# DOCUMENTOS DE TRABAJO

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N° 842 Septiembre 2019

BANCO CENTRAL DE CHILE

CENTRAL BANK OF CHILE

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The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy[∗]

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Abstract

We study the transmission of monetary policy shocks in a model in which realistic heterogeneity in price rigidity interacts with heterogeneity in sectoral size and input-output linkages, and derive conditions under which these heterogeneities generate large real effects. Quantitatively, heterogeneity in the frequency of price adjustment is the most important driver behind large real effects. Heterogeneity in input-output linkages and consumption shares contribute only marginally to real effects but alter substantially the identity and contribution of the most important sectors to the transmission of monetary shocks. In the model and data, reducing the number of sectors decreases monetary non-neutrality with a similar impact response of inflation. Hence, the initial response of inflation to monetary shocks is not sufficient to discriminate across models and for the real effects of nominal shocks and ignoring heterogeneous consumption shares and input-output linkages identifies the wrong sectors from which the real effects originate.

Resumen

Este artículo estudia la transmisión de la política monetaria en una economía donde interactúan tres dimensiones de heterogeneidad entre sectores que son empíricamente relevantes: rigideces nominales, tamaño y relaciones insumo-producto. Se obtienen condiciones en que estas tres formas de heterogeneidad amplifican el efecto real de la política monetaria. De ellas, la heterogeneidad en la frecuencia de cambios de precios es la más relevante cuantitativamente; heterogeneidad en tamaño y relaciones insumo-producto sólo tienen un impacto marginal. Tanto en el modelo como en el análisis empírico, reducir el nivel de desagregación reduce el efecto real de la política monetaria manteniendo un efecto similar sobre la inflación. Por lo tanto, la práctica usual de enfocarse en el efecto de la política monetaria sobre los precios no provee información suficiente para deducir su

efecto sobre la economía real. Finalmente, aunque la heterogeneidad de tamaño y relaciones insumo producto tiene un efecto menor sobre la actividad agregada, si tienen gran importancia para determinar los sectores sobre los cuales la política monetaria actúa con mayor fuerza.

I Introduction

Understanding how monetary policy transmits to the real economy and why nominal shocks have real effects are vital questions in monetary economics. The literature identified heterogeneity in price rigidities as a central driver behind the real effects of monetary shocks (see, e.g., Carvalho (2006) and Nakamura and Steinsson (2008)) but a recent literature suggests other heterogeneities on the production side might also be important for aggregate fluctuations. Sectors differ in size and different sectors use different intermediate input mixes to produce output. Gabaix (2011) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) derive conditions under which these heterogeneties can generate aggregate fluctuations from idiosyncratic or sectoral real shocks invalidating the diversification argument of Lucas (1977) and Ozdagli and Weber (2017) argue production networks shape the stock market response to monetary shocks. But most of the existing literature has studied how heterogeneities in sector size, input-output structure, and price stickiness shape aggregate fluctuations in isolation.

In this paper, we present new theoretical insights into the transmission of monetary policy shocks in an economy in which all three heterogeneities are present and interact with each other. First, we show real effects of nominal shocks are bigger if the share of intermediate inputs is high or if sticky-price sectors are important suppliers to the rest of the economy, to large sectors and to flexible-price sectors on impact, but to sticky price sectors following the shock.[1] Second, the level of disaggregation is central for the real effects of monetary policy. More granular economies result in larger real effects with similar price responses on impact. Third, the importance of specific sectors for the transmission of monetary policy shocks depends on which heterogeneities are present, and how they interact.

On the quantitative side, our contribution lies in the calibration of a detailed model of the U.S. economy to study the quantitative importance of the different types of heterogeneities. We calibrate a 341-sector version of the model to the input-output (I/O) tables from the Bureau of Economic Analysis (BEA) and the micro-data underlying the producer price index (PPI) from the Bureau of Labor Statistics (BLS). First, we find heterogeneity in price stickiness is the main driver of real output effects: It increases real output effects relative to an economy with homogeneous price stickiness by 70%. Additionally allowing for heterogeneity in consumption shares or size or both only has a marginal effect on impact and on cumulative real output effects.

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[1]Some of the results are well known. The dynamic prediction in the network setting is most distinctly new (see Basu (1995), Huang and Liu (2001, 2004), Shamloo (2010)), and Bouakez, Cardia, and Ruge-Murcia (2014))

Second, the choice of disaggregation plays an important role quantitatively. A 341-sector economy has a 7% (46%) larger cumulative real effect of monetary policy shocks than a less granular 58-(7-)sector model. However, across choices of aggregation, the response of inflation to the monetary policy shock is similar on impact and on average during the first few periods. The large differences in real output effects with similar impact responses of inflation across different levels of aggregation caution against drawing inference for the conduct of monetary policy from the initial response of inflation to monetary policy shocks.

Third, heterogeneity in price rigidity is key in determining which sectors are the most important contributors to the transmission of monetary shocks. In an economy with homogeneous price stickiness all sectors respond equally to a common monetary policy shock independent of their size or I/O structure. Once we introduce different price stickiness across sectors, sectoral output responses of the 10 most important sectors increase by 350%. Hence, heterogeneous price stickiness is central for differential sectoral real effects, but this result does not mean that heterogeneities in sector size and I/O structure does not matter for sectoral responses. Our baseline economy with all three heterogeneities present doubles the real effects of the ten most important sectors relative to the economy with homogeneous sector size and I/O structure and totally scrambles the identities of the 10 most important, contractionary responses. Thus, even though heterogeneity in I/O linkages or size only has a marginal effect on the aggregate real output responses, which sector transmits the monetary policy shock the most depends crucially on our exact specification of heterogeneities. As we remove heterogeneities, the distribution of responses also becomes much more compressed.

Notably, heterogeneity in price rigidity also changes the sign of the response for the least contractionary responses: In fact, the 10 least contractionary sectoral responses are positive in all combinations that include heterogeneity in price rigidity. As we remove heterogeneities from the baseline, responses also become more compressed and smaller – but only negative when price rigidity becomes homogeneous. The flip in sign is due to the fact that the 10 least contractionary, expansionary responses are concentrated in the most flexible sectors. These sectors can gain market share from lowering their relative prices more quickly than stickier-price sectors.

Taken together, these results show (i) Heterogeneous price stickiness is the central force for the real effects of nominal shocks; heterogeneity in intermediate input usage and in the I/O structure is less important; (ii) disaggregation matters for the real effects of monetary policy shocks but leaves the impact response of inflation largely unchanged; (iii) price stickiness that differs across sectors changes the identity and importance of the

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most important sectors for the transmission of monetary shocks; (iv) heterogeneous sector size and I/O structures further change the identity of the most important sectors for the real effects of monetary shocks and increase their importance, and hence, the effective granularity of the economy increases.

What mechanisms drive these results? In the model, firms set prices as a markup over a weighted average of future marginal costs. We identify four distinct channels through which I/O linkages and the heterogeneities of sector size and price stickiness affect the marginal-cost process. First, marginal costs of final-goods producers depend directly on the sector-specific input price index. Second, sector-specific wages depend indirectly on I/O linkages because the optimal mix of inputs depends on the relative price of intermediate inputs and labor. Third and fourth, the heterogeneities across sectors in total production, value-added, and intermediate inputs create wedges between sectoral participation in total output, production, and total GDP that feed back into marginal costs. These channels interact in shaping the response to nominal shocks in a very intuitive way: How important is the output of a given sector for final-goods production? How flexible are the output prices of the goods the sector uses in production? How important is the sector as a producer for total consumption?

We develop further, analytical intuition for the interaction of the three heterogeneities in a simplified model. In this economy, we gradually add each heterogeneity, and prove results analytically when possible. We start with an economy that features I/O linkages that can be homogeneous or heterogeneous across sectors. Key to this step is that price rigidity is homogeneous across sectors, and sectoral participation in GDP equals sectoral participation in total production. I/O linkages per se amplify the real effects of monetary policy, as in Nakamura and Steinsson (2008). However, heterogeneity in consumption shares and I/O linkages does not matter, because sectoral production and consumption shares do not produce wedges.

We then add heterogeneity in Calvo parameters. This addition generates a hump-shaped response in consumption, because flexible-price firms compete with sticky-price firms. Firms with flexible prices adjust prices in a staggered fashion and by less on impact than in a model with homogeneous Calvo rates across sectors. The dispersion of price stickiness amplifies cumulative real effects following an identical impact of consumption as in Carvalho (2006), Carvalho and Schwartzman (2015), and Alvarez et al. (2016). Heterogeneity in I/O linkages and consumption shares does not affect the impact response relative to an economy with homogeneous price stickiness and also does not have any systematic effect following the impact response.

Last, we allow for fully unrestricted heterogeneity in sector weights in GDP and in I/O

linkages. This additional degree of heterogeneity results in wedges between consumption prices and sectoral intermediate input prices, which influence sectoral marginal costs. Heterogeneity in I/O linkages can amplify or dampen the output response. For example, the economy may resemble more of a flexible-price economy or a sticky-price economy, depending on the interaction of sector size, the importance of sectors as suppliers to other sectors, and sectoral price stickiness. We characterize the interactions and their influence on real effects of monetary policy by three relations: (i) first-order out-degrees to sector size, (ii) first-order outdegrees adjusted by average flexibility to sector size, and (iii) covariances between sectoral linkages and size with price stickiness.

A. Literature review

Our paper contributes new insights to the literature on the transmission of monetary policy shocks in a network economy. Basu (1995) shows a roundabout production structure can magnify the importance of price rigidities through its effect on marginal costs, and results in larger welfare losses of demand-driven business cycles. Huang and Liu (2001, 2004) study the persistence of monetary shocks in a multi-sector model with roundabout production and fixed contract length. Aggregate output becomes more persistent in the their setup the higher the number of production stages and the share of intermediates. Their work theoretically shows that intermediate inputs amplify the importance of rigid prices with no impact on wage stickiness. Nakamura and Steinsson (2010) develop a multi-sector menu-cost model and show in a calibration of a six-sector version that heterogeneity in price stickiness together with I/O linkages can explain persistent real effects of nominal shocks with moderate degrees of price stickiness. Carvalho and Lee (2011) show a multi-sector Calvo model with intermediate inputs can reconcile why firms adjust more quickly to idiosyncratic shocks than to aggregate shocks (see also Boivin et al. (2009) and Shamloo (2010)). Bouakez, Cardia, and Ruge-Murcia (2014) estimate a multi-sector Calvo model with production networks using aggregate and sectoral data, and find evidence of heterogeneity in frequencies of price adjustments across sectors.

We contribute several new insights to this literature. Our most important quantitative innovation is to study the importance of networks on the propagation of nominal shocks in a detailed, 341-sector calibration of the U.S. economy. Second, we show both theoretically and quantitatively that reducing the number of sectors in our model decreases monetary non-neutrality. By contrast, across calibrations the impact response of inflation is similar across aggregation choices, and hence is not a sufficient statistic for

monetary non-neutrality. Finally, we point out a new identity effect: Heterogeneity in price rigidity is key in determining which sectors are the most important contributors to the transmission of monetary shocks but heterogeneity in sector size and I/O structure can change the sectoral identify substantially, and makes important contributors even more important and down-weighs the contribution of less important sectors. A few sectors with flexible prices can also increase their output following a contractionary shock, given their fall in relative price.

A high degree of specialization is a general, key feature of modern production economies. Gabaix (2011) and Acemoglu et al. (2012) show theoretically the network structure and the firm-size distribution are potentially important propagation mechanisms for aggregate fluctuations originating from firm and industry shocks. Acemoglu, Akcigit, and Kerr (2015) and Barrot and Sauvagnat (2016) show empirical evidence for the propagation of idiosyncratic supply shocks through the I/O structure. Carvalho (2014) provides an overview of this fast-growing literature. Idiosyncratic shocks propagate through changes in prices. In a companion paper (see Pasten, Schoenle, and Weber (2018)), we study how price rigidities affect the importance of idiosyncratic shocks as an origin of aggregate fluctuations.

Other recent applications of production networks in different areas of macroeconomics include Bigio and Lao (2017) who study the amplification of financial frictions through production networks, and Ozdagli and Weber (2017), who empirically show I/O linkages are a key propagation channel of monetary policy to the stock market. Additionally, Kelly, Lustig, and Van Nieuwerburgh (2013) study the joined dynamics of the firm-size distribution and stock return volatilities. Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) and Herskovic (2018) study asset-pricing implications of production networks.

II   Model

This section presents our full blown New Keynesian model. We highlight in particular how heterogeneities in price rigidities, sectoral size, and I/O linkages enter the model.

A.   Firms

A continuum of monopolistically competitive firms j operates in different sectors. We index firms by their sector, $k = 1, ..., K$, and by $j \in [0, 1]$. The set of consumption goods is partitioned into a sequence of subsets $\{\Im_k\}_{k=1}^{K}$ with measure $\{n_k\}_{k=1}^{K}$ such that $\sum_{k=1}^{K} n_k = 1$.

The first, real heterogeneity – heterogeneity in sectoral I/O linkages – enters via the production function of firm j in sector k

$$Y_{kjt} = L_{kjt}^{1-\delta} Z_{kjt}^{\delta}, \tag{1}$$

where L_{kjt} is labor and Z_{kjt} is an aggregator of intermediate inputs

$$Z_{kjt} \equiv \left[\sum_{r=1}^{K} \omega_{kr}^{\frac{1}{\eta}} Z_{kjt}(r)^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{2}$$

Here, $Z_{kjt}(r)$ denotes the intermediate input use by firm j in sector k in period t. The aggregator weights $\{\omega_{kr}\}_{k,r}$ satisfy $\sum_{r=1}^{K} \omega_{kr} = 1$ for all sectors k. We allow these weights to differ across sectors, which is a central ingredient of our analysis.

In turn, $Z_{kjt}(r)$ is an aggregator of goods produced in sector r,

$$Z_{kjt}(r) \equiv \left[n_r^{-1/\theta} \int_{\Im_r} Z_{kjt}(r,j')^{1-\frac{1}{\theta}} \, dj' \right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

$Z_{kjt}(r,j')$ is the amount of goods firm j' in sector r produces that firm k,j uses as input.

Demand for intermediate inputs $Z_{kjt}(r)$ and $Z_{kjt}(r,j')$ is given by the following demand equations

$$
\begin{aligned}
Z_{kjt}(r) &= \omega_{kr} \left(\frac{P_{rt}}{\mathrm{P}_{kt}} \right)^{-\eta} Z_{kjt}, \\
Z_{kjt}(r,j') &= \frac{1}{n_r} \left(\frac{P_{rj't}}{P_{rt}} \right)^{-\theta} Z_{kjt}(r).
\end{aligned}
$$

$P_{rj't}$ is the price firm j' in sector r charges, P_{rt} is a sectoral price index, and P_{kt} is an input-price index; we define both price indices below. In steady state, all prices are identical, and $\{\omega_{kr}\}_{r=1}^{K}$ is the share of costs that firm k,j spends on inputs from sector r and, hence, equals cell k,r in the I/O Tables (see online appendix). We refer to $\{\omega_{kr}\}_{r=1}^{K}$ as "I/O linkages." As a result, in steady state, all n_r firms in sector r share the demand of firm k,j for goods that sector r produces equally.

Away from steady state, a gap exists between the price index of sector r, P_{rt}, and the input price index, P_{kt}, that is relevant for firms in sector k. It distorts the share of sector r in the costs of firms in sector k. Similarly, price dispersion across firms within sector r determines the dispersion of demand of firms in sector k for goods in sector r.

Price indices relevant for the demand of intermediate inputs across sectors are defined as

$$P_{kt} = \left[\sum_{r=1}^{K} \omega_{kr} P_{rt}^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

$$P_{rt} = \left[\frac{1}{n_r} \int_{\Im_r} P_{rj't}^{1-\theta} dj'\right]^{\frac{1}{1-\theta}}.$$

Our second heterogeneity – heterogeneity of price rigidity – originates from our assumption about price setting. Firms set prices as in Calvo (1983), but we allow for differences in Calvo rates across sectors, $\{\alpha_k\}_{k=1}^{K}$. That is, the objective of firm j, k is given

$$\max_{P_{kjt}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k^s \left[P_{kjt} Y_{kjt+s} - MC_{kt+s} Y_{kjt+s}\right], \tag{4}$$

where $MC_{kt} = \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta}\right)^{-\delta} W_{kt}^{1-\delta} \left(P_t^k\right)^{\delta}$ are marginal costs after imposing the optimal mix of labor and intermediate inputs

$$\delta W_{kt} L_{kjt} = (1-\delta) P_t^k Z_{kjt}. \tag{5}$$

The optimal pricing problem takes the standard form

$$\sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k^s Y_{kjt+s} \left[P_{kt}^* - \frac{\theta}{\theta-1} MC_{kjt+s}\right] = 0. \tag{6}$$

Y_{kjt+s} is the total output of firm k, j at period $t+s$, $Q_{t,t+s}$ is the stochastic discount factor between period t and $t + s$, and θ is the elasticity of substitution within sector.

The optimal price for all adjusting firms within a given sector is identical, P_{kt}^*, allowing simple aggregation. Hence, the law of motion for sectoral prices is

$$P_{kt} = \left[(1-\alpha_k) P_{kt}^{*1-\theta} + \alpha_k P_{kt-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \ \forall k. \tag{7}$$

B.   Households

A large number of infinitely lived households exist. Households have a love for variety, and derive utility from consumption and leisure. Households supply all different types of labor. The representative household has additively separable utility in consumption and

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leisure and maximizes

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \sum_{k=1}^{K} \int_{\Im_k} g_k \frac{L_{kjt}^{1+\varphi}}{1 + \varphi} dj \right) \tag{8}$$

subject to

$$PC_t C_t = \sum_{k=1}^{K} W_{kt} \int_{\Im_k} L_{kjt} dj + \sum_{k=1}^{K} \Pi_{kt} + I_{t-1} B_{t-1} - B_t. \tag{9}$$

The budget constraint states nominal expenditure equals nominal household income. C_t and PC_t are aggregate consumption and prices, which we define below. L_{kjt} and W_{kt} are labor employed and wages paid by firm j in sector k. Households own firms and receive net income, Π_{kt}, as dividends. Bonds, B_t, pay a nominal gross interest rate of I_{t-1}. $\{g_k\}_{k=1}^{K}$ are parameters that we choose to ensure a symmetric steady state across all firms.

