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Immigration in Emerging Countries: A Macroeconomic Perspective*

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Abstract

Roughly one third of migrants worldwide reside in developing countries, yet most papers on the macroeconomic effects of immigration focus on advanced economies. We investigate the medium- and long-term effects of immigration in an emerging country, considering a salient feature of this type of economies: the importance of labor informality. We build an overlapping generations model featuring 30 cohorts, an informal sector, and households with heterogeneous skill levels, among other features, that help us match key demographic and economic characteristics of Chile, an emerging country that has recently experienced an important immigration flow. An immigration wave increases the supply of labor, creating downward pressure on wages in the formal sector. Workers, especially those with lower skills, respond by reallocating labor effort to the informal sector, which allows them to mitigate the decline in consumption triggered by lower formal-sector wages. Our model, thus, constitutes a framework for the quantitative analysis of immigration in emerging countries.

Resumen

Alrededor de una tercera parte de los migrantes del mundo reside en países en desarrollo. Sin embargo, la mayoría de investigaciones sobre los efectos macroeconómicos de la inmigración se enfocan en países avanzados. En este artículo investigamos los efectos de mediano y largo plazo de la inmigración en un país emergente, considerando una característica clave de este tipo de economías, la importancia del trabajo informal. Para esto, construimos un modelo de generaciones traslapadas con 30 cohortes,

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un sector informal, y hogares con niveles de habilidad heterogéneos, entre otras características, que nos permiten reproducir características demográficas y económicas clave de Chile, un país emergente que recientemente ha experimentado una ola inmigratoria importante. En nuestras simulaciones, una ola inmigratoria aumenta la oferta de trabajo, generando presión a la baja en los salarios del sector formal. Los trabajadores, especialmente aquellos con menor nivel de habilidad, responden reasignando trabajo hacia el sector informal, lo que les permite mitigar la caída en el consumo por trabajador asociada a los salarios más bajos en el sector formal. De este modo, el modelo constituye un marco para el análisis cuantitativo de la inmigración en países emergentes.

1 Introduction

Roughly one third of migrants worldwide reside in developing countries (OECD/ILO, 2018), yet most of the research on the macroeconomic effects of immigration focus on advanced economies. In this paper, we investigate the medium- and long-term effects of immigration in an emerging country, considering a salient feature of this type of economies: the importance of labor informality.

To address this, we build an overlapping generations model (OLG) featuring 30 cohorts, an informal sector, and households with heterogeneous skill levels, among other features, that help us match key demographic and economic characteristics of Chile, an emerging country that has recently experienced an important immigration flow. Specifically, we match the income distribution, saving rates, and informality across income quintiles. Following much of the literature, we model informality as self-employment. In our framework, an hour worked in the informal sector generates more disutility than an hour worked in the formal sector. The additional disutility of working in the informal sector captures, for example, the lower benefits and job security associated to this type of employment. Despite this feature, informal labor allows low-skilled workers to mitigate the decline in consumption triggered by the immigration wave. In this sense, the informal sector acts as a buffer against income fluctuations, which is consistent with the evidence on informality (see, for example, Loayza and Rigolini, 2011). An immigration wave increases the supply of labor, creating downward pressure on wages in the formal sector, so workers respond by reallocating labor effort to the informal sector.

Motivated by the immigration flow recently experienced by Chile, we model immigrants that, despite having similar skills to those of natives, experience a temporary underemployment spell—a period during which they cannot fully exercise the productivity associated to their skill level. This underemployment spell amplifies the distributional effects of the immigration flow, exacerbating the decline in wages of low-skilled workers and mitigating the decline in wages of high-skilled workers. Intuitively, this transitory underemployment spell can be justified by the time it takes a foreign worker to adapt to the host country's labor market.[1] This transitory phenomenon, related to adaptation, differs from the approach in other articles, which assume immigrants have permanently lower productivity, perhaps due to lower education quality in their home country.[2] In our simulations, therefore, immigration generates transitory effects on per capita variables and factor prices, though these effects are persistent. Since the productivity of immigrants eventually returns to that associated to their skill level, i.e., the underemployment spell ends, and immigrants and natives have similar skills, the effects of the immigration flow eventually die out.

In addition to the underemployment spell, our simulation of an immigration wave considers the fact that immigrants are much younger than natives. Taking the difference in the age

[1]See Lubotsky (2007) for a longitudinal analysis of immigrant's earnings in the United States, and Barker (2021a) for SVAR- and DSGE-based analyses of immigrant underemployment in Canada.

[2]See, for example, Canova and Ravn (2000) for an analysis of German unification modeled as the migration, from East to West Germany, of individuals with permanently lower skill levels.

structure of immigrants into account is quantitatively important, as we show below, highlighting the usefulness of an OLG model in this context.

To sum up, our simulations consider immigrants that are mostly young adults, and that experience an underemployment spell upon arrival. Immigrants are otherwise assumed to be indistinguishable from natives. This assumption not only simplifies the model, but is supported by some of the characteristics of immigrants in Chile. Aldunate, Contreras, de la Huerta, and Tapia (2019b) document that, in addition to having similar education levels to those of natives, immigrants have a similar distribution of employment across industries (they are not concentrated on a handful of industries), and have similar employment status (they are not disproportionately informal). There are, however, special characteristics of immigrants that we ignore, such as the higher unemployment rates they seem to face upon arrival (again, see Aldunate et al., 2019b), or the possibility that they bring little wealth, or that they remit an important fraction of disposable income to their home countries.

The paper lies at the intersection of two strands of the literature. The first strand considers the macroeconomic effects of immigration on advanced economies, and typically studies immigrants with lower skills than the native population.[3] The second strand of literature studies the role of labor informality in emerging countries. It finds that the informal sector acts as a buffer that helps sustain household consumption during bad times, and can explain, for instance, why aggregate employment is more volatile in advanced economies than in emerging countries.[4,5]

By bridging the gap between the literatures on immigration in advanced economies and on labor informality in emerging countries, this paper is the first comprehensive study of the macroeconomic effects of immigration in emerging countries, where informal labor is pervasive.

The paper is organized as follows. Section 2 presents the evidence that motivates the key features of our OLG model. Section 3 spells out the details of the model. Section 4 discusses the calibration of the model and the design of our simulations. Section 5 presents our baseline results, as well as an analysis of their sensitivity to our assumptions on the duration and intensity of the underemployment spell, as well as on the long-run skill level of immigrants. Section 6 concludes.[6]

2 Motivating Evidence

The design of the OLG model and our simulation exercises is motivated by features of the Chilean economy and the recent immigration wave it has experienced.

[3] See, for example, Boldrin and Montes (2015); Izquierdo, Jimeno, and Rojas (2010); Wilson (2003); Mandelman and Zlate (2012); and Holler and Schuster (2018).

[4] Most advanced economies, of course, offer unemployment insurance schemes that allow the unemployed to search for jobs. Unemployment insurance schemes are rare in emerging countries, so the unemployed cannot afford to remain without a job; informality offers an alternative.

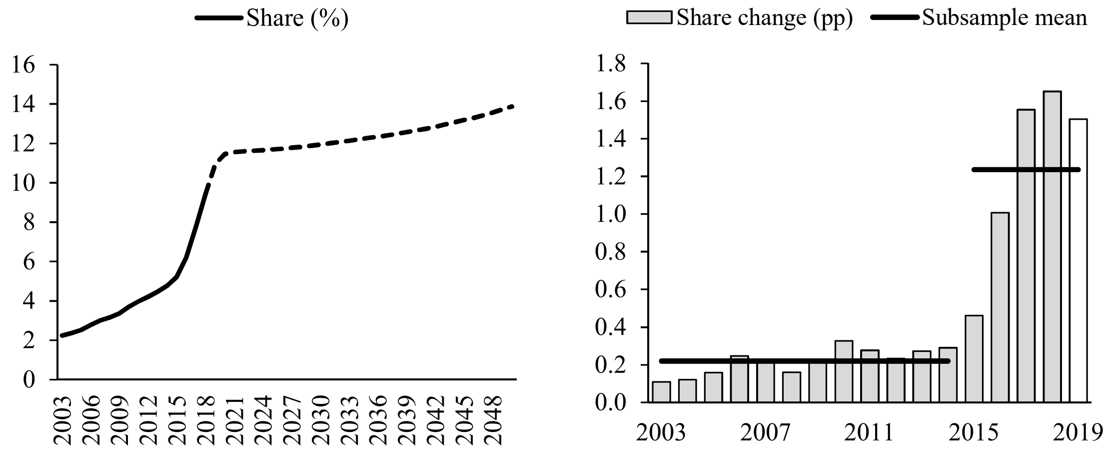
[5] See, for example, Fernández and Meza (2015); Finkelstein Shapiro and Mandelman (2016); and Restrepo-Echavarria (2014).

[6] One of the appendices shows how the model can be extended to study the effects of a statutory reduction in working hours.

2.1 The Immigration Flow

Chile has experienced a substantial immigration flow. As shown in the left panel of figure 1, the share of immigrants in the labor force nearly doubled in three years, jumping from 5.2% in 2015 to 9.4% in 2018. Prior to 2015, the share of immigrants in the labor force was increasing by about 0.2 percentage points per year, on average. During 2015–2019, this metric increases by a factor of six (right panel). We model this immigration wave as exogenous from the perspective of Chile.[7] There is nothing special about Chile during this period that could account for such a surge in immigration; certainly not economic growth, which was sluggish . Instead, the inflow of migrants is mainly related to the Venezuelan crisis and the massive emigration it has generated. Therefore, we believe it is reasonable to model this event as exogenous from the perspective of Chile.[8]

Figure 1: Share of immigrants in the labor force



Source: National Statistics Institute and Aldunate, Bullano, Canales, Contreras, Fernández, Fornero, García, García, Pena, Tapia, and Zúniga (2019a). Note: Projections start in 2019.

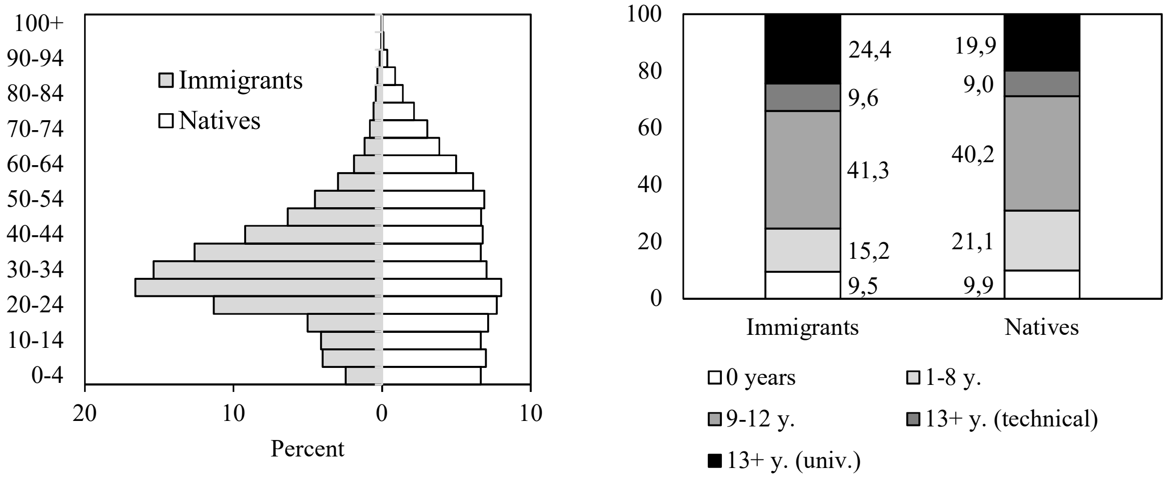
For the purposes of our analysis, two key features of Chile's immigration wave are that immigrants: (i) are primarily young adults, and (ii) have slightly higher education levels than natives. Figure 2 shows the age structure and education level of immigrants and natives. The fact that immigrants are primarily young adults implies that the immigration wave has not only generated higher growth in the total population, but even higher growth in the working-age population. Since immigrants have higher participation rates than natives, as documented in Aldunate et al. (2019b), the immigration-induced growth in the labor force is also higher than that of the total population. Our simulations, therefore, will consider an immigration wave that alters the age structure, reducing the ratio of passive to active individuals.

Following the evidence on the education level of immigrants, our baseline simulation will further assume that immigrants and natives have the same skill distribution. We will also study the sensitivity of our results to this assumption by allowing for immigrants to be slightly more skilled than natives. Although immigrants and natives have similar education levels, we will allow for an adjustment period during which immigrants are underemployed, as we discuss next.

[7]See, e.g., Mandelman and Zlate (2012) for an endogenous analysis of immigration.

[8]See Barker (2021b) for a general equilibrium analysis of the Venezuelan migration.

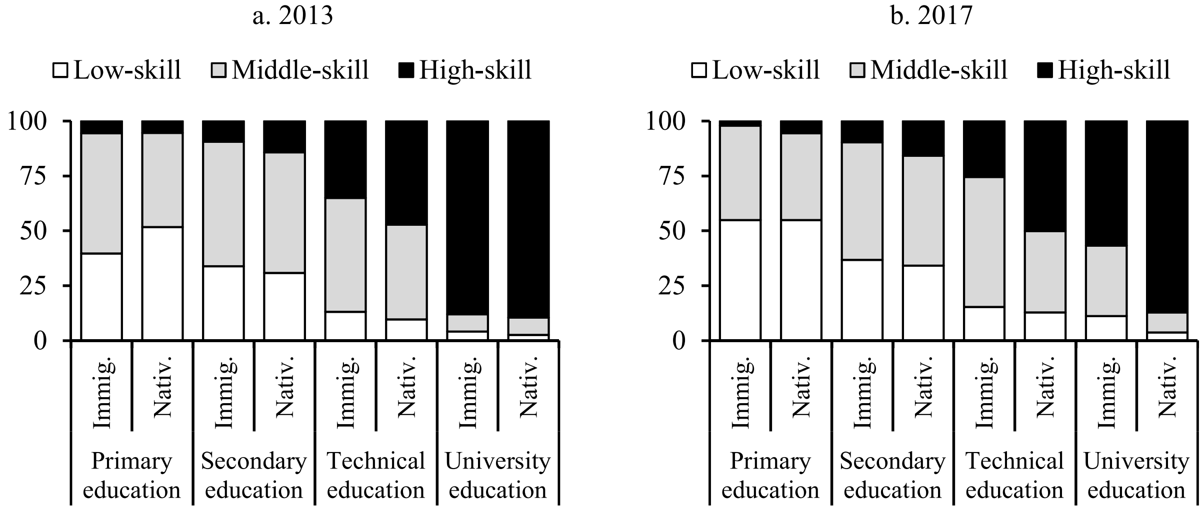
Figure 2: Age and education of immigrants



Source: Aldunate et al. (2019b).

It seems that immigrants experience an "underemployment spell" upon arrival, a period during which they cannot fully exercise the productivity associated to their skill level. Figure 3 presents some evidence in favor of this conclusion. It shows employment by education and skill level of the occupation, for natives and immigrants, and for the years 2013 (panel a on the left) and 2017 (panel b on the right). This evidence is based on data from the Socioeconomic Characterization Survey (Encuesta de Caracterización Socioeconómica Nacional; CASEN), which is conducted every two years in Chile. The recent immigration wave began around 2015, so the evidence from 2013 is prior to it, whereas the evidence from 2017 is gathered during the wave. Among individuals with higher education, in 2017 immigrants hold a substantially smaller proportion of high-skill occupations compared to natives; this was not the case in 2013. For example, in 2017, only 57% of immigrants with university studies worked in a high-skill occupation, whereas 87% of natives with that education level did. In 2013, prior to the recent immigration wave, immigrants and natives with university studies held almost the same (high) proportion of high-skill occupations. A similar divergence is visible for people with technical education: in 2013, the difference in the proportion of high-skill occupations held by immigrants and natives is 12 percentage points (pp), but in 2017 this difference increases to 25 pp. To sum up, during the immigration wave, immigrants hold a substantially smaller proportion of high-skill jobs than natives, even when both groups have a similar education level. This evidence is imperfect, as it only shows a snapshot at two points in time. Unfortunately, there is no data, to the extent of our knowledge, that would allow us to characterize the adaptation *process* of immigrants in the labor market. Nevertheless, we will interpret this evidence as reflecting a transitory underemployment spell experienced by immigrants, and not that immigrants are permanently less productive than natives, perhaps due to lower education quality in their home country, as in, for example, Canova and Ravn (2000).

