the diversification argument of Lucas (1977) might even apply in an economy with price rigidities – whereas the granular hypothesis of Gabaix (2011) and Acemoglu et al. (2012) applies in the frictionless counterpart.

Ultimately, whether our theoretical insights are quantitatively important is an empirical question. We show they are. We calibrate the multi-sector New Keynesian model featuring Calvo pricing to the sectoral GDP shares and input-output linkages from the BEA Input-Output Tables in the US, sectoral frequencies of price changes which we calculate based on the micro data underlying the Producer Price Index (PPI) from the BLS and sectoral TFP shocks that we estimate using the same PPI micro data. We calibrate to the finest degree of disaggregation possible, which results in 341 sectors.

In a series of counterfactual exercises, we reach a number of novel results. First, our baseline calibration assigns a relevant role of heterogeneous price stickiness across sectors for aggregate fluctuations: Heterogeneity in the frequency of price adjustment together with heterogeneous sector size and I/O linkages generate a level of GDP volatility two times higher than in an otherwise identical economy where prices are equally sticky across sectors. Our baseline calibration also generates a level of GDP volatility in the model that is of similar size as in the data: The implied monthly standard deviation of GDP in our calibration is 0.41% compared to 0.52% in the data.

Second, disciplining sectoral shocks using micro data is a key ingredient in our calibration to generate large GDP volatility from sectoral shocks. Our micro data suggest idiosyncratic shocks are much larger than aggregate shocks. On average, we estimate a monthly standard deviation of sectoral productivity shocks of 9.08% which is similar to the estimates of Midrigan (2011) to explain highly volatile price processes at the goods level. In order to perfectly match the data, we would need a monthly standard deviation of 11.47%. Plausibly sized aggregate productivity shocks, by contrast, do not produce realistic aggregate fluctuations in the presence of price rigidity, either homogeneous or heterogeneous across sectors. This insight emphasizes the importance of microeconomic shocks for macroeconomic modeling.

Third, heterogeneity in price rigidity generates large changes in sectoral multipliers and changes the identity of the most important sectors for aggregate fluctuations. Consistent with our simple analytical model, we find the change in the ranking of most important sectors for aggregate volatility from sectoral shocks does not only respond to the degree of price rigidity of a sector relative to the price stickiness of other sectors, but also to the overall differences between the physical production network and the effective,"friction-adjusted" production network that takes into account the sectoral degree of price rigidity. For example, sector 211000 (Oil and Gas Extraction) ranks thirty-third in importance for aggregate volatility under homogeneous pricing frictions, but it is the seventh most important sector under heterogeneous pricing frictions. Underlying this effect is a change in the distributions of effective size and network centrality: some physically large sectors become effectively small, and some central sectors effectively non-central in the network and vice versa.

Finally, our results continue to hold under a variety of alternative, empirically plausible monetary policy rules: a nominal GDP target, and Taylor rules that feature strict inflation targeting, output growth targeting, as well as interest rate smoothing. The monthly standard deviations of aggregate output fluctuations range from 0.32% to 0.57% across these monetary policy rules, compared to 0.52% in the data. In the context of these rules, we also consider the monthly standard deviation of aggregate prices. We show it ranges from 0.27% to 1.71%. To study a more active, but possibly less empirically plausible, policy rule, we also consider strict price-level targeting. Under price-level targeting, aggregate fluctuations from sectoral shocks are larger than empirically plausible. This finding may even suggest the necessity of pricing or other frictions to get a plausible level of fluctuations.

A. Literature Review

Connecting heterogeneous real features of the economy and idiosyncratic shocks as drivers of aggregate fluctuations is one central ingredient of our framework. Long and Plosser (1983) pioneer this microeconomic origin of aggregate fluctuations, and Horvath (1998, 2000) push this literature forward. Dupor (1999) follows Lucas (1977) and argues microeconomic shocks matter only due to poor disaggregation. Gabaix (2011) and Acemoglu et al. (2012) provide a frictionless benchmark for the power of heterogeneous real features of the economy to propagate idiosyncratic shocks to the aggregate, and re-establish the importance of microeconomic shocks by respectively invoking the empirical firm-size distribution and the sectoral network structure of the US economy. Barrot and Sauvagnat (2016), Acemoglu et al. (2016), and Carvalho et al. (2016) provide empirical evidence for the aggregate relevance of idiosyncratic shocks, and Carvalho (2014) synthesizes this literature.

We introduce heterogeneity in price rigidity as a new focus to the above analyses, adding to the long-standing question about the role of price rigidity for aggregate fluctuations. In fact, the distortionary role of frictions, and price rigidity in particular, is at the core of the business-cycle literature that conceptualizes aggregate shocks as the driver of aggregate fluctuations, including the New Keynesian literature. This literature has too many contributions to name (see Gali (2015) for a recent textbook treatment).

However, to the best of our knowledge, our paper is the first to analytically and quantitatively study the role of heterogeneous pricing frictions when aggregate fluctuations originate from microeconomic productivity shocks, and interact with heterogeneous real features of the economy. The degree of disaggregation of our quantitative analysis is also a central novel feature our paper, in particular the ability to measure price rigidity and idiosyncratic productivity shocks directly in the data at a highly disaggregated level. In studying the quantitative importance of our proposed mechanism we are the first to point that realistic, large idiosyncratic productivity shocks can generate economically significant aggregate fluctuations – while an aggregate productivity shock does not.

On the theory side our paper is related to a few recent contributions that study the microeconomic origin of aggregate fluctuations and include frictions in their analyses but study different questions. Bigio and La'O (2017) study the aggregate effects of the tightening of financial frictions in a production network. Baqaee (2018) shows entry and exit of firms coupled with CES preferences may amplify the aggregate effect of microeconomic shocks. Baqaee and Farhi (2019) decompose the effect of shocks into "direct" and "allocative efficiency" effects due to reduced-form wedges, with a focus on the first-order implications for aggregate TFP. We share with them our finding that the Hulten theorem does not apply in economies with frictions: Total sales of firms/ sectors are no longer a sufficient statistics for their effect on GDP.

In relation to these papers, our work differs in several dimensions. First, we study pricing as a measurable friction. This approach allows us to pursue quantitative assessments, explore dynamic effects of the friction, and to quantify the implications of a number of relevant model ingredients. Our micro data in particular allow us to make inference about the size of idiosyncratic shocks which are a key ingredient to inform modeling efforts. Our focus also goes beyond aggregate output. We focus on the effect on price stability as well, which is central for the mandate of many central banks around the world, but is absent from the existing literature entirely. Finally, pricing frictions are a very special friction: prices are key and first-order to the transmission of shocks. While other frictions may also affect how production adjusts to shocks, that reaction always depends on the ability of prices to pass through the relevant information.

Important antecedents to our work are Carvalho (2006) and Nakamura and Steinsson (2010). Carvalho (2006) focuses on the role of heterogeneity in the frequency of price changes for the aggregate effects of monetary policy shocks. Nakamura and Steinsson (2010) investigate monetary non-neutrality in a multi-sector menu cost model in which sectors are linked by input-output relationships. They both show that in the presence of strategic complementarities, aggregate prices move less and aggregate quantities react more than in the absence of sectoral heterogeneity in price rigidity. By contrast, our paper puts the heterogeneity of idiosyncratic productivity shocks to the forefront – and their interaction with heterogeneous model features like the heterogeneity in the frequency of price changes. Carvalho (2006) and Nakamura and Steinsson (2010) study the effect of a common monetary policy shock and hence a very different transmission mechanism focused on front-loading in the price response to the same shock.

On the quantitative side, most closely related to our work is Bouakez, Cardia, and Ruge-Murcia (2014) that also features heterogeneity in price rigidity but focus on different questions. The main related result in Bouakez et al. (2014) is the role of sectoral productivity shocks in a 30-sector economy in explaining aggregate variables, but the quantitative importance is modest compared to the role monetary policy and other shocks. Moreover, Bouakez et al. (2014) estimate that nearly half of their 30 sectors are flex-price sectors so that only service prices are sticky. Atalay (2017) pioneers the study of aggregate fluctuations from microeconomic shocks in a model with flexible prices but heterogeneous elasticities of substitution. Data limitations lead Atalay (2017) to focus on coarser levels of disaggregation. Baley and Blanco (2019) show that variations in the dispersion of the degree of price flexibility, driven by idiosyncratic uncertainty shocks, have important implications for monetary non-neutrality in a menu cost model. Castro-Cienfuegos (2019) instead studies the relevance of sectoral heterogeneity in models of production networks for the conduct of monetary policy.

Our model also relates to other previous work studying pricing frictions in production networks. Basu (1995) shows nominal price rigidity introduces misallocation resulting in nominal demand shocks looking like aggregate productivity shocks. Basu and De Leo (2016) study optimal monetary policy in a model of durable and non-durable goods in the presence of price rigidity and input-output linkages. Nakamura and Steinsson (2010), Midrigan (2011), and Alvarez, Le Bihan, and Lippi (2016), among many others, study monetary non-neutrality in models of endogenous price changes. Computational burden makes using such approaches infeasible at highly disaggregated levels. In a companion paper (Pasten, Schoenle, and Weber (2016)), we use our model to study the effect of disaggregation on monetary non-neutrality. We generally build on some of this work, relax assumptions on the production structure of the economy, and answer a different set of questions in a more disaggregated quantitative setting.

II Model

Our multi-sector model has households supplying labor and demanding goods for final consumption, firms operating under monopolistic competition producing varieties of goods using labor and intermediate inputs, and a monetary authority. Sectors are heterogeneous in the amount of final goods they produce, input-output linkages, and the frequency of price changes.

A. Households

The representative household solves

$$\max_{\{C_t, L_{kt}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{k=1}^{K} g_k \frac{L_{kt}^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

subject to

$$\begin{aligned} \sum_{k=1}^{K} W_{kt} L_{kt} + \sum_{k=1}^{K} \Pi_{kt} + I_{t-1} B_{t-1} - B_t &= P_t^c C_t \\ \sum_{k=1}^{K} L_{kt} &\leq 1, \end{aligned}$$

where C_t and P_t^c are aggregate consumption and aggregate prices, respectively. We can also interpret C_t as GDP and P_t^c as the consumer price index or the GDP deflator given we study a closed economy model with no investment and government spending. L_{kt} and W_{kt} are labor employed and wages paid in sector $k = 1, ..., K$. Households own firms and receive net income, Π_{kt}, as dividends. Bonds, B_{t-1}, pay a nominal gross interest rate of I_{t-1}. Total labor supply is normalized to 1.

GDP C_t aggregates from sectoral GDP C_{kt} and in turn from households' final demand for each goods C_{jkt} according to

$$C_t \equiv \left[\sum_{k=1}^{K} \omega_{ck}^{\frac{1}{\eta}} C_{kt}^{1-\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

$$C_{kt} \equiv \left[n_k^{-1/\theta} \int_{\Im_k} C_{jkt}^{1-\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

A continuum of goods indexed by $j \in [0,1]$ exists with total measure 1. Each good belongs to one of the K sectors in the economy. Mathematically, the set of goods is partitioned into K subsets $\{\Im_k\}_{k=1}^{K}$ with associated measures $\{n_k\}_{k=1}^{K}$ such that $\sum_{k=1}^{K} n_k = 1$.[3] We allow the elasticity of substitution across sectors η to differ from the elasticity of substitution within sectors θ.

The first key ingredient of our model is the vector of weights $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]$ in equation (2). Households' sectoral demand

$$C_{kt} = \omega_{ck} \left(\frac{P_{kt}}{P_t^c}\right)^{-\eta} C_t \tag{4}$$

determines the interpretation as sectoral GDP shares because in steady state all prices are identical, so $\omega_{ck} \equiv \frac{C_k}{C}$ where variables without a time subscript denote steady-state levels. Thus, the vector Ω_c satisfies $\Omega_c'\iota = 1$, where ι denotes a column-vector of 1s. Away from steady state, sectoral GDP shares depend on the gap between sectoral prices and the aggregate price index, P_t^c

$$P_t^c = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{5}$$

Household demand for goods within a sector is given by

$$C_{jkt} = \frac{1}{n_k} \left(\frac{P_{jkt}}{P_{kt}}\right)^{-\theta} C_{kt} \text{ for } k = 1, ..., K. \tag{6}$$

Firms within a sector equally share the production of goods in steady state. Away from steady state, the gap between a firm's price, P_{jkt}, and the sectoral price, P_{kt}, distorts the demand for goods within a sector.

Sectoral price is defined by

$$P_{kt} = \left[\frac{1}{n_k} \int_{\Im_k} P_{jkt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{7}$$

[3] The sectoral subindex is redundant, but it clarifies exposition. We can interpret n_k as the sectoral share in gross output.

The household first-order conditions determine labor supply and the Euler equation

$$\frac{W_{kt}}{P_t^c} = g_k L_{kt}^{\varphi} C_t^{\sigma} \text{ for all } k, j, \tag{8}$$

$$1 = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} I_t \frac{P_t^c}{P_{t+1}^c} \right]. \tag{9}$$

We implicitly assume sectoral segmentation of labor markets, so labor supply in equation (8) holds for a sector-specific wage $\{W_{kt}\}_{k=1}^{K}$. We choose the parameters $\{g_k\}_{k=1}^{K}$ to ensure a symmetric steady state across all firms.

B. Firms

A continuum of monopolistically competitive firms exists, each one producing a single good. To facilitate exposition, firms are indexed by the good $j \in [0,1]$ they produce and the sector, $k = 1, ...K$ they belong to. The production function is

$$Y_{jkt} = e^{a_{kt}} L_{jkt}^{1-\delta} Z_{jkt}^{\delta}, \tag{10}$$

where a_{kt} is an i.i.d. productivity shock to sector k with $\mathbb{E}[a_{kt}] = 0$ and $\mathbb{V}[a_{kt}] = v^2$ for all k, L_{jkt} is labor, and Z_{jkt} is an aggregator of intermediate inputs

$$Z_{jkt} \equiv \left[\sum_{k'=1}^{K} \omega_{kk'}^{\frac{1}{\eta}} Z_{jk} \left(k'\right)^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{11}$$

$Z_{kjt}(r)$ is the amount of goods firm jk demands as intermediate inputs from sector r in period t.

The second key ingredient of our model is heterogeneity in aggregator weights $\{\omega_{kk'}\}_{k,k'}$. We denote these weights in matrix notation as Ω, satisfying $\Omega\iota = \iota$. The demand of firm jk for goods produced in sector k' is given by

$$Z_{jkt}\left(k'\right) = \omega_{kk'} \left(\frac{P_{k't}}{P_t^k} \right)^{-\eta} Z_{jkt}. \tag{12}$$

We interpret $\omega_{kk'}$ as the steady-state share of goods from sector k' in the intermediate input use of sector k, which determines the input-output linkages across sectors in steady state. Away from the steady state, the gap between the price of goods in sector k' and the aggregate price relevant for a firm in sector k, P_t^k, distorts input-output linkages

$$P_t^k = \left[\sum_{k'=1}^{K} \omega_{kk'} P_{k't}^{1-\eta}\right]^{\frac{1}{1-\eta}} \quad \text{for } k = 1, ..., K. \tag{13}$$

P_t^k uses the sector-specific steady-state input-output linkages to aggregate sectoral prices.

The aggregator $Z_{jk}(k')$ gives the demand of firm jk for goods in sector k'

$$Z_{jk}(k') \equiv \left[n_{k'}^{-1/\theta} \int_{\Im_{k'}} Z_{jkt}(j', k')^{1-\frac{1}{\theta}} \, dj'\right]^{\frac{\theta}{\theta-1}}. \tag{14}$$

Firm jk's demand for an arbitrary good j' from sector k' is

$$Z_{jkt}(j', k') = \frac{1}{n_{k'}} \left(\frac{P_{j'k't}}{P_{k't}}\right)^{-\theta} Z_{jk}(k'). \tag{15}$$

In steady state, all firms within a sector share the intermediate input demand of other sectors equally. Away from steady state, the gap between a firm's price and the price index of the sector it belongs to (see equation (7)) distorts the firm's share in the production of intermediate inputs. Our economy has $K + 1$ different aggregate prices, one for the household sector and one for each of the K sectors. By contrast, the household sector and all sectors face unique sectoral prices.