Aggregate consumption is

$$C_t \equiv \left[\sum_{k=1}^{K} \omega_{ck}^{\frac{1}{\eta}} C_{kt}^{1 - \frac{1}{\eta}} \right]^{\frac{\eta}{\eta - 1}}, \tag{10}$$

where C_{kt} is the aggregation of sectoral consumption

$$C_{kt} \equiv \left[n_k^{-1/\theta} \int_{\Im_k} C_{kjt}^{1 - \frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta - 1}}. \tag{11}$$

C_{kjt} is the consumption of goods that firm j in sector k produces.

We allow the elasticity of substitution across sectors η to differ from the elasticity of substitution within sectors θ. We also allow the consumption weights $\{\omega_{ck}\}$ to differ across sectors, which is the third heterogeneity across sectors in our model. The weights satisfy $\sum_{k=1}^{K} \omega_{ck} = 1$.

Households' demand for sectoral goods C_{kt} and firm goods C_{kjt} is

$$C_{kt} = \omega_{ck} \left(\frac{P_{kt}}{PC_t} \right)^{-\eta} C_t,$$

$$C_{kjt} = \frac{1}{n_k} \left(\frac{P_{kjt}}{P_{kt}} \right)^{-\theta} C_{kt}.$$

We solve in the online appendix for the steady state of the economy. We show the consumption weights $\{\omega_{ck}\}_{r=1}^{K}$ determine the steady-state shares of sectors in total consumption (or value-added production). In the following, we refer to $\{\omega_{ck}\}_{r=1}^{K}$ as

"consumption shares." Heterogeneity in sectoral size enters through these shares. Away from steady state, a gap between sectoral prices, $\{P_{rt}\}_{r=1}^{K}$, and aggregate consumption prices, PC_t, distorts the share of sectors in aggregate consumption.[2]

The consumption price index PC_t is given by

$$PC_t = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{12}$$

and sectoral prices follow

$$P_{kt} = \left[\frac{1}{n_k} \int_{\Im_k} P_{kjt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \tag{13}$$

C. Monetary policy

The monetary authority sets the short-term nominal interest rate, I_t, according to a Taylor rule

$$I_t = \frac{1}{\beta} \left(\frac{PC_t}{PC_{t-1}} \right)^{\phi_\pi} \left(\frac{C_t}{\overline{C}} \right)^{\phi_y} e^{\mu_t}. \tag{14}$$

μ_t is a monetary shock following an AR(1) process with persistence ρ_μ. Thus, monetary policy reacts to aggregate consumption inflation and aggregate consumption.

In our quantitative exercises, we also study economies with interest rates smoothing similar to Coibion and Gorodnichenko (2012).

D. Equilibrium conditions and definitions

$$B_t = 0, \tag{15}$$

$$L_{kt} = \int_{\Im_k} L_{kjt} dj, \tag{16}$$

$$W_t \equiv \sum_{k=1}^{K} n_k W_{kt}, \tag{17}$$

$$L_t \equiv \sum_{k=1}^{K} L_{kt}, \tag{18}$$

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[2]The measure of firms in sector k, n_k, and the consumption shares are related in equilibrium (see online appendix).

$$Y_{kjt} = C_{kjt} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{k'j't}(k,j) \, dj'. \tag{19}$$

Equation (15) is the market-clearing condition in bond markets. Equation (16) defines aggregate labor in sector k. Equations (17) and (18) give aggregate wage (which is a weighted average of sectoral wages) and aggregate labor (which linearly sums up hours worked in all sectors). Equation (19) is Walras' law for the output of firm j in sector k.

III Heterogeneities and Marginal Costs

The dynamic behavior of marginal costs is crucial for understanding the response of the economy to a monetary policy shock. This section develops intuition for the effects of heterogeneity in price stickiness, I/O linkages, and sectoral size on marginal costs, and the real effects of monetary policy in a log-linearized system that we detail in the online appendix. In the following, small letters denote log deviations from steady state. We focus on the role of heterogeneity in I/O linkages. Our main, analytical propositions in the next section, however, do not require reading this section first.

We highlight how I/O linkages affect marginal costs and demand through four distinct channels. In particular, a new wedge emerges that drives marginal costs: I/O linkages create a difference between the aggregate consumption price index and intermediate input price indices that affects marginal costs.

The reduced-form system that embeds marginal costs has $K+1$ equations and unknowns: value-added production c_t and K sectoral prices $\{p_{kt}\}_{k=1}^{K}$. The first equation is

$$\sigma \mathbb{E}_t[c_{t+1}] - (\sigma + \phi_c) c_t + \mathbb{E}_t[pc_{t+1}] - (1 + \phi_\pi) pc_t + \phi_\pi pc_{t-1} = \mu_t, \tag{20}$$

which is a combination of the household Euler equation and the Taylor rule. The equation describes how variations in value-added production, c_t, and aggregate consumption prices, pc_t, respond to the monetary policy shock, μ_t.

Note pc_t is given by

$$pc_t = \sum_{k=1}^{K} \omega_{ck} p_{kt}. \tag{21}$$

In addition, K equations governing sectoral prices

$$\beta \mathbb{E}_t[p_{kt+1}] - (1+\beta) p_{kt} + p_{kt-1} = \kappa_k (p_{kt} - mc_{kt}), \tag{22}$$

complete the system, where $\kappa_k \equiv (1-\alpha_k)(1-\alpha_k\beta)/\alpha_k$ measures the degree of price

flexibility.

A. The effect of I/O linkages on marginal cost

Here, we show that I/O linkages crucially affect marginal costs. We distinguish between the use of intermediate inputs per se (i.e., $\delta > 0$) and heterogeneous usage of intermediate inputs across sectors (i.e., $\omega_{kr} \neq \omega_{k'r} \ \forall k, \forall k' \neq k$, and $\forall r$). Although our focus is on I/O linkages, heterogeneity in sectoral size ω_{ck} and in pricing frictions is also present through κ_k. We first derive how I/O linkages affect several intermediate, key variables.

First, I/O linkages affect the measure of sectors, $\{n_k\}_{k=1}^{K}$. The measure reflects the weighted average of the consumption share of sector k, ω_{ck}, and the importance of sector k as a supplier to the economy, ζ_k

$$n_k = (1 - \psi)\omega_{ck} + \psi\zeta_k, \tag{23}$$

where

$$\zeta_k \equiv \sum_{k'=1}^{K} n_{k't}\omega_{k'k}. \tag{24}$$

We refer to ζ_k as the "outdegree" of sector k, analogous to Acemoglu et al. (2012). The outdegree of sector k is the weighted sum of intermediate input use from sector k by all other sectors $\omega_{k'k}$, with weights $n_{k't}$. In steady state, all firms are identical and we can interpret n_k as the size of sector k.

Without intermediate inputs ($\delta = 0$), $\psi \equiv \delta (\theta - 1) / \theta = 0$, and only consumption shares determine sector size. By contrast, when firms use intermediate inputs for production ($\delta > 0$), heterogeneity in I/O linkages results in additional heterogeneities in sector size. The outdegree of sector k is higher when sector k is a supplier to many sectors or is a supplier of large sectors.

The vector $\aleph$ of sector sizes $\{n_k\}_{k=1}^{K}$ solves

$$\aleph = (1 - \psi) \left[\mathbb{I}_K - \psi\Omega'\right]^{-1} \Omega^C, \tag{25}$$

where $\mathbb{I}_K$ is the identity matrix of dimension K, Ω is the I/O matrix in steady state with elements $\{\omega_{kk'}\}$, and Ω^C is the vector of consumption shares, $\{\omega_{ck}\}$.

Second, heterogeneity in I/O linkages implies each sector faces a different intermediate input price index

$$\mathrm{p}_{kt} = \sum_{k'=1}^{K} \omega_{kk'}p_{k't}. \tag{26}$$

In particular, the sector-k intermediate input price index responds more to variation in the prices of another sector k' when that sector is a large supplier to sector k.

A.1 Direct effect on sectoral marginal costs

With intermediate inputs in production ($\delta > 0$), sectoral marginal costs are a weighted average of sectoral wages, but also sectoral intermediate input price indices

$$mc_{kt} = (1 - \delta) \, w_{kt} + \delta \mathrm{p}_{kt}. \tag{27}$$

The sectoral intermediate input price index, p_{kt}, reflects heterogeneity in I/O linkages. All else equal, an increase in the price of another sector k' implies higher costs of the intermediate input mix. Heterogeneity in I/O linkages allows this channel to differ across sectors.

A.2 Indirect effect through sectoral wages

I/O linkages also affect sectoral wages $\{w_{kt}\}$ indirectly because the efficient mix of labor and intermediate inputs in equation (5) depends on relative input prices. The production function implicitly defines sectoral labor demand for a given level of production y_{kt}

$$y_{kt} = l_{kt} + \delta \, (w_{kt} - \mathrm{p}_{kt}). \tag{28}$$

In a model without I/O linkages ($\delta = 0$), sectoral labor demand is inelastic after conditioning on sectoral production y_{kt}. Here, I/O linkages ($\delta > 0$) imply labor demand depends negatively on wages, because higher wages lead firms to substitute labor for intermediate inputs.

Combining the production function and sectoral labor supply yields

$$w_{kt} = \frac{1}{1 + \delta\varphi} \left[\varphi y_{kt} + \sigma c_t + \delta\varphi \, (\mathrm{p}_{kt} - pc_t) \right] + pc_t. \tag{29}$$

Thus, the optimal choice implies a wedge between sectoral intermediate input prices and aggregate consumption prices, $(\mathrm{p}_{kt} - pc_t)$.

What is the role of this wedge? In a model without I/O linkages ($\delta = 0$), wages respond one to one to variations in aggregate consumption prices pc_t through their effect on labor supply. An increase in sector k' prices positively affects wages in sector k. The relevant elasticity is tied to the consumption share of sector k', $\omega_{ck'}$. This effect is captured by the last term of equation (29).

In the presence of I/O linkages ($\delta > 0$), this last term continues to affect wages. However, the wedge ($\mathrm{p}_{kt} - pc_t$) now additionally comes into play: An increase in sector k' prices has an additional, positive effect on sector k wages when the share of sector k' as a supplier of sector k is larger than its consumption share, that is, when $\omega_{kk'} > \omega_{ck'}$. Intuitively, if sector k' is a large supplier to sector k, a positive variation in $p_{k't}$ has a larger effect on increasing the cost of intermediate inputs for firms in sector k. As a result, firms in sector k increase the demand for labor, and sector k wages go up.

A.3   Effect on sectoral demand

Next, we show I/O linkages can heterogeneously affect how variations in aggregate demand y_t transmit into sectoral demand, $\{y_{kt}\}_{k=1}^{K}$. This follows because sectoral demand is given by

$$y_{kt} = y_t - \eta \left[p_{kt} - (1 - \psi) \, pc_t - \psi \widetilde{p}_t \right], \tag{30}$$

where

$$\widetilde{p}_t \equiv \sum_{k=1}^{K} n_k \mathrm{p}_{kt}. \tag{31}$$

Sectoral demand depends on the sectoral relative price, p_{kt}, relative to a weighted average of aggregate consumption prices, pc_t, and an "average sector-relevant" price, $\widetilde{p}_t$. The latter weights sector-relevant aggregate prices by the size of sectors. We can write it as

$$\widetilde{p}_t = \sum_{k=1}^{K} \zeta_k p_{kt}, \tag{32}$$

that is, the sum of variations in sectoral prices weighted by their outdegrees $\{\zeta_k\}_{k=1}^{K}$.

Following an increase in prices of another sector k', the share of sector k in total demand increases in the outdegree of that other sector. This increase is stronger than the increase in an economy without intermediate inputs if that sector is a big supplier in the whole economy: $\zeta_{k'} > \omega_{ck'}$.

A.4   Effect on total demand

Finally, aggregate demand, y_t, also interacts with heterogeneity in I/O linkages. Aggregating Walras' law across all industries yields

$$y_t = (1 - \psi) \, c_t + \psi z_t, \tag{33}$$

where z_t is the total amount of intermediate inputs. The pure presence of intermediate inputs creates a wedge between total production, y_t, and value-added production, c_t. The dynamics of z_t around the steady state depend on the heterogeneity in I/O linkages across sectors.

We solve for z_t, combining Walras' law, the aggregate production function, aggregate labor supply, and the aggregation of efficient mixes between labor and intermediate inputs,

$$z_t = \frac{[(1+\varphi)(1-\psi) + \sigma(1-\delta)]c_t - (1-\delta)(\widetilde{p}_t - pc_t)}{(1-\psi) + \varphi(\delta - \psi)}. \tag{34}$$

In an economy with no I/O linkages ($\delta = 0$, $\psi = 0$), output equals consumption, $y_t = c_t$. With intermediate inputs ($\delta > 0$), z_t varies positively with c_t: More value-added production requires more intermediate inputs. This channel shows up as the first term in the numerator of equation (34). At the same time, an increase in prices of a given sector k' has a negative effect on z_t when that sector is central in the economy. This second effect is captured by the wedge $(\widetilde{p}_t - pc_t)$, the second term in the numerator of equation (34), equivalent to the condition that sectors are relatively more central than their GDP share implies: $\zeta_{k'} > \omega_{ck'}$. Then, an increase in prices of big suppliers in the economy results in higher prices for intermediate inputs for many sectors and/ or bigger sectors. These sectors then substitute intermediate inputs for labor, and the aggregate demand for intermediate inputs decreases.

To simplify exposition, we write the relationship between y_t and c_t as

$$y_t = (1 + \psi\Gamma_c)c_t - \psi\Gamma_p(\widetilde{p}_t - pc_t), \tag{35}$$

where $\Gamma_c \equiv \frac{(1-\delta)(\sigma+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)}$.

B. Overall solution for log-linearized marginal costs

We combine equations that we derived in the previous subsections to express marginal costs in terms of value-added production and sectoral prices

$$\begin{aligned} mc_{kt} &= \left[1 + \frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right]pc_t + \delta\frac{1+\varphi}{1+\delta\varphi}(p_{kt} - pc_t) + (1-\delta)\frac{\varphi\psi(\eta - \Gamma_p)}{1+\delta\varphi}(\widetilde{p}_t - pc_t) \\ &\quad - \frac{(1-\delta)\varphi\eta}{1+\delta\varphi}p_{kt} + \frac{1-\delta}{1+\delta\varphi}[\sigma + \varphi(1+\psi\Gamma_c)]c_t. \end{aligned} \tag{36}$$

By contrast, in an otherwise identical economy with no I/O linkages ($\delta = 0$), marginal

costs are given by

$$mc_{kt}^{\delta=0} = (1 + \varphi\eta)\, pc_t - \varphi\eta p_{kt} + (\sigma + \varphi)\, c_t. \tag{37}$$

In such an economy, an increase in prices of other sectors, $p_{k't}$, increases marginal costs. This effect uniformly depends on elasticities $1 + \varphi\eta$, and specifically, on the heterogeneity in consumption shares $\omega_{ck'}$.

In our setting, new effects arise. The first line of equation (36) shows how sectoral prices affect sectoral marginal costs in an economy with I/O linkages ($\delta > 0$). The effect of prices of other sectors on sector k marginal costs – contained in aggregate consumption prices via the first term – continues to be present, but is now mitigated because $1 + (1 - \delta)\varphi\eta/(1 + \delta\varphi) < (1 + \varphi\eta)$. At the same time, I/O linkages create new channels. In particular, prices of another sector $p_{k't}$ have a stronger effect on mc_{kt} if (i) sector k' is a big supplier to sector k; that is, $\omega_{kk'} > \omega_{ck'}$ so that the wedge $(\mathrm{p}_{kt} - pc_t) > 0$ (second term on the right-hand side of equation (36)); and (ii) sector k' is a big supplier in the whole economy; that is, $\zeta_{k'} > \omega_{ck'}$ so that the wedge $(\tilde{p}_t - pc_t) > 0$ (third term on the right-hand side of equation (36)). The overall direct effect of variations in $p_{k't}$ on mc_{kt} is

$$\frac{(1 - \delta)\left[1 + (1 - \psi)\varphi\eta + \psi\varphi\Gamma_p\right]}{1 + \delta\varphi}\omega_{ck'} + \delta\frac{1 + \varphi}{1 + \delta\varphi}\omega_{kk'} + (1 - \delta)\frac{\varphi\psi\left(\eta - \Gamma_p\right)}{1 + \delta\varphi}\zeta_{k'}. \tag{38}$$

The last two terms of equation (36) are standard. The fourth term on the right-hand side of equation (36) shows sector k marginal costs decrease in sector k prices. The demand for sectoral output is a decreasing function of its price, and hence, wages in sector k. The fifth term on the right-hand side of equation (36) shows marginal costs increase in value-added production c_t.

IV Theoretical Results

Here, we present new, closed-form results for the response of inflation and consumption to a monetary policy shock. In doing so, we benchmark our economy with heterogeneity in price rigidity against an economy in which prices are homogeneously rigid. We highlight how the I/O structure interacts with the pricing frictions and heterogeneity in sectoral size and shapes our results. We show that the identity effect – which sector contributes the most to monetary transmission – can be crucially affected by the interaction of heterogeneities. Also, the level of aggregation can be key for the degree of monetary non-neutrality. The latter two insights provide important guidance for monetary policymakers trying to correctly assess the most important sectors for the transmission of

monetary policy shocks to output and inflation.

A. Monetary non-neutrality in the simplified model

We start by introducing three assumptions that allow us to obtain results in closed form. First, household utility is log in consumption, $\sigma = 1$, and linear in leisure, $\varphi = 0$, such that

$$w_t = c_t + pc_t. \tag{39}$$

Second, the central bank targets a given level m_t of nominal aggregate demand,

$$m_t = c_t + pc_t, \tag{40}$$

where $pc_t \equiv \sum_{k=1}^{K} \omega_{ck} p_{kt}$.

Third, firms fully discount the future when adjusting prices ($\beta = 0$), so

$$p_{kt}^* = mc_{kt}. \tag{41}$$

Combining all these equations with the sectoral aggregation of prices

$$p_{kt} = (1 - \alpha_k)\, p_{kt}^* + \alpha_k p_{kt-1}, \tag{42}$$

yields

$$p_{kt} = (1 - \alpha_k)\left[(1 - \delta)\, m_t + \delta \mathrm{p}_{kt}\right] + \alpha_k p_{kt-1} \text{ for } k = 1, ..., K \tag{43}$$

with solution for the sectoral vector of prices

$$p_t = (1 - \delta) \sum_{\tau=0}^{\infty} \left([\mathbb{I} - \delta\, (\mathbb{I} - A)\, \Omega]^{-1} A \right)^{\tau} [\mathbb{I} - \delta\, (\mathbb{I} - A)\, \Omega]^{-1} (\mathbb{I} - A)\, \iota m_{t-\tau}. \tag{44}$$

where Ω denotes the matrix of I/O weights, A the diagonal matrix of α_k, and ι a unit vector of suitable dimension.

