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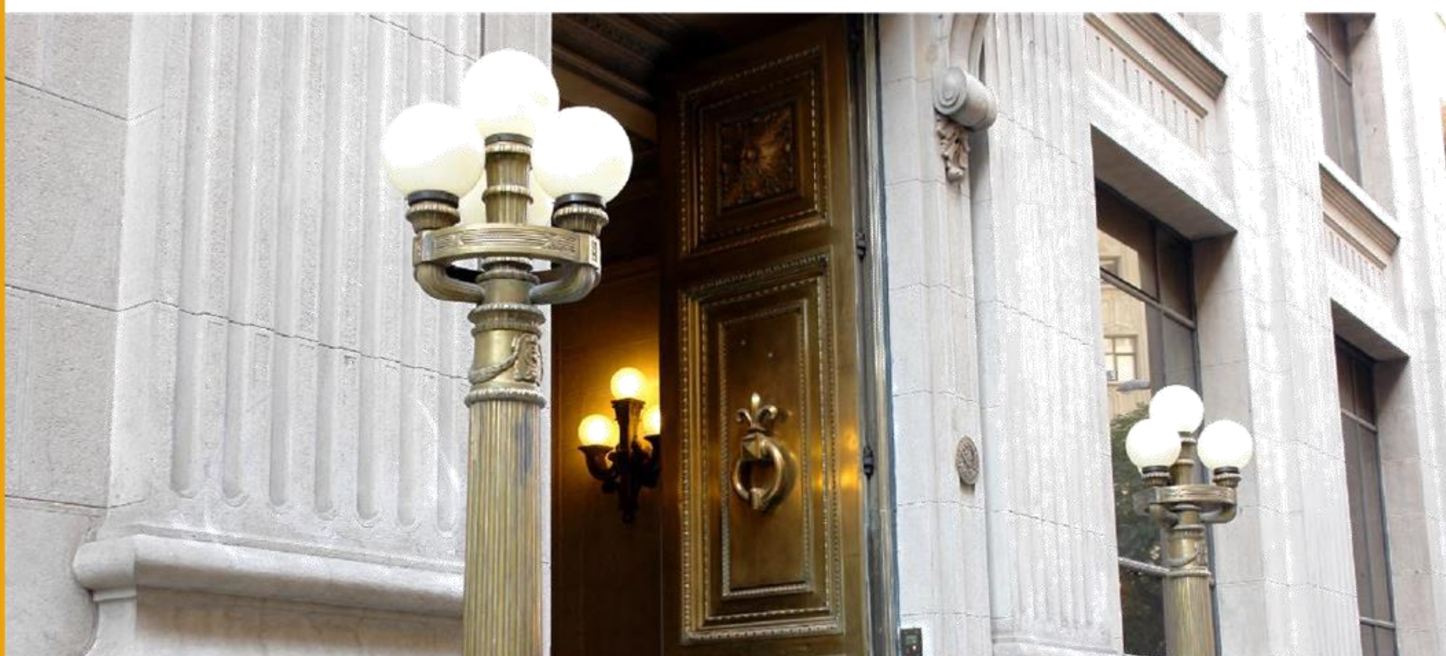

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Immigration in Emerging Countries: A Macroeconomic Perspective*

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Resumen

Los países en desarrollo albergan cerca de un tercio de la población migrante mundial, pero los análisis macroeconómicos sobre inmigración se han centrado principalmente en economías avanzadas. Este trabajo estudia los efectos macroeconómicos de la inmigración en países emergentes, donde la informalidad laboral desempeña un papel central. Desarrollamos un modelo de generaciones traslapadas con un sector informal y trabajadores calificados y no calificados, en el que los inmigrantes y los nativos con el mismo nivel de habilidades son sustitutos imperfectos. Un choque inmigratorio reduce los salarios formales de los inmigrantes, lo que provoca una reasignación de su esfuerzo laboral hacia el sector informal. Debido a efectos de congestión en la informalidad, este cambio reduce los ingresos informales de todos los trabajadores, incentivando a los nativos a reasignarse al sector formal. Aunque los salarios formales de los nativos también disminuyen—principalmente por el aumento en la oferta laboral—la caída es menor que la reducción en los ingresos informales. El modelo incorpora un desajuste ocupacional transitorio, que permite a los inmigrantes aceptar empleos por debajo de su nivel de habilidad, lo que amplifica las consecuencias distributivas de la inmigración al aumentar la oferta relativa de trabajo no calificado. El modelo se calibra para Chile, una economía emergente que ha experimentado recientemente una ola migratoria significativa, con el fin de cuantificar estas dinámicas.

Abstract

Developing countries host nearly one-third of the world's migrant population, yet macroeconomic analyses of immigration have largely focused on advanced economies. We study the macroeconomic effects of immigration in emerging countries, where labor informality plays a central role. We develop an overlapping generations model with an informal sector and high- and low-skilled workers, in which immigrants and natives of the same skill level are imperfect substitutes. An immigration shock depresses formal wages for immigrants, prompting a reallocation of their labor effort toward the informal sector. Due to congestion effects in informality, this shift reduces informal income for all workers, incentivizing native workers to reallocate toward the formal sector. While formal wages for natives also decline—primarily due to increased labor supply—the reduction is smaller than the drop in informal income. The model incorporates transitory downgrading, allowing immigrants to accept jobs below their skill level, which amplifies the distributional consequences of immigration by increasing the relative supply of low-skilled labor. The model is calibrated to Chile, an emerging economy that has recently experienced a significant immigration wave, to quantify these dynamics.

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1 Introduction

Developing countries host nearly one-third of the world's migrant population (OECD/ILO, 2018), yet macroeconomic analyses of immigration have largely focused on advanced economies, overlooking this substantial dimension of global migration. We address this gap by studying the macroeconomic effects of immigration in emerging countries, where labor informality plays a central role. In emerging countries, a substantial fraction of employment relations does not conform to labor legislation, evading taxes and social security contributions. Informal jobs are typically less productive than formal jobs, and lack access to social protections such as severance payments, health insurance, or pensions. However, it has been widely documented that informality acts as a buffer against income fluctuations during economic downturns, when formal jobs are scarce (see, for example, Loayza and Rigolini, 2011). Since unemployment insurance schemes are highly limited in emerging countries, a significant fraction of workers cannot afford to conduct lengthy job searches during downturns, so they settle for an informal job. We examine how labor informality shapes the macroeconomic response to an immigration shock.

To this end, we develop an overlapping generations (OLG) model with an informal sector and high- and low-skilled workers, in which immigrants and natives of the same skill level are imperfect substitutes. There are four types of workers—high- and low-skilled natives and high- and low-skilled immigrants. The dynamics of formal wages are determined by the relative abundance of three types of production inputs—labor relative to capital, high- relative to low-skilled labor, and native relative to immigrant labor.

In this framework, a large immigration shock depresses formal wages for immigrants, who become relatively more abundant, prompting a reallocation of their labor effort toward the informal sector. Due to congestion effects in informality, this shift reduces informal income for all workers, incentivizing native workers to reallocate toward the formal sector. While formal wages for natives also decline—primarily due to increased labor supply—the reduction is smaller than the drop in informal income.

The degree of imperfect substitutability between natives and immigrants of the same skill level is crucial for the results. When natives and immigrants are more substitutable, formal wages of native workers decline more than in the baseline case, whereas wages for immigrants decline less than in the baseline case. In this type of scenario, the relative income from formal and informal work may lead low-skilled native workers to reallocate to the informal sector.

The feature of congestion in the informal sector, i.e., the fact that the marginal return from informality is inversely related to total hours per worker in that sector, is important to discipline reallocation between the two sectors in the model and is intuitive if we think of informality as, for example, work on the street, where a larger number of workers adversely affects marginal returns. This feature is also crucial for the results. In the absence of congestion, informal income does not decline but remains constant, which mitigates the incentive of natives to reallocate to the formal sector. In fact, low-skilled natives reallocate to the informal sector. Furthermore, since the informal sector is larger in this scenario, the reduced labor supply in the formal sector results in higher formal wages than in the baseline case.

The model incorporates transitory downgrading, allowing immigrants to accept jobs below their skill level. This feature is motivated by the fact that it takes time for foreign workers to adapt to the local labor market.[1] When they leave their countries, immigrants may lose job- or country-specific skills that push them down the job ladder in the host country, facing a gradual recovery. Immigrants may also face regulatory barriers to finding adequate matches. Lawyers, physicians, and nurses, for example, face severe challenges if they want to practice these professions in a foreign country.

To capture this phenomenon, we model downgrading as an exogenous process, whereby a fixed fraction of high-skilled immigrants are temporarily treated as low-skilled upon arrival. This feature amplifies the distributional consequences of immigration by increasing the relative supply of low-skilled labor. Therefore, the decline in formal wages of low-skilled workers is larger, and the decline in wages of high-skilled workers smaller, than in a counterfactual without downgrading.[2]

Our exogenous approach to modeling downgrading contrasts with endogenous mechanisms in the underemployment literature. For instance, Barnichon and Zylberberg (2019) model a setting in which high-skilled workers compete with low-skilled workers to avoid congestion in high-skilled jobs during downturns, and firms prefer the former. In contrast, our framework treats downgraded high-skilled immigrants as equivalent to low-skilled immigrants, simplifying the analysis while capturing key distributional effects.

The OLG structure is crucial, because the age distribution of immigrants and natives differs substantially. Indeed, working-age individuals represent the bulk of most immigration waves. When immigrants are substantially younger than natives, the expansion in labor supply is larger than in a counterfactual of homogeneous age distributions. In addition to larger effects, younger immigrants generate more persistent effects of the expansion in labor supply, since it takes longer for them to reach retirement. Differences in the age distribution of immigrants and natives have also been found in advanced economies, such as the United States.[3] However, this feature is mostly ignored in the literature on the macroeconomic effects of immigration.[4]

The model is calibrated to match key demographic and economic characteristics of Chile, an emerging economy that has recently experienced a significant immigration wave, to quantify these dynamics. The Chilean case displays the three features previously highlighted: labor informality is substantial, the evidence suggests immigrants experience a downgrading spell, and immigrants are much younger than natives.

Our simulations of an immigration shock in Chile display highly persistent but not permanent effects on factor prices and variables in per capita terms. The reason is that immigrants eventually retire and die, and we assume their children are indistinguishable from natives. We also assume that, after experiencing a downgrading spell, the skill distribution of immigrants converges to that of the native population. This assumption follows from a similar distribution of years of education of immigrants and natives. However, the model could be easily re-calibrated to study an economy where the skill distribution of immigrants and natives differs permanently, as would be the case, for example, if the bulk of immigrants were low-skilled workers. Finally, we abstract from return migration and assume all immigrants stay permanently in the

[1]See Lubotsky (2007) for a longitudinal analysis of immigrant's earnings in the United States, and Barker (2021a) for SVAR- and DSGE-based analyses of immigrant underemployment in Canada.

[2]Lebow (2024) finds a similar result on the distributional effect of downgrading.

[3]Card (2001) shows that U.S. immigrants that arrived in the 1985–1990 period are substantially younger than natives, as well as previously arrived immigrants.

[4]Heterogeneity in age structure is typically considered in studies of the long-term fiscal impact of immigration, e.g., National Academies of Sciences and Medicine (2017). It is also considered in the related literature on population aging, e.g., Myers (2012) and Peri (2020).

host country.

Regarding the generalizability of our results to other emerging countries, we emphasize that the quantitative effects of immigration depend critically on the specific characteristics of natives, immigrants, and the host economy. In particular, the degree of imperfect substitutability between native and immigrant labor plays a central role. A lower elasticity of substitution tends to induce reallocation of low-skilled native workers from the informal to the formal sector, as the relative decline in informal income outweighs the decline in formal wages, whereas a higher elasticity reverses this pattern. Our contribution, therefore, lies in identifying and analyzing key channels—such as labor informality, imperfect substitutability, and transitory skill downgrading—that are especially relevant for emerging economies but remain underexplored in the macroeconomic literature, which has largely focused on advanced economies.

We abstract from two important dimensions of immigration that, while relevant, fall outside the scope of our analysis. The first concerns intergenerational effects. For example, young natives may accumulate human capital to insure against the possible adverse effects of immigration, while older and future generations may benefit from the contributions of mostly young working-age immigrants to financially stressed pay-as-you-go (PAYG) pension systems.[5] More broadly, immigration can affect fiscal sustainability, as shown in Storesletten (2000). However, our focus is on the medium-run effects of immigration on the more current generation of natives in emerging countries. We abstract from human capital accumulation, pension systems, and fiscal policy to maintain tractability. Future research could explore how intergenerational effects of immigration differ in emerging countries, and an OLG model like ours would be well-suited for such analysis.[6]

The second aspect that we abstract from is related to possible heterogeneity in fertility rates among natives and immigrants. Higher fertility rates among immigrant women could generate more prolonged effects, and vice versa. Storesletten (1995) estimates that, in the United States, high-skilled immigrants have lower fertility rates than natives, but medium- and especially low-skilled immigrants have higher fertility rates. We abstract from fertility differentials between natives and immigrants, though these could influence the long-run demographic and macroeconomic effects of immigration.

The paper lies at the intersection of two strands of the literature. The first considers the macroeconomic effects of immigration. This literature has primarily focused on advanced economies, and typically studies immigrants with lower skills than the native population (see, for example, Boldrin and Montes, 2015; Izquierdo, Jimeno, and Rojas, 2010; Wilson, 2003; Mandelman and Zlate, 2012; and Holler and Schuster, 2018). The second strand of the literature studies the role of informality in the labor market of emerging countries. It finds that the informal sector acts as a buffer that helps sustain household consumption during economic downturns, and can explain, for instance, why aggregate employment is more volatile in advanced economies (see, for example, Fernández and Meza, 2015; and Finkelstein Shapiro and Mandelman, 2016).

By bridging the gap between the literature on immigration in advanced economies and the literature on labor informality in emerging countries, this paper is the first comprehensive study of the macroeconomic effects of immigration in emerging countries, where informal labor is pervasive.

The paper is organized as follows. Section 2 discusses the evidence that motivates the key features

[5] Boldrin and Montes (2015) consider human capital accumulation in a three-cohort OLG model of immigration for Spain.

[6] Although Chile is a representative emerging economy in many respects, its pension system—predominantly fully funded and defined-contribution—differs from the PAYG systems common in other emerging countries.

of our OLG model. Section 3 describes the model, which introduces a tractable way to incorporate informality and downgrading into an OLG framework for the analysis of immigration shocks. Section 4 presents the calibration and the design of our simulations. Section 5 presents our baseline results, as well as an analysis of their sensitivity to different features and assumptions, such as the elasticity of substitution between immigrants and natives, the possibility of reallocating labor effort to the informal sector, as well as congestion in that sector, and the downgrading spell that affects immigrants. Section 6 concludes.

2 Motivating Evidence

This section discusses the evidence that motivates the key features of our OLG model and the design of our simulations. We describe the immigration wave that Chile has recently experienced, since we will calibrate the model to match key demographic and economic characteristics of this episode. The Chilean case is not unique, however, so we also provide references on other economies that have had similar experiences with immigration.

2.1 The Immigration Shock

We study the macroeconomic effects of an unexpected and substantial immigration flow that is exogenous from the perspective of the host country. The Chilean case of the late 2010s, which we describe below, supports this assumption, but other countries, of all sizes and states of development, have also experienced unexpected immigration shocks.[7] The possibility of sudden population displacement in the future, for example due to armed conflict, such as the Syrian refugee crisis and the Russian invasion of Ukraine, or due to events associated with climate change (Engler, Honjo, MacDonald, Piazza, and Sher, 2020), highlights the relevance of a continuous understanding of the effects of large migration shocks. Our focus on exogenous immigration shocks differs from the literature on endogenous migration decisions.[8]

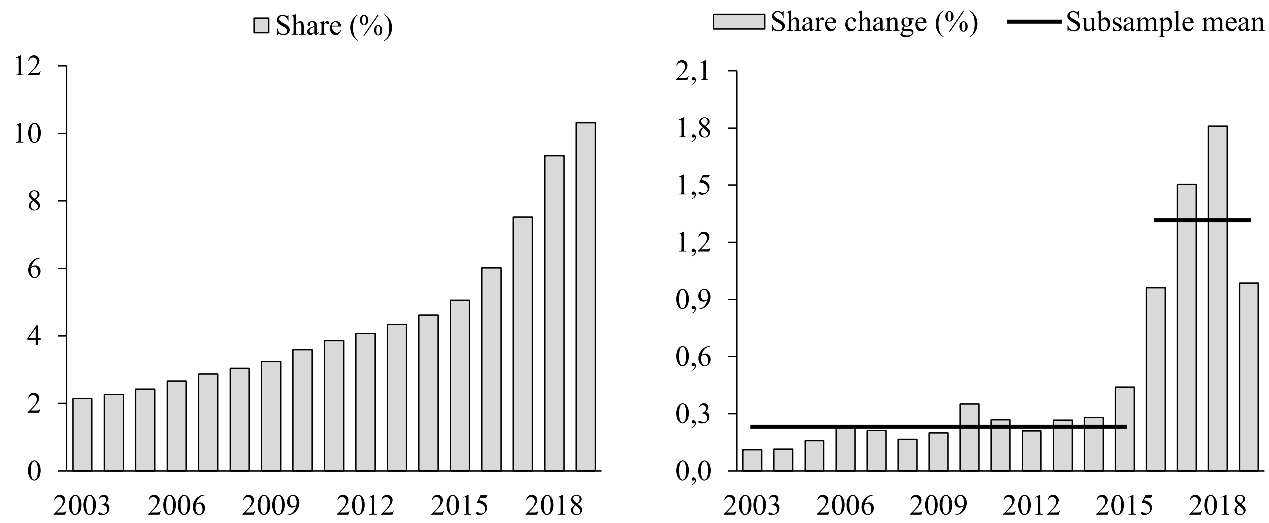
We now focus on Chile's immigration shock. As shown in the left panel of figure 1, the share of immigrants in the labor force nearly doubled in four years, jumping from 5.1% in 2015 to 10.3% in 2019. The right panel of the figure shows that, prior to 2015, the share of immigrants in the labor force increased by about 0.2 percentage points per year, on average, whereas during 2015–2019, this metric increases by a factor of six. There is nothing special about Chile during this period that could account for such a surge in immigration; certainly not economic growth, which was sluggish. Instead, the inflow of migrants is mainly related to the Venezuelan crisis and the massive emigration it has generated. Indeed, data from the First National Survey of Migration in Chile show that 45% of immigrants aged 18 years and older in the 2016–2020 period are Venezuelan. Therefore, we believe it is reasonable to model this event as

[7] To name a few examples, Canova and Ravn (2000) model the unification of East and West Germany as an unexpected flow of low-skilled Eastern workers to the West, Saiz (2003) studies the migration shock of Cubans to Miami in 1980 and its effect on the housing market, and Gray, Montresor, and Wright (2020) analyze the effects of the 2004 expansion of the European Union, which included Eastern European countries, as an immigration shock that affected firm-level innovation.

[8] For example, Mandelman and Zlate (2012) study how emigration from Mexico to the U.S. depends on business cycle conditions in both countries. Furlanetto and Robstad (2019) study the macroeconomic effects of immigration in Norway using a structural vector autoregression framework that allows for endogenous responses of immigration, as well as for exogenous shocks.

exogenous from the perspective of Chile.[9]

Figure 1: Share of immigrants in the labor force



Note.— Source: National Statistics Institute.

For the purposes of our analysis, two key features of Chile's immigration wave are that immigrants: (i) are primarily young working-age adults, and (ii) have slightly higher education levels than natives. The left panel of figure 2 shows the substantial difference in the age structure of immigrants and natives. Whereas only 36% of natives are between ages 20 and 44, 65% of immigrants belong to this group. The fact that immigrants are primarily young adults implies that the immigration wave has not only generated higher growth in the total population, but even higher growth in the working-age population. Since immigrants have higher participation rates than natives, as documented in Aldunate, Contreras, de la Huerta, and Tapia (2019b), the immigration-induced growth in the labor force is also higher than that of the total population. Our simulations, therefore, will consider an immigration wave that alters the age structure, reducing the ratio of passive to active individuals. As we mentioned in the introduction, this feature has been observed in immigration episodes in other countries.[10] This heterogeneity in the age distribution of immigrants and natives motivates the OLG structure of our model.

The right panel of figure 2 shows that immigrants have, on average, slightly higher education levels than natives. The share with a university education is 24% among immigrants, slightly higher than the 20% among natives. Following this evidence, our baseline simulation will further assume that immigrants and natives have the same skill distribution. We will study the sensitivity of our results to this assumption by allowing for immigrants to be slightly more skilled than natives. Of course, it could also be the case that the *quality* of education of immigrants is lower than natives, such that immigrants have lower skills. Although we do not consider this possibility, the model could be easily re-calibrated to accommodate this

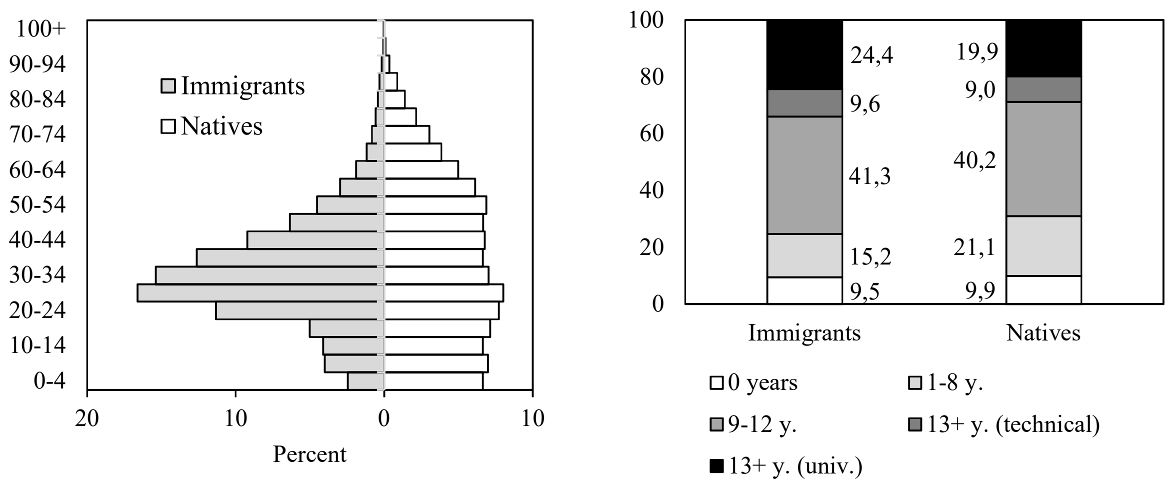
[9]See Barker (2021b) for a general equilibrium analysis of Venezuelan migration.

[10]Again, see Card (2001), National Academies of Sciences and Medicine (2017), Myers (2012), and Peri (2020).

type of scenarios.[11]

Our assumption that immigrants and natives have similar skills differentiates this paper from most of the literature, which is overwhelmingly concerned with the labor market effects of low-skilled immigration. This allows us to focus on less studied issues, such as the role of labor informality in the experience of emerging countries with immigration, the age structure of immigrants, and transitory downgrading. The assumption of similar skills is warranted for the case of Chile, where a substantial fraction of immigrants are high-skilled, as we have shown. Furthermore, the role of high-skilled immigrants is not unique to Chile's experience. Kerr (2017), for example, emphasizes that high-skilled immigration in the United States is both important and severely understudied.

Figure 2: Age and education of immigrants



Note.— The underlying data come from the 2017 population census and the CASEN survey. Source: Aldunate et al. (2019b).

It is important to note that, although the education of immigrants is similar to that of natives on average, there is substantial heterogeneity. For this purpose, table 1 shows data from the National Survey of Migration, which offers information on immigrants aged 18 and older that arrived between 2016 and 2020.[12] The table shows that five countries account for 89% of immigrants during this period—Venezuela, Haiti, Colombia, Peru and Bolivia, with Venezuela representing nearly half of immigrants. The share of immigrants with completed college (technical or university education) varies widely among these countries, especially among the most important sources of immigrants—Venezuela and Haiti. Nearly two thirds of adult Venezuelan immigrants have completed college, whereas one tenth of adult Haitian immigrants have attained this educational level. In our OLG model, we assume for simplicity that all immigrants have the same skill level. With respect to the age of immigrants, table 1 shows that there is substantially less heterogeneity among different countries of origin. Regardless of their origin, a substantial fraction of about 70%–80% of immigrants are aged 18–39. In our model simulations, immigrants are mostly young working-age adults.

[11] García and Guerra-Salas (2020) adjust the labor productivity of immigrants according to the quality of their education, proxied by standardized international test scores.

[12] The National Survey of Migration offers detailed information on immigrants, so it is useful to characterize their hetero-

Table 1: Heterogeneity among Immigrants of Different Origin (2016–2020)

Country of origin	Share of immigrants	Share completed college	Share age 18–39
Venezuela	45%	64.6%	71%
Haiti	19%	10.6%	78%
Colombia	10%	34.8%	67%
Peru	8%	23.1%	67%
Bolivia	7%	12.4%	79%

Notes.— Source: National Survey of Migration.

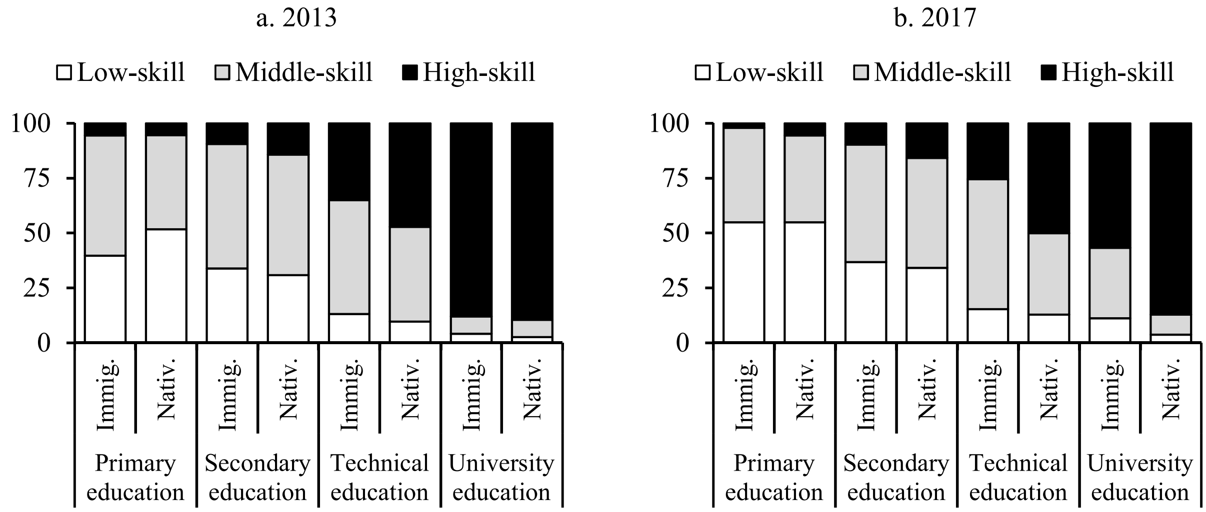
Although immigrants and natives have similar education levels, we model immigrants as experiencing a transitory downgrading spell, a period during which they cannot fully exercise the productivity associated with their education level because they hold jobs for which they are overqualified. Figure 3 shows employment by education and skill level of the occupation, for natives and immigrants, and for the years 2013 (panel a on the left) and 2017 (panel b on the right).[13] The recent immigration wave began around 2015, so the evidence from 2013 is prior to it, whereas the evidence from 2017 is gathered during the wave. During the immigration wave, when the fraction of recently-arrived immigrants significantly increased, immigrants held a substantially smaller proportion of high-skill occupations compared to natives of similar education level. This is not the case prior to the immigration wave, when recently-arrived immigrants represented a smaller fraction. Indeed, among individuals with higher education in 2017, only 57% of immigrants with university studies worked in a high-skill occupation, whereas 87% of natives with that education level did. In 2013, on the other hand, immigrants and natives with university studies held almost the same high proportion of high-skill occupations of about 90%. A similar divergence is visible for people with technical education: in 2013, the difference in the proportion of high-skill occupations held by immigrants and natives is 12 percentage points (pp), but in 2017 this difference increases to 25 pp.

To sum up, during the immigration wave, immigrants hold a substantially smaller proportion of high-skill jobs than natives, even when both groups have a similar education level. We acknowledge that this evidence is imperfect, because it refers to different cohorts of immigrants. Ideally, we would study panel data that allowed us to characterize the adaptation *process* of immigrants in the labor market. Unfortunately, these data are not available for Chile, to the best of our knowledge. Nevertheless, the phenomenon of occupational downgrading of immigrants has been observed in other episodes. Lebow (2024), for example, documents substantial downgrading of Venezuelan migrants to Colombia in the same period, whereas Dustmann, Schönberg, and Stuhler (2016) find downgrading of immigrants in the United States, United Kingdom, and Germany.

geneity. Tables such as 2 use other sources, because their purpose is to compare immigrants to natives.