The third key ingredient of our model is sectoral heterogeneity in price rigidity. For quantitative purposes, we model price rigidity à la Calvo with parameters $\{\alpha_k\}_{k=1}^K$ such that the pricing problem of firm jk is

$$\max_{P_{jkt}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k^{\tau} \left[P_{jkt} Y_{jkt+s} - MC_{kt+s} Y_{jkt+s}\right]. \tag{16}$$

Marginal costs are $MC_{kt} = \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta} e^{-a_{kt}} W_{kt}^{1-\delta} \left(P_t^k\right)^{\delta}$ after imposing the optimal mix of labor and intermediate inputs

$$\delta W_{kt} L_{jkt} = (1-\delta) P_t^k Z_{jkt}, \tag{17}$$

and $Q_{t,t+s}$ is the stochastic discount factor between periods t and $t + s$.[4] We assume the elasticities of substitution across and within sectors are the same for households and all firms.

[4]We choose Calvo pricing merely as an expository tool, and for computational reasons. We discuss details of choosing an endogenous price-adjustment technology at the end of Section III.

This assumption shuts down price discrimination among different customers, and firms choose a single price P^*_{kt}

$$\sum_{\tau=0}^{\infty} Q_{t,t+\tau} \alpha_k^s Y_{jkt+\tau} \left[P^*_{kt} - \frac{\theta}{\theta-1} MC_{kt+\tau} \right] = 0, \tag{18}$$

where $Y_{jkt+\tau}$ is the total production of firm jk in period $t+\tau$.

We define idiosyncratic shocks $\{a_{kt}\}_{k=1}^{K}$ at the sectoral level, and it follows the optimal price, P^*_{kt}, is the same for all firms in a given sector. Thus, aggregating among all prices within sector yields

$$P_{kt} = \left[(1-\alpha_k) P^{*1-\theta}_{kt} + \alpha_k P^{1-\theta}_{kt-1} \right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{19}$$

C. Monetary policy, equilibrium conditions, and definitions

The choice of monetary policy is crucial for the effects of real shocks on the economy, as Gali (1999) shows for technology shocks, or Woodford (2011) for government spending shocks. For pedagogical purposes, our simplified model in the next section assumes a nominal GDP target. This assumption allows us to obtain analytical expressions and build intuition. In our full-blown quantitative assessment, we instead use various empirically plausible monetary policy rules, with a flexible-inflation target Taylor rule as our baseline rule. We show that our key analytical results carry through in an empirically plausible setting for monetary policy.

In our quantitative exercises, our baseline Taylor rule takes the conventional form

$$I_t = \frac{1}{\beta} \left(\frac{P^c_t}{P^c_{t-1}} \right)^{\phi_\pi} \left(\frac{C_t}{C} \right)^{\phi_y}. \tag{20}$$

Monetary policy reacts to inflation, P^c_t/P^c_{t-1}, and deviations of GDP from the steady-state level, C_t/C.

Bonds are in zero net supply, $B_t = 0$, labor markets clear, and goods markets clear such that

$$Y_{jkt} = C_{jkt} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{j'k't}(j,k)\, dj', \tag{21}$$

implying a wedge between gross output Y_t and GDP C_t.

III Theoretical Results in a Simplified Model

We now make several simplifying assumptions that allow us to derive analytical results on how the interaction of heterogeneous price rigidities and heterogeneous real features of the economy can affect the propagation of idiosyncratic productivity shocks and to provide intuition. This illustration paves way for the subsequent section which establishes the quantitative aggregate relevance of our mechanism in a more realistic setting. We discuss the importance of several simplifying assumptions at the end of the section. A reader mainly interested in the quantitative evaluation may skip directly to Section VI.

We provide intuition by focusing first on the interacting of pricing frictions with the size distribution, and then with input-output linkages. We provide results both in terms of volatility of GDP and price stability, and the rate of convergence relative to the central limit theorem, two ways of looking at the aggregate relevance of idiosyncratic shocks. Introducing price stability is an additional novelty of our analysis relative to a literature that mainly focuses on GDP fluctuations.

A. Simplified Setup

All variables in lower cases denote log-linear deviations from steady state. We make the following simplifying assumptions:

(i) Households have log utility, $\sigma = 1$, and linear disutility of labor, $\varphi = 0$. Thus,

$$w_{kt} = p_t^c + c_t; \tag{22}$$

that is, the labor market is integrated, labor supply is infinitely elastic and nominal wages are proportional to nominal GDP.

(ii) Monetary policy targets steady state nominal GDP so

$$p_t^c + c_t = 0; \tag{23}$$

thus, wages remain invariant to shocks.

(iii) We replace Calvo price stickiness by a simple form of price rigidity: all prices are flexible, but with probability λ_k, a firm in sector k has to set its price before observing shocks. Thus,

$$P_{jkt} = \begin{cases} \mathbb{E}_{t-1}\left[P^*_{jkt}\right] & \text{with probability } \lambda_k, \\ P^*_{jkt} & \text{with probability } 1-\lambda_k, \end{cases} \tag{24}$$

where $\mathbb{E}_{t-1}$ is the expectation operator conditional on the $t-1$ information set. This price-setting technology takes a simple form to capture the key ingredient in our model: sectoral heterogeneity in the responsiveness of prices to idiosyncratic shocks.

Solution We show in the Online Appendix under assumptions (i), (ii), and (iii) that GDP solves

$$c_t = \chi' a_t, \tag{25}$$

where

$$\chi \equiv (\mathbb{I} - \Lambda) \left[\mathbb{I} - \delta\Omega' (\mathbb{I} - \Lambda)\right]^{-1} \Omega_c, \tag{26}$$

Λ is a diagonal matrix with price-rigidity probabilities $[\lambda_1, ..., \lambda_K]$ as entries, Ω_c and Ω are steady-state sectoral GDP shares and input-output linkages, respectively, and δ is the intermediate input share. Similarly, our specification of monetary policy implies the aggregate price level is pinned down by

$$p_t^c = -c_t = -\chi' a_t. \tag{27}$$

A linear combination of sectoral shocks describes the log-deviation of GDP and the price level from its steady state up to first-order. Thus, volatility of GDP and the aggregate price level are

$$v_c = v_p = v\sqrt{\sum_{k=1}^{K} \chi_k^2} = \|\chi\|_2 \, v, \tag{28}$$

because all sectoral shocks have the same volatility in the model of this section; that is, $\mathbb{V}[a_{kt}] = v^2$ for all k. $\|\chi\|_2$ denotes the Euclidean norm of χ. We refer to χ and $\|\chi\|_2$ as the *vector of sectoral multipliers* and the *total multiplier*, respectively, mapping sectoral shocks into the volatility of GDP and aggregate prices. We explore alternative monetary policy rules below in which the volatility of GDP and aggregate price no longer coincide.

We study the effect of the sectoral distribution of price rigidity on aggregate fluctuations from two perspectives: first, in a cross-sectional sense for a given finite number of sectors K; and second, with respect to distributional properties as the economy becomes increasingly more disaggregated, $K \to \infty$. We use these two perspectives to study in two sections the interaction between price rigidity and two real features of the economy embodied in (26): steady-state sectoral GDP shares Ω_c and steady-state input-output linkages Ω.

B. Price Rigidity and Sectoral GDP

This section shows how heterogeneity in price rigidity interacts with heterogeneity in sectoral size in propagating sectoral shocks. We provide intuition while abstracting from input-output linkages across sectors ($\delta = 0$). We first establish as a benchmark the case of interactions with homogeneous price rigidity across sectors. Then, we contrast results with the case of heterogeneous price rigidity across sectors. Our analysis nests the flexible-price results in Gabaix

(2011).

Absent input-output linkages ($\delta = 0$), the vector of sectoral multipliers χ in equation (26) solves

$$\chi = (\mathbb{I} - \Lambda)\,\Omega_c, \tag{29}$$

or, simply, $\chi_k = (1 - \lambda_k)\,\omega_{ck}$ for all k where $\omega_{ck} \equiv C_k/C$. This expression immediately implies steady-state GDP shares ω_{ck} fully determine sectoral multipliers only when prices are flexible ($\lambda_k = 0$). In general, sectoral multipliers depend on the joint sectoral distribution of size with price rigidity, which we subsequently also denote by "effective size distribution."[5]

B.1 Homogeneous Price Rigidity and Sectoral GDP

When price rigidity is homogeneous across sectors, that is, $\lambda_k = \lambda$, it affects output volatility and price stability as well as the rate of convergence in a straight-forward way. It can reduce the output and price volatility, while it has no effect on the rate of convergence. The results follow directly from the insight that the effective size distribution $(1-\lambda)\omega_{ck}$ has the same shape as the physical size distribution ω_{ck}, just rescaled by a constant $(1 - \lambda)$.

Lemma 1 *When all prices have the same degree of rigidity, $\lambda_k = \lambda\ \forall k$,*

$$v_c = v_p \frac{(1-\lambda)\,v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\left(C_k\right) + \overline{C}_k^2},$$

where $\overline{C}_k$ and $\mathbb{V}(\cdot)$ are the sample mean and sample variance of $\{C_k\}_{k=1}^K$ and no input-output linkages are present ($\delta = 0$).[6]

This lemma says the variance of the physical size distribution determines the volatility of GDP and prices, and price rigidity reduces it by a constant. The term $1-\lambda$ denotes the degree of price flexibility throughout the economy. Its presence implies a standard result that carries over to a multi-sector setting: The more flexible prices are, the more prices and quantities can adjust after a productivity shock. Thus, whenever price rigidity decreases, sectors become effectively large and shocks can pass through and propagate more strongly. Note that in the extreme case of perfectly sticky prices, no effect on aggregate volatility exists regardless the volatility of productivity shocks (up to a first order approximation). While productivity may change, wages remain constant due to our assumed monetary policy targeting nominal GDP and, given the

[5] Physical sales are no longer a sufficient statistic for the importance of sectors for aggregate volatility. This result breaks the Hulten (1978) result in the Gabaix (2011) framework for the case of no intermediate inputs.

[6] We define $\mathbb{V}(X_k)$ of a sequence $\{X_k\}_{k=1}^K$ as $\mathbb{V}(X_k) \equiv \frac{1}{K}\sum_{k=1}^K \left(X_k - \overline{X}\right)^2$. The definition of the sample mean is standard.

infinitely elastic labor supply, households work less.[7]

Homogeneous price rigidity has no impact on the aggregate decay properties.

Proposition 1 *If $\delta = 0$, $\lambda_k = \lambda$ for all k, and $\{C_k\}_{k=1}^K$ follows a power-law distribution with shape parameter $\beta_c \geq 1$, then*

$$v_c = v_p \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} v & \text{for } \beta_c = 1, \end{cases}$$

where u_0 is a random variable independent of K and v.

Proof. See Online Appendix. ■

Proposition 1 shows homogeneous price rigidity plays no role for the rate of decay in output and price volatility. A constant degree of price rigidity, bounded between 0 and 1, does not change the fatness of the tails of the convolved, effective distribution – the physical size distribution is only multiplied by a constant and its shape parameter is unaffected by the degree of price rigidity.

B.2 Heterogeneous Price Rigidity and Sectoral GDP

We next consider the effect of heterogeneity in price rigidity. Now the joint, non-trivially convolved distribution of price rigidity and size matters, and hence their covariance, with multipliers $\chi_k = (1 - \lambda_k)\omega_{ck}$. Economically, some sectors may effectively become larger or smaller than the average sector. As a result, the variance of GDP and prices can increase or decrease relative to the case of homogeneous price rigidities while the fatness of the tails in the distribution can similarly change and hence, the rate of convergence.

First, we study the cross-sectional effect of heterogeneous price rigidity.

Lemma 2 *When price rigidity is heterogeneous across sectors*

$$v_c = v_p = \frac{v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\left(\left(1-\lambda_k\right) C_k\right) + \left[\left(1-\overline{\lambda}\right) \overline{C}_k - \mathbb{COV}\left(\lambda_k, C_k\right)\right]^2},$$

where $\overline{\lambda}$ is the sample mean of $\{\lambda_k\}_{k=1}^K$ and $\mathbb{COV}(\cdot)$ is the sample covariance of $\{\lambda_k\}_{k=1}^K$ and $\{C_k\}_{k=1}^K$ and no input-output linkages are present ($\delta = 0$). [8]

[7] This result depends on the choice of monetary policy. An active optimal monetary policy would undo the effect of distortionary, homogeneous price rigidity. Below we study quantitatively the relevance of these results for generating sizable aggregate fluctuations.

[8] We define $\mathbb{COV}(X_k, Q_k)$ of sequences $\{X_k\}_{k=1}^K$ and $\{Q_k\}_{k=1}^K$ as $\mathbb{COV}(X_k, Q_k) \equiv \frac{1}{K}\sum_{k=1}^K \left(X_k - \overline{X}\right)\left(Q_k - \overline{Q}\right)$.

Lemma 2 states the volatility of output and prices depends on the sectoral dispersion of the convolved variables and their covariance terms. While trivial statistically, this lemma points to the key insight of the paper: heterogeneity of price rigidity has the power to increase or decrease aggregate volatility of output and prices because it changes the effective size distribution of the economy, beyond a simple re-scaling.

Depending on the cross-sectional dispersion of price rigidity, sectoral shocks may even generate sizable GDP volatility when all sectors have equal physical size. To see this point, consider $C_k = C/K$ for all k. Lemma 2 implies

$$v_c = v_p \frac{v}{K^{1/2}} \sqrt{\mathbb{V}\left(1-\lambda_k\right) + \left(1-\overline{\lambda}\right)^2}. \tag{30}$$

This result highlights the potential of heterogeneous price rigidity to become a "frictional" force that increases the propagation of sectoral productivity shocks even in settings in which no other heterogeneities are present.

By contrast, heterogeneity in price rigidity does not have the power by itself to affect the rate of decay. The effect on the rate of decay depends on what happens to the fatness of the tails of the joint distribution as we show in the following propositions.

Proposition 2 *If $\delta = 0$, $C_k/C = 1/K$, and the distribution of λ_k satisfies*

$$\Pr\left[1-\lambda_k > y\right] = \frac{y^{-\beta_\lambda} - 1}{y_0^{-\beta_\lambda} - 1} \text{ for } y \in \left[y_0, 1\right], \beta_\lambda > 0,$$

then $v_c = v_p \sim v/K^{1/2}$.

Proof. Given the distribution of $(1-\lambda_k), \mathbb{E}[(1-\lambda_k)^2]$ exists, so the Central Limit Theorem applies. ■

This proposition shows if all sectors have equal size, then the Central Limit Theorem governs the rate at which shocks die out, independent of the distribution of price rigidity. This result is due to the boundedness of price rigidity. If λ_k followed a Pareto distribution with support unbounded below, the rate of decay would depend on the shape parameter β_λ exactly as in Proposition 1. However, the bound at $\lambda_k = 0$ implies that the second moment of the distribution exists for any shape parameter.

The next proposition shows heterogeneity of price rigidity still does not matter for the rate of decay of GDP and price level volatility when it is independent of heterogeneity in the size distribution.

Proposition 3 *If $\delta = 0$, λ_k and C_k are independently distributed, the distribution of λ_k satisfies*

$$\Pr\left[1-\lambda_k > y\right] = \frac{y^{-\beta_\lambda}-1}{y_0^{-\beta_\lambda}-1} \text{ for } y \in \left[y_0, 1\right], \beta_\lambda > 0,$$

and C_k follows a power-law distribution with shape parameter $\beta_c \geq 1$, then

$$v_c = v_p \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} & \text{for } \beta_c = 1, \end{cases}$$

where u_0 is a random variable independent of K and v.