In the following, we use equations (40) and (43) to build intuition on the determinants of monetary non-neutrality in the model. In particular, we assume the economy is in steady state when a permanent monetary shock hits at period t^* such that $m_t = m$ for all $t \geq t^*$ and $m_t = 0$ for all $t < t^*$. We focus on characterizing the response of the aggregate consumption price index pc_t to the shock in three cases. In this simplified model, the

aggregate price response is a sufficient statistic for the real output response.

First, we consider the case in which price stickiness is homogeneous across sectors. This assumption sets up a useful benchmark for the following cases that feature heterogeneity in price stickiness, as well as our subsequent discussion of the effect of sectoral aggregation. At the same time, we are not placing any restrictions on the sectoral GDP shares ω_{ck} and the I/O structure $\{\omega_{kk'}\}$ yet.

Proposition 1 *When price stickiness is homogeneous across sectors, $\alpha_k = \alpha$ for all k, the response of aggregate consumption prices to a permanent monetary policy shock is*

$$pc_t(\alpha) = \left[1 - \left(\frac{\alpha}{1 - \delta(1-\alpha)}\right)^{t-t^*+1}\right] m \quad for \quad t \geq t^*, \tag{45}$$

such that

(1) $pc_t(\alpha)$ is decreasing in δ for any $t \geq t^*$, and

(2) heterogeneity of consumption shares $\{\omega_{ck}\}_{k=1}^{K}$ and I/O linkages $\{\omega_{kk'}\}_{k,k'=1}^{K}$ is irrelevant for the response of aggregate consumption prices to the monetary shock.

Proof. See online appendix. ■

The proposition presents two insights. First, the stickiness of marginal costs increases in δ; hence, the responsiveness of the aggregate consumption price index to the monetary policy shock decreases in δ. As a result, a lesser price response means stronger monetary non-neutrality. This result mimics the insights of the network multiplier in Basu (1995). Second, heterogeneity of consumption shares and I/O linkages are irrelevant for monetary non-neutrality with homogeneous price stickiness across sectors.

What happens if we allow for heterogeneity in price stickiness? The next proposition shows heterogeneity of price rigidity amplifies (mitigates) the response of output (prices) in all periods except upon impact – when it has no effect. This result follows, as a first step, from a simplified I/O structure. We fully relax this assumption in the subsequent proposition.

Proposition 2 *In an economy in which price stickiness is heterogeneous across sectors and I/O linkages are identical to consumption shares, $\omega_{kk'} = \omega_{ck'}$ for all k, k', the response of aggregate consumption prices to a permanent monetary policy shock is*

$$pc_t = \frac{1-\delta}{1-\delta(1-\overline{\alpha})}\left(1 - \sum_{k=1}^{K}\omega_{ck}\alpha_k^{t-t^*+1}\right) m + \frac{\delta}{1-\delta(1-\overline{\alpha})}\sum_{\tau=1}^{t-t^*}\left(\sum_{k=1}^{K}\omega_{ck}\alpha_k^{\tau}(1-\alpha_k)\right)pc_{t-\tau} \quad if \ t \geq t^*, \tag{46}$$

where $\overline{\alpha} \equiv \sum_{k=1}^{K} \omega_{ck}\alpha_k$, *such that*

(1) The sectoral heterogeneity of price stickiness and consumption shares are irrelevant for the response of output to the monetary shock on impact.

(2) $pc_t \leq pc_t(\overline{\alpha})$ *for* $t > t^*$. *The response of the aggregate consumption prices for* $t \geq t^*$ *is weakly decreasing in the dispersion of price stickiness, and depends on heterogeneities.*

Proof. See online appendix ■

Proposition 2 studies a simplified steady-state network economy in which sectoral output is used in equal proportions by consumers and other sectors. In this economy, sectoral heterogeneity in price stickiness amplifies monetary non-neutrality, as in Carvalho (2006) and Carvalho and Schwartzman (2015). In particular, sectoral heterogeneity of price stickiness does not affect the impact response, but increases the persistence of monetary non-neutrality. However, in an economy in which firms set prices in a forward-looking manner ($\beta > 0$), the increased persistence of monetary non-neutrality would also imply stronger monetary non-neutrality on impact.

What happens in the fully unrestricted case when price stickiness, sectoral size, and I/O linkages are heterogeneous across sectors?

Proposition 3 *Let* pc_t *denote the response of the aggregate consumption price index to a permanent monetary shock in an economy with no restrictions on the sectoral heterogeneity of price stickiness* $\{\alpha_k\}$, *sectoral size* $\{\omega_{ck}\}$, *and I/O linkages* $\{\omega_{kk'}\}$. *In this economy,*

$$pc_t = (1-\delta)\left(1 - \sum_{k=1}^{K} \omega_{ck}\alpha_k^{t-t^*+1}\right) m + \delta \sum_{\tau=0}^{t-t^*} \sum_{k=1}^{K} \left(\sum_{k'=1}^{K} \omega_{ck'}\alpha_{k'}^{\tau}(1-\alpha_{k'})\omega_{k'k}\right) p_{kt-\tau} \quad for \quad t \geq t^*,$$

(47)

with $p_{kt-\tau} = 0$ *if* $t < t^*$ *such that*

(1) The response of pc_t *is weaker on impact than in Proposition 2, when* $u_k \equiv \sum_{k'=1}^{K} \omega_{ck'}(1-\alpha_{k'})\omega_{k'k} > (1-\overline{\alpha})\omega_{ck}$ *for the sectors with the stickiest prices.*

(2) The response of pc_t *for* $t > t^*$ *is more persistent than in Proposition 2, when for sectors with the stickiest prices, either of the following conditions hold: (i)* $\overline{\omega}_k \equiv \frac{1}{K}\sum_{k'=1}^{K} \omega_{k'k} > \omega_{ck}$, *(ii)* $u_k > (1-\overline{\alpha})\omega_{ck}$, *(iii)* $COV(\omega_{ck'}\alpha_{k'}^{\tau}(1-\alpha_{k'}), \omega_{k'k}) > 0$.

Proof. See online appendix. ■

The fully unrestricted interaction creates an even richer transmission of monetary policy shocks. In doing so, the exact interaction of nominal and real heterogeneities is crucial for understanding the effects of a monetary shock on output and prices. The implications we find are completely new to the literature.

First, upon impact, the price effect is weaker – and hence monetary non-neutrality larger – than under the restricted heterogeneity of I/O linkages in Proposition 2. This effect happens when the largest sectors with the stickiest prices are also important suppliers to the largest, most flexible sectors. Second, in subsequent periods, aggregate price changes become more persistent given the three conditions in the second part of the proposition. In conjunction with the first result, this increased persistence means more persistence and larger monetary non-neutrality than under restricted heterogeneity.[3]

In particular, a corollary of these results is a novel identity effect: The extent to which a sector transmits monetary policy shocks depends on the exact interaction of heterogeneity in pricing frictions and heterogeneity in sectoral size and I/O linkages. The following corollaries summarize the contribution of each sector to the path of aggregate prices, first upon impact only, and then for all subsequent periods.

Corollary 1 *Upon impact, each sectoral contribution to the path of aggregate prices is given*

(1) independently of heterogeneity in I/O linkages under homogeneous price rigidity, by

$$\left[1 - \left(\frac{(1-\delta)(1-\alpha)}{1 - \delta\,(1-\alpha)} \right) \right] \omega_{ck} m, \ \ and \tag{48}$$

(2) by a function of heterogeneities under heterogeneous price rigidity,

$$e_k' \Omega^c \, (1-\delta) \left[\mathbb{I} - \delta\,(\mathbb{I} - A)\,\Omega \right]^{-1} (\mathbb{I} - A)\, \iota m, \tag{49}$$

where e_k is the kth basis vector, Ω the matrix of I/O weights, Ω^c the vector of consumption weights, A the diagonal matrix of α_k, and ι a unit vector. In the special case of $\omega_{ck'} = \omega_{kk'}$, the kth element equals $\left[1 - \left(\frac{(1-\delta)(1-\alpha_k)}{1-\delta(1-\bar{\alpha})} \right) \right] \omega_{ck} m$.

Corollary 2 *In subsequent periods, $t > t^*$, each sectoral contribution to the path of aggregate prices is given*

(1) independently of heterogeneity in I/O linkages under homogeneous price rigidity,

$$\left[1 - \left(\frac{\alpha}{1 - \delta\,(1-\alpha)} \right)^{t-t^*+1} \right] \omega_{ck} m, \ \ and \tag{50}$$

---

[3]Note we have left a contemporaneous term in the second term on the right-hand side of the proposition to make it more comparable to Proposition 2. The online appendix contains an explicit solution in terms of parameters and the monetary shock only.

(2) by the interaction of the heterogeneities under heterogeneous price rigidity,

$$e_k' \Omega^c (1 - \delta) \sum_{\tau=0}^{t-t^*} \left([\mathbb{I} - \delta (\mathbb{I} - A) \Omega]^{-1} A \right)^\tau [\mathbb{I} - \delta (\mathbb{I} - A) \Omega]^{-1} (\mathbb{I} - A) \iota m_{t-\tau}, \qquad (51)$$

where e_k is the kth basis vector, Ω the matrix of input-output weights, Ω^c the vector of consumption weights and A the diagonal matrix of α_k.

Proof. See online appendix. ∎

The importance of each sector to the monetary transmission mechanism crucially depends on the interaction of heterogeneities. Timing also plays an important role, whether we consider the impact response, or subsequent responses. The ranking of importance across the different cases can clearly change. For example, sectors that are important upon impact may be less important later on. Similarly, heterogeneity in I/O linkages can influence the importance of a sector. Ultimately, the extent to which the interaction of heterogeneities can affect the relative importance of sectors for aggregate fluctuations is a quantitative question.

We have not placed any restrictions on the total effect of monetary policy. Monetary policy shocks could easily generate the same real effects under different assumptions about which heterogeneities are present, whereas sectors are differently important in generating the same real effects. For example, consider a two-sector economy that generates a particular path of real output and prices following a monetary policy shocks. Now, if we simply flip all parameter values for the two sectors, the aggregate paths of output and prices remain identical. At the same time, the loading of monetary policy transmission flips. This identity effect may be important, for example, when optimally targeting monetary policy.

B. The effect of sectoral aggregation

Most of the literature so far studies models with only a limited number of sectors such as 6 or 30. In this subsection, we study in the simplified model whether the choice of aggregation matters for the effect of monetary policy shocks on key macroeconomic aggregates. In particular, we compare the effects of a permanent monetary policy shock on aggregate consumption prices at two levels of disaggregation: one with K sectors (denoted by pc_t), and another in which random pairs of sectors are merged log-linearly, so it has $K/2$ sectors (denoted by $\overline{pc}_t$). For simplicity, we assume K is even. In mathematical

terms, we compare

$$pc_t \equiv \sum_{k=1}^{K} \omega_{ck} p_{kt},$$

$$\overline{pc}_t \equiv \sum_{k'=1}^{K/2} \overline{\omega}_{ck'} \overline{p}_{k't}$$

such that

$$\overline{\omega}_{ck'} \equiv \omega_{c2k'-1} + \omega_{c2k'},$$

$$\overline{p}_{k't} \equiv \lambda_{k'} p_{2k'-1} + (1 - \lambda_{k'}) p_{2k'},$$

$$\lambda_{k'} = \frac{\omega_{c2k'-1}}{\omega_{c2k'-1} + \omega_{c2k'}}.$$

for $k' = 1, ..., K/2$. This specification is without loss of generality when merging two consecutive sectors, because the ordering of sectors is arbitrary.

In addition, Calvo parameters are aggregated among merged sectors by

$$\overline{\alpha}_{k'} \equiv \lambda_{k'} \alpha_{2k'-1} + (1 - \lambda_{k'}) \alpha_{2k'}.$$

and their I/O linkages as

$$\omega_{k's'} = \xi_{k'} \left(\omega_{2k'-1,2s'-1} + \omega_{2k'-1,2s'} \right) + (1 - \xi_{k'}) \left(\omega_{2k',2s'-1} + \omega_{2k',2s'} \right)$$

for $k', s' = 1, ..., K/2$. The weights $\xi_{k'}$ equal the shares of sectors in total intermediate input use of the merged sectors.

First, we show that monetary non-neutrality is higher in a more disaggregated economy in the absence of I/O linkages ($\delta = 0$).

Proposition 4 *When $\delta = 0$, the difference in the response of consumption prices to a permanent monetary shock at the two levels of disaggregation is given by*

$$\overline{pc}_t - pc_t = \sum_{k'=1}^{K/2} \overline{\omega}_{ck'} \left[\lambda_{k'} \alpha_{2k'-1}^{t-t^*+1} + (1 - \lambda_{k'}) \alpha_{2k'}^{t-t^*+1} - \overline{\alpha}_{k'}^{t-t^*+1} \right] m \qquad (52)$$

such that (i) $pc_t = \overline{pc}_t$ for $t = t^*$, (ii) $pc_t < \overline{pc}_t$ for $t > t^*$, (iii) $\overline{pc}_t - pc_t$ is increasing in the dispersion of Calvo parameters among merged sectors, and (iv) $\overline{pc}_t - pc_t$ is increasing in the consumption shares of merged sectors with the highest dispersion of Calvo parameters among merged sectors.

Proof. See online appendix. ∎

This proposition is an application of Jensen's inequality. The larger is the dispersion in frequencies of price changes among merged sectors, the smaller is the monetary non-neutrality as the level of disaggregation becomes increasingly more coarse relative to a more finely disaggregated economy. The difference in monetary non-neutrality across the two levels of disaggregation increases as time passes after the impact response when both are identical. The intuition for this result is the same as in our analysis above: Aggregating sectors overstates the response of prices to a monetary policy shock, because the measure of first-time responders is higher when the two sectors have the same frequency of price changes than when they exhibit different frequencies but with the same mean.

Next, we show a new result to the literature: Further disaggregation also leads to more monetary non-neutrality by overstating the amplification introduced by intermediate inputs. We show this result under restricted heterogeneity of the production network ($\omega_{sk} = \omega_{ck}$).

Proposition 5 *When $\delta \in (0,1)$ and $\omega_{sk} = \omega_{ck}$ for all $s, k = 1, .., K$, the difference in the response of consumption prices to a permanent monetary shock at the two levels of disaggregation is given by*

$$
\begin{aligned}
\overline{pc}_t - pc_t \;=\; & \frac{1-\delta}{1-\delta\,(1-\overline{\alpha})} \sum_{k'=1}^{K/2} \overline{\omega}_{ck'} \left(\lambda_{k'} \alpha_{2k'-1}^{t-t^*+1} + (1-\lambda_{k'}) \alpha_{2k'}^{t-t^*+1} - \overline{\alpha}_{k'}^{t-t^*+1} \right) m \\
& - \frac{\delta}{1-\delta\,(1-\overline{\alpha})} \sum_{\tau=1}^{\infty} \left(\sum_{k'=1}^{K/2} \overline{\omega}_{ck'} \left[\begin{array}{c} \lambda_{k'}\left(1-\alpha_{2k'-1}\right)\alpha_{2k'-1}^{\tau} + (1-\lambda_{k'})\left(1-\alpha_{2k'}\right)\alpha_{2k'}^{\tau} \\ - (1-\overline{\alpha}_{k'})\overline{\alpha}_{k'}^{\tau} \end{array} \right] \right) pc_{-\tau}, \quad (53)
\end{aligned}
$$

where $pc_t = 0$ if $t < t^*$. Results (i) through (iv) in the previous proposition continue to hold and are amplified by the intermediate input channel.

Proof. See online appendix. ∎

Intermediate inputs introduce strategic complementarity in the response of prices to a monetary shock, which implies consumption prices are more persistent at a finer level of disaggregation. This effect is stronger when merged sectors with more heterogeneous frequencies of price changes are also sectors with more homogeneous consumption shares, for a stronger short-term response of aggregate prices. The effect also increases when merged sectors with higher consumption shares have lower frequencies of price changes.

Third, the next proposition relaxes any restriction in the I/O structure of the production network. Now, the effect of aggregation on the amplification effect of monetary non-neutrality introduced by intermediate inputs is an intricate combination of the sectoral distributions of the frequency of price changes, consumption weights, and I/O linkages.

Proposition 6 *When* $\delta \in (0,1)$ *and I/O linkages are unrestricted, the difference in the response of consumption prices to a permanent monetary shock at the two levels of disaggregation is given by*

$$\overline{pc}_t - pc_t = (1-\delta)\left[\sum_{k'=1}^{K/2}\overline{\omega}_{ck'}\left(\lambda_{k'}\alpha_{2k'-1}^{t-t^*+1} + (1-\lambda_{k'})\alpha_{2k'}^{t-t^*+1} - \overline{\alpha}_{k'}^{t-t^*+1}\right)\right]m$$

$$-\delta\sum_{\tau=0}^{t-t^*}\sum_{k'=1}^{K/2}\sum_{s'=1}^{K/2}\overline{\omega}_{cs'}\left[\begin{pmatrix}\lambda_{s'}\alpha_{2s'-1}^{\tau}\left(1-\alpha_{2s'-1}^{\tau}\right)\frac{\omega_{2s-1,2k'-1}}{\omega_{c2k'-1}}\frac{\overline{\omega}_{ck'}}{\overline{\omega}_{s'k'}}\\ +(1-\lambda_{s'})\alpha_{2s'}^{\tau}\left(1-\alpha_{2s'}^{\tau}\right)\frac{\omega_{2s,2k'-1}}{\omega_{c2k'-1}}\frac{\overline{\omega}_{ck'}}{\overline{\omega}_{s'k'}}\\ -\overline{\alpha}_{s'}^{\tau}\left(1-\overline{\alpha}_{s'}\right)\end{pmatrix}\frac{\omega_{c2k'-1}\overline{\omega}_{s'k'}}{\overline{\omega}_{ck'}}p_{2k'-1,t-\tau}\\ +\begin{pmatrix}\lambda_{k'}\alpha_{2s'-1}^{\tau}\left(1-\alpha_{2s'-1}^{\tau}\right)\frac{\omega_{2s-1,2k'}}{\omega_{c2k'}}\frac{\overline{\omega}_{ck'}}{\overline{\omega}_{s'k'}}\\ +(1-\lambda_{s'})\alpha_{2s'}^{\tau}\left(1-\alpha_{2s'}^{\tau}\right)\frac{\omega_{2s,2k'}}{\omega_{c2k'}}\frac{\overline{\omega}_{ck'}}{\overline{\omega}_{s'k'}}\\ -\overline{\alpha}_{s'}^{\tau}\left(1-\overline{\alpha}_{s'}\right)\end{pmatrix}\frac{\omega_{c2k'}\overline{\omega}_{s'k'}}{\overline{\omega}_{ck'}}p_{2k',t-\tau}\end{array}\right].$$

Proof. See online appendix. ∎

So far, Jensen's inequality captures the whole effect of sectoral aggregation, because the response of aggregate prices to a monetary shock depends on the sum of non-linear functions of the sectoral frequency of price changes, $\{\alpha_k\}_{k=1}^K$. The same is true in this proposition for the first line on the RHS of the expression for $\overline{pc}_t - pc_t$. However, it is not true for the amplification term that is due to intermediate input use. In particular, for each of the large round parentheses in the second line in the above expression, now the difference between the two levels of aggregation depends non-trivially on the I/O linkages. Compared to Proposition 5, we now could have more or less monetary non-neutrality. In the fully heterogeneous setting, the exact quantitative effect will therefore depend on the joint distribution of heterogeneities, which we study quantitatively below.