[13] This evidence is based on data from the Socioeconomic Characterization Survey (*Encuesta de Caracterización Socioeconómica Nacional*; CASEN), which is conducted roughly every two years in Chile.

Figure 3: Employment by years of education and skill level of occupation



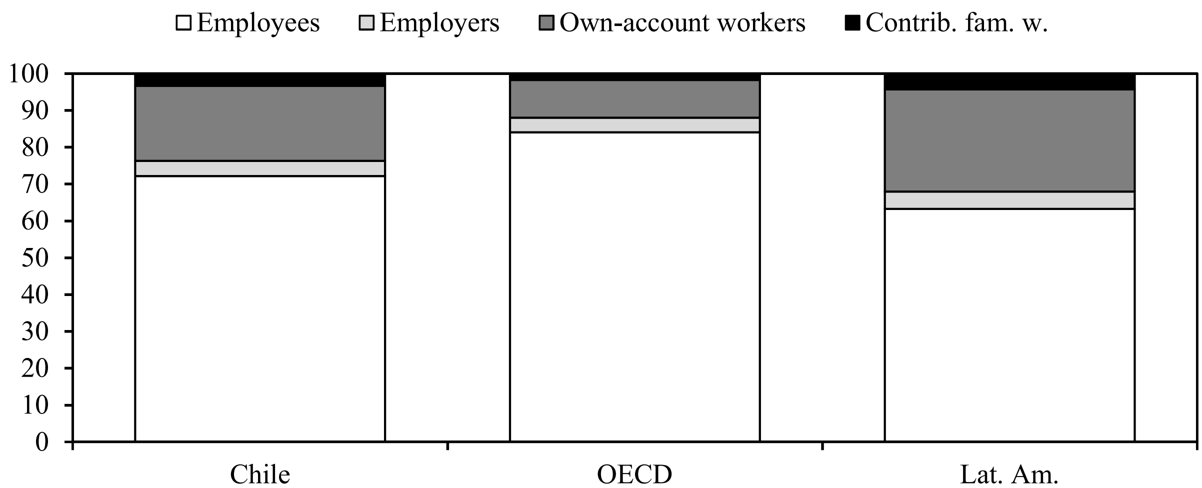
Note.— The underlying data come from the CASEN survey. The classification of occupations by skill level follows Lagakos et al. (2018). Source: Aldunate et al. (2019b).

2.2 Labor Informality

Emerging countries exhibit substantial labor informality. Figure 4 shows employment by category in Chile, a group of Latin American countries, and the OECD, a group of mostly advanced economies. There are several ways to define informality, but we focus on self-employment or "own-account" workers, following the literature that finds that this proxy is highly correlated with other definitions and offers higher data coverage.[14] The figure shows that the proportion of own-account workers is nearly three times higher in Latin American countries (28%) than in the OECD (10%), with Chile lying in the middle of these groups (20%).

[14]See, for example, Fernández and Meza (2015).

Figure 4: Employment by category

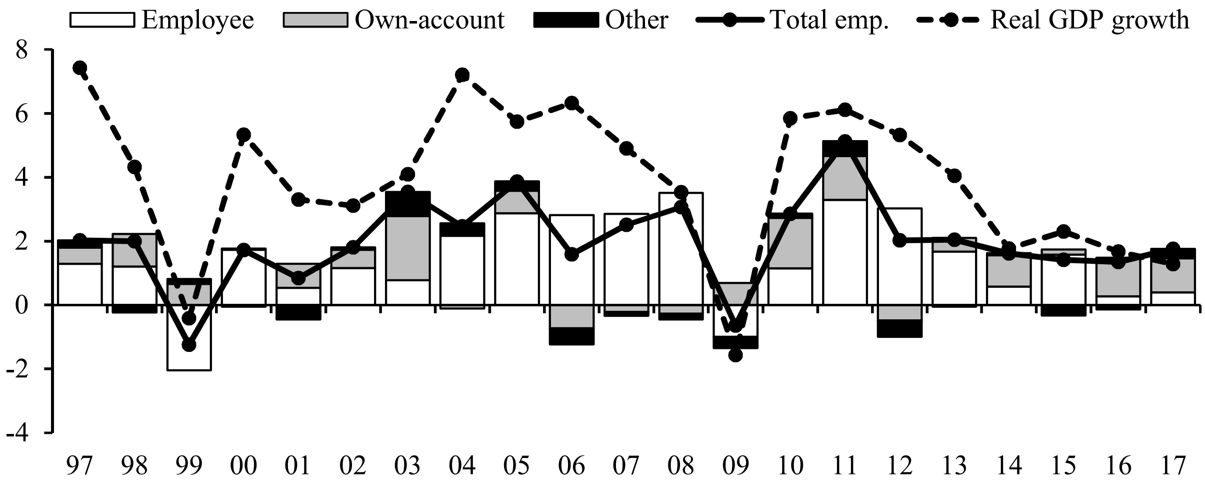


Note.— The underlying data come from the International Labour Organization. Categories of employment are: employees, employers, own-account workers, and contributing family workers. Source: Central Bank of Chile (2018).

The prevalence of informality in developing countries is a complex phenomenon with numerous causes and consequences for development (Ulyssea, 2020), but the key feature for our purposes is that informality acts as a buffer for income fluctuations (Loayza and Rigolini, 2011). Figure 5 reproduces results from Central Bank of Chile (2018) that show that employment growth is driven by formal work during economic expansions, and by informal work during contractions. Again, we focus on own-account workers as a proxy measure of informal labor, and on salaried employees as a proxy of formal labor. Chile grew strongly in the mid-to-late 2000s and in the aftermath of the Great Recession (2011-2013), periods during which employment growth was driven by formal work (white bars). On the other hand, the economy struggled in the aftermath of the Asian and Russian crises (up to 2003), and in 2014-2017, periods when informal work sustains job growth (grey bars).[15] Our OLG model will, therefore, feature an informal sector that acts as a buffer against income fluctuations.

[15]See also Parro G. and Reyes R. (2019), who find that in Chile, the share of self-employed increases in periods of low economic growth and vice versa, whereas the share of workers in formal work displays the opposite behavior (it increases in periods of *high* economic growth, and vice versa).

Figure 5: Employment growth by category



Note.— Source: Central Bank of Chile (2018).

In the next section, we spell out an OLG model consistent with the evidence presented in this section, as well as other demographic and economic features of the Chilean economy.

3 The Model

The model is based on the neoclassical growth models with overlapping generations pioneered by Samuelson (1958) and Diamond (1965). It is a real model with thirty generations, solved under perfect foresight.[16] We extend the standard OLG formulation in two key dimensions. First, we add an informal sector based on the work of Busato and Chiarini (2004), Busato, Chiarini, and Rey (2012) and Orsi, Raggi, and Turino (2014), with the purpose of capturing reallocation of labor supply between the formal and informal sectors. The informal sector is modeled as self-employment, which has been found to be a good proxy of informality (see , e.g., Fernández and Meza, 2015).[17,18] Second, we introduce worker heterogeneity along two aspects: skill level, which can be high or low, and birthplace, which can be domestic or foreign. Therefore, there are four types of workers in the model—high-skilled domestic, high-skilled foreign, low-skilled domestic and low-skilled foreign. In the formal sector, we allow for imperfect substitutability between immigrants and natives, even within a given skill level. Finally, we incorporate a rich set of exogenous shocks that allow us to match the changes in size, age, and skill composition of the population induced by an immigration shock.

In what follows, we describe the main features of the model. Appendix B offers a detailed derivation.

[16]Perfect foresight facilitates the solution of our non-linear model for studying medium- and long-term effects, at the cost of neglecting potential uncertainty effects.

[17]Recent estimates find that self-employment accounts for about 86% of informality in Chile; see INE (2023).

[18]McKiernan (2021) uses an OLG model with an informal sector and home production to study the 1980 pension reform in Chile. Both sectors, which generate similar effects, capture similar channels to our informal sector.

3.1 Population Dynamics

Individuals live for 30 periods, with the first 20 as active workers, and the last 10 as retirees. Death occurs with certainty after an individual has lived for 30 periods. Therefore, at any point in time, there are individuals from 30 generations or cohorts alive. A generation born at time t corresponds to the first cohort in period t, the second in $t+1$, and the k^{th} in $t+k$. Every generation consists of four types of workers, denoted by the indices ij, where $i = \{H, L\}$ denotes skill level (high or low), and $j = \{D, F\}$ denotes birthplace (domestic or foreign).

The size of a new generation born in this economy (N_t^1) relative to the size of the previous period's first generation (N_{t-1}^1) changes at the rate ($n^r + n_t^f$), i.e., $N_t^1 = \left(1 + n^r + n_t^f\right) N_{t-1}^1$; where n^r captures, in reduced form, the reproduction rate of the population, which we assume is constant, and n_t^f allows for time-varying increases of the first cohort due to immigration. Regarding cohorts 2 to 30, note that, absent immigration, the size of cohort g in period t is equal to the size of cohort $g-1$ in period $t-1$, but this may not be the case if there is immigration of individuals of cohort g in period t. Symbolically, we can describe this dynamic as follows: $N_t^g = (1 + f_t^g) N_{t-1}^{g-1}$, for $g \in [2, 30]$, where f_t^g denotes a change in the size of cohort g in period t due to immigration. We will use the exogenous variables n_t^f and f_t^g to simulate the change in population size and age structure induced by an immigration shock. The total population in period t is then defined as $N_t = \sum_{g=1}^{30} N_t^g$.[19]

We can keep track of the native (D) and immigrant (F) population within each cohort with the following laws of motion:

$$
\begin{aligned}
N_{Dt}^1 &= (1 + n_t^r)\, N_{t-1}^1, \\
N_{Ft}^1 &= n_t^f N_{t-1}^1, \\
N_{Dt}^g &= N_{Dt-1}^{g-1} = \ldots = (1 + n^r)\, N_{t-g}^1, \\
N_{Ft}^g &= N_{Ft-1}^{g-1} + f_t^g N_{t-1}^{g-1} = \ldots = n_{t-g+1}^f N_{t-g}^f + \sum_{s=1}^{g-1} f_{t-s+1}^{g-s+1} N_{t-s}^{g-s},
\end{aligned}
$$

and, after some algebra, we obtain that the size of the native population relative to the total one is given by $\frac{1+n_t^r}{1+n_t^r+n_t^f}$, whereas the relative size of the immigrant population is $\frac{n_t^f}{1+n_t^r+n_t^f}$.

The final step in the description of the population consists of distinguishing between high- and low-skilled natives and immigrants. This is crucial for two reasons: (i) natives and immigrants are imperfect substitutes, even within a given skill level, and (ii) a fraction of high-skilled immigrants experience a downgrading spell, which we model as a transitory decline in their skill level.

For the case of natives, we assume that a share $\lambda_{HD} \in (0, 1)$ are high-skilled. This share is constant across time and cohorts, so high-skilled natives of cohort g in period t are given by $N_{HDt}^g = \lambda_{HD} N_{Dt}^g$. Low-skilled natives are $N_{LDt}^g = (1 - \lambda_{HD}) N_{Dt}^g$. For the case of immigrants, the share of high-skilled varies across time and cohorts, and is given by $\lambda_{HFt}^g \in (0, 1)$, so high-skilled immigrants of cohort g in period t are given by $N_{HFt}^g = \lambda_{HFt}^g N_{Ft}^g$, whereas low-skilled immigrants are $N_{LFt}^g = (1 - \lambda_{HFt}^g) N_{Ft}^g$. We will use

[19] In steady state, $f^g = 0$ for all g, so we have that N, the total population in steady state, grows at the same rate as the first cohort at the steady state, N^1, namely $(n^r + n^f)$. Therefore, the immigration shock is transitory, but affects the population level permanently.

the time- and cohort-varying share of high-skilled immigrants to generate an exogenous downgrading spell that affects all immigrants, regardless of their age, for two model periods.[20] Each period corresponds to two years, as we explain in the calibration of the model.

3.2 Imperfect Substitution of Natives and Immigrants in the Formal Sector

The production process in the formal sector combines different types of labor and capital. Specifically, a representative firm demands capital, K_t, and a labor composite, L_t, to produce a homogeneous good, Y_t^F, used for consumption and investment, according to a standard Cobb-Douglas technology:[21]

$$Y_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha}, \tag{1}$$

where A_t denotes the labor-augmenting technology level, which grows at a constant rate z, so $A_t = (1+z)A_{t-1}$. The labor composite L_t, in turn, is a nested constant-elasticity-of-substitution (CES) bundle of formal labor supplied by workers of different skill and birthplace:[22]

$$L_t = \left[a_H \left(L_{Ht}\right)^{\rho_s} + a_L \left(L_{Lt}\right)^{\rho_s}\right]^{\frac{1}{\rho_s}}, \tag{2}$$

$$L_{it} = \left[a_{iD} \left(L_{iDt}\right)^{\rho_b} + a_{iF} \left(L_{iFt}\right)^{\rho_b}\right]^{\frac{1}{\rho_b}}, \quad i \in \{H, L\}. \tag{3}$$

Equation (2) is the CES bundle that combines high- and low-skilled labor, L_{Ht} and L_{Lt}, where a_H and a_L capture the productivity of these two types, and $\rho_s < 1$ governs the elasticity of substitution between workers of different skills, which is given by $1/(1-\rho_s)$. In turn, as shown in equations (3), high- and low-skilled labor, L_{Ht} and L_{Lt}, are themselves bundles that combine native and immigrant labor, L_{iDt} and L_{iFt}, for $i = \{H, L\}$. Analogously, a_{iD} and a_{iF} capture the productivity of natives and immigrants of skill level i, and $\rho_b < 1$ governs the elasticity of substitution between native and immigrant workers of a given skill level. As $\rho_k \to 1$ the different worker types are perfect substitutes, while as $\rho_k \to -\infty$ they become perfect complements, for $k = \{s, b\}$. Within a given skill level and birthplace, workers of different (active) cohorts are assumed to be perfect substitutes, so we have that:

$$L_{ijt} = \sum_{g=1}^{20} L_{ijt}^{g}, \quad i = \{H, L\}, \quad j = \{D, F\}.$$

The firm maximizes profits subject to its technology constraint, paying the different worker types and capital their marginal product, such that a zero-profit condition holds. The law of motion for capital is given by:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{4}$$

[20] Immigrants of all cohorts experience a two-period downgrading spell, which would seem to call for a share of high-skilled immigrants that is constant across cohorts. However, we simulate an immigration shock that occurs over several model periods, as we explain below, so we need this share to vary with cohort to keep proper track of immigrants' skills.

[21] Throughout, upper-case letters denote variables that are non-stationary due to population growth and productivity growth, whereas lower-case letters denote stationary variables.

[22] This formulation follows Ottaviano and Peri (2012).

where δ is the rate of depreciation and I_t denotes investment, which depends on households' consumption, saving and employment decisions. Specifically, investment is given by the portion of household disposable income that is not used for consumption:

$$I_t = Y_t^d + r_t S_{t-1} - C_t,$$

where Y_t^d denotes aggregate labor income, formal and informal, net of taxes, $r_t S_{t-1}$ is the aggregate gross return from assets, with $r_t = 1 + r_t^k - \delta$, where r_t^k is the marginal return to capital, and C_t is aggregate consumption. These variables are defined below or in the appendix.

3.3 Household Decisions and the Choice of Formal and Informal Work

We now describe the optimization problem of the workers in the model economy, with an emphasis on their choice of formal and informal work. Recall that there are four types of workers: high-skilled domestic, low-skilled domestic, high-skilled foreign, and low-skilled foreign, all of whom supply labor to the formal and informal sectors, facing the same trade-off.[23] To denote skill and birthplace, we use the subscripts $i = \{H, L\}$ and $j = \{D, F\}$, respectively.

Workers choose total labor supply, as in Brunner (1996) and Sommacal (2006), and how to allocate that labor across the formal and informal sectors. Additionally, they make consumption and savings decisions. Savings finance capital accumulation, which is rented to the perfectly competitive firm described previously, allowing agents to transfer wealth intertemporally. Retirees, in turn, only make decisions on consumption and savings, since they do not supply labor. Workers of all types born in period t make decisions on labor supply, consumption and savings to maximize their lifetime utility:

$$U_{ij,t} = \sum_{g=1}^{30} \beta^{g-1} \frac{\left(C_{ij,t+g-1}^g\right)^{1-\theta}}{1-\theta} - \sum_{g=1}^{20} \beta^{g-1} \Theta_{ij,t+g-1}^g \left(\frac{\left(l_{ij,t+g-1}^g + h_{ij,t+g-1}^g\right)^{\phi^l}}{\phi^l} + \frac{\kappa_{ij} \left(h_{ij,t+g-1}^g\right)^{\phi^h}}{\phi^h} \right), \quad (5)$$

where $C_{ij,t+g-1}^g$, $l_{ij,t+g-1}^g$ and $h_{i,t+g-1}^g$ denote consumption and labor supplied to the formal and informal sectors, respectively, when workers born in period t belong to cohort g in period $t+g-1$. $\theta > 0$ is the inverse elasticity of intertemporal substitution, ϕ^h, $\phi^l \geq 1$ govern the elasticity of labor supply, and $\beta \in (0,1)$ is the discount factor. The variable $\Theta_{ij,t}^g$ is an endogenous preference shifter based on Galí, Smets, and Wouters (2012) that allows us to abstract from the wealth effect on labor supply. $\kappa_{ij} > 0$ governs an additional specific utility cost of working in the informal sector. This parameter captures, in reduced form, limitations of informal work such as the lack of social benefits.

The intertemporal budget constraint (IBC) takes the following form:

$$\sum_{g=1}^{30} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1} C_{ij,t+g-1}^g = \sum_{g=1}^{20} \left(\prod_{n_g=1}^{g-1} r_{t+n_g} \right)^{-1} \left[(1-\tau) W_{ij,t+g-1} l_{ij,t+g-1}^g + f_{inf}(h_{ij,t+g-1}^g) \right],$$

[23] As we show below, we calibrate the model to match the observed participation of high- and low-skilled workers in the informal sector. Within each skill group, however, we do not differentiate between immigrants and natives in their participation in the informal sector. We justify this assumption when we discuss the calibration.

which states that the present value of lifetime consumption has to be equal to the present value of labor income, both formal and informal, over the individual's active life, where r_t denotes the gross real return on assets.

Formal labor income is determined by labor supplied to that sector, and the formal-sector wage net of a constant income tax rate, $(1-\tau)W_{ij,t+g-1}$. In turn, the wage is defined as the marginal product of formal labor:

$$W_{ijt} = (1-\alpha)\left(\frac{Y_t}{L_t}\right) a_i a_{ij} \left(\frac{L_t}{L_{it}}\right)^{1-\rho_s} \left(\frac{L_{it}}{L_{ijt}}\right)^{1-\rho_b}. \tag{6}$$

Production/income from the informal sector is given by the function $f_{inf}(\cdot)$, which uses labor as its only input, and is defined as:

$$f_{inf}(h^g_{ij,t}) = A_t \hat{b}_{ij} \frac{h^{\xi-1}_{pw,t}}{\eta} h^g_{ij,t}. \tag{7}$$

The marginal product of working in the informal sector grows with the underlying technological trend, A_t, is a function of the productivity of workers in the informal sector, $\hat{b}_{ij}$, and features "congestion" or decreasing returns in $h_{pw,t}$, total informal labor per worker, which households take as given when deciding on their informal labor supply. Additionally, the parameter ξ allows us to discipline the reallocation of labor across the formal and informal sectors in response to changes in the wage differential. Finally, η is a normalization parameter that simplifies the calibration of the model and the computation of its steady state.[24]

Note that taxation is only levied on formal labor, and that we will calibrate the parameters $\hat{b}_{ij}$ so that the informal sector is less productive than the formal sector.

Solving for the optimal labor decisions, as described in detail in Appendix B, we have that the intratemporal supply schedules to the formal and informal sectors, for $g \leq 20$, are respectively given by:

$$\Theta^g_{ij,t+g-1}\left(l^g_{ij,t+g-1} + h^g_{ij,t+g-1}\right)^{\phi^l-1} = \left(C^g_{ij,t+g-1}\right)^{-\theta}(1-\tau)W_{ij,t+g-1}, \tag{8}$$

$$\Theta^g_{ij,t+g-1}\left[\left(l^g_{ij,t+g-1} + h^g_{ij,t+g-1}\right)^{\phi^l-1} + \kappa_{ij}\left(h^g_{ij,t+g-1}\right)^{\phi^h-1}\right] = \left(C^g_{ij,t+g-1}\right)^{-\theta} A_{t+g-1}\hat{b}_{ij}\frac{h^{\xi-1}_{pw,t+g-1}}{\eta}, \tag{9}$$

which state that individuals supply labor to each sector, conditional on the optimal choice on labor supply to the other sector, up to the point where its marginal disutility equals the marginal utility.

To gain intuition on how workers' labor supply to the formal and informal sectors are related, we can simply combine these first order conditions and solve for labor supply to the informal sector to obtain the following schedule:

[24] Labor is the only input used in the informal sector, so it is akin to home production. This assumption is reasonable, because capital in the informal sector of emerging countries is known to represent a very small fraction of the total capital stock. For evidence on capital in the informal sector, see Lopez-Martin (2019); Porta and Shleifer (2008); and Busso, Fazio, and Levy Algazi (2012). It is, therefore, common to model an informal sector that does not use capital as an input; see Restrepo-Echavarria (2014).

$$h^g_{ij,t+g-1} = \begin{cases} \left[\left(C^g_{ij,t+g-1} \right)^{-\theta} \frac{\left[f'_{inf}(h^g_{ij,t+g-1}) - (1-\tau) W_{ij,t+g-1} \right]}{\Theta^g_{ij,t+g-1} \kappa_{ij}} \right]^{\frac{1}{\phi^h - 1}} & \text{if } f'_{inf}(h^g_{ij,t+g-1}) > (1-\tau) W^g_{ij,t+g-1}. \\ 0 & \text{otherwise.} \end{cases}$$

This expression shows that households supply labor to the informal sector as long as the marginal income from this activity exceeds the marginal income that they earn by working in the formal sector, net of taxes. In particular, households are compensated for the additional utility cost that informal work generates. Although this assumption is counterfactual, our calibration ensures that the marginal product of formal labor, i.e., the gross wage, is higher than the marginal income from informality. This structure captures the higher productivity of the formal sector while preserving incentives for informal work.[25]

3.4 Equilibrium in the Formal Labor Market

In equilibrium, labor supply equals labor demand for each worker type and cohort:

$$l^g_{ijt} N^g_{ijt} = L^g_{ijt}, \quad i = \{H, L\}, \quad j = \{D, F\}, \quad g = 1, 2, \ldots 20.$$

The appendix contains the derivation and the complete set of equilibrium conditions, as well as the stationary version of the model, and the derivation of the steady state.

4 Calibration and Simulation Design

This section describes the calibration of the model to the Chilean economy and the design of our simulations of an immigration shock. Table 2 summarizes the baseline calibration. The reproduction rate of the population, n^r, and the rate of change of the first cohort due to immigration at steady state, n^f, are calibrated to obtain an annual growth rate of 0.5% for the total population, and a share of immigrants in the labor force of 5% in steady state.[26]

Following Ottaviano and Peri (2012), high-skilled workers are those with at least some college education. As the right panel of figure 2 shows, 34% and 29% of immigrants and natives, respectively, belong to this category, so we set the share of high-skilled workers, for both immigrants and natives, to 30%.

The parameter ξ in equation (7) allows us to discipline the reallocation of labor effort across the formal and informal sectors. Following Joubert (2015), it is set to 0.57, so that a 5 percentage point increase in the income tax generates a 10% increase in informal labor.[27] The parameters that govern the efficiency of informal work, $\hat{b}_{ij}$ in equation (7), are calibrated to match participation rates in the informal sector of 15% and 30% for high- and low-skilled workers, respectively, based on data from the 2012 New Supplementary Income Survey.[28] We do not differentiate between immigrants and natives regarding participation in the

[25] For evidence on wage differentials between the formal and informal sectors in Chile, see Joubert (2015).

[26] The population growth rate comes from Albagli, Contreras, de la Huerta, Luttini, Naudon, and Pinto (2015). The share of immigrants in the labor force is approximately the one that prevailed before the immigration shock; see figure 1.

[27] Specifically, Joubert (2015) studies the effect of a 5 percentage point increase in payroll taxes to fund pensions in a partial equilibrium model for Chile.

[28] *Nueva Encuesta Suplementaria de Ingresos* (NESI). We classify as informal the following employment categories: employees without a contract, and self-employed individuals that work at home or on the street (excluding those with a

informal sector, assuming it is the same for both groups within a skill level. This assumption seems reasonable considering that about 20% of immigrants are self-employed, and the total participation in the informal sector under our calibration is similar at 25%.[29] This total participation in the informal sector is the weighted average of participation of high-skilled workers (15% with a weight of 30%) and low-skilled workers (30% with a weight of 70%). The parameters that determine the specific utility cost of informal work, κ_{ij} in equation (5), which govern the relative informal labor supply of workers, is set so that total labor supply for the four types of workers is similar. The parameter η is set so that the factor $(h_{pw,t}^{\xi-1}/\eta)$ in equation (7) equals 1 in steady state.

Our nested CES structure for the labor input, equations (2) and (3), requires parameterization of two elasticities of substitution (EoS). The first relates high- and low-skilled workers, whereas the second relates natives and immigrants within a given skill level. For the EoS between high- and low-skilled workers, given by $1/(1-\rho_s)$, we choose a value of 2.9, consistent with estimates by Ottaviano and Peri (2012), who use the same broad definition of skill level. The EoS between natives and immigrants within a skill level, given by $1/(1-\rho_b)$, is more difficult to calibrate. Our baseline results consider a value of 14 for this key parameter. Ottaviano and Peri (2012) estimate an EoS of 20 between natives and immigrants of the same *narrowly* defined skill and experience. In our model, the relevant EoS refers to natives and immigrants of the same *broadly* defined skill level and potentially different experience. In our setting, therefore, natives and immigrants are more different than in the context of Ottaviano and Peri (2012), so the EoS should be lower. We then study how results change when this elasticity takes a larger value of 50.

We calibrate the parameters that govern the productivity of high- and low-skilled workers, a_H and a_L in equation (2), so that 60% of income flows to high-skilled workers in steady state. According to the 2015 Socioeconomic Characterization Survey,[30] the distribution of total labor income across quintiles is [4.2%, 9.7%, 14.5%, 21.1%, 50.5%]. Assuming homogeneity within each quintile, the 30% of the population with the highest income would receive a share of 61.05% (50.5 + 21.1/2), which we round to 60% for high-skilled workers. Parameters a_{ij} in equation (3), which govern the productivity of workers of type ij, where $i \in \{H, L\}$ denotes skill and $i \in \{D, F\}$ denotes birthplace, are set so that formal wages of natives and immigrants of a given skill level are similar.

The rest of the parameters are more standard. The duration of a period, given by parameter T, is set to 2 years. This implies that individuals in the model work during the first 40 years of their lives, and are retired during the last 20 years. The discount factor, β, is calibrated to obtain aggregate savings of about 20% of GDP.[31] The technological growth rate, z, is set to 2%, in line with estimates and forecasts of trend growth for Chile obtained by Albagli et al. (2015) for the period 2016-2050. Following the same study, the labor share in production, $1-\alpha$, is set to 52%, which corresponds to the 2008-2013 average of the

university education, which are considered formal workers). The distribution of participation rates in informality across quintiles is [57%, 18%, 18%, 17%, 14%]. Assuming that high-skilled (low-skilled) workers participate the least (most) in the informal sector, and that participation within each of these bins is homogeneous, participation of high-skilled workers, who represent 30% of the population, would be 15% ($2/3 * 14\% + 1/3 * 17\%$; each quantile contains by definition 20% of the population). Similarly, participation of low-skilled workers, who represent 70% of the population, would be 30% ($2/7 * 57\% + 2/7 * 18\% + 2/7 * 18\% + 1/7 * 17\%$).

[29] Data on self-employment of immigrants comes from the National Survey of Migration, which offers information on immigrants aged 18 and older that arrived between 2016 and 2020. Self-employment is a good proxy of informality (Fernández and Meza, 2015).

[30] *Encuesta de Caracterización Socioeconómica Nacional* (CASEN).

[31] The source for this target is data from the World Bank.