Proof. See Online Appendix. ■

As in Proposition 2 this results is due to the lower bound on the support of price rigidity which implies the convolution $(1-\lambda_k)C_k$ follows a Pareto distribution with shape parameter β_c of the sectoral GDP Pareto distribution.

Two remarks about the economic importance of heterogeneous price rigidity are in order. First, at any rate of disaggregation, heterogeneity of price rigidity through its interaction with sector size changes the identity of sectors from which aggregate fluctuations originate. This observation is important given the emphasis of the recent granularity literature that aims at identifying the microeconomic origin of aggregate fluctuations, for example, for stabilization purposes. If shocks are idiosyncratic, re-weighting sectoral shocks of potentially opposite signs can easily change the sign of business cycles. The intuition is that a large (small) sector can become effectively small (large) if its highly rigid (flexible) prices reduces (increases) its *effective* size through the convolution of $(1-\lambda_k)C_k$.

We now move to the case when price rigidity and GDP shares are not independently distributed.

Proposition 4 *If $\delta = 0$, $\{(1-\lambda_k)\,C_k\}_{k=1}^{K}$ follows a power-law distribution with shape parameter $\beta_{\lambda c} \geq 1$, then*

$$v_c = v_p \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_{\lambda c}, 1/2\}}} v & \text{for } \beta_{\lambda c} > 1 \\ \frac{u_0}{\log K} v & \text{for } \beta_{\lambda c} = 1, \end{cases}$$

where u_0 is a random variable independent of K and v.

This proposition requires no proof as it is identical to Proposition 1. It simply states that, in the general case, if the convolution $(1-\lambda_k)\,C_k$ follows a Pareto distribution, the rate of decay of v_c as $K \to \infty$ depends on the shape parameter of $\beta_{\lambda c}$. When λ_k and C_k are not independent, price rigidity may have an effect on the rate of decay of v_c as $K \to \infty$ despite the bounded support of the price rigidity. We illustrate the breadth of the proposition in the Online Appendix.

C. Price Rigidity and Input-Output Linkages

We now study how heterogeneity in price rigidity and input-output linkages interact in propagating idiosyncratic productivity shocks. To show this point, we assume a positive intermediate input share but shut down the heterogeneity of sectoral GDP shares, that is, $C_k = C/K$.

The vector of multipliers χ in (26) now solves

$$\chi \equiv \frac{1}{K} (\mathbb{I} - \Lambda) \left[\mathbb{I} - \delta\Omega' (\mathbb{I} - \Lambda)\right]^{-1} \iota. \tag{31}$$

This expression shows a non-trivial interaction between price rigidity and input-output linkages exists across sectors.[9] To study this interaction, we follow Acemoglu et al. (2012) and use a second-order approximation of the vector of multipliers

$$\chi \simeq \frac{1}{K} (\mathbb{I} - \Lambda) \left[\mathbb{I}+\delta\Omega' (\mathbb{I} - \Lambda) + \delta^2 \left[\Omega' (\mathbb{I} - \Lambda)\right]^2\right] \iota. \tag{32}$$

C.1 Homogeneous Price Rigidity and Input-Output Linkages

When price rigidity is identical across sectors, an increase in rigidity uniformly lowers the effective centrality of all sectors and hence, aggregate volatility from idiosyncratic shocks falls. At the same time, because fatness of the entire centrality distribution is unchanged, the rate of convergence remains identical.

In the following, we measure centrality in the production network as in Acemoglu et al. (2012) using the following two definitions:

$$\begin{aligned} d_k &\equiv \sum_{k'=1}^{K} \omega_{k'k}, \\ q_k &\equiv \sum_{k'=1}^{K} d_{k'}\omega_{k'k}. \end{aligned}$$

where d_k measures the importance of sectors as supplier of intermediate inputs ("outdegree")

[9] This expression nests the solution for the "influence vector" in Acemoglu et al. (2012) when prices are fully flexible, that is, $\lambda_k = 0$ for all $k = 1, ..., K$. The only difference here is $\chi'\iota = 1/(1-\delta)$, because Acemoglu et al. (2012) normalize the scale of shocks such that the sum of the influence vector equals 1.

while q_k measures the importance of sectors as supplier of large suppliers of intermediate inputs ("second-order outdegree").

First, consider homogeneous price rigidity across sectors.

Lemma 3 *If price rigidity is homogeneous across sectors, $\lambda_k = \lambda$ for all k, then*

$$v_c = v_p \geq \frac{(1-\lambda)\, v}{K^{1/2}} \sqrt{\kappa + \delta'^2 \mathbb{V}(d_k) + 2\delta'^3 \mathbb{COV}(d_k, q_k) + \delta'^4 \mathbb{V}(q_k)}, \tag{33}$$

where $\kappa \equiv 1 + 2\delta' + 3\delta'^2 + 2\delta'^3 + \delta'^4$, $\delta' \equiv \delta(1-\lambda)$, $\mathbb{V}(\cdot)$ and $\mathbb{COV}(\cdot)$ are the sample variance and covariance statistics across sectors and input-output linkages are present ($\delta \in (0,1)$) while all sectors have equal size $\Omega_c = \frac{1}{K}\iota$.

Lemma 3 follows from equation (28), $d = \Omega'\iota$ and $q = \Omega'^2\iota$. The inequality holds because the exact solution for χ is strictly larger than the approximation. Note upstream effects through demand of intermediate inputs do not play any role here due to our focus on GDP, and our assumptions that imply unresponsiveness of wages to shocks.

Lemma 3 establishes two important insights. First, homogeneous price rigidity adjusts the effective centrality downwards and thereby, aggregate volatility. When sectors can pass through less of the shock through prices, volatility gets dampened. Physical first-order and second-order outdegrees are not sufficient to characterize network propagation. Second, price rigidity penalizes more strongly the quantitative effect of heterogeneity in second-order outdegrees than in first-order outdegrees. This result is important because the flexible-price analysis of Acemoglu et al. (2012) stresses second-order outdegrees contribute more to the aggregate fluctuations from idiosyncratic shocks than first-oder outdegrees. In general, inter-connections of order τ are penalized by a factor $(1-\lambda)^\tau$.

The next proposition shows results for the rate of decay of v_c as $K \to \infty$, still under the assumption of homogeneous price rigidity.

Proposition 5 *If $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, $\Omega_c = \frac{1}{K}\iota$, the distribution of outdegrees $\{d_k\}$, second-order outdegrees $\{q_k\}$, and the product of outdegrees $\{z_k = d_k q_k\}$ follow power-law distributions with respective shape parameters $\beta_d, \beta_q, \beta_z > 1$ such that $\beta_z \geq \frac{1}{2}\min\{\beta_d, \beta_q\}$, then*

$$v_c = v_p \geq \begin{cases} \frac{u_3}{K^{1/2}} v & \text{for } \min\{\beta_d, \beta_q\} \geq 2, \\ \frac{u_3}{K^{1-1/\min\{\beta_d,\beta_q\}}} v & \text{for } \min\{\beta_d, \beta_q\} \in (1,2), \end{cases}$$

where u_3 is a random variable independent of K and v.

Proof. See Online Appendix. ■

Proposition 5 shows homogeneous price rigidity has no impact on the rate of decay. The intuition from the previous section continues to hold: multiplying the distribution of outdegrees with a constant does not change the fatness of the tails. Therefore, the rate of decay only depends on the shape parameters of measures of network centrality, d and q, as in Acemoglu et al. (2012). The fattest tail among these distributions determines the rate of decay.[10]

C.2 Heterogeneous Price Rigidity and Input-Output Linkages

Next, we study the interaction of heterogeneous price rigidity with heterogeneous input-output linkages. Again, the entire joint distribution of our centrality measures and price rigidity matters. We now define the **modified outdegrees** and **modified second-order outdegrees**, respectively, for all $k = 1, ..., K$ as

$$\begin{aligned} \widetilde{d}_k &\equiv \left(1-\overline{\lambda}\right) \sum_{k'=1}^{K} \left(\frac{1-\lambda_{k'}}{1-\overline{\lambda}}\right) \omega_{k'k}, \\ \widetilde{q}_k &\equiv \left(1-\overline{\lambda}\right) \sum_{k'=1}^{K} \left(\frac{1-\lambda_{k'}}{1-\overline{\lambda}}\right) \widetilde{d}_{k'} \omega_{k'k}. \end{aligned}$$

Our first result concerns aggregate cross-sectional volatilities

Lemma 4 *If price rigidity and input-output linkages are heterogeneous across sectors, then*

$$v_c = v_p \geq \frac{v}{K^{1/2}} \left[\begin{array}{c} \left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[\widetilde{\kappa} + \delta^2 \mathbb{V}\left(\widetilde{d}_k\right) + 2\delta'^3 \mathbb{COV}\left(\widetilde{d}_k, \widetilde{q}_k\right) + \delta'^4 \mathbb{V}\left(\widetilde{q}_k\right)\right] \\ -\left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[2\delta^2\left(1+\widetilde{\delta}+\widetilde{\delta}^2\right)\mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) + \delta^4 \mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right)^2\right] \\ +\mathbb{COV}\left((1-\lambda_k)^2, \left(1+\delta\widetilde{d}_k + \delta^2 \widetilde{q}_k\right)^2\right) \end{array} \right]^{\frac{1}{2}}, \tag{34}$$

where $\widetilde{\kappa} \equiv 1 + 2\widetilde{\delta} + 3\widetilde{\delta} + 2\widetilde{\delta} + \widetilde{\delta}$, $\widetilde{\delta} \equiv \delta\left(1-\overline{\lambda}\right)$, $\overline{\lambda}$ *is the sample mean of* $\{\lambda_k\}_{k=1}^{K}$, $\mathbb{V}(\cdot)$ *and* $\mathbb{COV}(\cdot)$ *are the sample variance and covariance statistics across sectors, and* $\delta \in (0,1)$ *while all sectors have equal size* $\Omega_c = \frac{1}{K}\iota$.

Lemma 4 embodies the central result in this section, derived directly from equation (28) with $\widetilde{d} = \Omega' (\mathbb{I} - \Lambda)\iota$ and $\widetilde{q} = [\Omega' (\mathbb{I} - \Lambda)]^2 \iota$. What matters now for aggregate volatility is the entire distribution of *effective* centralities of sectors – after adjusting nodes by their degrees of price rigidities. In particular, $\widetilde{d}_k$ measures the importance of sectors as large suppliers of sectors with highly flexible prices. Similarly, $\widetilde{q}_k$ measures the importance of sectors as large suppliers of highly flexible sectors that are large suppliers of highly flexible sectors.

[10] Acemoglu et al. (2012) document in the U.S. data that $\beta_d \approx 1.4$ and $\beta_q \approx 1.2$. We find very similar numbers. Thus, we abstract from the case $\min\{\beta_d, \beta_q\} = 1$ in Proposition 5.

A key implication of this result is that – depending on the cross-sectoral distribution of price rigidities – $\widetilde{d}$ and $\widetilde{q}$ may be heterogeneous across sectors even when the production network is perfectly symmetric. This result reinforces the importance of studying the interaction of nominal and real heterogeneous features of the economy.

Analyzing the effect of the heterogeneous price rigidity on the rate of decay of v_c as $K \to \infty$ is more intricate than in the case with no intermediate inputs ($\delta = 0$).

Proposition 6 *If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, price rigidity is heterogeneous across sectors, the distribution of modified outdegrees $\left\{\widetilde{d}_k\right\}$, modified second-order outdegrees $\{\widetilde{q}_k\}$, and the product $\left\{z_k = \widetilde{d}_k \widetilde{q}_k\right\}$ follow power-law distributions with respective shape parameter $\widetilde{\beta}_d, \widetilde{\beta}_q, \widetilde{\beta}_z > 1$ such that $\widetilde{\beta}_z \geq \frac{1}{2}\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\}$, then*

$$v_c \geq \begin{cases} \frac{u_4}{K^{1/2}} v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \geq 2, \\ \frac{u_4}{K^{1-1/\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\}}} v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \in (1,2), \end{cases}$$

where u_4 is a random variable independent of K and v.

Proof. See Online Appendix. ■

This result resembles Proposition 4 but in the context of production networks. Although heterogeneous price rigidity interacts in a more complicated way with heterogeneous input-output linkages than sector size, the fundamental intuition is similar. If sectors with the most rigid (flexible) prices are also the most central in the *effective*, price-rigidity-adjusted production network such that $\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} > (<) \min\{\beta_d, \beta_q\}$, then GDP volatility may decay at a faster (slower) rate than when price rigidity is homogeneous across sectors or is independent of network centrality. Just as for the interaction with sectoral size, heterogeneity in price rigidity does have an effect for a finite level of disaggregation as in Lemma 4. This result holds regardless of whether or not price rigidity affects the rate of decay.

Finally, price rigidity distorts the identity of sectors from which aggregate fluctuations originate when idiosyncratic shocks drive aggregate volatility through the network. Re-weighting sectoral shocks of potentially opposite signs can again easily change the sign of business cycles. What matters is the *effective* not the physical network structure.

D. Relaxing Simplifying Assumptions

We now discuss the implications of relaxing the simplifying modeling assumptions.

Elastic labor supply Our first analysis allows for a positive inverse-Frisch elasticity $\varphi > 0$. This possibility opens up new channels for GDP shares, input-output linkages, and price rigidity to interact and affect the propagation of sectoral shocks to the volatility of GDP and the aggregate price level, which we explore below. Labor supply and demand now jointly determine wages such that

$$w_{kt} = c_t + p_t^c + \varphi l_{kt}^d \tag{35}$$

becomes the log-linear counterpart to equation (8). Thus, with monetary policy targeting $c_t + p_t^c = 0$, it no longer holds that sectoral productivity shocks have no effect on wages.[11] Because the labor market is sectorally segmented, wages may differ across sectors. To see the sources of sectoral wage variation, we start from labor demand implied by the sectoral aggregation of the production function and the efficiency condition on the mix between labor and intermediate inputs,

$$l_{kt}^d = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k \right). \tag{36}$$

Conditioning on sectors' gross output y_{kt}, this equation shows that a positive productivity shock in sector k directly decrease demand for labor in the shocked sector by a_{kt} and indirectly in all sectors by the effect of the productivity shock on sector-specific aggregate prices of intermediate inputs, p_t^k. This latter effect is due to firms substituting labor for cheaper intermediate inputs, the price of which the steady-state I/O linkages of sectors with sector k determine.

To see the way that productivity shocks affect sectors' gross output y_{kt}, we use the log-linear expression for Walras law

$$y_{kt} = \frac{(1-\psi)\,\omega_{ck}}{n_k} c_{kt} + \frac{\psi}{n_k} \sum_{k'=1}^{K} n_{k'} \omega_{k'k} z_{k't}(k), \tag{37}$$

such that sectoral gross output depends on households' demand as final goods and all sectors demand as intermediate inputs. The $\{n_k\}_{k=1}^{\infty}$ are the steady-state shares of sectors in aggregate gross output

$$n_k = (1-\psi)\,\omega_{ck} + \psi \sum_{k'=1}^{K} n_{k'} \omega_{k'k} \text{ for all } k = 1, ..., K. \tag{38}$$

$\psi \equiv Z/Y$ is the fraction of total gross output used as intermediate input in steady state.

Log-linear demands from households and sectors on goods produced in sector k are given

[11] The Online Appendix contains details of the derivations.

by

$$\begin{aligned} c_{kt} &= c_t - \eta \left(p_{kt} - p_t^c \right), \\ z_{k't}(k) &= z_{k't} - \eta \left(p_{kt} - p_t^k \right) \text{ for } k' = 1, ..., K. \end{aligned}$$

Thus, when sector k has a positive productivity shock, its demand from households and firms increases in the extent the price of sector k decreases relative to the price of goods produced in other sectors. This force pushes wages up in the shocked sector and down in all other sectors as households and firms decrease demand for all sectors with no positive shock. The strength of this effect depends on steady-state GDP shares for households' demand and steady-state input-output linkages.