Figure 3: Employment by years of education and skill level of occupation

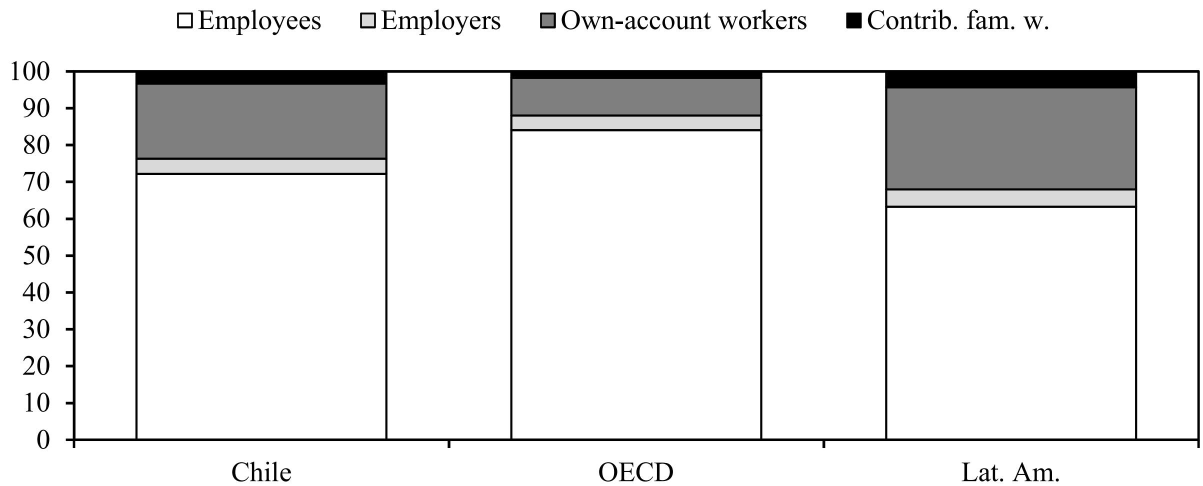


Source: Aldunate et al. (2019b). Note: The underlying data come from the CASEN survey. The classification of occupations by skill level follows Lagakos, Moll, Porzio, Qian, and Schoellman (2018).

2.2 Labor Informality

Emerging countries feature substantial labor informality, and Chile is no exception. Figure 4 shows employment by category in Chile, a group of Latin American countries, and the OECD. Following the literature, we focus on self-employment as a proxy measure of labor informality.[9] The proportion of self-employed (own-account workers) is much higher in Chile and Latin American countries (all emerging countries) than in the OECD, a group of mostly advanced economies.

Figure 4: Employment by category

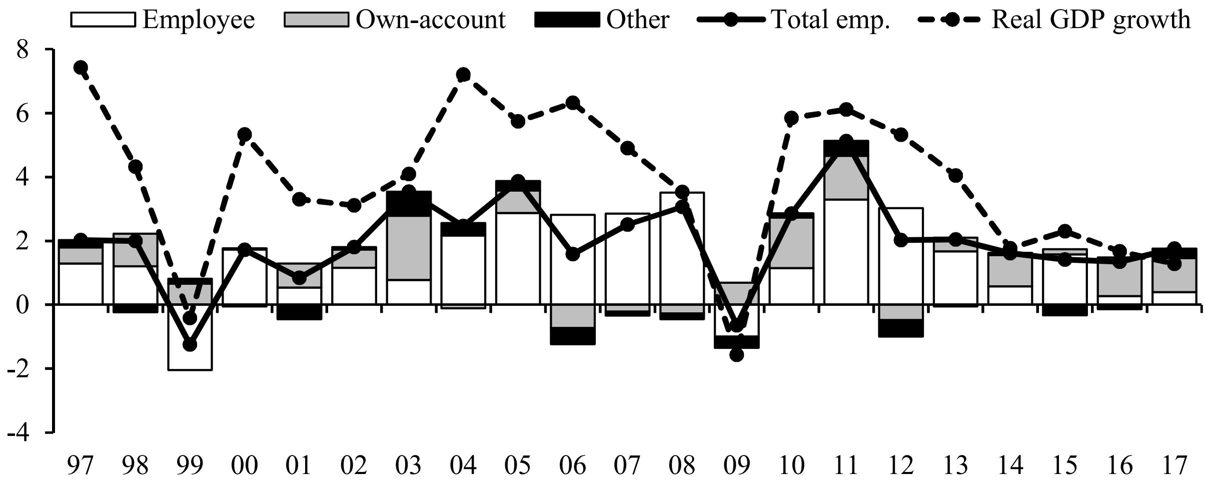


Source: Central Bank of Chile (2018). Note: The underlying data come from the International Labour Organization. Categories of employment are: employees, employers, own-account workers, and contributing family workers.

[9]See, for example, Fernández and Meza (2015).

Labor informality in Chile also conforms to results in the literature on emerging countries in that it acts as a buffer for income fluctuations. Figure 5 reproduces results from Central Bank of Chile (2018) that show that in good times, employment growth is driven by salaried (formal) work ("employees"; white bars), whereas in bad times, informal employment sustains job growth ("own-account" workers; grey bars). Chile grew strongly in the mid-to-late 2000s and in the aftermath of the Great Recession (2011-2013), periods during which employment growth was driven by formal work. On the other hand, the economy struggled in the aftermath of the Asian and Russian crises (up until 2003), and in 2014-2017, periods when informal work sustains job growth.[10] Our OLG model will, therefore, feature an informal sector that acts as a buffer against income fluctuations.

Figure 5: Employment growth by category



Source: Central Bank of Chile (2018).

In the next section, we spell out an OLG model that captures the evidence presented in this section, as well as other demographic and economic features of the Chilean economy.

3 The OLG Model

The model is based on the neoclassical growth models with overlapping generations pioneered by Samuelson (1958) and Diamond (1965). We add several features to an otherwise standard OLG model with thirty generations (or cohorts).[11] First, there are five types of workers with heterogeneous productivity levels that make optimal decisions on labor supply, as in Brunner (1996) and Sommacal (2006). This allows us to replicate the distribution of income across quintiles. Second, we introduce heterogeneous discount factors, which are decreasing in the income level, that allow us to match the heterogeneity of saving rates observed in the data. Third, we add an informal sector based on the work of Busato and Chiarini (2004), Busato, Chiarini,

[10]See also Parro G. and Reyes R. (2019), who find that in Chile, the share of self-employed increases in periods of low economic growth and vice versa, whereas the share of workers in formal work displays the opposite behavior (it increases in periods of *high* economic growth, and vice versa).

[11]A model with thirty cohorts allows us to study medium-term effects (each period is calibrated to last 2 years), while retaining reasonable computational efficiency.

and Rey (2012) and Orsi, Raggi, and Turino (2014), with the purpose of capturing reallocation of labor supply between the formal and informal sectors. The informal sector is modeled as self-employment, which has been found to be a good proxy of informality (see Fernández and Meza, 2015).[12] Fourth, we consider a financially open economy, in which foreigners own a fraction of the domestic capital stock, and national savers invest abroad.[13]

3.1 Demographics

Individuals have perfect foresight. They live for 30 periods, and therefore, in every period there are indivduals of 30 generations alive. A generation born at time t corresponds to the first cohort in period t, the second in $t+1$, and so on. Individuals spend the first 20 periods of their life working and then retire for the remaining 10 periods. Workers spend their labor income on consumption and save in one-period assets. These assets are transformed into physical capital that is rented to perfectly competitive (formal) firms. At the beginning of the following period, the remaining capital, net of depreciation, together with the gained rent is returned to the individuals, so that wealth is transferred intertemporally. Savings are used to finance consumption during retirement. Workers are divided into 5 different skill groups, denoted by the index $i = 1, \ldots, 5$, and ordered from least skilled ($i = 1$) to most skilled ($i = 5$).

The size of a new generation that is born into this economy (N_t^1) relative to the size of the previous period's first generation (N_{t-1}^1) changes at the rate $n^r + n_t^m$, i.e., $N_t^1 = N_{t-1}^1 \left(1 + n^r + n_t^m\right)$; where n^r denotes a constant growth rate used to accommodate the reproduction rate of the whole population, and n_t^m accounts for time-varying increases of the first cohort due to immigration. The size of older cohorts may vary over time due to migration only: $N_t^g = N_{t-1}^{g-1}(1 + m_t^g)$, for $g > 1$, where m_t^g denotes the rate of change of cohort $g > 1$ between periods $t-1$ and t. We will use n_t^m and the m_t^g to simulate the change in population size and age structure induced by an immigration wave. The total population in period t is then defined as $N_t = \sum_{g=1}^{30} N_t^g$.[14] We also assume that the distribution of skills for each generation can change over time, such that in period t there are $N_{i,t}^g = \lambda_{i,t}^g N_t^g$ individuals of skill group i in cohort g, where $\lambda_{i,t}^g \in (0,1)$ denotes the fraction of individuals of skill group i within cohort g in period t, and we have that $\sum_{i=1}^{5} \lambda_{i,t}^g = 1$. Therefore, both the size of every generation and the size of each skill group within each generation can change over time. We will use the $\lambda_{i,t}^g$ to simulate a change in the skill distribution induced by a wave of immigrants that experience an underemployment spell.

3.2 Firms

A representative firm demands capital, K_t, and labor, L_t, to produce a homogeneous good used for consumption and investment. The firm's production function is

$$Y_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha}. \tag{1}$$

[12] McKiernan (forthcoming) uses an OLG model with an informal sector to study pension reform in Chile.

[13] We abstract from human capital accumulation; Boldrin and Montes (2015) consider human capital accumulation in a three-cohort OLG model of immigration for Spain.

[14] At the steady state $m_{ss}^g = 0$ for all g, so we have that N_{ss}, the total population in steady state, grows at the same rate as the first cohort at the steady state, N_{ss}^1, namely $n_{ss}^r + n_{ss}^m$.

with $\alpha \in (0,1)$ and where A_t denotes the labor-augmenting technology level which is assumed to grow at a constant rate z, such that $A_t = A_{t-1}(1+z)$. The labor input L_t is a constant-elasticity-of-substitution (CES) bundle of labor of various skills:[15]

$$L_t = \left(\sum_{i=1}^{5} a_i L_{i,t}^{\rho}\right)^{1/\rho}, \tag{2}$$

where $L_{i,t}$ denotes the demand for workers from skill group i, the parameters $a_i > 0$ with $a_1 < a_2 < \cdots < a_5$ measure the productivities of the different worker types, and $\rho < 1$ governs the elasticity of substitution of workers of different skills, given by $1/(1-\rho)$. As $\rho \to 1$, workers of different skills are perfect substitutes, and as $\rho \to -\infty$ they are perfect complements.

The firm maximizes profits subject to (1) and (2), treating the rental rate of capital r_t^k and the skill-specific wage rates $W_{i,t}$ as given. Substituting the constraints out, the firm's optimization problem can be written as:

$$\max_{K_t, L_{i,t}} K_t^{\alpha} A_t^{1-\alpha} \left(\sum_{i=1}^{5} a_i L_{i,t}^{\rho}\right)^{\frac{1-\alpha}{\rho}} - r_t^k K_t - \sum_{i=1}^{5} W_{i,t} L_{i,t}, \tag{3}$$

yielding the following first-order conditions for capital and labor :

$$K_t: \qquad r_t^k = \alpha \frac{Y_t}{K_t}, \tag{4}$$

$$L_{i,t}: \qquad W_{i,t} = a_i W_t \left(\frac{L_t}{L_{i,t}}\right)^{1-\rho}, \tag{5}$$

where $W_t = \frac{(1-\alpha)Y_t}{L_t}$. These conditions imply that the zero-profit condition

$$Y_t = r_t^k K_t + \sum_{i=1}^{5} W_{i,t} L_{i,t} \tag{6}$$

is satisfied in equilibrium.

The law of motion for capital is given by,

$$K_{t+1} = (1-\delta)K_t + I_t + I_t^*, \tag{7}$$

where δ is the rate of depreciation, and I_t and I_t^* are domestic and foreign investment, respectively. Domestic investment I_t is given by a fraction $(1-\gamma)$ of household disposable income; the rest is invested abroad:

$$I_t = (1-\gamma)\left(Y_t^d + r_t S_{t-1} - C_t\right),$$

where Y_t^d aggregate labor income net of taxes, $r_t S_{t-1}$ is aggregate asset gross returns, and C_t is aggregate consumption; these variables will be defined below.

Foreign investment I_t^* is given by $I_t^* = \gamma^* A_t N_t$, a constant $\gamma^* > 0$ that grows due to

[15] Throughout, upper-case letters denote variables that are non-stationary due to population growth, productivity growth, or both, whereas lower-case letters denote stationary variables.

population growth. We calibrate the parameters γ and γ^*, which govern the economy's financial openness, to match data from Chile, as we explain below.

3.3 Workers of Type i

Utility of a worker of type i is a function of consumption, $C^g_{i,t}$, both when active ($g \leq 20$) and when retired ($g > 20$), and of labor effort (hours) supplied to the formal, $l^g_{i,t}$, and informal, $h^g_{i,t}$, sectors, when active. The present value of lifetime utility of a worker of skill i is then given by:

$$U_{i,t} = \sum_{g=1}^{30} \beta_i^{g-1} \frac{\left(C^g_{i,t+g-1}\right)^{1-\theta}}{1-\theta} - \sum_{g=1}^{20} \beta_i^{g-1} \Theta^g_{i,t+g-1} \frac{\left(l^g_{i,t+g-1} + h^g_{i,t+g-1}\right)^{\phi} + \kappa \left(h^g_{i,t+g-1}\right)^{\phi}}{\phi}, \tag{8}$$

where $\theta > 0$ is the inverse of the elasticity of intertemporal substitution, $\phi \geq 1$ determines the elasticity of labor supply, $\beta_i \in (0,1)$ is the subjective discount factor of each a worker of skill i, and $\kappa > 0$ is a specific utility cost of informal labor.[16] The variable $\Theta^g_{i,t}$ is an endogenous preference shifter based on Galí, Smets, and Wouters (2012) that is taken as given by the workers and satisfies

$$\Theta^g_{i,t+g-1} = \chi^g_i A_t^{1-\theta} \left[\left(\frac{C^g_{i,t+g-1}}{A_t} \right)^{-\theta} \right]^{\upsilon}, \tag{9}$$

with $\upsilon \in \{0,1\}$ and where $\chi^g_i > 0$ determines the disutility of work for skill group i and age group g. The purpose of this preference shifter is to limit the wealth effect on labor supply. When $\upsilon = 0$, we obtain $\Theta^g_{i,t+g-1} = \chi^g_i$, i.e., the standard constant relative risk aversion (CRRA) utility function implying a positive wealth effect, whereas the wealth effect is cancelled when $\upsilon = 1$.[17]

We model the informal sector as self-employment, which is conducted through a linear production function, with labor as the only input, and productivity $A_t b_i \frac{h_t^{\xi-1}}{\eta} a_i$, with $b_1 < b_2 < \cdots < b_5$, where h_t is the total size of informal labor. This formulation of self-employment productivity allows us to match the size of the informal sector and the extent to which households reallocate labor effort across sectors, as we explain below.[18]

Letting $S^g_{i,t}$ denote the stock of assets of generation g in period t, a worker's sequence of budget constraints for periods t, $t+1$, ... $t+30-1$ is given by:

$$C^1_{i,t} = (1-\tau) W_{i,t} l^1_{i,t} + A_t b_i \left(\frac{h_t^{\xi-1}}{\eta} \right) a_i h^1_{i,t} - S^1_{i,t},$$

[16]This specific utility cost of informality captures in reduced-form, for example, the lack of health insurance and other benefits, or the higher job insecurity in the informal sector.