Table 2: Baseline calibration

Parameter	Description	Value	Source / Target
n^r	Growth rate, first cohort: natives	-0.04	Population growth rate of 0.5%;
n^f	Growth rate, first cohort: immigrants (steady state)	0.05	immigrant labor force share of 5%.*
H/N	Share of high-skilled in labor force	0.3	Share with some college education
ξ	Congestion parameter in informality	0.57	Reallocation formal-informal: Joubert (2015)
$\hat{b}_{HD}$	Productivity in informal sector	0.45	Informal participation: 15% (high-skilled), 30% (low-skilled)
$\hat{b}_{HF}$		0.45	
$\hat{b}_{LD}$		0.2	
$\hat{b}_{LF}$		0.2	
κ_{HD}	Additional disutility of informal work	0.18	Similar labor supply across worker types
κ_{HF}		0.06	
κ_{LD}		0.15	
κ_{LF}		0.12	
η	Congestion factor parameter in informality	2.5	$\left(\frac{h_{pw,t}^{\xi-1}}{\eta}\right) = 1$ at steady state
ρ_s	Governs EoS between high and low skill	0.65	EoS of 2.9, Ottaviano and Peri (2012)
ρ_b	Governs EoS between natives and immigrants	0.928	EoS of 13.9
a_H	Formal sector productivity of high- and low-skilled workers	1.2	60% of income to high-skilled workers in steady state
a_L		0.6	
a_{HD}	Worker-type idiosyncratic productivity	0.55	Formal wages of natives and immigrants of a given skill level are similar
a_{HF}		0.45	
a_{LD}		0.55	
a_{LF}		0.45	
T	Duration of a period (years)	2	-
$\beta^{-1/T}-1$	Annual discount rate	0.82	Aggregate savings of 20%
z	Annual technological growth rate	2%	Albagli et al. (2015)
α	Capital share	0.48	Albagli et al. (2015)
θ	Inverse intertemporal EoS	1	Literature
ϕ^l	Governs labor supply elasticity	5	Orsi et al. (2014)
ϕ^h	Governs informal labor supply elasticity	3	Orsi et al. (2014)
$1-(1-\delta)^{1/T}$	Annual depreciation rate	4%	Literature
υ	Preference shifter parameter	1	No wealth effect on labor supply

Note.— *: rates are relative to the previous cohort size, so a negative value does not imply the population declines.

ratio of salaries paid by the corporate sector to the value added of that sector, net of taxes, according to national accounts data.[32] We take standard values for the inverse intertemporal elasticity of substitution, θ, which we set to 1, and the parameter that governs the elasticity of formal labor supply, ϕ^l, which we set to 5. The elasticity of informal labor supply with respect to the return differential between the formal and informal sector, $f'_{inf}(h^g_{ij,t+g-1}) - (1-\tau)W_{ij,t+g-1}$, is set to 3. The capital depreciation rate, δ, is set to 4% per year, as is standard in the literature. The parameter υ, which governs the strength of the wealth effect on labor supply, is set to 1. This specification favors the class of preferences that have a zero wealth effect, similar to those proposed by Greenwood, Hercowitz, and Huffman (1988). Consequently, changes in the quantity of labor supplied are only due to fluctuations in wages.[33]

We now describe the design of our simulation exercises, which are based on the recent experience of Chile. The first ingredient of our simulations is demographic. We simulate a change in the size of the labor force using the data shown in figure 1 for the period 2015–2050, which is based on historical and projected population data from the National Statistics Institute, and on estimates and forecasts of participation rates due to Aldunate, Bullano, Canales, Contreras, Fernández, Fornero, García, García, Pena, Tapia, and Zúniga (2019a).[34] Our simulations match population data and forecasts from 2015; this

[32]The underlying source of these data is the Central Bank of Chile.

[33]Correia, Neves, and Rebelo (1995) show that these preferences are useful for real small open economy models to match features of aggregate fluctuations.

[34]The dynamics of the labor force are based on estimates and projections for the total population, the working-age

is the year in which the large immigration wave begins. The shocks that alter the size of the labor force are concentrated in the first three model periods, which reflect the six-year period from 2015 to 2020. Thereafter, the demographic shocks are much smaller, which reflects the National Statistics Institute's more normal projections of immigration flows beyond 2020. The immigration flow in the first three periods of our simulations represents about 8.5% of the total labor force prior to the beginning of the wave (about 700,000 immigrants between 2015-2020 and a labor force of roughly 8.5 million in 2014).

The upper-left graph in figure 6 depicts the higher growth rate of the total population generated by the flow of immigrants entering the model economy.[35] The first vertical line in that graph marks the first period in which immigrant workers that arrive in the first three periods of the simulation are retired, whereas the second vertical line marks the first period in which these immigrants are deceased.In addition to altering the size of the labor force, and as shown on the right-hand side panels of figure 6, immigration alters the age structure of the population—indeed, the age structure of immigrants is skewed towards younger working-age adults. Since individuals in the model are active for 40 years (20 model periods) and retired for 20 years (10 periods), and since we abstract from human capital accumulation, our simulations match the age distribution of immigrants in the 20-80 range of age. In the model, the native population is assumed to grow at a constant rate, so the age distribution of natives is smoother than in the data. The lower-left graph of figure 6 shows that the ratio of retired to active individuals declines due to the immigration of mostly younger workers.

The second key ingredient of our simulations is related to the skills of immigrants and our assumption of a downgrading spell. As mentioned in subsection 2.1, our baseline results assume that the long-run skill distribution of immigrants is identical to that of the native population. However, in line with our assumption that immigrants experience a downgrading spell before they can fully integrate to the labor market, we impose an adverse distribution of skills on immigrants during the first two periods they live in this economy. This transitory shock to the skill distribution of immigrants lowers their productivity. Recall that in steady state, 30% of workers, both natives and immigrants, are high-skilled. In our baseline simulation, the skill distribution of immigrants is adversely skewed for two model periods (4 years), such that 90% of immigrants have the productivity associated with low-skilled workers. After the downgrading spell is over, immigrants recover their productivity immediately and return to the steady state skill distribution.

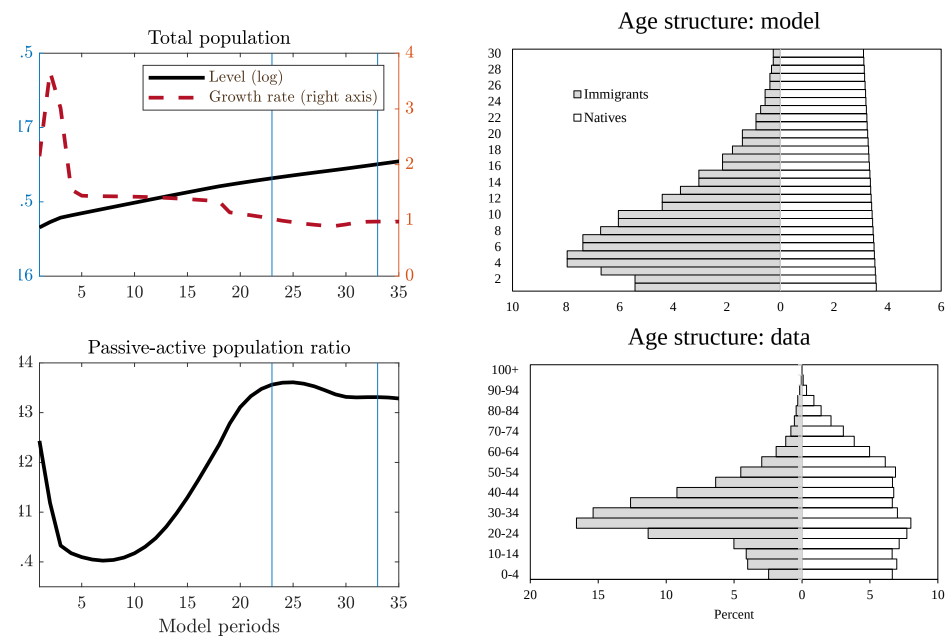
5 Results

This section presents the distributional, sectoral, and aggregate results of our baseline simulation of an immigration shock. We also highlight the role of imperfect substitutability between natives and immigrants, the informal sector, the downgrading spell, and heterogeneity in the age structure of immigrants and natives in driving these results.

population, and participation rates. The population data come from the National Statistics Institute (December 2018). The participation rates used to pin down the labor force come from Aldunate et al. (2019a), who construct forecasts of the labor force as an input for forecasts of trend growth in Chile.

[35] Since the model abstracts from the decision to participate in the labor market, we use the terms "labor force" and "total population" interchangeably, but as previously mentioned, the simulation design uses data on the labor force.

Figure 6: Population dynamics in simulations



Note.— The two vertical lines on the left-hand-side graphs mark the first period in which: (i) immigrant workers that arrive in the first three periods (2015–2020) retire, and (ii) these immigrants die. Each period in the model lasts 2 years. Source: Aldunate et al. (2019b) and authors' calculations.

5.1 Baseline Simulation

Figure 7 shows the evolution of formal wages and labor in the formal and informal sectors as a consequence of the immigration shock. Each column refers to one of the four types of workers in the model economy—high-skilled domestic (HD), high-skilled foreign (HF), low-skilled domestic (LD), and low-skilled foreign (LF).

As immigrants enter the economy, labor supply increases, putting downward pressure on the wages of all workers. However, the generalized decline in wages masks a complex dynamic—arising, in particular, from imperfect substitutability between labor inputs. This complexity is reflected in the varied wage responses across the different types of workers. Notably, immigrant wages fall significantly more than those of natives. As we will discuss in more detail, this sharper decline is mainly due to the limited substitutability between native and immigrant labor. With the large immigration shock, foreign labor becomes substantially more abundant relative to domestic labor, intensifying the downward pressure on immigrant wages.

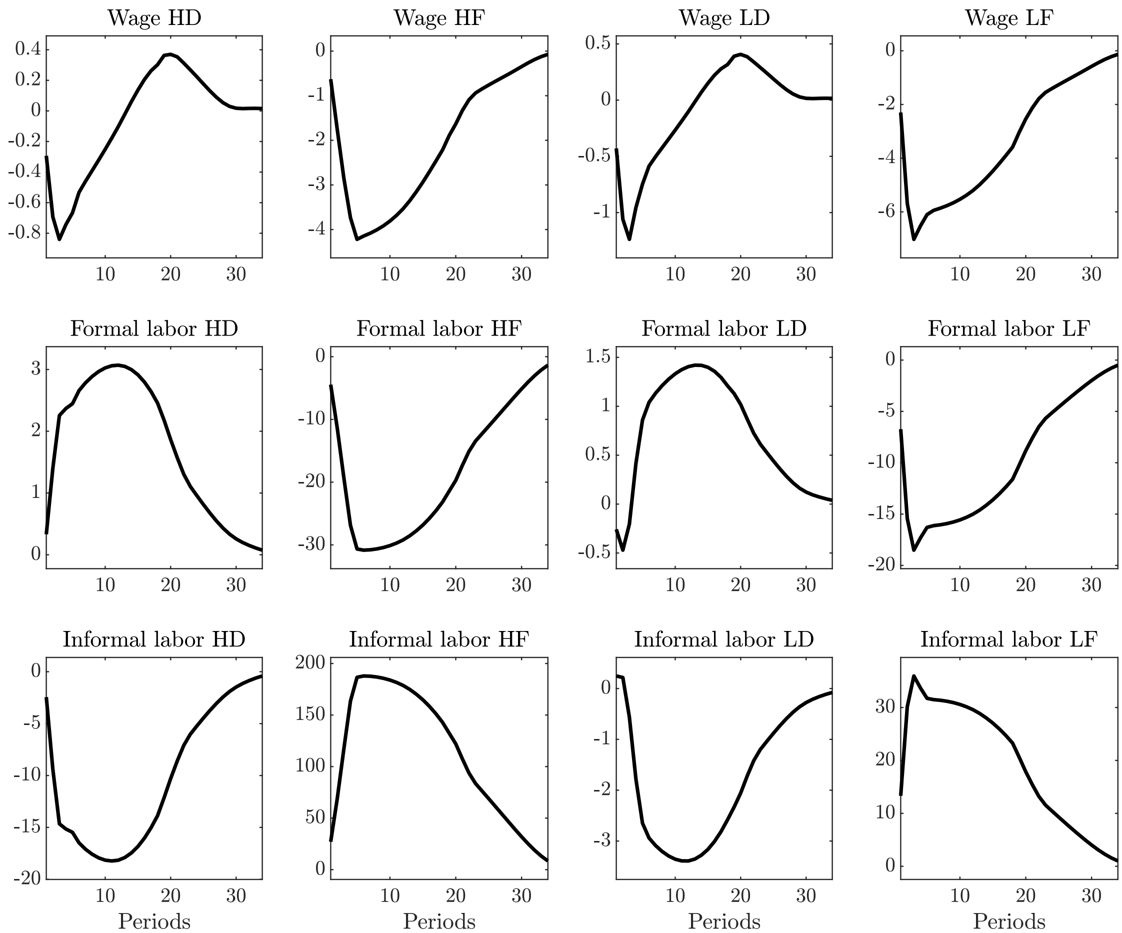
The large decline in immigrant wages makes the formal sector relatively less attractive than the informal sector, so high- and low-skilled immigrants respond by reallocating labor effort from the formal to the informal sector.[36] This substantial reallocation of immigrants to the informal sector generates, in turn, downward pressure on informal income due to congestion in that sector, i.e., the fact that the marginal return from informality is inversely related to total hours per worker. Congestion in informality plays a key role in disciplining reallocation between sectors in the model, and is intuitive if we think of informality as, for example, work on the street, where a larger number of workers adversely affects marginal returns.

Now, since informal income is common across worker types, its decline also affects natives. In particular, lower informal income makes the informal sector relatively less attractive for natives, who respond by reallocating labor effort from the informal to the formal sector.

Natives reallocate to the formal sector despite a decline in their wages during the first 15 periods. As we show below, the decline in informal income is larger than the decline in formal wages for natives. This differential creates the incentive for natives to reallocate to the formal sector.

[36] In figure 7, formal and informal labor of the different types of workers refers to that of the tenth cohort in every period.

Figure 7: Baseline simulation: Wages and sectoral labor



Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Percent deviations from steady state. Formal and informal labor refer to the tenth cohort in each period. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

To better understand the forces driving wage dynamics in the model economy, we ~~now~~ turn to the determinants of formal wages. The nested CES structure of formal labor described in equations (2) and (3)—which aggregates labor first by skill and then by birthplace—results in a wage for each worker of type ij, where $i \in \{H, L\}$ denotes skill and $j \in \{D, F\}$ denotes birthplace, that, after detrending, can be written as the product of three main factors:

$$w_{ijt} = \underbrace{(1-\alpha)\,(y_t/l_t)}_{\text{Labor supply effect}} \quad a_i a_{ij} \quad \underbrace{(l_t/l_{it})^{1-\varrho_s}}_{\text{Imperfect subst. skill}} \quad \underbrace{(l_{it}/l_{ijt})^{1-\varrho_b}}_{\text{Imperfect subst. birthplace}}.$$

The first factor, $(1-\alpha)\,(y_t/l_t)$, which we call the labor supply effect, captures the downward pressure of increased aggregate labor supply on wages. This component exerts uniform downward pressure across all worker types and is the only factor that would drive wages if a single type of worker existed. The second factor, $(l_t/l_{it})^{1-\varrho_s}$, captures the effect of imperfect substitutability between high- and low-skilled labor on wages. When labor of a given skill level becomes more abundant, workers of that skill level experience downward pressure on their wage. At the same time, because this abundance implies a relative scarcity of workers with the other skill level, demand for the latter increases, putting upward pressure on their wage. The intensity of this effect is governed by parameter ϱ_s, which in turn governs the elasticity of substitution (EoS) between labor of different skill, given by $1/(1-\varrho_s)$. As ϱ_s approaches unity, high- and low-skilled labor become perfect substitutes in (formal) production, with an EoS that approaches infinity, and this effect vanishes. Lower values indicate imperfect substitutability. The third factor, $(l_{it}/l_{ijt})^{1-\varrho_b}$, captures the effect of imperfect substitutability between native and immigrant labor on wages. Analogously, when labor of a given birthplace becomes more abundant (scarce), workers of that birthplace experience downward (upward) pressure on their wage, with parameter ϱ_b governing the intensity of this effect.[37]

Figure 8 shows the evolution of these three forces for the four worker types. The black line displays the labor supply effect, which generates downward pressure on wages. As previously mentioned, it is common to all workers.[38] The red line displays the effect of imperfect substitutability between high- and low-skilled labor. The downgrading spell amplifies this force by temporarily increasing the relative supply of low-skilled labor, thereby intensifying downward pressure on their wages. Simultaneously, the downgrading spell puts upward pressure on wages of high-skilled workers, because it makes this group relatively scarcer. Finally, the blue line captures imperfect substitutability between natives and immigrants of a given skill level. This force exerts downward pressure on immigrant wages, since the immigration shock makes immigrants more abundant relative to natives, and upward pressure on wages of natives, since natives become relatively scarcer with respect to immigrants. This effect is larger for immigrants due to their small initial share of the labor force, which makes the relative increase in their supply much larger.

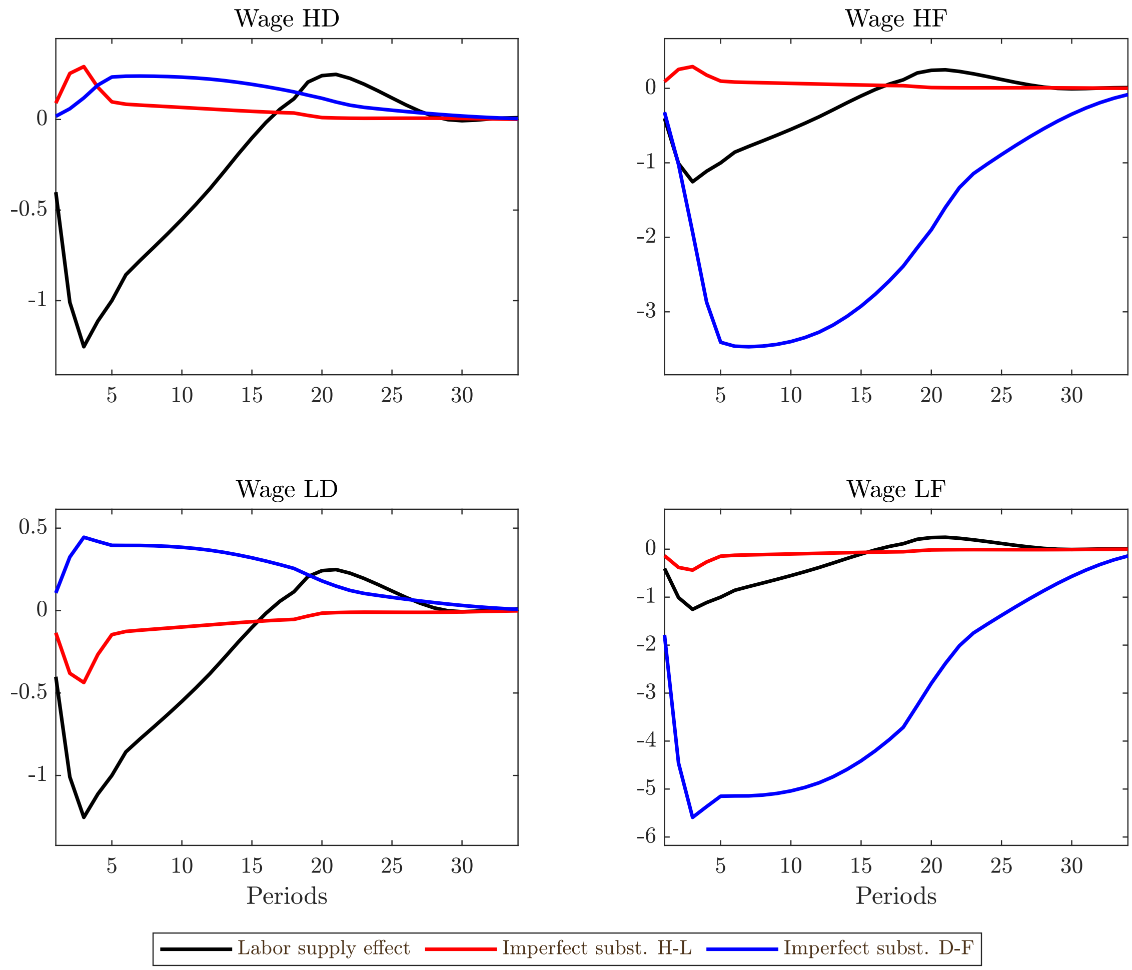
The figure confirms that the dominant force behind the decline in immigrant wages is the imperfect substitutability with natives, particularly given the large relative increase in immigrant labor. For low-skilled immigrants, the other two forces, labor supply and imperfect substitutability between workers of different skill, also put downward pressure on the wage. For high-skilled immigrants, imperfect substitutability of workers of different skill exerts upward pressure on the wage, but this effect is dwarfed by the downward pressure of the other two forces.

For native workers, the initial decline in formal-sector wages is primarily driven by the labor supply effect (black line). For low-skilled natives, imperfect substitutability of skill also generates initial downward pressure on the wage, whereas imperfect substitutability between immigrants and natives generates upward pressure. For high-skilled workers, in turn, the labor supply effect dominates the other two forces, both of which generate upward pressure on the wage. After approximately 15 periods, the labor supply effect begins to exert upward pressure on wages, likely due to capital accumulation catching up and the gradual retirement of earlier immigrant cohorts.

[37] a_i and a_{ij} are productivity parameters, which are held constant in this decomposition.

[38] The labor supply effect, $(1-\alpha)y_t/l_t$, also includes the role of the unitary elasticity of substitution between capital and labor on wages, which reflects the Cobb-Douglas production function assumed for the formal sector; see equation (1).

Figure 8: Baseline simulation: Drivers of formal wages



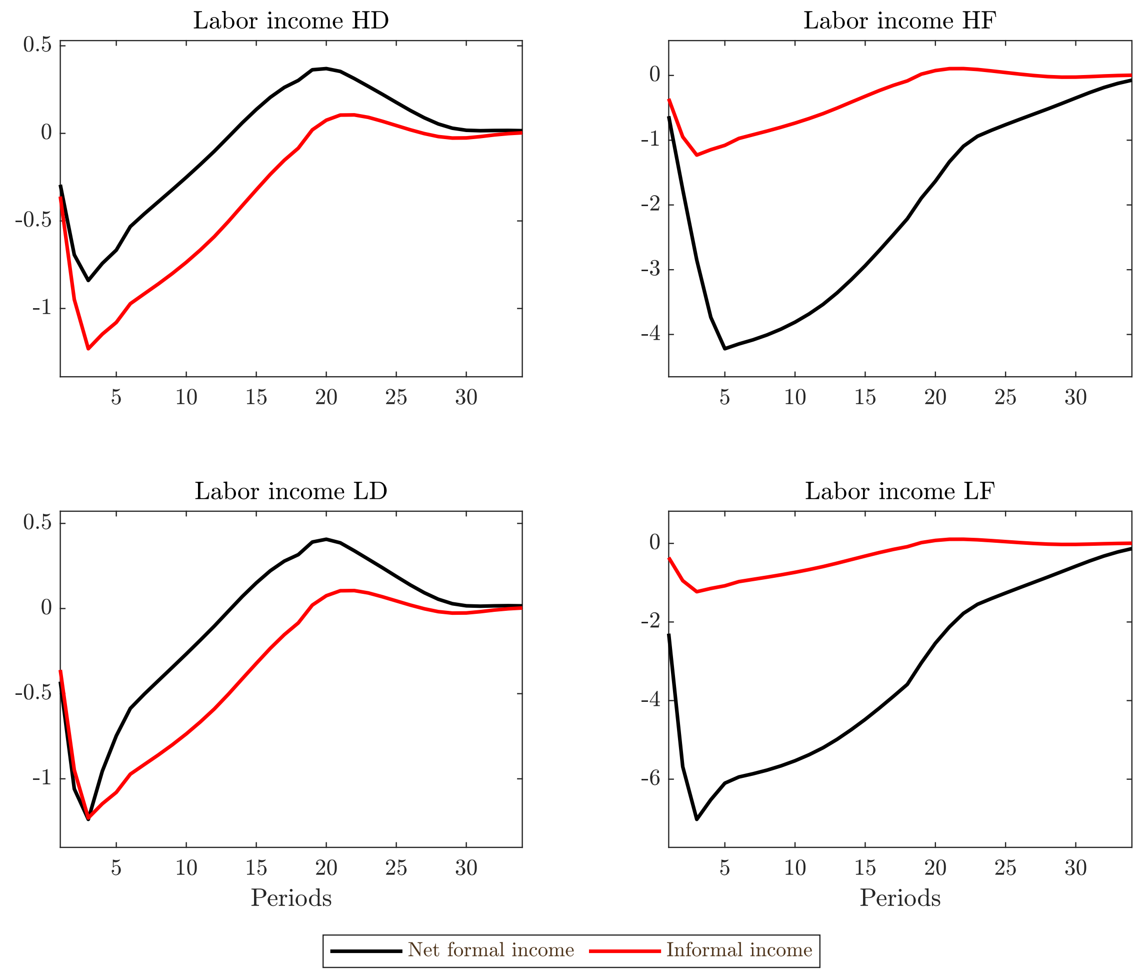
Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). The wage of worker of type ij, where $i \in \{H, L\}$ denotes skill and $j \in \{D, F\}$ denotes birthplace is given, after detrending, by $w_{ijt} = (1-\alpha)\,(y_t/l_t)\,a_i a_{ij}\,(l_t/l_{it})^{1-\varrho_s}\,(l_{it}/l_{ijt})^{1-\varrho_b}$. The black line (labor supply effect) displays the factor $(1-\alpha)\,(y_t/l_t)$, the red line (imperfect substitutability of labor of different skill) displays the factor $(l_t/l_{it})^{1-\varrho_s}$, and the blue line (imperfect substitutability of natives and immigrants) the factor $(l_{it}/l_{ijt})^{1-\varrho_b}$. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

Labor reallocation across sectors is governed by the relative evolution of formal and informal income. Figure 9 shows the evolution of the formal wage, net of income taxes, and informal income for the four types of workers. Since informal income declines uniformly across worker types, as we mentioned previously, differences in net formal wages drive the observed heterogeneity in sectoral reallocation.

Both high- and low-skilled natives reallocate labor effort from the informal to the formal sector. This pattern reflects the fact that informal income declines more sharply than formal wages.

For immigrants, formal wages decline more sharply than informal income. This makes the informal sector relatively more attractive, prompting immigrants to reallocate labor effort accordingly.

Figure 9: Baseline simulation: Formal and informal income

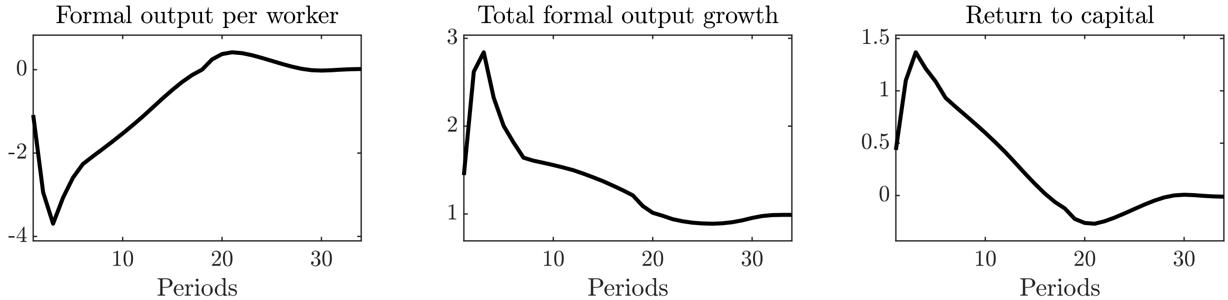


Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

We conclude the analysis of the baseline simulation by examining the aggregate implications of the immigration shock. Figure 10 shows that formal output per worker declines, although total output in the economy expands, since the size of the economy increases with immigration. Capital is predetermined, so the economy-wide ratio of capital to (formal) labor declines. Consequently, the return to capital increases, which incentivizes investment.[39]

[39]Note that capital is only used in the formal sector, so the relevant measure of scarcity relates it to formal labor.