Summing up, these effects create interdependence in the determination of wages. For $\varphi > 0$, wages solve

$$w_t = \Theta^{-1} \left[\theta_c c_t + \theta_p p_t - \varphi a_t \right], \tag{39}$$

where w_t is the vector of sectoral wages, and the parameters are

$$\begin{aligned} \Theta &\equiv (1 + \delta\varphi) \mathbb{I} - (1 + \varphi) \psi D^{-1} \Omega D; \\ \theta_c &\equiv \left[\mathbb{I} - \psi D^{-1} \Omega' D \right] \iota + \varphi (1 - \psi) D^{-1} \Omega_c; \\ \theta_p &\equiv \left[\mathbb{I} - \psi D^{-1} \Omega' D \right] \iota \Omega_c' - \varphi\eta \left[\mathbb{I} - (1 - \psi) D^{-1} \Omega_c \Omega_c' \right] \\ &\quad + \varphi \left[(\eta - 1) \psi D^{-1} \Omega' D \Omega - \delta \Omega \right], \end{aligned}$$

where $\mathbb{I}$ is a $K \times K$ identity matrix, D is a $K \times K$ matrix with vector $[n_k]_{k=1}^K$ on its diagonal, Ω_c is a column-vector of GDP shares $\{\omega_{ck}\}_{k=1}^K$, and Ω is the matrix $[\omega_{k'k}]_{k',k=1}^K$ with steady state input-output linkages across sectors.

This expression collapses to $w_{kt} = c_t + p_t^c$ when $\varphi = 0$. In the special case when $\delta = 0$ (i.e., no intermediate inputs), sectoral wages solve

$$w_t = (1 + \varphi) \iota c_t + \left[(1 + \varphi\eta) \iota \Omega_c' - \varphi\eta \mathbb{I} \right] p_t - \varphi a_t. \tag{40}$$

Although the interaction between price rigidity and sector size and I/O linkages is more involved, as these effects jointly create an interdependence of labor demand across sectors, our key insight in the previous subsection remains: heterogeneity in price rigidity affects the propagation of idiosyncratic sectoral productivity shocks by affecting the responsiveness of sectoral prices to these shocks together with GDP shares and input output linkages.

The general solution for χ' when $\delta > 0$ now becomes

$$\chi' = \Omega_c' \left[\mathbb{I} - \delta\left(\mathbb{I} - \Lambda\right)\Omega - \left(1-\delta\right)\left(\mathbb{I} - \Lambda\right)\Theta^{-1}\left(\theta_p - \theta_c\Omega_c'\right)\right]^{-1}\left(\mathbb{I} - \Lambda\right)\left[\mathbb{I} + \left(1-\delta\right)\varphi\Theta^{-1}\right]. \quad (41)$$

Although now functional forms are more involved, sectoral price rigidity affects the aggregate propagation of sectoral productivity shocks through distorting the effect of the distribution of GDP shares and input-ouput linkages.

From a different angle, to further explore the effect of elastic labor supply, consider the special case of no input-output linkages ($\delta = 0$), so

$$\chi' = \Omega_c'\left(\mathbb{I} - \Phi\right), \quad (42)$$

where Φ is a diagonal matrix with entries

$$\frac{1-\lambda_k}{1+\varphi\eta\left(1-\lambda_k\right)}\left[1 - \varphi\left(\eta - 1\right)\sum_{k'=1}^{K}\frac{\omega_{ck'}\left(1-\lambda_{k'}\right)}{1+\varphi\eta\left(1-\lambda_{k'}\right)}\right]^{-1}, \quad (43)$$

for $k = 1, ..., K$ on its diagonal. Note $\Phi = \Lambda$ when $\varphi = 0$. According to equation (28), the inverse-Frisch elasticity φ has two opposite effects on the capacity of price rigidity to generate aggregate volatility from sectoral productivity shocks. On the one hand, if sector k has more flexible prices, its demand responds by more to its own productivity shocks, so wages in the shocked sector respond by more. This effect is captured by the denominator of the term outside the brackets. On the other hand, the response of prices in the shocked sector has an effect on the demand of other sectors. This effect is captured by the term in brackets which is common to all sectors. Thus, in the absence of input-output linkages ($\delta = 0$), more elastic labor supply reduces the quantitative importance of price rigidity to generate fluctuations. However, because both effects operate through sectoral demand, the effect of φ depends on the elasticity of substitution across sectors, η. Quantitatively, empirical estimates suggest η is small (see Atalay (2017) and Feenstra, Luck, Obstfeld, and Russ (2018)).

Active Monetary Policy The specification of monetary policy crucially affects the response of GDP and aggregate prices to technology shocks in the presence of price rigidities. Because our exogenous nominal demand specification aims at providing intuition, we perform a detailed analysis of various monetary policy specification in the quantitative section. Here, we address one specific aspect of the choice of monetary policy: we show active monetary policy can eliminate the effect of changing the average level of price rigidity, while not directly affecting the effect of heterogeneity in pricing frictions in the propagation of shocks.

Consider the general solution for log-linear deviations of GDP for an arbitrary monetary policy assuming the monetary policy instrument is money supply

$$c_t = \left(1 - \chi'\iota\right) m_t + \chi' a_t, \tag{44}$$

where m_t is the log-linear deviation of money supply from steady state. This expression is valid for any assumption on the inverse-Frisch elasticity, $\varphi \geq 0$, and the joint distribution of sectoral GDP, input-output linkages, and price rigidity.

We can directly see from equation (44) monetary policy does not interact with the effect of heterogeneous price rigidity for the aggregate implication of sectoral shocks; it only introduces a scale effect regardless of whether shocks are aggregate or sector-specific. As the negative sign in the first term indicates, this effect partially offsets the effect due to changing average levels of price rigidity in the economy.

For a more concrete intuition, consider an extreme version of a price-level targeting such that monetary policy stabilizes the GDP deflator, by setting $p_t^c = 0$. In this case,

$$c_t = \frac{\chi' a_t}{\chi'\iota}. \tag{45}$$

Price-level targeting reproduces the flexible-price response of the economy to aggregate productivity shocks for any joint distribution of sectoral GDP, input-output linkages, and price rigidity. As a result, this monetary policy rule perfectly offsets the effect of changing average levels of price rigidity. Under alternative, inefficient monetary policy rules, the effect is not completely offset, because the joint distribution of sectoral GDP, input-output linkages, and price rigidity does affect the response of c_t to aggregate productivity shocks. The interactions of heterogeneity in nominal and real features will continue to affect the GDP deflator. We do not aim to study optimal policy but consider various empirically plausible monetary policy specifications in the quantitative section below.

The result that price level fluctuations are the mirror image of GDP fluctuations above is valid only for the monetary policy rule ensuring constant nominal GDP in steady state. Given the solution of c_t in equation (44), the price level solves

$$p_t^c = \chi' \left(\iota m_t - a_t\right). \tag{46}$$

Thus, the response of m_t to shocks affect the volatility of aggregate prices differently than of

GDP. An extreme example is price level targeting, when $p_t^c = 0$ by construction and c_t solves equation (45).

Pricing Friction Our analytical model uses a simplified form of price-setting. However, modeling price rigidities à la Calvo does not change our analytical results. We study the exact quantitative effects in our subsequent calibrations. Calvo pricing adds serial correlation in the response of prices even when shocks are i.i.d.

$$p_{kt} = (1 + \beta + \xi_k)^{-1} \left[\xi_k mc_{kt} + \beta \mathbb{E}\left[p_{kt+1}\right] + p_{kt-1}\right] \text{for} k = 1, ..., K, \tag{47}$$

where $\xi_k \equiv (1 - \alpha_k)(1 - \beta\alpha_k)/\alpha_k$.

The exact monetary policy specification will determine how much of the serial correlation the Calvo friction introduces transmits to the response of output. Parallel to our analytical results above, we adjust the definitions of the multipliers to capture possible serial correlation for our calibrations in the next section. Using a general representation of the solution for c_t,

$$c_t = \sum_{\tau=0}^{\infty} \sum_{k=1}^{K} \rho_{k\tau} a_{kt-\tau}, \tag{48}$$

we redefine multipliers as

$$\chi_k \equiv \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}, \tag{49}$$

such that $v_c = \|\chi\|_2 v$ still holds.

Endogenous Price Adjustment In our setup, price rigidity is exogenous, and thus the degree of price rigidity and the process of sectoral shocks are unrelated. With an endogenous pricing friction, a link could exist between price flexibility and the volatility of shocks, or other moments of the distribution of shocks, or model elements that pin down the sS bands. However, it is not clear ex-ante how these considerations would affect our results.

First, the key *theoretical* mechanism of our model would qualitatively remain unchanged. Again, the effective not the physical importance of a sector would determine the propagation of shocks. The effective importance would now only be differently defined: it would depend on the interaction of the real features with the primitives that pin down the width of the sS bands, such as menu costs, elasticities, or shock volatilities.

Second, this theoretical possibility thus comes with several degrees of freedom. A priori in which direction – if any – it would change our subsequent quantitative results is unclear. We cannot quantify such effects without implementing a state-dependent pricing model. Empirically, no strong relationship exists between the volatility of shocks and the frequency of price changes for manufacturing sectors. A regression of the frequency of price changes on the volatility of productivity shocks results in an R^2 of less than 2%, which shows these shocks explain very little of the variation in the frequency of price changes in our data.[12]

Third, such an analysis would face major computational challenges. Our level of disaggregation makes endogenous pricing models such as menu-cost models computationally infeasible because the intermediate input price indices represent a large number of state variables for each sector. For example, in our subsequent calibration, we work with 341 sectors. With not only heterogeneity in pricing frictions across sectors, but also heterogeneous input-output linkages, the firms in each of the 341 sectors have to solve a different dynamic programming problem with 340 different input price indices. This large number of state variables poses a challenge beyond currently available computational methods. Therefore, we leave a formal analysis of the implications of endogenous price rigidity for future research, and our results should be seen as a first-order approximation.

Independent of these considerations, differences in pass-through of shocks – however micro-founded – will change the effective real structure of the economy.

IV Data

This section describes the data we use to construct the input-output linkages, and sectoral GDP, and the micro-pricing data we use to construct sectoral measures of price stickiness and the standard deviation of productivity shocks.

A. Input-Output Linkages and Sectoral Consumption Shares

The BEA produces input-output tables detailing the dollar flows between all producers and purchasers in the US Producers include all industrial and service sectors, as well as household production. Purchasers include industrial sectors, households, and government entities. The BEA constructs the input-output tables using Census data that are collected every five years beginning in 1982. The input-output tables are based on NAICS industry codes. Prior to 1997, the input-output tables were based on SIC codes.

The input-output tables consist of two basic national-accounting tables: a “make” table and a “use” table. The make table shows the production of commodities by industry. Rows

[12] We detail the data we use and the construction of the variables in Section IV.

present industries, and columns present the commodities each industry produces. Looking across columns for a given row, we see all the commodities a given industry produces. The sum of the entries comprises industry output. Looking across rows for a given column, we see all industries producing a given commodity. The sum of the entries is the output of a commodity. The use table contains the uses of commodities by intermediate and final users. The rows in the use table contain the commodities, and the columns show the industries and final users that utilize them. The sum of the entries in a row is the output of that commodity. The columns document the products each industry uses as inputs and the three components of value added: compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. The sum of the entries in a column is industry output.

We utilize the input-output tables for 2002 to create an industry network of trade flows. The BEA defines industries at two levels of aggregation: detailed and summary accounts. We use the detailed levels of aggregation to create industry-by-industry trade flows. The BEA also provides the data to calibrate sectoral GDP shares.

The BEA provides concordance tables between NAICS codes and input-output industry codes. We follow the BEA's input-output classifications with minor modifications to create our industry classifications. We account for duplicates when NAICS codes are not as detailed as input-output codes. In some cases, an identical set of NAICS codes defines different input-output industry codes. We aggregate industries with overlapping NAICS codes to remove duplicates.

We combine the make and use tables to construct an industry-by-industry matrix that details how much of an industry's inputs other industries produce. We use the make table ($MAKE$) to determine the share of each commodity each industry k produces. We define the market share ($SHARE$) of industry k's production of commodities as

$$SHARE = MAKE \odot (\mathbb{I} - MAKE)^{-1}_{k,k'} .$$

We multiply the share and use tables (USE) to calculate the dollar amount industry k' sells to industry k. We label this matrix revenue share ($REVSHARE$), which is a supplier industry-by-consumer industry matrix,

$$REVSHARE = SHARE \times USE.$$

We then use the revenue-share matrix to calculate the percentage of industry k' inputs purchased from industry k, and label the resulting matrix $SUPPSHARE$

$$SUPPSHARE = REVSHARE \odot \left((\mathbb{I} - MAKE)^{-1}_{k,k'} \right)'. \tag{50}$$

The input-share matrix in this equation is an industry-by-industry matrix and therefore consistently maps into our model.[13]

B. Frequencies of Price Adjustments

We use the confidential microdata underlying the PPI from the BLS to calculate the frequency of price adjustment at the industry level.[14] The PPI measures changes in prices from the perspective of producers, and tracks prices of all goods-producing industries, such as mining, manufacturing, and gas and electricity, as well as the service sector. The BLS started sampling prices for the service sector in 2005. The PPI covers about 75% of the service-sector output. Our sample ranges from 2005 to 2011.

The BLS applies a three-stage procedure to determine the sample of goods. First, to construct the universe of all establishments in the US, the BLS compiles a list of all firms filing with the Unemployment Insurance system. In the second and third stages, the BLS probabilistically selects sample establishments and goods based on either the total value of shipments or the number of employees. The BLS collects prices from about 25,000 establishments for approximately 100,000 individual items on a monthly basis. The BLS defines PPI prices as "net revenue accruing to a specified producing establishment from a specified kind of buyer for a specified product shipped under specified transaction terms on a specified day of the month." Prices are collected via a survey that is emailed or faxed to participating establishments. Individual establishments remain in the sample for an average of seven years until a new sample is selected to account for changes in the industry structure.

We calculate the frequency of price changes at the goods level, FPA, as the ratio of the number of price changes to the number of sample months. For example, if an observed price path is \$10 for two months and then \$15 for another three months, one price change occurs during five months, and the frequency is 1/5. We aggregate goods-based frequencies to the BEA industry classification.

The overall mean monthly frequency of price adjustment is 16.78%, which implies an average duration, $-1/\log(1 - FPA)$, of 5.44 months. Substantial heterogeneity is present in

[13] Ozdagli and Weber (2016) follow a similar approach.

[14] The data have been used before in Nakamura and Steinsson (2008), Goldberg and Hellerstein (2011), Bhattarai and Schoenle (2014), Gorodnichenko and Weber (2016), Gilchrist, Schoenle, Sim, and Zakrajšek (2017), Weber (2015), and D'Acunto, Liu, Pflueger, and Weber (2016), among others.

the frequency across sectors, ranging from as low as 2.74% for the semiconductor manufacturing sector (duration of 35.96 months) to 96.47% for dairy production (duration of 0.30 months).

C. Volatility of Idiosyncratic Productivity Shocks

We also use the PPI micro price data to show that firms are subject to highly volatile idiosyncratic TFP movements – rather than aggregate shocks. In addition to heterogeneity in price rigidity, highly volatile idiosyncratic shocks are a key ingredient to generate our quantitative results.

We estimate the standard deviation of productivity shocks $\sigma^2_{a_k}$ faced by firms j in sector k as as the difference between the volatility of price changes $\sigma^2_{\Delta p_{kj}}$ in a sector k and any aggregate time trend α

$$\sigma^2_{a_k} = \sigma^2_{\Delta p_{kj}} - \alpha. \tag{51}$$

The idea behind this approach is a price adjustment reflects idiosyncratic shocks after netting out any movements in aggregate TFP.

