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Shifting Inflation Expectations and Monetary Policy*

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Abstract

The idea of "anchored" inflation expectations is often understood as a situation in which long-run expected inflation does not significantly respond to new information. Furthermore, expectations are thought to become "unanchored" only after a long enough sequence of inflation surprises. In this paper we conceptualize this idea in a monetary DSGE model with a time-varying learning mechanism, in which the sensitivity of agents to incoming data depends on accumulated inflation forecast errors. The latter affect the learning gain that agents use to update their beliefs on future inflation. We show how this mechanism improves the fit of the model to macroeconomic data, including expected inflation, for the Chilean inflation targeting period. In particular, we show that observed episodes with anchored and unanchored expectations are well captured by the estimated time-varying learning gain. We then use the estimated model to assess the role of monetary policy to anchor inflation expectations over time.

Resumen

La idea de expectativas de inflación ancladas es generalmente entendida como una situación en la cual las expectativas de inflación de largo plazo no responden significativamente a nueva información. Asimismo, se considera que se produce un desanclaje de expectativas solo después de una secuencia lo suficientemente larga de sorpresas de inflación. En este trabajo conceptualizamos esta idea en un modelo monetario DSGE con un mecanismo de aprendizaje que varía en el tiempo y donde la sensibilidad de los agentes a los datos entrantes depende de los errores de inflación acumulados. Estos últimos afectan la tasa de ganancia de aprendizaje que los agentes utilizan para actualizar sus creencias sobre la inflación futura. En el trabajo mostramos como este mecanismo mejora el ajuste del modelo a los datos, incluidos datos de expectativas de inflación, durante el periodo en el que el Banco Central de Chile adopta una meta de inflación. En particular, mostramos que la tasa de ganancia variable en el tiempo es capaz de capturar correctamente tanto episodios de desanclaje como de anclaje de expectativas de inflación. Finalmente, utilizamos el modelo para evaluar el rol de la política monetaria en el anclaje de expectativas de inflación.

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* The views expressed are those of the authors and do not necessarily represent official positions of the Central Bank of Chile or its Board members.
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1 Introduction

Inflation-targeting central banks tend to pay special attention to whether inflation expectations are well anchored, as this "ensures that temporary movements in inflation do not feed into wages and prices and hence become permanent" (Draghi, 2014). However, standard monetary models with rational expectations, such as the ones mostly used at central banks, are not well suited for analyzing this matter. The reason is that under rational expectations agents have perfect knowledge about the structure of the economy, including the central bank's objective function. Therefore, agents trust that the long-run equilibrium inflation rate will equal the central bank's inflation target. In contrast, inflation expectations in actual economies are not perfectly anchored, or the extent to which they are anchored can change depending on economic developments and the conduct of monetary policy. This paper addresses this issue by developing a monetary model where inflation expectations can shift over time (and may become unanchored).

The idea that the extent to which inflation expectations are anchored can change over time, depending on inflation surprises, was once discussed by the former chairman of the Federal Reserve Ben Bernanke, who used the term "anchored" to mean relatively insensitive to incoming data. Accordingly, if the public experiences a spell of inflation higher (lower) than they had expected, but their long-run expectation of inflation changes little as a result, then inflation expectations are well anchored. On the other hand, if the public reacts to a short period of higher (lower) than expected inflation by marking up (down) their long-run expectation considerably, then expectations are poorly anchored (see Bernanke, 2007). Similar ideas about the role of anchored expectations have regularly been expressed by other central bankers (see, for instance, King, 2005; Trichet, 2005; De Gregorio, 2008; Yellen, 2009, 2015, 2017; Draghi, 2014, 2017).

According to Bernanke (2007), while the possibility that inflation expectations may become unanchored is not easily handled in a rational expectations framework, this possibility fits quite naturally into a framework where agents form their expectations in an adaptive way. In fact, several studies in the literature on learning in macroeconomics have analyzed the implications of adaptive learning for inflation dynamics and monetary policy; for a review, see Evans and Honkapohja (2009). These studies have usually examined the learnability (or expectational stability) of rational expectations equilibria (see, e.g., Bullard and Mitra, 2002, 2007; Evans and Honkapohja, 2003, 2006), or optimal monetary policy when agents form their expectations using adaptive learning (see, e.g., Gaspar, Smets, and Vestin, 2006; Orphanides and Williams, 2007, 2008, 2009; Chen and Kulthanavit, 2008).

However, few existing studies on monetary policy under learning allow for shifting sensitivities of agents' expectations to incoming data that would match Bernanke's (2007) definition of (un)anchored expectations. In particular, under a standard ordinary least squares (OLS) learning scheme, agents update the parameters of their forecasting models (loosely referred to as beliefs) by putting equal weights on all data. Another commonly updating scheme, commonly used in empirical works, called constant-gain learning, sets more weight on newer data, but this weight is fixed over time. The possibility of a time-varying learning mechanism has been explored by some studies following Marcet and Nicolini (2003), where the learning scheme switches between OLS and constant-gain learning. A recent study implementing such a mechanism in a partial equilibrium framework for inflation is Carvalho, Eusepi, Moench, and Preston (2017). However, to our knowledge no study exists that analyzes (optimal) monetary policy in a general equilibrium model with a time-varying learning mechanism;[1] that is, a framework that would capture the interactions and feedback effects between expectations and inflation dynamics on one side and monetary policy on the other. Related to the previous point, another characteristic that is shared by most analysis of monetary policy under learning is that they are based on relatively small New Keynesian models. This contrasts with analysis of optimal policy under rational expectations in medium-scale Dynamic Stochastic General Equilibrium (DSGE) models (see, e.g., Schmitt-Grohé and Uribe, 2005, 2007). This kind of model, which might allow a more complete assessment of the actual structure of the economy and trade-offs faced by policymakers, is used at most central banks for regular policy analysis. Moreover, several studies have found an important role for learning in quantitative DSGE frameworks designed to fit macroeconomic data (see, e.g., Milani, 2007; Slobodyan and Wouters, 2012a,b).[2] These studies point towards the potential usefulness of allowing for learning in DSGE models used for monetary policy analysis. Hence, in this paper we develop a monetary DSGE model with a time-varying learning mechanism for inflation expectations. Our starting framework is a standard, medium-sized quantitative New Keynesian model, which shares many of the features of the models currently used at central banks (including different nominal and real rigidities). The model also incorporates a number of features that are introduced to fit Chilean macroeconomic data, a point to which we return below. Based on this model, we make two major modifications: first, we allow for adaptive learning by the agents about all forward-looking variables in the model; and second, we allow for shifting sensitivities to incoming data in the scheme used by the agents to updated their beliefs on future inflation. The latter is implemented through a time-varying learning gain parameter for inflation expectations, whose evolution depends on accumulated inflation forecast errors over

[1]One study, Murray (2008), has analyzed the Great Moderation hypothesis in a model with a time-varying learning mechanism.

[2]Milani (2007) shows that an estimated DSGE model with learning and a standard set of structural nominal and real rigidities matches inflation persistence in the data better than its rational expectations counterpart. Slobodyan and Wouters (2012a,b) obtain similar findings, including that a monetary DSGE model with learning can explain the observed decline in the mean and the volatility of inflation as well as Phillips curve flattening.

a specific horizon. Through these modifications, we attempt to conceptualize the idea that the extent to which inflation expectations are anchored can change over time, in line with Bernanke (2007), as discussed above.

The precise mechanism is as follows. Agents are assumed to form expectations using estimated reduced-form models, which they re-estimate each period as new information becomes available, using a given learning gain. In addition, if they face inflation surprises, agents change the estimation strategy they use to update their inflation forecasting models by putting more weight on more recent forecast errors, through a larger gain. The underlying motivation for such a time-varying learning mechanism is that agents may respond to the possibility that the economic environment may have changed, by using a learning process that produces better forecasts in such a situation; see Marcet and Nicolini (2003). This switch generates a larger sensitivity of inflation expectations to incoming data. The change is gradually reverted as the estimated reduced-form models recover a better degree of forecasting performance.

The evolution of inflation expectations in Chile is an interesting case of study in this context. The Central Bank of Chile is conducting an economic expectations survey since the consolidation of its inflation targeting regime in 2001, which has a 3% target for annual inflation to be achieved over a two-year horizon.[3] Long-run inflation expectations from that survey (measured by the median of two-year ahead expectations) have equaled the inflation target most of the time since 2001. However, during some episodes, long-run expectations deviated from the target. According to the hypothesis that we study in this paper, these episodes may have been associated with higher or lower-than-expected inflation. To test this hypothesis, we conduct a simple empirical exercise based on binary regressions on the outcomes from the inflation survey. This exercise shows that the observed deviations from the central bank's inflation target can be explained by accumulated forecast errors: these were relatively large (small) when expectations deviated from (equaled) the target.[4] Thus, this exercise establishes a key stylized fact, namely, that the probability that inflation expectations may become unanchored increases with accumulated inflation surprises, which is consistent with the learning mechanism in our DSGE model.

We estimate the DSGE model and its rational expectations counterpart on Chilean macroeconomic data from 2001Q4 to 2016Q4, including one-year and two-year ahead inflation expectations

[3]Chile was among the first countries in the world to adopt an inflation targeting regime. After enacting new central bank legislation in 1989, which gave independence to the monetary authority and mandated price stability as a primary objective, the central bank announced its first annual inflation objective in September 1990. The inflation objective was gradually lowered during the 1990s towards the current 3% target (with a tolerance range of 2 to 4%). The latter exists since 2001, together with a floating exchange rate and the short-term nominal interest rate as the main instrument for monetary policy (see Morandé and Schmidt-Hebbel, 2001; Mishkin and Savastano, 2001; Morandé, 2002; Central Bank of Chile, 2007).

[4]The level of inflation itself is not found to be a statistically significant factor explaining the probability that inflation expectations deviate from the target, once accumulated forecast errors are controlled for.

among the set of observed variables. In particular, the parameters that regulate the scheme that agents use to update their beliefs are estimated together with the remaining deep parameters of the model, disciplined by the observed measures of expectations.

Based on the estimated model, we conduct a goodness-of-fit comparison with the rational expectations version of the model, in order to assess the importance of the learning mechanism implemented in our model. This comparison shows that the presence of adaptive learning improves the fit of the model to the data, in part due to the time-varying learning mechanism. Also, we show that the observed unanchoring episodes are well captured by the estimated time-varying learning gain, which increases around those episodes.

Next, we use the model to analyze the transmission of inflationary shocks and changes in monetary policy. This analysis shows that the responses of inflation expectations and effective inflation are relatively larger when the gain is relatively high (i.e., after a sequence of inflation surprises). In addition, we find that monetary policy is relatively more effective in affecting inflation outcomes when the gain is relatively high.

As a final step, we analyze the role of monetary policy in order to anchor inflation expectations, following some of the previous literature on the design of monetary policy rules under uncertainty and learning. In particular, we assess alternative (simple, implementable) monetary policy rules and ask whether a stronger response of the monetary authority to inflation, or a response to agents' forecast errors, can be useful to face the monetary authority's output-inflation trade-off, expressed according to a quadratic loss function. We find that the optimal monetary policy rule under rational expectations performs badly under learning, relative to the optimal rule under learning, which requires a stronger response to inflation or expected inflation. In addition, we find that the loss is reduced when the monetary authority responds more aggressively to inflation or expected inflation when the learning gain for inflation expectations is relatively high, since such a policy reduces the incidence of unanchoring episodes.

The finding that monetary policy should respond more aggressively to inflation or expected inflation when agents form their expectations using adaptive learning confirms previous results from the literature on monetary policy under learning (see, e.g., Orphanides and Williams, 2008, 2009). However, our model adds a new dimension to this finding, which is that increasing the monetary authority's response to inflation can reduce the incidence of unanchoring episodes. Thereby, the gains from a relatively strict inflation targeting regime are estimated to be larger than one would obtain in a rational expectations framework or a framework with a standard learning mechanism.

Overall, the contribution of our paper is two-fold. First, we develop and estimate a medium-sized monetary DSGE model with adaptive learning and a time-varying learning mechanism for inflation. Second, we use this framework to analyze the implications of shifting inflation expectations for monetary policy. Our analysis is especially relevant for inflation-targeting central banks that pay attention to whether inflation expectations are well anchored.

The rest of the paper is structured as follows. Section 2 contains the case study of inflation expectations, where we conduct binary regressions to explain episodes of anchored and unanchored inflation expectations in Chilean survey data. Section 3 describes our monetary DSGE model with the time-varying learning mechanism for inflation expectations, as well as its estimation on Chilean macroeconomic data. Section 4 presents the results, focusing on the goodness-of-fit of the model compared to its rational expectations counterpart, the time-varying transmission of inflationary shocks implied by the model, and the analysis of the role of monetary policy in order to anchor inflation expectations. Finally, Section 5 concludes.

2 Case Study: Shifting Inflation Expectations in Chile

In this section we present a simple empirical exercise aimed at providing insight into the link between agents' forecast errors and the anchoring of inflation expectations. One of the main building blocks of our model is the mechanism by which agents' inflation expectations may become un-anchored and this exercise helps justify how it is ultimately modeled.

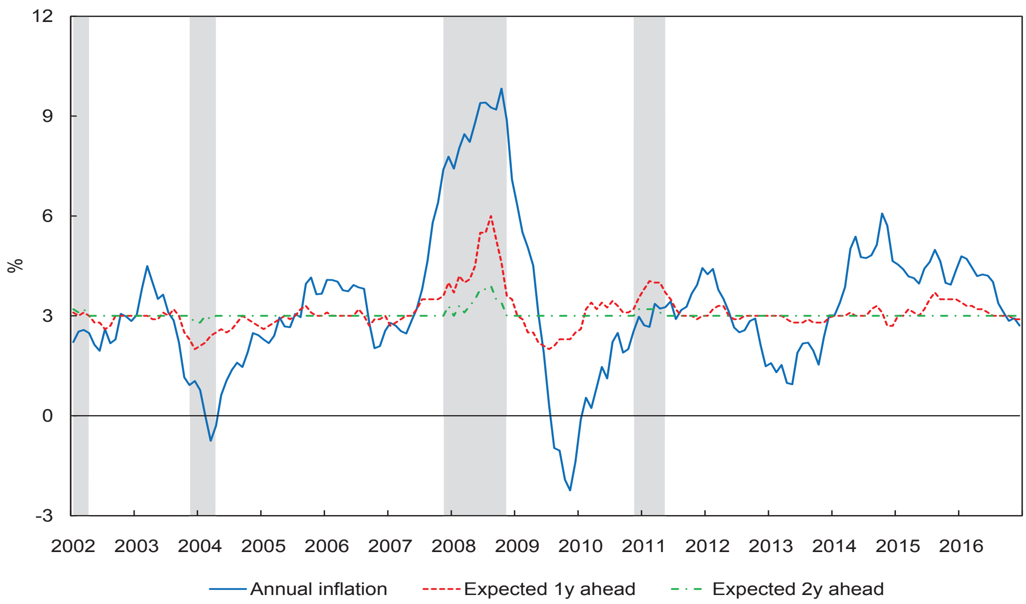

Figure 1: Effective and expected CPI inflation

Figure 1 plots annual inflation and one and two years ahead inflation expectations for Chile since the introduction of the 3% inflation target.[5] The figure also depicts, using grey bars, the periods in which the two years-ahead inflation forecast deviates from 3%. Following an informal working consensus, these deviations are associated with (potential) inflation unanchoring events.[6] Since 2002, one can observe several of this events taking place as well as other periods in which inflation rose or fell but in which the two years ahead inflation expectations remained at target. This suggests that the inflation level is not a main determinant of unanchoring, if at all.

Figure 2, in turn, depicts the mean absolute one-month ahead inflation expectation error (MAE), using a 12 months moving average. It shows how relatively large forecast errors tend to be associated with unanchoring episodes, while relatively small ones tend to be observed when inflation expectations are anchored.