V Data and Calibration

A detailed calibration to the U.S. economy is one of the contributions of this paper. The data we use can potentially provide the basis for many other model evaluations, including detailed policy analyses. The main data contribution lies in pinning down three measurable sources of heterogeneity: different combinations of intermediate inputs for production, different sectoral sizes, and heterogeneous Calvo rates. In making these choices, the granularity of the I/O data determines the definition of sectors for the PPI data. We now describe the data we use to construct the I/O linkages, measures of sectoral size, and price stickiness.

A. Input-Output tables

The BEA produces I/O tables detailing the dollar flows between all producers and purchasers in the U.S. Producers include all industrial and service sectors. Purchasers include industrial sectors, households, and government entities. The BEA constructs the I/O tables using Census data that are collected every five years. The BEA has published I/O tables every five years beginning in 1982 and ending with the most recent tables in 2012. The I/O tables are based on NAICS industry codes. Prior to 1997, the I/O tables were based on SIC codes.

The I/O tables consist of two basic national-accounting tables: a "make" table and a "use" table. The make table shows the production of commodities by industry. Rows present industries, and columns present the commodities each industry produces. Looking across columns for a given row, we see all the commodities a given industry produces. The sum of the entries comprises industry output. Looking across rows for a given column, we see all industries producing a given commodity. The sum of the entries adds up to the output of a commodity. The use table contains the uses of commodities by intermediate and final users. The rows in the use table contain the commodities, and the columns show the industries and final users that utilize them. The sum of the entries in a row is the output of that commodity. The columns document the products each industry uses as inputs and the three components of "value added": compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. The sum of the entries in a column adds up to industry output.

We utilize the I/O tables for 2002 to create an industry network of trade flows. The BEA defines industries at two levels of aggregation: detailed and summary accounts. We use both levels of aggregation to create industry-by-industry trade flows. The tables also pin down sectoral size.

The BEA provides concordance tables between NAICS codes and I/O industry codes. We follow the BEA's I/O classifications with minor modifications to create our industry classifications. We account for duplicates when NAICS codes are not as detailed as I/O codes. In some cases, an identical set of NAICS codes defines different I/O industry codes. We aggregate industries with overlapping NAICS codes to remove duplicates.

We combine the make and use tables to construct an industry-by-industry matrix that details how much of an industry's inputs other industries produce (see section III of the online appendix for details).

B. Price stickiness data

We use the confidential microdata underlying the producer price data (PPI) from the BLS to calculate the frequency of price adjustment at the industry level.[4] The PPI measures changes in selling prices from the perspective of producers, and tracks prices of all goods-producing industries, such as mining, manufacturing, and gas and electricity, as well as the service sector.[5]

The BLS applies a three-stage procedure to determine the sample of individual goods. In the first stage, to construct the universe of all establishments in the U.S., the BLS compiles a list of all firms filing with the Unemployment Insurance system. In the second and third stages, the BLS probabilistically selects sample establishments and goods based on either the total value of shipments or the number of employees. The BLS collects prices from about 25,000 establishments for approximately 100,000 individual items on a monthly basis. The BLS defines PPI prices as "net revenue accruing to a specified producing establishment from a specified kind of buyer for a specified product shipped under specified transaction terms on a specified day of the month." Prices are collected via a survey that is emailed or faxed to participating establishments. Individual establishments remain in the sample for an average of seven years until a new sample is selected to account for changes in the industry structure.

We calculate the frequency of price changes at the goods level, FPA, as the ratio of the number of price changes to the number of sample months. For example, if an observed price path is \$10 for two months and then \$15 for another three months, one price change occurs during five months, and the frequency is 1/5. We aggregate goods-based frequencies to the BEA industry classification.

The overall mean monthly frequency of price adjustment is 16.78%, which implies an average duration, $-1/\log(1 - FPA)$, of 6.15 months. Substantial heterogeneity is present in the frequency across sectors, ranging from as low as 4.01% for the semiconductor manufacturing sector (duration of 56.26 months) to 93.75% for dairy production (duration of 0.83 months).

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[4]The data have been used before in Nakamura and Steinsson (2008), Goldberg and Hellerstein (2011), Bhattarai and Schoenle (2014), Gilchrist, Schoenle, Sim, and Zakrajšek (2015), Gorodnichenko and Weber (2016), Weber (2015), and D'Acunto, Liu, Pflueger, and Weber (2016).

[5]The BLS started sampling prices for the service sector in 2005. The PPI covers about 75% of the service sector output. Our sample ranges from 2005 to 2011.

C. Parameter Calibration

We calibrate our model at different levels of detail to analyze monetary non-neutrality. One contribution is the calibration of a highly disaggregated 341-sector economy, which we discuss in section VI, and contrast key results with more aggregated economies.

We calibrate the model at the monthly frequency using standard parameter values in the literature (see Table 1). The coefficient of relative risk aversion σ is 1, and $\beta = 0.9975$, implying an annual risk-free interest rate of 3%. We set $\varphi = 2$, implying a Frisch elasticity of labor supply of 0.5. We set δ, the average share of inputs in the production function, to 0.5, in line with Basu (1995) and empirical estimates. We set the within-sector elasticity of substitution θ to 6, implying a steady-state markup of 20%, and the across-sector elasticity of substitution η to 2, in line with Carvalho and Lee (2011). We set the parameters in the Taylor rule to standard values of $\phi_\pi = 1.24$ and $\phi_c = 0.33/12$ (see Rudebusch (2002)) with a persistence parameter of monetary shocks of $\rho = 0.9$. We also study a calibration with interest rate smoothing as in Coibion and Gorodnichenko (2012).[6]

We investigate the robustness of our findings to permutations in parameter values in section IV of the online appendix. Overall, the main conclusions remain unaffected by variations in assumptions.

VI Quantitative Results

We now study the importance of different heterogeneities for the real effects of monetary shocks in detailed calibration of the model, for the identity of the sectors that are the biggest contributors to real effects, and how sectoral aggregation affects real effects of monetary policy shocks.

A. Monetary policy shocks and monetary non-neutrality

We now present our first quantitative result: Heterogeneity in price stickiness is the main driver behind real effects of monetary policy. At the same time, the interaction of heterogeneous price stickiness, sector size, and I/O linkages can lower or amplify real effects, but only by small amounts. This result, however, depends on the level of granularity. Heterogeneity in the frequency of price changes is also the main driver behind the response of inflation, and heterogeneity in sector size or I/O linkages contribute little.

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[6]When Coibion and Gorodnichenko (2012) estimate a Taylor rule without interest rate smooth but persistent shocks, they find estimates of the autoregressive parameter of monetary policy shocks equal to 0.96.

We proceed by studying the response of consumption, inflation, and real marginal costs to a 1% monetary policy shock. The benchmark economy is a fully heterogeneous economy in which price stickiness, size, and I/O linkages differ across sectors. We then shut down one heterogeneity at a time to develop step-wise intuition analogous to section IV.

We calibrate five different cases to arrive at our results.[7] Table 2 lists the different combinations of frequencies of price adjustments across sectors, sector sizes, and I/O linkages we study. Table 3 and Figure 1 show our results. We discuss the detailed results of the different cases in the online appendix.

In summary, our quantitative model suggests the following conclusion: Heterogeneity in price stickiness is the main driver of the real effects of monetary policy shocks in our calibration of a 341-sector economy to the empirical distribution of price stickiness from the BLS, and sectoral size and the I/O structure from the BEA. I/O linkages and heterogeneity in sectoral size have some effect as the different cases show, but these effects are much smaller than the effects of heterogeneity in price stickiness. These findings suggest no strong systematic relationship between price flexibility and the importance of sectors as suppliers of flexible sectors, or the economy as a whole. Empirically, the correlation of price stickiness with consumption weights is 0.05, with first-order outdegrees 0.47, and the correlation of outdegrees with sector size is 0.01.

Our baseline studies a model with Taylor rule and persistent shocks. Instead, we also consider the sensitivities of our findings to a Taylor rule with interest-rate smoothing. Coibion and Gorodnichenko (2012) show that interest-rate smoothing is a better description for persistent target rates compared to persistent shocks. We follow their specification of the Taylor rule and vary the degree of policy smoothing from 0.1 to 0.9.

Figure 2 reports the results. We scale the shocks so that the impact response for consumption is identical across all specifications of the Taylor rule. When we compare the response of consumption, inflation, and real marginal costs of an economy with a Taylor rule and persistent shocks (solid line) to an economy with a high degree of interest-rate smoothing which Coibion and Gorodnichenko (2012) argue is the empirically relevant case ($\rho = 0.9$), we find similar impact responses but also cumulative responses across both calibrations. A lower degree of interest rates smoothing instead results in substantially larger real effects but also price responses consistent with the intuition that a demand

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[7]We also studied additional economies in which heterogeneous I/O linkages equal consumption shares ($(\omega_{kk'} = \omega_{ck})$ to mirror some special cases of section IV. Results are similar to the cases we discuss in detail in the appendix.

shock affects prices and quantities in the same direction and to a larger extent if monetary policy does not buffer the shock.

B. Heterogeneity across sectors, and identity effects

So far, we find heterogeneous price stickiness is the key driver behind large effects of monetary policy shocks, whereas heterogeneous sector size or I/O linkages seem to play a secondary role. We will see below, however, that such a conclusion would be premature. Our analysis shows substantial heterogeneity in the sectoral responses to the common, monetary policy shock. Which sector transmits the monetary policy shock the most depends crucially on our specification of heterogeneities. This finding presents another new and important result of our paper, especially for policymakers. Heterogeneity in markup responses reflects heterogeneity in real output and identity effects.

We present these results by focusing on the 10 most and least contractionary sectoral output contributors to real output effects of monetary shocks. Table 4 reports the respective cumulative real effects of monetary policy shocks in Panels A and B for our different cases.

We know from the discussion in section IV that all sectors are equally responsive in models with homogeneous flexible or sticky prices. The actual response in an economy in which sectors differ in their degree of price stickiness, sector size, and I/O linkages differs markedly across sectors. We see in Table A that the 10 least responsive contractionary sectors have a large and positive response to a contractionary monetary policy shock. The positive response can happen if these sectors are substantially more flexible than the average sector in the data which is indeed the case. Sectors with a positive consumption response have an average frequency of price adjustment that is larger by a factor of 3 relative to the average sector. In panel B, instead, we see very large negative responses among the 10 most responsive sectors to a contractionary shock.

We see in column (2) and (3) that shutting down either heterogeneity in sector size or I/O linkages reduces the effective granularity substantially. Both in panel A and B, we see a large compression in the responses. The most responsive sectors in columns (2) and (3) respond by only 25% of the response of the most responsive sector in column (1) with all heterogeneities presents and the range of the responses between sector 1 and 10 shrinks by a factor of 5.

When we shut down both heterogeneity in sector size and I/O linkages and only focus on differences in price stickiness across sectors, we see an additional compression to the mean both among the least and most responsive sectors. When we compare the

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response across columns (1) to (4), we see (i) heterogeneous price stickiness is central for a differential response across sectors to a common monetary policy shock; (ii) heterogeneity in sector size and I/O linkages by themselves add to the granularity of the economy relative to column (4); (iii) it is the inter-linkages between heterogeneity in sector size, price stickiness, and I/O linkages that have a big effects on the contribution of the most and least responsive sectors to a common monetary policy shock.

The results in Table 4 show that the intricacies in which different heterogeneities interact play a crucial role for the relative contribution of different sectors to the aggregate real effects of monetary policy. Purely focusing on the average impact and cumulative effects masks substantial heterogeneity across sectoral responses and policy makers that would aim to stabilize certain sectors would possibly commit policy mistakes by only focusing on the heterogeneity in price stickiness across sections which is a classical result in the literature (see Aoki (2001)).

Figure 3 graphically illustrates the identity effects across all 341 sectors when we go from case 1 in which all heterogeneities are present to case 4 with only heterogeneous price stickiness across sectors. Rankings change substantially across sectors with some sectors changing up to 300 ranks. Hence, the exact choice of heterogeneities is clearly important for the identity of sectors in the transmission mechanism.

Heterogeneity in markups reflects the heterogeneity in real output effects. Price markups are of independent importance and interest, because they measure the inefficiency in the economy and are equivalent to a countercyclical labor wedge (see Gali et al. (2007)). In our setting, the product market wedge is the sole driver of the labor wedge which is consistent with recent empirical work by Bils, Klenow, and Malin (2014).[8] The level of markups in the full model is higher than in the homogeneous benchmark case, and markups display a rich, dynamic pattern. We report these findings in Figure A.4 of the online appendix.

The effect of fully interacted heterogeneities in our model becomes clear in comparison to the completely homogeneous economy. The markup responses of the homogeneous economy are summarized in Panel (a) of Figure A.4. All sectoral responses are fast-decaying and identical across all percentiles. The markup response is more than 4.5% on impact, with a half-life of eight periods.

By contrast, two differential facts emerge for the full model (case 1): First, the median sectoral response is substantially larger. The initial median markup response increases to approximately 6%. The dashed, thick blue line summarizes the median response. The

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[8]Shimer (2009) stresses the lack of work on heterogeneity in the product market, a channel we are putting forward and expanding upon by allowing for interactions of different heterogeneities.

half-life of the median response is twice as long as in the homogeneous case.

Second, substantial dispersion exists in the markup response. The top 5th percentile of markups increases to over 10%; the bottom 5th percentile does not increase above 4%. The sectoral markups also show very different dynamic patterns: The top percentiles show a hump-shaped response that is very persistent, with a half-life of more than 15 periods. At the same time, the lowest percentiles decay exponentially with a half-life of less than ten periods. These very different price-markup responses directly result from the convolutions of the different underlying heterogeneities. They open up new avenues to study how interactions of different heterogeneities shape inefficiencies in the economy.

C. Sectoral Aggregation and Real Effects

In this section, we study the response of our model economy to monetary policy shocks for different levels of disaggregation keeping constant the average degree of price stickiness across levels of aggregation. The choice of aggregation results in large differences in real effects across model economies. We arrive at this conclusion in two steps.

First, we compare the two levels of granularity published by the BEA: detail (effectively 341 sectors), summary (56 sectors), and an even coarser aggregation with only 7 sectors. The left panel of Figure 4 and Table 5 report our findings. Cumulative real effects of monetary policy are 6% larger in the more disaggregated 341-sector economy than in the 56-sector calibration but almost 50% larger than in an economy with only 7 sectors which has been the approximate number of sectors in many calibrations in the literature.

The inflation response is interesting: Upon impact, the inflation response is very similar across different levels of disaggregation, whereas already large differences exists in the consumption response across calibrations. Moreover, the inflation response of the 56 sector economy is larger on impact than the response for the 7 sector economy but the consumption response on impact is also larger for the more disaggregated model. This finding cautions against drawing inference for monetary policy from the impact response of inflation to monetary policy shocks.

Second, motivated by these findings as well as our theoretical results in section IV – that the degree of granularity can matter for the real effects of monetary policy – we now systematically show the importance of granularity when we aggregate sectors by size instead of following the BEA aggregation.

In many models, sector size is a good proxy for sector technology. The right panel of Figure 4 and Table A.2 in the online appendix show the results. The real effects

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of monetary policy are now dramatically affected. They are more than 40% larger on impact for the most disaggregated economy compared to the less disaggregated economy with only 56 sectors, even though the impact response of inflation is again similar. Real effects are monotonically increasing in the granularity of the economy. Figure A.2 in the online appendix shows the dispersion in the frequency of price adjustment shrinks for less granular economies.

What mechanisms are driving these aggregation effects? We find the interactions of heterogeneities are more important than the convexification of price rigidities in creating aggregation effects, but they act with some delay. We show the relative importance by computing the total consumption price paths in a 341 and an 7 sector economy, as well as the two components from equation (47), due to convexification and interactions. Figure A.3 in the online appendix illustrates how the two channels contribute to the total gap.

VII   Concluding Remarks

We present new theoretical and quantitative insights into the transmission of monetary policy shocks when heterogeneity in price stickiness, the I/O structure, and sector size interact.

Although rich theoretical predictions exist for how the interaction of these heterogeneities shapes the real output effects of monetary shocks, we find in our calibration to the US economy that heterogeneity in price stickiness is the central mechanism for generating large and persistent real output effects. Heterogeneity in I/O linkages, instead, only plays a marginal role. In addition, we document that small-scale models might substantially underestimate output effects – even though the impact response of inflation is almost identical across different levels of granularity. We also find that heterogeneity in price rigidity is key in determining which sectors are the most important contributors to the transmission of monetary shocks. Finally, while heterogeneity in sector size and I/O linkages only play a minor role in shaping the aggregate real output effects of monetary shocks, we find that they jointly increase the effective granularity in the economy.

Our results have important policy implications. First, the impact response of inflation to a monetary policy shock is not sufficient for the real effects of monetary shocks. Second, the real effects of more granular economies are substantially larger compared to economies with only a small number of sectors. And finally, all heterogeneities we study are important for the identity and contribution of the most important sectors to the real effects of monetary policy. In particular, sectoral price stickiness is not the only determinant for the contribution to aggregate real effects. Despite mattering only marginally for the overall

real effects, heterogeneity in sector size and I/O linkages are important contributors and generate an increase in the effective granularity: the most important sectors contributing to aggregate real effects become even more important, whereas the least important sectors become even less important. The latter result suggest that central bank should no longer only focus on stabilizing the prices of the most sticky-price sectors but should jointly focus on the interaction of sector size, I/O linkages, and price stickiness.