Of course, markups may adjust as well, absorbing part of the shocks. In fact, a large literature on exchange rate pass-through shows, prices respond only incomplete to shocks even in the long run. Hence, our estimates are likely lower bounds to the true productivity processes.

Infrequent price adjustment might introduce a counteracting measurement problem. Infrequent price changes imply that we overstate the size of the monthly shock processes, mistaking the accumulation of several unobserved small changes as large monthly change conditional on observing adjustment. To address this concern, we only consider sectors where on average at least 80% of prices in a six-digit NAICS sector adjust every month. This is equivalent to an average price duration of approximately 1.25 months. As a result, the conditional price changes we consider should closely reflect the underlying monthly shocks. We also adjust the estimated standard deviations downwards by the square root of the expected duration of price spells, given by $1/FPA_k$.[15] These two adjustments, together with the underestimation of pass-through due to markup absorption give us confidence in the estimates of the size of shocks that drive the price adjustment process.

The data suggest sectoral productivity shocks, based on the PPI micro data, are extremely volatile, especially compared to aggregate shocks: As Table 1 and Figure 1 show, the estimated mean sectoral volatility in the full sample is 9.92% with a median of 8.40%. In our preferred subsample with large frequencies of price adjustment, it is 9.08%. Our estimates are in line with

[15] If underlying monthly innovations are i.i.d. and productivity follows a random walk, then the variance of observed changes is the sum of individual variances: $\sigma^2 = K\sigma^2_k$. As a result, monthly standard deviations are given by $\sigma^2_k = \sigma^2/K$.

the implication of Midrigan (2011) who requires a monthly standard deviation of idiosyncratic shocks between 8% and 11.2% to explain highly volatile price processes at the goods level. By contrast, the standard deviation of aggregate PPI inflation is only 0.34%. To the extent we miss parts of the shocks due to absorbing markup movements, then our estimates represent a lower bound.

V Calibration

We calibrate the steady-state input-output linkages of our model, Ω, to the US input-output tables in 2002. The same 2002 BEA data allow us to calibrate steady-state sectoral GDP shares, Ω_c. The Calvo parameters match the industry-average frequency of price adjustments between 2005 and 2011, using the micro data underlying the PPI from the BLS, for the same classification of industries used by the BEA.[16] When we consider calibrations with homogeneous price rigidity, we use the weighted mean, while we use shares of 1/341 for homogeneous size or input-output shares. We calibrate the monthly standard deviation of the idiosyncratic shocks to match 8.75%, also computed from BLS PPI microdata (see Section IV). After we merge the input-output and the frequency-of-price-adjustment data, we end up with 341 industries, which we refer as "sectors."

The 2002 BEA data has 407 unique sectors. We lose some of them for three reasons. First, some sectors produce almost exclusively final goods, so the BLS data do not contain enough observations to compute reliable average frequencies of price adjustment. Second, the goods some sectors produce do not trade in a formal market, so the BLS has no prices to record. Examples of missing sectors are (with input-output industry codes in parentheses) "Military armored vehicle, tank, and tank component manufacturing" (336992) or "Religious organizations" (813100). Third, the BEA data for some sectors are not available at the six-digit level.

All our calibrations are at a monthly frequency, so the discount factor is $\beta = 0.9967$ (4% annual risk-free interest rate). The elasticity of substitution across sectors is $\eta = 2$. No estimates of this elasticity exist at the level of disaggregation we require. Atalay (2017) reports an average elasticity of substitution of .15 for 30 industries; presumably a higher level of disaggregation implies higher elasticity of substitution across sectors. The elasticity of substitution within sectors is $\theta = 6$ in our baseline calibration, which implies a markup of 20%. We report robustness checks for elasticity values below. We set $\delta = 0.5$ so the intermediate-input share in steady state is $\delta * (\theta - 1)/\theta = 0.42$, which matches the 2002 BEA data. As discussed above, we assume

[16]We interpret the frequencies of price adjustments as the probability a sector can adjust prices after the shock.

sectoral productivity shocks are i.i.d. and serially uncorrelated with monthly standard deviation $\sigma_{a_k} = 9.08\%$ while the aggregate productivity shock is also serially uncorrelated with standard deviation $\sigma_a = 0.34\%$.

In our most preferred calibration we furthermore set the inverse-Frisch elasticity to $\varphi = 2$. Monetary policy follows a Taylor rule with parameters $\phi_c = 0.33/12 = 0.0275$ and $\phi_\pi = 1.34$. When we consider the output growth rate as an additional target, we assume a parameter of $\phi_{gc} = 1.5$. In the case of interest rate smoothing, we assume a persistence parameter of 0.9 following Coibion and Gorodnichenko (2012).

VI Quantitative Results

This section provides quantitative evidence for the importance of the interaction of heterogeneity in price rigidity with heterogeneity in real features of the economy for the propagation of idiosyncratic shocks. We show sectoral TFP shocks can generate sizable aggregate fluctuations of both GDP and prices. We also demonstrate the distortionary role price rigidities have for the identity of which sectors are important for aggregate fluctuations.

To highlight the mechanisms at work, we compare model economies with heterogeneous and homogeneous price rigidities on the nominal side while on the real side, we allow sectors to be either equally large with equal intermediate input consumption across sectors, or fully heterogeneous as in the data. These comparisons are analogous to the exposition in our simple model. This section also demonstrates the importance of large idiosyncratic shocks relative to aggregate shocks as an ingredient to generate our results. Finally, while different empirically plausible monetary policy rules affect the exact level of aggregate fluctuations, heterogeneity in price rigidity and large idiosyncratic productivity shocks generate large aggregate fluctuations under all empirically plausible monetary policy frameworks we consider.

A. Main Results

Heterogeneity of price rigidity plays a key role in amplifying the propagation of idiosyncratic productivity shocks to the aggregate economy. We show this result by comparing a version of our calibrated economy where we go from homogeneous pricing frictions to heterogeneous pricing frictions across sectors. Our baseline calibration otherwise features fully heterogeneous sector size and input-output linkages, and monetary policy follows a flexible-inflation targeting Taylor rule. Labor is supplied elastically. We use this baseline model to study if our theoretical results from the simplified model in Section III are empirically relevant or not in a more realistic model.

The theoretical propositions matter quantitatively: allowing for fully heterogeneous price rigidity across sectors doubles the effect of idiosyncratic shocks on aggregate fluctuations. In fact, we can generate a monthly standard deviation of aggregate consumption of 0.41%, which is close to a realistic target of 0.51% used in calibrations of RBC models (see for example, Sims (2017).[17] We see in row 1 of Table 2 the standard deviation under homogeneous price rigidity is only 0.22% and approximately doubles to 0.41% when we allow for heterogeneous price rigidities.

What mechanism is behind these results? First, at an abstract level, the identity of which sectors are important for aggregate fluctuations changes due to the interaction of heterogeneous size and I/O structure with heterogeneous pricing frictions. Hence, some sectors may become effectively larger or effectively more central, and hence the overall propagation capacity in the economy may increase. We summarize this change in the importance of sectors for aggregate fluctuations in Figure 2 where we go from homogeneous pricing frictions to fully heterogeneous pricing frictions, with heterogeneous I/O linkages and sectoral size in both cases. We plot the ranks of sectoral contributions to aggregate fluctuations. A rank of 2 for example means that the share of GDP volatility generated by this sector is the second largest share. If there was no role for heterogeneous pricing frictions, sectors would line up along the 45 degree line.

Notably, the introduction of sectoral heterogeneity in pricing frictions changes the identity of sectors that drive aggregate fluctuations. For example, sector 211000 (Oil and Gas Extraction) ranks thirty-third under homogeneous pricing frictions, but it is the seventh most important sector under heterogeneous pricing frictions.

The simple model, in particular Lemma 2 and Lemma 4, suggests the exact results of the interaction of real features with heterogeneity in pricing frictions depend in a complicated fashion on the exact variance-covariance matrices. Two plots illustrate that the underlying *effective* size and centrality distributions indeed change due to the presence of heterogeneous pricing frictions, which is what our analytical section suggests. The x-axis of Figure 3 plots the ranks of sectors according to their physical size (ω_{ck}). The y-axis plots the ranks according to effective size ($\omega_{ck} * (1 - \lambda_k)$). Similarly, Figure 4 illustrates the change in the distribution of network centrality to effective network centrality. The figures suggest changes in *effective* size and centrality distributions are behind the changes in propagation capacity, we next demonstrate in detail the quantitative contributions of each model ingredient.

In particular, we demonstrate that each heterogeneous real feature of the economy has a larger effect on aggregate fluctuations when it interacts with heterogeneous pricing frictions,

[17] We convert the conventional quarterly target of 0.9% to a monthly number via $0.51\% = 0.9\%/3^{0.5}$.

rather than when pricing frictions are homogeneous across sectors. We also show that heterogeneity in pricing frictions by itself can generate substantial fluctuations from sectoral shocks. Yet, only the full interaction of all nominal and real heterogeneities generates large aggregate fluctuations. We present these results in rows (2) through (5) of Table 2. Relative to our baseline, we first remove heterogeneity in input-output linkages and assume each sector buys $1/341^{th}$ of its inputs equally from all sectors, including itself ("hom. Ω"). Now, the model generates a monthly standard deviation of 0.31% under heterogeneous pricing frictions, and of 0.16% under homogeneous pricing frictions. When we instead assume all sectors have equal size ("hom. Ω_C"), the model generates aggregate volatility of 0.1% under heterogeneous pricing frictions, and 0.02% under homogeneous pricing frictions. These two comparisons show heterogeneity in size has a stronger interaction effect with heterogeneity in pricing frictions in propagating idiosyncratic shocks than the convolution of heterogeneous I/O structure and price rigidities.

The last two rows of the Table 2 show under homogeneous size and homogeneous I/O linkages, or no intermediate input consumption at all ($\delta = 0$), heterogeneity in pricing frictions still substantially increases aggregate volatility. The monthly standard deviation increases from 0.06% to 0.12%, and from 0.01% to 0.14%, respectively, as we move from homogeneous to heterogeneous price rigidities. However, as the model predicts, we see in Table 2 only the full interaction of heterogeneous price rigidities with heterogeneity in size and I/O linkages creates sizable aggregate fluctuations. We provide further details in Online Appendix Section III*B.* on how the simple model predicts the results for the various combinations of heterogeneities across rows of Table 2.

Large idiosyncratic productivity shocks – relative to aggregate shocks – are a further key ingredient to generate empirically realistic aggregate fluctuations, an important result of our paper. Table 3 illustrates the importance of this assumption and assumes idiosyncratic shocks, while still i.i.d. across sectors, have the same standard deviation as aggregate productivity shocks ($\sigma_{a_k} = \sigma_a$). Columns (1) and (2) report the monthly consumption standard deviation in a calibration with idiosyncratic shocks, whereas columns (3) and (4) report the monthly consumption standard deviation in a calibration with aggregate shocks. Now, the model generates a level of aggregate fluctuations that is too low compared to the fluctuations we observe in the data. The predicted standard deviation of value added is a mere 0.02% in our fully heterogeneous baseline economy and only about half under homogeneous pricing friction (row (1), columns (1) and (2)). Rows (2) to (5) show aggregate volatility from sectoral shocks is even lower in other model calibrations.

While it is widely understood in the empirical literature on price setting that microeconomic shocks can be very large, these results emphasize the importance of this assumption for macroeconomic modeling. To better understand the magnitude of sectoral shocks needed to match aggregate fluctuations in our baseline model, we use simulations to find the idiosyncratic volatility required to match an aggregate volatility of 0.52%. We find sectoral productivity shocks need to have an average standard deviation of 11.47% in our baseline model, close to the number of 9.08% we use in our baseline calibration.

This number contrasts with a monthly aggregate productivity shock standard deviation of about 0.34% in the data that we use in the calibration of Table 3. An aggregate shock of this magnitude only generates a standard deviation of 0.052% under homogeneous and of 0.106% under heterogeneous price rigidities (see row (1), columns (3) and (4)). The simulations of the model show we would require an aggregate productivity shock with a monthly volatility of 1.66% in the fully heterogeneous economy to match aggregate volatility, fives-times bigger than empirical estimates and of 3.3% in an economy with homogeneous price rigidity, ten times the empirical estimates. The results for estimated and simulated aggregate shocks are in stark contrast to the estimates of sectoral shock volatility of 9.08% and the simulated one of 11.47% which are of similar magnitude.

Independent of the specific measurement of sectoral shocks using PPI micro data, the magnitude of sectoral shock volatility provides a useful benchmark. Idiosyncratic shocks need to be more volatile than aggregate shocks to generate a sufficient level of aggregate volatility in our baseline calibration. To be precise, they need to be larger by a factor of 11.47/1.66=7 based on the calibrations. The conventional wisdom – productivity shocks cannot generate sufficient aggregate volatility when prices are sticky – is not true if price rigidity is heterogeneous and if shocks are idiosyncratic and have an empirically realistic magnitude. It is true, however, that aggregate shocks with plausible volatility do not generate realistic aggregate fluctuations in the presence of price stickiness, independent of whether price rigidities are homogeneous or heterogeneous across sectors.

B. The Role of Monetary Policy

While different empirically plausible monetary policy rules affect the exact level of aggregate fluctuations, heterogeneity in price rigidity, size, input-output linkages and large idiosyncratic productivity shocks generate large aggregate fluctuations under all empirically plausible monetary policy frameworks we consider.

We show this result by using different, empirically relevant monetary policy rules instead of the flexible inflation targeting Taylor rule in the baseline: a nominal GDP target ($p_t^c + c_t = 0$), and Taylor rules that feature strict inflation targeting ($i_t = \phi_\pi \pi_t$), GDP growth targeting ($i_t = \phi_\pi \pi_t + \phi_c c_t + \phi_{gc} gc_t$), as well as interest rate smoothing ($i_t = \rho_i i_{t-1} + (1-\rho_i) * (\phi_\pi \pi_t + \phi_c c_t + \phi_{gc} gc_t)$). Finally, to study a more active policy rule, we consider strict price level targeting ($p_t^c = 0$).

We find that under all policy rules, our calibrated model generates aggregate fluctuations of sizable magnitude. The monthly standard deviations of aggregate output fluctuations range from 0.32% to 0.57% across these monetary policy rules, compared to 0.52% in the data. Specifically, a Taylor that features strict inflation targeting generates an aggregate standard deviation of consumption of 0.57%. Nominal GDP targeting generates a standard deviation of 0.34%. Taylor rules that also put weight on GDP growth, and additionally, also feature interest rate smoothing both generate a volatility of 0.32%. Column (2) of Table 4 summarizes in detail the effects of the different monetary policy rules.

By contrast, under price-level targeting, our model generates a monthly standard deviation of GDP of 3.59% substantially larger than the monthly target of 0.52% which is no surprise given the active nature of the policy rule. It can be interpreted in several ways. First, it leaves room for a smaller required standard deviation of the underlying productivity shocks to generate aggregate fluctuations. Second, one may think that this monetary policy rule is not empirically as plausible as the other monetary policy rules. Third, allowing for some degree of passivity in the monetary rule under pricing frictions may even be necessary to get the level of fluctuations right – that is, low enough to be near a 0.52% target.

Considering these alternative monetary policy rules also allows us to study the effects on price stability.[18] Columns (3) and (4) of Table 4 show the result. Heterogeneity in pricing frictions increases the standard deviation of not only GDP, but also the price level, as our simple model predicts. A Taylor rule that features flexible inflation targeting in a model with heterogeneity in pricing frictions generates a standard deviation of 1.71% relative to 1.65% under homogeneous pricing frictions. Under a Taylor rule that also features interest rate smoothing, the monthly standard deviation of prices is 0.27% under homogeneous price rigidities, and 0.37% under heterogeneous price rigidities.