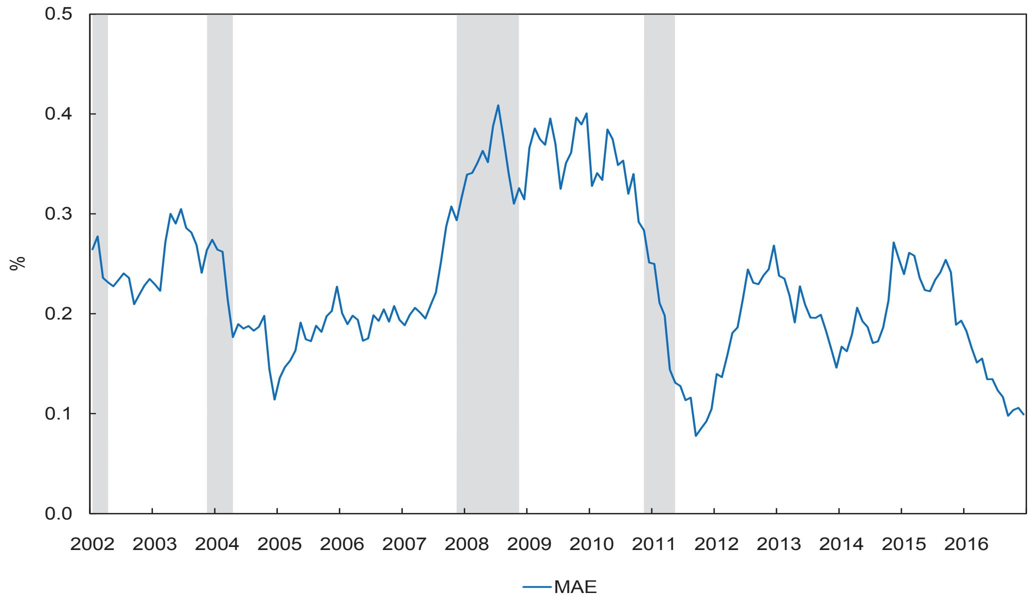

Figure 2: Mean absolute inflation forecast error

Both these graphs suggest that when agents accumulate a number of relatively large forecast errors (i.e. after facing a series of short-term inflation surprises), long-run inflation expectations tend to deviate from target, becoming, potentially, un-anchored. This happened in a particularly strong fashion during 2008.

[5] The former is constructed and provided by the Instituto Nacional de Estadística of Chile, while the latter are collected by the Encuesta de Expectativas Económicas survey, which is carried out by the Central Bank of Chile.

[6] This is an informal definition of unanchoring, which is not precisely the one we conceptualize in the DSGE model later on (i.e. the one Bernanke suggests), but that serves as a working approximation.

In order to test this hypothesis and to provide more formal evidence supporting this relation, we run a series of Probit and Logit regressions with different specifications and present the results. Let $y_t = 0$ denote the case in which the two years-ahead inflation expectation is equal to 3% and $y_t = 1$ the case when it is different from it (this precisely captures the gray bars in the graphs). We regress the so constructed binary variable on the mean absolute one-month ahead inflation expectation errors, over the last 12 months, and on absolute deviations of the monthly inflation rate from its implicit target of 3/12%. Table 1 summarizes the results.

	Probit		Logit	
Constant	-2.61	-2.67	-4.85	-5.00
	(-0.45)	(-0.46)	(-0.93)	(-0.96)
	[0.000]	[0.000]	[0.000]	[0.000]
MAE	5.36	7.76	10.45	15.28
	(-1.64)	(-3.13)	(-3.19)	(-5.87)
	[0.001]	[0.013]	[0.001]	[0.009]
MME(MAE)	2.12	3.07	2.60	3.80
Inflation		-1.53		-3.01
		(-1.71)		(-3.10)
		[0.372]		[0.332]
MME(Inflation)		-0.61		-0.75
R-squared	0.10	0.10	0.10	0.11
Obs.	180	180	180	180

Table 1: Notes: Standard errors (p-values) in round (squared) brackets, MME = mean marginal effects.

As expected, we find that the mean absolute error has a significant effect on the "unanchoring" probability of inflation expectations. In fact, by computing its mean marginal effect, we can estimate that an increase of the mean absolute error by 0.1 percentage points increases the probability of becoming un-anchored by between 0.21 and 0.38 percentage points. In addition, we also find that the level of inflation is not significant for the deviations of the two years ahead inflation expectations from target, our proxy for "unanchoring" episodes. This is particularly interesting, since it is not an intuitive result. We find similar results when replacing the MAE by other measures such as the root mean square error (RMSE) or when introducing lags.

Plotting the probability response curves, figure 3, we can further observe that the probability of becoming un-anchored increases steeply once the forecast errors become larger than the mean, reaching substantially large levels, $70 - 75\%$, only for the largest observed errors.

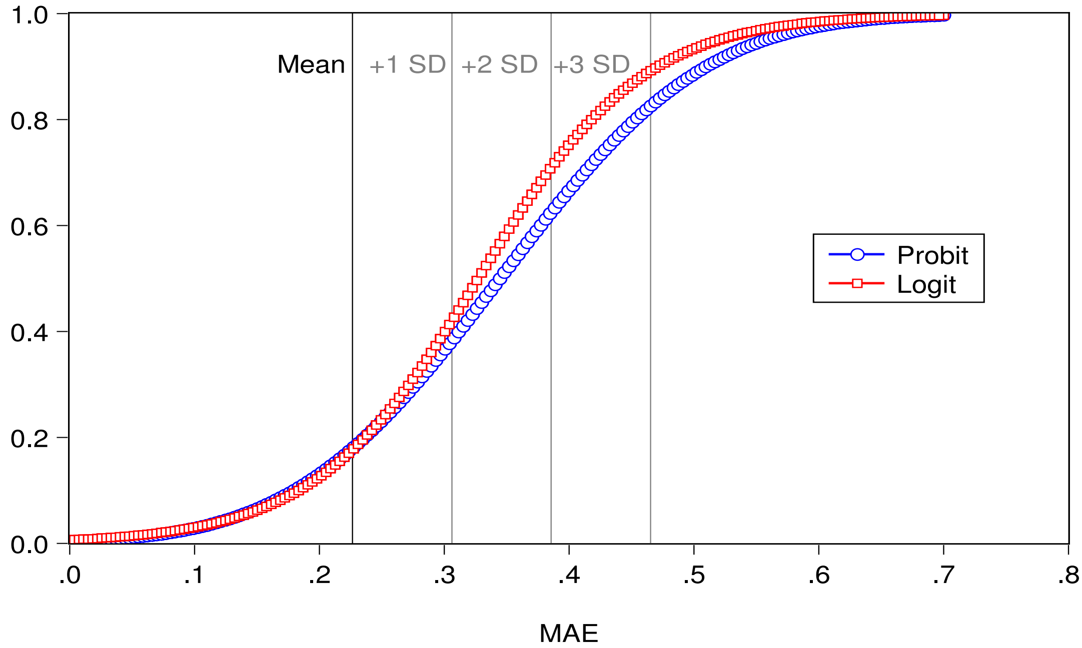

Figure 3: unanchoring probability as function of MAE

Following the evidence presented here and in line with the literature (Murray (2008), Carvalho, Eusepi, Moench, and Preston (2017)), the agents' unanchoring mechanism is made solely depended on agents' forecast errors. The specific modeling of the unanchoring mechanism is described later in the paper.

3 A Monetary DSGE Model with Shifting Inflation Expectations

Next, we provide a sketch of the Dynamic Stochastic General Equilibrium (DSGE) model and summarize its estimation. A detailed description of the model is provided in the appendix.

3.1 Basic framework

The model is based on a New Keynesian model for a small open economy. It consists of four main sectors: Households, Firms, Government and an external sector. In the first sector, a continuum of infinitely lived households of mass one, with identical asset endowments and identical preferences, seek to maximize their lifetime utility subject to a budget constraint; deciding on how much of the final good to consume (relative to an external habit component) and how many hours to work. Each period, they also decide how much to invest in capital, taking into account its adjustment costs, and how much to save and borrow by purchasing domestic currency denominated government bonds and trading foreign currency bonds, both being non-state-contingent assets. The second sector, the supply side of the economy, is composed by different types of firms that are all owned by the households. There is a set of monopolistically competitive firms producing different varieties of a home good, choosing labor and capital as inputs, and setting prices subject to a Calvo-type scheme with indexation; a set of monopolistically competitive importing firms, which import a homogeneous foreign good and transform it into different varieties, setting prices subject also to a Calvo-type scheme with indexation; and three groups of perfectly competitive firms that aggregate products: one combining different varieties of the home good into a composite home good, one packing the imported varieties into a composite foreign good and, finally, another one that bundles the composite home and foreign goods to create a final good. This final good is purchased by the households, both for consumption and investment, and by the government. In addition, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad, it follows an exogenous process and captures, primarily, the importance of the copper sector of Chile. The third sector, the government, follows a Ricardian fiscal policy and sets the short term nominal interest rate according to a Taylor-type monetary policy rule. Finally, the fourth sector, the rest of the world, demands home composite goods and buys the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the domestic economy is assumed to be small relative to the foreign economy, which means that the foreign producer price level is identical to the foreign consumption-based price index and that the foreign interest rate is taken as given. The relevant foreign nominal interest rate, however, depends on a country premium that increases with the economy's net foreign debt position.

The model also features a standard set of exogenous shocks including a preference shock, an investment specific technology shock, both permanent and transitory TFP shocks, a monetary policy shock, a government expenditure shock and an import price shock, as well as exogenous processes modeling the foreign aggregate demand, the foreign price inflation rate, the risk-free world interest rate, the country premium, and the real price and production of copper.

The model microfundations, its solution, steady state derivation and log-linearization are presented in the appendices A, B, C, and D. The model is finally completed by the expectations formation mechanism, which we introduce next.

3.2 Expectations formation

In order to take their decisions, agents in the model need to forecast a series of future variables, such as aggregate consumption, price inflation and the real exchange rate. Following the Adaptive Learning (AL) literature, we assume agents construct those forecasts by means of reduced-form models, which they (re-)estimate each period as new information becomes available.[7] There are several reasons that support such a deviation from the standard rational expectations (RE) assumption, ranging from the enormous demand RE places on agent's knowledge about how the economy works, to the improved data fit that AL generally delivers. However, the main reason why we adopt such a framework here responds, as it will become clear shortly and as Bernanke (2007) points out, to the natural way it allows us to accommodate the concept of anchored expectations. Indeed, in the model we assume that inflation variables, namely $\{\pi_t, \pi_t^H, \pi_t^F\}$, may become un-anchored. The remaining forward-looking variables, we assume are forecasted following a standard constant-gain scheme. There are tow main reasons for this distinction, first, we want to discipline the unanchoring dynamics with expectations data, and for Chile only expectations about inflation are available; and second, inflation is the variable Central Banks recurrently worry about when it comes to unanchoring.

Let us describe the expectation formation mechanism in more detail: agents are assumed to construct forecasts for every forward-looking variable appearing in the model, x_t^f, using estimated linear reduced-form models of the following form,

$$x_t^f = \beta' x_{t-1}^s + \varepsilon_t \tag{1}$$

where $x_t^f = \{c_t, \pi_t, \pi_t^H, \pi_t^F, i_t, rer_t, r_t^k, q_t\}$,[8] β is a vector of parameters, loosely referred to as beliefs, and x_{t-1}^s is the vector of all states appearing in the minimum-state-variable solution of the model under rational expectations.[9] Agents re-estimate these reduced-form models every

[7] For a detailed introduction to Adaptive Learning see Evans and Honkapohja (2001).

[8] The listed variables are in order, consumption, aggregate inflation, home good inflation, foreign good inflation, investment, real exchange rate, the return on capital and the price of capital.

[9] Note that since the agents' forecasting models have the same form as the MSV solution, the rational expec-

period, as new information becomes available. They do so, by using the following recursive updating equations,

$$\hat{\beta}_t = \hat{\beta}_{t-1} + g_t R_t^{-1} x_{t-1}^s \left(x_t^f - \hat{\beta}_{t-1}' x_{t-1}^s \right) \quad (2)$$

$$R_t = R_{t-1} + g_t \left(x_{t-1}^s x_{t-1}^{s}{}' - R_{t-1} \right) \quad (3)$$

which adjust beliefs in light of the last forecast error and where g_t, called the gain parameter, regulates the rate of that adjustment; or, in other words, the sensitivity of agents' beliefs to new available data.[10] And it is precisely through this gain parameter that adaptive learning allows to accommodate the unanchoring of expectations in a natural way. As expectations become more (un-)anchored, agents should become (more) less sensitive to incoming information and, therefore, g_t should (increase) decrease. Building on Murray (2008) and Carvalho, Eusepi, Moench, and Preston (2017), and keeping in line with the results of section 2, we propose to capture this mechanism with the following algorithm for g_t,[11]

$$g_t = \begin{cases} g \cdot \left(\frac{\frac{1}{J}\Sigma_{i=0}^{J-1} |\hat{E}_{t-1-i}(x_{t-i}) - x_{t-i}|}{\nu \cdot RMSE_t} \right)^{\nu^h} & , if\ \frac{1}{J}\Sigma_{i=0}^{J-1} |\hat{E}_{t-1-i}(x_{t-i}) - x_{t-i}| \geq \nu \cdot RMSE_t \\ g \cdot \left(\frac{\frac{1}{J}\Sigma_{i=0}^{J-1} |\hat{E}_{t-1-i}(x_{t-i}) - x_{t-i}|}{\nu \cdot RMSE_t} \right)^{\nu^l} & , if\ otherwise \end{cases}$$

where $\nu > 0$ determines the switch-triggering threshold and $\nu_h \geq 0$ and $\nu_l \geq 0$ allow for non-linearities.

By construction, then, as the average forecast error over the last J periods increases, so does the gain and, therefore, so does the agents' sensitivity to incoming data. Furthermore, these increases are continuous; notice, in particular, that when the inequality condition holds with equality, both r.h.s. terms are equal to g. Moreover, notice that this unanchoring algorithm nests the case of a constant gain, a case in which the sensitivity to incoming data is constant or, in order terms, a case where there is no unanchoring ($\nu_h = \nu_l = 0$). This is of particular

tations solution is nested in this setup. This functional form is a standard assumption that provides a relative restricted deviation from RE. Other functional forms found in the literature include simple AR(2) models, as in Slobodyan and Wouters (2012a), or small VARs, as in Jääskelä and McKibbin (2010) and Orphanides and Williams (2008).

[10] Notice that if g_t is set to $\frac{1}{t}$, then equations (2) and (3) become the recursive formulation of ordinary least squares, where R_t is the covariance matrix of the states.

[11] Given the specificities of the DSGE model at hand, in particular its size, the discontinuity of unanchoring mechanisms more in line with Carvalho, Eusepi, Moench, and Preston (2017) proved to be very problematic for the estimation. Therefore, we constructed the continuous unanchoring scheme proposed here. Kushner and Yin (2003) is the only other continuous algorithm for a time-varying gain that we are aware of.

relevance as the estimations of the model will allow the data to neglect the unanchoring dynamics in inflation expectations.

As mentioned before, for all other non-inflation forward-looking variables, we adopt the standard assumption in the empirical learning literature and set g_t to a small positive constant.[12] Under this assumption agents discount past information more heavily than recent one, reflecting their wish to remain flexible to potential changes in the economy. Notwithstanding, it is important to notice that under this updating scheme the sensitivity of beliefs to new data is constant over time.

Finally, agents construct their forecasts as,

$$\hat{E}_t x^f_{t+1} = \hat{\beta}'_{t-1} x^s_t \tag{4}$$

where $\hat{E}$ denotes subjective expectations.[13]

3.3 Calibration and estimation

Most of the model's parameters are estimated using a Bayesian approach, while others are calibrated or estimated separately. The latter include some parameters that are drawn from related studies for Chile, some that are chosen to match sample averages or long-run ratios for the Chilean economy, and the parameters of the exogenous processes for which we have a data counterpart, such as the ones for foreign inflation, foreign interest rate and the international price of copper. Table 5, in appendix E, lists those parameters.