Figure 10: Baseline simulation: Aggregate effects



Note.— Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

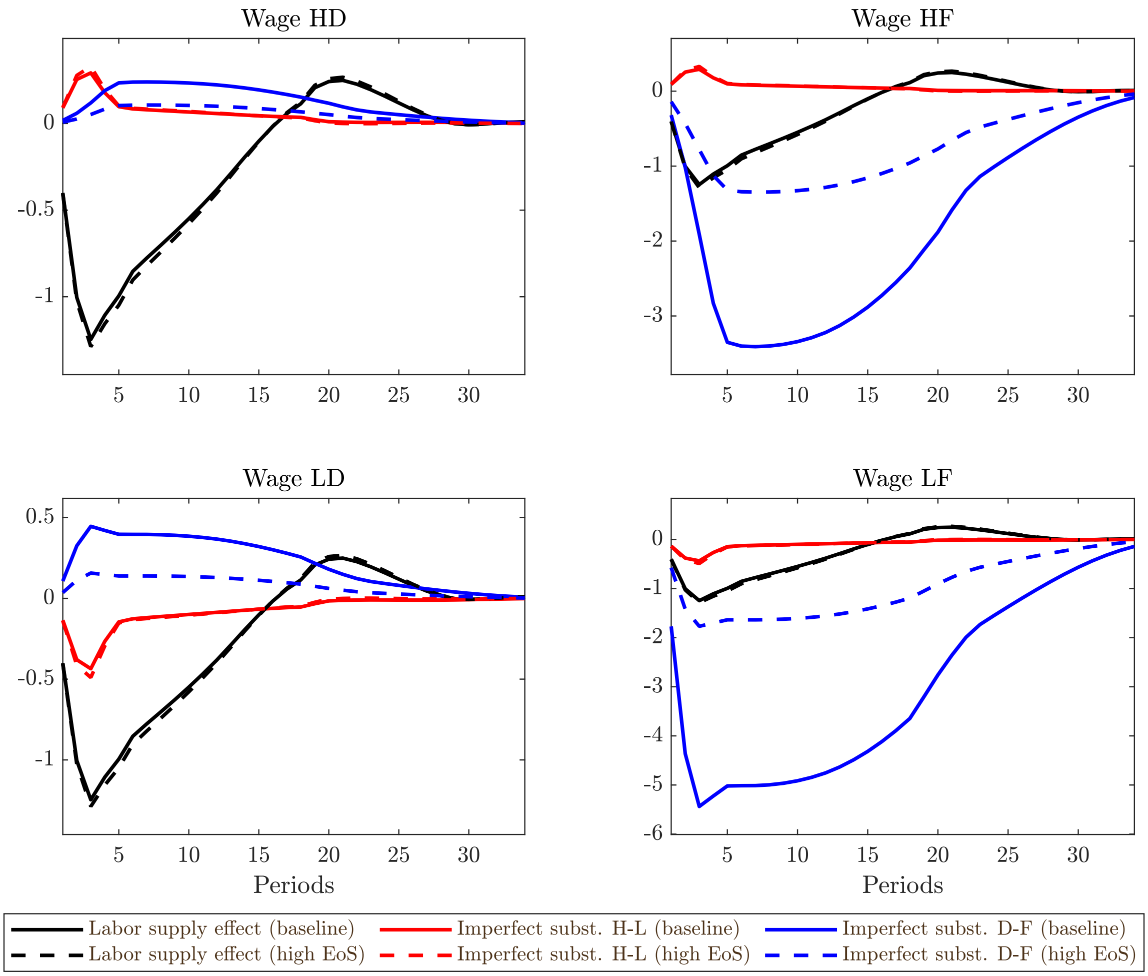
5.2 The Role of Imperfect Substitutability Between Natives and Immigrants

The elasticity of substitution (EoS) governs how easily firms can replace native with immigrant labor in production, and thus shapes the distributional consequences of immigration. In the model, imperfect substitutability affects the economy primarily through its effect on formal-sector wages. Figure 11 compares the key drivers of wages in the baseline simulation with a case in which the EoS is more than three times larger—50 versus 14.[40]

A higher EoS between native and immigrant labor mitigates the effect of imperfect substitutability on wages. Specifically, the upward pressure on wages of natives, who are scarcer relative to immigrants after the shock, is reduced. Similarly, the downward pressure on wages of immigrants, who are more abundant relative to natives after the shock, is also reduced. The contributions of the other two forces—labor supply and skill substitutability—remain virtually unchanged. Therefore, formal wages of natives decline more than in the baseline case, whereas wages of immigrants decline less than in the baseline case, as figure 12 shows.

[40] Parameter ρ_b, which governs the EoS, given by $1/(1-\rho_b)$, takes a value of 0.98 in this alternative scenario, in which we alter the EoS but leave the steady state unchanged. To this end, we modify the formal wage equilibrium equations (equations (38) in Appendix B), and the equations that determine the formal labor composites l_{Ht} and l_{Lt} (equations (50) in Appendix B) in order to keep the same steady state as in the baseline simulation. These six equations are the only ones in the equilibrium system that contain the parameter ρ_b. Since the model is highly non-linear, its response to shocks is state-dependent. This adjustment preserves the functional form of the model while holding the steady state constant, which allows us to focus on the changes in the dynamics that are a consequence of the change in the EoS while abstracting from effects that would arise solely from changes in the steady state.

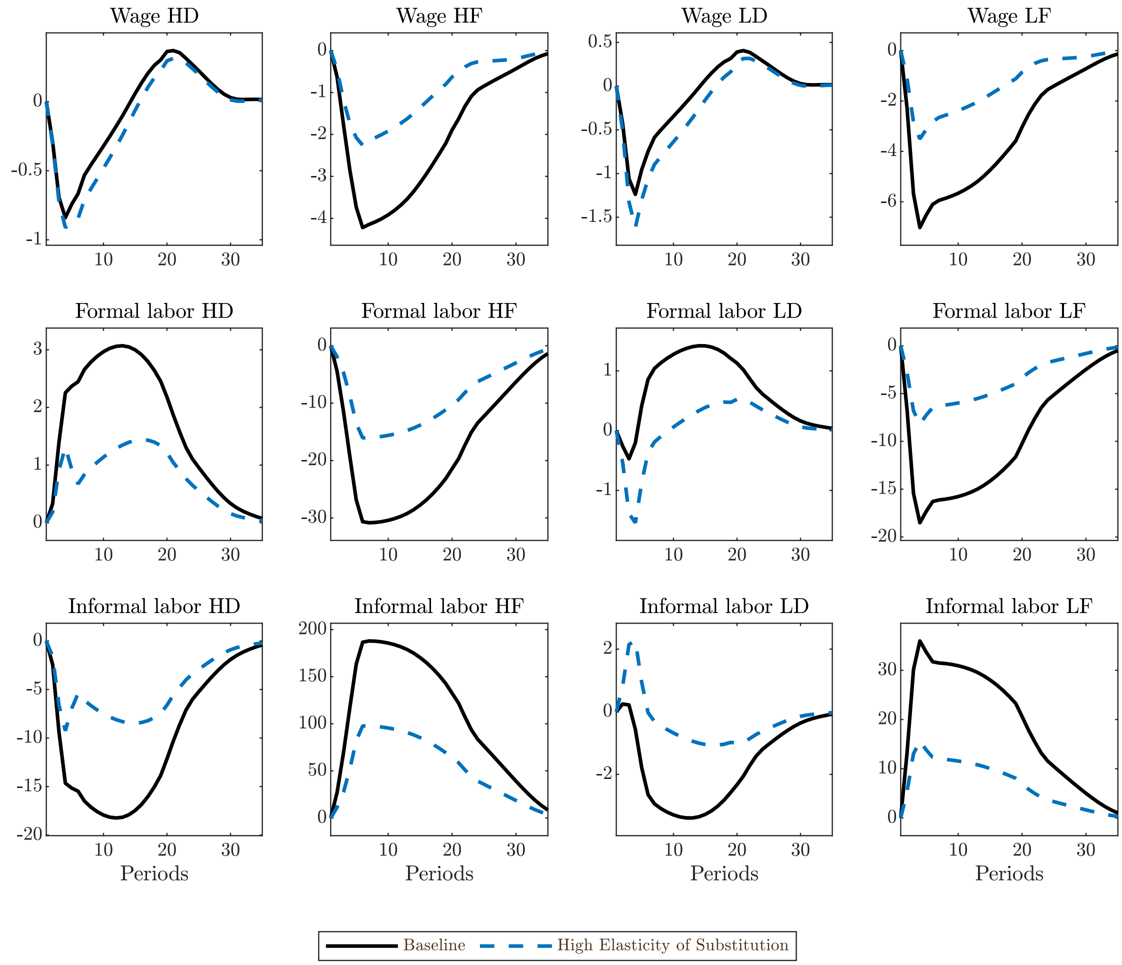
Figure 11: Substitutability between natives and immigrants: Drivers of formal wages



Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which the elasticity of substitution between natives and immigrants, ρ_b, takes a value of 50, instead of 14 in the baseline results. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

Because immigrant wages decline by only about half as much as in the baseline case, they have a lower incentive to reallocate to the informal sector. For natives, a lower formal wage reduces the incentive to reallocate to the formal sector. As figure 13 shows, informal income is slightly higher than in the baseline case, which also reduces the incentive to reallocate to the formal sector. In fact, under higher substitutability between natives and immigrants, the informal sector becomes relatively more attractive for low-skilled native workers, who reallocate labor effort from the formal to the informal sector during the first 5 periods, due to the combination of lower formal wages and a more modest decline in informal income.

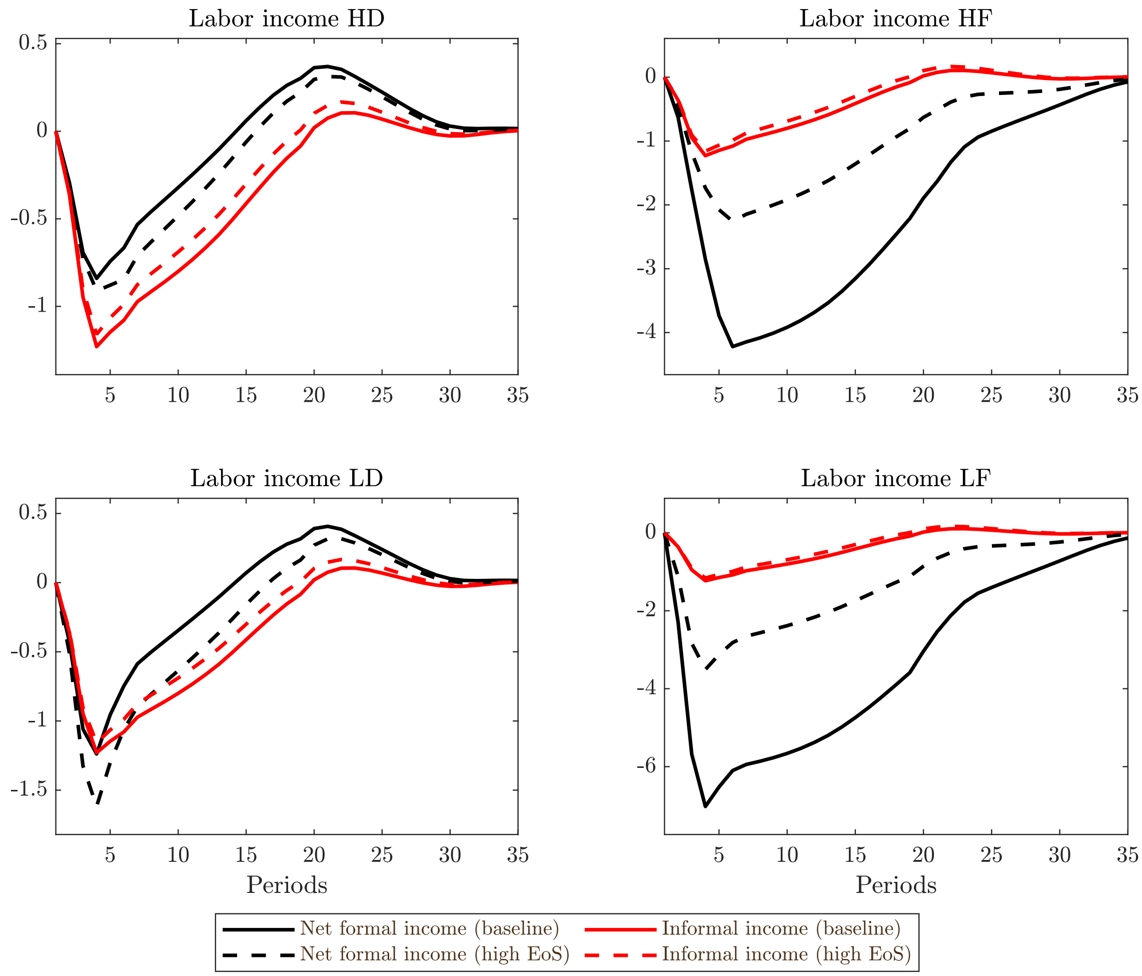
Figure 12: Substitutability between natives and immigrants: Wages and sectoral labor



Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which the elasticity of substitution between natives and immigrants, ρ_b, takes a value of 50, instead of 14 in the baseline results. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

Informal income is only marginally higher in this scenario relative to the baseline. Immigrants put less downward pressure on informal income due to substantially lower reallocation to that sector, but this is partially offset by increased native participation in the informal sector, which exerts downward pressure on informal income.

Figure 13: Substitutability between natives and immigrants: Formal and informal income

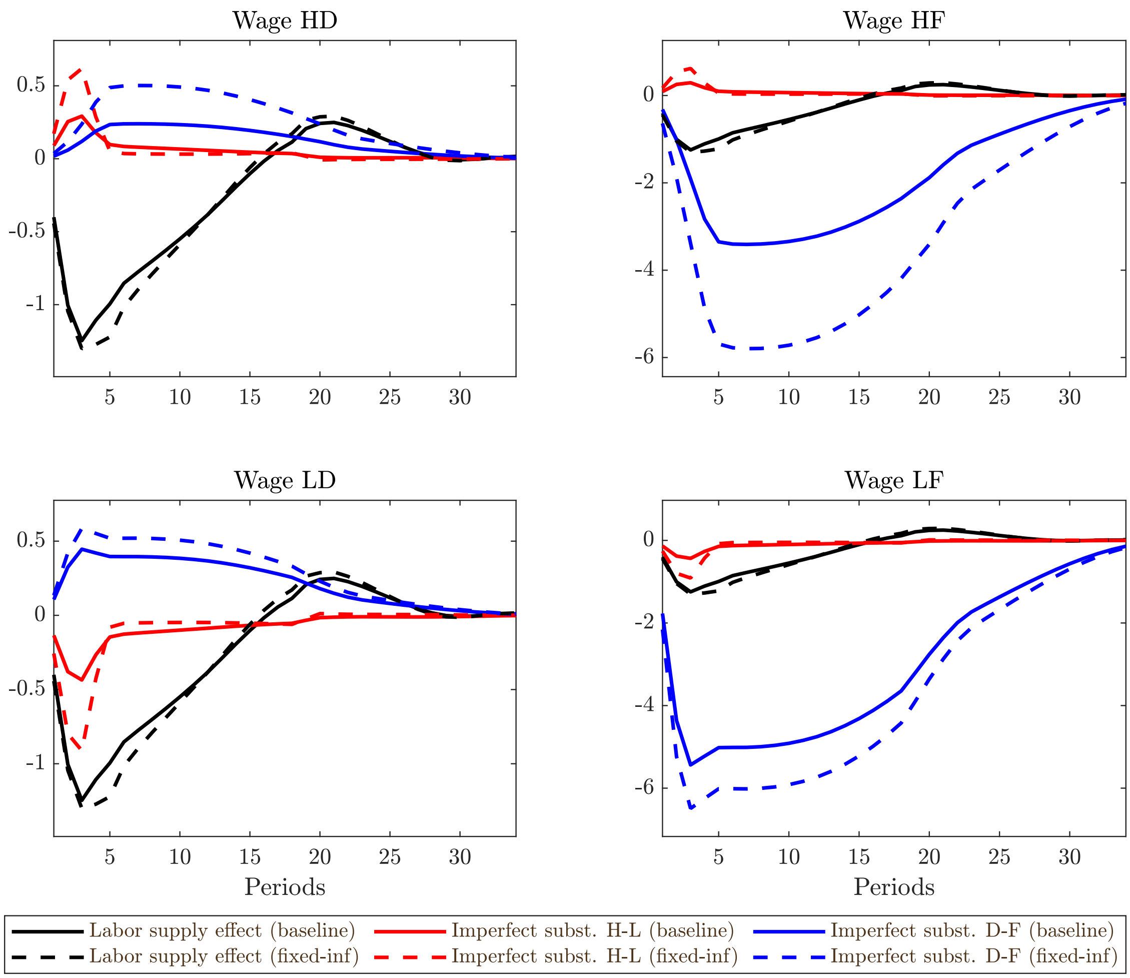


Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which the elasticity of substitution between natives and immigrants, ρ_b, takes a value of 50, instead of 14 in the baseline results. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

5.3 The Role of the Informal Sector

To isolate the role of labor reallocation between the formal and informal sectors, we fix the number of hours worked in the informal sector at the steady state level throughout the simulation. Figure 14 shows that, when workers cannot reallocate labor effort across sectors in response to changes in their relative income, the three forces that affect formal wages are amplified. Because workers cannot shift to the sector offering relatively higher income, the economy cannot absorb the shock through reallocation, leading to sharper wage adjustments. This dynamic results in a net positive effect on the high-skilled native wage, which is higher than in the baseline case, and a mixed effect on the wage of low-skilled natives, which is initially lower than in the baseline case, but subsequently higher. For immigrant workers, formal wages are lower than in the baseline case, mainly due to a larger effect of imperfect substitutability between native and immigrant labor.

Figure 14: The role of the informal sector: Drivers of formal wages



Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which hours worked in the informal sector are fixed at their steady state. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

High-skilled natives are the only group whose wages exceed baseline levels throughout the simulation. This is due to the higher upward pressure of imperfect substitutability between native and immigrant labor. Since high-skilled immigrants remain in the formal sector, the relative scarcity of high-skilled natives increases, intensifying the upward wage pressure from imperfect substitutability. Initially, the wage of low-skilled natives falls more than in the baseline due to a stronger skill-substitutability effect, since low-skilled workers are relatively more abundant in the formal sector during the first few periods, but after the downgrading spell ends, the upward pressure from birthplace substitutability dominates.

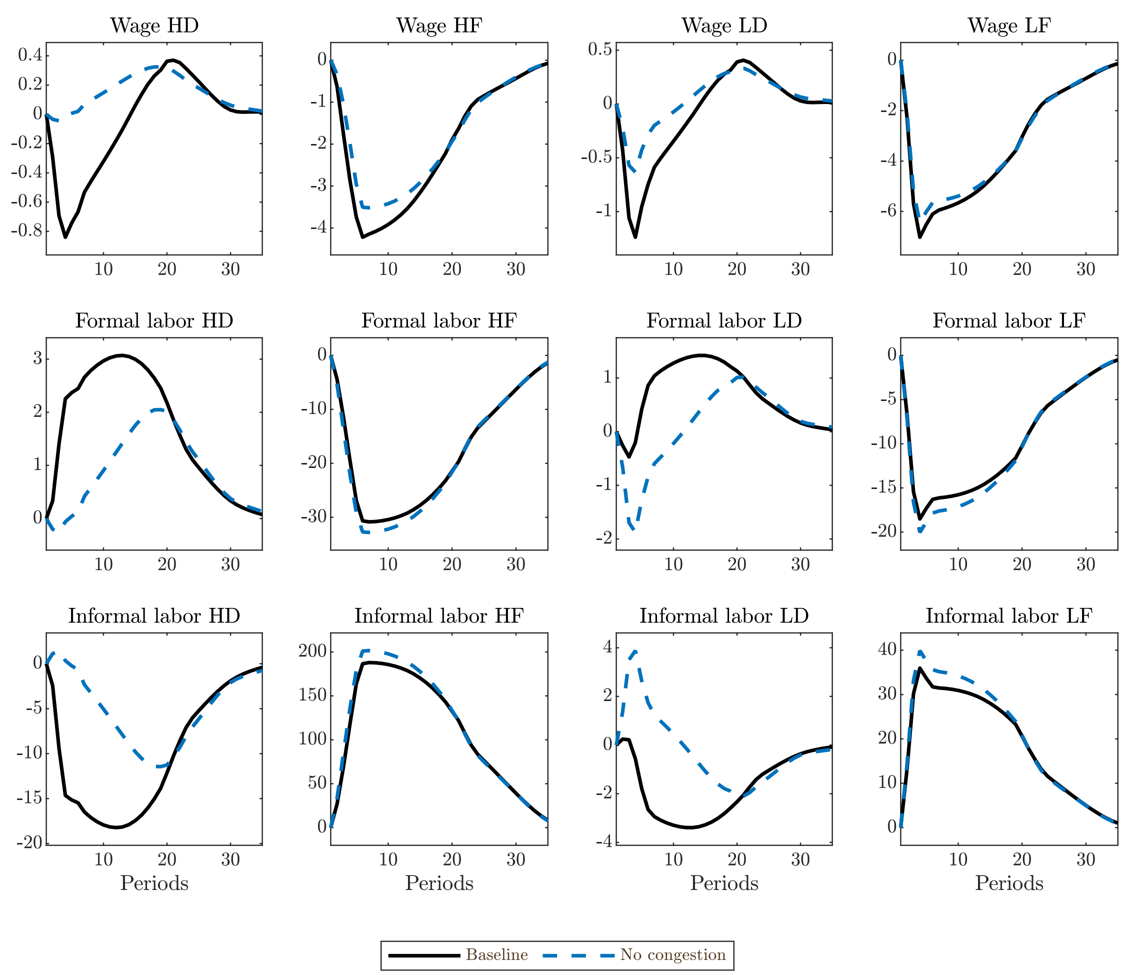
The wage of immigrants is lower for both high-skilled and low-skilled workers. With informal labor fixed at its steady state, immigrants cannot reallocate to the informal sector, so they are relatively more abundant than natives in the formal sector. Thus, imperfect substitutability between native and immigrant labor is the main driver of the decline in immigrants' wages.

For brevity, we do not show figures on sectoral employment or formal versus informal income. They are available in appendix A. Finally, note that by fixing informal labor, we also eliminate the congestion mechanism that is crucial for sectoral reallocation in the baseline simulation. We isolate the congestion mechanism in the next subsection.

5.4 The Role of Congestion in the Informal Sector

In the baseline simulation, the informal sector features congestion, with parameter ξ governing the extent to which informal income responds to total informal employment per worker, thereby allowing us to discipline labor reallocation between the formal and informal sectors.

Figure 15: Congestion in the informal sector: Wages and sectoral labor

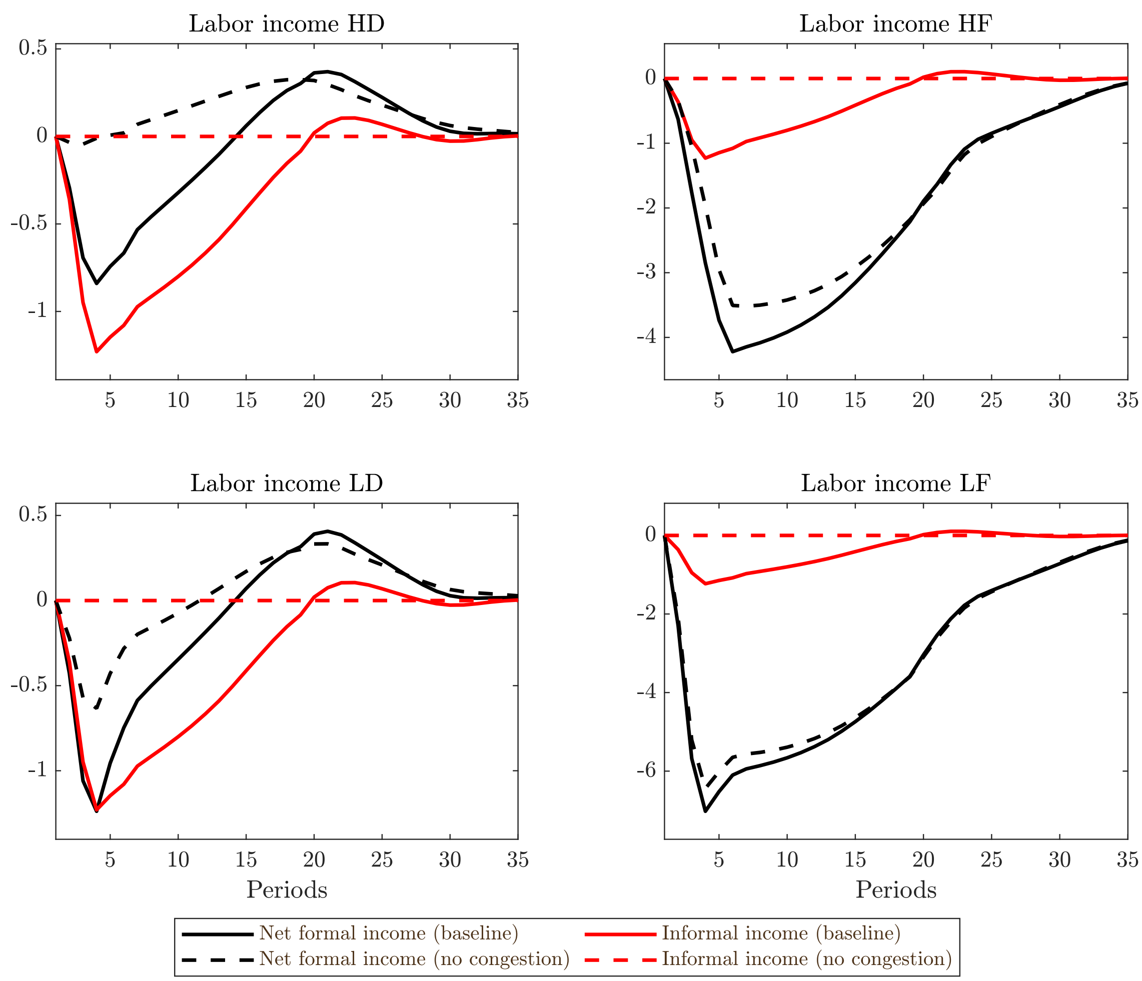


Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which congestion in the informal sector is turned off, so informal income is constant. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

We compare the baseline results to a scenario in which congestion is shut down, so informal income is constant and independent of total informal employment per worker. In this alternative scenario, the congestion parameter ξ takes a value of 1, instead of 0.57 in the baseline case.[41] Setting $\xi = 1$ implies that informal income is fully inelastic to changes in informal labor supply, effectively removing congestion.

Figure 15 shows that in this case, immigrants' reallocation of labor effort to the informal sector is similar to the baseline case. However, this dynamic does not generate a decline in informal income, as it does in the baseline case due to congestion.

Figure 16: Congestion in the informal sector: Formal and informal income



Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which congestion in the informal sector is turned off, so informal income is constant. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

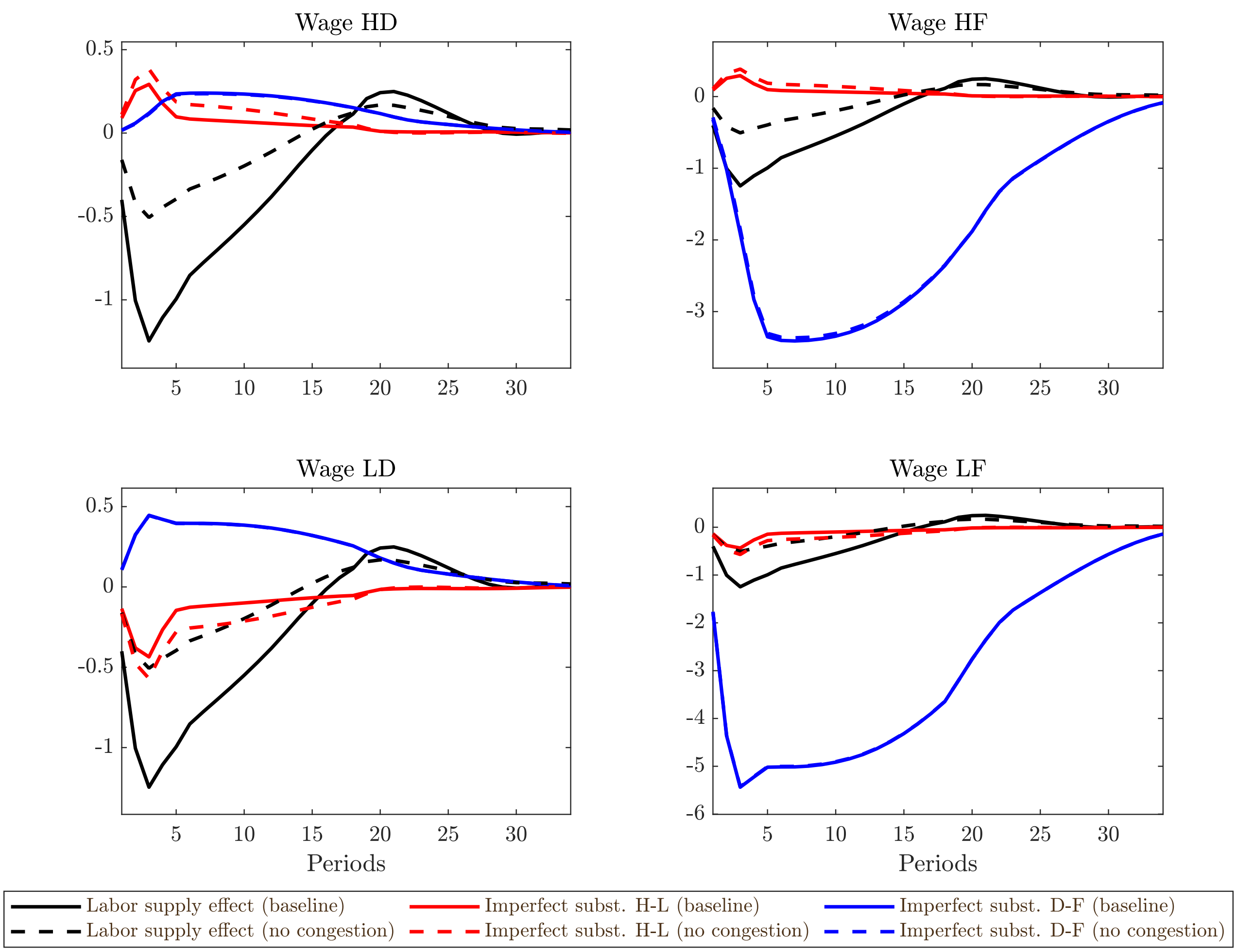
Without the decline in informal income observed in the baseline simulation, natives have a smaller incentive to reallocate labor effort to the formal sector. Indeed, low-skilled natives reallocate labor effort toward the informal sector during the first 11 periods, when the net formal wage is lower than informal income, as figure 16 shows. In the baseline simulation, congestion in the informal sector reduces informal

[41] Following the logic behind the scenario with a different elasticity of substitution (subsection 5.2), we adjust the model in order to leave the steady state unchanged when we set $\xi = 1$. Specifically, we adjust equations (7), which govern informal income and are a function of parameter ξ.

income, creating an incentive for natives to shift toward formal employment. Removing this mechanism alters that incentive structure.