While we study a rich set of heterogeneities at the sectoral level that are we can directly map into data, we leave out other important differences such as the degree of durability, differences in intermediate input shares, or heterogeneity in the elasticities of demand. We hypothesize that more downstream sectors with stickier prices might also produce more durable goods compared to commodities sectors with flexible prices and higher elasticities of demand which would possibly amplify real output effects of monetary shocks. We believe these heterogeneities are important to study but leave a detailed analysis to future research.

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Figure 1: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock**
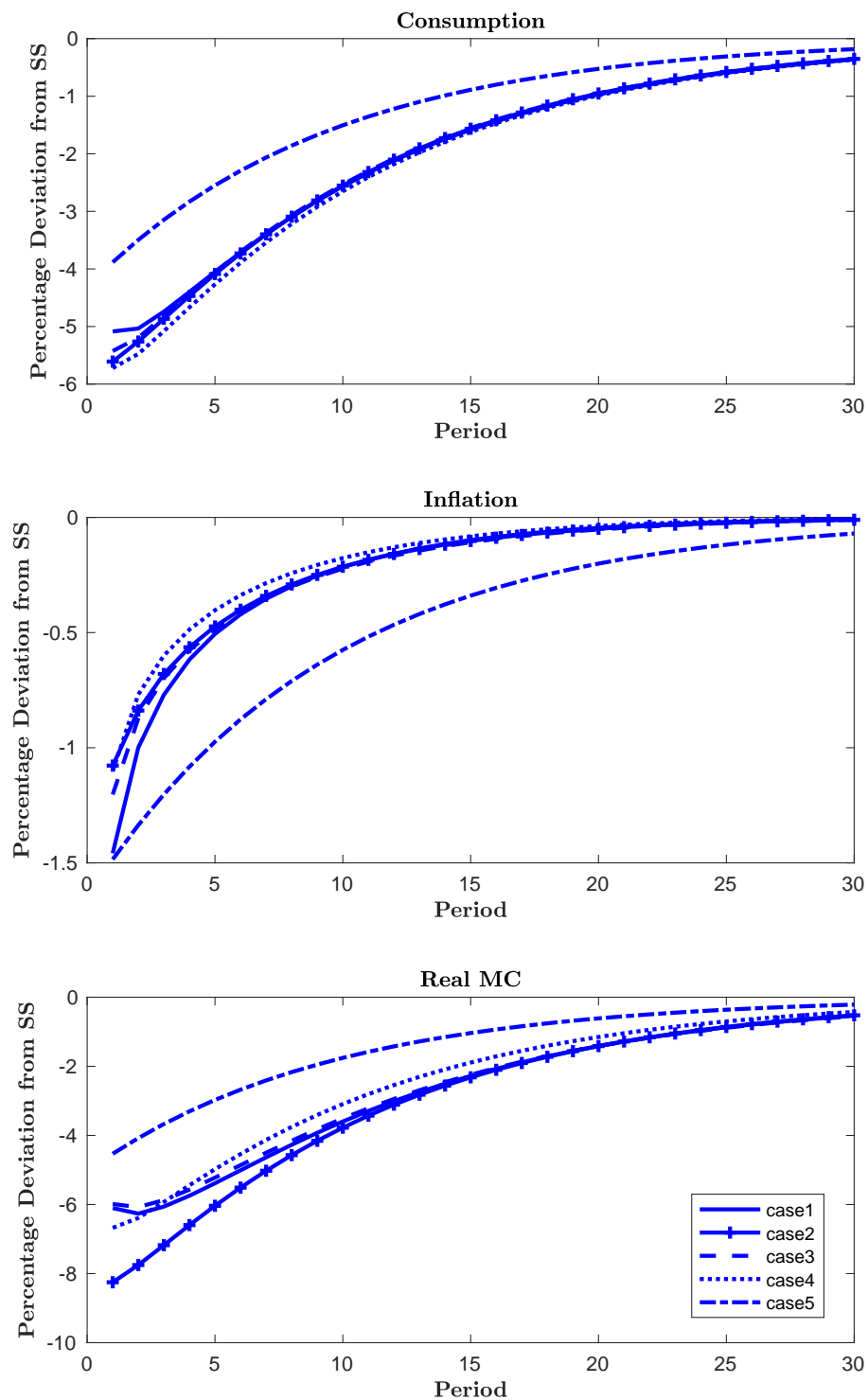
This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard-deviation monetary policy shock for a 341-sector model for different cases (see Table 2 for a description of the different cases).

Figure 2: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock: Interest-rate Smoothing**
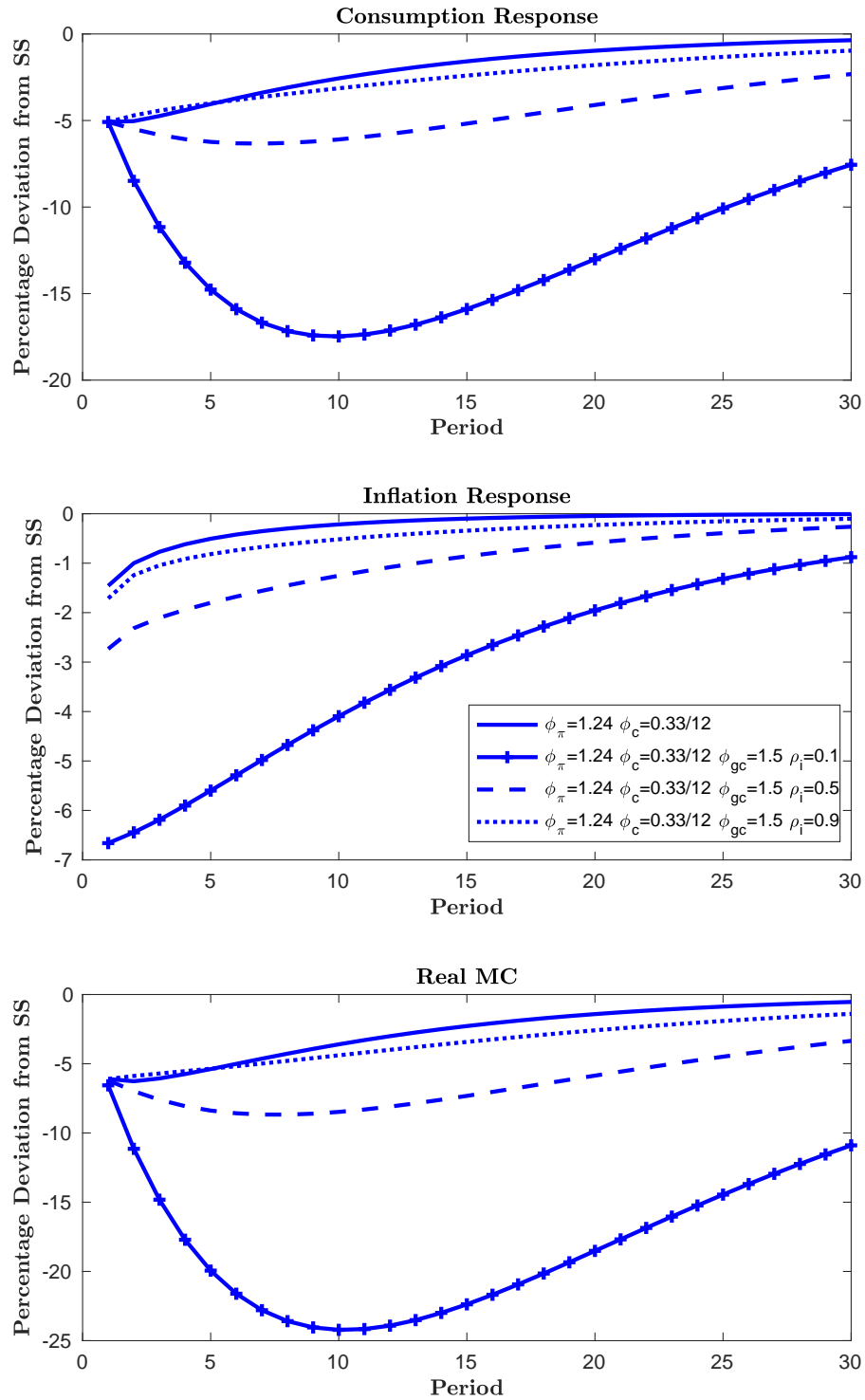
This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard-deviation monetary policy shock for a 341-sector model for different cases (see Table 2 for a description of the different cases) for different Taylor rules with interest-rate smoothing.

Figure 3: **Ranking of Sectors, Cases 1 and 4**
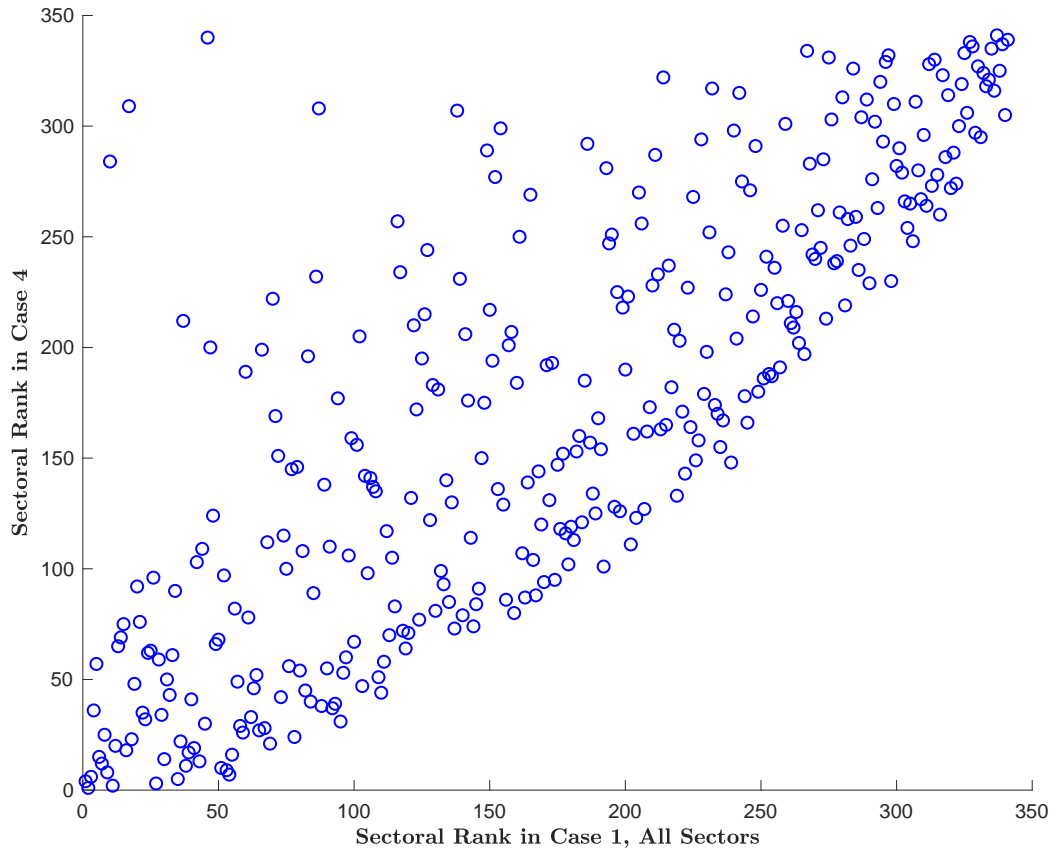
This figure plots the sectoral rankings for the cumulative IRF for case 1 against case 4 (see Table 2 for a description of the different cases).

Figure 4: **Response of Real Consumption and Inflation: Aggregation**

Industry-Code Aggregation
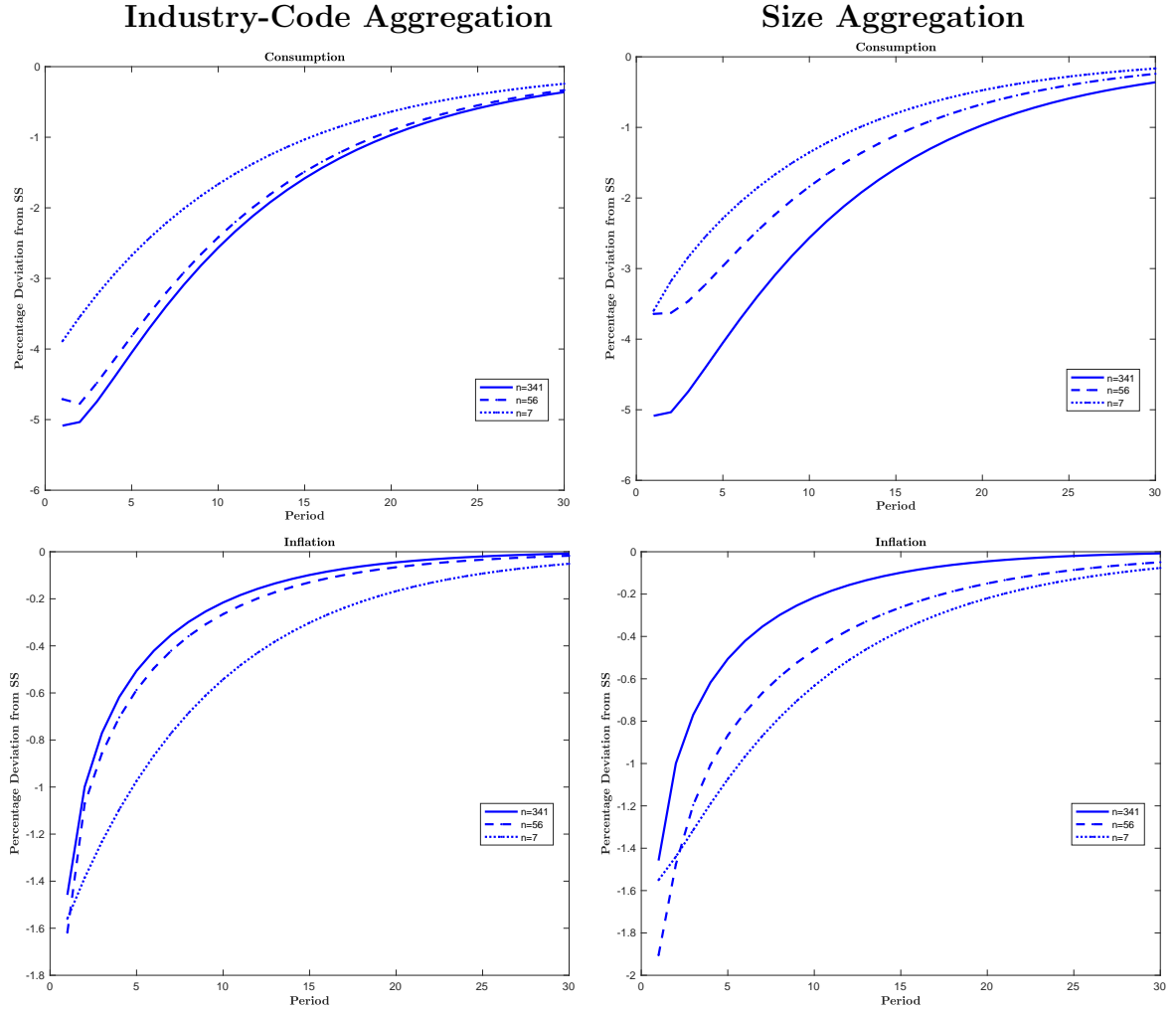
Size Aggregation

This figure plots the impulse response function of real consumption and inflation to a one-standard-deviation monetary policy shock for different levels of aggregation. In the left panel, we aggregate sectors by industry codes and in the right panels, we aggregate based on the sector size always keeping the average frequency of price adjustment constant.

Table 1: Calibration Parameters

This table reports the parameter values of the calibration of the model developed in section IV.

β	0.9975	Monthly discount factor
σ	1	Relative risk aversion
φ	2	Inverse of Frisch elasticity
δ	0.5	Average inputs share in production function
η	2	Elasticity of substitution across sectors
θ	6	Elasticity of substitution within sectors
ϕ_π	1.24	Responsiveness of monetary policy to consumption inflation
ϕ_c	0.33/12	Responsiveness of monetary policy to output variations
ρ	0.9	Persistence of monetary policy shock

Table 2: **Overview of Calibration Cases**

This table details the assumptions on frequencies, consumption weights, and input-output linkages for the different cases employed in the calibration.

	Frequencies	Consumption Weights	Input-Output Linkages
Case 1	sticky, heterogeneous	heterogeneous	heterogeneous
Case 2	sticky, heterogeneous	heterogeneous	homogeneous
Case 3	sticky, heterogeneous	homogeneous	heterogeneous
Case 4	sticky, heterogeneous	homogeneous	homogeneous
Case 5	sticky, homogeneous	homogeneous	homogeneous

Table 3: **Response to Monetary Policy Shock**

This table reports the impact response, the cumulative impulse response, and the persistence of the response defined as AR(1) coefficient due to a one-percent monetary policy shock for consumption (Panel A), inflation (Panel B), and real marginal costs (Panel C) for a 350-sector economy for different cases (see Table 2 for a description of the different cases).

	Case 1	Case 2	Case 3	Case 4	Case 5
Panel A. Consumption					
Impact	−5.09	−5.61	−5.43	−5.72	−3.88
Cumulative IRF	−60.40	−61.11	−60.49	−63.42	−37.19
Persistence	0.92	0.90	0.93	0.90	0.87
Panel B. Inflation					
Impact	−1.46	−1.08	−1.20	−1.10	−1.48
Cumulative IRF	−7.10	−6.40	−6.72	−5.60	−14.22
Persistence	0.88	0.88	0.89	0.87	0.87
Panel C. Real Marginal Costs					
Impact	−6.10	−8.25	−5.99	−6.68	−4.53
Cumulative IRF	−82.09	−90.32	−80.44	−73.99	−43.39
Persistence	0.92	0.89	0.91	0.89	0.87

Table 4: **Response to Monetary Policy Shock: Sorted by Cumulative Response**

This table reports the cumulative real consumption response to a one-percent contractionary monetary policy shock for a 341-sector economy for different cases (see Table 2 for a description of the different cases). Panel A reports the response of the least responsive contractionary sectors and Panel B reports the response of the most responsive contractionary sectors.