For the estimation, the model is first log-linearized around its non-stochastic steady sate and then brought into state-space form,

$$x_t = A(\theta, \beta_{t-1})x_{t-1} + B(\theta, \beta_{t-1})\varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma_\varepsilon) \tag{5}$$

$$obs_t = F(\theta, \beta_{t-1})x_t + \omega_t, \quad \omega_t \sim \mathcal{N}(0, \Sigma_\omega) \tag{6}$$

The first equation - the transition equation - characterizes the laws of motion of the endogenous variables in the model and the second equation - the observation equation - maps the model's endogenous variables to the observables. In particular, x_t denotes the vector of the model's endogenous variables, θ is the vector of parameters to be estimated, ε_t is the vector of innovations

[12] This type of Adaptive Learning is known as constant-gain or perpetual learning.

[13] Notice that the forecast agents use in their beliefs updating equation, (2), is slightly different from the forecast they use in their decision problems. This is a technical simplicity used to avoid simultaneity between x^f_t and $\hat{\beta}_t$, see Evans and Honkapohja (2001) (footnote in page 200).

to the exogenous shocks in the model, and $A(\theta, \beta_{t-1})$, $B(\theta, \beta_{t-1})$ and $F(\theta, \beta_{t-1})$ are real matrices of appropriate size, which depend on the parameters and on agents' beliefs. obs_t is the vector of observables that includes fifteen Chilean and foreign macroeconomic variables ranging from the fourth quarter of 2001 to the fourth quarter of 2016: GDP, investment, private and government consumption, copper production, CPI inflation, import price inflation, the short-term nominal interest rate, the real exchange rate, a trade-weighted GDP as a proxy for foreign aggregate demand, external CPI inflation, the LIBOR as a proxy for the foreign interest rate, the real copper price and the expected CPI inflation at one and two ahead.[14] And ω_t is a vector collecting the measurements errors used for output, consumption, investment, inflation and the real exchange rate; all of which are calibrated to 10% of the corresponding sample variance. Finally, we include measurement errors for the one year and two years ahead inflation expectation series, but these are estimated.

Then, the likelihood can be computed with the Kalman Filter and, given a prior, the posterior distribution of the parameters is explored and generated using a Metropolis-Hastings algorithm. In particular, the parameters that regulate the scheme that agents use to update their beliefs are estimated together with the remaining deep parameters of the model and disciplined by the observed measures of expectations. [15] The estimation results, together with the prior specification is presented in the appendix.

4 Results

In this section we present our results based on the estimated model. First, we analyze the unanchoring dynamics of inflation expectations and, in order to assess its importance, compare the goodness-of-fit of the model featuring this setup with the rational expectations version of the model. Next, we use the model to study the transmission of inflationary shocks and changes in monetary policy. And finalize by assessing the role of monetary policy in order to anchor inflation expectations, following some of the literature on the design of monetary policy rules under uncertainty and learning.

[14] The inflation expectations time series correspond to the *Encuesta de Expectativas Económicas*, a survey carried out by the Central Bank of Chile.

[15] Adaptive learning introduces non-linearities in an otherwise log-linear model. Not only the beliefs updating equation are non-linear - see equations (2) and (3) -, but also agents' expectations, $\hat{E}_t x^f_{t+1} = \hat{\beta}'_{t-1} x^s_t$. This is because the parameters of the reduced-form models used to forecast forward variables, and that under rational expectations were constants, are now dynamic states. In principle, Bayesian estimation of such a model would have to relay on some non-linear filter, however, these types of filters are too costly computationally. Therefore, following much of the empirical learning literature, we abstract of all uncertainty in the beliefs updating equations and treat β_t as time varying parameters.

4.1 Unanchoring of inflation expectations

Figure 4, depicts the evolution of g_t for the inflation variables over the sample for Chile. The gain dynamic shows that the model is able to capture the episodes generally associated with the unanchoring of inflation expectations, as well as the episodes in which inflation expectations are thought to have been anchored.

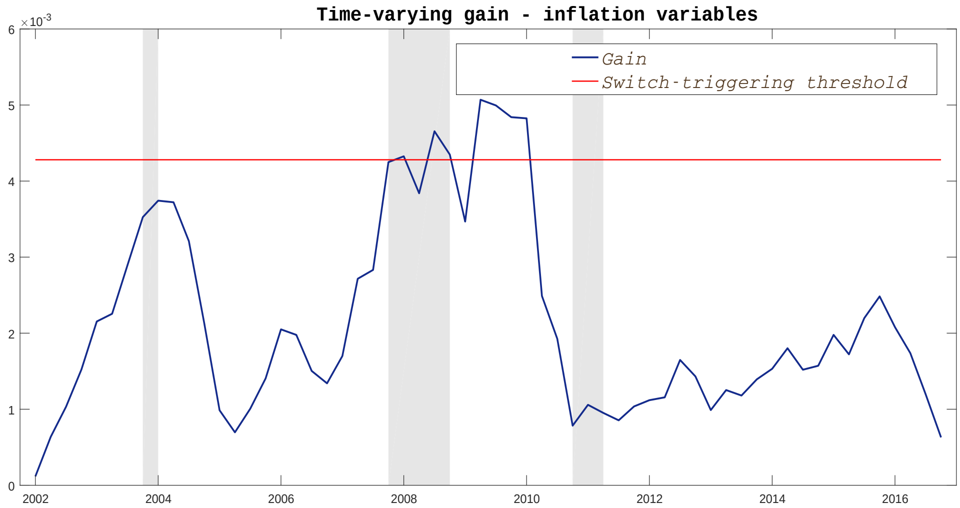

Figure 4: unanchoring dynamic for inflation variables, g_t

The gain attains its largest values when the two years ahead inflation expectations are not at target, except for the 2011 episode, and becomes relatively low otherwise, which is in line with the observed forecast errors. In particular, in 2008, the gain exceeded the switch-triggering threshold and the agents' sensitivity to incoming data became largest.[16] According to the model, expectations were well anchored during the 2011 episode even though the two years ahead inflation expectations were not at target. This underscores the conceptualization of anchoring presented here which differs from what some economist tend to understand as unanchoring. Namely, the unanchoring of expectations is an increase in agents' sensitivity to short-term news and not a deviation of agents' long-term expectations from the monetary authority's target.

Interestingly, the model also suggests a significant increase in the degree of unanchoring towards 2015, but that did not reach the levels of previous episodes, despite the increase in inflation. During this period worries about a possible unanchoring were substantial among economists in Chile. It is also worth noticing that the gain tends to increase gradually over time. This is an appealing feature of this setup, which provides an early-warning mechanism to which the

[16]Given the continuous nature of our unanchoring mechanism, it is more sensible to talk about the degree of unanchoring or anchoring instead of unanchoring episodes, though one could consider as episodes of unanchoring whenever the gain for the inflation variables becomes larger than the switch-triggering threshold.

monetary policy could react.

4.2 Goodness-of-fit analysis

Next, we compare the fit with the data of the model featuring our adaptive learning scheme that allows for unanchoring with the rational expectations version of the model. This helps asses the importance of the learning mechanism implemented in our model.

	RE	AL-unanchoring
$p(\theta \vert Y)$	-1303.7	-1289.7
$p(Y)$	-1387.5	-1382.0

Table 2: Log posteriors and marginal likelihoods.

Table 2 presents the log posterior estimates in the second row. Our model improves the fit to the data, upon the version under rational expectations, in about 5.5 log points. Following the scales proposed by Jeffreys (1961) and Kass and Raftery (1995), this constitutes substantial evidence in favor of the model with adaptive learning and unanchoring expectations.

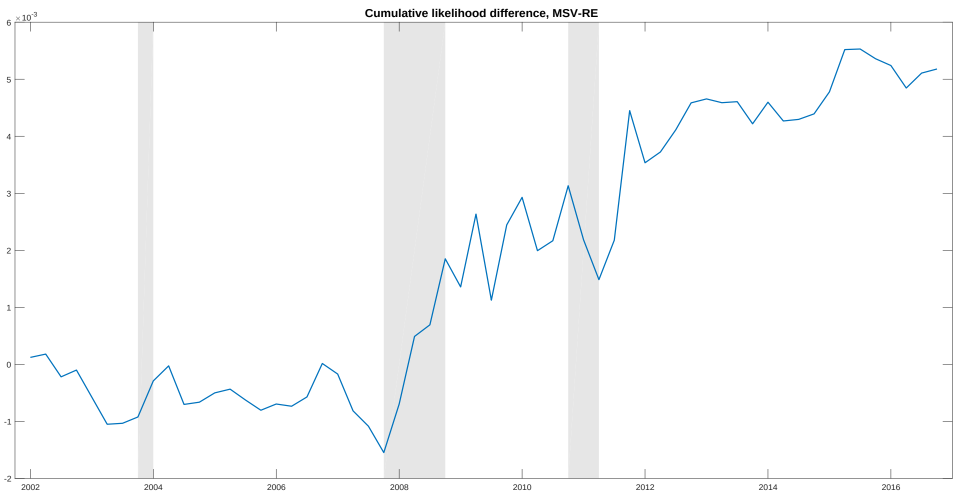

Figure 5: Cumulative likelihood difference, MSV - RE

Looking more closely into the in-sample fit of the model under both expectations formation mechanisms, we can observe how the major improvement of the model feature unanchoring relative to its rational expectations counterpart takes place during the 2008 episode, in which the 2 years-ahead inflation expectations deviated from target. More generally, as expected, unanchoring appears to help the fit particularly when the degree of unanchoring increases.

4.3 Effects of inflationary shocks

A crucial aspect of the model that needs to be checked, is whether the sensitivity of inflation expectations to short-term news actually increases when expectations become un-anchored relative to when they are not. In order to see this, we look at the impulse response functions (IRF) of inflation expectations during both these types of episodes. Figures 6 and 7 plot the cases for two inflationary shocks, a government expenditure shock and a foreign demand shock respectively. The graphs show the reactions during a period in which expectations were well anchored, the fourth quarter of 2001 (in black), and during the worst unanchoring episode in the sample, the third quarter of 2008 (in blue). We also plot the IRFs corresponding to the model under rational expectations as a reference, but using the parametrization estimated under the model that incorporates unanchoring (in red).

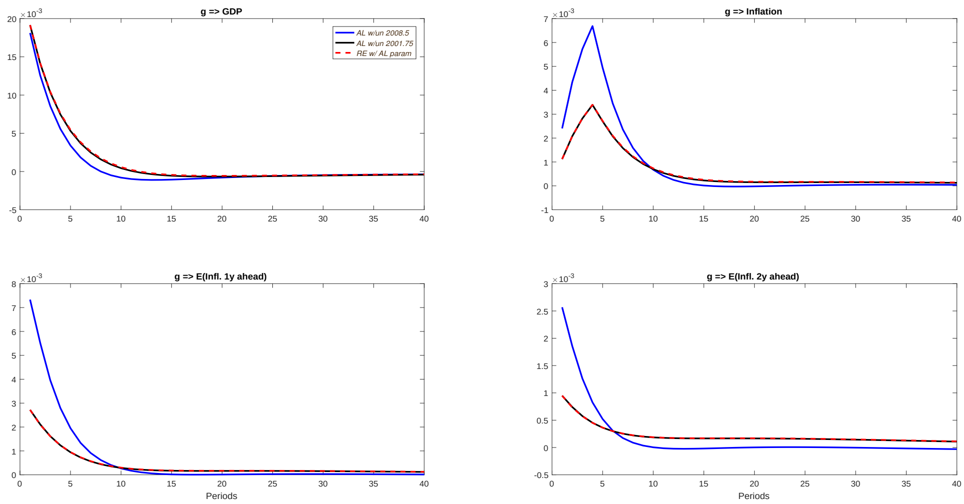

Figure 6: Impulse responses to a government spending shock (25 bp)

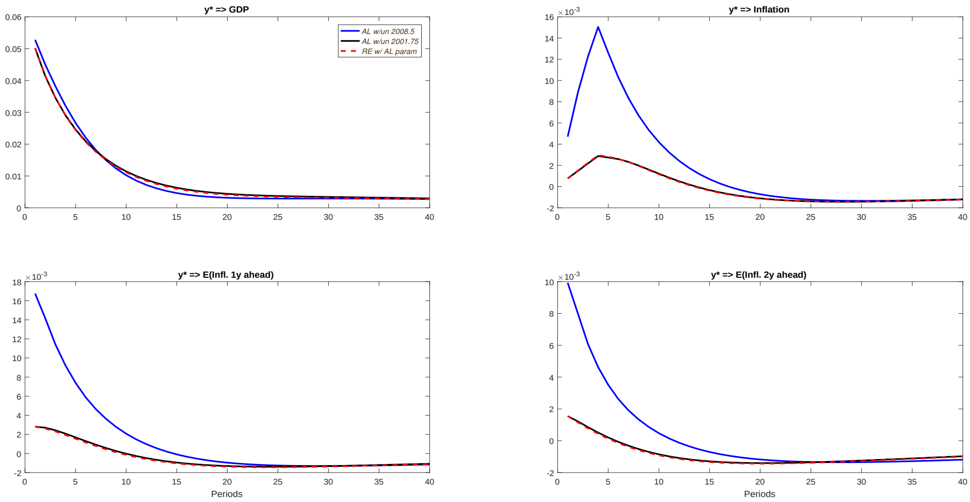

Figure 7: Impulse responses to a foreign demand shock (25 bp)

The IRFs show, indeed, that the inflation expectations responses become several times larger when inflation expectations are un-anchored than when they are well anchored. Thus, the proposed mechanism included in the model correctly conceptualizes the unanchoring of expectations as defined by Bernanke (2007). Also, in turn, the effect on inflation is significantly larger under unanchoring as it incorporates the larger effect on expectations. Is interesting to observe that when inflation expectations are well anchored, the responses of the economy to these shocks does not differ significantly from the model under rational expectations - though differences do exist in the responses of other variables to other shocks.

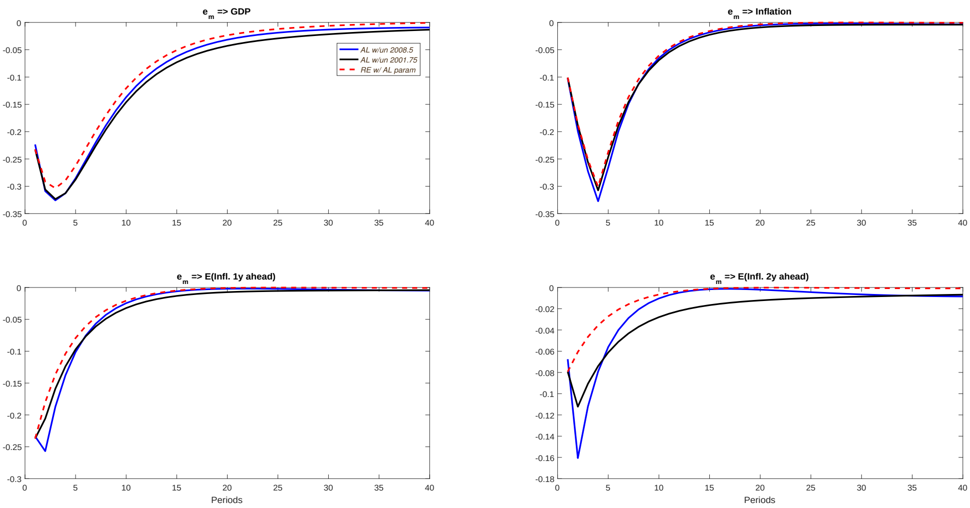

Figure 8: Impulse responses to a monetary policy shock (25 bp)

We also show the IRFs to a monetary policy shock. Besides the larger effect on inflation expectations that can be observed under unanchoring, the graphs show that the effect of monetary policy has on annual inflation, at least in the short run, increases with agents' sensitivity (see figure 8).

4.4 The role of monetary policy

In this final section we analyze the role of monetary policy in the context of unanchoring inflation expectations. In particular, we study the effects that unanchoring has on the coefficients of the monetary policy rule assumed in the model, were these to be chosen optimally. We also take a look into possible variations of monetary policy that may help reduce the unanchoring episodes, both in quantity and quality.