Formal wages are higher than in the baseline case for all types of workers. Figure 17 shows that this result is driven by the labor supply effect (black line). When the informal sector does not feature congestion, equilibrium labor allocation shifts toward the informal sector, reducing labor supply in the formal sector. A lower level of labor in the formal sector reduces the downward pressure of the labor supply effect on wages. Notably, the wage of high-skilled domestic workers rises above steady state levels almost immediately in this case, since the forces of imperfect substitutability of skill and birthplace, both of which exert upward pressure on the high-skill wage, dominate the labor supply effect. This analysis highlights the role of the informal sector as a pressure valve: when it absorbs more labor without reducing income, it alleviates pressure on the formal sector, raising equilibrium wages.

Figure 17: Congestion in the informal sector: Drivers of formal wages

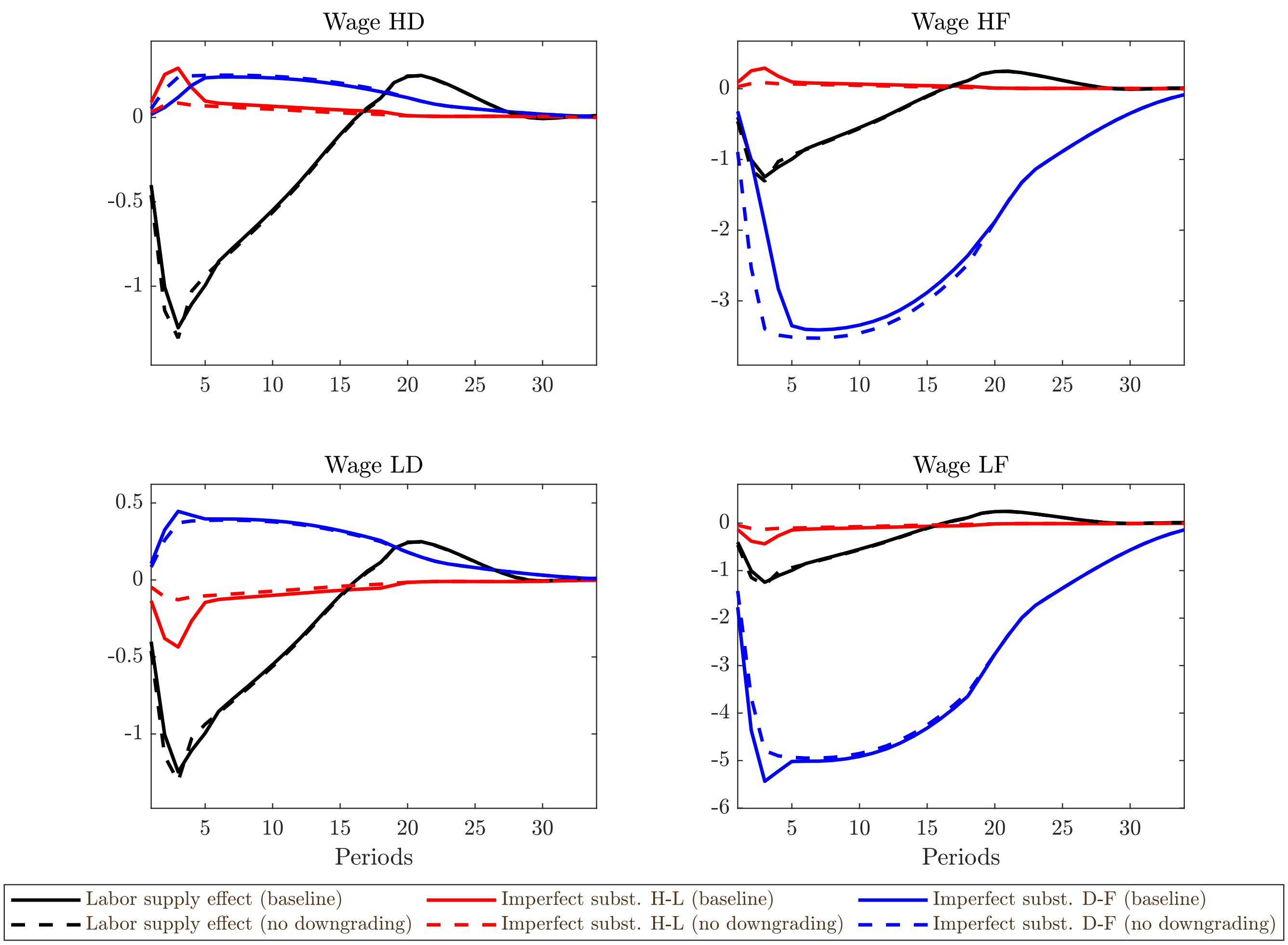


Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which congestion in the informal sector is turned off, so informal income is constant. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

5.5 The Role of the Downgrading Spell

To study the role of the downgrading spell, figure 18 compares the forces that drive formal wages in the baseline results to a simulation that does not assume immigrants experience a downgrading spell. As immigrants enter this economy they immediately find jobs matching their abilities.

Figure 18: The role of the downgrading spell



Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which immigrants do not experience a downgrading spell. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

The downgrading spell induces a larger relative increase of low-skilled formal labor supply that intensifies the downward wage pressure on low-skilled workers through the channel of imperfect substitutability between skill types. The opposite effect operates on high-skilled workers. The downgrading spell reduces the relative supply of high-skilled labor, making them relatively scarcer and thus raising their wage. Figure 18 shows that when the transitory downgrading spell is absent, the force of imperfect substitutability of skill is closer to neutral—it generates less upward pressure on the wage of high-skilled workers, and less downward pressure on the wage of low-skilled workers.

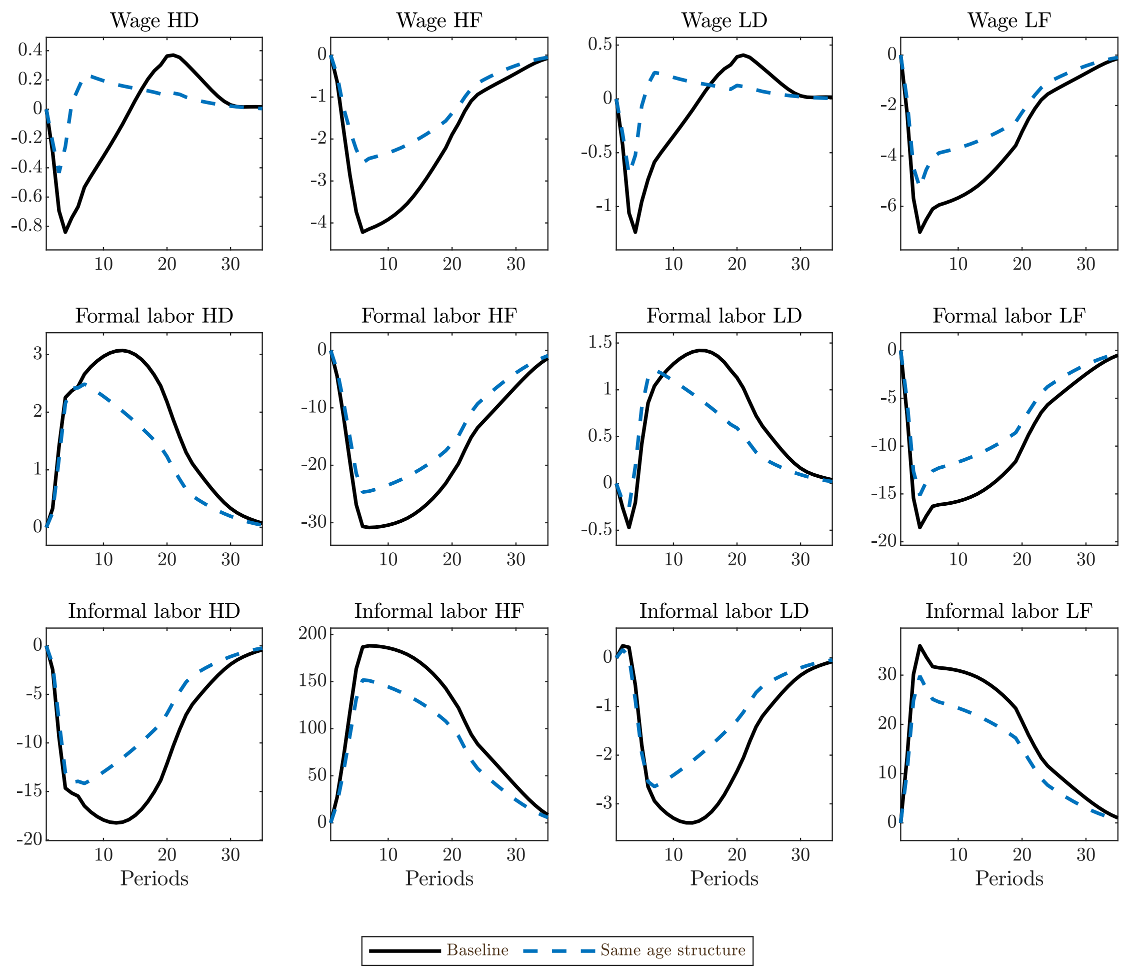
When the transitory downgrading spell is absent, the wages of high- and low-skilled domestic workers behave more similarly, with an initial decline of about 1%. In the baseline case, the wage of low-skilled domestic workers reaches a trough of about 1.25% below steady state, whereas the wage of high-skilled domestic workers reaches a trough of about 0.75%. Thus, the downgrading spell accounts for approximately 0.5 percentage points of the wage gap between high- and low-skilled natives in the short run. For brevity, we do not show figures on sectoral employment or formal versus informal income. They are available in appendix A.

5.6 The Role of the Age Structure of Immigrants

To conclude this section, we examine the importance of accounting for differences in the age structure between immigrants and natives—an aspect that motivates the use of an overlapping generations (OLG) model. Figure 19 compares the baseline results with a simulation in which immigrants and natives have the same age structure. In this case, the magnitude of the effects of an immigration shock are substantially smaller and less persistent. Since the immigrant population is older than in the baseline simulation, fewer immigrants are active in the labor force, and those who are retire sooner, leading to a smaller and shorter-lived increase in labor supply. This, in turn, reduces the magnitude and persistence of wage and reallocation effects. The decline in wages is smaller and less persistent, mainly due to milder labor supply and birthplace-substitutability effects.[42] The trough in wages is approximately half of that in the baseline simulation. Sectoral reallocation is also smaller than in the baseline simulation. These results underscore the importance of accounting for demographic heterogeneity in immigration models. Ignoring age structure can lead to significant underestimation of the macroeconomic and distributional effects of immigration. This finding is particularly relevant for emerging economies experiencing immigration waves dominated by young workers, as is the case in Chile. For brevity, we do not show figures on wage drivers or formal versus informal income. They are available in appendix A.

[42] The skill-substitutability effect on wages is similar to the baseline case. It is mainly driven by the transitory downgrading spell, as we previously explained. Downgrading lasts only two model periods, so it is less sensitive to changes in the age structure.

Figure 19: The role of age structure



Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which the age structure of immigrants is identical to that of natives (they still experience a downgrading spell). Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

6 Conclusion

Emerging countries receive a substantial share of global migration, yet the macroeconomic literature on immigration has largely focused on advanced economies. This paper fills that gap by examining how immigration affects economies where labor informality is pervasive—a defining feature of many emerging markets. We develop an overlapping generations model that incorporates labor informality, imperfect substitutability between natives and immigrants, and transitory skill downgrading. Calibrated to the Chilean economy, the model reveals that immigration can trigger complex reallocation dynamics between the formal and informal sectors, with distributional consequences that depend on labor market structure and immigrant characteristics. In particular, the degree of substitutability between native and immigrant labor plays a central role in shaping wage and employment outcomes. While our analysis is positive in nature, the results underscore the importance of accounting for informality and demographic heterogeneity when evaluating immigration policy in emerging economies. Future research could extend this framework to explore intergenerational effects, fiscal implications, and the role of human capital accumulation.

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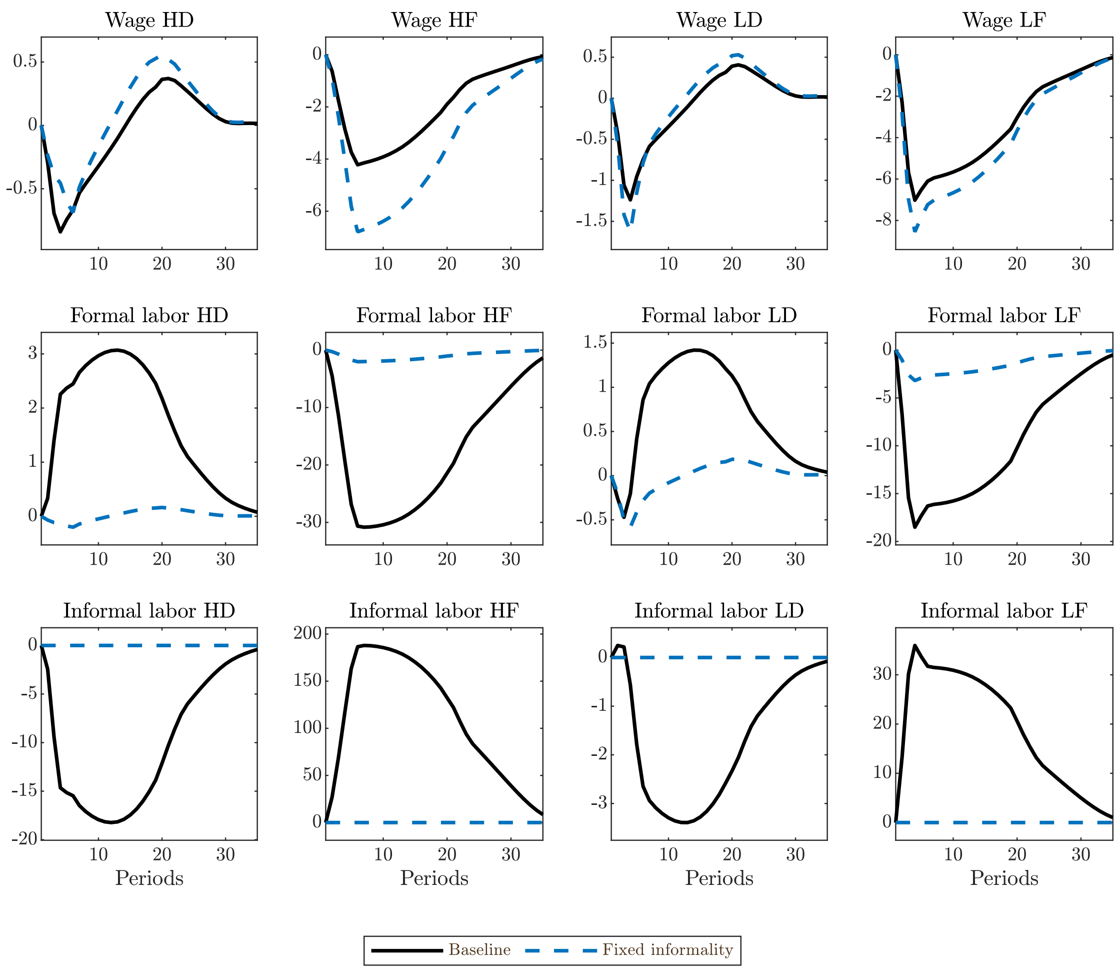
Appendix

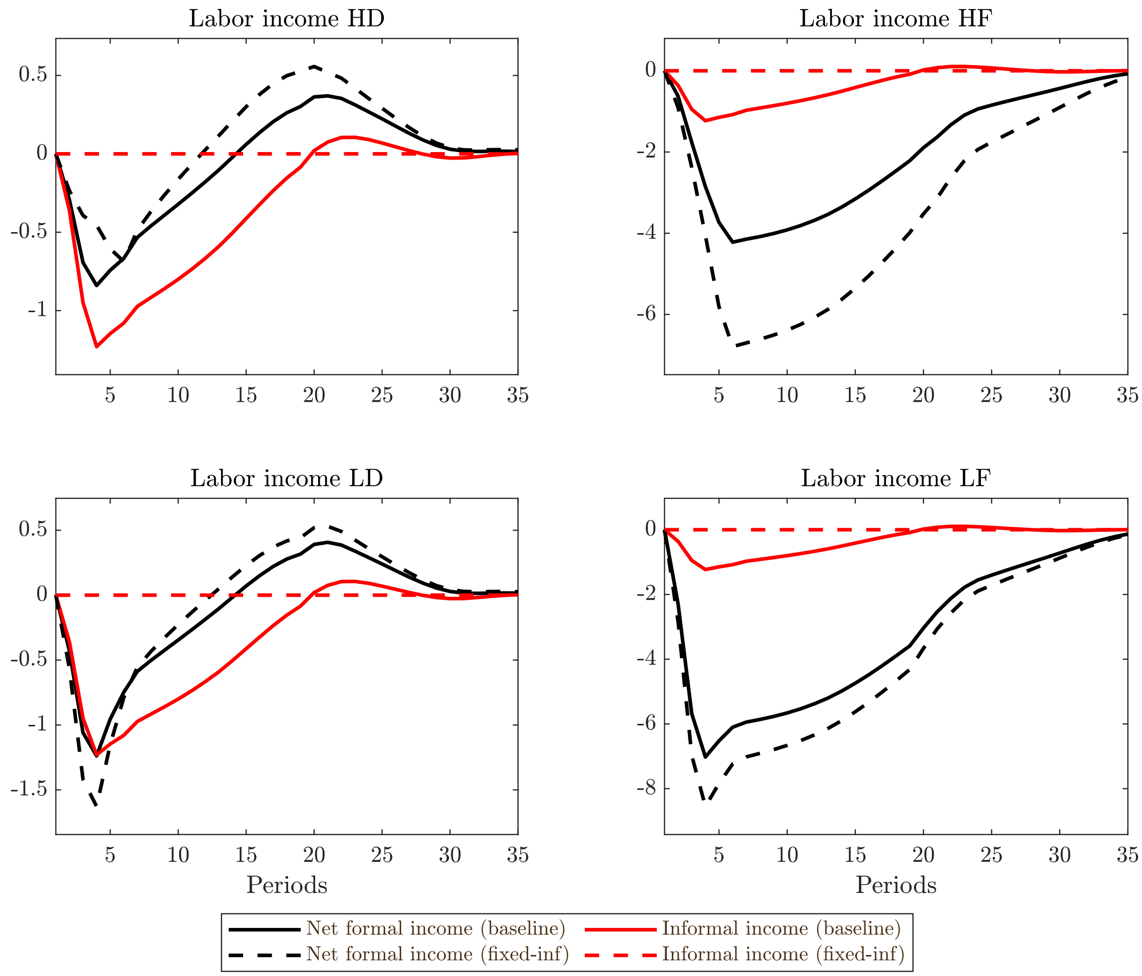
A Complementary Results

Figure 20: The role of the informal sector: Wages and sectoral labor



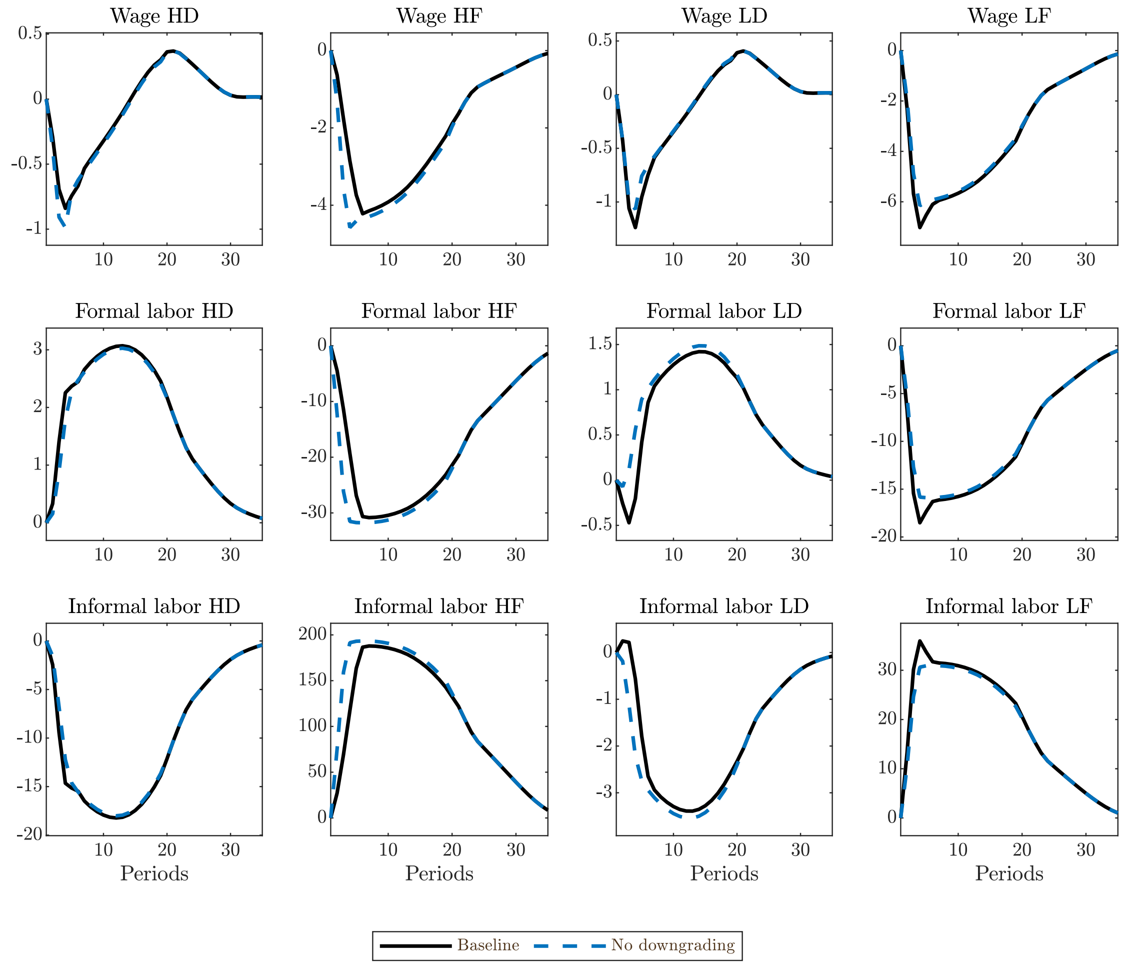
Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which hours worked in the informal sector are fixed at their steady state. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

Figure 21: The role of the informal sector: Formal and informal income



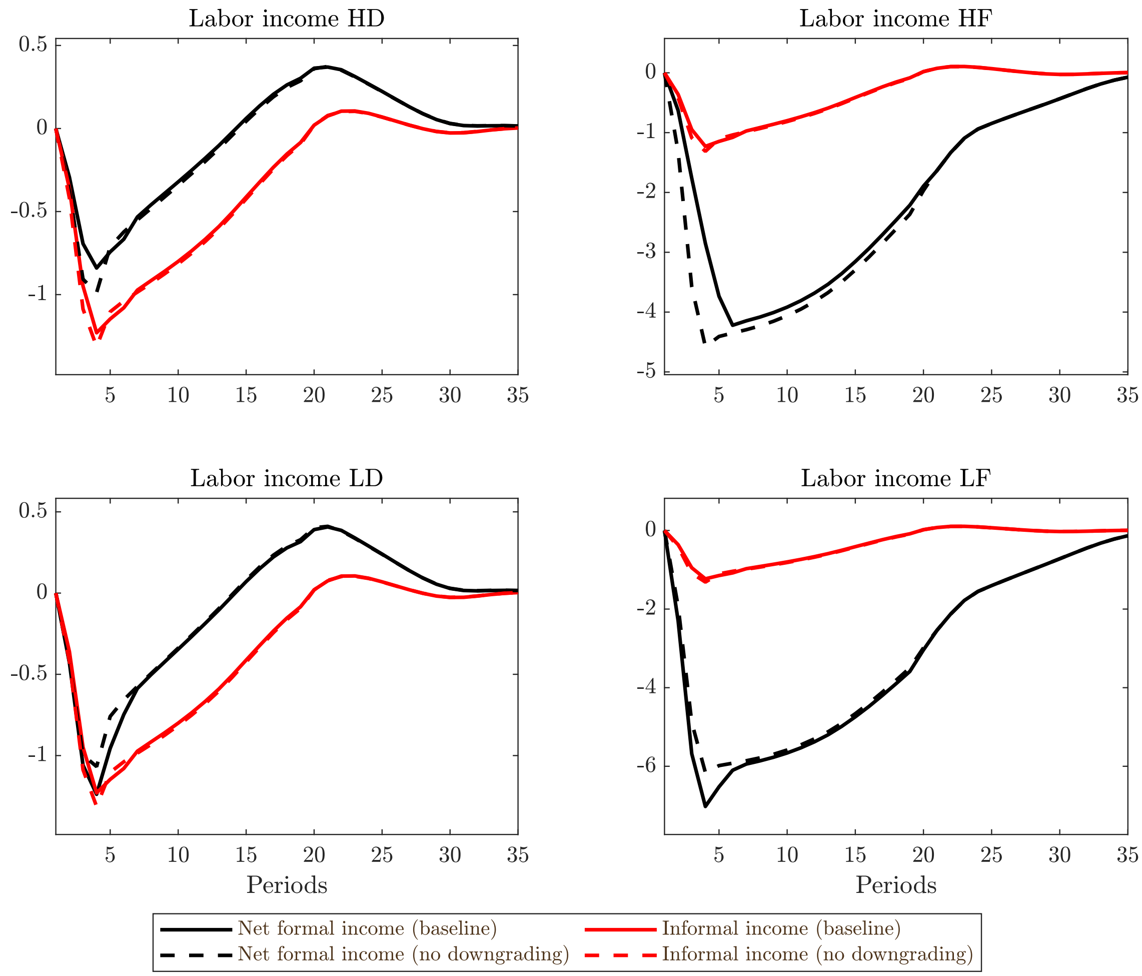
Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which hours worked in the informal sector are fixed at their steady state. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