		Case 1	Case 2	Case 3	Case 4	Case 5
		Panel A. Cumulative Consumption Response: Least Responsive				
Least responsive	1	59.50	46.92	43.56	8.85	−37.19
	2	61.10	48.53	44.14	8.97	−37.19
	3	75.30	49.34	44.31	9.06	−37.19
	4	81.80	54.96	45.30	9.24	−37.19
	5	85.80	56.11	45.76	9.65	−37.19
	6	86.10	62.45	46.09	9.71	−37.19
	7	86.90	63.78	51.70	9.88	−37.19
	8	187.60	71.27	64.32	10.02	−37.19
	9	256.70	84.83	67.59	10.02	−37.19
	10	259.40	87.58	75.72	10.40	−37.19
		Panel B. Cumulative Consumption Response: Most Responsive				
Most responsive	1	−1070.60	−277.98	−206.25	−193.45	−37.19
	2	−634.50	−201.46	−194.31	−177.44	−37.19
	3	−222.40	−183.41	−186.57	−172.31	−37.19
	4	−210.40	−179.29	−186.26	−167.40	−37.19
	5	−210.10	−176.74	−177.95	−165.90	−37.19
	6	−199.30	−168.34	−173.84	−161.73	−37.19
	7	−193.70	−166.74	−173.65	−161.06	−37.19
	8	−191.20	−161.37	−173.04	−157.72	−37.19
	9	−184.90	−159.18	−172.92	−156.97	−37.19
	10	−184.00	−158.37	−172.81	−155.28	−37.19

Table 5: **Response to Monetary Policy Shock (341 vs 56 vs 7 sector economy)**

This table reports the impact response, the cumulative impulse response, and the persistence of the response defined as AR(1) coefficient due to a one-percent monetary policy shock for consumption (Panel A), inflation (Panel B), and real marginal costs (Panel C) for a 341-sector economy, a 56-sector economy and a 7-sector economy for case 1 (see Table 2 for a description of the different cases).

	341 Sectors	56 Sectors	7 Sectors
Panel A. Consumption			
Impact	−5.09	−4.71	−3.89
Cumulative IRF	−60.40	−56.79	−40.56
Persistence	0.92	0.90	0.94
Panel B. Inflation			
Impact	−1.46	−1.62	−1.56
Cumulative IRF	−7.10	−8.34	−13.65
Persistence	0.88	0.88	0.90
Panel C. Real Marginal Costs			
Impact	−6.10	−4.21	−4.82
Cumulative IRF	−82.09	−53.94	−58.02
Persistence	0.92	0.90	0.88

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Online Appendix:
Production Networks and the Propagation of
Monetary Policy Shocks

Ernesto Pasten, Raphael Schoenle, and Michael Weber

Not for Publication

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I Log-linearized System

A. Steady-State Solution

The steady state equilibrium sets the same prices for all firms

$$PC = \mathrm{P}_k = P_k = P \tag{A.1}$$

such that

$$
\begin{aligned}
C_k &= \omega_{ck}C, \\
C_{jk} &= \frac{1}{n_k}C_k, \\
Z_{jk}\left(k'\right) &= \omega_{kk'}Z, \\
Z_{jk}\left(j',k'\right) &= \frac{1}{n_{k'}}Z_{jk}\left(k'\right).
\end{aligned}
$$

All firms produce the quantity, so $Y_{jk} = Y$, and

$$
\begin{aligned}
Y_{jk} &= C_{jk} + \sum_{k'=1}^{K}\int_{\Im_{k'}} Z_{j'k'}\left(j,k\right)dj', \\
Y_{kt} &= C_{kt} + \sum_{k'=1}^{K} Z_{k'}\left(k\right), \\
Y_t &= C_t + Z_t
\end{aligned}
$$

given that the labor and intermediate inputs share are the same for all firms in steady

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state,

$$
\begin{aligned}
Y_{jk} &= Y; \\
L_{jk} &= L; \\
Z_{jk} &= Z.
\end{aligned}
$$

By aggregation, it holds that

$$
\begin{aligned}
C &= \sum_{k=1}^{K} \int_{\Im_k} C_{jk} dj, \\
Z &= \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{jk}\left(j', k'\right) dj'.
\end{aligned}
$$

so,

$$
\begin{aligned}
Y &= C + Z, \\
n_k &= (1 - \psi)\,\omega_{ck} + \sum_{k'=1}^{K} n_{k'} \omega_{k'k}.
\end{aligned}
$$

In turn, steady-state labor supply from equation is

$$
\frac{W_k}{P} = g_k L_k^{\varphi} C^{\sigma}. \tag{A.2}
$$

In a symmetric steady state, $L_k = n_k L$, so this steady state exists if $g_k = n_k^{-\varphi}$ such that $W_k = W$ for all k. Thus, steady-state labor supply is given by

$$
\frac{W}{P} = L^{\varphi} C^{\sigma}. \tag{A.3}
$$

Households' budget constraint, firms' profits, production function, efficiency of

production (from equation (5)) and optimal prices in steady state are, respectively,

$$CP = WL + \Pi \tag{A.4}$$

$$\Pi = PY - WL - PZ \tag{A.5}$$

$$Y = L^{1-\delta} Z^{\delta} \tag{A.6}$$

$$\delta WL = (1 - \delta) PZ \tag{A.7}$$

$$sP = \frac{\theta}{\theta - 1} \xi W^{1-\delta} P^{\delta} \tag{A.8}$$

for $\xi \equiv \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta}$.

B. Log-linear System

B.1 Demand

Households' demands for goods for all $k = 1, ..., K$ become

$$
\begin{aligned}
c_{kt} &= c_t - \eta \left(p_{kt} pc_t \right), \\
c_{jkt} &= c_{kt} - \theta \left(p_{jkt} - p_{kt} \right).
\end{aligned}
\tag{A.9}
$$

In turn, firm jk's demands for goods for all $k, r = 1, ..., K$,

$$
\begin{aligned}
z_{jkt} \left(k' \right) - z_{jkt} &= \eta \left(p_{k't} - p_{kt} \right), \\
z_{jkt} \left(j', k' \right) &= z_{jkt} \left(k' \right) - \theta \left(p_{j'k't} - p_{k't} \right).
\end{aligned}
\tag{A.10}
$$

Firms' gross output satisfies Walras' law,

$$y_{jkt} = (1 - \psi) \omega_{ck} c_{jkt} + \psi \sum_{k'=1}^{K} \omega_{k'k} \int_{\Im_{k'}} z_{j'k't} (j, k) \, dj' \tag{A.11}$$

Aggregating at the sectoral level using $x_{kt} = \frac{1}{n_k} \int_{\Im_k} x_{jkt} dj$ for $x = y, c$ and $z_{k't}(k) = \frac{1}{n_k n_{k'}} \int_{\Im_k} \int_{\Im_{k'}} z_{j'k't} (j, k) \, dj' dj$,

$$n_{kt} y_{kt} = (1 - \psi) \omega_{ck} c_{kt} + \psi \sum_{k'=1}^{K} n_{k'} \omega_{k'k} z_{k't} (k), \tag{A.12}$$

3

and, at the aggregate level using $y_t = \sum_k n_k y_{kt}$, $c_t = \sum_k \omega_{ck} c_{kt}$ and $z_t = \sum_{k=1}^{K} \sum_{k'=1}^{K} \omega_{k'k} n_{k'} z_{k't}(k)$,

$$y_t = (1 - \psi) c_t + \psi z_t. \tag{A.13}$$

B.2 IS and Labor Supply

The household Euler equation becomes

$$c_t = \mathbb{E}_t [c_{t+1}] - \sigma^{-1} \{i_t - (\mathbb{E}_t [pc_{t+1}] - p_t)\}. \tag{A.14}$$

The labor supply condition is

$$w_{kt} - pc_t = \varphi l_{kt} + \sigma c_t. \tag{A.15}$$

B.3 Firms

Production function:
$$y_{jkt} = a_{kt} + (1 - \delta) l_{jkt} + \delta z_{jkt}. \tag{A.16}$$

Efficiency condition:
$$w_{kt} - \mathrm{p}_{kt} = z_{jkt} - l_{jkt}. \tag{A.17}$$

Marginal costs:
$$mc_{kt} = (1 - \delta) w_{kt} + \delta \mathrm{p}_{kt} - a_{kt}. \tag{A.18}$$

Optimal reset price:
$$p_{kt}^* = (1 - \alpha_k \beta) mc_{kt} + \alpha_k \beta \mathbb{E}_t \left[p_{kt+1}^* \right]. \tag{A.19}$$

Sectoral prices:
$$p_{kt} = (1 - \alpha_k) p_{kt}^* + \alpha_k p_{kt-1}. \tag{A.20}$$

The monetary policy rule must be added to close the model. In the main text sometimes we use money supply assuming

$$c_t + pc_t = m_t \tag{A.21}$$

and, when specified, we use a Taylor rule,

$$i_t = \phi_\pi \left(pc_t - pc_{t-1} \right) + \phi_c c_t. \tag{A.22}$$

II Proofs

Proposition 1

Proof. When $\alpha_k = \alpha$ for all k, we aggregate equations (43) using consumption weights to obtain

$$pc_t \left(\alpha \right) = \frac{\left(1 - \delta\right)\left(1 - \alpha\right)}{1 - \delta\left(1 - \alpha\right)} m_t + \frac{\alpha}{1 - \delta\left(1 - \alpha\right)} pc_{t-1} \left(\alpha \right).$$

Equation (45) follows by using the previous expression to solve for $pc_t \left(\alpha \right)$ when a permanent monetary shock m hits at t^*. (1) and (2) follow directly from equation (45). ■

Proposition 2

Proof. The restriction $\omega_{kk'} = \omega_{ck'}$ for all k, k' implies $\mathrm{p}_{kt} = pc_t$ for all k. Thus, according to equation (43), sectoral prices solve

$$p_{kt} = \left(1 - \alpha_k\right) \left[\left(1 - \delta\right) m_t + \delta pc_t \right] + \alpha_k p_{kt-1}.$$

Iterating backwards and using $pc_t = \sum_{k=1}^{K} \omega_{ck} p_{kt}$ yields equation (46).

(1) follows from setting $p_{kt} = 0$ for $t < t^*$ and for $t = t^*$, $\left(\sum_k \omega_{ck} \alpha_k - \bar{\alpha} \right) = 0$.

For (2), denote $\widetilde{pc}_t \left(\alpha \right)$ the solution for aggregate consumption prices in Proposition 1. Further assume $pc_{t-s} = \widetilde{pc}_{t-s} \left(\alpha \right)$. Hence,

$$pc_t - \widetilde{pc}_t \left(\overline{\alpha} \right) = -\frac{\left(1 - \delta\right) m}{1 - \delta\left(1 - \overline{\alpha}\right)} \left(\sum_{k=1}^{K} \omega_{ck} \alpha_k^{t-t^*+1} - \overline{\alpha}^{t-t^*+1} \right)$$

$$+ \frac{\delta}{1 - \delta\left(1 - \overline{\alpha}\right)} \sum_{\tau=1}^{t-t^*} \left(\sum_{k=1}^{K} \omega_{ck} \alpha_k^\tau \left(1 - \alpha_k\right) - \overline{\alpha}^\tau \left(1 - \overline{\alpha}\right) \right) pc_{t-\tau}.$$

Jensen's inequality, $\widehat{pc}_t - \widetilde{pc}_t \left(\overline{\alpha} \right) \leq 0$ and decreases in the dispersion of price stickiness across sectors, $\{\alpha_k\}_{k=1}^{k}$. In addition, $pc_{t-s} \leq \widetilde{pc}_{t-s}$ for all $s \geq 1$, which reinforces the result. ■

Proposition 3

Proof. Let $\widehat{pc}_t$ denote the response of aggregate consumption prices in Proposition 2. Assume $p_{kt-s} = \widehat{p}_{kt-s}$ for all $s \geq 1$. Subtracting equation (46) from equation (47) yields

$$pc_t - \widehat{pc}_t = \delta \sum_{k=1}^{K} \left(u_k p_{kt} - (1-\overline{\alpha}) \omega_{ck} \widetilde{p}_{kt} \right) + \delta \sum_{\tau=1}^{t-t^*} \left(\sum_{k'=1}^{K} \omega_{ck'} \alpha_{k'}^{\tau} (1-\alpha_{k'}) \right) \sum_{k=1}^{K} (\overline{\omega}_k - \omega_{ck}) p_{kt-\tau}$$
$$+ \delta K \sum_{\tau=1}^{t-t^*} \sum_{k=1}^{K} COV \left(\omega_{ck'} \alpha_{k'}^{\tau} (1-\alpha_{k'}), \omega_{k'k} \right) p_{kt-\tau},$$

(A.23)

for $u_k \equiv \sum_{k'=1}^{K} \omega_{ck'} (1-\alpha_{k'}) \omega_{k'k}$ and $\overline{\omega}_k \equiv \frac{1}{K} \sum_{k'=1}^{K} \omega_{k'k}$.

For (1), we use $p_{kt} = 0$ for $t < t^*$, so $pc_t < \widehat{pc}_t$ when $\sum_{k=1}^{K} \left(u_k p_{kt} - (1-\overline{\alpha}) \omega_{ck} \widetilde{p}_{kt} \right) < 0$. This inequality holds when for the sectors with most sticky prices $u_k > (1-\overline{\alpha}) \omega_{ck}$.

For (2), two additional terms come to play. The second term of equation (A.23) is negative when for the sectors with most sticky prices $\overline{\omega}_k > \omega_{ck}$. The third term of equation (A.23) is negative when the covariance term is positive for the most sticky sectors and negative for the most flexible sectors. Relaxing the assumption that $p_{kt-s} = \widehat{p}_{kt-s}$ reinforces these results.

Finally, we can derive an expression that only contains parameters and the monetary policy shock on the right-hand side. We have that

$$p_{kt} = (1-\delta)(1-\alpha_k) m + \delta (1-\alpha_k) \mathrm{p}_{kt} + \alpha_k p_{kt-1}$$

(A.24)

which in matrix form using the sectoral price vector p_t becomes

$$p_t = [\mathbb{I} - \delta (\mathbb{I} - A) \Omega]^{-1} [(1-\delta)(\mathbb{I} - A) \iota m + A p_{t-1}]$$

(A.25)

where ι is a vector of ones and A is a diagonal matrix of α_k. Solving backwards yields

$$e_k \Omega^C (1-\delta) \left[\mathbb{I} - [\mathbb{I} - \delta (\mathbb{I} - A) \Omega]^{-1} (\mathbb{I} - A) \right]^{-1} \left[\mathbb{I} - \left([\mathbb{I} - \delta (\mathbb{I} - A) \Omega]^{-1} (\mathbb{I} - A) \right)^{t-t^*+1} \right]$$
$$[\mathbb{I} - \delta (\mathbb{I} - A) \Omega]^{-1} (\mathbb{I} - A) \iota m.$$

(A.26)

∎

Proposition 4

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Proof. When $\delta = 0$,

$$p_{kt} = (1 - \alpha_k) m_t + \alpha_k p_{kt-1} \quad \text{for all} \quad k \tag{A.27}$$

such that

$$pc_t = \left(1 - \sum_{k=1}^{K} \omega_{ck} \alpha_k^{t-t^*+1} \right) m,$$

$$\overline{pc}_t = \left(1 - \sum_{k'=1}^{K/2} \overline{\omega}_{ck'} \overline{\alpha}_{k'}^{t-t^*+1} \right) m$$

Thus,

$$\overline{pc}_t - pc_t = \sum_{k'=1}^{K/2} \overline{\omega}_{ck'} \left[\lambda_{k'} \alpha_{2k'-1}^{t-t^*+1} + (1 - \lambda_{k'}) \alpha_{2k'}^{t-t^*+1} - \overline{\alpha}_{k'}^{t-t^*+1} \right] \tag{A.28}$$

for any period $t \geq t^*$.

It follows from Jensen's inequality

$$\lambda_{k'} \alpha_{2k'-1}^{t-t^*+1} + (1 - \lambda_{k'}) \alpha_{2k'}^{t-t^*+1} - \overline{\alpha}_{k'}^{t-t^*+1} > 0 \tag{A.29}$$

for $\alpha_{2k'-1} \neq \alpha_{2k'}$ and $t > t^*$. The previous expression decreases in $|\alpha_{2k'-1} - \alpha_{2k'}|$ and reaches its minimum for

$$\lambda^*(t) = \frac{\left[\frac{1}{t-t^*+1} \left(\alpha_{2k'-1}^{t-t^*+1} - \alpha_{2k'}^{t-t^*+1} \right) \right]^{\frac{1}{t-t^*}}}{\left(\alpha_{2k'-1} - \alpha_{2k'} \right)^{\frac{t-t^*+1}{t-t^*}}} - \frac{\alpha_{2k'}}{\alpha_{2k'-1} - \alpha_{2k'}}, \tag{A.30}$$

given $\alpha_{2k'-1}$ and $\alpha_{2k'}$. ∎

Proposition 5

Proof. Sectoral prices now solve

$$p_{kt} = (1 - \alpha_k) \left[(1 - \delta) m_t + \delta pc_t \right] + \alpha_k p_{kt-1} \quad \text{for all } k \tag{A.31}$$

such that

$$pc_t = \frac{1-\delta}{1-\delta(1-\overline{\alpha})}\left(1-\sum_{k=1}^{K}\omega_{ck}\alpha_k^{t-t^*+1}\right)m + \frac{\delta}{1-\delta(1-\overline{\alpha})}\sum_{\tau=1}^{t-t^*}\left(\sum_{k=1}^{K}\omega_{ck}\alpha_k^{\tau}(1-\alpha_k)\right)pc_{t-\tau},$$

$$\overline{pc}_t = \frac{1-\delta}{1-\delta(1-\overline{\alpha})}\left(1-\sum_{k'=1}^{K/2}\overline{\omega}_{ck'}\overline{\alpha}_{k'}^{t-t^*+1}\right)m + \frac{\delta}{1-\delta(1-\overline{\alpha})}\sum_{\tau=1}^{t-t^*}\left(\sum_{k'=1}^{K/2}\overline{\omega}_{ck'}\overline{\alpha}_k^{\tau}(1-\overline{\alpha}_k)\right)\overline{pc}_{t-\tau}.$$

for any period $t \geq t^*$.

Assume that $p_{t-\tau}^c = \overline{p}_{t-\tau}^c$ for all $\tau \geq 1$, so

$$pc_t - \overline{pc}_t = -\frac{1-\delta}{1-\delta(1-\overline{\alpha})}\sum_{k'=1}^{K/2}\overline{\omega}_{ck'}\left(\lambda_{k'}\alpha_{2k'-1}^{t-t^*+1} + (1-\lambda_{k'})\alpha_{2k'}^{t-t^*+1} - \overline{\alpha}_{k'}^{t-t^*+1}\right)m$$

$$+\frac{\delta}{1-\delta(1-\overline{\alpha})}\sum_{\tau=1}^{t-t^*}\left(\sum_{k'=1}^{K/2}\overline{\omega}_{ck'}\left[\begin{array}{c}\lambda_{k'}(1-\alpha_{2k'-1})\alpha_{2k'-1}^{\tau} + (1-\lambda_{k'})(1-\alpha_{2k'})\alpha_{2k'}^{\tau}\\ -(1-\overline{\alpha}_{k'})\overline{\alpha}_{k'}^{\tau}\end{array}\right]\right) \tag{A.32}$$

The difference $pc_t - \overline{pc}_t$ is a sum of two terms. The first term in equation (A.32) is a rescaled analogue of the preceeding price stickiness convexification so results (i)–(iv) still apply. The second term is a measure of the amplification of monetary non-neutrality due to strategic complementary introduced by intermediate inputs. The higher this term, the smaller is pc_t relative to $\overline{pc}_t$.