C. Robustness

We show our results are robust to assuming different parameter values, and to allowing for full heterogeneity in idiosyncratic productivity shocks across sectors. One might be concerned a low elasticity of substitution within sector might partially drive our findings. Table 5 shows in the

[18] Under a price-level target, this effect is not possible by definition.

left two columns results when we increase the within-sectors elasticity of substitution, θ, from a baseline value of 6 to 11 which is equivalent to reducing the markup from 20% to 10%. Columns (3) and (4) show what happens when the elasticity of substitution across sectors η is raised from 2 to 3.

When we increase θ to a value of 11, we find idiosyncratic shocks generate a 0.50% standard deviation of GDP rather than 0.41% under the baseline Taylor rule (compare row (1) column (2) of Table 5 to row (1) column (2) of Table 4), matching the data even (0.52%) better. Other monetary policy rules also generate sizable aggregate volatility from sectoral shocks which are always larger under heterogeneous price rigidities than under homogeneous price rigidities with the exception of price level targeting. When we instead increase η to a value of 3, we find a somewhat lower aggregate volatility of 0.38% in our fully heterogeneous calibration, which is still close to the baseline estimate of 0.41 (compare row (1) column (4) of Table 5 to row (1) column (2) of Table 4).

Finally, we study the results of allowing for a fully heterogeneous distribution of TFP shock volatility, where each sector receives shocks from a distribution with a different standard deviation that we estimate empirically. Overall, we see in Table 6, our model generates a very similar level of aggregate fluctuations. Allowing for heterogeneous instead of homogeneous pricing frictions increases consumption volatility from 0.17% to 0.38%, similar to the results in Table 2. The other rows in Table 6 show a similarly important effect of heterogeneity in pricing frictions for propagating idiosyncratic productivity shocks across different calibrations.

VII Concluding Remarks

To date, the implications of price rigidity, and frictions in general, for the importance of idiosyncratic shocks for aggregate fluctuations remain largely unexplored. We study the effect of price rigidity on the potential of sectoral shocks to drive aggregate fluctuations. We do so theoretically and quantitatively in a calibrated 341-sector New Keynesian model with heterogeneity in sector size, sector input-output linkages, and output-price stickiness. Our analysis suggests price rigidity has direct and important implications for the modeling and understanding of business cycles. In conjunction with realistically calibrated productivity shocks, it can generate large, realistically sized aggregate fluctuations. The interaction between different heterogeneities also has important implications for the conduct of monetary policy.

A central bank that aims to stabilize sectoral prices of "big" or "central" sectors might make systematic policy mistakes if it does not take into account the "frictional" origin of aggregate fluctuations. Our results hold under different assumptions on the conduct of monetary policy but we find price-level targeting even generates too much aggregate volatility from sectoral shocks which might suggest the necessity of pricing frictions to get empirically plausible aggregate fluctuations from micro-level shocks. Although beyond the scope of this paper, future work could explore the design of optimal monetary policy in a heterogeneous production economy.

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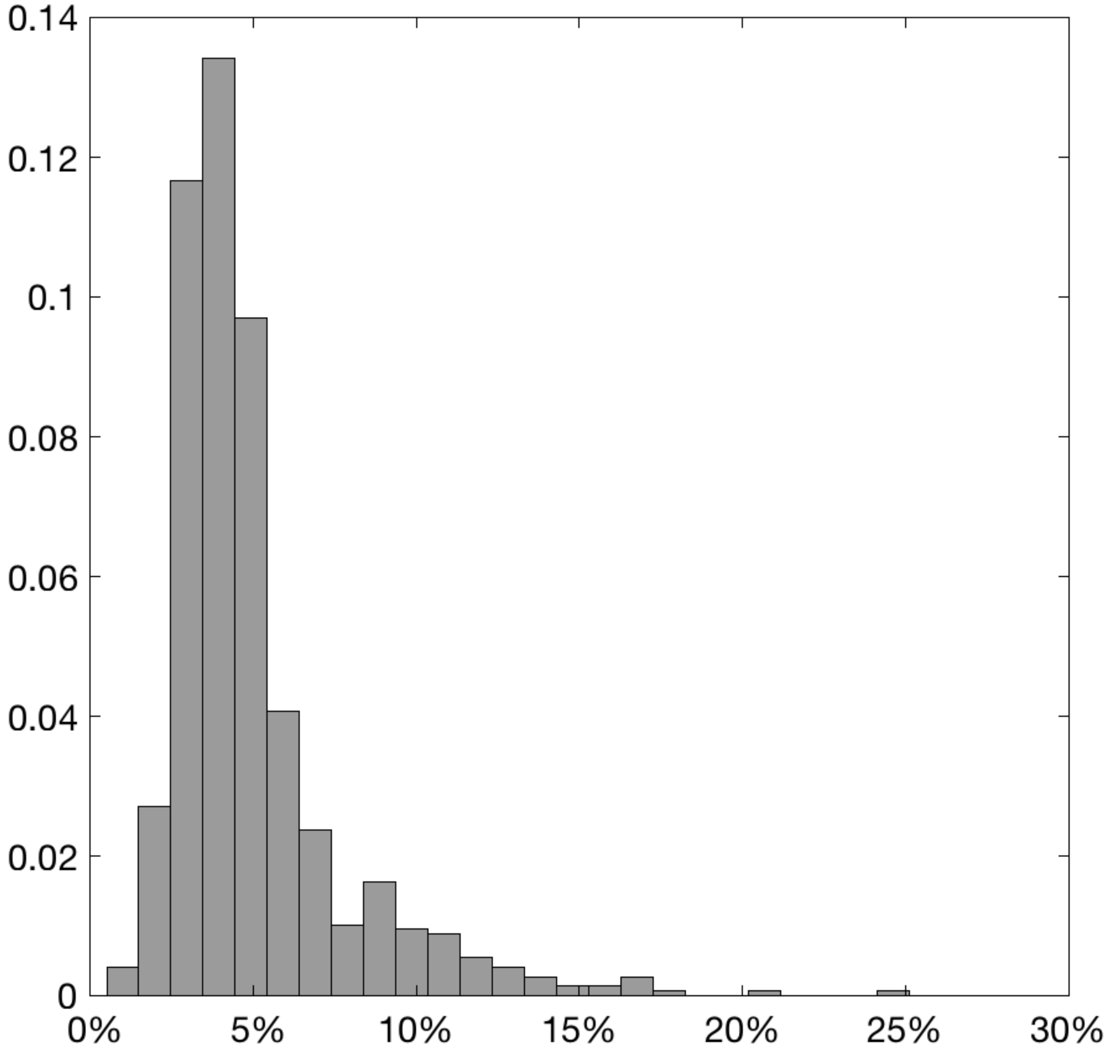
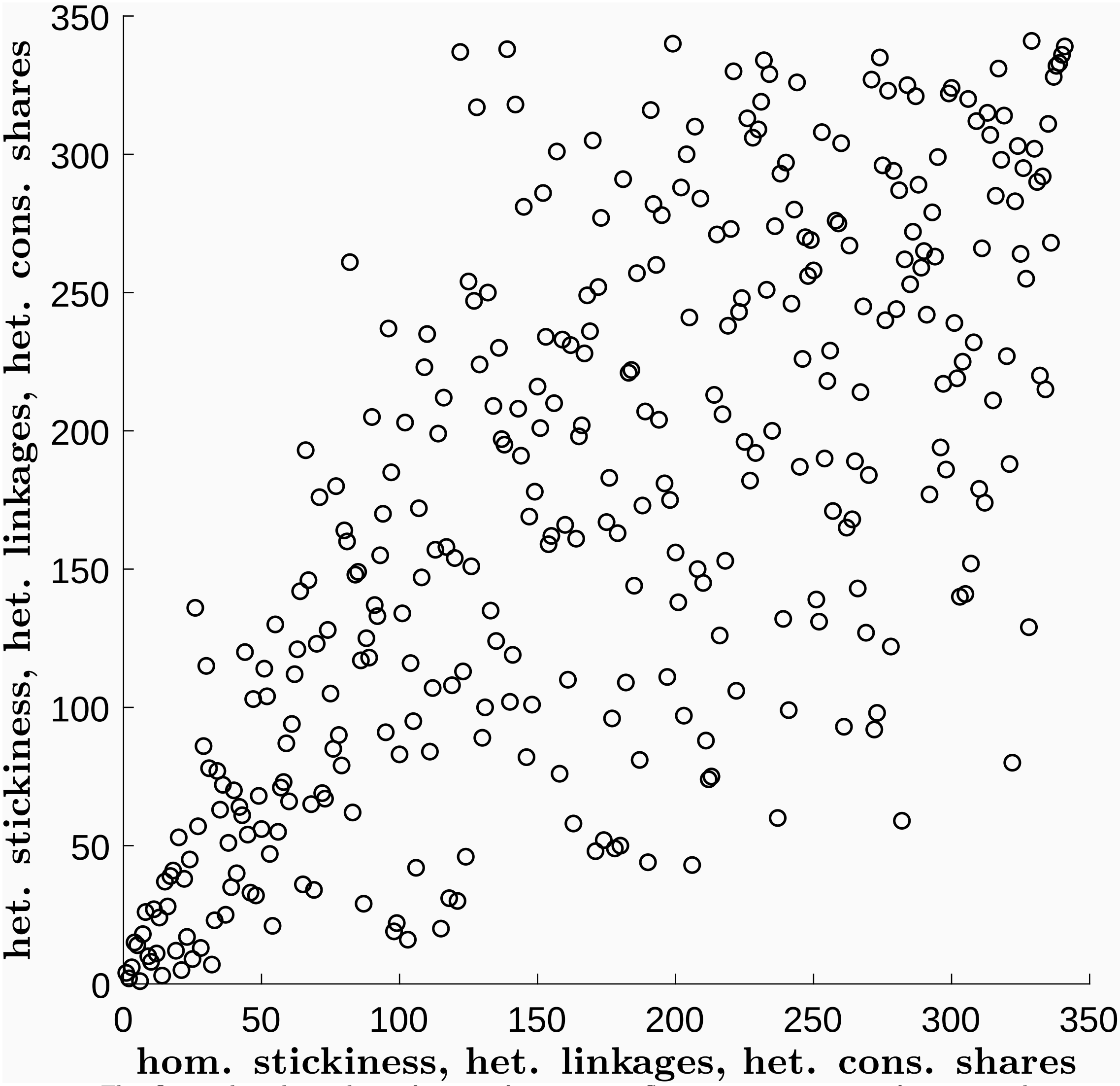
Figure 1: **Estimated Standard Deviation of Sectoral Shocks: Histogram**



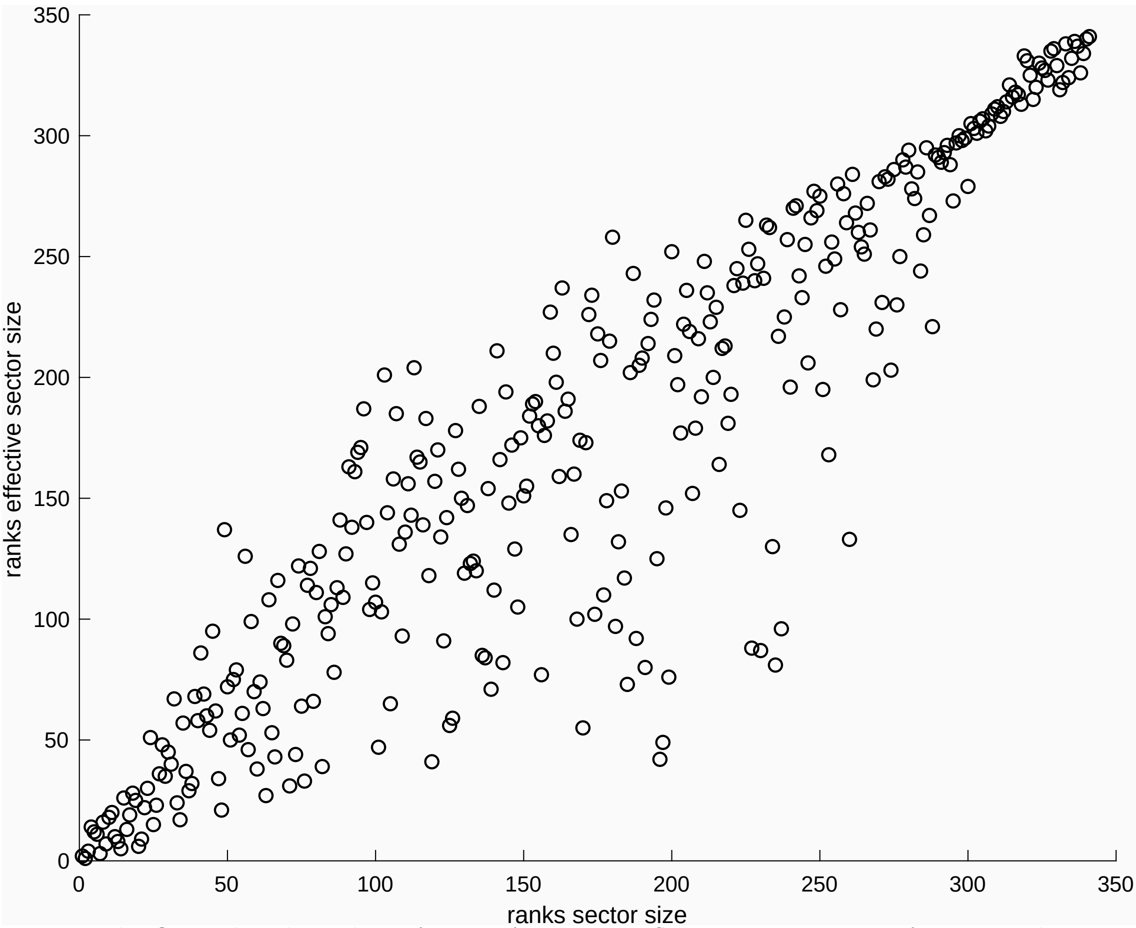
This figure plots the histogram of estimated monthly standard deviations of productivity shocks at the 341 sector level determined by the BEA using micro data underlying the producer price index at the Bureau of Labor Statistics. See Section IV for details of the data construction.