Figure 22: The role of the downgrading spell: Wages and sectoral labor



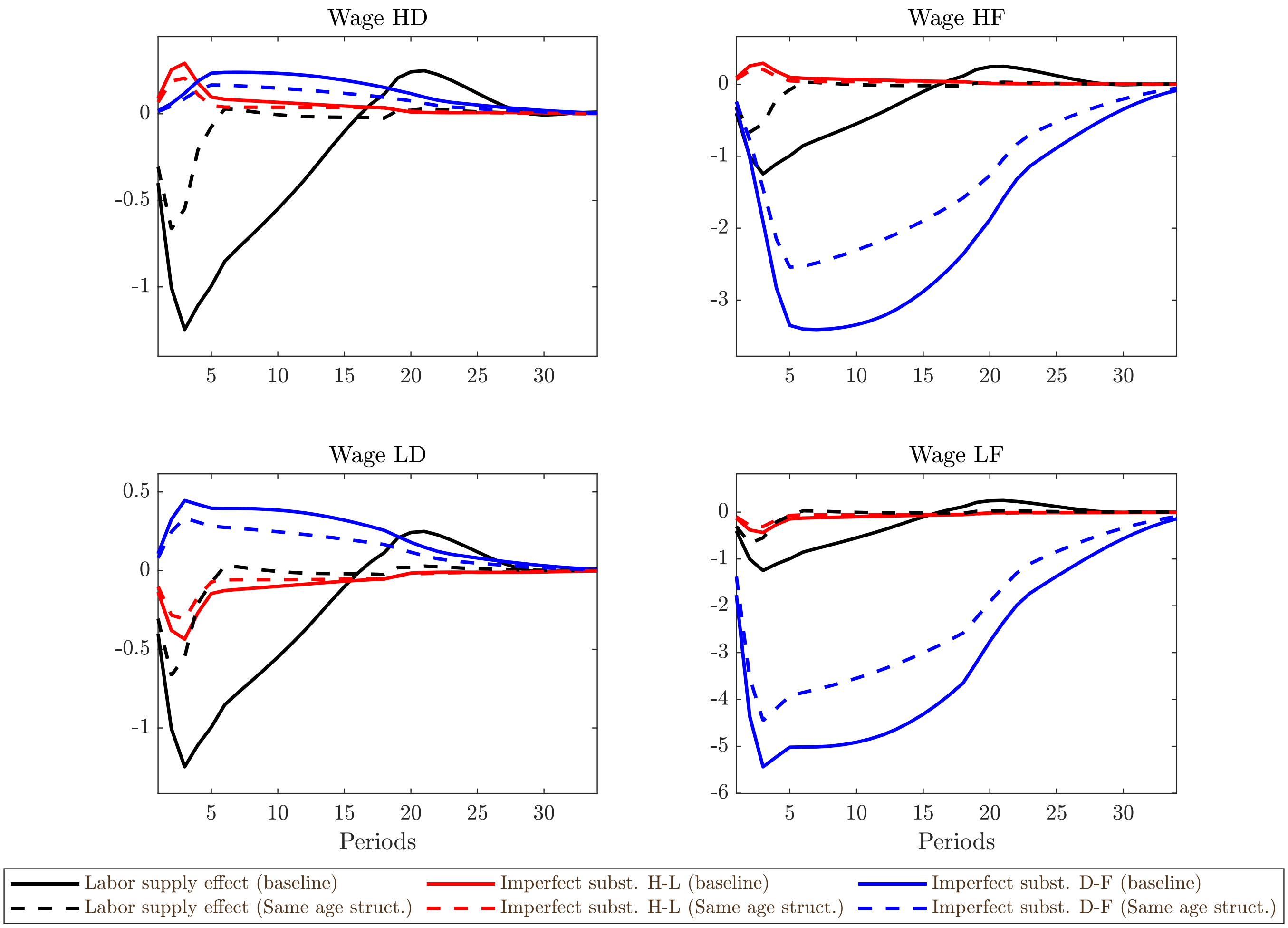
Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which immigrants do not experience a downgrading spell. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

Figure 23: The role of the downgrading spell: Formal and informal income



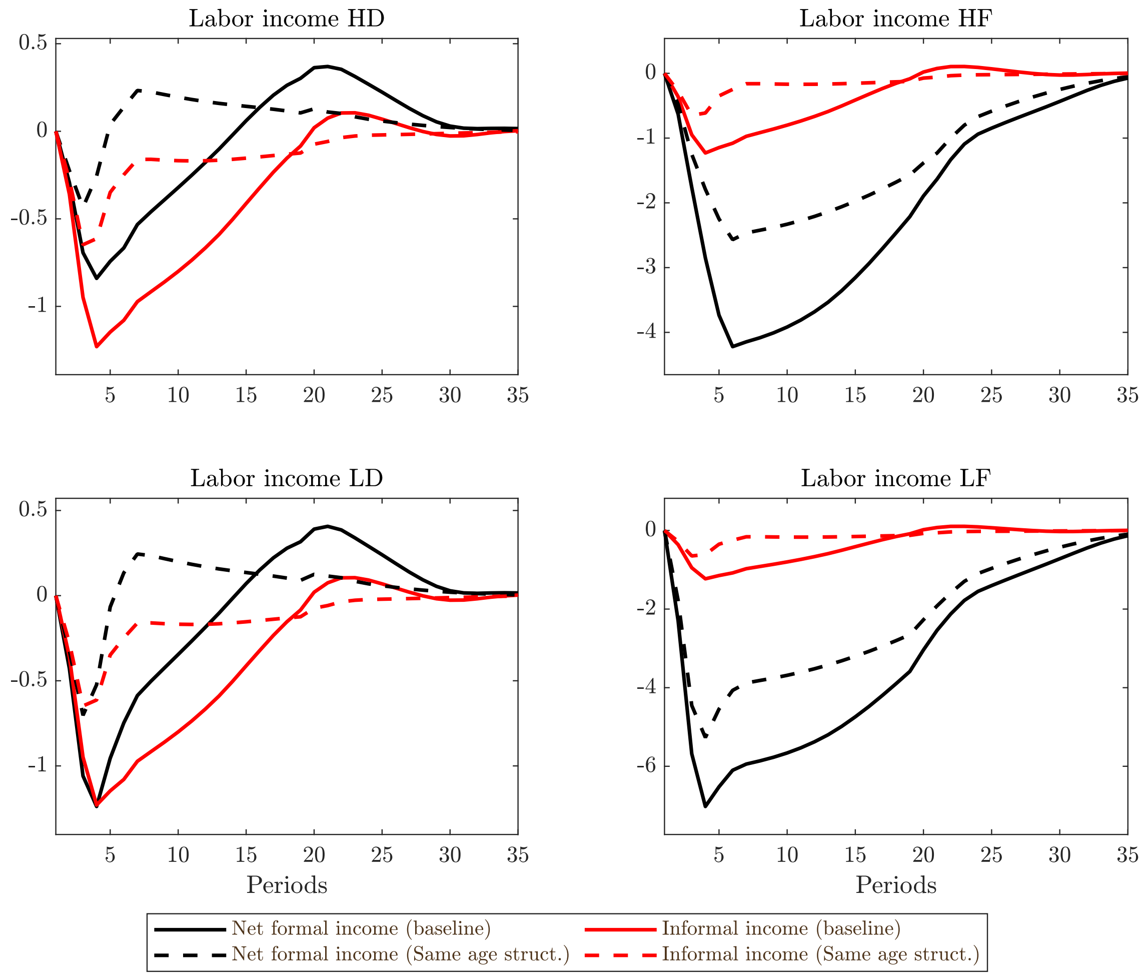
Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which immigrants do not experience a downgrading spell. Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

Figure 24: The role of the age structure: Drivers of formal wages



Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which the age structure of immigrants is identical to that of natives (they still experience a downgrading spell). Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

Figure 25: The role of the age structure: Formal and informal income



Note.— The four types of workers are denoted HD (high-skilled domestic), HF (high-skilled foreign), LD (low-skilled domestic), and LF (low-skilled foreign). Dashed lines show results from a simulation in which the age structure of immigrants is identical to that of natives (they still experience a downgrading spell). Percent deviations from steady state. Individuals live for 30 periods (20 active; 10 retired). Simulations conducted under perfect foresight.

B Model derivation

B.1 Firms

A representative firm demands capital, K_t, and labor, L_t, to produce a homogeneous good used for consumption and investment. The firm's production function is

$$Y_t^F = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha}. \tag{10}$$

with $\alpha \in (0,1)$ and where A_t denotes the labor-augmenting technology level which is assumed to grow at a constant rate z, such that $A_t = A_{t-1}(1+z)$. The labor input L_t is a constant-elasticity-of-substitution

(CES) bundle of labor of various skills:[43]

$$L_t = \left[a_H \left(L_{Ht}\right)^{\varrho_s} + a_L \left(L_{Lt}\right)^{\varrho_s}\right]^{\frac{1}{\varrho_s}}$$
$$L_{it} = \left[a_{iD} \left(L_{iDt}\right)^{\varrho_b} + a_{iF} \left(L_{iFt}\right)^{\varrho_b}\right]^{\frac{1}{\varrho_b}} \quad \text{for} \quad i = \{H, L\},$$

where $L_{i,t}$ denotes the demand for workers from skill group i, the parameters $a_i, a_{ij} > 0$ measure the productivities of the different worker types, and $\varrho_s, \varrho_b < 1$ govern the elasticity of substitution of workers of different skills and origin. As $\varrho_n \to 1$, workers of different skills are perfect substitutes, and as $\varrho_n \to -\infty$ they are perfect complements. Also, $L_{ijt} = \sum_{g=1}^{g_r-1} L_{ijt}^g$ aggregates labor demand across active cohorts.

The firm maximizes profits subject to (10) and the labor composite, treating the rental rate of capital r_t^k and the skill-specific wage rates $W_{i,t}$ as given. Substituting the constraints out, the firm's optimization problem can be written as:

$$\max_{K_t, L_{ijt}} K_t^\alpha A_t^{1-\alpha} L_t^{1-\alpha} - r_t^k K_t - \sum_{i \in \{H,L\}} \sum_{j \in \{D,F\}} \sum_{g=1}^{20} W_{ijt} L_{ijt}^g, \tag{11}$$

yielding the following first-order conditions for capital and labor :

$$K_t : \qquad r_t^k = \alpha \frac{Y_t^F}{K_t}, \tag{12}$$

$$L_{ijt}^g : \qquad W_{ijt}^g = a_i a_{ij} W_t \left(\frac{L_t}{L_{it}}\right)^{1-\varrho_s} \left(\frac{L_{it}}{L_{ijt}}\right)^{1-\varrho_b}, \tag{13}$$

where $W_t = \frac{(1-\alpha)Y_t^F}{L_t}$. These conditions imply that the zero-profit condition

$$Y_t^F = r_t^k K_t + \sum_{i \in \{H,L\}} \sum_{j \in \{D,F\}} \sum_{g=1}^{20} W_{ijt} L_{ijt}^g \tag{14}$$

is satisfied in equilibrium. The law of motion for capital is given by,

$$K_{t+1} = (1-\delta) K_t + I_t^{Net}, \tag{15}$$

where δ is the rate of depreciation and I_t^{Net} denotes net investment.

B.2 Workers of Type *ij*

Utility of a worker of type ij is a function of consumption, $C_{i,j,t}^g$, both when active ($g \leq 20$) and when retired ($g > 20$), and of labor effort (hours) supplied to the formal, $l_{i,j,t}^g$, and informal, $h_{i,j,t}^g$, sectors, when

[43] Throughout, upper-case letters denote variables that are non-stationary due to population growth, productivity growth, or both, whereas lower-case letters denote stationary variables.

active; for $i = \{H, L\}$ and $j = \{D, F\}$. The present value of lifetime utility of a worker of skill i and origin j is then given by:

$$U_{ij,t} = \sum_{g=1}^{30} \beta^{g-1} \frac{\left(C_{ij,t+g-1}^{g}\right)^{1-\theta}}{1-\theta} - \sum_{g=1}^{20} \beta^{g-1} \Theta_{ij,t+g-1}^{g} \left(\frac{\left(l_{ij,t+g-1}^{g} + h_{ij,t+g-1}^{g}\right)^{\phi^l}}{\phi^l} + \frac{\kappa_{ij} \left(h_{ij,t+g-1}^{g}\right)^{\phi^h}}{\phi^h} \right),$$

where $\theta > 0$ is the inverse of the elasticity of intertemporal substitution, ϕ^l, $\phi^h \geq 1$ determines the elasticity of labor supply, $\beta \in (0,1)$ is the subjective discount factor, and $\kappa_{ij} > 0$ is a specific utility cost of informal labor.[44] The variable Θ_{ijt}^{g} is an endogenous preference shifter based on Galí et al. (2012) that is taken as given by the workers and satisfies

$$\Theta_{ijt+g-1}^{g} = \chi_{ij}^{g} A_t^{1-\theta} \left[\left(\frac{C_{ijt+g-1}^{g}}{A_t} \right)^{-\theta} \right]^{\upsilon}$$

with $\upsilon \in \{0, 1\}$ and where $\chi_{ij}^{g} > 0$. The purpose of this preference shifter is to limit the wealth effect on labor supply. When $\upsilon = 0$, we obtain $\Theta_{ijt+g-1}^{g} = \chi_{ij}^{g}$, i.e., the standard constant relative risk aversion (CRRA) utility function implying a positive wealth effect, whereas the wealth effect is cancelled when $\upsilon = 1$. We assume this later case in the calibration.[45]

We model the informal sector as self-employment, which is conducted through a linear production function, with labor as the only input, and with marginal product equal to $A_t \hat{b}_{ij} \frac{h_{pw,t}^{\xi-1}}{\eta}$, where h_t is the total size of informal labor and is used to account for *congestion* effects in informality.[46] This formulation of self-employment productivity allows us to match the size of the informal sector and the extent to which households reallocate labor effort across sectors, as we explain below.[47] At times we will use the following notation,

$$f_{inf}(h_{ij,t}^{g}) = A_t \hat{b}_{ij} \frac{h_{pw,t}^{\xi-1}}{\eta} h_{ij,t}^{g}$$

for $i = \{H, L\}$ and $j = \{D, F\}$. Letting S_{ijt}^{g} denote the stock of savings an individual of cohort g and type ij has in period t, for $i = \{H, L\}$ and $j = \{D, F\}$, a worker's sequence of budget constraints for periods t, $t+1$, ... $t + 30 - 1$ is given by:

$$C_{ij,t}^{1} = (1-\tau) W_{ij,t} l_{i,t}^{1} + A_t \hat{b}_{ij} \left(\frac{h_{pw,t}^{\xi-1}}{\eta} \right) h_{ij,t}^{1} - S_{ij,t}^{1},$$

$$C_{ij,t+1}^{2} = (1-\tau) W_{ij,t+1} l_{ij,t+1}^{2} + A_{t+1} \hat{b}_{ij} \left(\frac{h_{pw,t+1}^{\xi-1}}{\eta} \right) h_{ij,t+1}^{2} + r_{t+1} S_{ij,t}^{1} - S_{ij,t+1}^{2},$$

[44] This specific utility cost of informality captures in reduced-form, for example, the lack of health insurance and other benefits, or the higher job insecurity in the informal sector.

[45] The disutility of work is assumed to grow with the factor $A_t^{1-\theta}$ so that the model has a balanced growth path when $\theta \neq 1$.

[46] $\hat{b}_{ij} = b_{ij} a_i a_{ij}$ for $i = \{H, L\}$ and $j = \{D, F\}$.

[47] Labor is the only input used in self-employment, so it is akin to home production. This assumption is reasonable, because capital in the informal sector of emerging countries is known to represent a very small fraction of the total capital stock. For evidence on capital in the informal sector, see Lopez-Martin (2019); Porta and Shleifer (2008); and Busso et al. (2012). It is, therefore, common to model an informal sector that does not use capital as an input; see

$$\dots$$

$$C^{20}_{ij,t+19} = (1-\tau)W_{ij,t+19} l^{20}_{ij,t+19} + A_{t+19}\hat{b}_{ij}\left(\frac{h^{\xi-1}_{pw,t+19}}{\eta}\right) h^{20}_{ij,t+19} + r_{t+19} S^{19}_{ij,t+18} - S^{20}_{ij,t+19},$$

$$C^{21}_{ij,t+20} = r_{t+20} S^{21}_{ij,t+19} - S^{21}_{ij,t+20},$$

$$\dots$$

$$C^{30}_{ij,t+29} = r_{t+19} S^{29}_{ij,t+28},$$

where $r_t = 1 + r^{Tk}_t - \delta$ denotes the gross return on assets. The period-by-period budget constraints can be combined to obtain an intertemporal budget constraint (IBC):

$$\sum_{g=1}^{30}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1} C^g_{ij,t+g-1} = \sum_{g=1}^{20}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1}\left[(1-\tau)W_{ij,t+g-1} l^g_{ij,t+g-1} + f_{inf}(h^g_{ij,t+g-1})\right],$$

The first generation at time t, then, faces the problem of maximizing utility (**??**) subject to the IBC (B.2), taking $W_{ijt+g-1}$ and r_{t+g-1} as given. This problem can be represented by the following Lagrangian:

$$\begin{aligned}
\mathcal{L}_{ijt} = &\sum_{g=1}^{30}\beta^{g-1}\frac{\left(C^g_{ijt+g-1}\right)^{1-\theta}}{1-\theta} \\
&-\sum_{g=1}^{20}\beta^{g-1}\Theta^g_{ijt+g-1}\left(\frac{\left(l^g_{ij,t+g-1} + h^g_{ij,t+g-1}\right)^{\phi^l}}{\phi^l} + \frac{\kappa_{ij}\left(h^g_{ij,t+g-1}\right)^{\phi^h}}{\phi^h}\right) \\
&+\Psi_{ijt}\left(\sum_{g=1}^{20}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1}\left[(1-\tau)W_{ijt+g-1} l^g_{ijt+g-1} + f_{inf}(h^g_{ijt+g-1})\right]\right. \\
&\left. -\sum_{g=1}^{30}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1} C^g_{ijt+g-1}\right)
\end{aligned}$$

where $\Psi_{i,t}$ denotes the Lagrange multiplier associated with (B.2). Then, for $g = 1, ..., 30$ the first-order conditions are:

$$C^g_{ij,t+g-1}: \quad 0 = \beta^{g-1}\left(C^g_{ij,t+g-1}\right)^{-\theta} - \Psi_{ij,t}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1}. \tag{16}$$

For $g = 1, ..., 20$, i.e., when individuals are in the workforce, we also have their labor supply to both the formal and to the informal labor markets, i.e.,

$$l^g_{ijt+g-1}: \quad 0 = -\beta^{g-1}\Theta^g_{ijt+g-1}\left(l^g_{ijt+g-1} + h^g_{ijt+g-1}\right)^{\phi^l-1} + \Psi_{ijt}(1-\tau)W_{ijt+g-1}\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1}, \tag{17}$$

$$h^g_{ijt+g-1}: \quad 0 = -\beta^{g-1}\Theta^g_{ijt+g-1}\left[\left(l^g_{ijt+g-1} + h^g_{ijt+g-1}\right)^{\phi^l-1} + \kappa_{ij}\left(h^g_{ijt+g-1}\right)^{\phi^h-1}\right] + \Psi_{ijt}A_{t+g-1}b_i\left(\frac{h^{\xi-1}_{pw,t+g-1}}{\eta}\right)\left(\prod_{n_g=1}^{g-1} r_{t+n_g}\right)^{-1}. \tag{18}$$

From (16), we obtain the set of Euler equations for consumption for any two different periods, i.e. for $g_1 < g_2 \leq 30$:

$$\left(C^{g_1}_{ijt+g_1-1}\right)^{-\theta} = \beta^{g_2-g_1}\left(\prod_{n_g=g_1}^{g_2-1} r_{t+n_g}\right)\left(C^{g_2}_{ijt+g_2-1}\right)^{-\theta}. \tag{19}$$

Solving for Ψ_{ijt} in (16) and using the result in (17) yields the intratemporal formal-sector labor supply schedules, $g \leq 20$:

$$\Theta^g_{ijt+g-1}\left(l^g_{ijt+g-1} + h^g_{ijt+g-1}\right)^{\phi-1} = \left(C^g_{ijt+g-1}\right)^{-\theta}(1-\tau)W_{ijt+g-1}. \tag{20}$$

Similarly, from (18) we obtain the informal-sector labor supply schedules, $g \leq 20$:

$$\Theta^g_{ijt+g-1}\left[\left(l^g_{ijt+g-1} + h^g_{ijt+g-1}\right)^{\phi^l-1} + \kappa_{ij}\left(h^g_{ijt+g-1}\right)^{\phi^h-1}\right] = \left(C^g_{ijt+g-1}\right)^{-\theta}A_{t+g-1}b_i\left(\frac{h^{\xi-1}_{pw,t+g-1}}{\eta}\right). \tag{21}$$

To gain intuition, it is useful to combine (20) with (21), and to solve the resulting equation for $h^g_{ijt+g-1}$, which yields:

$$h^g_{ijt+g-1} = \begin{cases} \left[\left(C^g_{ijt+g-1}\right)^{-\theta}\frac{\left[f'_{inf}(h^g_{ijt+g-1})-(1-\tau)W_{ijt+g-1}\right]}{\Theta^g_{ijt+g-1}\kappa_{ij}}\right]^{\frac{1}{\phi-1}} & \text{if } f'_{inf}(h^g_{ijt+g-1}) > (1-\tau)W^g_{ijt+g-1}, \\ 0 & \text{otherwise.} \end{cases}$$

This expression shows that households supply labor to the informal sector as long as the marginal income from this activity exceeds the marginal income that they earn by working in the formal sector, net of taxes. In particular, households are compensated for the additional utility cost that informal work generates. By driving a wedge between the gross and net income from formal work, the tax rate on formal labor income allows the model to reproduce the fact that even though the informal sector is less productive than the formal one, workers have an incentive to work in it.[48]

We can use the intertemporal budget contraint to re-write the consumption equations as follows

[48] For evidence on wage differentials between the formal and informal sectors in Chile, see Joubert (2015).

(noting $S^0_{ijt-1} = 0$):

For $g_3 \leq 20$

$$C^{g_3}_{ijt+g_3-1} = \left(\sum_{g=g_3}^{30} \left(\prod_{n_g=g_3}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta^{g-g_3}\right)^{\frac{1}{\theta}} \right)^{-1} \tag{22}$$

$$\left\{ r_{t+g_3-1} S^{g_3-1}_{ijt+g_3-2} + \sum_{g=g_3}^{20} \left(\prod_{n_g=g_3}^{g-1} r_{t+n_g} \right)^{-1} \left[(1-\tau) W_{ij,t+g-1} l^g_{ijt+g-1} + f_{inf}(h^g_{ijt+g-1}) \right] \right\},$$

and for $g_3 > 20$

$$C^{g_3}_{ijt+g_3-1} = \left(\sum_{g=g_3}^{30} \left(\prod_{n_g=g_3}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta^{g-g_3}\right)^{\frac{1}{\theta}} \right)^{-1} r_{t+g_3-1} S^{g_3-1}_{ijt+g_3-2}. \tag{23}$$

Finally, we define some aggregate variables,

$$Y^d_t = \sum_{i\in\{H,L\}} \sum_{j\in\{D,F\}} \sum_{g=1}^{20} (1-\tau) W_{ijt} L^g_{ijt} + A_t \hat{b}_{ij} \left(\frac{h_{pw,t}}{\eta} \right)^{\xi-1} h^g_{ijt} N^g_{ijt}$$

$$C_t = \sum_{i\in\{H,L\}} \sum_{j\in\{D,F\}} \sum_{g=1}^{30} C^g_{ijt} N^g_{ijt}$$

$$r_t S_{t-1} = r_t \sum_{i\in\{H,L\}} \sum_{j\in\{D,F\}} \sum_{g=1}^{29} S^g_{ijt} N^g_{ijt}$$

$$Y^T_t = Y^F_t + Y^I_t = K^\alpha_t \left(A_t L_t\right)^{1-\alpha} + \sum_{i\in\{H,L\}} \sum_{j\in\{D,F\}} \sum_{g=1}^{20} A_t \hat{b}_{ij} \left(\frac{h_{pw,t}}{\eta} \right)^{\xi-1} h^g_{ijt} N^g_{ijt}$$

Since we introduce a tax, τ , to obtain a wedge between net and gross income that allows us to model an incentive for agents to work in informality despite their lower productivity, we need to determine what it is done with those taxes. We assume the government finances running costs and that it runs a balance budget, i.e. $G_t = T_t$, where

$$T_t = \tau \sum_{i\in\{H,L\}} \sum_{j\in\{D,F\}} \sum_{g=1}^{20} W^g_{ijt} L^g_{ijt}$$

We have a market clearing condition

$$Y^T_t = C_t + I^{Net}_t + G_t$$

where

$$I^{Net}_t = \Delta K_{t+1} + \delta K_t$$

B.3 Labor Market Equilibrium

Equilibrium in the labor market requires that demand for each skill group equal supply. Recall the firm's labor input is a CES bundle composed of labor of each skill, $L_{i,t}$. The labor input from skill group i at time t includes individuals of all active cohorts from that skill group:

$$L_{ijt} = \sum_{g=1}^{20} L_{ijt}^{g}, \tag{24}$$

where L_{ijt}^{g} is given by

$$L_{ijt}^{g} = l_{ijt}^{g} N_{ij,t}^{g}, \tag{25}$$

i.e., hours worked by workers of type ij - for $i = \{H, L\}$ and $j = \{D, F\}$ - and cohort g (l_{ijt}^{g}), multiplied by the number of workers (N_{ijt}^{g}).

To conclude note that the model is non-stationary due to technological progress and population growth. Next we present the complete set of equilibrium conditions, as well as the detrended version of the model, and the derivation of the steady state.