From Jensen's inequality

$$\lambda_{k'}(1-\alpha_{2k'-1})\alpha_{2k'-1}^{\tau} + (1-\lambda_{k'})(1-\alpha_{2k'})\alpha_{2k'}^{\tau} - (1-\overline{\alpha}_{k'})\overline{\alpha}_{k'}^{\tau} < 0 \tag{A.33}$$

for $\alpha_{2k'-1} \neq \alpha_{2k'}$ and $t > t^*$.

Thus, the difference in the consumption price response between baseline and merged economies, $\overline{pc}_t - pc_t$, is larger when the difference in frequencies, α, is higher for merged sectors with higher combined weight, $\overline{\omega}_{ck'}$. Relaxing the assumption $pc_{t-\tau} = \overline{pc}_{t-\tau}$ for all $\tau \geq 1$ reinforces the result regarding frequencies. ■

Proposition 6

Proof. This proposition follows entirely analogously to the preceeding ones, allowing for unrestricted heterogeneity in I/O linkages and given the pricing expression in Proposition 3. ■

III   Industry by Industry Input-Output Matrix

We use the make table ($MAKE$) to determine the share of each commodity c that each industry i produces. We define the market share ("$SHARE$") of industry i's production of commodity c as

$$SHARE = MAKE \odot (\mathbb{I} \times MAKE)_{i,j}^{-1}, \tag{A.34}$$

where $\mathbb{I}$ is a matrix of 1s with suitable dimensions.

We multiply the share and use tables (USE) to calculate the dollar amount that industry i sells to industry j. We label this matrix revenue share ($REVSHARE$), which is a supplier industry-by-consumer industry matrix,

$$REVSHARE = (SHARE \times USE). \tag{A.35}$$

We use the revenue share matrix to calculate the percentage of industry j's inputs purchased from industry i and label the resulting matrix $SUPPSHARE$:

$$SUPPSHARE = REVSHARE \odot ((USE \times \mathbb{I})_{i,j}^{-1})^{\top}. \tag{A.36}$$

The input-share matrix in equation (A.36) is an industry-by-industry matrix and therefore consistently maps into our model.

IV   Robustness

In this subsection, we first show how the specification of monetary policy affects our conclusions. We then analyze how changes in parameters and tails of the frequency distribution affect our findings.

As the first set of robustness checks, we vary the parameters of the Taylor rule. We also study a specification with exogenous nominal demand, rather than closing the model with a Taylor rule. We report the results in Figure A.5 and Table A.3 for the same five cases.

As our first experiment, we increase ϕ_π, the systematic response of monetary policy to inflation in the Taylor rule, from a baseline value of 1.24 to 2.5.

We see in Figure A.5 a similar response of inflation independent of whether we study heterogeneous or homogeneous price stickiness, sector size, and I/O structure. The impact response of inflation is, however, roughly cut to one-third compared to our baseline in Figure 1, which comes from weaker demand effects. The inflation response tends to be slightly more persistent with all three forms of heterogeneity, but the cumulative inflation responses in an economy with $\phi_\pi = 2.5$ are still smaller than in an economy with $\phi_\pi = 1.24$ (compare panels B of Table 3 and Table A.3).

The higher systematic response to inflation in the Taylor rule reduces the impact response of consumption by a factor of up to four across different cases (compare panels A of Table 3 and Table A.3). A model with heterogeneous price stickiness but homogeneous sector size and I/O structure has an almost twice as large impact response compared to an economy in which all three forms of heterogeneities interact (case 3 vs. case 1). A higher weight on inflation stabilization in the Taylor rule for a given demand shock results in a larger stabilization of output in the standard New Keynesian model. Once we allow for heterogeneous price stickiness, the cumulative real effects of monetary policy shocks contract by 40%, with a more stringent response to inflationary pressure in the Taylor rule. We see similar results once we add heterogeneity in sector size and I/O structure. We see in Panels A and C of Table 3 and Table A.3 that the different forms of heterogeneity introduce a slightly more sluggish and persistent response in consumption and real marginal costs, which explains the large real effects of demand shocks, despite the smaller effects on impact. This finding is reminiscent of the responder-nonresponder framework discussed in Carvalho (2006) and the selection effect of Carvalho and Schwartzman (2015).

Changes in the systematic response to output growth have little impact on the response of real consumption, inflation, or real marginal costs (not tabulated).

In our second experiment, we close the model by positing exogenous nominal demand. Figure A.6 reports our findings. The dashed blue line represents our baseline response with the Taylor rule, and the dash-dotted blue line represents the response for a model with exogenous nominal demand. Real marginal costs barely move in the model with exogenous demand, resulting in a small and transient impact response of inflation and a one-percentage-point response of consumption on impact. The impact response of consumption is smaller by a factor of five than the impact response with a Taylor rule.

As a second set of robustness checks, we analyze how changes in parameters and tails

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of the frequency distribution affect our findings. First, we study the effect of changes in risk aversion, σ, the Frisch elasticity, φ, the average input share in production, δ, the elasticity of substitution within and across sectors, η, θ, and the persistence of monetary policy shocks, ρ, in our full-blown model (case 1 in Table 2). Specifically, we set (baseline parameters in parentheses) $\sigma = 2(1)$, $\varphi = 1(2)$, $\delta = 0.7(0.5)$, $\eta = 6(2)$, $\theta = 10(6)$, and $\rho = 0.95(0.90)$. Figure A.7 and Table A.4 report our findings.

Overall, we see our results in the baseline calibration of Table 3 and Figure 1 are robust to variations in parameter values. The only exception is the increase of the coefficient of relative risk aversion, σ, from a baseline value of 1 to 2. The intratemporal rate of substitution from leisure to consumption determines the real wage. The drop in consumption results in a drop in the real wage, which increases in σ. Lower real wages lower the response of real marginal costs and the overall demand pressure.

V Proofs

In our first case, the full set of heterogeneities is present and firms in different sectors differ by their degree of price stickiness and intermediate input usage and sectors differ in size. We see in Table 3 and Figure 1 large real effect of monetary policy on impact that are persistent over time. Real effects are large compared to a New Keynesian model with fully flexible prices in which prices fully absorb the monetary policy shock, and we do not see any effect on real consumption or marginal costs.

Case 2 is similar to case 1 but imposes intermediate input shares that are homogeneous and equal across sectors. This case corresponds to Proposition 3 in the simplified model, where we allow consumption and I/O weights to differ. In the calibration, we find imposing homogeneous I/O linkages results in real effects both on impact and in total that are similar to our baseline calibration in which we allow for all heterogeneities to interact.

In case 3 instead, we impose homogeneous sector size but heterogeneous I/O linkages and case 4 imposes both homogeneous sector size and I/O linkages. Under the assumptions of case 4, however, sectoral weights of all aggregate prices are the same, so that $\mathrm{p}_{kt} = pc_t = \widetilde{p}_t$ for all sectors k, and the wedges between price indices that affect marginal cost are absent. We see relative to case 1 only small variations, and real output effects, the

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inflation response or how marginal costs react are similar across the different cases.

Finally in case 5, we introduce homogeneity in price stickiness and keep sectoral size and I/O linkages homogeneous. We see in Figure 1 that now real effects drop substantially and inflation responds more immediately. The reason for the larger real effects in economies with heterogeneous price stickiness is a negative selection effect – older prices are less likely to change, and real output effects are larger – an effect that dominates any of the other factors that influence markups, and real effects of monetary policy are now substantially larger and the price effect is muted.

In calibrations with heterogeneous price stickiness, we see the real effects of monetary policy both on impact and cumulatively more than double compared to an economy with homogeneous but equal average price stickiness (see Table 3). The inflation response, instead, is substantially muted (see blue dashed line in Figure 1). Our results confirm the intuition for economies with heterogeneity in price setting of Carvalho and Schwartzman (2015) and Nakamura and Steinsson (2010) and Proposition 2 in our simplified model. Relative to the model predictions, the impact difference in the price response relative to the previous case also highlights the role of forward-looking price setters.

Figure A.1: **Distribution of the Frequency of Price Adjustment**
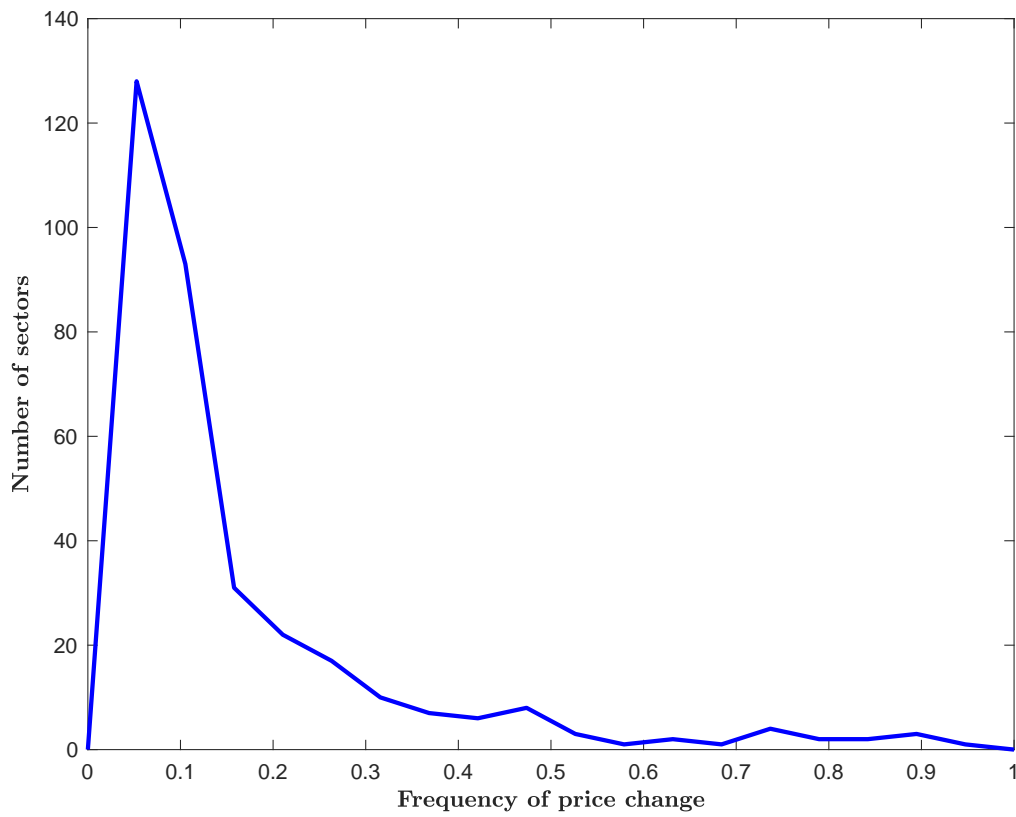
This figure plots the distribution of the frequency of price adjustment for a 341-sector model using the microdata underlying the PPI from the BLS.

Figure A.2: **Variance of Frequency of Price adjustment: Size Aggregation**
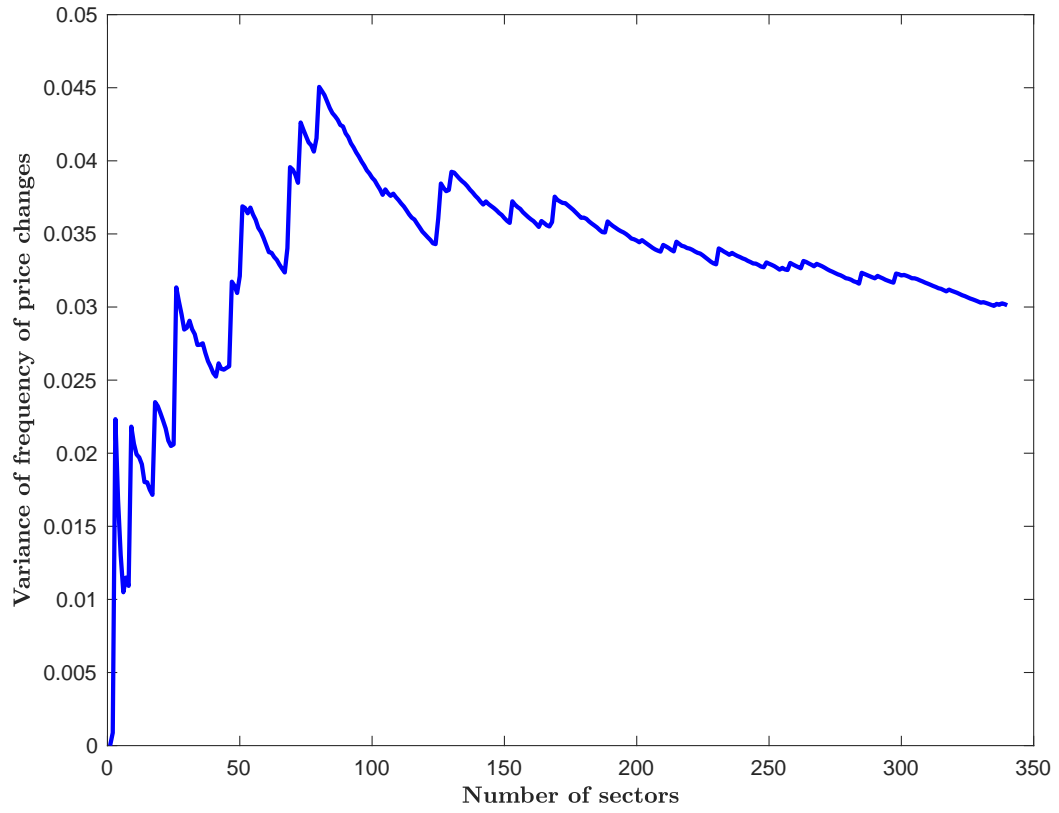
This figure plots the variance of the frequency of price adjustments for different levels of aggregation. We aggregate sectors by the sector size keeping the average frequency of price adjustment constant.

Figure A.3: **Price Gap and Components, 341 and 7 Sectors**
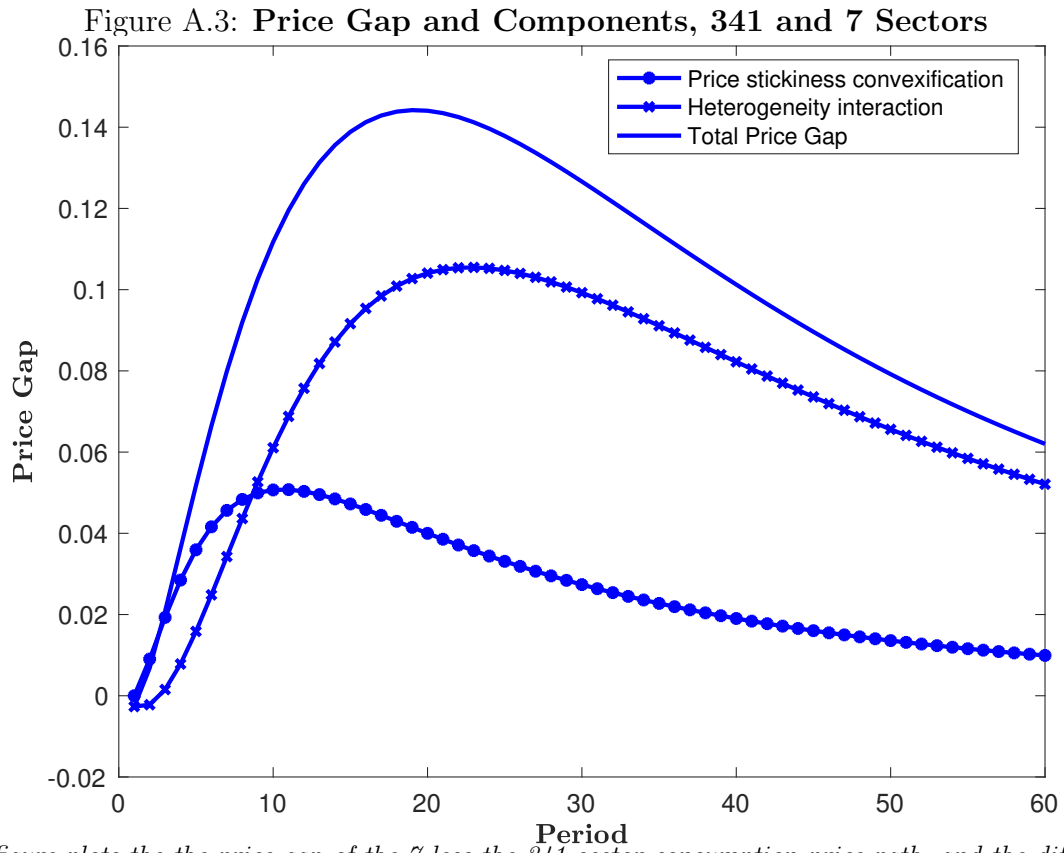
This figure plots the the price gap of the 7 less the 341 sector consumption price path, and the difference in the convexification and interaction components implied by equation (47) that make up the price paths.