Figure 2: **Ranking of Sectors: Heterogeneous versus Homogeneous Price Stickiness**



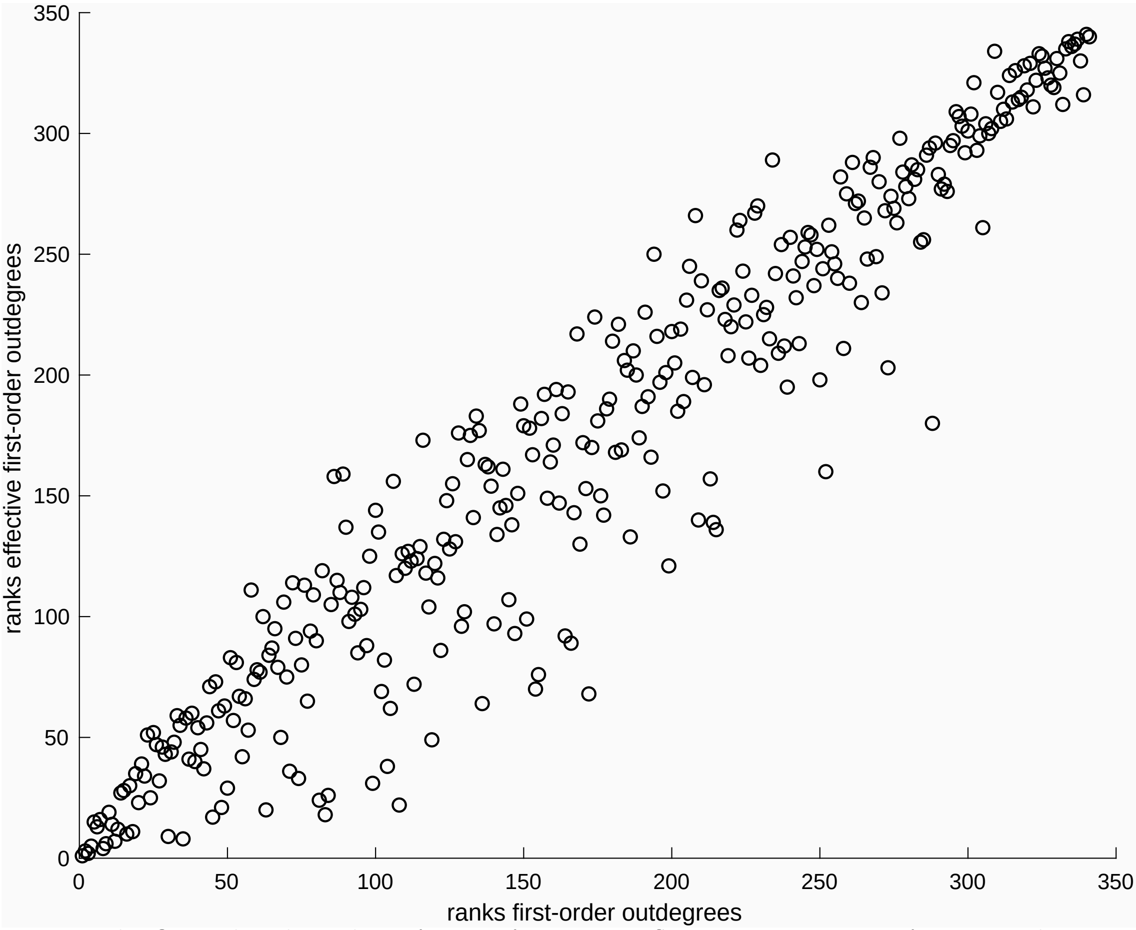
This figure plots the ranking of sectors for aggregate fluctuations originating from sectoral shocks for an economy with heterogeneous price stickiness across sectors (y-axis) and an economy with identical price stickiness for all sectors (x-axis). We assume heterogeneous GDP shares and input-output linkages calibrated to the US in both cases.

Figure 3: **Ranking of Sectors: Effective versus Physical Size Distribution**



This figure plots the ranking of sectors for aggregate fluctuations originating from sectoral shocks for an economy with heterogeneous price stickiness across sectors (y-axis) and an economy with identical price stickiness for all sectors (x-axis). We assume heterogeneous GDP shares and input-output linkages calibrated to the US in both cases.

Figure 4: **Ranking of Sectors: Effective versus Physical Centrality Distribution**



This figure plots the ranking of sectors for aggregate fluctuations originating from sectoral shocks for an economy with heterogeneous price stickiness across sectors (y-axis) and an economy with identical price stickiness for all sectors (x-axis). We assume heterogeneous GDP shares and input-output linkages calibrated to the US in both cases.

Table 1: **Estimated Standard Deviation of Sectoral Shocks: Distribution**

This table reports the moments of estimated monthly standard deviations of productivity shocks at the 341 sector level determined by the BEA using micro data underlying the producer price index at the Bureau of Labor Statistics. See Section IV for details of the data construction.

Mean	Std	Mean (FPA>0.8)	Adj Mean	P5	P25	Median	P75	P95
9.92%	5.47%	9.08%	3.67%	4.67%	6.70%	8.39%	10.91%	21.33%

Table 2: **Aggregate Volatility from Sectoral Shocks: Baseline**

This table reports the aggregate monthly GDP volatility from sectoral shocks. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index (PPI) from the BLS, and the average sectoral shock volatility that we also calculate using the PPI micro data. Monetary policy follows a Taylor rule.

			Homogeneous Calvo (1)	Heterogeneous Calvo (2)
(1)	het Ω_c	het Ω	0.22%	0.41%
(2)	het Ω_c	hom Ω	0.16%	0.31%
(3)	hom Ω_c	het Ω	0.02%	0.10%
(4)	hom Ω_c	hom Ω	0.06%	0.12%
(5)	hom Ω_c	$\delta = 0$	0.01%	0.14%

Table 3: **Aggregate Volatility from Sectoral and Aggregate Shocks: Equal-Shock Size**

This table reports the aggregate monthly GDP volatility from sectoral shocks. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index PPI from the BLS, and sectoral shocks (columns (1) and (2)) equal in size to aggregate shocks (columns (3) and (4)). Monetary policy follows a Taylor rule.

			Hom Calvo	Het Calvo	Hom Calvo	Het Calvo
			(1)	(2)	(3)	(4)
			Sectoral Shocks		Aggregate Shocks	
(1)	het Ω_c	het Ω	0.01%	0.02%	0.05%	0.11%
(2)	het Ω_c	hom Ω	0.01%	0.01%	0.04%	0.03%
(3)	hom Ω_c	het Ω	0.00%	0.00%	0.01%	0.01%
(4)	hom Ω_c	hom Ω	0.00%	0.00%	0.04%	0.05%
(5)	hom Ω_c	$\delta = 0$	0.00%	0.01%	0.01%	0.04%
(6)	Shock size to match data		21.32%	11.47%	3.30%	1.66%

Table 4: **Aggregate Volatility from Sectoral Shocks: Monetary Policy**

This table reports the aggregate monthly GDP and price-level volatility from sectoral shocks. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share for different monetary policy implementations. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index (PPI) from the BLS, and the average sectoral shock volatility that we also calculate using the PPI micro data.

		Hom Calvo	Het Calvo	Hom Calvo	Het Calvo
		(1)	(2)	(3)	(4)
		GDP Volatility		Price Volatility	
(1)	Flexible Inflation Targeting	0.22%	0.41%	1.65%	1.71%
(2)	Price Level Targeting	3.59%	3.59%	0.00%	0.00%
(3)	Nominal GDP Targeting	0.23%	0.34%	0.23%	0.34%
(4)	Strict Inflation Targeting	0.41%	0.57%	4.49%	5.05%
(5)	Output Growth in Taylor Rule	0.20%	0.32%	0.26%	0.36%
(6)	Interest Rate Smoothing	0.21%	0.32%	0.27%	0.37%

Table 5: **Aggregate Volatility from Sectoral Shocks: Variations in Parameters**

This table reports the aggregate monthly GDP volatility from sectoral shocks. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share for different monetary policy implementations. We also change the within-sectors elasticity of substitution, θ, from a baseline value of 6 to 11 and the elasticity of substitution across sectors, η, from 2 to 3. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index (PPI) from the BLS, and the average sectoral shock volatility that we also calculate using the PPI micro data.

		Hom Calvo	Het Calvo	Hom Calvo	Het Calvo
		(1)	(2)	(3)	(4)
		$\theta = 11$		$\eta = 3$	
(1)	Flexible Inflation Targeting	0.25%	0.50%	0.25%	0.38%
(2)	Price Level Targeting	2.93%	2.85%	3.58%	3.63%
(3)	Nominal GDP Targeting	0.25%	0.39%	1.69%	2.38%
(4)	Strict Inflation Targeting	0.26%	0.51%	0.11%	0.26%
(5)	Output Growth in Taylor Rule	0.23%	0.37%	0.25%	0.36%
(6)	Interest Rate Smoothing	0.23%	0.37%	0.25%	0.37%

Table 6: **Aggregate Volatility from Sectoral Shocks: Heterogeneous Shocks**

This table reports the aggregate quarterly GDP volatility from sectoral shocks. Ω_c represents the vector of GDP shares, Ω the matrix of input-output linkages, and δ the intermediate input share. We calibrate a 341-sector version of our model to the input-output tables and sector size from the BEA, the frequencies of price adjustment from the micro data underlying the Producer Price Index (PPI) from the BLS, and the sector-specific shock volatility that we also calculate using the PPI micro data. Monetary policy follows a Taylor rule.

			Homogeneous Calvo (1)	Heterogeneous Calvo (2)
(1)	het Ω_c	het Ω	0.17%	0.38%
(2)	het Ω_c	hom Ω	0.13%	0.32%
(3)	hom Ω_c	het Ω	0.02%	0.09%
(4)	hom Ω_c	hom Ω	0.03%	0.11%
(5)	hom Ω_c	$\delta = 0$	0.00%	0.11%

Online Appendix:
Price Rigidities and the Granular Origins of Aggregate Fluctuations

Ernesto Pasten, Raphael Schoenle, and Michael Weber

Not for Publication

I Steady-State Solution and Log-linear System

A. Steady-State Solution

Without loss of generality, set $a_k = 0$. We show below conditions for the existence of a symmetric steady state across firms in which

$$W_k = W,\ Y_{jk} = Y,\ L_{jk} = L,\ Z_{jk} = Z,\ P_{jk} = P \text{ for all } j, k.$$

Symmetry in prices across all firms implies

$$P^c = P^k = P_k = P$$

such that, from equations (2), (3), (11), and (14) in the main body of the paper,

$$\begin{aligned} C_k &= \omega_{ck} C, \\ C_{jk} &= \frac{1}{n_k} C_k, \\ Z_{jk}\left(k'\right) &= \omega_{kk'} Z, \\ Z_{jk}\left(j', k'\right) &= \frac{1}{n_{k'}} Z_{jk}\left(k'\right). \end{aligned}$$

The vector $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]'$ represents steady-state sectoral shares in value-added C, $\Omega = \{\omega_{kk'}\}_{k,k'=1}^{K}$ is the matrix of input-output linkages across sectors, and $\aleph \equiv [n_1, ..., n_K]'$ is the vector of steady-state sectoral shares in gross output Y.

It also holds that

$$\begin{aligned} C &= \sum_{k=1}^{K} \int_{\Im_k} C_{jk} dj, \\ Z_{jk} &= \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{jk}\left(j', k'\right) dj' = Z. \end{aligned}$$

From Walras' law in equation (21) and symmetry across firms, it follows

$$Y = C + Z. \tag{A.1}$$

Walras' law also implies for all j, k

$$\begin{aligned} Y_{jk} &= C_{jk} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{j'k'}\left(j, k\right) dj', \\ Y &= \frac{\omega_{ck}}{n_k} C + \frac{1}{n_k} \left(\sum_{k'=1}^{K} n_{k'} \omega_{k'k} \right) Z, \end{aligned}$$

so $\aleph$ satisfies

$$\begin{aligned} n_k &= (1-\psi)\,\omega_{ck} + \psi \sum_{k'=1}^{K} n_{k'} \omega_{k'k}, \\ \aleph &= (1-\psi)\left[I - \psi \Omega'\right]^{-1} \Omega_c, \end{aligned}$$

for $\psi \equiv \frac{Z}{Y}$. Note by construction $\aleph' \iota = 1$, which must hold given the total measure of firms is 1.

Steady-state labor supply from equation (8) is

$$\frac{W_k}{P} = g_k L_k^{\varphi} C^{\sigma}.$$

In a symmetric steady state, $L_k = n_k L$, so this steady state exists if $g_k = n_k^{-\varphi}$ such that $W_k = W$ for all k. Thus, steady-state labor supply is given by

$$\frac{W}{P} = L^{\varphi} C^{\sigma}. \tag{A.2}$$

Households' budget constraint, firms' profits, production function, efficiency of production (from equation (17)) and optimal prices in steady state are, respectively,

$$CP = WL + \Pi \tag{A.3}$$

$$\Pi = PY - WL - PZ \tag{A.4}$$

$$Y = L^{1-\delta} Z^{\delta} \tag{A.5}$$

$$\delta WL = (1-\delta) PZ \tag{A.6}$$

$$sP = \frac{\theta}{\theta - 1} \xi W^{1-\delta} P^{\delta} \tag{A.7}$$

for $\xi \equiv \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta}$.

Equation (A.7) solves

$$\frac{W}{P} = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{1-\delta}}. \tag{A.8}$$

This latter result together with equations (A.5), (A.6), and (A.7) solves

$$\frac{\Pi}{P} = \frac{1}{\theta} Y.$$

Plugging the previous result in equation (A.4) and using equation (A.1) yields

$$\begin{aligned} C &= \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right] Y \\ Z &= \delta \left(\frac{\theta - 1}{\theta} \right) Y, \end{aligned} \tag{A.9}$$

such that $\psi \equiv \delta \left(\frac{\theta - 1}{\theta} \right)$.

This result and equation (A.7) gives

$$L = \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\delta}{1-\delta}} Y,$$

where Y from before together with equations (A.2), (A.9) and (A.8) solves

$$Y = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{(1-\delta)(\sigma+\varphi)}} \left[\delta \left(\frac{\theta - 1}{\theta} \right) \right]^{\frac{\delta \varphi}{(1-\delta)(\sigma+\varphi)}} \left[1 - \delta \left(\frac{\theta - 1}{\theta} \right) \right]^{-\frac{\sigma}{\sigma+\varphi}}.$$

B. Log-linear System

B.1 Aggregation

Aggregate and sectoral consumption which we interpret as value-added, given by equations (2) and (3), are

$$\begin{aligned} c_t &= \sum_{k=1}^{K} \omega_{ck} c_{kt}, \\ c_{kt} &= \frac{1}{n_k} \int_{\Im_k} c_{jkt} dj. \end{aligned} \tag{A.10}$$

Aggregate and sectoral production of intermediate inputs are

$$\begin{aligned} z_t &= \sum_{k=1}^{K} n_k z_{kt}, \\ z_{kt} &= \frac{1}{n_k} \int_{\Im_k} z_{jkt} dj, \end{aligned} \tag{A.11}$$

where equations (11) and (14) imply that $z_{jk} = \sum_{r=1}^{K} \omega_{kr} z_{jk}(r)$ and $z_{jk}(r) = \frac{1}{n_r} \int_{\Im_r} z_{jk}(j', r) \, dj'$.

