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Rodrigo Cifuentes

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Huérfanos 1175, primer piso.
Teléfono: (56-2) 6702475   Fax: (56-2) 6702231

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HOW DOES PENSION REFORM AFFECT SAVINGS AND WELFARE

Rodrigo Cifuentes
Economista Senior
Gerencia de Análisis Financiero
Banco Central de Chile

Resumen

Este trabajo estudia los efectos de una reforma de pensiones sobre el ahorro por precaución, la acumulación de riqueza y el bienestar. Se utilizan técnicas de programación dinámica para resolver el consumo óptimo de un trabajador que enfrenta incertidumbre en el ingreso del trabajo y en sus pensiones. Luego, con valores de parámetros representativos de EE.UU., se evalúa el impacto sobre trabajadores con distinto nivel de educación de dos políticas. La primera es la eliminación de la redistribución en la fórmula de beneficios actual. La segunda es el cambio de un sistema definido en beneficios (DB) por uno definido en contribuciones (DC). En el primer caso, los resultados muestran que la redistribución es valorada por todos los tipos de trabajadores, incluso los de mayor ingreso esperado que esperan pérdidas por la redistribución. El ahorro agregado aumenta en 4% como consecuencia de la mayor incertidumbre. La segunda política tiene las consecuencias contrarias. El bienestar aumenta al introducir un sistema DC ya que éste cuenta con propiedades de seguro superiores. Estas provienen del hecho de que los períodos con menor varianza en los ingresos tienen una mayor ponderación en el cálculo de los beneficios. El ahorro por precaución disminuye, con lo que el ahorro agregado cae en 1,4%.

Abstract

This paper explores the effects of pension reform on precautionary savings, wealth accumulation and welfare. The impact of pension programs on income uncertainty through life has been largely ignored in the literature of precautionary savings and pension reform. This paper uses dynamic programming techniques to solve for the optimal consumption of a worker that faces uncertainty on labor and retirement income. Subsequently, with parameter values for the U.S., I study the impact of two policies on workers with different on educational levels. One is the elimination of redistribution in the current benefits formula. The second is the change from system defined in benefits (DB) to one defined in contributions (DC). In the first case, results show that redistribution is valued positively by all types of agents, even by those who expect losses from it, given their expected income. As a consequence, workers increase their savings to prepare for the increased uncertainty. The increase in aggregate savings is in the order of 4%. The second case has the opposite consequences. Welfare increases with the adoption of the DC system because it has superior insurance properties. The advantage consists in that periods on which the variance of income is lower receive a higher weight in the calculation of benefits. Precautionary savings are reduced with a fall in aggregate savings of 1.4%.

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1. Introduction

An aging population and systemic maturity have brought much discussion about reforming the old-age pensions component of Social Security Systems in recent years. The amount of resources involved in this program in both developed and developing countries make this question an issue of first order importance in policy analysis. A central component of the debate on reforming social security is the advantages and disadvantages of moving towards a system defined in contributions (DC) with pre-funding of benefits. Advocates of this reform claim that a funded DC scheme will provide benefits such as an increase in aggregate private savings[1], reduced distortions in labor markets and improved return in workers' contributions.

This paper draws attention to two prevalent issues in the discussion. First, the positive impact on aggregate savings comes from the fact that during the transition towards the new system, a burden is imposed on some generations in the form of additional taxes or reduced benefits. This burden is a method of reducing the public debt that is implicit in a pay-as-you-go (PAYG) system. Therefore, the increase in aggregate savings ultimately comes from the reduction in the implicit debt in the PAYG system. A policy that reduces public debt with the result of increasing aggregate savings in the economy is always available, and therefore, the result is not inherent in pension reform[2]. If pension reform is done in such a way that promised benefits for all generations are respected and no new taxes are imposed, there is no increase in aggregate savings (Valdés-Prieto, 1997; Rangel, 1998).