The central bank is here assumed to minimize a weighted sum of the unconditional variances of the deviation of inflation from target, the deviation of output from its trend and the first difference of the nominal interest rate. This is a standard loss function used in the literature that captures the main objectives of monetary policy, see Orphanides and Williams (2009) and Woodford (2001) among others. We compute the minimization for both expectations formation mechanisms, the one we propose here, that allows for unanchoring, and the rational expectations benchmark. The validity of this exercise is based on the assumption that the estimations of the model, under both these expectations formation mechanisms, deliver the true deep parameters' values, respectively. The loss function can be written succinctly as

$$L = var\left(\pi\right) + \lambda var\left(y\right) + var\left(\Delta i\right) \tag{7}$$

We further consider two cases of this loss function, a first one, following Orphanides and Williams (2009), with $\lambda = 1$ that places equal weight on the stabilization of inflation and activity; and a second one, with $\lambda = 0.1$ that reflects a central bank that is more concerned about inflation than activity, as, for example, inflation targeting central banks. For both cases and both expectations formation mechanisms, we find that the optimal reactions to activity are close to zero, while the reactions to inflation are, as expected, larger when the monetary authority places relatively more weight on inflation variations, i.e. when $\lambda = 0.1$. Moreover, the reaction to inflation is found to be significantly larger for the optimal rules under unanchoring than under rational expectations. Similarly, when $\lambda = 1$, the optimal reaction coefficient to inflation is 1.09 while under unanchoring it rises to 1.32. For the case of $\lambda = 0.1$, these coefficients are found to be 2.27 and 3.90 respectively.

Consequent with these results, we find that the performance of the optimal policies derived under the assumption of rational expectations perform poorly if the actual world is better described with the model that allows for the unanchoring of expectations, see table 3. These results are in line with the ones reported in Orphanides and Williams (2008, 2009), where monetary policy derived under rational expectations is found to perform poorly under an adaptive learning world.

λ	Optimal MP under RE	Optimal MP under unanchoring
1	16.57	13.5
0.1	2.08	1.95

Table 3: Losses under the model that allows for unanchoring for optimal rules.

Next we conduct a simple exercise in which we introduce small variations to the Taylor rule of the model and see the effects that they have on the unanchoring dynamics of inflation expectations. More concretely, we consider two variations, first we replace the inflation term with the expectation of inflation the following period, $E_t(\pi_{t+1})$, a formulation that has proven not to be useful under RE; and second we consider a monetary policy rule that increases its response to inflation deviations from target whenever the degree of unanchoring increases beyond 80% of the switch-triggering threshold. This last policy rule aims at using the gain dynamic as a warning system that can trigger a change in policy and so prevent high unanchoring episodes.

	Unanchoring frequency (%)			Loss - $\lambda = 1$		
α_π	I	II	III	I	II	III
1.4	2.7	1.97	1.83	26.7	29.2	24.0
1.87	2.42	1.91	1.64	23.8	23.9	22.1
2.4	2.37	1.83	1.71	21.7	20.8	20.5
2.8	2.32	1.79	1.60	20.8	19.4	19.2
3.2	2.31	1.97	1.58	20.1	19.6	20.8

Table 4: Simulated dynamics under alternative monetary policy rules.

Table 4 shows how the frequency of unanchoring episodes, defined as those periods in which the gain g_t is above the switch triggering threshold, changes as the reaction to inflation increases for three monetary policy rules. First (I), the original Taylor rule assumed in the model, second (II), the one that replaces the current inflation term with the expectation of inflation the following period, and third (III), the rule that increases the response to inflation when the degree of unanchoring becomes to large - the response is multiplied by four in these cases. The second row of the table corresponds to $\alpha = 1.87$, which is the value estimated for the coefficient of inflation deviations of the model under unanchoring. Therefore, for the rule I, we obtain the actual Taylor rule use in the estimation, and 2.42% is the actual frequency of unanchoring events implied by the estimated model. As it can be observed, the frequency of unanchoring events decreases as the response to inflation, or to inflation expectations in the case of rule II, increases. However this is not always the case, for rule II, increasing the response from 2.8 to 3.2 actually increases the amount of periods in which inflation expectations become un-anchored. This has to do with the way the unanchoring mechanism works, namely, that too large inflation responses generate more forecast errors which enter in absolute value. The monetary policy that increases its response to inflation when the degree of unanchoring becomes large enough appears to have a good performance in terms of reducing the frequency of episodes. A similar pattern is found when looking the implied losses, larger responses to inflation tend to reduce the loss, though here the trade-offs between inflation stabilization and output and changes in the interest rate have a more direct effect. Again, the policy rule III, appears to have the dominant performance here.

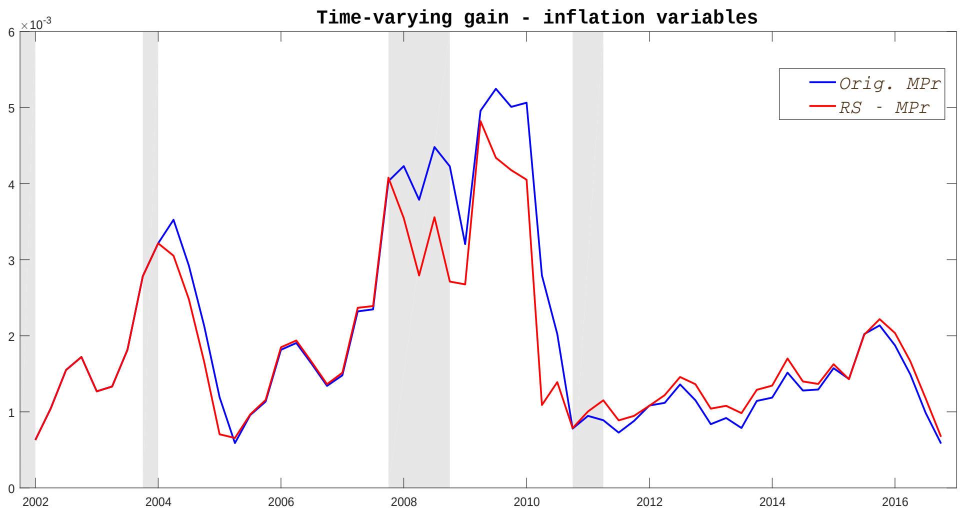

Figure 9: Counter factual exercise, higher responses during high sensitivity periods.

Finally, figure 9 presents the counterfactual dynamics for the gain under the alternative monetary policy III. The graph shows how this policy would have been able to reduce the sensitivity of agents' to incoming data during the unanchoring episodes of 2004 and 2008.

5 Conclusions

We propose a monetary model that features unanchoring of inflation expectations and test it for the Chilean economy, which stands as an relevant case-study for these types of events. Furthermore, this is the first example of a medium-size DSGE model to accommodate unanchoring of expectations. In the model unanchoring is defined as episodes in which inflation expectations become more sensitive to short-run surprises and is implement through a continuous time-varying gain that depends on accumulated forecast errors within an adaptive learning scheme. This setup conceptualizes unanchoring in the spirit suggested by Bernanke (2007) and follows the mechanisms proposed by Marcet and Nicolini (2003) and Carvalho, Eusepi, Moench, and Preston (2017). The estimated model improves the fit with the data upon its rational expectations counterpart. Moreover, the model does this while capturing well the anchored and unanchored inflation expectations episodes in Chile since 2002. As such, this type of model can be used to identify unanchoring episodes which may serve to trigger alternative monetary policy responses in order to prevent or mitigate their effects. In particular, we find that more aggressive monetary policy responses to inflation during these events can reduce their frequency as well as their intensity. And, unlike under rational expectations, it can be useful to respond to expected inflation in the monetary policy rule. Additionally, we show that the effectiveness of monetary policy on inflation increases (decreases) when agents become more (less) sensitive to short-term information. Finally, we find that the optimal monetary policy of the form proposed derived under rational expectations has a poor performance under the model with unanchoring, relative to the optimal policy derived under unanchoring. In turn, these optimal rules require stronger responses to inflation.

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Appendix

In what follows we provide complementary details of the model, including the complete model derivation in appendix A, its detrended version in appendix B, the derivation of the steady state in appendix C, the log-linear version of the model in appendix D and, finally, a list of those parameters that are calibrated or normalized and the values to which they are set.

A Detailed Description of the Model

A.1 Households

A continuum of infinitely lived households of mass one with identical asset endowments and identical preferences maximize their utility by deciding on consumption of a final good (C_t) - relative to external habits - and hours worked (h_t), for each period. Households save and borrow by purchasing domestic currency denominated government bonds (B_t) and by trading foreign currency bonds ($D_t^\star$, where a positive number expresses an amount borrowed), both being non-state-contingent assets. They also purchase an investment good (I_t) for accumulation of the physical capital stock (K_t). Households maximize expected discounted utility:

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \left[\xi_t^c \frac{\left(C_t - \phi_c \tilde{C}_{t-1}\right)^{1-\sigma}}{1-\sigma} - \Xi_t^h \frac{h_t^{1+\varphi}}{1+\varphi} \right]$$

subject to the period-by-period budget constraint

$$P_t C_t + P_t I_t + B_t + S_t D_{t-1}^\star R_{t-1}^\star + T_t = W_t h_t + R_t^k K_{t-1} + R_{t-1} B_{t-1} + S_t D_t^\star + \Omega_t \tag{8}$$

and the law of motion for capital

$$K_t = (1-\delta) K_{t-1} + \left[1 - \Gamma\left(\frac{I_t}{I_{t-1}}\right)\right] \xi_t^i I_t \tag{9}$$

where $\beta \in (0,1)$ is the discount factor, ξ_t^c and Ξ_t^h are preference shocks that affect the (dis)utility from consumption and hours worked, respectively, $\sigma > 0$ and $\varphi \geq 0$ denote the inverse of the intertemporal elasticity of substitution and the inverse elasticity of labour supply, $\delta \in (0,1]$ denotes the depreciation rate of capital, ξ_t^i is an investment shock that captures changes in the efficiency of the investment process, P_t denotes the price of the consumption and investment good, S_t denotes the nominal exchange rate (units of domestic currency per unit of foreign currency), $R_t^\star$ and R_t denote the foreign and domestic nominal returns on bonds, T_t denotes lump-sum taxes, W_t denotes the nominal wage, R_t^k denotes the nominal rental rate of capital and, finally, Ω_t denotes dividend income from the ownership of firms. The function

$\Gamma(\cdot)$ captures convex investment adjustment costs defined as

$$\Gamma\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2$$

with $\gamma \geq 0$ and $a \geq 1$ (where the latter denotes the long-run balanced growth rate).

Defining for convenience the multiplier on the budget constraint as $\beta^t \frac{\lambda_t A_t^{-\sigma}}{P_t}$ and the multiplier on the law of motion for capital as $\beta^t q_t \lambda_t A_t^{-\sigma}$, we can write the following first-order conditions:

$$\partial C_t: \ \lambda_t A_t^{-\sigma} = \xi_t^c \left(C_t - \phi_c \tilde{C}_{t-1}\right)^{-\sigma} \tag{10}$$

$$\partial h_t: \ \frac{W_t}{P_t} \lambda_t A_t^{-\sigma} = \Xi_t^h h_t^{\varphi} \tag{11}$$

$$\partial B_t: \ \lambda_t A_t^{-\sigma} = \beta R_t E_t \left\{ \frac{\lambda_{t+1}}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \tag{12}$$

$$\partial D_t^{\star}: \ \lambda_t A_t^{-\sigma} = \beta R_t^{\star} E_t \left\{ \frac{\lambda_{t+1} \pi_{t+1}^s}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \tag{13}$$

$$\partial K_t: \ q_t \lambda_t A_t^{-\sigma} = \beta E_t \left\{ \lambda_{t+1} A_{t+1}^{-\sigma} \left[\frac{R_{t+1}^k}{P_{t+1}} + q_{t+1}(1-\delta) \right] \right\} \tag{14}$$

$$\begin{aligned} \partial I_t: \quad 1 = {} & q_t \left[1 - \Gamma\left(\frac{I_t}{I_{t-1}}\right) - \Gamma'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}\right] \xi_t^i \\ & + \beta E_t \left\{ q_{t+1} \frac{\lambda_{t+1} A_{t+1}^{-\sigma}}{\lambda_t A_t^{-\sigma}} \Gamma'\left(\frac{I_{t+1}}{I_t}\right) \frac{I_{t+1}^2}{I_t^2} \xi_{t+1}^i \right\} \end{aligned} \tag{15}$$

In addition, we have that $\tilde{C}_t = C_t$ in equilibrium. The implied discount factor for nominal claims is, iterating upon (12):

$$r_{t,t+s} = \frac{1}{\prod_{i=0}^{s-1} R_{t+i}} = \beta^s \frac{\lambda_{t+s} A_{t+s}^{-\sigma}}{\lambda_t A_t^{-\sigma} \pi_{t+s}} \tag{16}$$

A.2 Production

The supply side of the economy is composed by different types of firms that are all owned by the households. There is a set of monopolistically competitive firms producing different varieties of a home good, X_{jt}^H, using labor and capital as inputs; a set of monopolistically competitive importing firms that import a homogeneous foreign good to transform it into varieties, X_{jt}^F; and three groups of perfectly competitive firms that aggregate products: one packing different varieties of the home good into a composite home good, X_t^H, one packing the imported varieties into a composite foreign good, X_t^F, and, finally, another one that bundles the composite home and foreign goods to create a final good, Y_t^C. This final good is purchased by households (C_t, I_t), and the government (G_t). In addition, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad (and which follows an exogenous process). The total mass of firms in each sector is normalized to one.

A.2.1 Final goods

A representative final goods firm demands composite home and foreign goods in the amounts X_t^H and X_t^F, respectively, and combines them according to the following technology:

$$Y_t^C = \left[\omega^{1/\eta}\left(X_t^H\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(X_t^F\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}} \tag{17}$$

where $\omega \in (0,1)$ is inversely related to the degree of home bias and $\eta > 0$ measures the substitutability between domestic and foreign goods. The selling price of this final good is denoted by P_t, while the prices of the domestic and foreign inputs are P_t^H and P_t^F, respectively. Subject to the technology constraint (17), the firm maximizes its profits over the inputs, taking prices as given:

$$\max_{X_t^H, X_t^F} \; P_t\left[\omega^{1/\eta}\left(X_t^H\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(X_t^F\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}} - P_t^H X_t^H - P_t^F X_t^F$$

The first-order conditions of this problem determine the optimal input demands:

$$X_t^H = \omega\left(\frac{P_t^H}{P_t}\right)^{-\eta} Y_t^C \tag{18}$$

$$X_t^F = (1-\omega)\left(\frac{P_t^F}{P_t}\right)^{-\eta} Y_t^C \tag{19}$$

Combining these optimality conditions and using that zero profits hold in equilibrium, we can write

$$P_t = \left[\omega\left(P_t^H\right)^{1-\eta} + (1-\omega)\left(P_t^F\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{20}$$

A.2.2 Home composite goods

A representative home composite goods firm demands home goods of all varieties $j \in [0,1]$ in amounts X_{jt}^H and combines them according to the technology

$$Y_t^H = \left[\int_0^1 \left(X_{jt}^H\right)^{\frac{\epsilon_H-1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H-1}} \tag{21}$$

with $\epsilon_H > 0$. Let P_{jt}^H denote the price of the home good of variety j. Subject to the technology constraint (21), the firm maximizes its profits $\Pi_t^H = P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj$ over the input demands X_{jt}^H taking prices as given:

$$\max_{X_{jt}^H} \; P_t^H\left[\int_0^1 \left(X_{jt}^H\right)^{\frac{\epsilon_H-1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H-1}} - \int_0^1 P_{jt}^H X_{jt}^H dj$$

This implies the following first-order conditions for all j:

$$\partial X_{jt}^H: \; P_t^H \left(Y_t^H\right)^{1/\epsilon_H}\left(X_{jt}^H\right)^{-1/\epsilon_H} - P_{jt}^H = 0$$

such that the input demand functions are

$$X_{jt}^H = \left(\frac{P_{jt}^H}{P_t^H} \right)^{-\epsilon_H} Y_t^H \tag{22}$$

Substituting (22) into (21) yields the price of home composite goods:

$$P_t^H = \left[\int_0^1 \left(P_{jt}^H \right)^{1-\epsilon_H} dj \right]^{\frac{1}{1-\epsilon_H}} \tag{23}$$