[17]The disutility of work is assumed to grow with the factor $A_t^{1-\theta}$ so that the model has a balanced growth path when $\theta \neq 1$.

[18]Labor is the only input used in self-employment, so it is akin to home production. This assumption is reasonable, because capital in the informal sector of emerging countries is known to represent a very small fraction of the total capital stock. For evidence on capital in the informal sector, see Lopez-Martin (2019); Porta and Shleifer (2008); and Busso, Fazio, and Levy (2012). It is, therefore, common to model an informal sector that does not use capital as an input; see Restrepo-Echavarria (2014).

$$C_{i,t+1}^{2} = (1-\tau)W_{i,t+1}l_{i,t+1}^{2} + A_{t+1}b_i\left(\frac{h_{t+1}^{\xi-1}}{\eta}\right)a_i h_{i,t+1}^{2} + r_{t+1}S_{i,t}^{1} - S_{i,t+1}^{2},$$

$$\dots$$

$$C_{i,t+19}^{20} = (1-\tau)W_{i,t+19}l_{i,t+19}^{20} + A_{t+19}b_i\left(\frac{h_{t+19}^{\xi-1}}{\eta}\right)a_i h_{i,t+19}^{20}$$
$$+ r_{t+19}S_{i,t+18}^{19} - S_{i,t+19}^{20},$$

$$C_{i,t+20}^{21} = r_{t+20}S_{i,t+19}^{21} - S_{i,t+20}^{21},$$

$$\dots$$

$$C_{i,t+29}^{30} = r_{t+19}S_{i,t+28}^{29},$$

where τ is a constant tax rate on formal income. The last two equations describe the simpler constraints faced by retirees. Note also that the constraint in the last period of life incorporates the simplifying assumption that individuals make no bequests, so assets are zero when they die.

The period-by-period budget constraints can be combined to obtain an intertemporal budget constraint (IBC):

$$\sum_{g=1}^{30}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1} C_{i,t+g-1}^{g} = \sum_{g=1}^{20}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1} \left[(1-\tau)W_{i,t+g-1}l_{i,t+g-1}^{g} + A_{t+g-1}b_i\left(\frac{h_{t+g-1}^{\xi-1}}{\eta}\right)a_i h_{i,t+g-1}^{g}\right], \tag{10}$$

where

$$r_t = 1 + (1-\gamma)\left(R_t^{Tk} - 1\right) + \gamma r^{\star} \tag{11}$$

denotes the gross return on assets. We introduce financial openness in a simple way assuming that an exogenous fraction $\gamma \in [0,1]$ of assets is invested abroad at a fixed net interest rate $r^{\star}$, while the remaining savings are invested in domestic capital with total gross return R_t^{Tk}, where $R_t^{Tk} = 1 + r_t^k - \delta$.

The first generation at time t, then, faces the problem of maximizing utility (8) subject to the IBC (10), taking $W_{i,t+g-1}^{g}$ and r_{t+g-1} as given. This problem can be represented by the following Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{i,t} = & \sum_{g=1}^{30} \beta_i^{g-1} \frac{\left(C_{i,t+g-1}^g\right)^{1-\theta}}{1-\theta} \\
& - \sum_{g=1}^{20} \beta_i^{g-1} \Theta_{i,t+g-1}^g \frac{\left(l_{i,t+g-1}^g + h_{i,t+g-1}^g\right)^\phi + \kappa \left(h_{i,t+g-1}^g\right)^\phi}{\phi} \\
& + \Psi_{i,t} \left(\sum_{g=1}^{20} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1} \left[(1-\tau) W_{i,t+g-1} l_{i,t+g-1}^g + A_{t+g-1} b_i \left(\frac{h_{t+g-1}^{\xi-1}}{\eta} \right) a_i h_{i,t+g-1}^g \right] \right. \\
& \left. - \sum_{g=1}^{30} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1} C_{i,t+g-1}^g \right) \qquad (12)
\end{aligned}
$$

where $\Psi_{i,t}$ denotes the Lagrange multiplier associated with (10). Then, for $g = 1, ..., 30$ the first-order conditions are:

$$
C_{i,t+g-1}^g : \quad 0 = \beta_i^{g-1} \left(C_{i,t+g-1}^g\right)^{-\theta} - \Psi_{i,t} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1}. \qquad (13)
$$

For $g = 1, ..., 20$, i.e., when individuals are in the workforce,

$$
\begin{aligned}
l_{i,t+g-1}^g : \quad 0 = & -\beta_i^{g-1} \Theta_{i,t+g-1}^g \left(l_{i,t+g-1}^g + h_{i,t+g-1}^g \right)^{\phi-1} \qquad (14) \\
& + \Psi_{i,t} (1-\tau) W_{i,t+g-1} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1},
\end{aligned}
$$

$$
\begin{aligned}
h_{i,t+g-1}^g : \quad 0 = & -\beta_i^{g-1} \Theta_{i,t+g-1}^g \left[\left(l_{i,t+g-1}^g + h_{i,t+g-1}^g \right)^{\phi-1} + \kappa \left(h_{i,t+g-1}^g \right)^{\phi-1} \right] \qquad (15) \\
& + \Psi_{i,t} A_{t+g-1} b_i \left(\frac{h_{t+g-1}^{\xi-1}}{\eta} \right) a_i \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1}.
\end{aligned}
$$

From (13), we obtain the set of Euler equations for consumption for any two different periods, i.e. for $g_1 < g_2 \leq 30$:

$$
\left(C_{i,t+g_1-1}^{g_1} \right)^{-\theta} = \beta_i^{g_2-g_1} \left(\prod_{n_g=g_1}^{g_2-1} r_{t+n_g} \right) \left(C_{i,t+g_2-1}^{g_2} \right)^{-\theta}. \qquad (16)
$$

Solving for $\Psi_{i,t}$ in (13) and using the result in (14) yields the intratemporal formal-sector labor supply schedules, $g \leq 20$:

$$
\Theta_{i,t+g-1}^g \left(l_{i,t+g-1}^g + h_{i,t+g-1}^g \right)^{\phi-1} = \left(C_{i,t+g-1}^g \right)^{-\theta} (1-\tau) W_{i,t+g-1}. \qquad (17)
$$

Similarly, from (15) we obtain the informal-sector labor supply schedules, $g \leq 20$:

$$\Theta^g_{i,t+g-1}\left[\left(l^g_{i,t+g-1}+h^g_{i,t+g-1}\right)^{\phi-1}+\kappa\left(h^g_{i,t+g-1}\right)^{\phi-1}\right]=\left(C^g_{i,t+g-1}\right)^{-\theta}A_{t+g-1}b_i\left(\frac{h^{\xi-1}_{t+g-1}}{\eta}\right)a_i. \tag{18}$$

To gain intuition, it is useful to combine (17) with (18), and to solve the resulting equation for $h^g_{i,t+g-1}$, which yields:

$$h^g_{i,t+g-1}=\begin{cases}\left[\left(C^g_{i,t+g-1}\right)^{-\theta}\frac{\left[A_{t+g-1}b_i\left(\frac{h^{\xi-1}_{t+g-1}}{\eta}\right)a_i-(1-\tau)W_{i,t+g-1}\right]}{\Theta^g_{i,t+g-1}\kappa}\right]^{\frac{1}{\phi-1}} & \text{if } A_{t+g-1}b_i\left(\frac{h^{\xi-1}_{t+g-1}}{\eta}\right)a_i>(1-\tau)W^g_{i,t+g-1} \\ 0 & \text{otherwise.}\end{cases}$$

This expression shows that households supply labor to the informal sector as long as the marginal income from this activity exceeds the marginal income that they earn by working in the formal sector, net of taxes. In particular, households are compensated for the additional utility cost that informal work generates. By driving a wedge between the gross and net income from formal work, the tax rate on formal labor income allows the model to reproduce the fact that the formal sector is more productive than the informal sector, while retaining an incentive to work in the informal sector.[19]

We can use the intertemporal budget contraint to re-write the consumption equations as follows (noting $S^0_{i,t-1}=0$):

For $g_3 \leq 20$

$$C^{g_3}_{i,t+g_3-1}=\left(\sum_{g=g_3}^{30}\left(\prod_{n_g=g_3}^{g-1}r_{t+n_g}\right)^{\frac{1-\theta}{\theta}}\left(\beta_i^{g-g_3}\right)^{\frac{1}{\theta}}\right)^{-1} \tag{19}$$
$$\left\{r_{t+g_3-1}S^{g_3-1}_{i,t+g_3-2}+\sum_{g=g_3}^{20}\left(\prod_{n_g=g_3}^{g-1}r_{t+n_g}\right)^{-1}\right.$$
$$\left.\left[(1-\tau)W_{i,t+g-1}l^g_{i,t+g-1}+A_{t+g-1}b_i\left(\frac{h^{\xi-1}_{t+g-1}}{\eta}\right)a_ih^g_{i,t+g-1}\right]\right\},$$

and for $g_3 > 20$

$$C^{g_3}_{i,t+g_3-1}=\left(\sum_{g=g_3}^{30}\left(\prod_{n_g=g_3}^{g-1}r_{t+n_g}\right)^{\frac{1-\theta}{\theta}}\left(\beta_i^{g-g_3}\right)^{\frac{1}{\theta}}\right)^{-1}r_{t+g_3-1}S^{g_3-1}_{i,t+g_3-2}. \tag{20}$$

Finally, we can now define the aggregate variables used to describe the evolution of the capital stock (7). Aggregate labor income net of taxes is given by $Y^d_t=\sum_{i=1}^5\sum_{g=1}^{20}(1-\tau)W_{i,t}l^g_{i,t}+b_i(h^{\xi-1}_t/\eta)a_ih^g_{i,t}$. Aggregate asset gross returns are given by $r_tS_{t-1}=r_t\sum_{g=1}^{29}S^g_{t-1}$. Finally, aggregate consumption is given by $C_t=\sum_{g=1}^{30}C^g_t$.

[19] For evidence on wage differentials between the formal and informal sectors in Chile, see Joubert (2015).

3.4 Labor Market Equilibrium

Equilibrium in the labor market requires that demand for each skill group equal supply. Recall the firm's labor input is a CES bundle composed of labor of each skill, $L_{i,t}$. The labor input from skill group i at time t includes individuals of all active cohorts from that skill group:

$$L_{i,t} = \sum_{g=1}^{20} L_{i,t}^{g}, \tag{21}$$

where $L_{i,t}^{g}$ is given by

$$L_{i,t}^{g} = l_{i,t}^{g} N_{i,t}^{g}, \tag{22}$$

i.e., hours worked by workers of type i and cohort g ($l_{i,t}^{g}$), multiplied by the number of workers ($N_{i,t}^{g}$).

To conclude the description of the model, note that it is non-stationary due to technological progress and population growth. The appendix contains the complete set of equilibrium conditions, as well as the detrended version of the model, and the derivation of the steady state.

4 Calibration and Simulation Design

The model is calibrated to the Chilean economy. Table 1 summarizes the baseline calibration. The duration of a period, given by parameter T, is set to 8 quarters. This implies that individuals in the model are retired for 20 years following a working life of 40 years. The 5 different skill groups in the economy represent income quintiles.[20] The subjective discount factors of the different skill groups, β_i for $i = 1, ..., 5$, in turn, are calibrated to replicate the average saving rates for each income quintile, which are computed from the 2012 Family Budget Survey (Encuesta de Presupuestos Familiares; EPF), using the methodology proposed by Madeira (2015, 2016). The matched saving rates are 9%, -0.1%, 4.2%, 8.2% and 17% for the first to fifth quintiles, respectively. These rates imply an aggregate saving rate, weighted by income, of 7.7%.[21] The parameter that governs the disutility of work, χ_i^g, is calibrated so as to set total labor supply (formal plus informal), equal across skill groups in the initial steady state.

The annual population growth rate, n^r, is set to 0.5%, and the technological growth rate, z, is set to 2%, in line with estimates and forecasts of trend growth for Chile obtained by Albagli, Contreras, de la Huerta, Luttini, Naudon, and Pinto (2015) for the period 2016-2050. Following the same study, the labor share in production, $1 - \alpha$, is set to 52%, which corresponds to the 2008-2013 average of the ratio of salaries paid by the corporate sector to the value added of that sector, net of taxes, according to national accounts data.[22] The capital depreciation rate, δ, is set to 4% per year.

The parameter υ, which governs the size of the wealth effect on labor supply is set to 1. This specification favors the class of preferences that have a zero wealth effect, similar to the preferences proposed by Greenwood, Hercowitz, and Huffman (1988). Consequently, changes in

[20]Quintile number 1 represents the lowest and least skilled group.

[21]This number is close to the aggregate saving rate for Chile that was estimated to be between 8.3% and 9.8% since 2010 by the OECD (https://data.oecd.org/hha/household-savings.htm).

[22]The source of these data is the Central Bank of Chile.

the quantity of labor supplied are only due to fluctuations in wages.[23]

The productivity parameters in the formal sector, a_i for $i = 1, ..., 5$, are calibrated to replicate the distribution of labor income (percentage of aggregate income corresponding to each quintile) as reported by the 2015 Socioeconomic Characterization Survey (Encuesta de Caracterización Socioeconómica Nacional; CASEN).[24]

The parameter that determines the specific utility cost of informal work, κ, is set to 1/35, so as to make the return from informal work lower than the gross wage and higher than the net wage in the formal sector. The parameter ξ, in turn, allows us to discipline the reallocation of labor effort across the formal and informal sectors. Following Joubert (2015), it is set to 0.53, so that a 5-percentage-point increase in the income tax generates a 10% increase in informal labor.[25] The parameter η is set so that the factor $(h_t^{\xi-1}/\eta)$ equals 1 in the initial steady state.

The parameters that govern the efficiency of informal work, b_i, are calibrated so as to match participations in the informal sector of 57%, 18%, 18%, 17% and 14%, for quintiles 1 to 5, respectively, which implies an aggregate participation of 25% in the informal sector. These statistics were computed using data from the 2012 New Supplementary Income Survey (Nueva Encuesta Suplementaria de Ingresos; NESI). We consider the following as informal workers: employees without a contract, and self-employed individuals that work at home or on the street (excluding those with a university education, which are considered formal workers). The parameter ρ, which determines the elasticity of substitution between the different skill groups, is set to 0.33, following Sommacal (2006). This value implies an elasticity of substitution between the different skill types of 1.5, in line with the empirical evidence for the U.S. reported by Ciccone and Peri (2005).

The fraction of domestic savings invested abroad, γ, is set to 40%. This number corresponds to the value of all foreign investment by Pension Fund Administrators (Administradoras de Fondos de Pensiones), as a fraction of total assets, according to August 2016 data from the Chilean Superintendency of Pensions. The foreign interest rate, r^*, is set to 3% per annum. In turn, the fraction of foreign investment in Chile, γ^*, is set to 145% of GDP at the initial steady state. This number, from national accounts, corresponds to the gross investment position of foreign entities in Chilean banks, other financial institutions, non-financial firms, and households, expressed as a ratio of GDP.

We now describe the design of our simulation exercises, which is based on the recent experience of Chile. The first ingredient of our simulations is demographic. We simulate a change in the size of the labor force using the data shown in figure 1 for the period 2015–2050, which is based on historical and projected population data from the National Statistics Institute, and on estimates and forecasts of participation rates due to Aldunate et al. (2019a).[26] Our simulations

[23] Correia, Neves, and Rebelo (1995) show that these preferences are useful for real small open economy models to match features of aggregate fluctuations.