The second issue is that most of the literature ignores a key dimension in the analysis of Social Insurance systems, which is the quality of the insurance provided by them. Most of the literature studies the problem in a context without uncertainty, and therefore is unable to provide an assessment of this dimension.

This paper contributes to the debate on social security reform by analyzing the problem in a context in which uncertainty is assumed explicitly. The importance of this is twofold. First, when uncertainty is considered, pension reform will have consequences on aggregate savings even in the case when promised benefits are paid to all generations and no new taxes are imposed on any generation. This arises from the fact that changes in the pension system alter the uncertainty that workers face with respect to their lifetime resources. Considering uncertainty in order to understand savings behavior seems to be fundamental, as recent empirical findings show (Carroll, 1997). Public pension systems, in turn, constitute a major source of income for a large fraction of the population in the U.S. Therefore, changes

[1] Feldstein and Samwick (1997) find a considerable impact of pension reform on aggregate savings in the case of the U.S. In their simulations, transition generations pay for the implicit debt in the old system.

[2] It is true, though, that pension reform allows the possibility of making a policy to reduce public debt to be more viable politically.

in the variance of remaining lifetime resources will change the savings behavior of a worker during her life, and thus, aggregate savings.

Second, including uncertainty explicitly allows us to assess which type of pension arrangement provides a better insurance to workers. Specifically, I measure the variance of pensions that a worker faces under different pension schemes and the impact that this has on her welfare.

This paper studies the effects on welfare and aggregate savings in the economy of a pension reform. The specific reform studied in this paper is the following. The current defined benefit system (DB) in the U.S. is replaced by a DC system in such a way that all promised benefits are paid and that no new taxes are raised to finance the reform. The appendix provides a proof that such a reform is feasible and that the rate of return in the new DC system has to be same as that implicit in the original DB system[3]. The reform is analyzed in two steps. The first step analyzes the elimination of redistribution in the DB system. The second analyzes the change in the type of system from a DB to a DC.

The analysis is done by comparing steady states in a partial equilibrium model. The specific reform considered here allows the transition to be characterized straightforwardly from this comparison. The choice of partial equilibrium is due to computational constraints. Current available computational capacity would make calculation of a general equilibrium model of this sort extremely costly in terms of time. The analysis focuses on the impact of idiosyncratic shocks, both transitory and permanent, to labor income. The extent to which Social Security systems can insure aggregate shocks is studied in Rangel and Zeckhauser (1999) and Demange and Laroque (1999).

The model presented here takes Social Security as given, that is, it does not arise endogenously from agents' optimization. One concern that sometimes arises about these kinds of models is that if there is not an explicit reason for having a social security system, then welfare analysis of pension reform will be biased towards the result that any reform that tends to eliminate Social Security will be positive. This criticism often mixes two different issues. One is a "correct" concern that if the forces that give origin to Social Security are not present in the analysis, then part of the benefits associated with it are being omitted from the analysis. I will address this issue in the next paragraph. But the issue is raised "incorrectly" in other situations. In particular, the absence of endogenously determined Social Security is raised against some analyses that find large positive benefits from reforming Social Security toward funded schemes. In several cases, as in Campbell *et al* (1999) for example, the problem arises from not considering how the transition costs are going to be financed. When this happens, large benefits from reform arise, but they hide the fact that in the transition several generations are being made worse off. Therefore, moves towards eliminating Social Security look good not because its benefits are not being

[3] A similar result can be found in Geanakoplos, Mitchell and Skinner (1998).

considered, but because a part of the costs are being left out. In this paper the costs of abandoning the initial PAYG system are explicitly taken into account in the analysis.