A.2.3 Home goods of variety j

Home goods of variety j are produced according to the technology

$$Y_{jt}^H = z_t K_{jt-1}^\alpha \left(A_t h_{jt} \right)^{1-\alpha} \tag{24}$$

with capital share $\alpha \in (0,1)$, and where z_t is an exogenous stationary technology shock, while A_t is a non-stationary technology disturbance, both common to all varieties. The firm producing variety j satisfies the demand given by (22) but it has monopoly power for its variety. As the price setting decision is independent of the remaining choices, the problem of firm j can also be represented in two stages. In the first stage, the firm hires labor and rents capital to minimize production costs subject to the technology constraint (24):

$$\min_{h_{jt}, K_{jt-1}} \; W_t h_{jt} + R_t^k K_{jt-1} \quad \text{s.t.} \quad Y_{jt}^H = z_t K_{jt-1}^\alpha \left(A_t h_{jt} \right)^{1-\alpha}$$

Letting $P_t^H mc_{jt}^H$ denote the multiplier on the technology constraint (i.e., nominal marginal costs expressed in home composite goods prices), we can rewrite the problem as follows:

$$\min_{h_{jt}, K_{jt-1}} \; W_t h_{jt} + R_t^k K_{jt-1} + P_t^H mc_{jt}^H \left[Y_{jt}^H - z_t K_{jt-1}^\alpha \left(A_t h_{jt} \right)^{1-\alpha} \right]$$

The corresponding first-order conditions are

$$\partial h_{jt}: \; W_t = P_t^H mc_{jt}^H \left(1 - \alpha \right) \frac{Y_{jt}^H}{h_{jt}} \tag{25}$$

$$\partial K_{jt-1}: \; R_t^k = P_t^H mc_{jt}^H \alpha \frac{Y_{jt}^H}{K_{jt-1}} \tag{26}$$

Substituting (25) and (26) into (24) yields the following expression (common for all j) for real marginal costs in units of the final domestic good:

$$mc_{jt}^H = mc_t^H = \frac{1}{\alpha^\alpha \left(1 - \alpha \right)^{1-\alpha}} \frac{(R_t^k)^\alpha W_t^{1-\alpha}}{z_t A_t^{1-\alpha} P_t^H} \tag{27}$$

In the second stage of firm j's problem, given nominal marginal costs $P_t^H mc_{jt}^H$, the firm chooses its price P_{jt}^H to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period the firm can change its price optimally with probability $1 - \theta_H$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\gamma_H \in [0,1]$ and $1-\gamma_H$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^H$ that maximizes the current market value of the profits generated until it can reoptimize again.[17] As the firms are owned by the households, they discount profits by the households' stochastic discount factor for nominal payoffs, $r_{t,t+s}$. A reoptimizing firm, therefore, solves the following problem:

$$\max_{\tilde{P}_{jt}^H} E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left(\tilde{P}_{jt}^H - P_{t+s}^H mc_{jt+s}^H \right) Y_{jt+s}^H \quad \text{s.t.} \quad Y_{jt+s}^H = X_{jt+s}^H = \left(\frac{\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H}}{P_{t+s}^H} \right)^{-\epsilon_H} Y_{t+s}^H$$

which can be rewritten as

$$\max_{\tilde{P}_{jt}^H} E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left[\left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \left(P_{t+s}^H \right)^{\epsilon_H} - mc_{jt+s}^H \left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \left(P_{t+s}^H \right)^{1+\epsilon_H} \right] Y_{t+s}^H$$

The first-order conditions determining the optimal price $\tilde{P}_t^H$ can be written as follows:[18]

$$\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Big[(1-\epsilon_H) \left(\tilde{P}_t^H \right)^{-\epsilon_H} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \left(P_{t+s}^H \right)^{\epsilon_H} \\
&\qquad + \epsilon_H mc_{t+s}^H \left(\tilde{P}_t^H \right)^{-\epsilon_H - 1} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \left(P_{t+s}^H \right)^{1+\epsilon_H} \Big] Y_{t+s}^H \\
\Leftrightarrow \quad 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \frac{\left(P_{t+s}^H \right)^{\epsilon_H}}{P_t^H} \\
&\qquad - mc_{t+s}^H \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \frac{\left(P_{t+s}^H \right)^{1+\epsilon_H}}{P_t^H} \Bigg] Y_{t+s}^H \\
\Leftrightarrow \quad 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H} \right)^{\epsilon_H} \\
&\qquad - mc_{t+s}^H \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H} \right)^{1+\epsilon_H} \Bigg] Y_{t+s}^H
\end{aligned}$$

where the second step follows from multiplying both sides by $-\tilde{P}_t^H / (P_t^H \epsilon_H)$, and the third by defining

[17] Therefore, the following relation holds:

$$P_{jt+s}^H = \tilde{P}_{jt}^H \pi_{t+1}^{I,H} \dots \pi_{t+s}^{I,H}$$

where

$$\pi_t^{I,H} = \left(\pi_{t-1}^H \right)^{\gamma_H} \left(\pi_t^T \right)^{1-\gamma_H}$$

and, in turn, $\pi_t^H = P_t^H / P_{t-1}^H$ and π_t^T denotes the inflation target in period t.

[18] Notice that the subscript j has been removed from $\tilde{P}_t^H$; this simplifies notation and underlines that the prices chosen by all firms j that reset prices optimally in a given period are equal as they face the same problem by (27).

$\tilde{p}_t^H = \tilde{P}_t^H / P_t^H$. The first-order condition can be rewritten in recursive form as follows, defining $F_t^{H_1}$ as

$$\begin{aligned}
F_t^{H_1} &= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H} Y_{t+s}^H \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{\epsilon_H} Y_{t+s+1}^H \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\pi_{t+1}^H\right)^{\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \right. \\
&\qquad \left. \times \left(\tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{\epsilon_H} Y_{t+s+1}^H \right\} \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\pi_{t+1}^H\right)^{\epsilon_H} F_{t+1}^{H_1} \right\} \qquad (28)
\end{aligned}$$

and, analogously, $F_t^{H_2}$ as

$$\begin{aligned}
F_t^{H_2} &= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} mc_{t+s}^H \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{1+\epsilon_H} Y_{t+s}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} mc_{t+s+1}^H \left(\tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{1+\epsilon_H} Y_{t+s+1}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H} \left(\pi_{t+1}^H\right)^{1+\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} mc_{t+s+1}^H \right. \\
&\qquad \left. \left(\tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{1+\epsilon_H} Y_{t+s+1}^H \right\} \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H} \left(\pi_{t+1}^H\right)^{1+\epsilon_H} F_{t+1}^{H_2} \right\} \qquad (29)
\end{aligned}$$

such that

$$F_t^{H_1} = F_t^{H_2} = F_t^H \qquad (30)$$

Using (23), we have

$$\begin{aligned}
1 &= \int_0^1 \left(\frac{P_{jt}^H}{P_t^H}\right)^{1-\epsilon_H} dj \\
&= \left(1 - \theta_H\right) \left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H \left(\frac{P_{t-1}^H \pi_t^{I,H}}{P_t^H}\right)^{1-\epsilon_H} \\
&= \left(1 - \theta_H\right) \left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{1-\epsilon_H} \qquad (31)
\end{aligned}$$

The second equality above follows from the fact that, under Calvo pricing, the distribution of prices among firms not reoptimizing in period t corresponds to the distribution of aggregate prices in period $t-1$, though with total mass reduced to θ_H.

A.2.4 Foreign composite goods

As in the case of home composite goods, a representative foreign composite goods firm demands foreign goods of all varieties $j \in [0,1]$ in amounts X_{jt}^F and combines them according to the technology

$$Y_t^F = \left[\int_0^1 (X_{jt}^F)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj \right]^{\frac{\epsilon_F}{\epsilon_F - 1}} \tag{32}$$

with $\epsilon_F > 0$. Let P_{jt}^F denote the price of the foreign good of variety j. Analogously to the case of home composite goods, profit maximization yields the input demand functions

$$X_{jt}^F = \left(\frac{P_{jt}^F}{P_t^F} \right)^{-\epsilon_F} Y_t^F \tag{33}$$

for all j, and substituting (33) into (32) yields the price of foreign composite goods:

$$P_t^F = \left[\int_0^1 (P_{jt}^F)^{1-\epsilon_F} dj \right]^{\frac{1}{1-\epsilon_F}} \tag{34}$$

A.2.5 Foreign goods of variety j

Importing firms buy an amount M_t of a homogeneous foreign good at the price $P_t^{M\star}$ abroad and convert this good into varieties Y_{jt}^F that are sold domestically, and where total imports are $\int_0^1 Y_{jt}^F dj$. We assume that the import price level $P_t^{M\star}$ cointegrates with the foreign producer price level $P_t^\star$, i.e., $P_t^{M\star} = P_t^\star \xi_t^m$, where ξ_t^m is a stationary exogenous process. The firm producing variety j satisfies the demand given by (33) but it has monopoly power for its variety. As it takes one unit of the foreign good to produce one unit of variety j, nominal marginal costs in terms of composite goods prices are

$$P_t^F mc_{jt}^F = P_t^F mc_t^F = S_t P_t^{M\star} = S_t P_t^\star \xi_t^m \tag{35}$$

Given marginal costs, the firm producing variety j chooses its price P_{jt}^F to maximize profits. In setting prices, the firm faces a Calvo-type problem similar to domestic firms, whereby each period the firm can change its price optimally with probability $1-\theta_F$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\gamma_F \in [0,1]$ and $1-\gamma_F$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^F$ that maximizes the current market value of the profits generated until it can reoptimize.[19] The solution to this problem is analogous to the case of domestic varieties, implying the first-order condition

$$F_t^{F_1} = F_t^{F_2} = F_t^F \tag{36}$$

[19] As in the home varieties case, the following relation holds:

$$P_{jt+s}^F = \tilde{P}_{jt}^F \pi_{t+1}^{I,F} \dots \pi_{t+s}^{I,F}$$

where

$$\pi_t^{I,F} = \left(\pi_{t-1}^F \right)^{\gamma_F} \left(\pi_t^T \right)^{1-\gamma_F}$$

and, in turn, $\pi_t^F = P_t^F / P_{t-1}^F$.

where, defining $\tilde{p}_t^F = \tilde{P}_t^F / P_t^F$,

$$F_t^{F_1} = \frac{\epsilon_F - 1}{\epsilon_F} \left(\tilde{p}_t^F\right)^{1-\epsilon_F} Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} \left(\pi_{t+1}^F\right)^{\epsilon_F} F_{t+1}^{F_1} \right\}$$

and

$$F_t^{F_2} = \left(\tilde{p}_t^F\right)^{-\epsilon_F} mc_t^F Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\pi_{t+1}^F\right)^{1+\epsilon_F} F_{t+1}^{F_2} \right\}$$

Using (34), we further have

$$1 = (1 - \theta_F) \left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F} \right)^{1-\epsilon_F} \tag{37}$$

A.2.6 Commodities

We assume the country has available an exogenous and stochastic endowment of commodities Y_t^{Co}. Moreover, these commodities are not consumed domestically but entirely exported. Therefore, the entire production is sold at a given international price $P_t^{Co\star}$, which is assumed to evolve exogenously. We further assume that the government receives a share $\chi \in [0, 1]$ of this income and the remaining share goes to foreign agents.

A.3 Fiscal and monetary policy

The government consumes an exogenous stream of final goods G_t, levies lump-sum taxes T_t, and issues one-period bonds B_t. Hence, the government satisfies the following period-by-period constraint,

$$T_t + B_t + \chi S_t P_t^{Co\star} Y_t^{Co} = P_t G_t + R_{t-1} B_{t-1} \tag{38}$$

In turn, monetary policy is carried out according to a Taylor-type rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\alpha_R} \left[\left(\frac{\pi_t}{\pi_t^T} \right)^{\alpha_\pi} \left(\frac{GDP_t / GDP_{t-1}}{a} \right)^{\alpha_y} \right]^{1-\alpha_R} e_t^m \tag{39}$$

where $\alpha_R \in [0, 1)$, $\alpha_\pi > 1$, $\alpha_y \geq 0$ and where π_t^T is an exogenous inflation target and e_t^m an i.i.d. shock that captures deviations form the rule.[20]

A.4 Rest of the world

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the

[20] We do not need a time-varying target, so we will set it to a constant.

domestic economy, but the domestic economy is assumed to be small relative to the foreign economy. The latter implies that the foreign producer price level $P_t^\star$ is identical to the foreign consumption-based price index. Further, let $P_t^{H\star}$ denote the price of home composite goods expressed in foreign currency. Given full tradeability and competitive export pricing, the law of one price holds separately for home composite goods and the commodity good, i.e. $P_t^H = S_t P_t^{H\star}$ and $P_t^{Co} = S_t P_t^{Co\star}$. That is, domestic and foreign prices of both goods are identical when expressed in the same currency. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods, i.e., $P_t^F mc_t^F = S_t P_t^\star \xi_t^m$ from (35). The real exchange rate rer_t therefore satisfies

$$rer_t = \frac{S_t P_t^\star}{P_t} = \frac{P_t^F}{P_t}\frac{mc_t^F}{\xi_t^m} \tag{40}$$

We also have the following relation

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{41}$$

where $\pi_t^s = S_t/S_{t-1}$. Foreign demand for the home composite good $X_t^{H\star}$ is given by

$$X_t^{H\star} = \left(\frac{P_t^H}{S_t P_t^\star}\right)^{-\eta^\star} Y_t^\star \tag{42}$$

with $\eta^\star > 0$ and where $Y_t^\star$ denotes foreign aggregate demand or GDP. Both $Y_t^\star$ and $\pi_t^\star$ evolve exogenously. The relevant foreign nominal interest rate is composed by an exogenous risk-free world interest rate R_t^W plus a country premium that increases with the economy's net foreign debt position (expressed as a ratio of nominal GDP):

$$R_t^\star = R_t^W \exp\left\{\phi_d\left(\frac{S_t D_t^\star}{GDPN_t} - \bar{d}\right)\right\}\xi_t^R \tag{43}$$

with $\phi_d > 0$ and where ξ_t^R is an exogenous shock to the country premium.

A.5 Aggregation

A.5.1 Goods market clearing

In the market for the final good, the clearing condition is

$$Y_t^C = C_t + I_t + G_t \tag{44}$$

In the market for the home and foreign composite goods we have, respectively,

$$Y_t^H = X_t^H + X_t^{H\star} \tag{45}$$

and

$$Y_t^F = X_t^F \tag{46}$$

while in the market for home and foreign varieties we have, respectively,

$$Y_{jt}^{H} = X_{jt}^{H} \tag{47}$$

and

$$Y_{jt}^{F} = X_{jt}^{F} \tag{48}$$

for all j.