[24] According to this survey, the fraction of aggregate income corresponding to each quintile is 4.2%, 9.7%, 14.5%, 21.1% and 50.5%.

[25] Specifically, Joubert (2015) studies the effect of a 5-percentage-point increase in payroll taxes to fund pensions in a partial equilibrium model for Chile.

[26] The dynamics of the labor force are based on estimates and projections for the total population, the working-age population, and participation rates. The population data come from the National Statistics Institute (December 2018). The participation rates used to pin down the labor force come from Aldunate et al. (2019a), who construct forecasts of the labor force as an input for forecasts of trend growth in Chile.

Table 1: Baseline calibration

Parameter	Description	Value/to match	Source/comments
T	Duration of a period	8 quarters	-
$\beta_i^{-1/T} - 1$	Annual discount rates	Saving rates	2012 Family Budget Survey
$\chi_i^g = \chi_i$	Disutility of labor	Total labor supply constant $\forall i$	-
n^r	Annual population growth rate	0.5%	Albagli et al. (2015)
z	Annual technological growth rate	2%	Albagli et al. (2015)
θ	Intertemp. elasticity of subs., inv	1	Literature
ϕ	Governs labor supply elasticity	6	Literature
α	Capital share	0.48	Albagli et al. (2015)
$1-(1-\delta)^{1/T}$	Annual depreciation	4%	Literature
κ	Disutility of informal work	1.3	-
ξ	Param. on informal output	0.53. Reallocation formal/inf	Joubert (2015)
η	Param. on informal output	$\left(\frac{h_t^{\xi-1}}{\eta}\right) = 1$ at initial ss	-
υ	Wealth effect	1	Greenwood et al. (1998)
ρ	Elasticity of subs. across skills	0.33	Ciccone and Peri (2005)
a_5	Productivity, Form. sector	1	Calibration targets:
a_4	Productivity param., formality.	0.42	Labor income quintiles
a_3	Productivity, Form. sector	0.28	(2015 CASEN)
a_2	Productivity, Form. sector	0.18	
a_1	Productivity, Form. sector	0.05	$[4.2, 9.7, 14.5, 21.1, 50.5]^*$
a_5b_5	Productivity param., informality.	1.68	Calibration targets: Aggregate
a_4b_4	Product. Inf. sector	0.72	participation in informal sector
a_3b_3	Product. Inf. sector	0.49	and distribution across quintiles.
a_2b_2	Product. Inf. sector	0.32	(2012 NESI)
a_1b_1	Product. Inf. sector	0.16	$[57, 18, 18, 17, 14]^*$

Note: *income quintiles 1 through 5 are ordered from lowest to highest income.

reproduce population data beginning in 2015, the year in which the immigration flow begins. The shocks that alter the size of the labor force are concentrated in the first three model periods, which reflect the six-year period 2015–2020. Thereafter, the demographic shocks are much smaller, which reflects the National Statistics Institute's more normal projections of immigration flows after 2020. The immigration flow in the first three periods of our simulations represents 8.7% of the total labor force prior to the beginning of the wave (about 700,000 immigrants between 2015-2020 and a labor force of about 8.5 million in 2014).

The upper-left graph in figure 6 depicts the higher growth rate of the total population generated by the flow of immigrants entering the model economy.[27] The first vertical line in that graph marks the first period in which immigrant workers that arrive in the first three periods of the simulation are retired, the second vertical line marks the first period in which these immigrants are deceased, and the third vertical line marks the period in which all immigrants are deceased and the economy reaches the steady state. In addition to altering the size of the labor force, and as shown in the right-hand side panels of figure 6, immigration alters the age structure of the population—indeed, the age structure of immigrants is skewed towards younger working-age adults. Since individuals in the model are active for 40 years (20 periods, each with duration of 8 quarters) and retired for 20 years (10 periods), and since we abstract from human capital accumulation, our simulations match the age distribution of immigrants in the 20-80 range of age. In the model, the native population is assumed to grow at a constant rate, so the age distribution of natives is smoother than in the data. The lower-left graph of Figure 5 shows that the ratio of retired to active individuals declines due to the immigration of mostly younger workers.

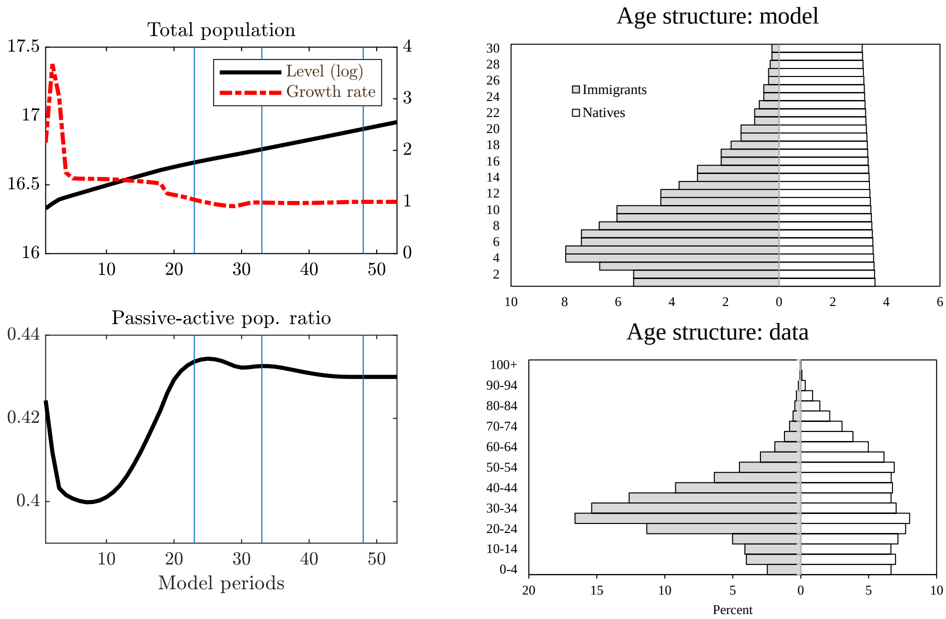
The second key ingredient of our simulations is related to the skills of immigrants and our assumption of an underemployment spell. As mentioned in subsection 2.1, our baseline results assume that the long-run skill distribution of immigrants is identical to that of the native population. However, in line with our assumption that immigrants experience an underemployment spell before they can fully integrate into the labor market, we impose a temporary *de facto* distribution of skills on immigrants during the first two periods they live in this economy. This transitory shock to the skill distribution of immigrants lowers their productivity. Recall that in steady state, the population is distributed evenly among the five skill groups. In our baseline scenario, the skill distribution of immigrants is adversely skewed for two model periods (4 years), such that 70% of immigrants have the productivity associated to the two groups of lowest skills, as shown in figure 7. After the underemployment spell is over, immigrants recover their productivity immediately and return to the steady state skill distribution.[28]

We also study the sensitivity of our baseline results to alternative assumptions on the intensity and duration of the underemployment spell. In one alternative scenario, the intensity of the underemployment spell is higher than in the baseline simulation—the skill distribution is more adversely skewed, such that 100% of immigrants have the productivity associated to the 2 groups of lowest skills. In another alternative scenario, the duration of the underemployment spell is

[27] Since the model abstracts from the decision to participate in the labor market, we use the terms "labor force" and "total population" interchangeably, but as previously mentioned, the simulation design uses data on the labor force.

[28] Note that in the calibration of the underemployment spell, we target the skill distribution of *all* immigrants, but the skill "loss" of individual immigrants that leads to this distribution is not unique.

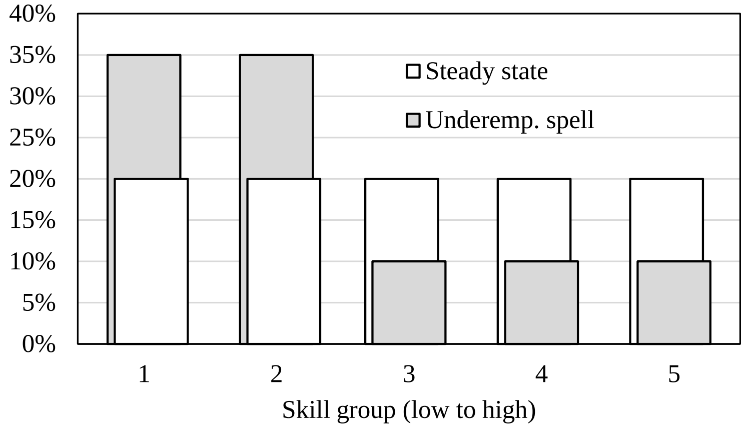
Figure 6: Population dynamics in simulations



Source: Aldunate et al. (2019b) and authors' calculations.
Note: The vertical lines in the left-hand-side graphs mark the first period in which: (i) all immigrant workers are retired, and (ii) all immigrant workers are deceased. Each period in the model lasts 2.5 years.

longer—it lasts 5 model periods (10 years), as opposed to 2 periods in the baseline simulation.

Figure 7: Distribution of skills



Source: Authors' calculations.

5 Results

This section presents the distributional, sectoral, and aggregate results of our baseline simulation of an immigration wave. We highlight the role of the informal sector, and study the sensitivity of the results to our assumptions on the duration and intensity of the underemployment spell that affects immigrants upon arrival, as well as their long-run skill level.

5.1 Baseline Simulation

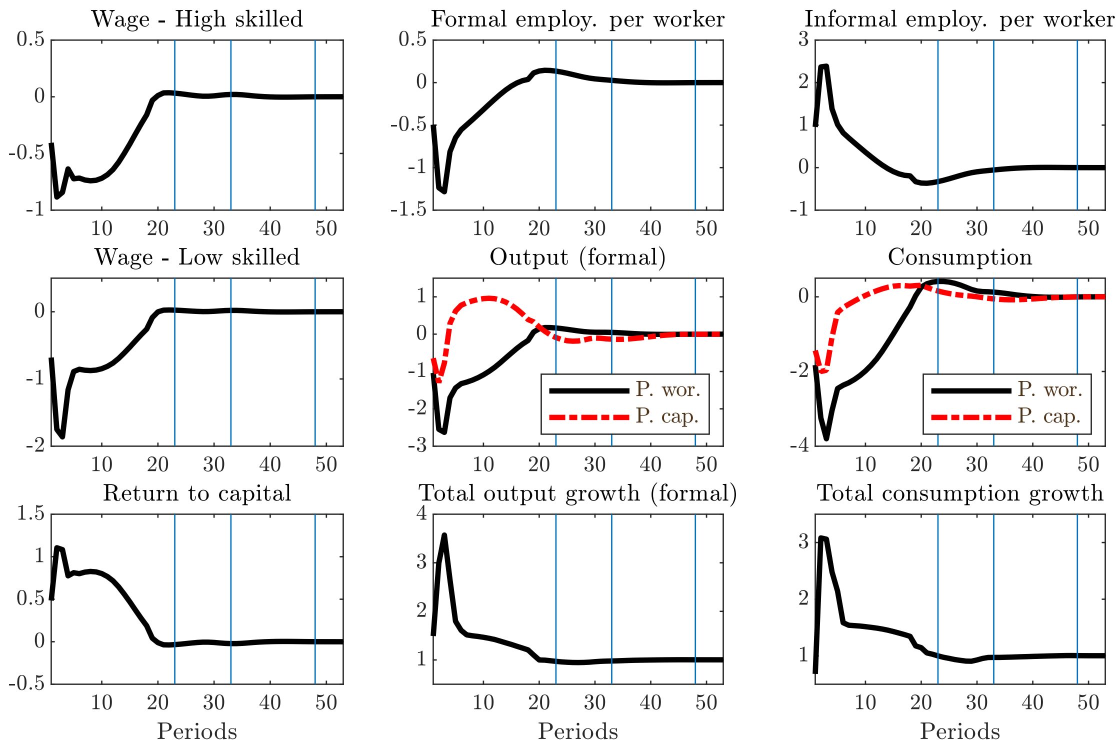
Figure 8 shows the results of our baseline simulation of an immigration wave. As mentioned in the previous section, the key ingredients of the simulation are shocks to population growth and the age structure, and the underemployment spell that immigrants experience. The immigration wave leads to an increase in labor supply of workers of all skill levels, which generates a generalized decline in wages. The decline in wages of low-skilled workers, however, is nearly twice as large as that of high-skilled workers. This distributional effect of the shock is primarily due to the underemployment spell that affects immigrants, which is modeled as a transitory deterioration in their skill distribution. On impact, therefore, the increase in supply of low-skilled labor is larger than that of high-skilled labor, so low-skilled wages fall more than high-skilled wages. We will elaborate on this point below. Capital is predetermined, so the economy-wide ratio of capital to (formal) labor declines. Consequently, the return to capital increases.[29]

The generalized decline in formal-sector wages induces active workers to reallocate labor effort to the informal sector: the average active worker devotes less hours to the formal sector and more hours to the informal sector. Reallocation to the informal sector is driven by the behavior of low-skilled workers, whose wages in the formal sector are highly affected.

Regarding the aggregate effects of the immigration wave, note that both (formal) output and consumption (of formal output and informal self-employment) per worker decline. In per capita terms, i.e. with respect to the total population, output and consumption fall less than in per worker terms, because the total population increases less than the labor force in the simulation, as in the data (recall immigrants are mostly young adults). Finally, although output and consumption decline in per worker and per capita terms, total output and consumption in the economy expand, since the size of the economy increases with immigration.

[29]Note that capital is only used in the formal sector, so the relevant measure of scarcity relates it to formal labor.

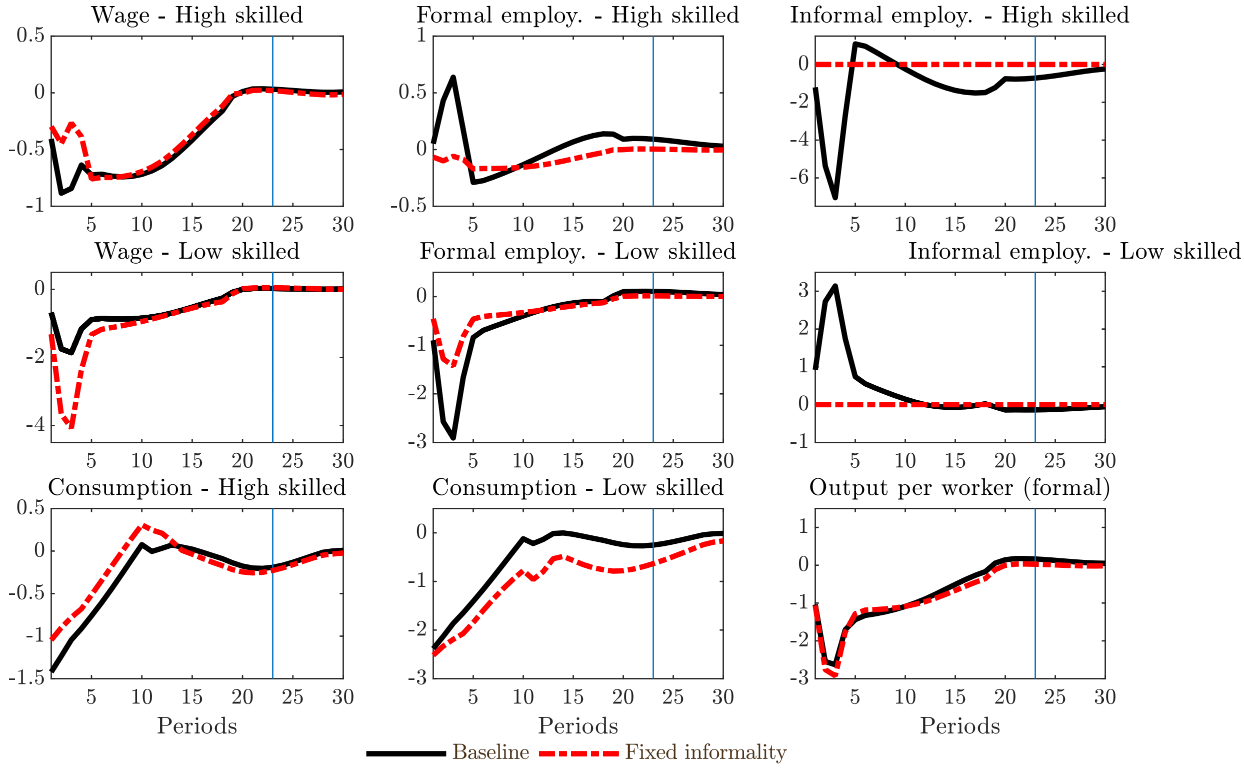
Figure 8: Baseline simulation



Note: Percent deviation from steady state, except for total output and consumption growth rates. The three vertical lines in each graph mark the first period in which: (i) immigrant workers that arrive in the first three periods retire, (ii) these immigrants die, and (iii) all immigrants die. Each period lasts 2 years. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perferct foresight.