With respect to the "correct" concern, two comments can be made. First, among the most accepted arguments explaining Social Security are intergenerational income redistribution and paternalism (Diamond, 1977). The former refers to the fact that the origin of Social Security in the U.S. is linked to the great depression. A decision was made to redistribute income in favor of some generations particularly damaged by this event at the expense of all future generations. If this is the case then there is no reason to have endogenously determined social security in the model. The system is simply part of the environment once the event that caused its creation is in the past, and the way to deal with it is simply by recognizing the costs and benefits that the system imposes to all generations. The paternalist argument says that society cares about some people saving too little for retirement when left to their own devices. Therefore it imposes a forced savings mechanism on the entire population. This can be embedded in the analysis assuming that there are heterogeneous preferences and that the political system imposes certain preferences on lifetime consumption in the entire population, as in Cifuentes and Valdés-Prieto (1997). In the context of the analysis in this paper, modeling a context consistent with paternalism would amount to adding more cases to the analysis (with different time preferences), but would not change qualitatively the results obtained.

Finally, as will be clear from the results, one of the contributions made by this paper is precisely the fact that when labor income uncertainty is introduced into the analysis the result that less social security is better is radically changed.

The paper is organized as follows. Section Two discusses the sources of uncertainty that affect the determination of pensions. Section Three presents the model, the parameters chosen, the solution techniques and the type of results obtained in the simulation. Section Four presents and discusses the effects on welfare and savings of the elimination of redistribution in the DB system. Section Five analyzes the effect on welfare and savings of a reform from a DB to a DC system. Section Six concludes.

2. Uncertainty in Pension Systems

A pensions program can be organized in many different ways. Here, I restrict attention to systems that relate pensions to the history of labor income. Even among these, there are multiple ways in which retirement benefits can be determined. The following expression summarizes different factors that can intervene in determining the level of retirement benefits:

$$p = a(k)\sum_{i=Y_W}^{Y_R} b_i(k)w_i\left(\prod_{j=i+1}^{Y_R} R_j(k)\right) \qquad (1)$$

where p is the retirement benefit that the worker receives in the form of an annuity. A worker is assumed to work from age Y_W to the age of retirement Y_R. The retirement benefit is determined as a function of a weighted sum of taxable wages w_i [4], where the weights are the factors b_i and a gross rate of return R_i, compounded between the time of the contribution and that of retirement. The variable k indexes different pension systems.

For example, a DB system that pays a fixed replacement rate over the average taxable wage over active life will have a equal to the replacement rate, b_i equal to $1/(Y_R-Y_W+1)$ and R_i equal to 1. If the benefit is determined over the average of the last, say, ten years, b_i will be zero for $i=Y_W,\ldots,Y_{R-10}$ and 1/10 for $i=Y_{R-9},\ldots,Y_R$. Replacement rate a may also vary with the level of the weighted sum of taxable wages. A DC system will have b_i equal to the contribution rate at each period and R_i equal to the gross return of the asset where contributions are invested. Coefficient a will be the annuity factor.

The formula makes clear that the effect of different sources of uncertainty varies across different pension systems. Therefore, systems differ in the variance of expected benefits at the beginning of life but also during the lifetime. For example, a DB system that averages all periods of active life gives a higher weight to realizations of labor income, as compared to a DC system. A DC system, in turn, introduces variability in the rate of return.

The formula also highlights the fact that there are other sources of uncertainty aside from wages and returns. In a DB system, a, the replacement rate, can be changed by the authority in response to financial problems. Also, the degree of redistribution can change according to changes in the preferences of the political majority.

In the same way that systems transmit shocks to income and to rates of returns to the value of retirement benefits, they can have mechanisms to insure against adverse idiosyncratic shocks. There are several ways to do this. In a DB system the replacement rate a can be a decreasing function of the weighted wage average. In a DC scheme, part of the contribution can be accumulated in a shared account that is used to supplement retirement income up to a certain target level in the case of poor income realizations.

It should be noted that the analysis here has been done without reference to whether the system is funded or not. The distinction between DB and DC systems refers to the way a worker accumulates benefits throughout her lifetime and not to the type of underlying asset that backs the promises.