A.5.2 Factor market clearing

In the market for labor, the clearing condition is

$$\int_0^1 h_{jt} dj = h_t \tag{49}$$

while in the market for capital we have

$$\int_0^1 K_{jt} dj = K_t \tag{50}$$

Combining (25) and (26), integrating over j, and using the previous factor market clearing conditions we obtain

$$\alpha W_t \int_0^1 h_{jt} dj = \alpha W_t h_t = (1-\alpha) R_t^k \int_0^1 K_{jt-1} dj = (1-\alpha) R_t^k K_{t-1}$$

which shows that the capital-labor ratio satisfies

$$\frac{K_{t-1}}{h_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} = \frac{K_{jt-1}}{h_{jt}} \tag{51}$$

A.5.3 Inflation and relative prices

The following holds for $j = H,\ F$:

$$p_t^j = \frac{P_t^j}{P_t}$$

and, also,

$$\frac{p_t^j}{p_{t-1}^j} = \frac{\pi_t^j}{\pi_t}$$

A.5.4 Aggregate supply

Integrating the productions of different varieties of home goods (24) over j and using the fact that capital-labor ratios are identical, see (51), as well as (49), we obtain

$$\int_0^1 Y_{jt}^H dj = z_t A_t^{1-\alpha} \left(\frac{K_{t-1}}{h_t} \right)^{\alpha} \int_0^1 h_t(j) dj = z_t K_{t-1}^{\alpha} \left(A_t h_t \right)^{1-\alpha}$$

Integrating (47) over j and using (22) then yields aggregate output of home goods as

$$\int_0^1 Y_{jt}^H dj = \int_0^1 X_{jt}^H dj = Y_t^H \int_0^1 \left(p_{jt}^H \right)^{-\epsilon_H} dj$$

or, combining the previous two equations,

$$Y_t^H \Xi_t^H = z_t K_{t-1}^{\alpha} \left(A_t h_t\right)^{1-\alpha}$$

where Ξ_t^H is a price dispersion term satisfying

$$\begin{aligned} \Xi_t^H &= \int_0^1 \left(\frac{P_{jt}^H}{P_t^H}\right)^{-\epsilon_H} dj \\ &= \left(1-\theta_H\right)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{-\epsilon_H} \Xi_{t-1}^H \end{aligned}$$

A.5.5 Aggregate demand

Aggregate demand or GDP is defined as the sum of domestic absorption and the trade balance. Domestic absorption is equal to $Y_t^C = C_t + I_t + G_t$. The nominal trade balance is defined as

$$TB_t = P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \tag{52}$$

Integrating (48) over j and using (33) shows that imports satisfy

$$M_t = \int_0^1 Y_{jt}^F dj = \int_0^1 X_{jt}^F dj = Y_t^F \int_0^1 \left(\frac{P_{jt}^F}{P_t^F}\right)^{-\epsilon_F} dj = Y_t^F \Xi_t^F$$

where Ξ_t^F is a price dispersion term satisfying

$$\Xi_t^F = \left(1-\theta_F\right)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F} \Xi_{t-1}^F$$

We then define real and nominal GDP, respectively, as

$$GDP_t = C_t + I_t + G_t + X_t^{H\star} + Y_t^{Co} - M_t$$

and

$$GDPN_t = P_t \left(C_t + I_t + G_t\right) + TB_t \tag{53}$$

Note that by combining (53) with the zero profit condition in the final goods sector, i.e., $P_t Y_t^C = P_t^H X_t^H + P_t^F X_t^F$, and using the market clearing conditions for final and composite goods, (44)-(45), GDP is seen to be equal to total value added (useful for the steady state):

$$\begin{aligned} GDPN_t &= P_t Y_t^C + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H X_t^H + P_t^F X_t^F + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H Y_t^H + S_t P_t^{Co\star} Y_t^{Co} + P_t^F X_t^F - S_t P_t^{M\star} M_t \end{aligned}$$

A.5.6 Balance of payments

Aggregate nominal profits are given by

$$
\begin{aligned}
\Omega_t &= \underbrace{P_t Y_t^C - P_t^H X_t^H - P_t^F X_t^F}_{\Pi_t^C} + \underbrace{P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj}_{\Pi_t^H} + \underbrace{P_t^F Y_t^F - \int_0^1 P_{jt}^F X_{jt}^F dj}_{\Pi_t^F} \\
&\quad + \underbrace{\int_0^1 \left(P_{jt}^H Y_{jt}^H - W_t h_{jt} - R_t^k K_{jt-1}\right) dj}_{\int_0^1 \Pi_{jt}^H dj} + \underbrace{\int_0^1 \left(P_{jt}^F Y_{jt}^F - S_t P_t^{M\star} Y_{jt}^F\right) dj}_{\int_0^1 \Pi_{jt}^F dj} \\
&= P_t \left(C_t + I_t + G_t\right) + P_t^H X_t^{H\star} - S_t P_t^{M\star} M_t - W_t h_t - R_t^k K_{t-1} \\
&= P_t \left(C_t + I_t + G_t\right) + TB_t - S_t P_t^{Co\star} Y_t^{Co} - W_t h_t - R_t^k K_{t-1}
\end{aligned}
$$

where the second equality uses the market clearing conditions (44)-(50), and the third equality uses the definition of the trade balance, (52). Substituting out Ω_t in the households' budget constraint (8) and using the government's budget constraint (38) to substitute out taxes T_t shows that the net foreign asset position evolves according to

$$
-S_t D_t^\star = TB_t - S_t D_{t-1}^\star R_{t-1}^\star - (1-\chi)\, S_t P_t^{Co\star} Y_t^{Co}
$$

A.6 Driving forces

We define $a_t = A_t/A_{t-1}$, $\xi_t^h = \Xi_t^h / A_t^{1-\sigma}$, $g_t = G_t/A_t$, $y_t^{Co} = Y_t^{Co}/A_t$, $y_t^\star = Y_t^\star/A_t$ and $p_t^{Co\star} = P_t^{Co\star}/P_t$, and we assume that each exogenous variable follows an AR(1) process:

$$
\log(x_t/x) = \rho_x \log(x_{t-1}/x) + e_t^x
$$

for $x = \left\{a, z, \xi^c, \xi^h, \xi^i, \xi^R, g, R^W, y^\star, \pi^\star, y^{Co}, p^{Co\star}\right\}$. We also have e_t^m. All disturbances are white noise.

B Stationary Equilibrium Conditions

In this section the model is brought into stationary form. For this, the following variables are defined: $rer_t = S_t P_t^\star / P_t$, $w_t = \frac{W_t}{A_t P_t}$, $r_t^k = \frac{R_t^k}{P_t}$, $tb_t = \frac{TB_t}{A_t P_t}$, $d_t^\star = \frac{D_t^\star}{A_t P_t^\star}$ and $gdpn_t = \frac{GDPN_t}{A_t P_t}$. In addition, all other upper case variables are divided by A_t and written as lower case variables. Finally, we also define $\Delta_t^{GDP} = \frac{GDP_t}{GDP_{t-1}}$, $\Delta_t^C = \frac{C_t}{C_{t-1}}$ and $\Delta_t^I = \frac{I_t}{I_{t-1}}$. The rational expectations equilibrium of the stationary version of the model is then the set of sequences

$$\{\lambda_t, c_t, w_t, h_t, r_t^k, k_t, i_t, q_t, R_t, R_t^\star, \pi_t, \pi_t^s, \pi_t^{I,H}, \pi_t^H, \pi_t^{I,F}, \pi_t^F, p_t^H, p_t^F, \tilde{p}_t^H, \tilde{p}_t^F, y_t^C, y_t^H, y_t^F, x_t^H, x_t^{H\star}, x_t^F,$$
$$mc_t^H, mc_t^F, f_t^H, f_t^F, rer_t, d_t^\star, gdp_t, gdpn_t, tb_t, m_t, \Xi_t^H, \Xi_t^F, \Delta_t^{GDP}, \Delta_t^C, \Delta_t^I\}_{t=0}^\infty$$

(41 variables) such that for a given set of initial values and exogenous processes

$$\{z_t, a_t, \xi_t^c, \xi_t^h, \xi_t^i, \xi_t^m, \xi_t^R, g_t, y_t^{Co}, \pi_t^\star, R_t^W, y_t^\star, p_t^{Co\star}, e_t^m\}_{t=0}^\infty$$

the following 41 conditions are satisfied:

$$\lambda_t = \xi_t^c \left(c_t - \phi_c \frac{c_{t-1}}{a_t} \right)^{-\sigma} \tag{54}$$

$$w_t \lambda_t = \xi_t^h h_t^\varphi \tag{55}$$

$$\lambda_t = \beta R_t E_t \left\{ \frac{\lambda_{t+1}}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \tag{56}$$

$$\lambda_t = \beta R_t^\star E_t \left\{ \frac{\lambda_{t+1} \pi_{t+1}^s}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \tag{57}$$

$$q_t \lambda_t = \beta E_t \left\{ \lambda_{t+1} a_{t+1}^{-\sigma} \left[r_{t+1}^k + q_{t+1}(1-\delta) \right] \right\} \tag{58}$$

$$\begin{aligned} 1 = {} & q_t \left[1 - \frac{\gamma}{2} \left(\frac{i_t}{i_{t-1}} a_t - a \right)^2 - \gamma \left(\frac{i_t}{i_{t-1}} a_t - a \right) \frac{i_t a_t}{i_{t-1}} \right] \xi_t^i \\ & + \beta \gamma E_t \left\{ q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} a_{t+1}^{-\sigma} \left(\frac{i_{t+1}}{i_t} a_{t+1} - a \right) \left(\frac{i_{t+1}}{i_t} a_{t+1} \right)^2 \xi_{t+1}^i \right\} \end{aligned} \tag{59}$$

$$k_t = (1-\delta) \frac{k_{t-1}}{a_t} + \left[1 - \frac{\gamma}{2} \left(\frac{i_t}{i_{t-1}} a_t - a \right)^2 \right] \xi_t^i i_t \tag{60}$$

$$y_t^C = \left[\omega^{1/\eta} \left(x_t^H \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(x_t^F \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} \tag{61}$$

$$x_t^F = (1-\omega) \left(p_t^F \right)^{-\eta} y_t^C \tag{62}$$

$$x_t^H = \omega \left(p_t^H \right)^{-\eta} y_t^C \tag{63}$$

$$p_t^H mc_t^H = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(r_t^k)^\alpha w_t^{1-\alpha}}{z_t} \tag{64}$$

$$frack_{t-1} h_t = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k} a_t \tag{65}$$

$$f_t^H = \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \right)^{1-\epsilon_H} y_t^H + \beta \theta_H E_t \left\{ \frac{\lambda_{t+1} a_{t+1}^{1-\sigma}}{\lambda_t \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} \left(\pi_{t+1}^H \right)^{\epsilon_H} f_{t+1}^H \right\} \tag{66}$$

$$f_t^H = \left(\tilde{p}_t^H \right)^{-\epsilon_H} mc_t^H y_t^H + \beta \theta_H E_t \left\{ \frac{\lambda_{t+1} a_{t+1}^{1-\sigma}}{\lambda_t \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left(\pi_{t+1}^H \right)^{1+\epsilon_H} f_{t+1}^H \right\} \tag{67}$$

$$1 = (1-\theta_H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{1-\epsilon_H} \tag{68}$$

$$\pi_t^{I,H} = \left(\pi_{t-1}^H\right)^{\gamma_H} \left(\pi^T\right)^{1-\gamma_H} \tag{69}$$

$$p_t^F mc_t^F = rer_t \xi_t^m \tag{70}$$

$$f_t^F = \frac{\epsilon_F - 1}{\epsilon_F}\left(\tilde{p}_t^F\right)^{1-\epsilon_F} y_t^F + \beta\theta_F E_t \left\{ \frac{\lambda_{t+1} a_{t+1}^{1-\sigma}}{\lambda_t \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F}\right)^{1-\epsilon_F} \left(\pi_{t+1}^F\right)^{\epsilon_F} f_{t+1}^F \right\} \tag{71}$$

$$f_t^F = \left(\tilde{p}_t^F\right)^{-\epsilon_F} mc_t^F y_t^F + \beta\theta_F E_t \left\{ \frac{\lambda_{t+1} a_{t+1}^{1-\sigma}}{\lambda_t \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F}\right)^{-\epsilon_F} \left(\pi_{t+1}^F\right)^{1+\epsilon_F} f_{t+1}^F \right\} \tag{72}$$

$$1 = (1-\theta_F)\left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{1-\epsilon_F} \tag{73}$$

$$\pi_t^{I,F} = \left(\pi_{t-1}^F\right)^{\gamma_F} \left(\pi_t^T\right)^{1-\gamma_F} \tag{74}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha_R} \left[\left(\frac{\pi_t}{\pi^T}\right)^{\alpha_\pi} \left(\frac{\Delta_t^{GDP}}{a}\right)^{\alpha_y}\right]^{1-\alpha_R} e_t^m \tag{75}$$

$$R_t^\star = R_t^W \exp\left\{\phi_d \left(\frac{rer_t d_t^\star}{gdpn_t} - \bar{d}\right)\right\} \xi_t^R \tag{76}$$

$$x_t^{H\star} = \left(\frac{p_t^H}{rer_t}\right)^{-\eta^\star} y_t^\star \tag{77}$$

$$y_t^C = c_t + i_t + g_t \tag{78}$$

$$y_t^H = x_t^H + x_t^{H\star} \tag{79}$$

$$y_t^F = x_t^F \tag{80}$$

$$y_t^H \Xi_t^H = z_t \left(\frac{k_{t-1}}{a_t}\right)^\alpha h_t^{1-\alpha} \tag{81}$$

$$\Xi_t^H = (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{-\epsilon_H} \Xi_{t-1}^H \tag{82}$$

$$\frac{p_t^H}{p_{t-1}^H} = \frac{\pi_t^H}{\pi_t} \tag{83}$$

$$\frac{p_t^F}{p_{t-1}^F} = \frac{\pi_t^F}{\pi_t} \tag{84}$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{85}$$

$$m_t = y_t^F \Xi_t^F \tag{86}$$

$$\Xi_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F} \Xi_{t-1}^F \tag{87}$$

$$gdp_t = c_t + i_t + g_t + x_t^{H\star} + y_t^{Co} - m_t \tag{88}$$

$$gdpn_t = c_t + i_t + g_t + tb_t \tag{89}$$

$$tb_t = p_t^H x_t^{H\star} + rer_t p_t^{Co\star} y_t^{Co} - rer_t \xi_t^m m_t \tag{90}$$

$$- rer_t d_t^\star = tb_t - \frac{rer_t}{a_t \pi_t^\star} d_{t-1}^\star R_{t-1}^\star - (1-\chi)\, rer_t p_t^{Co\star} y_t^{Co} \tag{91}$$

$$\Delta_t^{GDP} = \frac{gdp_t}{gdp_{t-1}} a_t \tag{92}$$

We further include two auxiliary variables to match observables:

$$\Delta_t^C = \frac{c_t}{c_{t-1}} a_t \tag{93}$$

$$\Delta_t^I = \frac{i_t}{i_{t-1}} a_t \tag{94}$$

The exogenous processes are:

$$\log(z_t/z) = \rho_z \log(z_{t-1}/z) + e_t^z$$

$$\log(a_t/a) = \rho_a \log(a_{t-1}/a) + e_t^a$$

$$\log(\xi_t^c/\xi^c) = \rho_{\xi^c} \log(\xi_{t-1}^c/\xi^c) + e_t^{\xi^c}$$

$$\log(\xi_t^h/\xi^h) = \rho_{\xi^h} \log(\xi_{t-1}^h/\xi^h) + e_t^{\xi^h}$$

$$\log(\xi_t^i/\xi^i) = \rho_{\xi^i} \log(\xi_{t-1}^i/\xi^i) + e_t^{\xi^i}$$

$$\log(\xi_t^R/\xi^R) = \rho_{\xi^R} \log(\xi_{t-1}^R/\xi^R) + e_t^{\xi^R}$$

$$\log(g_t/g) = \rho_g \log(g_{t-1}/g) + e_t^g$$

$$\log(y_t^{Co}/y^{Co}) = \rho_{y^{Co}} \log(y_{t-1}^{Co}/y^{Co}) + e_t^{y^{Co}}$$

$$\log(\pi_t^\star/\pi^\star) = \rho_{\pi^\star} \log(\pi_{t-1}^\star/\pi^\star) + e_t^{\pi^\star}$$

$$\log(R_t^W/R^W) = \rho_{R^W} \log(R_{t-1}^W/R^W) + e_t^{R^W}$$

$$\log(y_t^\star/y^\star) = \rho_{y^\star} \log(y_{t-1}^\star/y^\star) + e_t^{y^\star}$$

$$\log(p_t^{Co\star}/p^{Co\star}) = \rho_{p^{Co\star}} \log(p_{t-1}^{Co\star}/p^{Co\star}) + e_t^{p^{Co\star}}$$

$$\log(\xi_t^m/\xi^m) = \rho_{\xi^m} \log(\xi_{t-1}^m/\xi^m) + e_t^{\xi^m}$$

Finally, there is e_t^m. All disturbances are white noise.

C Steady State

In this section we show how to compute the steady state for a given value of all parameters and exogenous variables in the long run, except for β, R^W, $\pi^\star$, g, y^{Co}, $y^\star$ and ξ^h that are determined endogenously to get the following endogenous variables in steady state: R, π^s, p^H, $s^g = g/gdpn$, $s^{Co} = p^{Co\star} y^{Co} rer/gdpn$, $s^{tb} = tb/gdpn$ and h.