To study the role of reallocation to the informal sector in the baseline results, we fix the number of hours worked in the informal sector at the steady state level and rerun the simulation. Figure 9 shows that the main effect of the informal sector is that it mitigates the decline in consumption experienced by individuals with the lowest skills (middle of the third row). Since low-skilled individuals cannot reallocate labor effort to the informal sector, the low-skilled wage in the formal sector declines more than twice as in the baseline simulation (second row), exacerbating the distributional effects of the immigration wave. In sum, the informal sector allows workers with the lowest skills to cushion the downward pressure of immigration on their income and consumption.

Figure 9: Baseline simulation: The role of the informal sector

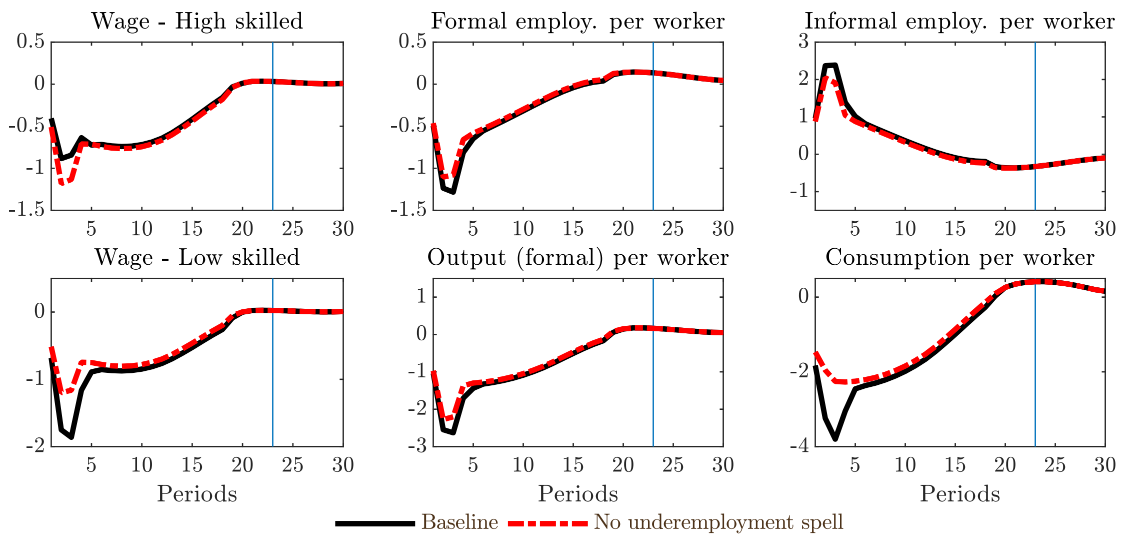


Note: The red lines show results from a simulation in which hours worked in the informal sector are fixed at their steady state. Percent deviation from steady state. The vertical line in each graph marks the first period in which immigrant workers that arrive in the first three periods retire.

To study the role of the underemployment spell, figure 10 compares the baseline results to a simulation that ignores this feature, i.e., a simulation that includes only demographic shocks that alter the size and age structure of the population, so that immigrants and natives have the same skill distribution. The two panels on left show that the main effect of the underemployment spell is distributional: in its absence, the decline in wages across workers of different skill levels is fairly similar at around 1% from steady state. When a fraction of immigrants are underemployed, as if they had lower skills, low-skilled labor is relatively more abundant, so low-skilled wages fall much more, and high-skilled wages fall less, than in the baseline simulation. Finally, and as one would expect, consumption per worker is also less sensitive to the immigration wave when immigrants are not affected by the underemployment spell.

To conclude this subsection, we study the importance of considering that the age structure of immigrants is very different than that of natives, i.e., the importance of using a model with overlapping generations. Figure 11 compares the baseline results with a simulation in which immigrants and natives have the same age structure. In this case, the magnitude of the effects of an immigration wave are substantially smaller and less persistent. Since the immigrant population is older than in the baseline simulation, retired immigrants represent a larger fraction of the total immigrant population, and active immigrants reach the retirement age sooner. The increase in labor supply is, thus, smaller and less persistent, and so is the downward pressure

Figure 10: Baseline simulation: The role of the underemployment spell



Note: The red lines show results from a simulation in which immigrants do not experience an underemployment spell, i.e., one that includes only demographic shocks that alter the size and age structure of the population. Percent deviation from steady state. The vertical line in each graph marks the first period in which immigrant workers that arrive in the first three periods retire.

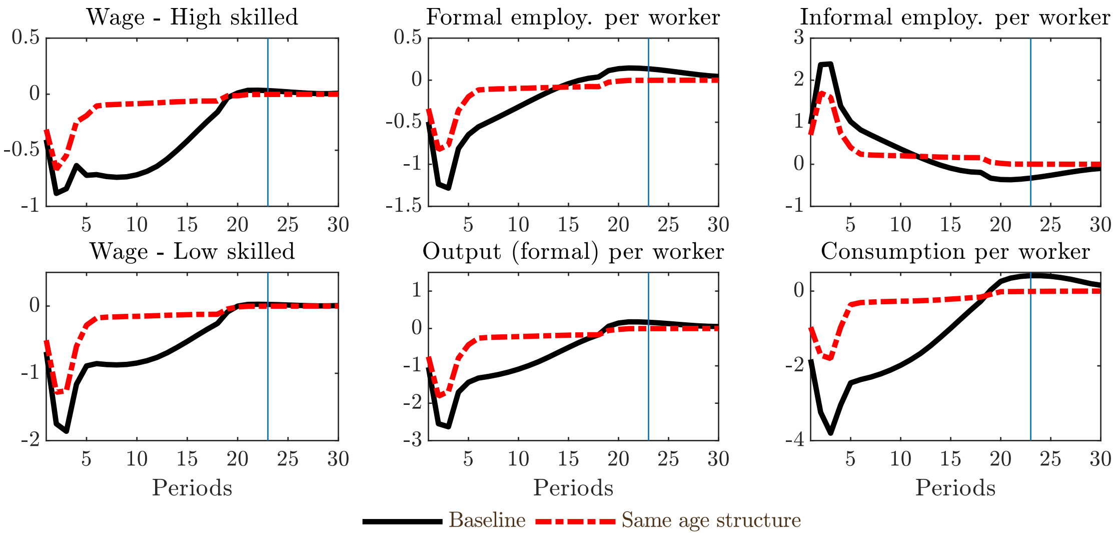
on wages. Reallocation to the informal sector is also smaller than in the baseline simulation. Naturally, the decline in output and consumption per worker is much smaller, reaching a trough of about half of that in the baseline simulation. These results support the use of an OLG model that is able to accommodate changes in the age structure of the population.

5.2 Sensitivity Analysis

We now study the sensitivity of the results to alternative assumptions on the duration and intensity of the underemployment spell experienced by immigrants, as well as on their long-run skill level. Figure 12 considers the case in which immigrants experience a longer underemployment spell, figure 13 considers the case in which the underemployment spell is more widespread, so that the skill distribution of immigrants is more adversely skewed, and figure 14 considers a scenario in which the long-run skill level of immigrants is slightly higher than natives, as in the data (right panel of figure 2).

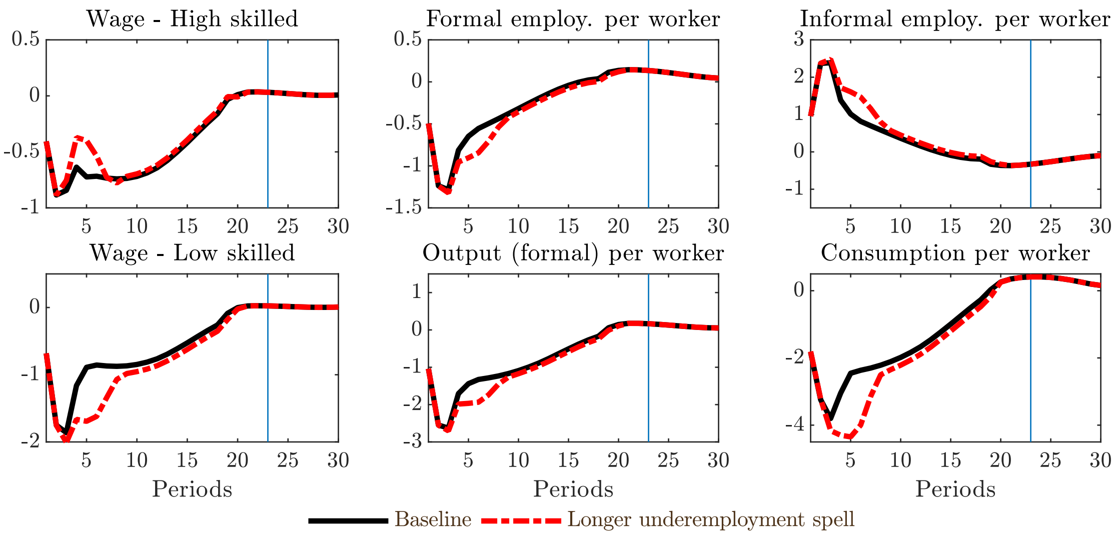
In our baseline simulation, immigrants experience an underemployment spell of 2 model periods (4 years). In figure 12, we compare these results with a scenario in which the adaptation of immigrants to the labor market requires substantially more time, so that the underemployment spell lasts 5 model periods (10 years). Under a longer underemployment spell, the decline in low-skilled wages is more persistent, which induces a larger reallocation of labor effort from the formal to the informal sector. Consequently, output and especially consumption per worker decline more persistently than in the baseline simulation.

Figure 11: Baseline simulation: The role of age structure



Note: The dashed red lines show results from a simulation in which the age structure of immigrants is identical to that of natives (they still experience an underemployment spell). Percent deviation from steady state. The vertical line in each graph marks the first period in which immigrant workers that arrive in the first three periods retire.

Figure 12: Longer underemployment spell

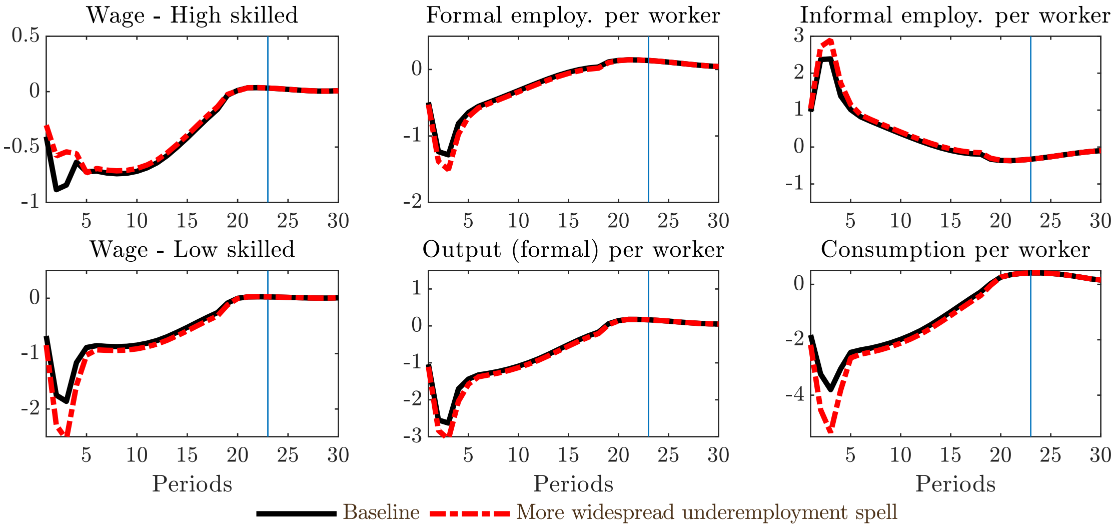


Note: The red lines show results from a simulation in which immigrants experience an underemployment spell of 5 periods (10 years); 2 periods in the baseline results. Percent deviation from steady state. The vertical line in each graph marks the first period in which immigrant workers that arrive in the first three periods retire.

Figure 13 studies the sensitivity of the baseline results to our assumption on the intensity of the underemployment spell. Recall that in the baseline simulation, 70% of immigrants are distributed in the two lowest skills. We compare the baseline results to a simulation in which

100% of immigrants are distributed in the two lowest skills.[30] The distributional effects of the shock are larger in this scenario, with the low-skilled wage falling more, and the high-skilled wage falling less, than in the baseline simulation. Since low-skilled workers are more susceptible to reallocation, informal employment per worker expands more than in the baseline simulation, formal employment per worker contracts more than in the baseline simulation, and thus, output and consumption per worker are more adversely affected.

Figure 13: Widespread underemployment spell



Note: The red lines show results from a simulation in which the skill distribution of immigrants more adversely skewed—all immigrants are evenly distributed in the two lowest skills. In the baseline simulation, the skill distribution of immigrants is [35%,35%,10%,10%,10%]. Percent deviation from steady state. The vertical line in each graph marks the first period in which immigrant workers that arrive in the first three periods retire.

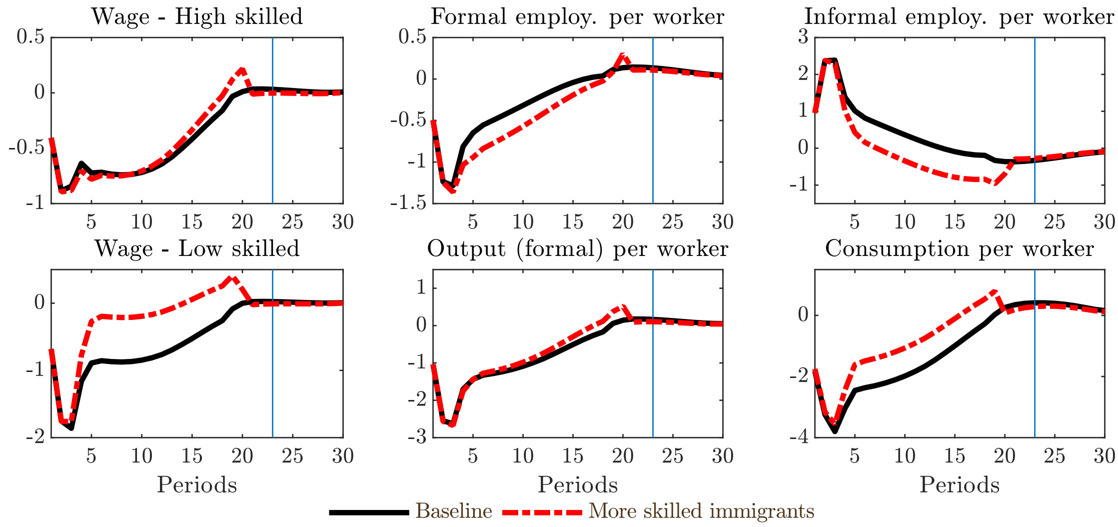
The final analysis of the sensitivity of our results concerns the long-run skill level of immigrants. In our baseline simulations, we assumed immigrants and natives have the same long-run skill level. However, as shown in the right panel of figure 2, the data suggest immigrants have, on average, a slightly higher education level than natives. If education levels were to map directly onto skills, immigrants would have a slightly higher skill level. Figure 14 considers this case—after experiencing an underemployment spell, immigrants adjust to the labor market and exercise a slightly higher skill level than natives.[31] We can see that after the first few periods of the simulation, once the underemployment spell is over, low-skilled wages recover much faster than in the baseline case. Indeed, by period 5, the low-skilled wage is very close to its steady state. Since the decline in the low-skilled wage is more transitory, the increase in informal labor is also more short-lived. A smaller deterioration in the labor market results in a faster recovery

[30] As in the baseline case, the underemployment spell lasts for 2 model periods.