B.4 Equilibrium Conditions

$$\begin{aligned}
\Big\{ & C_{HDt}^{1}, C_{HFt}^{1}, C_{LDt}^{1}, C_{LFt}^{1}, \ldots, C_{HDt}^{30}, C_{HFt}^{30}, C_{LDt}^{30}, C_{LFt}^{30}, \\
& S_{HDt}^{1}, S_{HFt}^{1}, S_{LDt}^{1}, S_{LFt}^{1}, \ldots, S_{HDt}^{29}, S_{HFt}^{29}, S_{LDt}^{29}, S_{LFt}^{29}, \\
& l_{HDt}^{1}, l_{HFt}^{1}, l_{LDt}^{1}, l_{LFt}^{1}, \ldots, l_{HDt}^{20}, l_{HFt}^{20}, l_{LDt}^{20}, l_{LFt}^{20} \\
& h_{HDt}^{1}, h_{HFt}^{1}, h_{LDt}^{1}, h_{LFt}^{1}, \ldots, h_{HDt}^{20}, h_{HFt}^{20}, h_{LDt}^{20}, h_{LFt}^{20} \\
& L_{HDt}^{1}, L_{HFt}^{1}, L_{LDt}^{1}, L_{LFt}^{1}, \ldots, L_{HDt}^{20}, L_{HFt}^{20}, L_{LDt}^{20}, L_{LFt}^{20}, \\
& L_{HDt}, L_{HFt}, L_{LDt}, L_{LFt}, L_{L,t}, L_{H,t}, W_{HDt}, W_{HFt}, W_{LDt}, W_{LFt}, N_{HDt}, \\
& N_{HFt}, N_{LDt}, N_{LFt}, N_{Dt}, N_{Ft}, N_{Dt}^{1}, N_{Ft}^{1}, ..., N_{Dt}^{30}, N_{Ft}^{30}, N_{t}^{1}, ..., N_{t}^{30}, \\
& N_{HDt}^{1}, N_{HFt}^{1}, N_{LDt}^{1}, N_{LFt}^{1}, \ldots, N_{HDt}^{30}, N_{HFt}^{30}, N_{LDt}^{30}, N_{LFt}^{30}, \\
& C_{t}^{1}, \ldots, C_{t}^{30}, S_{t}^{1}, \ldots, S_{t}^{29}, r_t, r_t^{k}, Y_t^{T}, Y_t^{F}, Y_t^{I}, \\
& C_t, S_t, L_t, H_t, W_t, K_{t+1}, I_t, I_t^{Net}, T_t, G_t, N_t, A_t \Big\}_{t=0}^{\infty},
\end{aligned}$$

such that for given initial values, the following conditions are satisfied for every t. For $0 < g \leq 20$, defining $S_{ij,t}^{0} = 0$, for $i \in \{H, L\}$ and $j \in \{D, F\}$,

$$C_{ijt}^{g} = (1-\tau) W_{ijt}^{g} l_{ijt}^{g} + A_t \hat{b}_{ij} \frac{h_{pw,t}^{\xi-1}}{\eta} h_{ij,t}^{g} + r_t S_{ij,t-1}^{g-1} - S_{ij,t}^{g},$$

for $20 < g < 30$ and $i \in \{H, L\}$ and $j \in \{D, F\}$:

$$C_{ij,t}^{g} = r_t S_{ij,t-1}^{g-1} - S_{ij,t}^{g},$$

and

$$C_{ij,t}^{30} = r_t S_{ij,t-1}^{29}.$$

For $0 < gg \leq 20$:

$$C_{ijt+gg-1}^{gg} = \left(\sum_{g=gg}^{30} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta^{g-gg}\right)^{\frac{1}{\theta}} \right)^{-1} \Bigg[r_{t+gg-1} S_{ijt+gg-2}^{gg-1} +$$

$$\sum_{g=gg}^{20} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{-1} \left[(1-\tau) W_{ijt+g-1}^{g} l_{ijt+g-1}^{g} + A_{t+g-1} \hat{b}_{ij} \frac{h_{pw,t+g-1}^{\xi-1}}{\eta} h_{ijt+g-1}^{g} \right] \Bigg],$$

for $20 < gg$, $i \in \{H, L\}$ and $j \in \{D, F\}$:

$$C_{ij,t+gg-1}^{gg} = \left(\sum_{g=gg}^{30} \left(\prod_{n_g=gg}^{g-1} r_{t+n_g} \right)^{\frac{1-\theta}{\theta}} \left(\beta^{g-gg}\right)^{\frac{1}{\theta}} \right)^{-1} r_{t+gg-1} S_{ij,t+gg-2}^{gg-1},$$

for $g = 1, ..., 30$:

$$C_t^g = \sum_{i \in \{H,L\}} \sum_{j \in \{D,F\}} C_{ij,t}^g N_{ij,t}^g,$$

for $0 < g \leq 20$ we have:

$$\chi_{ij}^g A_{t+1-g} \left(l_{ijt}^g + h_{ijt}^g \right)^{\phi^l - 1} = \left(\frac{C_{ijt}^g}{A_{t+1-g}} \right)^{-\theta(1-\upsilon)} (1-\tau) W_{ijt}^g$$

$$h_{ijt}^g = \max \left\{ \left[\left(C_{ijt}^g \right)^{-\theta(1-\upsilon)} \frac{\left(A_t \hat{b}_{ij} \frac{h_{pw,t}^{\xi-1}}{\eta} - (1-\tau) W_{ijt}^g \right)}{\chi_{ij}^g A_{t+1-g}^{1-\theta(1-\upsilon)} \kappa_{ij}} \right]^{\frac{1}{\phi^h - 1}}, 0 \right\}$$

$$W_{ijt} = W_{ijt}^g = a_i a_{ij} W_t \left(\frac{L_t}{L_{it}} \right)^{1-\varrho_s} \left(\frac{L_{it}}{L_{ijt}} \right)^{1-\varrho_b}$$

for $0 < g \leq 20$, $i \in \{H, L\}$ and $j \in \{D, F\}$ we have:

$$L_{ijt}^g = l_{ijt}^g N_{ijt}^g$$

for $i \in \{H, L\}$ and $j \in \{D, F\}$,

$$L_{ijt} = \sum_{g=1}^{20} L_{ijt}^g$$

$$H_{ijt} = \sum_{g=1}^{20} h_{ijt}^g N_{ijt}^g$$

and for $i \in \{H, L\}$,

$$L_{it} = \sum_{j \in \{F,J\}} L_{ijt}^g,$$

The population satisfies,

$$N_{Dt}^1 = (1 + n_t^r) N_{t-1}^1$$

$$N_{Ft}^{1} = n_t^f N_{t-1}^{1}$$

for $i < g \leq 30$

$$N_{Dt}^{g} = N_{Dt-1}^{g-1}$$

$$N_{Ft}^{g} = N_{Ft-1}^{g-1} + f_t^g N_{t-1}^{g-1}$$

Then,

$$N_{Dt} = \sum_{g=1}^{m} N_{Dt}^{g} = \sum_{g=1}^{m} N_{Dt-(g-1)}^{1} = \sum_{g=1}^{m} \left(1 + n_{t-g+1}^{r}\right) N_{t-g}^{1}$$

$$N_{Ft} = \sum_{g=1}^{m} N_{Ft}^{g} = N_t - N_{Dt}$$

for $g \leq 30$,

$$N_t^g = N_{Dt}^{g} + N_{Ft}^{g}$$

and high- and low-skilled individuals are determined by,

$$N_{HDt}^{g} = \lambda_{HD} N_{Dt}^{g}$$

$$N_{LDt}^{g} = (1 - \lambda_{HD}) N_{Dt}^{g}$$

$$N_{HFt}^{g} = \lambda_{HFt}^{g} N_{Ft}^{g}$$

$$N_{LFt}^{g} = (1 - \lambda_{HFt}^{g}) N_{Ft}^{g}$$

$$Y_t^F = K_t^{\alpha} (A_t L_t)^{1-\alpha},$$

$$L_t = \left[a_H (L_{Ht})^{\varrho_s} + a_L (L_{Lt})^{\varrho_s}\right]^{\frac{1}{\varrho_s}}$$

and for $i = \{H, L\}$,

$$L_{it} = \left[a_{iD} (L_{iDt})^{\varrho_b} + a_{iF} (L_{iFt})^{\varrho_b}\right]^{\frac{1}{\varrho_b}}$$

$$W_t = \frac{(1-\alpha) Y_t^F}{L_t},$$

$$r_t^k = \alpha \frac{Y_t^F}{K_t},$$

$$r_t = 1 + r_t^k - \delta$$

$$Y_t^I = \sum_{j \in \{D,F\}} \sum_{g=1}^{20} A_t \hat{b}_{ij} \frac{h_{pw,t}^{\xi-1}}{\eta} h_{ijt}^{g} N_{ijt}^{g},$$

$$Y_t^T = Y_t^F + Y_t^I,$$

for $0 < g \leq 29$:

$$S_t^g = \sum_{i \in \{H,L\}} \sum_{j \in \{D,F\}} S_{ijt}^g N_{ijt}^g$$

and

$$S_t = \sum_{g=1}^{29} S_t^g,$$

$$T_t = \tau \sum_{i \in \{H,L\}} \sum_{j \in \{D,F\}} \sum_{g=1}^{20} iW_{ijt}^g L_{Hjit}^g ii,$$

$$G_t = T_t,$$

$$C_t = \sum_{g=1}^{30} C_t^g,$$

$$Y_t^T = C_t + I_t^{Net} + G_t$$

$$I_t^{Net} = \Delta K_{t+1} + \delta K_t,$$

$$K_{t+1} = I_t,$$

$$A_t = A_{t-1}(1+z)$$

$$N_t^1 = N_{t-1}^1(1 + n^r + n_t^f),$$

$$N_t^g = N_{t-1}^{g-1}(1 + f_t^g), \quad g \in [2, 30]$$

Some complementary variables:

$$H_t = \sum_{i \in \{H,L\}} \sum_{j \in \{D,F\}} \sum_{g=1}^{20} h_{ijt}^g N_{ijt}^g$$

$$L_t^{total} = \sum_{g=1}^{20} \sum_{i \in \{H,L\}} \sum_{j \in \{D,F\}} l_{ij,t}^g N_{ij,t}^g.$$

B.5 Detranded Equilibrium Conditions

To re-write the model in stationary form , we define the stationary per-capita cariables: $c_{ij,t}^g = C_{ij,t}^g / A_{t+1-g}$, $s_{ij,t}^g = S_{ij,t}^g / A_{t+1-g}$, $w_{ij,t} = W_{ij,t}/A_t$, $l_{ij,t}^g = L_{ij,t}^g / N_{ij,t}^g$, $c_t = C_t / (A_t N_t^1)$, $c_t^g = C_t^g / (A_{t+1-g} N_t^g)$, $s_t = S_t / (A_t N_t^1)$, $s_t^g = S_t^g / (A_{t+1-g} N_t^g)$, $l_{ij,t} = L_{ij,t}/N_t^1$, $h_{ij,t} = H_{ij,t}/N_t^1$, $l_{i,t} = L_{i,t}/N_t^1$, $l_{i,t}^{total} = L_{i,t}^{total}/N_t^1$, $t_t = T_t / (A_t N_t^1)$, $g_t = G_t / (A_t N_t^1)$, $y_t^T = Y_t^T / (A_t N_t^1)$, $y_t^F = Y_t^F / (A_t N_t^1)$, $y_t^I = Y_t^I / (A_t N_t^1)$, $l_t = L_t / N_t^1$, $l_t^{total} = L_t^{total}/N_t^1$, $h_t = H_t/N_t^1$, $w_t = W_t/A_t$, $k_t = K_t / (A_{t-1} N_{t-1}^1)$, $i_t = I_t / (A_t N_t^1)$. The stationary competitive equilibrium of the model is then given by the set of sequences

$$\begin{aligned}
&\Big\{c^1_{HD,t}, c^1_{HF,t}, c^1_{LD,t}, c^1_{LF,t}, ..., c^m_{HD,t}, c^m_{HF,t}, c^m_{LD,t}, c^m_{LF,t},\\
&s^1_{HD,t}, s^1_{HF,t}, s^1_{LD,t}, s^1_{LF,t}, ..., s^{m-1}_{HD,t}, s^{m-1}_{HF,t}, s^{m-1}_{LD,t}, s^{m-1}_{LF,t},\\
&l^1_{HD,t}, l^1_{HF,t}, l^1_{LD,t}, l^1_{LF,t}, ..., l^{g_r-1}_{HD,t}, l^{g_r-1}_{HF,t}, l^{g_r-1}_{LD,t}, l^{g_r-1}_{LF,t},\\
&h^1_{LD,t}, h^1_{LF,t}, ..., h^{g_r-1}_{LD,t}, h^{g_r-1}_{LF,t}, \lambda_{HF,t}, n^r_t, n^f_t, f^2_t, ..., f^m_t,\\
&l_{HD,t}, l_{HF,t}, l_{LD,t}, l_{LF,t}, l_{L,t}, l_{H,t}, w_{HD,t}, w_{HF,t}, w_{LD,t}, w_{LF,t},\\
&h_{HD,t}, h_{HF,t}, h_{LD,t}, h_{LF,t}, s_{HD,t}, s_{HF,t}, s_{LD,t}, s_{LF,t},\\
&l^{total}_{L,t}, l^{total}_{H,t}, c^1_t, ..., c^m_t, s^1_t, ..., s^{m-1}_t, c_t, s_t, h^{total}_t, l_t, l^{total}_t, w_t,\\
&r_t, r^\star_t, r^k_t, y^T_t, y^F_t, y^I_t, ca_t, f_t, k_{t+1}, i_t, t_t, g_t\Big\}^\infty_{t=0}
\end{aligned}$$

such that for given initial values, the following conditions are satisfied for every t:[49]
for $1 \leq g \leq 20$ and $s^0_{ijt} = 0$

$$c^g_{ijt} = (1-\tau)(1+z)^{g-1} w_{ijt} l^g_{ijt} + (1+z)^{g-1} \hat{b}_{ij} \frac{h^{\xi-1}_{pw,t}}{\eta} h^g_{ijt} + r_t s^{g-1}_{ij,t-1} - s^g_{ijt} \tag{26}$$

for $20 < g < 30$

$$c^g_{ijt} = r_t s^{g-1}_{ij,t-1} - s^g_{ijt}, \tag{27}$$

and

$$c^{30}_{ijt} = r_t s^{29}_{ij,t-1}, \tag{28}$$

for $0 < gg \leq 20$, $i \in \{H, L\}$ and $j \in \{D, F\}$:

$$c^{gg}_{ijt} = \left(\sum_{g=gg}^{30} \left(\prod_{n_g=1}^{g-gg} r_{t+n_g}\right)^{\frac{1-\theta}{\theta}} \beta^{\frac{g-gg}{\theta}}\right)^{-1} \left(r_t s^{gg-1}_{ij,t-1} + \sum_{g=gg}^{20} \left(\prod_{n_g=1}^{g-gg} r_{t+n_g}\right)^{-1}\right. \tag{29}$$

$$\left.(1+z)^{g-1}\left[(1-\tau) w_{ij,t+g-gg} l^g_{ij,t+g-gg} + \hat{b}_{ij} \frac{h^{\xi-1}_{pw,t+g-gg}}{\eta} h^g_{ij,t+g-gg}\right]\right) \tag{30}$$

if $20 < gg$:

$$c^{gg}_{ijt} = \left(\sum_{g=gg}^{30} \left(\prod_{n_g=1}^{g-gg} r_{t+n_g}\right)^{\frac{1-\theta}{\theta}} \left(\beta^{g-gg}\right)^{\frac{1}{\theta}}\right)^{-1} r_t s^{gg-1}_{ij,t-1}, \tag{31}$$

[49]To cast this system into Dynare syntax for predetermined variables, k_{t+1} and k_t need to be written as k and $k(-1)$ respectively.

$$c_t^1 = \frac{\left(1+n_t^r\right)}{\left(1+n_t^r+n_t^f\right)}\left[\left(1-\lambda_{HD}\right)c_{LDt}^1+\lambda_{HD}c_{HDt}^1\right] \tag{32}$$

$$+\frac{n_t^f}{\left(1+n_t^r+n_t^f\right)}\left[\left(1-\lambda_{HFt}^1\right)c_{LFt}^1+\lambda_{HFt}^1 c_{HFt}^1\right] \tag{33}$$

for $g>1$

$$c_t^g = \frac{\left(1+n_{t-g+1}^r\right)}{\prod_{s=0}^{g-2}\left(1+f_{t-s}^{g-s}\right)\left(1+n_{t-g+1}^r+n_{t-g+1}^f\right)}\left[\left(1-\lambda_{HD}\right)c_{LDt}^g+\lambda_{HD}c_{HDt}^g\right] \tag{34}$$

$$+\left[\frac{n_{t-g+1}^f}{\prod_{s=0}^{g-2}\left(1+f_{t-s}^{g-s}\right)\left(1+n_{t-g+1}^r+n_{t-g+1}^f\right)}+\sum_{s=1}^{g-1}f_{t-s+1}^{g-s+1}\prod_{u=0}^{s-1}\left(1+f_{t-u}^{g-u}\right)^{-1}\right] \tag{35}$$

$$\left[\left(1-\lambda_{HFt}^g\right)c_{LFt}^g+\lambda_{HFt}^g c_{HFt}^g\right] \tag{36}$$

also

$$c_t = \sum_{g=1}^{30} c_t^g\left(1+z\right)^{1-g}\prod_{s=0}^{g-2}\frac{\left(1+f_{t-s}^{g-s}\right)}{\left(1+n_{t-s}^r+n_{t-s}^f\right)} \tag{37}$$

for $0<g\leq 20$, $i\in\{H,L\}$ and $j\in\{D,F\}$:

$$w_{ijt}^g = a_i a_{ij} w_t\left(\frac{l_t}{l_{it}}\right)^{1-\varrho_s}\left(\frac{l_{it}}{l_{ijt}}\right)^{1-\varrho_b}, \tag{38}$$

Aggregate labor - formal and informal - across cohorts for each agent type yields,

$$l_{LDt} = \sum_{g=1}^{20} l_{LDt}^g\frac{\left(1-\lambda_{HD}\right)\left(1+n_{t-g+1}^r\right)}{\prod_{s=0}^{g-1}\left(1+n_{t-s}^r+n_{t-s}^f\right)} \tag{39}$$

$$\begin{aligned} l_{LFt} &= l_{LF,t}^1\frac{\left(1-\lambda_{HFt}^1\right)n_t^f}{1+n_t^r+n_t^f}+l_{LF,t}^2\left(1-\lambda_{HFt}^2\right)\left[\frac{n_{t-1}^f}{\prod_{s=0}^{1}\left(1+n_{t-s}^r+n_{t-s}^f\right)}+\frac{f_t^2}{1+n_t^r+n_t^f}\right] \\ &+\sum_{g=3}^{20} l_{LF,t}^g\left(1-\lambda_{HFt}^g\right)\left[\frac{n_{t-g+1}^f}{\prod_{s=0}^{g-1}\left(1+n_{t-s}^r+n_{t-s}^f\right)}+\sum_{s=1}^{g-1}f_{t-s+1}^{g-s+1}\frac{\prod_{u=s}^{g-2}\left(1+f_{t-u}^{g-u}\right)}{\prod_{u=0}^{g-2}\left(1+n_{t-u}^r+n_{t-u}^f\right)}\right] \end{aligned} \tag{40}$$

$$l_{HDt} = \sum_{g=1}^{20} l_{HD,t}^g\frac{\lambda_{HD}\left(1+n_{t-g+1}^r\right)}{\prod_{s=0}^{g-1}\left(1+n_{t-s}^r+n_{t-s}^f\right)} \tag{41}$$

$$l_{HFt}=l^1_{HF,t}\frac{\lambda^1_{HFt}n^f_t}{1+n^r_t+n^f_t}+l^2_{HF,t}\lambda^2_{HFt}\left[\frac{n^f_{t-1}}{\prod_{s=0}^{1}\left(1+n^r_{t-s}+n^f_{t-s}\right)}+\frac{f^2_t}{1+n^r_t+n^f_t}\right] \tag{42}$$
$$+\sum_{g=3}^{20}l^g_{HF,t}\lambda^g_{HFt}\left[\frac{n^f_{t-g+1}}{\prod_{s=0}^{g-1}\left(1+n^r_{t-s}+n^f_{t-s}\right)}+\sum_{s=1}^{g-1}f^{g-s+1}_{t-s+1}\frac{\prod_{u=s}^{g-2}\left(1+f^{g-u}_{t-u}\right)}{\prod_{u=0}^{g-2}\left(1+n^r_{t-u}+n^f_{t-u}\right)}\right]$$

$$h_{LDt}=\sum_{g=1}^{20}h^g_{LD,t}\frac{(1-\lambda_{HD})\left(1+n^r_{t-g+1}\right)}{\prod_{s=0}^{g-1}\left(1+n^r_{t-s}+n^f_{t-s}\right)} \tag{43}$$

$$h_{LFt}=h^1_{LF,t}\frac{(1-\lambda^1_{HFt})\,n^f_t}{1+n^r_t+n^f_t}+h^2_{LF,t}\left(1-\lambda^2_{HFt}\right)\left[\frac{n^f_{t-1}}{\prod_{s=0}^{1}\left(1+n^r_{t-s}+n^f_{t-s}\right)}+\frac{f^2_t}{1+n^r_t+n^f_t}\right] \tag{44}$$
$$+\sum_{g=3}^{20}h^g_{LF,t}\left(1-\lambda^g_{HFt}\right)\left[\frac{n^f_{t-g+1}}{\prod_{s=0}^{g-1}\left(1+n^r_{t-s}+n^f_{t-s}\right)}+\sum_{s=1}^{g-1}f^{g-s+1}_{t-s+1}\frac{\prod_{u=s}^{g-2}\left(1+f^{g-u}_{t-u}\right)}{\prod_{u=0}^{g-2}\left(1+n^r_{t-u}+n^f_{t-u}\right)}\right]$$

$$h_{HDt}=\sum_{g=1}^{20}h^g_{HD,t}\frac{\lambda_{HD}\left(1+n^r_{t-g+1}\right)}{\prod_{s=0}^{g-1}\left(1+n^r_{t-s}+n^f_{t-s}\right)} \tag{45}$$

$$h_{HFt}=h^1_{HF,t}\frac{\lambda^1_{HFt}n^f_t}{1+n^r_t+n^f_t}+h^2_{HF,t}\lambda^2_{HFt}\left[\frac{n^f_{t-1}}{\prod_{s=0}^{1}\left(1+n^r_{t-s}+n^f_{t-s}\right)}+\frac{f^2_t}{1+n^r_t+n^f_t}\right] \tag{46}$$
$$+\sum_{g=3}^{20}h^g_{HF,t}\lambda^g_{HFt}\left[\frac{n^f_{t-g+1}}{\prod_{s=0}^{g-1}\left(1+n^r_{t-s}+n^f_{t-s}\right)}+\sum_{s=1}^{g-1}f^{g-s+1}_{t-s+1}\frac{\prod_{u=s}^{g-2}\left(1+f^{g-u}_{t-u}\right)}{\prod_{u=0}^{g-2}\left(1+n^r_{t-u}+n^f_{t-u}\right)}\right]$$

for $i\in\{H,L\}$:

$$l^{total}_{i,t}=\sum_{j\in\{D,F\}}l_{ij,t} \tag{47}$$

$$h_t=h^{total}_{H,t}+h^{total}_{L,t} \tag{48}$$

$$l_t=\left[a_H\left(l_{Ht}\right)^{\varrho_s}+a_L\left(l_{Lt}\right)^{\varrho_s}\right]^{\frac{1}{\varrho_s}} \tag{49}$$

for $i\in\{H,L\}$:

$$l_{it}=\left[a_{iD}\left(l_{iDt}\right)^{\varrho_b}+a_{iF}\left(l_{iFt}\right)^{\varrho_b}\right]^{\frac{1}{\varrho_b}} \tag{50}$$

for $i\in\{H,L\}$ and $j\in\{D,F\}$:

$$\chi_{ij}\left(l^g_{ijt}+h^g_{ijt}\right)^{\phi^l-1}=\left(c^g_{ijt}\right)^{-\theta(1-\upsilon)}\left(1-\tau\right)w_{ijt}\left(1+z\right)^{g-1}, \tag{51}$$

$$h_{ijit}^{g}=\begin{cases}\left[\left(c_{ijt}^{g}\right)^{-\theta(1-\upsilon)}\dfrac{\left(\hat{b}_{Lj}\frac{h_{pw,t}^{\xi-1}}{\eta}-(1-\tau)w_{ijt}\right)}{(1+z)^{1-g}\chi_{ij}\kappa_{ij}}\right]^{\frac{1}{\phi^{h}-1}} & if\quad \hat{b}_{ij}\frac{h_{pw,t}^{\xi-1}}{\eta}>(1-\tau)\,w_{ijt}\\ 0 & otherwise\end{cases}\tag{52}$$

$$y_t^F=\left(\frac{k_t}{(1+z)\left(1+n_t^r+n_t^f\right)}\right)^{\alpha}(l_t)^{1-\alpha}\tag{53}$$

$$w_t=\frac{(1-\alpha)\,y_t^F}{l_t}\tag{54}$$

$$r_t^k=\alpha\frac{y_t^F}{k_t}(1+z)\left(1+n_t^r+n_t^f\right)\tag{55}$$

$$r_t=1+r_t^k-\delta\tag{56}$$

$$\begin{aligned}y_t^I=&\frac{h_{pw,t}^{\xi-1}}{\eta}\sum_{g=1}^{g_r-1}\frac{\left(1+n_{t-g+1}^r\right)}{\prod_{s=0}^{g-1}\left(1+n_{t-s}^r+n_{t-s}^f\right)}\left[\hat{b}_{LD}h_{LD,t}^{g}\left(1-\lambda_{HD}\right)+\hat{b}_{HD}h_{HD,t}^{g}\lambda_{HD}\right]\\&+\frac{h_{pw,t}^{\xi-1}}{\eta}\left[\hat{b}_{LF}h_{LF,t}^{1}\left(1-\lambda_{HF,t}^{1}\right)+\hat{b}_{HF}h_{HF,t}^{1}\lambda_{HFt}^{1}\right]\frac{n_t^f}{1+n_t^r+n_t^f}\\&+\frac{h_{pw,t}^{\xi-1}}{\eta}\left[\hat{b}_{LF}h_{LF,t}^{2}\left(1-\lambda_{HF,t}^{2}\right)+\hat{b}_{HF}h_{HF,t}^{2}\lambda_{HFt}^{2}\right]\left[\frac{n_{t-1}^f}{\prod_{s=0}^{1}\left(1+n_{t-s}^r+n_{t-s}^f\right)}+\frac{f_t^2}{1+n_t^r+n_t^f}\right]\\&+\frac{h_{pw,t}^{\xi-1}}{\eta}\sum_{g=3}^{g_r-1}\left[\hat{b}_{LF}h_{LF,t}^{g}\left(1-\lambda_{HF,t}^{g}\right)+\hat{b}_{HF}h_{HF,t}^{g}\lambda_{HFt}^{g}\right]\\&\left[\frac{n_{t-g+1}^f}{\prod_{s=0}^{g-1}\left(1+n_{t-s}^r+n_{t-s}^f\right)}+\sum_{s=1}^{g-1}f_{t-s+1}^{g-s+1}\frac{\prod_{u=s}^{g-2}\left(1+f_{t-u}^{g-u}\right)}{\prod_{u=0}^{g-2}\left(1+n_{t-u}^r+n_{t-u}^f\right)}\right]\end{aligned}\tag{57}$$

$$y_t^T=y_t^F+y_t^I\tag{58}$$

$$\begin{aligned}s_t^1=&\frac{(1+n_t^r)}{\left(1+n_t^r+n_t^f\right)}\left[(1-\lambda_{HD})\,s_{LDt}^1+\lambda_{HD}s_{HDt}^1\right]\\&+\frac{n_t^f}{\left(1+n_t^r+n_t^f\right)}\left[\left(1-\lambda_{HFt}^1\right)s_{LFt}^1+\lambda_{HFt}^1 s_{HFt}^1\right]\end{aligned}\tag{59}$$

for $1 < g < 29$

$$s_t^g = \frac{\left(1 + n_{t-g+1}^r\right)}{\left(1 + n_{t-g+1}^r + n_{t-g+1}^f\right) \prod_{s=0}^{g-2} \left(1 + f_{t-s}^{g-s}\right)} \left[\left(1 - \lambda_{HD}\right) s_{LDt}^g + \lambda_{HD} s_{HDt}^g\right] \tag{60}$$

$$+ \left[\frac{n_{t-g+1}^f}{\left(1 + n_{t-g+1}^r + n_{t-g+1}^f\right) \prod_{s=0}^{g-2} \left(1 + f_{t-s}^{g-s}\right)} + \sum_{s=1}^{g-1} f_{t-s+1}^{g-s+1} \prod_{u=0}^{s-1} \left(1 + f_{t-u}^{g-u}\right)^{-1}\right]$$
$$\left[\left(1 - \lambda_{HFt}^g\right) s_{LFt}^g + \lambda_{HFt}^g s_{HFt}^g\right]$$

$$s_t = \sum_{g=1}^{m-1} s_t^g \left(1 + z\right)^{1-g} \prod_{s=0}^{g-2} \frac{\left(1 + f_{t-s}^{g-s}\right)}{\left(1 + n_{t-s}^r + n_{t-s}^f\right)} \tag{61}$$

$$g_t = t_t \tag{62}$$

$$t_t = \tau \sum_{g=1}^{g_r-1} \frac{\left(1 + n_{t-g+1}^r\right)}{\prod_{s=0}^{g-1} \left(1 + n_{t-s}^r + n_{t-s}^f\right)} \left[\left(1 - \lambda_{HD}\right) w_{LD,t} l_{LD,t}^g + \lambda_{HD} w_{HD,t} l_{HD,t}^g\right] \tag{63}$$

$$+ \tau \left[\left(1 - \lambda_{HFt}^1\right) w_{LF,t} l_{LF,t}^1 + \lambda_{HFt}^1 w_{HF,t} l_{HF,t}^1\right] \frac{n_t^f}{1 + n_t^r + n_t^f}$$

$$+ \tau \left[\left(1 - \lambda_{HFt}^2\right) w_{LF,t} l_{LF,t}^2 + \lambda_{HFt}^2 w_{HF,t} l_{HF,t}^2\right] \left[\frac{n_{t-1}^f}{\prod_{s=0}^{1} \left(1 + n_{t-s}^r + n_{t-s}^f\right)} + \frac{f_t^2}{1 + n_t^r + n_t^f}\right]$$

$$+ \tau \sum_{g=3}^{g_r-1} \left[\frac{n_{t-g+1}^f}{\prod_{s=0}^{g-1} \left(1 + n_{t-s}^r + n_{t-s}^f\right)} + \sum_{s=1}^{g-1} f_{t-s+1}^{g-s+1} \frac{\prod_{u=s}^{g-2} \left(1 + f_{t-u}^{g-u}\right)}{\prod_{j=0}^{g-2} \left(1 + n_{t-j}^r + n_{t-j}^f\right)}\right]$$
$$\left[\left(1 - \lambda_{HFt}^g\right) w_{LF,t} l_{LF,t}^g + \lambda_{HFt}^g w_{HF,t} l_{HF,t}^g\right]$$

$$i_t^{Net} = k_{t+1} - \frac{\left(1 - \delta\right) k_t}{\left(1 + z\right) \left(1 + n_t^r + n_t^f\right)} \tag{64}$$

$$y_t^T = c_t + i_t^{Net} + g_t \tag{65}$$

$$l_t^{total} = l_{H,t}^{total} + l_{L,t}^{total} \tag{66}$$

for $x = T, \ , F, \ , I$, output per capita is

$$y_t^{x,pc} = \frac{y_t^x}{1 + \sum_{g=2}^{30} \prod_{s=0}^{g-2} \frac{\left(1 + f_{t-s}^{g-s}\right)}{\left(1 + n_{t-s}^r + n_{t-s}^f\right)}} \tag{67}$$

and output per active capita,

$$y_t^{x,pc} = \frac{y_t^x}{1 + \sum_{g=2}^{20} \prod_{s=0}^{g-2} \frac{\left(1+f_{t-s}^{g-s}\right)}{\left(1+n_{t-s}^r+n_{t-s}^f\right)}} \tag{68}$$

C Steady State

C.1 Homptopy

We solve for the steady state of our model by means of the homotopy approach available in Dynare. This strategy allows for the computation of the steady state of a complex nonlinear model by departing from a more simple one for which a solution can be - more easily - found. The sequential approach progressively transforms the more simple model into the actual model of interest and, at each step, computes the steady state of the increasingly more complex model by searching for a solution close to the one found in the previous step.