Sectoral and aggregate prices are (equations (5), (7), and (13)),

$$\begin{aligned} p_{kt} &= \int_{\Im_k} p_{jk} dj \text{ for } k = 1, ..., K \\ p_t^c &= \sum_{k=1}^{K} \omega_{ck} p_{kt}, \\ p_t^k &= \sum_{k'=1}^{K} \omega_{kk'} p_{k't}. \end{aligned}$$

Aggregation of labor is

$$\begin{aligned} l_t &= \sum_{k=1}^{K} l_{kt}, \\ l_{kt} &= \int_{\Im_k} l_{jkt} dj. \end{aligned} \tag{A.12}$$

B.2 Demand

Households' demands for goods in equations (4) and (6) for all $k = 1, ..., K$ become

$$\begin{aligned} c_{kt} - c_t &= \eta \left(p_t^c - p_{kt} \right), \\ c_{jkt} - c_{kt} &= \theta \left(p_{kt} - p_{jkt} \right). \end{aligned} \tag{A.13}$$

In turn, firm jk's demands for goods in equation (12) and (15) for all $k, r = 1, ..., K$,

$$\begin{aligned} z_{jkt}\left(k'\right) - z_{jkt} &= \eta \left(p_t^k - p_{k't} \right), \\ z_{jkt}\left(j', k'\right) - z_{jkt}\left(k'\right) &= \theta \left(p_{k't} - p_{j'k't} \right). \end{aligned} \tag{A.14}$$

Firms' gross output satisfies Walras' law,

$$y_{jkt} = (1 - \psi)\, c_{jkt} + \psi \sum_{k'=1}^{K} \int_{\Im_{k'}} z_{j'k't}\,(j, k)\, dj'. \tag{A.15}$$

Total gross output follows from the aggregation of equations (21),

$$y_t = (1 - \psi)\, c_t + \psi z_t. \tag{A.16}$$

B.3 IS and Labor Supply

The household Euler equation in equation (9) becomes

$$c_t = \mathbb{E}_t \left[c_{t+1} \right] - \sigma^{-1} \left\{ i_t - \left(\mathbb{E}_t \left[p_{t+1}^c \right] - p_t \right) \right\}.$$

The labor supply condition in equation (8) is

$$w_{kt} - p_t^c = \varphi l_{kt} + \sigma c_t. \tag{A.17}$$

B.4 Firms

Production function:

$$y_{jkt} = a_{kt} + (1 - \delta)\, l_{jkt} + \delta z_{jkt} \tag{A.18}$$

Efficiency condition:

$$w_{kt} - p_t^k = z_{jkt} - l_{jkt} \tag{A.19}$$

Marginal costs:

$$mc_{kt} = (1 - \delta)\, w_{kt} + \delta p_t^k - a_{kt} \tag{A.20}$$

Optimal reset price:

$$p_{kt}^* = (1 - \alpha_k \beta)\, mc_{kt} + \alpha_k \beta \mathbb{E}_t \left[p_{kt+1}^* \right]$$

Sectoral prices:

$$p_{kt} = (1 - \alpha_k)\, p_{kt}^* + \alpha_k p_{kt-1}$$

B.5 Taylor Rule:

$$i_t = \phi_\pi \left(p_t^c - p_{t-1}^c \right) + \phi_c c_t$$

II Solution of Key Equations in Section III

A. Solution of Equation (26)

Setting $\sigma = 1$ and $\varphi = 0$ in equation (A.17) yields

$$w_{kt} = c_t + p_t^c = 0,$$

where the equality follows from the assumed monetary policy rule, so equation (A.20) becomes

$$mc_{kt} = \delta p_t^k - a_{kt}.$$

Here, sectoral prices for all $k = 1, ..., K$ are governed by

$$\begin{aligned} p_{kt} &= (1 - \lambda_k)\, mc_{kt} \\ &= \delta (1 - \lambda_k)\, p_t^k - (1 - \lambda_k)\, a_{kt}, \end{aligned}$$

which in matrix form solves

$$p_t = -\left[\mathbb{I} - \delta (\mathbb{I} - \Lambda)\, \Omega\right]^{-1} (\mathbb{I} - \Lambda)\, a_t.$$

$p_t \equiv [p_{1t}, ..., p_{Kt}]'$ is the vector of sectoral prices, Λ is a diagonal matrix with the vector $[\lambda_1, ..., \lambda_K]'$ on its diagonal, Ω is the matrix of input-output linkages, and $a_t \equiv [a_{1t}, ..., a_{Kt}]'$ is the vector of realizations of sectoral technology shocks.

The monetary policy rule implies $c_t = -p_t^c$, so

$$c_t = \left(\mathbb{I} - \Lambda'\right) \left[\mathbb{I} - \delta \left(\mathbb{I} - \Lambda'\right) \Omega'\right]^{-1} \Omega_c' a_t.$$

III Further Theoretical Results

A. Illustrative examples for Proposition 4

Proposition 4 states that, when there is no need of intermediate inputs for production, i.e., when $\delta = 0$, the asymptotic rate of decay of GDP volatility v_c depends on the convolved distribution of $\chi_k = (1 - \lambda_k) C_k$. Previous propositions show that sectorial price rigidity λ_k plays no role for the rate of decay of v_c as $K \to \infty$ when λ_k and sectoral GDP C_k are independently distributed. This result may not hold when independency is relaxed. To show this, we rely on an illustrative example.

Assume that C_k is Pareto distribution with shape parameter β_c and

$$\lambda_k = \max\left\{0, 1 - \phi C_k^{\mu}\right\} \tag{A.21}$$

for $\mu \in (-1, 0)$ and $\phi > 0$. This relationship implies that larger sectors measured by GDP have more rigid prices. In this case $(1 - \lambda_k) C_k$ is Pareto distributed with shape parameter $\beta_{\lambda c} = \beta_c / (1 + \mu)$. From Proposition 4 follows that price rigidity in this case accelerates the rate of decay of v_c distribution relative to a frictionless economy or one with homogeneous price rigidity. It is also possible that conditions for granularity are satisfied in an economy with flexible prices while the Central Limit Theorem applies in an economy with rigid prices.

An example of the converse case, when price rigidity slows down the rate of decay of v_c, is not workeable in closed form. An illustrative, although weaker, example in closed form is obtained by assuming $\mu \in (0, 1)$ and the same functional relationship between λ_k and C_k than above. Now any sector with GDP higher than $\phi^{-1/\mu}$ has fully flexible prices. In other words, price rigidity and sectoral GDP become independent for sectors larger than $\phi^{-1/\mu}$. As a result, $(1 - \lambda_k) C_k$ follows a Pareto distribution with shape parameter $\beta_{\lambda c} = \beta_c / (1 + \mu)$ for $C_k < \phi^{-1/\mu}$ and β_c for $C_k \geq \phi^{-1/\mu}$. Following the same steps in the proof of Proposition 4, the rate of decay of v_c depends on $\beta_c / (1 + \mu)$ for $K < K^*$ and on β_c for $K \geq K^*$ with $K^* \equiv \left(x_0 \phi^{1/\mu}\right)^{-\beta_c}$. Intuitively, when the number of sectors is large enough, $K > K^*$, sectors with fully flexible prices dominate the upper tail of the sectoral GDP distribution, so the rate of decay of aggregate volatility is the same as in a frictionless economy.

This result is weaker than when $\mu \in (-1, 0)$, but this example is still appealing if K^* is large or, in other words, for a level of disaggregation where fully flexible sectors do not dominate in the upper tail of the sectoral GDP distribution. In the data we exploit for our quantitative exercises below, the finest degree of disaggregation available which contains 341 sectors, there is no a single sector with fully flexible prices.

Although we have assumed examples with a deterministic relationship between sectoral GDP and price rigidity for expositional convenience, the same argument could be extended when this relationship is stochastic. Similarly to the above, a stochastic relationship between price rigidity and sectoral GDP implies that price rigidity has an effect on the identity of the most important sectors for GDP volatility.

B. Intuition for quantitative results

We find in Section VI across calibrations that (i) heterogeneity in price stickiness alone can generate large aggregate fluctuations from idiosyncratic shocks, (ii) homogeneous input-output linkages mute the size effect relative to an economy without intermediate input use, and (iii) introducing heterogeneous price rigidity across sectors in an economy with heterogeneity in sectors' sizes and homogeneous input-output linkages increases GDP volatility relative to the same economy with only aggregate shocks. We now briefly elaborate in these three special cases using the simplified model in III.

Consider the economy of Section III with sectors of potentially different sizes, input-output linkages, and price rigidities. In this general case, the vector of multipliers up to second-order terms has elements

$$\chi_k \geq (1-\lambda_k)\left[\omega_{ck} + \delta \widehat{d}_k + \delta^2 \widehat{q}_k\right]. \tag{A.22}$$

As before, ω_{ck} denotes the size of sector k, $\widehat{d}_k$ the **generalized outdegree** of sector k, and $\widehat{q}_k$ the **generalized second-order outdegree** of sector k:

$$\begin{aligned} \widehat{d}_k &\equiv \sum_{k'=1}^{K} \omega_{ck'} \left(1-\lambda_{k'}\right) \omega_{k'k}, \qquad \text{(A.23)} \\ \widehat{q}_k &= \sum_{k'=1}^{K} \widehat{d}_{k'} \left(1-\lambda_{k'}\right) \omega_{k'k}. \end{aligned}$$

These two terms embody the effects of heterogeneity of GDP shares, input-output linkages, and price rigidity across sectors.

CASE 1. With these expressions, we start with a special case when the only source of heterogeneity across sectors is price rigidity. If input-output linkages and sector sizes are homogeneous across sectors, that is, $\omega_{ck} = \omega_{kk'} = 1/K$ for all k, k', but price stickiness λ_k is heterogeneous, then

$$\|\chi\|_2 = \frac{1}{K\left(1-\delta\left(1-\overline{\lambda}\right)\right)}\sqrt{\sum_{k=1}^{K}\left(1-\lambda_k\right)^2},$$

where $\overline{\lambda} \equiv \frac{1}{K}\sum_{k=1}^{K}\lambda_k$.

Thus, GDP volatility is increasing in the dispersion of price stickiness across sectors. As shown in Section III, the decay of GDP volatility is $N^0.5$ for any distribution of sectorial price rigidity, but for finite N is still possible that GDP volatility is sizable.

CASE 2. The next case is when only input-output linkages are restricted to be homogeneous. If input-output linkages are homogeneous across sectors, that is, $\omega_{kk'} = 1/K$ for all k, k', but sector size ω_{ck} is unrestricted and prices are frictionless ($\lambda_k = 0$), then given var$(\omega_{ck}) \geq var(1/K)$, then

$$\frac{\sqrt{\sum_{k=1}^{K}\left(\omega_{ck}+\frac{\delta/K}{1-\delta}\right)^2}}{\left(1-\delta\right)^{-1}} \leq \sqrt{\sum_{k=1}^{K}\omega_{ck}^2},$$

Thus, GDP volatility is ceteris paribus smaller compared to an economy without input-output linkages, that is, ($\delta = 0$). This is because the homogeneity in input-output linkages introduces an additive component that is the same across all multiplies and thus reducing the cross sectional dispersion among them.

CASE 3. Allowing for price stickiness ($\lambda_k > 0$) is an important case in the calibrations. We study this case next. Relative to the previous proposition, we find the introduction of heterogeneous price rigidity leads to an overall increase in the relative multiplier.

If input-output linkages are homogeneous across sectors, that is, $\omega_{kk'} = 1/K$ for all k, k', but sector size ω_{ck} is unrestricted, and price rigidity is heterogeneous ($\lambda_k > 0$), then given $\overline{\lambda} \equiv \frac{1}{K}\sum_{k=1}^{K}\lambda_k$, $\overline{\overline{\lambda}} \equiv \frac{1}{K}\sum_{k=1}^{K}\omega_{ck}\lambda_k$ the multiplier of idiosyncratic shocks relative to the aggregate multiplier,

$$\frac{\|\chi\|_2}{\sum_{k=1}^{K} \chi_k} = \frac{\sqrt{\sum_{k=1}^{K} (1-\lambda_k)^2 \left(\omega_{ck} + \frac{\delta\left(1-\overline{\overline{\lambda}}\right)/K}{1-\delta\left(1-\overline{\lambda}\right)} \right)^2}}{\left(1-\overline{\overline{\lambda}}\right) / \left[1-\delta\left(1-\overline{\lambda}\right)\right]},$$

Thus, GDP volatility (relative to the same economy with only aggregate shocks) is increasing in the simple and weighted average of heterogeneous price rigidity across sectors, $1-\overline{\lambda}$, and $1-\overline{\overline{\lambda}}$, in the covariance of price rigidity and sector size, $cov((1-\lambda_k, \omega_{ck})$, and in the dispersion of sectoral price flexibility, λ_k. The first effect is due to the average degree of price rigidity in the economy. The last two effects and capture the incidence of heterogeneity in the interaction with sector size and the dispersion of price flexibility.

IV Proofs

Most proofs below are modifications of the arguments in Gabaix (2011), Proposition 2, which rely heavily on the Levy's Theorem (as in Theorem 3.7.2 in Durrett (2013) on p. 138).

Theorem 5 (Levy's Theorem) *Suppose $X_1, ..., X_K$ are i.i.d. with a distribution that satisfies*

(i) $\lim_{x\to\infty} \Pr[X_1 > x] / \Pr[|X_1| > x] = \theta \in (0,1)$

(ii) $\Pr[|X_1| > x] = x^{-\zeta} L(x)$ *with* $\zeta < 2$ *and* $L(x)$ *satisfies* $\lim_{x\to\infty} L(tx)/L(x) = 1$.

Let $S_K = \sum_{k=1}^{K} X_k$,

$$a_K = \inf\{x : \Pr[|X_1| > x] \le 1/K\} \text{ and } b_K = K\mathbb{E}\left[X_1 \mathbf{1}_{|X_1| \le a_K}\right], \tag{A.24}$$

As $K \to \infty$, $(S_K - b_K)/a_K \xrightarrow{d} u$, *where* u *has a nondegenerated distribution.*

A. Proof of Proposition 1

In the following proofs, we go through three cases: first, when both first and second moments exist, second, when only the first moment exists, and third, when neither first nor second moments exist.

Generally, when there are no intermediate inputs ($\delta = 0$) and price rigidity is homogeneous across sectors ($\lambda_k = \lambda$ for all k),

$$\|\chi\|_2 = \frac{1-\lambda}{K^{1/2}\overline{C}_k} \sqrt{\frac{1}{K} \sum_{k=1}^{K} C_k^2}. \tag{A.25}$$

Given the power-law distribution of C_k, the first and second moments of C_k exist when $\beta_c > 2$, so

$$K^{1/2} \|\chi\|_2 \longrightarrow \frac{\sqrt{\mathbb{E}\left[C_k^2\right]}}{\mathbb{E}\left[C_k\right]}. \tag{A.26}$$

In contrast, when $\beta_c \in (1,2)$, only the first moment exists. In such cases, by the Levy's theorem,

$$K^{-2/\beta_c} \sum_{k=1}^{K} C_k^2 \xrightarrow{d} u_0^2, \tag{A.27}$$

where u_0^2 is a random variable following a Levy's distribution with exponent $\beta_c/2$ since $\Pr\left[C_k^2 > x\right] = x_0^{\beta} x^{-\beta_c/2}$.

Thus,

$$K^{1-1/\beta_c}\left\|\chi\right\|_2 \xrightarrow{d} \frac{u_0}{\mathbb{E}\left[C_k\right]}. \tag{A.28}$$

When $\beta_c = 1$, the first and second moments of C_k do not exist. For the first moment, by Levy's theorem,

$$\left(\overline{C}_k - \log K\right) \xrightarrow{d} g, \tag{A.29}$$

where g is a random variable following a Levy distribution.

The second moment is equivalent to the result above and hence

$$\left(\log K\right)\left\|\chi\right\|_2 \xrightarrow{d} u'. \tag{A.30}$$

B. Proof of Proposition 3

Let λ_k and C_k be two independent random variables distributed as specified in the Proposition, the counter-cumulative distribution of $z_k = (1-\lambda_k)\,C_k$ is given by

$$f_Z\left(z\right) = \int_z^{z/y_0} f_{C_k}\left(z/y\right) f_{1-\lambda_k}\left(y\right) dy, \tag{A.31}$$

which follows a Pareto distribution with shape parameter β_c for $z \geq 1$. The proof of the Proposition then follows the proof of Proposition 1 for

$$\left\|\chi\right\|_2 = \frac{1}{K^{1/2}\overline{C}_k}\sqrt{\frac{1}{K}z_k^2} \text{ for } z \geq 1. \tag{A.32}$$

C. Proof of Proposition 5

When $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, and $\Omega_c = \frac{1}{K}\iota$, we know

$$\begin{aligned}
\left\|\chi\right\|_2 &\geq \frac{1-\lambda}{K}\sqrt{\sum_{k=1}^{K}\left[1+\delta' d_k + \delta'^2 q_k\right]^2} \\
&\geq (1-\lambda)\sqrt{\frac{1+2\delta'+2\delta'^2}{K} + \frac{\delta'^2}{K^2}\sum_{k=1}^{K}\left[d_k^2 + 2\delta' d_k q_k + \delta'^2 q_k^2\right]}.
\end{aligned}$$

Following the same argument as in Proposition 2,

$$K^{-2/\beta_d} \sum_{k=1}^{K} d_k^2 \longrightarrow u_d^2,$$
$$K^{-2/\beta_q} \sum_{k=1}^{K} q_k^2 \longrightarrow u_q^2,$$
$$K^{-1/\beta_z} \sum_{k=1}^{K} d_k q_k \longrightarrow u_z^2,$$

where u_d^2, u_q^2 and u_z^2 are random variables. Thus, if $\beta_z \geq 2\min\{\beta_d, \beta_q\}$,

$$v_c \geq \frac{u_3}{K^{1-1/\min\{\beta_d,\beta_q\}}} v \tag{A.33}$$

where u_3^2 is a random variable.

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