[31] To consider immigrants with a higher distribution of skills than natives, and since we do not wish to introduce specific productivity parameters for immigrants, we simply redefine the size of each skill group of immigrants to capture the desired distribution. Under this scenario, immigrants have the following long-run skill distribution (from high to low): [0.25, 0.21, 0.25, 0.10, 0.19].

of consumption per worker.[32]

Figure 14: More skilled immigrants



Note: The red lines show results from a simulation in which, after the underemployment spell, immigrants have slightly higher skills than natives. In the baseline simulation, their long-run skill level is identical. Percent deviation from steady state. The vertical line in each graph marks the first period in which immigrant workers that arrive in the first three periods retire.

6 Conclusion

Emerging countries receive substantial immigration flows, yet the literature on the macroeconomic effects of immigration has largely ignored them. This paper fills this gap, considering the importance of labor informality in emerging countries. The informal sector acts as a buffer that allows low-skilled workers to mitigate the decline in consumption generated by an immigration wave. An overlapping generations model (OLG) calibrated to Chilean demographic and economic data suggests that immigration increases labor supply, putting downward pressure on formal-sector wages, and triggering a reallocation of labor effort to the informal sector, which helps workers, especially those with lower skills, sustain income and consumption.

Our simulations highlight the importance of modeling a rich demographic structure that allows for immigrants with a different age structure than natives, as well as the importance of the adaptation process of immigrants to the labor market. The longer this process takes, the larger the impact of immigration on the economy. The model then provides important quantitative references for the macroeconomic effects of immigration in emerging countries.

We find that an immigration flow puts downward pressure on wages, but this result should not be interpreted in a normative sense, as suggesting immigration is undesirable. There are many channels, which we do not explore, through which migration could contribute to macroeconomic stability and long-run growth. To name a few, labor mobility can mitigate macroeconomic

[32] Although immigrants have a higher long-run skill level, the economy converges to the initial steady state after immigrants die. We are effectively assuming that immigrants' descendants do not inherit their parents' higher skill level.

fluctuations (Mandelman and Zlate, 2012), promote innovation by increasing diversity (Akcigit, Grigsby, and Nicholas, 2017), and contribute to long-run growth by facilitating the allocation of resources to the most productive regions (Peri, 2012).[33] Likewise, we find that informality acts as a buffer that mitigates the decline in consumption of low-skilled workers, but this should not be interpreted in a normative sense, as suggesting that informality is beneficial. It is well known that informality is a powerful drag on emerging countries' long-run growth prospects (Docquier, Müller, and Naval, 2017).

Our analysis of the macroeconomic effects of immigration in an emerging country is based on simulations of an OLG model calibrated to match Chilean data. It would certainly be desirable to complement these results with those of an empirical identification of the effects of an immigration shock, as in Furlanetto and Robstad (2019), who estimate Bayesian vector autoregressions on data for Norway, but data limitations prevent this type of analysis for an economy such as Chile, which lacks quarterly data on net immigration flows. Moreover, substantial immigration is a very recent phenomenon; it began around 2015. Even if better data were available, it would be challenging to observe enough variation in immigration to identify the effect of shocks.

Finally, the OLG model we build in this paper could be extended to analyze other issues. For example, including a fiscal block would allow an analysis of the effect of immigration on public finances and pension systems.

[33] Immigration could also benefit the pay-as-you-go pension system of the host country; see Lacomba and Lagos, 2010.

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Appendices

A Equilibrium Conditions

In equilibrium, labor supply of each worker type and cohort equals labor demand:

$$l_{i,t}^{g} N_{i,t}^{g} = L_{i,t}^{g}, \quad i = 1, ..., 5 \;\; g = 1, ..., 20$$

Aggregate consumption is given by

$$C_t^g = \sum_{i=1}^{5} C_{i,t}^{g} N_{i,t}^{g}.$$

Aggregate savings satisfy

$$S_t = \sum_{g=1}^{29} S_t^g,$$

where

$$S_t^g = \sum_{i=1}^{5} S_{i,t}^{g} N_{i,t}^{g}.$$

A fraction $1 - \gamma$ of total savings, S_t, is then invested - into capital - to transfer the savings into the following period:

$$I_t = (1 - \gamma) S_t + \gamma^{\star} N_t^1, \tag{23}$$

where we allow for the possibility that foreign agents finance some fraction $\gamma^{\star} \in [0, 1]$ of the capital stock.[34]

The (non-stationary) competitive equilibrium of the model is then given by the set of sequences

$$\begin{aligned} \{&C_{1,t}^{1}, ..., C_{5,t}^{1}, ..., C_{1,t}^{30}, ..., C_{5,t}^{30}, S_{1,t}^{1}, ..., S_{5,t}^{1}, ..., S_{1,t}^{29}, ..., S_{5,t}^{29}, \\ &l_{1,t}^{1}, ..., l_{5,t}^{1}, ..., l_{1,t}^{20}, ..., l_{5,t}^{20}, h_{1,t}^{1}, ..., h_{5,t}^{1}, ..., h_{1,t}^{20}, ..., h_{5,t}^{20}, \\ &W_{1,t}^{1}, ..., W_{5,t}^{1}, ..., W_{1,t}^{20}, ..., W_{5,t}^{20}, L_{1,t}, ..., L_{5,t}, L_{1,t}^{1}, ..., L_{5,t}^{1}, \\ &..., L_{1,t}^{20}, ..., L_{5,t}^{20}, N_{1,t}, ..., N_{5,t}, N_t^1, ..., N_t^{30}, r_t^k, R_t^{Tk}, r_t, Y_t, L_t, \\ &W_t, C_t, C_t^1, ..., C_t^{30}, S_t, S_t^1, ..., S_t^{29}, K_{t+1}, I_t, N_t\}_{t=0}^{\infty}, \end{aligned}$$

such that for given initial values, the following conditions are satisfied for every t. For $0 < g \leq 20$, defining $S_{i,t}^{0} = 0$,

$$C_{i,t}^{g} = W_{i,t}^{g} l_{i,t}^{g} + A_t b_i \frac{h_t^{\xi - 1}}{\eta} a_i h_{i,t}^{g} + r_t S_{i,t-1}^{g-1} - S_{i,t}^{g},$$

for $20 < g < 30$:

$$C_{i,t}^{g} = r_t S_{i,t-1}^{g-1} - S_{i,t}^{g},$$

[34]That fraction is assumed to grow with the trend N_t^1 so as to ensure a balanced growth path.

and

$$C_{i,t}^{30} = r_t S_{i,t-1}^{29}.$$

For $0 < gg \leq 20$:

$$C_{i,t+gg-1}^{gg} = \left(\sum_{g=gg}^{30} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} \left[r_{t+gg-1} S_{i,t+gg-2}^{gg-1} + \right.$$
$$\left. \sum_{g=gg}^{20} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{-1} \left[W_{i,t+g-1}^{g} l_{i,t+g-1}^{g} + A_{t+g-1} b_i \frac{h_{t+g-1}^{\xi-1}}{\eta} a_i h_{i,t+g-1}^{g} \right] \right],$$

for $20 < gg$:

$$C_{i,t+gg-1}^{gg} = \left(\sum_{g=gg}^{30} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} r_{t+gg-1} S_{i,t+gg-2}^{gg-1},$$

for $g = 1, ..., 30$:

$$C_t^g = \sum_{i=1}^{5} C_{i,t}^g N_{i,t}^g,$$

for $0 < g \leq 20$:

$$\chi_i^g A_{t+1-g} \left(l_{i,t}^g + h_{i,t}^g \right)^{\phi-1} = \left(\frac{C_{i,t}^g}{A_{t+1-g}} \right)^{-\theta(1-\upsilon)} W_{i,t}^g,$$

$$h_{i,t}^g = \max \left\{ \left[\left(C_{i,t}^g \right)^{-\theta(1-\upsilon)} \frac{\left(A_t b_i \frac{h_t^{\xi-1}}{\eta} a_i - W_{i,t}^g \right)}{\chi_i^g \kappa} \right]^{\frac{1}{\phi-1}}, 0 \right\},$$

$$W_{i,t}^g = a_i W_t \left(\frac{L_t}{L_{i,t}} \right)^{1-\rho},$$

$$L_{i,t} = \sum_{g=1}^{20} L_{i,t}^g,$$

$$l_{i,t}^g N_{i,t}^g = L_{i,t}^g,$$

$$N_{i,t}^g = \lambda_{i,t}^g N_t^g,$$

$$Y_t = K_t^\alpha \left(A_t L_t \right)^{1-\alpha},$$

$$L_t = \left(\sum_{i=1}^{5} a_i L_{i,t}^\rho \right)^{1/\rho},$$

$$W_t = \frac{(1-\alpha) Y_t}{L_t},$$

$$r_t^k = \alpha \frac{Y_t}{K_t},$$

$$R_t^{Tk} = r_t^k + (1-\delta),$$

$$r_t = 1 + (1-\gamma)\left(R_t^{Tk} - 1\right) + \gamma r^{\star},$$

$$S_t = \sum_{g=1}^{29} S_t^g,$$

for $0 < g \leq 29$:

$$S_t^g = \sum_{i=1}^{5} S_{i,t}^g N_{i,t}^g.$$

$$C_t = \sum_{g=1}^{30} C_t^g,$$

$$I_t = (1-\gamma) S_t + \gamma^{\star} A_t N_t^1,$$

$$K_{t+1} = I_t,$$

$$A_t = A_{t-1}(1+z)$$

$$N_t^1 = N_{t-1}^1 (1 + n^r + n_t^m),$$

$$N_t^g = N_{t-1}^g (1 + m_t^g) \quad \forall g > 1.$$

Some complementary variables:

$$H_t = \sum_{g=1}^{20} \sum_{i=1}^{5} h_{i,t}^g N_{i,t}^g, \tag{24}$$

$$L_t^{total} = \sum_{g=1}^{20} \sum_{i=1}^{5} l_{i,t}^g N_{i,t}^g. \tag{25}$$

To rewrite the model in stationary form, we define the stationary per-capita cariables: $c_{i,t}^g = C_{i,t}^g / A_{t+1-g}$, $c_t^g = C_t^g / (A_{t+1-g} N_t^g)$, $s_{i,t}^g = S_{i,t}^g / A_{t+1-g}$, $s_t^g = S_t^g / (A_{t+1-g} N_t^g)$, $s_t = S_t / \left(A_t N_t^1\right)$, $w_{i,t}^g = W_{i,t}^g / A_{t+1-g}$, $w_t = W_t / A_t$, $l_{i,t} = L_{i,t} / N_t^1$, $l_{i,t}^g = L_{i,t}^g / N_{i,t}^g$, $l_t = L_t / N_t^1$,$k_t = K_t / \left(A_{t-1} N_{t-1}^1\right)$, $i_t = I_t / \left(A_t N_t^1\right)$,$y_t = Y_t / \left(A_t N_t^1\right)$, $c_t = C_t / \left(A_t N_t^1\right)$.

The stationary competitive equilibrium of the model is then given by the set of sequences

$$\begin{aligned} \{c_{1,t}^1, ..., c_{5,t}^1, ..., c_{1,t}^{30}, ..., c_{5,t}^{30}, s_{1,t}^1, ..., s_{5,t}^1, ..., s_{1,t}^{29}, ..., s_{5,t}^{29}, \\ l_{1,t}^1, ..., l_{5,t}^1, ..., l_{1,t}^{20}, ..., l_{5,t}^{20}, h_{1,t}^1, ..., h_{5,t}^1, ..., h_{1,t}^{20}, ..., h_{5,t}^{20}, \\ w_{1,t}^1, ..., w_{5,t}^1, ..., w_{1,t}^{20}, ..., w_{5,t}^{20}, l_{1,t}, ..., l_{5,t}, \\ r_t^k, R_t^{Tk}, r_t, y_t, l_t, h_t, l_t^{total} \\ w_t, c_t, c_t^1, ..., c_t^{30}, s_t, s_t^1, ..., s_t^{29}, k_{t+1}, i_t\}_{t=0}^{\infty}, \end{aligned}$$

such that for given initial values, the following conditions are satisfied for every t:[35]

$$c_{i,t}^{1} = w_{i,t}^{1} l_{i,t}^{1} + b_i \frac{h_t^{\xi-1}}{\eta} a_i h_{i,t}^{1} - s_{i,t}^{1}, \tag{26}$$

for $1 < g \leq 20$

$$c_{i,t}^{g} = w_{i,t}^{g} l_{i,t}^{g} + (1+z)^{g-1} b_i a_i h_{i,t}^{g} + r_t s_{i,t-1}^{g-1} - s_{i,t}^{g}, \tag{27}$$

for $20 < g \leq 30$

$$c_{i,t}^{g} = r_t s_{i,t-1}^{g-1} - s_{i,t}^{g}, \tag{28}$$

and

$$c_{i,t}^{30} = r_t s_{i,t-1}^{29}, \tag{29}$$

for $0 < gg \leq 20$ and defining $s_{i,t-1}^{0} = 0$:

$$\begin{aligned} c_{i,t+gg-1}^{gg} = {} & \left(\sum_{g=gg}^{30} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} \\ & \left\{ r_{t+gg-1} s_{i,t+gg-2}^{gg-1} + \sum_{g=gg}^{20} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{-1} \right. \\ & \left. \left[w_{i,t+g-1}^{g} l_{i,t+g-1}^{g} + (1+z)^{g-1} b_i \frac{h_{t+g-1}^{\xi-1}}{\eta} a_i h_{i,t+g-1}^{g} \right] \right\}, \\ & t = s - gg + 1, \end{aligned} \tag{30}$$

$$\begin{aligned} c_{i,t}^{gg} = {} & \left(\sum_{g=gg}^{30} \left(\prod_{n_g=1}^{g-gg} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} \\ & \left\{ r_t s_{i,t-1}^{gg-1} + \sum_{g=gg}^{20} \left(\prod_{n_g=1}^{g-gg} r_{t+n_g} \right)^{-1} \right. \\ & \left. \left[w_{i,t+g-gg}^{g} l_{i,t+g-gg}^{g} + (1+z)^{g-gg} b_i \frac{h_{t+g-gg}^{\xi-1}}{\eta} a_i h_{i,t+g-gg}^{g} \right] \right\}, \end{aligned}$$

if $20 < gg$:

$$c_{i,t+gg-1}^{gg} = \left(\sum_{g=gg}^{30} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} r_{t+gg-1} s_{i,t+gg-2}^{gg-1}, \tag{31}$$

[35] To cast this system into Dynare syntax for predetermined variables, k_{t+1} and k_t need to be written as k and $k\,(-1)$ respectively.