3. The Model and Parameters

3.1 Model

This paper works with a model of consumption under uncertainty with a set up similar to that of Zeldes (1989) and Carroll (1992). I assume that workers are active between ages 25 and *H*, an age known with certainty, at which time they retire. I assume the workers will die with certainty at age *D*. *H* and *D* vary for different types of workers. Workers supply labor inelastically during their active lives. I assume that preferences take the standard additively separable expected utility form such that the intertemporal maximization problem for the worker is:

$$\max_{C_t} u(C_t) + E_t \sum_{s=t+1}^{T} b^{(s-t)} u(C_s) \tag{2}$$

$$s.t. \quad \tilde{X}_{t+1} = R(X_t - C_t) + \tilde{Y}_{t+1} \tag{3}$$

where *u(.)* is an instantaneous utility function, C_t is consumption, $b = 1/(1+d)$ is the discount factor with δ the discount rate, X_t is cash-on-hand (income in period *t* plus financial wealth in the same period), *R* is the gross one-period return on savings and Y_t is income. The instantaneous utility function is assumed to be of the Constant Relative Risk Aversion (CRRA) form:

$$u(C) = \frac{C^{(1-r)}}{1-r} \tag{4}$$

The income process is described by equations (5) through (9):

$$\tilde{Y}_t = \tilde{P}_t \tilde{V}_t \qquad for\ t = 25,\ldots,65 \tag{5}$$

$$Y_t = B_{65} \qquad for\ t = 66,\ldots,82 \tag{6}$$

$$\tilde{P}_t = G_t P_{t-1} \tilde{N}_t \tag{7}$$

where P_t is the permanent income at age *t*, V_t is a transitory shock to income, B_t is the retirement benefit, G_t is the growth factor of permanent income due to age and N_t is a permanent shock to income. The transitory and permanent shocks to income are assumed to have a lognormal distribution with parameters:

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[4] Taxable wage is w_i=min(total wage, maximum taxable wage).

$$\log\left(\tilde{N}\right) \sim N\left(-s^{2}_{\log \tilde{N}} / 2, s^{2}_{\log \tilde{N}}\right) \tag{8}$$

$$\log\left(\tilde{V}\right) \sim N\left(-s^{2}_{\log V} / 2, s^{2}_{\log V}\right) \tag{9}$$

The mean of the lognormal distribution ensures that the mean of *N* and *V* are equal to 1, given that,

$$E_t \tilde{N}_{t+1} = \exp\left[m_{\log N} + s^{2}_{\log N} / 2\right] = \exp\left[-s^{2}_{\log N} / 2 + s^{2}_{\log N} / 2\right] = \exp[0] = 1$$

where *m* represents the mean of the normal distribution.

Equation (5) says that the retirement benefit is an annuity. Once the uncertainty about the level of retirement benefits is resolved, its value is certain for the rest of the retirement period. It should be noted that despite being a certain variable *during* retirement, it is a random variable *before* retirement starts. The moment at which uncertainty about its level is resolved will vary across systems.

The variable B_t is defined as the level of retirement benefit that is known with certainty at time *t*. In other words, B_t is the part of the retirement benefit whose determinants have already been realized. In systems where benefits are determined as a function of the level of income realized, B_t will be the retirement benefit that the worker will receive if her labor income were to be zero in all remaining periods until retirement. In this paper I compare two systems: A DB system in which the benefit is defined as a fraction of the average wage over all active periods, and a DC system in which the benefit is defined on the basis of the accumulated individual contributions. In the case of the DB system, the evolution of B_t is described by:

$$\tilde{B}^{DB}_{t} = B^{DB}_{t-1} + \frac{\min(\tilde{Y}_t, Y^{\max})}{41} y \tag{10}$$

where y is the replacement rate and Y^{max} is the maximum taxable wage. In the case of the DC system, the evolution of B_t is given by:

$$\tilde{B}^{DC}_{t} = B^{DC}_{t-1} + q \min(\tilde{Y}_t, Y^{\max})\left(\prod_{j=t+1}^{Y_R} R^{SS}_{j}\right) f \tag{11}$$

where q is the rate of contribution, R^{SS} is the gross return paid by the social security system on the individual accounts and f is the annuity factor, i.e. the factor that converts the stock accumulated in the individual accounts into an annuity. Equation (11) is specific to the case where the return R^{SS} is certain[5].