Then, from (75), (56), (57), (76) and (85):

$$\pi = \pi^T, \quad \beta = \frac{\pi a^\sigma}{R}, \quad R^\star = \frac{R}{\pi^s}, \quad R^W = \frac{R^\star}{\xi^R}, \quad \pi^\star = \frac{\pi}{\pi^s}$$

From (83), (84), (69) and (74) and using $\pi = \pi^T$:

$$\pi^H = \pi^F = \pi^{I,H} = \pi^{I,F} = \pi$$

From (59) and (58):

$$q = 1/\xi^i, \quad r^k = q\left(\frac{a^\sigma}{\beta} - 1 + \delta\right)$$

From (69), (73), (67), (66), (71), (72), (82) and (87):

$$\tilde{p}^H = \tilde{p}^F = 1, \quad mc^H = \frac{\epsilon_H - 1}{\epsilon_H}, \quad mc^F = \frac{\epsilon_F - 1}{\epsilon_F}, \quad \Xi^H = \Xi^F = 1$$

From (64), (65), (60) and (81):

$$w = \left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha} p^H mc^H z}{(r^k)^\alpha}\right]^{\frac{1}{1-\alpha}}$$

$$k = \frac{\alpha}{1-\alpha} h \frac{w}{r^k} a, \quad i = k\left[\frac{1-(1-\delta)/a}{\xi^i}\right]$$

$$y^H = z\left(\frac{k}{a}\right)^\alpha h^{1-\alpha}/\Xi^H$$

Using that $1 = \left[\omega(p^H)^{1-\eta} + (1-\omega)(p_t^F)^{1-\eta}\right]^{\frac{1}{1-\eta}}$:

$$p^F = \left[\frac{1-\omega(p^H)^{1-\eta}}{1-\omega}\right]^{\frac{1}{1-\eta}}$$

Then, from (70):

$$rer = mc^F p^F/\xi^m$$

From $gdpn$ equal to value added ($gdpn = p^H y^H + s^{Co} gdpn + p^F y^F - rer\ \xi^m m$) and the relations $y^F = x^F = (1-\omega)(p^F)^{-\eta} y^C = (1-\omega)(p^F)^{-\eta}(gdpn - tb) = (1-\omega)(p^F)^{-\eta}\left(1 - s^{tb}\right) gdpn$ and $m = y^F \Xi^F$ (see below):

$$gdpn = \frac{p^H y^H}{1 - s^{Co} - (p^F - rer\ \xi^m \Xi^F)(1-\omega)(p^F)^{-\eta}(1 - s^{tb})}$$

Thus, from their definitions:

$$tb = s^{tb} gdpn, \quad g = s^g gdpn, \quad y^{Co} = \frac{s^{Co} gdpn}{p^{Co\star} rer}$$

From (78), (88), (62), (63), (79), (77), (80) and (86):

$$y^C = gdpn - tb, \quad x^F = (1-\omega)(p^F)^{-\eta} y^C, \quad x^H = \omega (p^H)^{-\eta} y^C$$

$$x^{H\star} = y^H - x^H, \quad y^\star = x^{H\star} \left(\frac{p^H}{rer}\right)^{\eta\star}, \quad y^F = x^F, \quad m = y^F \Xi^F$$

From (78), (90), (91) and (78):

$$c = y^C - g - i$$

$$gdp = c + i + g + x^{H\star} + y^{Co} - m$$

$$d^\star = \left[tb - (1-\chi) p^{Co\star} y^{Co} rer\right] \left[rer \left(\frac{R^\star}{a\pi^\star} - 1\right)\right]^{-1}, \quad \bar{d} = \frac{rer\ d^\star}{gdpn}$$

From (67) and (72):

$$f^H = \frac{(\tilde{p}^H)^{-\epsilon_H} y^H mc^H}{1 - \beta\theta_H a^{1-\sigma}}, \quad f^F = \frac{(\tilde{p}^F)^{-\epsilon_F} y^F mc^F}{1 - \beta\theta_F a^{1-\sigma}}$$

From (54), (55), (92) and (93):

$$\lambda = \xi^c \left(c - \phi_c \frac{c}{a}\right)^{-\sigma}, \quad \xi^h = \frac{w\lambda}{h^\varphi}$$

$$\Delta^{GDP} = \Delta^C = \Delta^I = a$$

D Linear Model

In this section we provide the (log-)linearized version of the model. For this, let $\hat{x}_t = \log(x_t/x)$ and $\check{x}_t = x_t - x$. The linear system of equilibrium equations is then as follows.

$$\hat{\lambda}_t = \hat{\xi}^c_t - \sigma (\lambda/\xi^c)^{1/\sigma} \left[c\hat{c}_t + \phi_c \frac{c}{a} (\hat{a}_t - \hat{c}_{t-1})\right]$$

$$\hat{w}_t + \hat{\lambda}_t = \hat{\xi}^h_t + \varphi \hat{h}_t$$

$$\hat{\lambda}_t = \hat{R}_t + E_t \left\{\hat{\lambda}_{t+1} - \hat{\pi}_{t+1} - \sigma \hat{a}_{t+1}\right\}$$

$$\hat{\lambda}_t = \hat{R}^\star_t + E_t \left\{\hat{\lambda}_{t+1} + \hat{\pi}^s_{t+1} - \hat{\pi}_{t+1} - \sigma \hat{a}_{t+1}\right\}$$

$$\hat{q}_t + \hat{\lambda}_t = E_t \left\{\hat{\lambda}_{t+1} - \sigma \hat{a}_{t+1} + \frac{r^k}{r^k + q(1-\delta)} \hat{r}^k_{t+1} + \frac{q(1-\delta)}{r^k + q(1-\delta)} \hat{q}_{t+1}\right\}$$

$$\gamma a^2 (\hat{\imath}_t + \hat{a}_t - \hat{\imath}_{t-1}) = \hat{q}_t + \hat{\xi}^i_t + \beta\gamma a^{3-\sigma} (E_t \{\hat{\imath}_{t+1} + \hat{a}_{t+1}\} - \hat{\imath}_t)$$

$$k\hat{k}_t = (1-\delta)\frac{k}{a}\left(\hat{k}_{t-1} - \hat{a}_t\right) + \xi^i i\left(\hat{\xi}^i_t + \hat{\imath}_t\right)$$

$$\hat{y}^C_t = \left(y^C\right)^{1/\eta - 1}\left[\omega^{1/\eta}\left(x^H\right)^{1-1/\eta}\hat{x}^H_t + (1-\omega)^{1/\eta}\left(x^F\right)^{1-1/\eta}\hat{x}^F_t\right]$$

$$\hat{x}^F_t = \hat{y}^C_t - \eta\hat{p}^F_t$$

$$\hat{x}^H_t = \hat{y}^C_t - \eta\hat{p}^H_t$$

$$\hat{p}^H_t + \widehat{mc}^H_t = \alpha\hat{r}^k_t + (1-\alpha)\hat{w}_t - \hat{z}_t$$

$$\hat{k}_{t-1} - \hat{h}_t = \hat{w}_t - \hat{r}^k_t + \hat{a}_t$$

$$\begin{aligned} f^H\hat{f}^H_t \quad = \quad & \frac{\epsilon_H - 1}{\epsilon_H}\left(\tilde{p}^H\right)^{1-\epsilon_H} y^H\left[(1-\epsilon_H)\widehat{\tilde{p}}^H_t + \hat{y}^H_t\right] \\ & + \beta\theta_H\frac{a^{1-\sigma}}{\pi}\left(\pi^{I,H}\right)^{1-\epsilon_H}\left(\pi^H\right)^{\epsilon_H} f^H\left[\begin{array}{c} (1-\epsilon_H)\widehat{\tilde{p}}^H_t - \hat{\lambda}_t + E_t\left\{\hat{\lambda}_{t+1} + (1-\sigma)\hat{a}_{t+1} - \hat{\pi}_{t+1}\right\} \\ +E_t\left\{(1-\epsilon_H)\left(\hat{\pi}^{I,H}_{t+1} - \widehat{\tilde{p}}^H_{t+1}\right) + \epsilon_H\hat{\pi}^H_{t+1} + \hat{f}^H_{t+1}\right\} \end{array}\right] \end{aligned}$$

$$\begin{aligned} f^H\hat{f}^H_t \quad = \quad & \left(\tilde{p}^H\right)^{-\epsilon_H} mc^H y^H\left[-\epsilon_H\widehat{\tilde{p}}^H_t + \widehat{mc}^H_t + \hat{y}^H_t\right] \\ & + \beta\theta_H\frac{a^{1-\sigma}}{\pi}\left(\pi^{I,H}\right)^{-\epsilon_H}\left(\pi^H\right)^{1+\epsilon_H} f^H\left[\begin{array}{c} -\epsilon_H\widehat{\tilde{p}}^H_t - \hat{\lambda}_t + E_t\left\{\hat{\lambda}_{t+1} + (1-\sigma)\hat{a}_{t+1} - \hat{\pi}_{t+1}\right\} \\ +E_t\left\{-\epsilon_H\left(\hat{\pi}^{I,H}_{t+1} - \widehat{\tilde{p}}^H_{t+1}\right) + (1+\epsilon_H)\hat{\pi}^H_{t+1} + \hat{f}^H_{t+1}\right\} \end{array}\right] \end{aligned}$$

$$0 = (1-\theta_H)\left(\tilde{p}^H\right)^{1-\epsilon_H}\widehat{\tilde{p}}^H_t + \theta_H\left(\frac{\pi^{I,H}}{\pi^H}\right)^{1-\epsilon_H}\left(\hat{\pi}^{I,H}_t - \hat{\pi}^H_t\right)$$

$$\hat{\pi}^{I,H}_t = \gamma_H\hat{\pi}^H_{t-1}$$

$$\hat{p}^F_t + \widehat{mc}^F_t = \widehat{rer}_t + \hat{\xi}^m_t$$

$$\begin{aligned} f^F\hat{f}^F_t \quad = \quad & \frac{\epsilon_F - 1}{\epsilon_F}\left(\tilde{p}^F\right)^{1-\epsilon_F} y^F\left[(1-\epsilon_F)\widehat{\tilde{p}}^F_t + \hat{y}^F_t\right] \\ & + \beta\theta_F\frac{a^{1-\sigma}}{\pi}\left(\pi^{I,F}\right)^{1-\epsilon_F}\left(\pi^F\right)^{\epsilon_F} f^F\left[\begin{array}{c} (1-\epsilon_F)\widehat{\tilde{p}}^F_t - \hat{\lambda}_t + E_t\left\{\hat{\lambda}_{t+1} + (1-\sigma)\hat{a}_{t+1} - \hat{\pi}_{t+1}\right\} \\ +E_t\left\{(1-\epsilon_F)\left(\hat{\pi}^{I,F}_{t+1} - \widehat{\tilde{p}}^F_{t+1}\right) + \epsilon_F\hat{\pi}^F_{t+1} + \hat{f}^F_{t+1}\right\} \end{array}\right] \end{aligned}$$

$$\begin{aligned} f^F\hat{f}^F_t \quad = \quad & \left(\tilde{p}^F\right)^{-\epsilon_F} mc^F y^F\left[-\epsilon_F\widehat{\tilde{p}}^F_t + \widehat{mc}^F_t + \hat{y}^F_t\right] \\ & + \beta\theta_F\frac{a^{1-\sigma}}{\pi}\left(\pi^{I,F}\right)^{-\epsilon_F}\left(\pi^F\right)^{1+\epsilon_F} f^F\left[\begin{array}{c} -\epsilon_F\widehat{\tilde{p}}^F_t - \hat{\lambda}_t + E_t\left\{\hat{\lambda}_{t+1} + (1-\sigma)\hat{a}_{t+1} - \hat{\pi}_{t+1}\right\} \\ +E_t\left\{-\epsilon_F\left(\hat{\pi}^{I,F}_{t+1} - \widehat{\tilde{p}}^F_{t+1}\right) + (1+\epsilon_F)\hat{\pi}^F_{t+1} + \hat{f}^F_{t+1}\right\} \end{array}\right] \end{aligned}$$

$$0 = (1-\theta_F)\left(\tilde{p}^F\right)^{1-\epsilon_F}\widehat{\tilde{p}}^F_t + \theta_F\left(\frac{\pi^{I,F}}{\pi^F}\right)^{1-\epsilon_F}\left(\hat{\pi}^{I,F}_t - \hat{\pi}^F_t\right)$$

$$\hat{\pi}^{I,F}_t = \gamma_F\hat{\pi}^F_{t-1}$$

$$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\left[\alpha_\pi \hat{\pi}_t + \alpha_y \hat{\Delta}_t^{GDP}\right] + e_t^m$$

$$\hat{R}_t^\star = \hat{R}_t^W + \phi_d \bar{d}\left(\widehat{rer}_t + \frac{\check{d}_t^\star}{d^\star} - \widehat{gdpn}_t\right) + \hat{\xi}_t^R$$

$$\hat{x}_t^{H\star} = \hat{y}_t^\star - \eta^\star\left(\hat{p}_t^H - \widehat{rer}_t\right)$$

$$y^C \hat{y}_t^C = c\hat{c}_t + i\hat{\imath}_t + g\hat{g}_t$$

$$y^H \hat{y}_t^H = x^H \hat{x}_t^H + x^{H\star}\hat{x}_t^{H\star}$$

$$\hat{y}_t^F = \hat{x}_t^F$$

$$\hat{y}_t^H + \hat{\Xi}_t^H = \hat{z}_t + \alpha\left(\hat{k}_{t-1} - \hat{a}_t\right) + (1-\alpha)\hat{h}_t$$

$$\Xi^H \hat{\Xi}_t^H = -\epsilon_H(1-\theta_H)\left(\tilde{p}^H\right)^{-\epsilon_H}\widehat{\tilde{p}}_t^H + \theta_H\left(\frac{\pi^{I,H}}{\pi^H}\right)^{-\epsilon_H}\Xi^H\left[-\epsilon_H\left(\hat{\pi}_t^{I,H} - \hat{\pi}_t^H\right) + \hat{\Xi}_{t-1}^H\right]$$

$$\hat{p}_t^H - \hat{p}_{t-1}^H = \hat{\pi}_t^H - \hat{\pi}_t$$

$$\hat{p}_t^F - \hat{p}_{t-1}^F = \hat{\pi}_t^F - \hat{\pi}_t$$

$$\widehat{rer}_t - \widehat{rer}_{t-1} = \hat{\pi}_t^s + \hat{\pi}_t^\star - \hat{\pi}_t$$

$$\hat{m}_t = \hat{y}_t^F + \hat{\Xi}_t^F$$

$$\Xi^F \hat{\Xi}_t^F = -\epsilon_F(1-\theta_F)\left(\tilde{p}^F\right)^{-\epsilon_F}\widehat{\tilde{p}}_t^F + \theta_F\left(\frac{\pi^{I,F}}{\pi^F}\right)^{-\epsilon_F}\Xi^F\left[-\epsilon_F\left(\hat{\pi}_t^{I,F} - \hat{\pi}_t^F\right) + \hat{\Xi}_{t-1}^F\right]$$

$$gdp\ \widehat{gdp}_t = c\hat{c}_t + i\hat{\imath}_t + g\hat{g}_t + x^{H\star}\hat{x}_t^{H\star} + y^{Co}\hat{y}_t^{Co} - m\hat{m}_t$$

$$gdpn\ \widehat{gdpn}_t = c\hat{c}_t + i\hat{\imath}_t + g\hat{g}_t + t\check{b}_t$$

$$t\check{b}_t = p^H x^{H\star}\left(\hat{p}_t^H + \hat{x}_t^{H\star}\right) + rer\ p^{Co\star}y^{Co}\left(\widehat{rer}_t + \hat{p}_t^{Co\star} + \hat{y}_t^{Co}\right) - rer\xi^m m\left(\widehat{rer}_t + \hat{\xi}_t^m + \hat{m}_t\right)$$

$$\begin{aligned} -\left(rer\ d^\star\widehat{rer}_t + rer\ \check{d}_t^\star\right) \quad &= \quad t\check{b}_t - \frac{rer}{a\pi^\star}d^\star R^\star\left(\widehat{rer}_t - \hat{a}_t - \hat{\pi}_t^\star + \frac{\check{d}_{t-1}^\star}{d^\star} + \hat{R}_{t-1}^\star\right) \\ &\quad -(1-\chi)rer\ p^{Co\star}y^{Co}\left(\widehat{rer}_t + \hat{p}_t^{Co\star} + \hat{y}_t^{Co}\right) \end{aligned}$$