$$c_{i,t}^{gg} = \left(\sum_{g=gg}^{30} \left(\prod_{n_g=1}^{g-gg} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-gg} \right)^{\frac{1}{\theta}} \right)^{-1} r_t s_{i,t-1}^{gg-1},$$

for $g = 1, ..., 30$:

$$c_t^g = \sum_{i=1}^{5} c_{i,t}^g \lambda_{i,t}^g, \tag{32}$$

for $0 < g \leq 20$:

$$\chi_i^g \left(l_{i,t}^g + h_{i,t}^g \right)^{\phi-1} = \left(c_{i,t}^g \right)^{-\theta(1-\upsilon)} w_{i,t}^g, \tag{33}$$

$$h_{i,t}^g = \max \left\{ \left[\left(c_{i,t}^g \right)^{-\theta(1-\upsilon)} \frac{\left((1+z)^{g-1} b_i \frac{h_t^{\xi-1}}{\eta} a_i - w_{i,t}^g \right)}{\chi_i^g \kappa} \right]^{\frac{1}{\phi-1}}, 0 \right\}, \tag{34}$$

$$w_{i,t}^g = (1+z)^{g-1} a_i w_t \left(\frac{l_t}{l_{i,t}} \right)^{1-\rho}, \tag{35}$$

$$l_{i,t} = \sum_{g=1}^{20} l_{i,t}^g \lambda_{i,t}^g \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}, \tag{36}$$

$$y_t = \left(\frac{k_t}{(1+z)(1+n^r+n_t^m)} \right)^{\alpha} (l_t)^{1-\alpha}, \tag{37}$$

$$l_t = \left(\sum_{i=1}^{5} a_i (l_{i,t})^{\rho} \right)^{1/\rho}, \tag{38}$$

$$w_t = \frac{(1-\alpha) y_t}{l_t}, \tag{39}$$

$$r_t^k = \alpha \frac{y_t}{k_t} (1+z)(1+n^r+n_t^m), \tag{40}$$

$$R_t^{Tk} = r_t^k + 1 - \delta, \tag{41}$$

$$r_t = 1 + (1-\gamma) \left(R_t^{Tk} - 1 \right) + \gamma r^{\star}, \tag{42}$$

$$s_t = \sum_{g=1}^{29} s_t^g (1+z)^{1-g} \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}, \tag{43}$$

for $0 < g \leq 29$

$$s_t^g = \sum_{i=1}^{5} s_{i,t}^g \lambda_{i,t}^g, \tag{44}$$

$$c_t = \sum_{g=1}^{30} c_t^g (1+z)^{1-g} \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}, \tag{45}$$

$$i_t = (1-\gamma) s_t + \gamma^{\star}, \tag{46}$$

$$k_{t+1} = i_t, \tag{47}$$

Some complementary variables:

$$h_t = \sum_{g=1}^{20} \sum_{i=1}^{5} h_{i,t}^g \lambda_{i,t}^g \prod_{j=0}^{g-2} \frac{1 + m_{t-j}^{g-j}}{1 + n^r + n_{t-j}^m}, \tag{48}$$

$$l_t^{total} = \sum_{g=1}^{20} \sum_{i=1}^{5} l_{i,t}^g \lambda_{i,t}^g \prod_{j=0}^{g-2} \frac{1 + m_{t-j}^{g-j}}{1 + n^r + n_{t-j}^m}, \tag{49}$$

$$0 = y_t - r_t^k k_t (1+z)^{-1} (1 + n^r + n_t^m)^{-1} - \sum_{i=1}^{5} \sum_{g=1}^{20} w_{i,t}^g l_{i,t}^g \lambda_{i,t}^g (1+z)^{1-g} \prod_{j=0}^{g-2} \frac{1 + m_{t-j}^{g-j}}{1 + n^r + n_{t-j}^m}. \tag{50}$$

Output per capita:

$$\begin{aligned}
\frac{Y_t}{A_t \sum_{g=1}^{30} N_t^g} &= y_t \frac{N_t^1}{\sum_{g=1}^{30} N_t^g} \\
&= y_t \frac{1}{\sum_{g=1}^{30} \frac{N_t^g}{N_t^1}} \\
&= y_t \frac{1}{\sum_{g=1}^{30} \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}} \\
&= \frac{y_t}{1 + \sum_{g=2}^{30} \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}}.
\end{aligned} \tag{51}$$

Capital per capita:

$$\begin{aligned}
\frac{K_t}{\sum_{g=1}^{30} N_t^g} &= k_t \frac{A_{t-1} N_{t-1}^1}{A_t \sum_{g=1}^{30} N_t^g} \\
&= k_t \frac{1}{\sum_{g=1}^{30} \frac{N_t^g}{N_{t-1}^1}} \\
&= k_t \frac{1}{\sum_{g=1}^{30} (1 + n^r + n_t^m) \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}} \\
&= \frac{k_t (1 + n^r + n_t^m)^{-1}}{1 + \sum_{g=2}^{30} \prod_{j=0}^{g-2} \frac{1+m_{t-j}^{g-j}}{1+n^r+n_{t-j}^m}}.
\end{aligned} \tag{52}$$

B Steady State

Let variables without time subscript denote steady state values. We solve for the steady state using numerical methods. As starting values for the numerical solver we use the analytical steady state solution for the special case when $\chi_i^g = \chi_j^g$ and $\beta_i = \beta_j$ for all i and j, $\theta = \rho = 1$, $\upsilon = 1$, $\gamma = \gamma^\star = 0$ and $\lambda_{i,t}^g = \lambda$. We further have that in steady state, $n_{ss}^r = n^r$, and there is no migration, i.e. $n_{ss}^m = 0$ and $m_{ss}^g = 0$. Then, from (10)

$$\sum_{g=1}^{30} r^{30-g} c_i^g = \sum_{g=1}^{20} r^{30-g} \left[w_i^g l_i^g + (1+z)^{g-1} b_i \frac{h^{\xi-1}}{\eta} a_i h_i^g \right]$$

and using (16)

$$c_i^{g_1} r^{30-g_1} \sum_{g=1}^{30} \beta_i^{g-g_1} = \sum_{g=1}^{20} r^{30-g} \left[w_i^g l_i^g + (1+z)^{g-1} b_i \frac{h^{\xi-1}}{\eta} a_i h_i^g \right]$$

or

$$c_i^{g_1} = \left(\sum_{g=1}^{30} \beta_i^{g-g_1} \right)^{-1} \sum_{g=1}^{20} r^{g_1-g} \left[w_i^g l_i^g + (1+z)^{g-1} e^{I_{g<g_{old}}} b_i a_i h_i^g \right] \tag{53}$$

$$c_i^{g_2} \beta_i^{g_1-g_2} r^{g_1-g_2} = c_i^{g_1} \tag{54}$$

From (35), and since $\rho = 1$,

$$w_i^g = (1+z)^{g-1} a_i w \tag{55}$$

Thus, for g_1, g_2 we have

$$w_i^{g_1} = (1+z)^{g_1-g_2} w_i^{g_2}$$

From (34), and as long as $\chi_i^{g_1} = \chi_i^{g_2}$, for all g_1 and g_2, $\theta = 1$ and $\upsilon = 1$,

$$h_i^g = \max \left\{ \left[\frac{(1+z)^{g-1} a_i \left(\left(\frac{h^{\xi-1}}{\eta} \right) b_i - w \right)}{\chi_i^g \kappa} \right]^{\frac{1}{\phi-1}}, 0 \right\} \tag{56}$$

if $b_i < w$ we have that for $g_1 < g_2 \leq 20$

$$h_i^{g_1} = 0 = h_i^{g_2}$$

else,

$$\begin{aligned} h_i^{g_1} &= \left[\frac{(1+z)^{g_1-1} a_i \left(\left(\frac{h^{\xi-1}}{\eta} \right) b_i - w \right)}{\chi_i^{g_1} \kappa} \right]^{\frac{1}{\phi-1}} \\ &= (1+z)^{\frac{g_1-g_2}{\phi-1}} \left[\frac{(1+z)^{g_2-1} a_i \left(\left(\frac{h^{\xi-1}}{\eta} \right) b_i - w \right)}{\chi_i^{g_2} \kappa} \right]^{\frac{1}{\phi-1}} \\ &= (1+z)^{\frac{g_1-g_2}{\phi-1}} h_i^{g_2} \end{aligned} \tag{57}$$

We further choose b_i such that

$$l_i^g = b_i \left(\frac{h^{\xi-1}}{\eta} \right) \frac{h_i^g}{w} \tag{58}$$

From (33), using (**??**), (58)

$$l_i^{g_1} = \left(1 + \frac{w}{b_i\left(\frac{h_i^{\xi-1}}{\eta_i}\right)}\right)^{-1} \left(\frac{(1+z)^{g_1-1}\,a_i}{\chi_i^{g_1}}w\right)^{\frac{1}{\phi-1}}$$

and

$$l_i^{g_2} = \left(1 + \frac{w}{b_i\left(\frac{h_i^{\xi-1}}{\eta_i}\right)}\right)^{-1} \left(\frac{(1+z)^{g_2-1}\,a_i}{\chi_i^{g_2}}w\right)^{\frac{1}{\phi-1}} \tag{59}$$

Then, if $\phi \neq 1$, we have

$$l_i^{g_1} = (1+z)^{\frac{g_1-g_2}{\phi-1}}\, l_i^{g_2} \tag{60}$$

Therefore, formal work of two generations satisfy the same proportion as the informal work of the same two generations, see (57) and (60). Then, from (36), (60) and (59),

$$\begin{aligned} l_i &= \sum_{g=1}^{20} l_i^g \lambda\,(1+n)^{1-g} \\ &= l_i^{g_2} \sum_{g=1}^{20} \lambda\,(1+n)^{1-g}\,(1+z)^{\frac{g-g_2}{\phi-1}} \\ &= \left(1 + \frac{w}{b_i\left(\frac{h_i^{\xi-1}}{\eta_i}\right)}\right)^{-1} \left(\frac{(1+z)^{g_2-1}\,a_i}{\chi_i^{g_2}}w\right)^{\frac{1}{\phi-1}} \\ &\sum_{g=1}^{20} \lambda\,(1+n)^{1-g}\,(1+z)^{\frac{g-g_2}{\phi-1}} \end{aligned} \tag{61}$$

To normalize l_i to 1 - in this steady state - one needs to set, for any g_3 and $g_2 < gr$,

$$\chi_i^{g_3} = \left[\sum_{g=1}^{20} \lambda\,(1+n)^{1-g}\,(1+z)^{\frac{g-g_2}{\phi-1}} \left(1 + \frac{w}{b_i\left(\frac{h_i^{\xi-1}}{\eta_i}\right)}\right)^{-1} \left((1+z)^{g_2-1}\,a_i w\right)^{\frac{1}{\phi-1}}\right]^{\phi-1}$$

From (38):

$$l = \sum_{i=1}^{n_l} a_i l_i = \sum_{i=1}^{n_l} a_i \tag{62}$$

Then, using (58), (55), (60) in (53), we can write

$$c_i^1 = \left(1 + \sum_{g=2}^{30} \beta_i^{g-g_1}\right)^{-1} 2a_i w l_i^{g_1}\,(1+z)^{\frac{1-\phi-g_1}{\phi-1}} \sum_{g=1}^{20} r^{1-g}\,(1+z)^{\frac{g\phi^l}{\phi-1}} \tag{63}$$

Using the above equations sequentially we can obtain expressions for saving, i.e.
from (26)

$$c_i^{g_3} = \left(1 + \sum_{g=g_3+1}^{30} \beta_i^{g-g_3}\right)^{-1} \left[r s_i^{g_3-1} + 2a_i w l_i^{g_1} (1+z)^{\frac{1-\phi-g_1}{\phi-1}} \sum_{g=g_3}^{20} r^{g_3-g} (1+z)^{\frac{g\phi}{\phi-1}}\right] \tag{64}$$

$$s_i^1 = 2a_i l_i^1 w - c_i^1$$

$$s_i^1 = \left\{1 - \left(1 + \sum_{g=2}^{30} \left(r^{g-1}\right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-1}\right)^{\frac{1}{\theta}}\right)^{-1} \sum_{g=1}^{20} r^{1-g} (1+z)^{\frac{\phi(g-1)}{\phi-1}}\right\} 2a_i w l_i^1$$

$\Rightarrow$

$$c_i^2 = \left(1 + \sum_{g=3}^{30} \left(r^{g-2}\right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-2}\right)^{\frac{1}{\theta}}\right)^{-1} \left[r s_i^1 + 2a_i w l_i^{g_1} \sum_{g=2}^{20} r^{2-g} (1+z)^{g-1+\frac{g-g_1}{\phi-1}}\right]$$

$$c_i^2 = 2a_i w l_i^1 \left(1 + \sum_{g=3}^{30} \left(r^{g-2}\right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-2}\right)^{\frac{1}{\theta}}\right)^{-1} \left[r + \left(1 - \left(1 + \sum_{g=2}^{m} \left(r^{g-1}\right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-1}\right)^{\frac{1}{\theta}}\right)^{-1}\right) \right.$$
$$\left. \sum_{g=2}^{20} r^{2-g} (1+z)^{\frac{\phi(g-1)}{\phi-1}}\right]$$

$$l_i^{g_1} = (1+z)^{\frac{g_1-g_2}{\phi-1}} l_i^{g_2}$$

$$l_i^2 = (1+z)^{\frac{1}{\phi-1}} l_i^1$$

$$s_i^2 = 2(1+z) a_i (1+z)^{\frac{1}{\phi-1}} l_i^1 w + r_t s_i^1 - c_i^2$$

$$s_i^2 = 2a_i w l_i^1$$
$$(1+z)(1+z)^{\frac{1}{\phi-1}} + r\left(1 - \left(1 + \sum_{g=2}^{30} \left(r^{g-1}\right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-1}\right)^{\frac{1}{\theta}}\right)^{-1} - \left(1 + \sum_{g=3}^{30} \left(r^{g-2}\right)^{\frac{1-\theta}{\theta}} \beta_i^{g-2\frac{1}{\theta}}\right)^{-1}\right)$$
$$+ \left\{\left(1 - \left(1 + \sum_{g=2}^{30} \left(r^{g-1}\right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-1}\right)^{\frac{1}{\theta}}\right)^{-1}\right)\left(1 - \left(1 + \sum_{g=3}^{30} \left(r^{g-2}\right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-2}\right)^{\frac{1}{\theta}}\right)^{-1}\right) - 1\right\}$$
$$\sum_{g=2}^{20} r^{2-g} (1+z)^{\frac{\phi(g-1)}{\phi-1}}$$

then, for $1 < g_3 < 21$

$$s_i^g = 2(1+z)^{g-1} a_i l_i^g w + r_t s_i^{g-1} - c_i^g$$

$$c_i^{g_3} = \left(1 + \sum_{g=g_3+1}^{30} \left(r^{g-g_3}\right)^{\frac{1-\theta}{\theta}} \left(\beta_i^{g-g_3}\right)^{\frac{1}{\theta}}\right)^{-1} \left[r s_i^{g_3-1} + 2a_i w l_i^{g_1} \sum_{g=g_3}^{20} r^{g_3-g} (1+z)^{g-1+\frac{g-g_1}{\phi-1}}\right]$$