This is usually done by setting certain parameters to particular values that render the model more simple. In other words we depart from a special calibration that allows us to compute the steady state and the approach moves the parameter values to our desired calibration. The following table shows the parameters that enter our homotopy approach together with their initial and final values,

Parameter	Values in simplified model		Values in calibrated model
ϕ_h	5	$\longrightarrow$	3.00
β	0.9	$\longrightarrow$	0.82
b_{HD}	1.11$*$	$\longrightarrow$	0.67
b_{HF}	1.11$*$	$\longrightarrow$	0.82
b_{LD}	1.11$*$	$\longrightarrow$	0.62
b_{LF}	1.11$*$	$\longrightarrow$	0.75
κ_{HD}	1	$\longrightarrow$	0.18
κ_{HF}	1	$\longrightarrow$	0.06
κ_{LD}	1	$\longrightarrow$	0.15
κ_{LF}	1	$\longrightarrow$	0.12
ρ_s	1	$\longrightarrow$	0.65
ρ_b	1	$\longrightarrow$	0.93
η	$3.99^{\xi-1}$	$\longrightarrow$	2.51
τ	0.01	$\longrightarrow$	0.25

This table shows the transition of parameter values from the simplified model to the calibrated model used in the homotopy approach.$*$ this value is computed as described below, $\hat{b}_{ij}$.

In particular, note that in our initial simple case, the parameter that governs the labor supply elasticity in the formal market is equal to the one in informality, b_{ij} is homogenous across groups and set as described below, the parameter that determines the additional disutility of working in the informal sector, κ_{ij}, is homogeneous and equal to 1, both natives and foreigners and high- and low- skilled workers are perfect substitutes, η is set so that in this special case the congestion factor equals 1 in steady state, and, finally, the tax rate is set to a very small number. Additionaly, $\upsilon = 1$ to shut down wealth effects. We further have that in steady state n_{ss}^r and n_{ss}^f are constant, and there is no migration wave, i.e. f_{ss}^g.

C.2 Steady State Derivation

Let us now derive the steady state for the above special calibration, that is, the one that takes the values in the midle column of the previous table. Variables without time subscript denote steady state values.

Departing from the IBC - for $j \in \{D, F\}$ and $i \in \{H, L\}$ - we can write

$$\sum_{g=1}^{30} r^{1-g} c_{ij}^{g} = \sum_{g=1}^{20} r^{1-g} (1+z)^{g-1} \left[(1-\tau) w_{ij} l_{ij}^{g} + \hat{b}_{ij} h_{ij}^{g} \right] \tag{69}$$

then using the Euler equation $c_{ij}^{g_1} \left(\beta_i^{g-g_1} r^{g-g_1} \right)^{\frac{1}{\theta}} = c_{ij}^{g} \;\Rightarrow$

$$c_{ij}^{g_1} = \left(\sum_{g=1}^{30} r^{\frac{g-g_1}{\theta} - g} \beta^{\frac{g-g_1}{\theta}} \right)^{-1} \sum_{g=1}^{20} r^{-g} (1+z)^{g-1} \left[(1-\tau) w_{ij} l_{ij}^{g} + \hat{b}_{ij} \frac{h^{\xi-1}}{\eta} h_{ij}^{g} \right] \tag{70}$$

for $1 < g_1 \leq 20$, $j \in \{D, F\}$ and defining as usual $s_4 ij^0 = 0$,

$$\begin{aligned} c_{ij}^{g_1} = & \left(\sum_{g=g_1}^{30} \left(r^{g-g_1} \right)^{\frac{1-\theta}{\theta}} \beta^{\frac{g-g_1}{\theta}} \right)^{-1} \\ & \left(r s_{ij}^{g_1-1} + \sum_{g=g_1}^{20} r^{g_1-g} (1+z)^{g-1} \left[(1-\tau) w_{ij} l_{ij}^{g} + \hat{b}_{ij} \frac{h^{\xi-1}}{\eta} h_{ij}^{g} \right] \right) \end{aligned} \tag{71}$$

then,

$$\begin{aligned} s_{ij}^{g_1-1} &= \left(\sum_{g=g_1}^{30} \left(r^{g-g_1} \right)^{\frac{1-\theta}{\theta}} \beta^{\frac{g-g_1}{\theta}} \right) \frac{c_{ij}^{g_1}}{r} - \sum_{g=g_1}^{20} r^{g_1-g-1} (1+z)^{g-1} \left[(1-\tau) w_{ij} l_{ij}^{g} + \hat{b}_{ij} \frac{h^{\xi-1}}{\eta} h_{ij}^{g} \right] \\ &= \left(\sum_{g=g_1}^{30} \left(r^{g-g_1} \right)^{\frac{1-\theta}{\theta}} \beta^{\frac{g-g_1}{\theta}} \right) \left(\sum_{g=1}^{30} r^{\frac{g-g_1}{\theta}+1-g} \beta^{\frac{g-g_1}{\theta}} \right)^{-1} \sum_{g=1}^{20} r^{-g} (1+z)^{g-1} \left[(1-\tau) w_{ij} l_{ij}^{g} + \hat{b}_{ij} \frac{h^{\xi-1}}{\eta} h_{ij}^{g} \right] \\ &- \sum_{g=g_1}^{20} r^{g_1-g-1} (1+z)^{g-1} \left[(1-\tau) w_{ij} l_{ij}^{g} + \hat{b}_{ij} \frac{h^{\xi-1}}{\eta} h_{ij}^{g} \right], \end{aligned} \tag{72}$$

for $20 \leq g_1$, we have

$$s_{ij}^{g_1-1} = \left(\sum_{g=g_1}^{30} \left(r^{g-g_1} \right)^{\frac{1-\theta}{\theta}} \beta_i^{\frac{g-g_1}{\theta}} \right) \frac{c_{ij}^{g_1}}{r} \tag{73}$$

then,

$$\begin{aligned} s_{ij}^{g_1-1} &= \left(\sum_{g=g_1}^{30} \left(r^{g-g_1} \right)^{\frac{1-\theta}{\theta}} \beta^{\frac{g-g_1}{\theta}} \right) \frac{c_{ij}^{g_1}}{r} \\ &= \left(\sum_{g=g_1}^{30} \left(r^{g-g_1} \right)^{\frac{1-\theta}{\theta}} \beta^{\frac{g-g_1}{\theta}} \right) \left(\sum_{g=1}^{m} r^{\frac{g-g_1}{\theta}+1-g} \beta^{\frac{g-g_1}{\theta}} \right)^{-1} \\ & \sum_{g=1}^{20} r^{-g} (1+z)^{g-1} \left[(1-\tau) w_{ij} l_{ij}^{g} + \hat{b}_{ij} \frac{h^{\xi-1}}{\eta} h_{ij}^{g} \right], \end{aligned} \tag{74}$$

Next, setting $\frac{h^{\xi-1}}{\eta} = 1$, we have

$$w_{ij} = a_i a_{ij} w \tag{75}$$

and

$$h_{ij}^{g}=\begin{cases}\left[\frac{\left(\hat{b}_{ij}-(1-\tau)w_{ij}\right)}{(1+z)^{1-g}\chi_i\kappa_{ij}}\right]^{\frac{1}{\phi^{h}-1}} & if \quad \hat{b}_{ij}>(1-\tau)\,w_{ij}\\ 0 & otherwise\end{cases} \tag{76}$$

if $\hat{b}_{ij}>(1-\tau)\,w_{ij}$ we further have,

$$\begin{aligned} h_{ij}^{g_1} &= \left[\frac{\left(\hat{b}_{ij}-(1-\tau)\,w_{ij}\right)}{(1+z)^{1-g_1}\,\chi_i\kappa_{ij}}\right]^{\frac{1}{\phi^{h}-1}} \\ &= (1+z)^{\frac{g_1-g_2}{\phi^{h}-1}}\left[\frac{\left(\hat{b}_{ij}-(1-\tau)\,w_{ij}\right)}{(1+z)^{1-g_2}\,\chi_i\kappa_{ij}}\right]^{\frac{1}{\phi^{h}-1}} \\ &= (1+z)^{\frac{g_1-g_2}{\phi^{h}-1}}\,h_{ij}^{g_2} \end{aligned} \tag{77}$$

else, if $\hat{b}_{ij}\leq(1-\tau)\,w_{ij}$

$$h_{ij}^{g_1}=0=(1+z)^{\frac{g_1-g_2}{\phi^{h}-1}}\,h_{ij}^{g_2} \tag{78}$$

We next choose $\hat{b}_{ij}$ such that

$$l_{ij}^{g}=\hat{b}_{ij}\frac{h_{ij}^{g}}{(1-\tau)\,w_{ij}} \tag{79}$$

This assumption will allow us to solve for the capital-labor ratio. Ex post, we will proof that such a $\hat{b}_{ij}$ exists. We then, can write

$$\begin{aligned} l_{ij}^{g_1} &= \left(\frac{(1-\tau)}{\chi_i}w_{ij}\,(1+z)^{g_1-1}\right)^{\frac{1}{\phi^{l}-1}}-h_{ij}^{g_1} \\ &= (1+z)^{\frac{g_1-g_2}{\phi^{l}-1}}\left(\frac{(1-\tau)}{\chi_i}w_{ij}\,(1+z)^{g_2-1}\right)^{\frac{1}{\phi^{l}-1}}-(1+z)^{\frac{g_1-g_2}{\phi^{h}-1}}\,h_{ij}^{g_2} \\ &= (1+z)^{\frac{g_1-g_2}{\phi-1}}\,l_{ij}^{g_2} \end{aligned} \tag{80}$$

$$
\begin{aligned}
l_{LD} &= \sum_{g=1}^{g_r-1} l_{LD}^{g} \frac{(1-\lambda_{HD})(1+n^r)}{(1+n^r+n^f)^g} \\
&= l_{LD}^{g_2} \sum_{g=1}^{g_r-1} (1+z)^{\frac{g-g_2}{\phi-1}} \frac{(1-\lambda_{HD})(1+n^r)}{(1+n^r+n^f)^g} \\
&= l_{LD}^{g_2} (1-\lambda_{HD})(1+n^r)(1+z)^{\frac{g_2}{1-\phi}} \sum_{g=1}^{g_r-1} \frac{(1+z)^{\frac{g}{\phi-1}}}{(1+n^r+n^f)^g} \\
&= l_{LD}^{g_2} (1-\lambda_{HD})(1+n^r)(1+z)^{\frac{g_2}{1-\phi}} \sum_{g=1}^{g_r-1} \left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right)^g \\
&= l_{LD}^{g_2} (1-\lambda_{HD})(1+n^r)(1+z)^{\frac{g_2}{1-\phi}} \frac{\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right) - \left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right)^{g_r}}{1-\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right)}
\end{aligned} \tag{81}
$$

$$
\begin{aligned}
l_{LF} &= \sum_{g=1}^{g_r-1} l_{LF}^{g} \frac{(1-\lambda_{HF})\, n^f}{(1+n^r+n^f)^g} \\
&= l_{LF}^{g_2} \sum_{g=1}^{g_r-1} (1+z)^{\frac{g-g_2}{\phi-1}} \frac{(1-\lambda_{HF})\, n^f}{(1+n^r+n^f)^g} \\
&= l_{LF}^{g_2} (1-\lambda_{HF})\, n^f (1+z)^{\frac{g_2}{1-\phi}} \sum_{g=1}^{g_r-1} \left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right)^g \\
&= l_{LF}^{g_2} (1-\lambda_{HF})\, n^f (1+z)^{\frac{g_2}{1-\phi}} \frac{\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right) - \left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right)^{g_r}}{1-\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right)}
\end{aligned} \tag{82}
$$

$$
\begin{aligned}
l_{HD} &= \sum_{g=1}^{g_r-1} l_{HD}^{g} \frac{\lambda_{HD}(1+n^r)}{(1+n^r+n^f)^g} \\
&= l_{HD}^{g_2} \sum_{g=1}^{g_r-1} (1+z)^{\frac{g-g_2}{\phi-1}} \frac{\lambda_{HD}(1+n^r)}{(1+n^r+n^f)^g} \\
&= l_{HD}^{g_2} \lambda_{HD}(1+n^r)(1+z)^{\frac{g_2}{1-\phi}} \sum_{g=1}^{g_r-1} \left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right)^g \\
&= l_{HD}^{g_2} \lambda_{HD}(1+n^r)(1+z)^{\frac{g_2}{1-\phi}} \frac{\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right) - \left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right)^{g_r}}{1-\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)}\right)}
\end{aligned} \tag{83}
$$

$$\begin{aligned}
l_{HF} &= \sum_{g=1}^{g_r-1} l_{HF}^{g} \frac{\lambda_{HF} n^f}{(1+n^r+n^f)^g} \\
&= l_{HF}^{g_2} \sum_{g=1}^{g_r-1} (1+z)^{\frac{g-g_2}{\phi-1}} \frac{\lambda_{HF} n^f}{(1+n^r+n^f)^g} \\
&= l_{HF}^{g_2} \lambda_{HF} n^f (1+z)^{\frac{g_2}{1-\phi}} \sum_{g=1}^{g_r-1} \left(\frac{(1+z)^{\frac{1}{\phi-1}}}{(1+n^r+n^f)} \right)^g \\
&= l_{HF}^{g_2} \lambda_{HF} n^f (1+z)^{\frac{g_2}{1-\phi}} \frac{\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{\left(1+n^r+n^f\right)}\right) - \left(\frac{(1+z)^{\frac{1}{\phi-1}}}{\left(1+n^r+n^f\right)}\right)^{g_r}}{1-\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{\left(1+n^r+n^f\right)}\right)}
\end{aligned} \tag{84}$$

using $l_{ij}^g = \hat{b}_{ij} \frac{h_{ij}^g}{(1-\tau) w_{ij}}$ and the above expressions in (72) and (74) we get for $1 < g_1 20$

$$\begin{aligned}
s_{ij}^{g_1-1} &= 2(1-\tau) a_i a_{ij} w \left[\frac{\sum_{g=g_1}^{30} \left(r^{g-g_1}\right)^{\frac{1-\theta}{\theta}} \beta^{\frac{g-g_1}{\theta}}}{\sum_{g=1}^{30} r^{\frac{g-g_1}{\theta}+1-g} \beta^{\frac{g-g_1}{\theta}}} \sum_{g=1}^{20} r^{-g} (1+z)^{g-1} l_{ij}^g - \sum_{g=g_1}^{20} r^{g_1-g-1} (1+z)^{g-1} l_{ij}^g \right] \\
&= 2(1-\tau) a_i a_{ij} w l_{ij}^1 \underbrace{\left[\frac{\sum_{g=g_1}^{30} \left(r^{g-g_1}\right)^{\frac{1-\theta}{\theta}} \beta^{\frac{g-g_1}{\theta}}}{\sum_{g=1}^{30} r^{\frac{g-g_1}{\theta}+1-g} \beta^{\frac{g-g_1}{\theta}}} \sum_{g=1}^{20} r^{-g} (1+z)^{\frac{(g-1)\phi}{\phi-1}} - \sum_{g=g_1}^{20} r^{g_1-g-1} (1+z)^{\frac{(g-1)\phi}{\phi-1}} \right]}_{\Theta(g_1,\beta)} \\
&= 2(1-\tau) a_i a_{ij} w l_{ij}^1 \Theta(g_1,\beta) \\
&= 2(1-\tau) a_i a_{ij} w l_{ij} \Theta(g_1,\beta) \Gamma_{ij} \mathit{\Lambda}
\end{aligned} \tag{85}$$

where

$$\begin{aligned}
\Theta(g_1,\beta) = & \frac{r^{g_1-1} (1+z)^{\frac{\phi}{1-\phi}}}{1-r^{-1}(1+z)^{\frac{\phi}{\phi-1}}} \\
& \left\{ \left(1 - \frac{r^{\frac{1-\theta}{\theta}} \beta^{\frac{1}{\theta}} - \left(r^{\frac{1-\theta}{\theta}} \beta^{\frac{1}{\theta}}\right)^{g_1}}{r^{\frac{1-\theta}{\theta}} \beta^{\frac{1}{\theta}} - \left(r^{\frac{1-\theta}{\theta}} \beta^{\frac{1}{\theta}}\right)^{m+1}} \right) \left(r^{-1}(1+z)^{\frac{\phi}{\phi-1}} - \left(r^{-1}(1+z)^{\frac{\phi}{\phi-1}}\right)^{g_r} \right) \right. \\
& \left. - \left(\left(r^{-1}(1+z)^{\frac{\phi}{\phi-1}}\right)^{g_1} - \left(r^{-1}(1+z)^{\frac{\phi}{\phi-1}}\right)^{g_r} \right) \right\}
\end{aligned} \tag{86}$$

$\Gamma_{LD} = ((1-\lambda_{HD})(1+n^r))^{-1}$, $\Gamma_{LF} = \left((1-\lambda_{HF}) n^f\right)^{-1}$, $\Gamma_{HD} = (\lambda_{HD}(1+n^r))^{-1}$, $\Gamma_{HF} = \left(\lambda_{HF} n^f\right)^{-1}$, and $\mathit{\Lambda} = \left((1+z)^{\frac{1}{1-\phi}} \frac{\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{\left(1+n^r+n^f\right)}\right) - \left(\frac{(1+z)^{\frac{1}{\phi-1}}}{\left(1+n^r+n^f\right)}\right)^{g_r}}{1-\left(\frac{(1+z)^{\frac{1}{\phi-1}}}{\left(1+n^r+n^f\right)}\right)} \right)^{-1}$. For $20 < g_1$

$$
\begin{aligned}
s_{ij}^{g_1-1} &= 2\left(1-\tau\right)a_i a_{ij} w \frac{\sum_{g=g_1}^{30}\left(r^{g-g_1}\right)^{\frac{1-\theta}{\theta}}\beta^{\frac{g-g_1}{\theta}}}{\sum_{g=1}^{30} r^{\frac{g-g_1}{\theta}+1-g}\beta^{\frac{g-g_1}{\theta}}}\sum_{g=1}^{20} r^{-g}\left(1+z\right)^{g-1} l_{ij}^{g} \\
&= 2\left(1-\tau\right)a_i a_{ij} w l_{ij}^{1} \frac{\sum_{g=g_1}^{30}\left(r^{g-g_1}\right)^{\frac{1-\theta}{\theta}}\beta^{\frac{g-g_1}{\theta}}}{\sum_{g=1}^{30} r^{\frac{g-g_1}{\theta}+1-g}\beta^{\frac{g-g_1}{\theta}}}\sum_{g=1}^{20} r^{-g}\left(1+z\right)^{\frac{(g-1)\phi}{\phi-1}} \\
&= 2\left(1-\tau\right)a_i a_{ij} w l_{ij}^{1} \underbrace{\frac{\sum_{g=g_1}^{30}\left(r^{g-g_1}\right)^{\frac{1-\theta}{\theta}}\beta^{\frac{g-g_1}{\theta}}}{\sum_{g=1}^{30} r^{\frac{g-g_1}{\theta}+1-g}\beta^{\frac{g-g_1}{\theta}}}\sum_{g=1}^{20} r^{-g}\left(1+z\right)^{\frac{(g-1)\phi}{\phi-1}}}_{\Omega(g_1,\beta)} \\
&= 2\left(1-\tau\right)a_i a_{ij} w l_{ij}\Omega\left(g_1,\beta\right)\Gamma_{ij}\mathit{\Lambda} \qquad (87)
\end{aligned}
$$

where

$$
\begin{aligned}
\Omega\left(g_1,\beta\right) = r^{g_1-1}\left(1-\frac{r^{\frac{1-\theta}{\theta}}\beta^{\frac{1}{\theta}}-\left(r^{\frac{1-\theta}{\theta}}\beta^{\frac{1}{\theta}}\right)^{g_1}}{r^{\frac{1-\theta}{\theta}}\beta^{\frac{1}{\theta}}-\left(r^{\frac{1-\theta}{\theta}}\beta^{\frac{1}{\theta}}\right)^{31}}\right)\left(1+z\right)^{\frac{\phi}{1-\phi}} \\
\frac{r^{-1}\left(1+z\right)^{\frac{\phi}{\phi-1}}-\left(r^{-1}\left(1+z\right)^{\frac{\phi}{\phi-1}}\right)^{21}}{1-r^{-1}\left(1+z\right)^{\frac{\phi}{\phi-1}}} \qquad (88)
\end{aligned}
$$

Then we have,

$$
\begin{aligned}
& k = s \\
& \overset{(61)}{\overbrace{=}} \sum_{g=1}^{m-1} s^{g}\left(1+z\right)^{1-g}\left(1+n^{r}+n^{f}\right)^{1-g} \\
& \overset{(60)}{\overbrace{=}} \sum_{g=1}^{m-1}\frac{\left(1+n^{r}\right)\left[\left(1-\lambda_{HD}\right)s_{LD}^{g}+\lambda_{HD}s_{HD}^{g}\right]+n_{t}^{f}\left[\left(1-\lambda_{HF}\right)s_{LF}^{g}+\lambda_{HF}s_{HF}^{g}\right]}{\left(1+z\right)^{g-1}\left(1+n^{r}+n^{f}\right)^{-g}} \\
& \overset{(85)(87)}{\overbrace{=}} 2\left(1-\tau\right)w\mathit{\Lambda}\left(\sum_{g=1}^{g_r-1}\Theta\left(g+1,\beta\right)\frac{a_L\left(a_{LD}l_{LD}+a_{LF}l_{LF}\right)+a_H\left(a_{HD}l_{HD}+a_{HF}l_{HF}\right)}{\left(1+z\right)^{g-1}\left(1+n^{r}+n^{f}\right)^{-g}}\right. \\
& \left.+\sum_{g=g_r}^{m-1}\Omega\left(g+1,\beta\right)\frac{a_L\left(a_{LD}l_{LD}+a_{LF}l_{LF}\right)+a_H\left(a_{HD}l_{HD}+a_{HF}l_{HF}\right)}{\left(1+z\right)^{g-1}\left(1+n^{r}+n^{f}\right)^{-g}}\right) \\
& \overset{(50)}{\overbrace{=}} 2\left(1-\tau\right)w\mathit{\Lambda}\left(\sum_{g=1}^{g_r-1}\frac{\Theta\left(g+1,\beta\right)\left(a_L l_L+a_H l_H\right)}{\left(1+z\right)^{g-1}\left(1+n^{r}+n^{f}\right)^{-g}}+\sum_{g=g_r}^{m-1}\frac{\Omega\left(g+1,\beta\right)\left(a_L l_L+a_H l_H\right)}{\left(1+z\right)^{g-1}\left(1+n^{r}+n^{f}\right)^{-g}}\right) \\
& \overset{(49)}{\overbrace{=}} 2\left(1-\tau\right)wl\mathit{\Lambda}\left(\sum_{g=1}^{g_r-1}\frac{\Theta\left(g+1,\beta\right)}{\left(1+z\right)^{g-1}\left(1+n^{r}+n^{f}\right)^{-g}}+\sum_{g=g_r}^{m-1}\frac{\Omega\left(g+1,\beta\right)}{\left(1+z\right)^{g-1}\left(1+n^{r}+n^{f}\right)^{-g}}\right)
\end{aligned}
$$

using

$$
w = \frac{\left(1-\alpha\right)y^{F}}{l} = \left(1-\alpha\right)\left(\frac{k/l}{\left(1+z\right)\left(1+n^{r}+n^{f}\right)}\right)^{\alpha} \qquad (89)
$$

and

$$
\begin{aligned}
r &= 1 + r^k - \delta \\
&= 1 + \alpha \frac{y^F}{k} (1+z) \left(1 + n^r + n^f\right) - \delta \\
&= 1 + \alpha \left(\frac{k/l}{(1+z)(1+n^r+n^f)} \right)^{\alpha-1} - \delta
\end{aligned} \tag{90}
$$

we have

$$
\begin{aligned}
\left(\frac{k}{l}\right)^{1-\alpha} &= 2(1-\tau)(1-\alpha)\left((1+z)\left(1+n^r+n^f\right)\right)^{-\alpha} \Lambda \times \\
&\left(\sum_{g=1}^{g_r-1} \frac{\Theta(g+1,\beta)}{(1+z)^{g-1}(1+n^r+n^f)^{-g}} + \sum_{g=g_r}^{m-1} \frac{\Omega(g+1,\beta)}{(1+z)^{g-1}(1+n^r+n^f)^{-g}} \right)
\end{aligned} \tag{91}
$$

This is an equation in r and the capital-labor ratio, k/l, since $r = 1+\alpha\left(\frac{k/l}{(1+z)\left(1+n^r+n^f\right)}\right)^{\alpha-1} - \delta$, this is an equation only on the capital-labor ratio. Solving that non-linear equation, we can obtain a capital-labor ratio and then proceed as in the open economy case. We solve this equation with a non-linear solver. Let us finish by determining the $\hat{b}_{ij}$ needed for the previous deviation

$$
l_{ij}^g = \hat{b}_{ij} \frac{h_{ij}^g}{(1-\tau) w_{ij}}
$$

then

$$
(1-\tau) a_i a_{ij} w l_{ij}^g = \hat{b}_{ij} h_{ij}^g
$$

using

$$
\begin{aligned}
l_{ij}^g + h_{Lj}^g &= \left((1-\tau)(1+z)^{g-1} \frac{w_{ij}}{\chi_i} \right)^{\frac{1}{\phi-1}} \\
&= \left((1-\tau)(1+z)^{g-1} \frac{a_i a_{ij} w}{\chi_i} \right)^{\frac{1}{\phi-1}}
\end{aligned}
$$

we have,

$$
\begin{aligned}
l_{lj}^g &= \left(1 + (1-\tau)\frac{w_{ij}}{\hat{b}_{ij}}\right)^{-1} \left((1-\tau)(1+z)^{g-1} \frac{w_{ij}}{\chi_i} \right)^{\frac{1}{\phi^l-1}} \\
&= \left(1 + (1-\tau)\frac{w_{ij}}{\hat{b}_{ij}}\right)^{-1} \left((1-\tau)(1+z)^{g-1} \frac{a_i a_{ij} w}{\chi_i} \right)^{\frac{1}{\phi^l-1}}
\end{aligned}
$$

Putting everything together, we have (we guess here that in ss $\hat{b}_{ij} > (1-\tau) w_{ij}$, we verify this ex-post)

$$
\begin{aligned}
(1-\tau) a_i a_{ij} w l_{ij}^g &= (1-\tau) a_i a_{ij} w \left(1 + (1-\tau) \frac{w_{ij}}{\hat{b}_{ij}}\right)^{-1} \left((1-\tau)(1+z)^{g-1} \frac{a_i a_{ij} w}{\chi_i}\right)^{\frac{1}{\phi-1}} \\
&= \hat{b}_{ij} \left[\frac{\left(\hat{b}_{ij} - (1-\tau) w_{ij}\right)}{(1+z)^{1-g} \chi_i \kappa_{ij}}\right]^{\frac{1}{\phi-1}} = \hat{b}_{ij} h_{ij}^g
\end{aligned}
$$

$$
(1-\tau) a_i a_{ij} w \left(1 + (1-\tau) \frac{a_i a_{ij} w}{\hat{b}_{ij}}\right)^{-1} \left((1-\tau) a_i a_{ij} w\right)^{\frac{1}{\phi-1}} = \hat{b}_{ij} \left[\frac{\left(\hat{b}_{ij} - (1-\tau) a_i a_{ij} w\right)}{\kappa_{ij}}\right]^{\frac{1}{\phi-1}}
$$

since $\hat{b}_{ij} = a_i a_{ij} b_{ij}$ we have

$$
\left((1-\tau) w\right)^{\frac{\phi}{\phi-1}} \kappa_{ij} = \left(b_{ij} - (1-\tau) w\right) \left(b_{ij} + (1-\tau) w\right)^{\phi-1} \tag{92}
$$

The rest follows by solving the equations of the equilibrium, for example,

$$
w = \frac{(1-\alpha) y^F}{l} = (1-\alpha) \left(\frac{k/l}{(1+z)(1+n^r+n^f)}\right)^{\alpha}
$$

and

$$
r^k = \alpha \left(\frac{k/l}{(1+z)(1+n^r+n^f)}\right)^{\alpha-1}
$$

The individual consumption and saving choices have to be determined sequentially and alternating between consumption and saving,i.e. c_{ij}^1, s_{ij}^1, c_{ij}^2, s_{ij}^2,....

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