$$\hat{\Delta}_t^{GDP} = \widehat{gdp}_t - \widehat{gdp}_{t-1} + \hat{a}_t$$

$$\hat{\Delta}_t^C = \hat{c}_t - \hat{c}_{t-1} + \hat{a}_t$$

$$\hat{\Delta}_t^I = \hat{\imath}_t - \hat{\imath}_{t-1} + \hat{a}_t$$

The model can now be reduced to a more compact system of equations by eliminating $\hat{f}_t^H$, $\hat{f}_t^F$, $\widehat{\tilde{p}}_t^H$, $\widehat{\tilde{p}}_t^F$, $\hat{\pi}_t^{I,H}$, $\hat{\pi}_t^{I,F}$, $\hat{\Xi}_t^H$, $\hat{\Xi}_t^F$ and $\hat{\lambda}_t$, using the steady state solutions for λ, i, r^k, $\tilde{p}^H$, $\tilde{p}^F$, $\pi^{I,H}$, $\pi^{I,F}$, π^H, π^F, Ξ^H

and Ξ^F, as well as the relations $E_t\left\{\hat{\xi}^c_{t+1}\right\} = \rho_{\xi^c}\hat{\xi}^c_t$, $E_t\left\{\hat{a}_{t+1}\right\} = \rho_a \hat{a}_t$ and $\hat{\Xi}^H_{-1} = \hat{\Xi}^F_{-1} = 0$:

$$\hat{h}_t = \frac{1}{\varphi}\left\{\hat{w}_t - \frac{\sigma}{1-\phi_c/a}\left[\hat{c}_t + \frac{\phi_c}{a}\left(\hat{a}_t - \hat{c}_{t-1}\right)\right] + \hat{\xi}^c_t - \hat{\xi}^h_t\right\}$$

$$\hat{c}_t = \frac{1}{1+\phi_c/a}\left\{\left[E_t\left\{\hat{c}_{t+1}\right\} + \frac{\phi_c}{a}\hat{c}_{t-1} + \left(\rho_a - \frac{\phi_c}{a}\right)\hat{a}_t\right] - \frac{1-\phi_c/a}{\sigma}\left[\hat{R}_t - E_t\left\{\hat{\pi}_{t+1}\right\} - \left(1-\rho_{\xi^c}\right)\hat{\xi}^c_t\right]\right\}$$

$$\hat{R}^\star_t + E_t\left\{\hat{\pi}^s_{t+1}\right\} = \hat{R}_t$$

$$\hat{q}_t + \hat{R}_t - E_t\left\{\hat{\pi}_{t+1}\right\} = \beta a^{-\sigma} E_t\left\{\left(\frac{a^\sigma}{\beta} - 1 + \delta\right)\hat{r}^k_{t+1} + (1-\delta)\,\hat{q}_{t+1}\right\}$$

$$\gamma a^2\left(\hat{\imath}_t + \hat{a}_t - \hat{\imath}_{t-1}\right) = \hat{q}_t + \hat{\xi}^i_t + \beta\gamma a^{3-\sigma}\left(E_t\left\{\hat{\imath}_{t+1} + \hat{a}_{t+1}\right\} - \hat{\imath}_t\right)$$

$$\hat{k}_t = \frac{1-\delta}{a}\left(\hat{k}_{t-1} - \hat{a}_t\right) + \left(1 - \frac{1-\delta}{a}\right)\left(\hat{\xi}^i_t + \hat{\imath}_t\right)$$

$$\hat{y}^C_t = \left(y^C\right)^{1/\eta-1}\left[\omega^{1/\eta}\left(x^H\right)^{1-1/\eta}\hat{x}^H_t + (1-\omega)^{1/\eta}\left(x^F\right)^{1-1/\eta}\hat{x}^F_t\right]$$

$$\hat{x}^F_t = \hat{y}^C_t - \eta\hat{p}^F_t$$

$$\hat{x}^H_t = \hat{y}^C_t - \eta\hat{p}^H_t$$

$$\hat{p}^H_t + \widehat{mc}^H_t = \alpha\hat{r}^k_t + (1-\alpha)\,\hat{w}_t - \hat{z}_t$$

$$\hat{k}_{t-1} - \hat{h}_t = \hat{w}_t - \hat{r}^k_t + \hat{a}_t$$

$$\hat{\pi}^H_t = \frac{(1-\theta_H)\left(1-\beta\theta_H a^{1-\sigma}\right)}{\theta_H\left(1+\beta a^{1-\sigma}\gamma_H\right)}\widehat{mc}^H_t + \frac{\gamma_H}{1+\beta a^{1-\sigma}\gamma_H}\hat{\pi}^H_{t-1} + \frac{\beta a^{1-\sigma}}{1+\beta a^{1-\sigma}\gamma_H}E_t\left\{\hat{\pi}^H_{t+1}\right\}$$

$$\hat{p}^F_t + \widehat{mc}^F_t = \widehat{rer}_t + \hat{\xi}^m_t$$

$$\hat{\pi}^F_t = \frac{(1-\theta_F)\left(1-\beta\theta_F a^{1-\sigma}\right)}{\theta_F\left(1+\beta a^{1-\sigma}\gamma_F\right)}\widehat{mc}^F_t + \frac{\gamma_F}{1+\beta a^{1-\sigma}\gamma_F}\hat{\pi}^F_{t-1} + \frac{\beta a^{1-\sigma}}{1+\beta a^{1-\sigma}\gamma_F}E_t\left\{\hat{\pi}^F_{t+1}\right\}$$

$$\hat{R}_t = \alpha_R\hat{R}_{t-1} + (1-\alpha_R)\left[\alpha_\pi\hat{\pi}_t + \alpha_y\hat{\Delta}^{GDP}_t\right] + e^m_t$$

$$\hat{R}^\star_t = \hat{R}^W_t + \phi_d\bar{d}\left(\widehat{rer}_t + \frac{\check{d}^\star_t}{d^\star} - \widehat{gdpn}_t\right) + \hat{\xi}^R_t$$

$$\hat{x}^{H\star}_t = \hat{y}^\star_t - \eta^\star\left(\hat{p}^H_t - \widehat{rer}_t\right)$$

$$y^C\hat{y}^C_t = c\hat{c}_t + i\hat{\imath}_t + g\hat{g}_t$$

$$y^H\hat{y}^H_t = x^H\hat{x}^H_t + x^{H\star}\hat{x}^{H\star}_t$$

$$\hat{y}^F_t = \hat{x}^F_t$$

$$\hat{y}^H_t = \hat{z}_t + \alpha\left(\hat{k}_{t-1} - \hat{a}_t\right) + (1-\alpha)\,\hat{h}_t$$

$$\hat{p}^H_t - \hat{p}^H_{t-1} = \hat{\pi}^H_t - \hat{\pi}_t$$

$$\hat{p}^F_t - \hat{p}^F_{t-1} = \hat{\pi}^F_t - \hat{\pi}_t$$

$$\widehat{rer}_t - \widehat{rer}_{t-1} = \hat{\pi}^s_t + \hat{\pi}^\star_t - \hat{\pi}_t$$

$$\hat{m}_t = \hat{y}^F_t$$

$$gdp\ \widehat{gdp}_t = c\hat{c}_t + i\hat{\imath}_t + g\hat{g}_t + x^{H\star}\hat{x}^{H\star}_t + y^{Co}\hat{y}^{Co}_t - m\hat{m}_t$$

$$gdpn\ \widehat{gdpn}_t = c\hat{c}_t + i\hat{\imath}_t + g\hat{g}_t + t\check{b}_t$$

$$t\check{b}_t = p^H x^{H\star}\left(\hat{p}^H_t + \hat{x}^{H\star}_t\right) + rer\ p^{Co\star}y^{Co}\left(\widehat{rer}_t + \hat{p}^{Co\star}_t + \hat{y}^{Co}_t\right) - rer\xi^m m\left(\widehat{rer}_t + \hat{\xi}^m_t + \hat{m}_t\right)$$

$$\begin{aligned} -\left(rer\ d^\star\widehat{rer}_t + rer\ \check{d}^\star_t\right) \quad &= \quad t\check{b}_t - \frac{rer}{a\pi^\star}d^\star R^\star\left(\widehat{rer}_t - \hat{a}_t - \hat{\pi}^\star_t + \frac{\check{d}^\star_{t-1}}{d^\star} + \hat{R}^\star_{t-1}\right) \\ &\quad -(1-\chi)rer\ p^{Co\star}y^{Co}\left(\widehat{rer}_t + \hat{p}^{Co\star}_t + \hat{y}^{Co}_t\right) \end{aligned}$$

$$\hat{\Delta}^{GDP}_t = \widehat{gdp}_t - \widehat{gdp}_{t-1} + \hat{a}_t$$

$$\hat{\Delta}^C_t = \hat{c}_t - \hat{c}_{t-1} + \hat{a}_t$$

$$\hat{\Delta}^I_t = \hat{\imath}_t - \hat{\imath}_{t-1} + \hat{a}_t$$

The exogenous processes are:

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + e^z_t$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + e^a_t$$

$$\hat{\xi}^c_t = \rho_{\xi^c}\hat{\xi}^c_{t-1} + e^{\xi^c}_t$$

$$\hat{\xi}^h_t = \rho_{\xi^h}\hat{\xi}^h_{t-1} + e^{\xi^h}_t$$

$$\hat{\xi}^i_t = \rho_{\xi^i}\hat{\xi}^i_{t-1} + e^{\xi^i}_t$$

$$\hat{\xi}^R_t = \rho_{\xi^R}\hat{\xi}^R_{t-1} + e^{\xi^R}_t$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + e^g_t$$

$$\hat{y}^{Co}_t = \rho_{y^{Co}}\hat{y}^{Co}_{t-1} + e^{y^{Co}}_t$$

$$\hat{\pi}^\star_t = \rho_{\pi^\star}\hat{\pi}^\star_{t-1} + e^{\pi^\star}_t$$

$$\hat{R}^W_t = \rho_{R^W}\hat{R}^W_{t-1} + e^{R^W}_t$$

$$\hat{y}^\star_t = \rho_{y^\star}\hat{y}^\star_{t-1} + e^{y^\star}_t$$

$$\hat{p}^{Co\star}_t = \rho_{p^{Co\star}}\hat{p}^{Co\star}_{t-1} + e^{p^{Co\star}}_t$$

$$\hat{\xi}^m_t = \rho_{\xi^m}\hat{\xi}^m_{t-1} + e^{\xi^m}_t$$

E Calibration and estimation

Table 5 lists the parameters of the model that are calibrated, as well as some variables normalizations and steady state values that are targeted. Table 6 lists the maximum likelihood estimates of the exogenous processes for which we have a direct counterpart in the data and that are used in the model.

Parameter	Description	Value
σ	Inverse of the elasticity of intertemporal substitution	1
φ	Inverse Frisch elasticity	1
ω	Home bias parameter	0.67
χ	Government share of copper exports	0.56
δ	Quarterly depreciation rate	0.02
α	Share of labor in wholesale domestic goods	0.33
ϵ_H	Elast. of substitution domestic varieties	11
ϵ_F	Elast. of substitution imported varieties	11
π^T	Annual inflation target	0.03
β	Annual discount factor	0.9995
π^S	Annual nominal exchange rate depreciation	0
ξ^R	Annual country premium, net rate	0.015
$\bar{d}$	Foreign debt-to-nominal GDP ratio threshold	0.56
s^{Co}	Copper to GDP ratio	0.11
s^g	Government consumption to GDP ratio	0.12
h	Hours per worker	1/3
p^H	Home good price	1
p^{Co}	Domestic copper price	1

Table 5: List of calibrated parameters and steady state values.

Parameter	Description	Value
$AR(1)$ coefficients		
ρ_{R^W}	Exogenous risk-free world interest rate|.	0.97
$\rho_{y^\star}$	Foreign aggregate GDP.	0.88
$\rho_{\pi^\star}$	Foreign relevant inflation.	0.42
ρ_{ξ^m}	Import price shock.	0.54
$\rho_{y^{Co}}$	Copper production.	0.70
$\rho_{p^{Co}}$	Foreign copper price.	0.96
ρ_g	Government consumption.	0.79
Innovations std		
$100\sigma_{R^W}$	Exogenous risk-free world interest rate.	0.09
$100\sigma_{y^\star}$	Foreign aggregate GDP.	0.53
$10\sigma_{\pi^\star}$	Foreign relevant inflation.	0.26
$10\sigma_{\xi^m}$	Import price shock.	0.24
$10\sigma_{y^{Co}}$	Copper production.	0.27
$\sigma_{p^{Co}}$	Foreign copper price.	0.13
σ_g	Government consumption.	0.13

Table 6: List of exogenous processes estimated outside the model.

Table 7, in turn, presents the estimation results for the parameters of the model. In particular, the last two columns, presents the estimation of the model assuming rational expectations instead of adaptive learning plus the unanchoring of inflation expectations.

Table 7: Prior and posterior distributions. Deep parameters.

Parameter	Description	Initial Prior			Posterior			
					AL-unanchoring		RE	
		Distribution	mean	std	mode	std	mode	std
ϕ_c	Habit formation.	B	0.7	0.1	0.76		0.78	
γ	Investment adjustment costs.	N	4	1.5	4.52		4.87	
η	Elast. of subst. H and F in final good.	N	1.5	0.25	1.78		1.81	
$\eta^\star$	Elast. of subst. H and F in foreign demand.	N	0.25	0.05	0.31		0.32	
θ_H	Calvo probability domestic prices.	B	0.8	0.08	0.90		0.89	
γ_H	Indexation domestic prices.	B	0.5	0.15	0.07		0.08	
θ_F	Calvo probability import prices.	B	0.8	0.08	0.79		0.78	
γ_F	Indexation import prices.	B	0.5	0.15	0.12		0.11	
ϕ_d	Debt to GDP position threshold.	Γ^{-1}	0.01	∞	0.01		0.01	
α_R	Taylor rule inertia parameter.	B	0.75	0.1	0.80		0.81	
α_π	Taylor rule inflation response.	N	1.5	0.1	1.87		1.82	
α_y	Taylor rule output response.	B	0.13	0.05	0.39		0.39	
Exogenous processes								
$AR(1)$ coefficients								
ρ_z	Transitory technology shock.	B	0.75	0.1	0.42		0.43	
ρ_a	Unit root technology shock.	B	0.75/2	0.1	0.84		0.84	
ρ_{ξ_c}	Preference shock.	B	0.75	0.1	0.92		0.91	
ρ_{ξ_i}	Investment specific technology shock.	B	0.75	0.1	0.67		0.68	
ρ_{ξ_R}	Country premium shock.	B	0.75	0.1	0.72		0.66	
Innovations std								
$10\sigma_z$	Transitory technology shock.	Γ^{-1}	$0.01^\star$	∞	0.93		0.84	
$100\sigma_a$	Unit root technology shock.	Γ^{-1}	0.01	∞	0.78		0.85	
$10\sigma_{\xi_c}$	Preference shock.	Γ^{-1}	0.01	∞	0.21		0.23	
$10\sigma_{\xi_i}$	Investment specific technology shock.	Γ^{-1}	0.01	∞	0.51		0.54	
$100\sigma_{\xi_R}$	Country premium shock.	Γ^{-1}	0.01	∞	0.84		1.02	
$100\sigma_{\xi_m}$	Monetary policy shock.	Γ^{-1}	0.01	∞	0.16		0.16	
Expectations parameters								
$\sigma^2_{EEE_{11}}$	1 year ahead expectations me. error.	B	$0.030^\star$	0.022	0.079		0.068	
$100\sigma^2_{EEE_{23}}$	2 years ahead expectations me. error.	B	0.002	0.001	0.03		0.03	
ρ_1	Non-inflation variables gain.	Γ^{-1}	0.018	0.015	0.01		-	
$100\rho_2$	Inflation variables gain-base.	Γ^{-1}	0.018	0.015	0.43		-	
ν_h	Unanchoring function parameter.	N	1	0.5	0.94		-	
ν_l	Unanchoring function parameter.	N	1	0.5	1.12		-	
ν	Switch-triggering threshold.	N	1.5	0.5	1.47		-	

Note: * Priors are expressed in terms of the parameter, without the pre-multiplying factor.

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