for $30 > g_3 \geq 21$

$$s_i^{g_3} = r s_i^{g_3-1} - c_i^{g_3}$$

Starting from (46) and (47) we can write

$$
\begin{aligned}
k &= (1-\gamma)\, s \\
&\overbrace{=}^{(43)} (1-\gamma) \sum_{g=1}^{29} s^g (1+n)^{1-g} (1+z)^{1-g} \\
&\overbrace{=}^{(44)} (1-\gamma) \sum_{i=1}^{n_l} \overbrace{\lambda_i}^{\lambda_i=\lambda} \left(\sum_{g=1}^{29} s_i^g (1+n)^{1-g} (1+z)^{1-g} \right) \\
&\overbrace{=}^{(??)} \sum_{i=1}^{n_l} \lambda \left(\begin{array}{c} (1-\gamma) \sum_{g=1}^{29} (1+n)^{1-g} (1+z)^{1-g} \\ \left(\left(\sum_{g_3=g+1}^{30} \beta_i^{g_3-g-1} \right) \left(\sum_{g_3=1}^{30} \beta_i^{g_3-g-1} \right)^{-1} \right. \\ \sum_{g_3=1}^{20} r^{g+1-g_3} (1+z)^{g_3-1} \left(\frac{\beta_i^{g_2-g_3} r^{g_2-g_3}}{(1+z)^{g_2-g_3}} \right)^{\frac{1}{\phi-1}} \\ \left. - \sum_{g_3=g+1}^{20} r^{g+1-g_3} (1+z)^{g_3-1} \left(\frac{\beta_i^{g_2-g_3} r^{g_2-g_3}}{(1+z)^{g_2-g_3}} \right)^{\frac{1}{\phi-1}} \right) \end{array} \right) \frac{1+b_i}{r} a_i w l_i^{g_2} \\
&\overbrace{=}^{\beta_i=\beta_j} \frac{w}{r} \left(\begin{array}{c} (1-\gamma) \sum_{g=1}^{29} (1+n)^{1-g} (1+z)^{1-g} \\ \left(\left(\sum_{g_3=g+1}^{30} \beta_i^{g_3-g-1} \right) \left(\sum_{g_3=1}^{30} \beta_i^{g_3-g-1} \right)^{-1} \right. \\ \sum_{g_3=1}^{20} r^{g+1-g_3} (1+z)^{g_3-1} \left(\frac{\beta_i^{g_2-g_3} r^{g_2-g_3}}{(1+z)^{g_2-g_3}} \right)^{\frac{1}{\phi-1}} \\ \left. - \sum_{g_3=g+1}^{20} r^{g+1-g_3} (1+z)^{g_3-1} \left(\frac{\beta_i^{g_2-g_3} r^{g_2-g_3}}{(1+z)^{g_2-g_3}} \right)^{\frac{1}{\phi-1}} \right) \end{array} \right) \sum_{i=1}^{n_l} \lambda (1+b_i)\, a_i l_i^{g_2} \\
&\overbrace{=}^{b_i=b_j} \frac{w}{r} (1+b_i) \left(\begin{array}{c} (1-\gamma) \sum_{g=1}^{29} (1+n)^{1-g} (1+z)^{1-g} \\ \left(\left(\sum_{g_3=g+1}^{30} \beta_i^{g_3-g-1} \right) \left(\sum_{g_3=1}^{30} \beta_i^{g_3-g-1} \right)^{-1} \right. \\ \sum_{g_3=1}^{20} r^{g+1-g_3} (1+z)^{g_3-1} \left(\frac{\beta_i^{g_2-g_3} r^{g_2-g_3}}{(1+ag)^{g_2-g_3}} \right)^{\frac{1}{\phi-1}} \\ \left. - \sum_{g_3=g+1}^{20} r^{g+1-g_3} (1+z)^{g_3-1} \left(\frac{\beta_i^{g_2-g_3} r^{g_2-g_3}}{(1+ag)^{g_2-g_3}} \right)^{\frac{1}{\phi-1}} \right) \end{array} \right) \sum_{i=1}^{n_l} \lambda a_i l_i^{g_2} \\
&\overbrace{=}^{(61)} \frac{w}{r} (1+b_i) \left(\sum_{g=1}^{20} \lambda (1+n)^{1-g} \left(\frac{\beta_i^{g_2-g} r^{g_2-g}}{(1+z)^{g_2-g}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \\
&\left(\begin{array}{c} (1-\gamma) \sum_{g=1}^{29} (1+n)^{1-g} (1+z)^{1-g} \\ \left(\left(\sum_{g_3=g+1}^{30} \beta_i^{g_3-g-1} \right) \left(\sum_{g_3=1}^{30} \beta_i^{g_3-g-1} \right)^{-1} \right. \\ \sum_{g_3=1}^{20} r^{g+1-g_3} (1+z)^{g_3-1} \left(\frac{\beta_i^{g_2-g_3} r^{g_2-g_3}}{(1+z)^{g_2-g_3}} \right)^{\frac{1}{\phi-1}} \\ \left. - \sum_{g_3=g+1}^{20} r^{g+1-g_3} (1+z)^{g_3-1} \left(\frac{\beta_i^{g_2-g_3} r^{g_2-g_3}}{(1+z)^{g_2-g_3}} \right)^{\frac{1}{\phi-1}} \right) \end{array} \right) \sum_{i=1}^{n_l} \lambda a_i l_i
\end{aligned}
$$

$$\overset{(62)}{\widehat{=}} \frac{1+b_i}{r}\left(\sum_{g=1}^{20}(1+n)^{1-g}\left(\frac{\beta_i^{g_2-g}r^{g_2-g}}{(1+z)^{g_2-g}}\right)^{\frac{1}{\phi-1}}\right)^{-1}$$

$$\left(\begin{array}{c} (1-\gamma)\sum_{g=1}^{29}(1+n)^{1-g}(1+z)^{1-g} \\ \left(\left(\sum_{g_3=g+1}^{30}\beta_i^{g_3-g-1}\right)\left(\sum_{g_3=1}^{30}\beta_i^{g_3-g-1}\right)^{-1}\right. \\ \sum_{g_3=1}^{20}r^{g+1-g_3}(1+z)^{g_3-1}\left(\frac{\beta_i^{g_2-g_3}r^{g_2-g_3}}{(1+z)^{g_2-g_3}}\right)^{\frac{1}{\phi-1}} \\ \left.-\sum_{g_3=g+1}^{20}r^{g+1-g_3}(1+z)^{g_3-1}\left(\frac{\beta_i^{g_2-g_3}r^{g_2-g_3}}{(1+z)^{g_2-g_3}}\right)^{\frac{1}{\phi-1}}\right) \end{array}\right) wl$$

$$\overset{(37),(39)}{\widehat{=}} \frac{1+b_i}{r}\left(\sum_{g=1}^{20}(1+n)^{1-g}\left(\frac{\beta_i^{g_2-g}r^{g_2-g}}{(1+z)^{g_2-g}}\right)^{\frac{1}{\phi-1}}\right)^{-1}$$

$$\left(\begin{array}{c} (1-\gamma)\sum_{g=1}^{29}(1+n)^{1-g}(1+z)^{1-g} \\ \left(\left(\sum_{g_3=g+1}^{30}\beta_i^{g_3-g-1}\right)\left(\sum_{g_3=1}^{30}\beta_i^{g_3-g-1}\right)^{-1}\right. \\ \sum_{g_3=1}^{20}r^{g+1-g_3}(1+z)^{g_3-1}\left(\frac{\beta_i^{g_2-g_3}r^{g_2-g_3}}{(1+ag)^{g_2-g_3}}\right)^{\frac{1}{\phi-1}} \\ \left.-\sum_{g_3=g+1}^{20}r^{g+1-g_3}(1+z)^{g_3-1}\left(\frac{\beta_i^{g_2-g_3}r^{g_2-g_3}}{(1+z)^{g_2-g_3}}\right)^{\frac{1}{\phi-1}}\right) \end{array}\right) \frac{(1-\alpha)}{(1+z)^{\alpha}(1+n)^{\alpha}}\left(\frac{k}{l}\right)^{\alpha} l$$

Then, the capital labor ratio is then a non linear function of r, and the model parameters,

$$\frac{k}{l} = \left(\begin{array}{c} \frac{1+b_i}{r}\left(\sum_{g=1}^{20}(1+n)^{1-g}\left(\frac{\beta_i^{-g}r^{-g}}{(1+z)^{-g}}\right)^{\frac{1}{\phi-1}}\right)^{-1} \\ \frac{(1-\alpha)(1-\gamma)}{(1+z)^{\alpha}(1+n)^{\alpha}}\sum_{g=1}^{29}(1+n)^{1-g}(1+z)^{1-g} \\ \left(\left(\sum_{g_3=g+1}^{30}\beta_i^{g_3-g-1}\right)\left(\sum_{g_3=1}^{30}\beta_i^{g_3-g-1}\right)^{-1}\right. \\ \sum_{g_3=1}^{20}r^{g+1-g_3}(1+z)^{g_3-1}\left(\frac{\beta_i^{-g_3}r^{-g_3}}{(1+z)^{-g_3}}\right)^{\frac{1}{\phi-1}} \\ \left.-\sum_{g_3=g+1}^{20}r^{g+1-g_3}(1+z)^{g_3-1}\left(\frac{\beta_i^{-g_3}r^{-g_3}}{(1+z)^{-g_3}}\right)^{\frac{1}{\phi-1}}\right) \end{array}\right)^{\frac{1}{1-\alpha}} \tag{65}$$

and thus, using (62), we can obtain the steady state value for k,

$$k = \frac{k}{l}l \tag{66}$$

From (39) and (37), then,

$$w = \frac{(1-\alpha)}{(1+z)^{\alpha}(1+n)^{\alpha}}\left(\frac{k}{l}\right)^{\alpha} \tag{67}$$

Next we derive the b_i that ensures that (58) is satisfied, and show that this b_i is common for all i. For this, we depart from equations (58) and use (56), (59) as well as (54), (60) and (**??**),

$$wl_i^g = h_i^g = \left[\frac{(1+z)^{g-1}a_i(b_i-w)}{c_i^g\chi_i^g\kappa}\right]^{\frac{1}{\phi-1}}$$

(**??**), (60)⇒

$$wl_i^{g_1} = \left[\frac{(1+z)^{g_1-1} a_i (b_i - w)}{a_i w (1+b_i) l_i^{g_2} \left(\sum_{g=1}^{30} \beta_i^{g-g_1}\right)^{-1} \chi_i^{g_1} \kappa} \right.$$
$$\left. \left(\sum_{g=1}^{20} r^{g_1-g} (1+z)^{g-1} \left(\frac{\beta_i^{g_2-g} r^{g_2-g}}{(1+z)^{g_2-g}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right]^{\frac{1}{\phi-1}}$$
$$= \left[\frac{(1+z)^{g_1-1} (b_i - w) \left(\sum_{g=1}^{30} \beta_i^{g-g_1}\right)}{w (1+b_i) l_i^{g_1} \left(\frac{\beta_i^{g_2-g_1} r^{g_2-g_1}}{(1+z)^{g_2-g_1}} \right)^{\frac{1}{1-\phi}} \chi_i^{g_1} \kappa} \right.$$
$$\left. \left(\sum_{g=1}^{20} r^{g_1-g} (1+z)^{g-1} \left(\frac{\beta_i^{g_2-g} r^{g_2-g}}{(1+z)^{g_2-g}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right]^{\frac{1}{\phi-1}}$$

⇒

$$(wl_i^{g_1})^{\phi} = \frac{(1+z)^{g_1-1} (b_i - w) \left(\sum_{g=1}^{30} \beta_i^{g-g_1}\right)}{(1+b_i) \left(\frac{\beta_i^{g_2-g_1} r^{g_2-g_1}}{(1+z)^{g_2-g_1}} \right)^{\frac{1}{1-\phi}} \chi_i^{g_1} \kappa}$$
$$\left. \left(\sum_{g=1}^{20} r^{g_1-g} (1+z)^{g-1} \left(\frac{\beta_i^{g_2-g} r^{g_2-g}}{(1+z)^{g_2-g}} \right)^{\frac{1}{\phi-1}} \right)^{-1} \right]$$

(59)⇒

$$b_i = \kappa w^{\phi} (1+w)^{1-\phi} + w \tag{68}$$

for all i and for all g. Therefore, there exists ab_i that ensures that (58) is satisfied. In addition, one can observe, that since $w > 0$ by definition, and $\kappa > 0$, $b_i > w$. The remaining steady state values are then as follows, from (37),

$$y = \left(\frac{k_t}{(1+z)(1+n)} \right)^{\alpha} l^{1-\alpha} \tag{69}$$

From (47)

$$k = i \tag{70}$$

From (40),

$$r^k = \alpha \frac{y}{k} (1+z)(1+n) \tag{71}$$

From (41)

$$R^{Tk} = r^k + 1 - \delta \tag{72}$$

From (42),

$$
\begin{aligned}
r &= 1 + (1-\gamma)\left(R^{Tk} - 1\right) + \gamma r^{\star} \\
&= 1 + (1-\gamma)\left(\alpha \frac{y}{k}(1+z)(1+n) - \delta\right) + \gamma r^{\star}
\end{aligned}
\tag{73}
$$

Using (67) and (73) to express w and r in terms of the capital labor ratio, (65) and (68) constitute a non-linear system of two equations and two unknowns, b_i and $\frac{\kappa}{l}$. Once we solve for this unknowns, all other variables can be determined. From (44)

$$
s^g = \sum_{i=1}^{n_l} s_i^g \lambda_i \tag{74}
$$

From (43),

$$
s = \sum_{g=1}^{29} s^g (1+n)^{1-g} (1+z)^{1-g} \tag{75}
$$

From (32),

$$
c^g = \sum_{i=1}^{n_l} c_i^g \lambda_i \tag{76}
$$

From (45)

$$
c = \sum_{g=1}^{30} c^g (1+n)^{1-g} (1+z)^{1-g} \tag{77}
$$

C Workweek Reduction

This appendix shows one way in which our model could be modified in order to study a scenario in which working hours are statutorily reduced, e.g., reducing the work-week from 45 to 40 hours.

A reduction in working hours could be interpreted as a reduction in the effective labor input entering the representative firm's production function, for a given number of workers. We would introduce the parameter $l_i^{adj} \in (0,1)$ in equation (21), such that $L_{i,t} = \sum_{g=1}^{20} l_i^{adj} L_{i,t}^g$. This would effectively generate a reduction in labor demand, consistent with an increase in labor costs, as the modified equations for profit maximization and optimal labor demand show:

$$
\max_{K_t, L_{i,t}^g} K_t^{\alpha} A_t^{1-\alpha} \left(\sum_{i=1}^{5} a_i \left(\sum_{g=1}^{20} l_i^{adj} L_{i,t}^g \right)^{\rho} \right)^{\frac{1-\alpha}{\rho}} - r_t^k K_t - \sum_{i=1}^{5} \sum_{g=1}^{20} W_{i,t}^g L_{i,t}^g,
$$

$$
L_{i,t}^g : \quad W_{i,t}^g = a_i W_t l_i^{adj} \left(\frac{L_t}{L_{i,t}} \right)^{1-\rho}.
$$

Ignoring changes in population due to immigration, we could reinterpret the problem facing each individual of skill level i, who decides how many hours to work in the formal and informal sectors, as a problem facing each *household* of skill level i, which would decide how many members to allocate to the formal and informal sectors. Under this interpretation, the problem of the household could include the lower disutility that working fewer hours in the formal sector would entail. This extension would allow labor supply to compensate to some extent the decline in hours. We introduce the factor $(l_i^{adj})^{\frac{1}{\phi^{adj}}}$ in the household's optimization problem:

$$
\begin{aligned}
U_{i,t} = & \sum_{g=1}^{30} \beta_i^{g-1} \frac{\left(C_{i,t+g-1}^{g}\right)^{1-\theta}}{1-\theta} \\
& - \sum_{g=1}^{20} \beta_i^{g-1} \Theta_{i,t+g-1}^{g} \frac{\left((l_i^{adj})^{\frac{1}{\phi^{adj}}} l_{i,t+g-1}^{g} + h_{i,t+g-1}^{g}\right)^{\phi} + \kappa \left(h_{i,t+g-1}^{g}\right)^{\phi}}{\phi},
\end{aligned}
$$

where the exponent $\frac{1}{\phi^{adj}}$ regulates the extent to which labor supply would compensate the decline in total hours. When $\phi^{adj} = 1$, the compensation of labor supply is complete in the sense that family income remains unchanged. When $\phi^{adj} \to \infty$, labor supply does not compensate the decline in hours.

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