Figure A.4: **Response of Markups to Monetary Policy Shock**
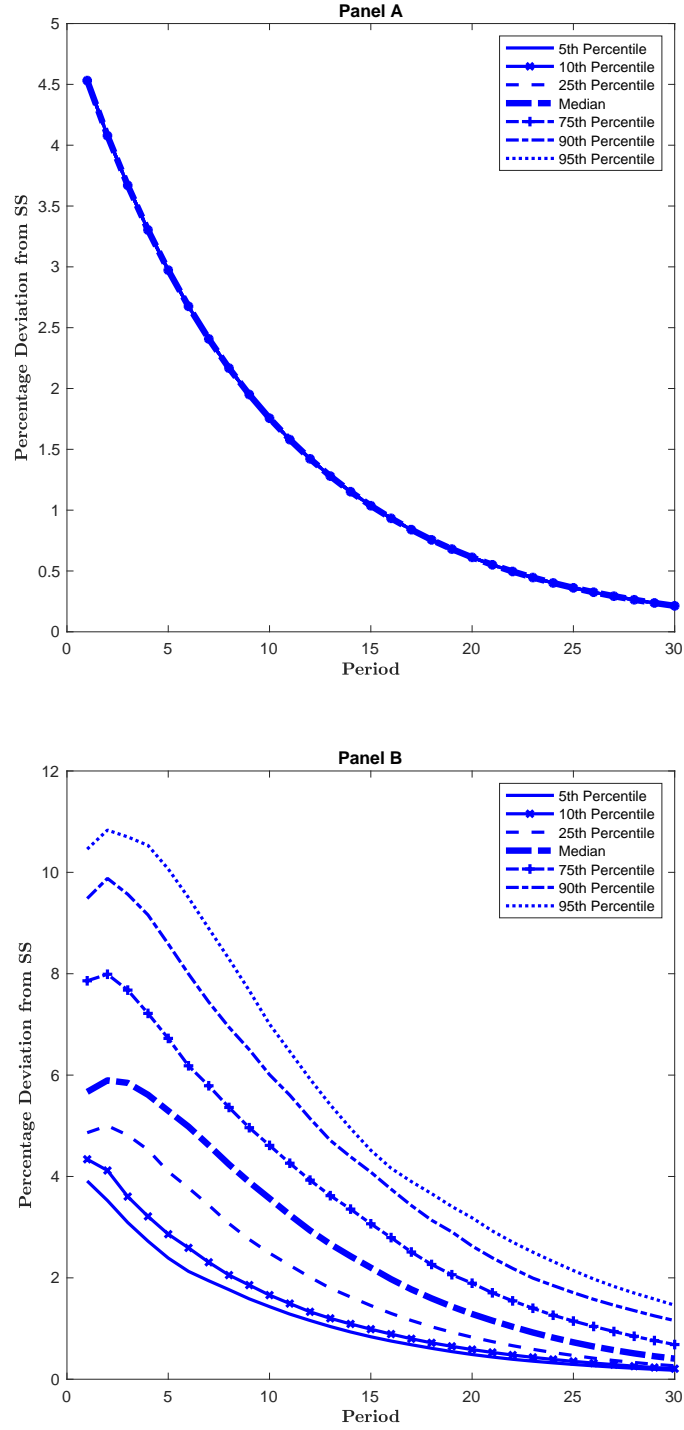
This figure plots the impulse response function of markups to a one-standard-deviation monetary policy shock for a 341-sector model for case 5 in Panel A and case 1 in Panel B (see Table 2 for a description of the different cases).

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Figure A.5: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock** ($\phi_\pi = 2.5$)
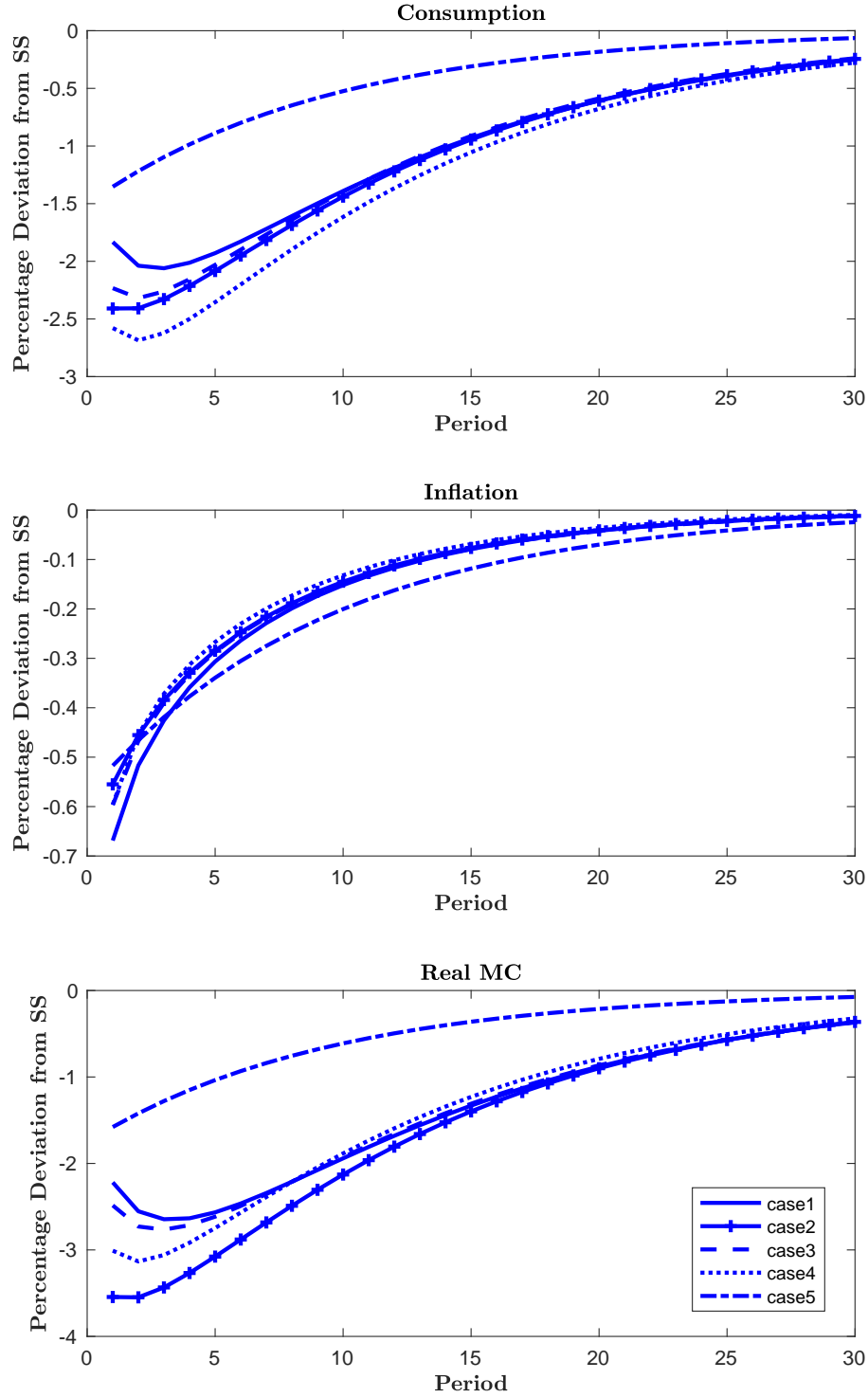
This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard-deviation monetary policy shock for a 341-sector model for different cases (see Table 2 for a description of the different cases) with a coefficient on inflation in the Taylor rule of $\phi_\pi = 2.5$.

Figure A.6: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock (exogenous nominal demand)**
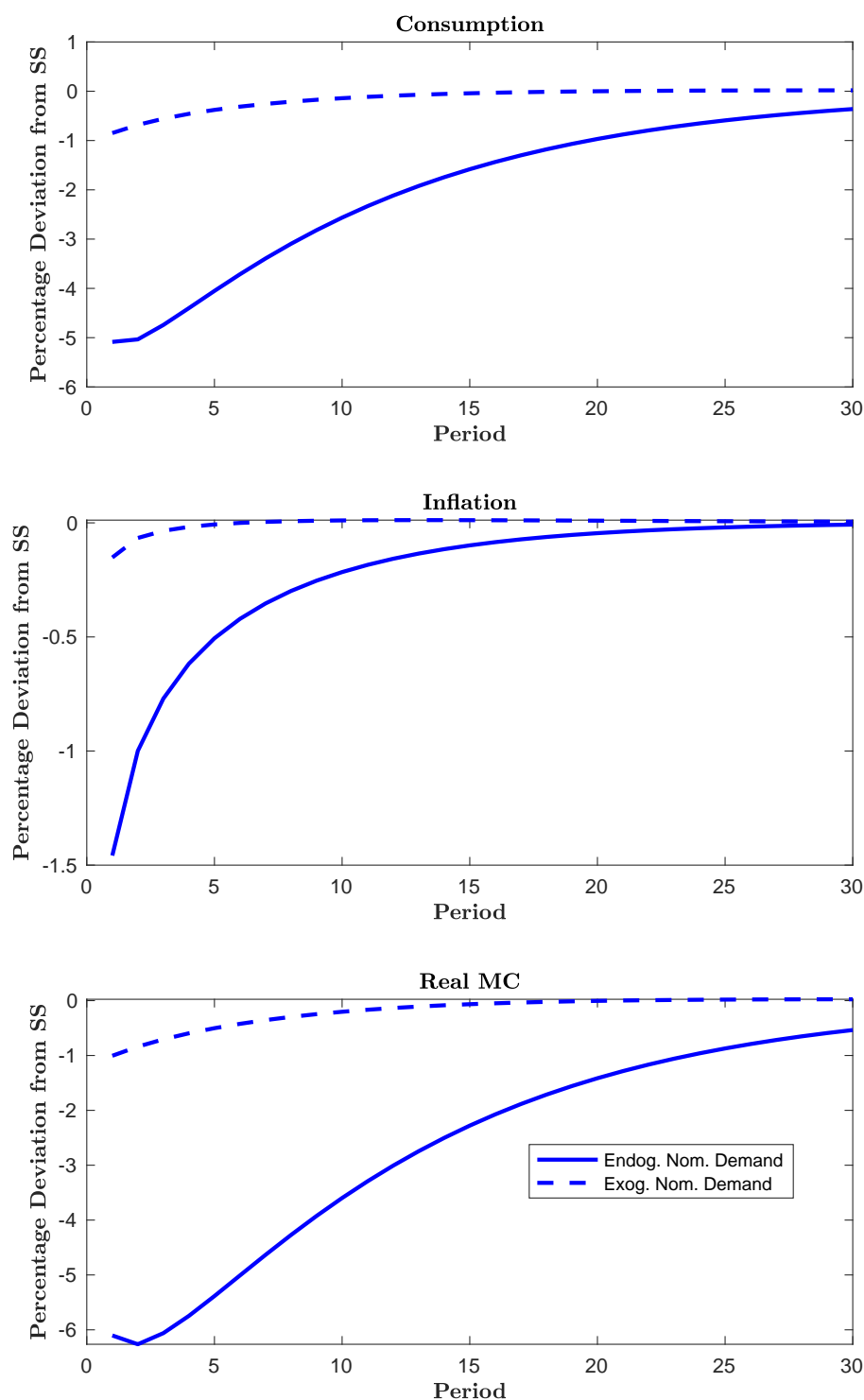
This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard-deviation monetary policy shock for a 341-sector model for case 1 (see Table 2 for a description of the different cases), closing the model with positing exogenous nominal demand.

Figure A.7: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock (variations in parameters)**
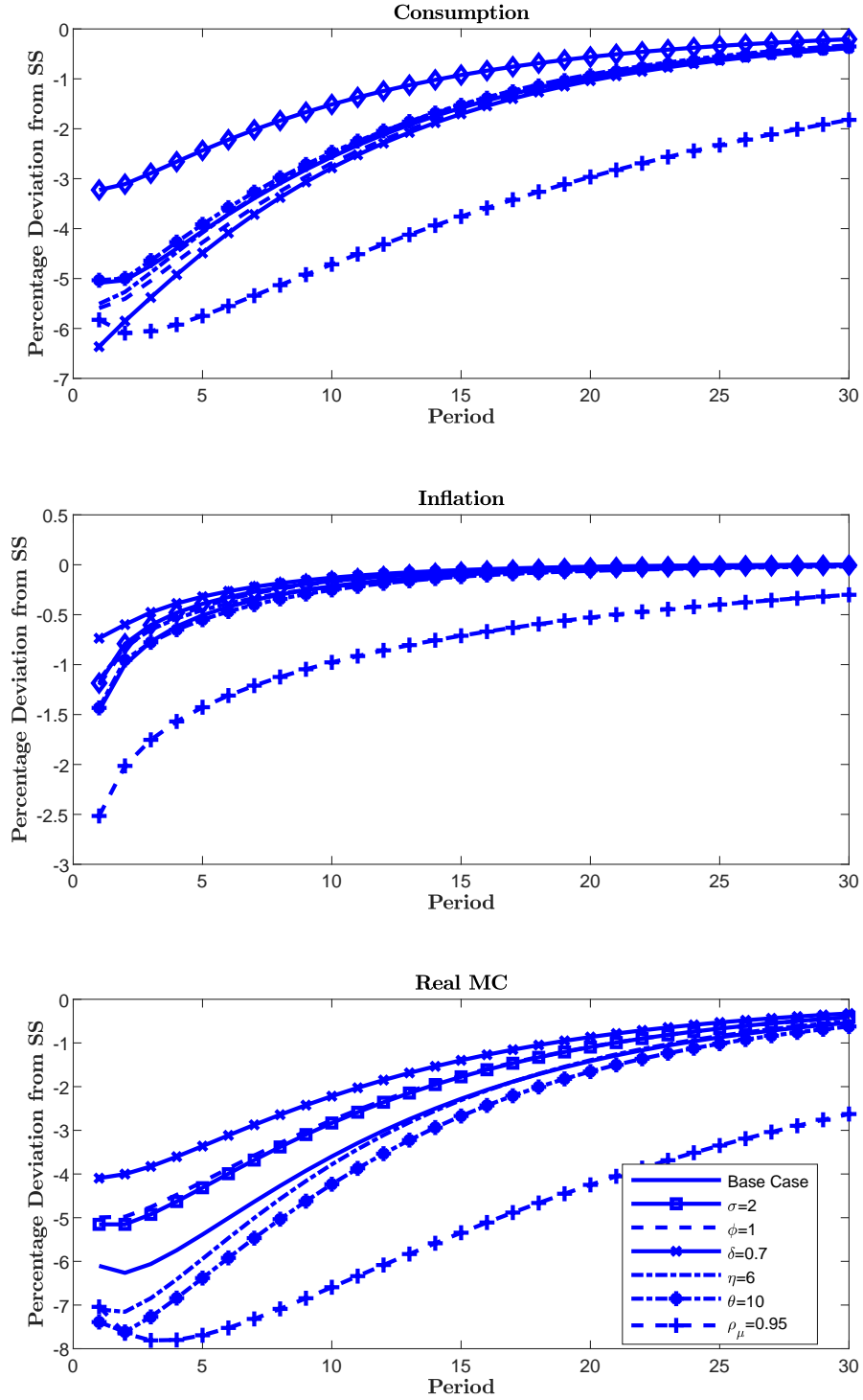
This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard-deviation monetary policy shock for a 341-sector model for case 1 (see Table 2) for different values of structural parameters.

Table A.1: Descriptive Statistics

The table reports the moments of the frequency of price adjustment, FPA, distribution for a 7-sector model in panel A , 56-sector model in panel B and a 341-sector model in panel C using the microdata underlying the PPI from the BLS.

	Mean	Median	Std	25th Pct	75th Pct
Panel A. 7 Sector Economy					
FPA	0.17	0.10	0.17	0.07	0.19
Panel B. 56 Sector Economy					
FPA	0.18	0.13	0.19	0.08	0.24
Panel C. 341 Sector Economy					
FPA	0.15	0.17	0.22	0.08	0.21

Table A.2: **Response to Monetary Policy Shock: Size Aggregation (341 vs 56 vs 7 sector economy)**

This table reports the impact response, the cumulative impulse response, and the persistence of the response defined as AR(1) coefficient due to a one-percent monetary policy shock for consumption (Panel A), inflation (Panel B), and real marginal costs (Panel C) for a 341-sector economy, a 56-sector economy and a 7-sector economy for case 1 (see Table 2 for a description of the different cases). We aggregate sectors by size.

	341 Sectors	56 Sectors	7 Sectors
Panel A. Consumption			
Impact	−5.09	−3.64	−3.60
Cumulative IRF	−60.40	−43.21	−33.62
Persistence	0.92	0.93	0.83
Panel B. Inflation			
Impact	−1.46	−1.90	−1.55
Cumulative IRF	−7.10	−12.93	−15.51
Persistence	0.88	0.92	0.85
Panel C. Real Marginal Costs			
Impact	−6.10	−1.80	−5.69
Cumulative IRF	−82.09	−42.28	−50.61
Persistence	0.92	0.94	0.85

Table A.3: **Response to Monetary Policy Shock ($\phi_\pi = 2.5$)**

This table reports the impact response, the cumulative impulse response, and the persistence of the response defined as AR(1) coefficient due to a one-percent monetary policy shock for consumption (Panel A), inflation (Panel B), and real marginal costs (Panel C) for a 341-sector economy for different cases (see Table 2 for a description of the different cases) with a coefficient on inflation in the Taylor rule of $\phi_\pi = 2.5$.

	Case 1	Case 2	Case 3	Case 4	Case 5
Panel A. Consumption					
Impact	−1.83	−2.41	−2.23	−2.58	−1.35
Cumulative IRF	−30.92	−32.99	−31.91	−36.92	−12.96
Persistence	0.94	0.92	0.94	0.93	0.87
Panel B. Inflation					
Impact	−0.67	−0.55	−0.60	−0.60	−0.52
Cumulative IRF	−4.31	−3.96	−4.07	−3.76	−4.95
Persistence	0.92	0.91	0.92	0.90	0.87
Panel C. Real Marginal Costs					
Impact	−2.22	−3.54	−2.49	−3.01	−1.58
Cumulative IRF	−42.34	−48.77	−42.80	−43.08	−15.12
Persistence	0.94	0.91	0.94	0.92	0.87

Table A.4: **Response to Monetary Policy Shock (variations in parameters)**

This table reports the impact response, the cumulative impulse response, and the persistence of the response defined as AR(1) coefficient due to a one-percent monetary policy shock for consumption (Panel A), inflation (Panel B), and real marginal costs (Panel C) for a 341-sector economy for case 1 (see Table 2) for different values of structural parameters.

	Base	$\sigma = 2$	$\phi = 1$	$\delta = 0.7$	$\eta = 6$	$\theta = 10$	$\rho = 0.95$
				Panel A. Consumption			
Impact	-5.09	-3.23	-5.59	-6.36	-5.51	-5.03	-5.83
Cumulative IRF	-60.40	-36.09	-63.90	-67.14	-60.04	-58.70	-115.22
Persistence	0.92	0.91	0.91	0.91	0.92	0.92	0.94
				Panel B. Inflation			
Impact	-1.46	-1.19	-1.20	-0.74	-1.15	-1.43	-2.52
Cumulative IRF	-7.10	-5.52	-5.69	-4.07	-6.57	-7.59	-25.89
Persistence	0.88	0.87	0.88	0.88	0.85	0.89	0.91
				Panel C. Real Marginal Costs			
Impact	-6.10	-5.15	-4.99	-4.10	-7.10	-7.39	-7.04
Cumulative IRF	-82.09	-65.32	-63.85	-51.20	-87.61	-97.21	-158.88
Persistence	0.92	0.91	0.91	0.91	0.91	0.91	0.94

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