Additionally, I assume explicit borrowing constraints which implies that[6]:

$$X_t \geq 0 \qquad \forall t \tag{12}$$

An alternative to explicit liquidity constraints is to assume an environment such that consumers *choose* never to borrow as a result of their optimization. This will be obtained if there is a positive probability of labor income being zero in every period, which can be the result of unemployment, for example[7]. This, combined with the fact that marginal utility at zero consumption is infinite with a CRRA utility function leads to the result that consumers will always want to keep a positive stock of net assets in order to avoid being left without resources in the event of zero labor income. Therefore, they will never borrow. It should be noted, however, that the logic behind this result lies in the positive probability of income being zero not only in one period, but in all remaining periods[8]. Given the presence of formal and informal insurance mechanisms to secure a minimum consumption floor (i.e. Social Security and family, respectively), this argument does not seem to provide a better explanation for borrowing constraints than those arguments that come from the structure of financial markets.

3.2 Parameters

Preferences

Different scenarios are simulated with different values for preference parameters. The chosen values cover the range estimated in the literature. Lawrance (1991) estimates discount rates with data from the Panel Study on Income Dynamics (PSID) on consumption using Euler equations. She finds considerable heterogeneity among income groups, with estimates ranging from 12% for groups with high

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[5] When return R is not certain, equation (11) becomes: $\tilde{B}_t^{DC} = B_{t-1}^{DC}(1+\tilde{R}^{SS}) + q\,\min(\tilde{Y}_t, Y^{\max})f$

[6] Note that this prevents consuming retirement benefits before retirement. In some cases, available resources late in life (income plus financial wealth) can be lower than a retirement benefit known with certainty. This will imply that consumption will jump upwards at retirement.

[7] This approach is taken by an important faction of the recent literature of consumption under uncertainty, including Zeldes (1989), Carroll (1992, 1997) and Gourinchas and Parker (1997), among others.

[8] The argument goes as follows. Given that income can be zero in the last period, the worker will chose to get to that period with some positive amount of savings. Therefore, she does not incur debt in the next to last period. Applying the same reasoning successively to all previous periods implies that it will never be optimal to borrow.

levels of labor income to 19% for groups with low labor income. A different approach for estimation has appeared in recent consumption literature related to the theory of consumption under uncertainty. This approach consists of simulating profiles of consumption or wealth over the life cycle, and matching them with those profiles found in the data. Estimates for underlying parameters in preferences (discount rate and the parameter of risk aversion) are those that generate in simulations the profiles closest to the data. Estimates obtained under this method are in general highly sensitive to the parameters assumed in the simulations. Gourinchas and Parker (1997) estimate discount rates that range from 3.3% to 4.3% according to the level of education. They use data from the Consumers Expenditures Survey (CEX) on consumption and income. Their estimates are sensitive to assumptions on other parameters, particularly the interest rate. Their estimates seem to show a discount rate that is one percentage point above the market interest rate. Carroll and Samwick (1997) use PSID data on consumption and income. Their point estimates for the discount rate are highly sensitive to other parameters chosen and range from 5% to 14%. Moreover, confidence intervals are large. Samwick (1997) estimate distributions of the discount rate using data on wealth from the Survey of Consumer Finances. The median discount rate varies with assumptions of the parameter of risk aversion, the definition of financial wealth and the initial level of assets. In most scenarios chosen, the median discount rate is in the range 3%-5%, although in many cases the median value lies between 7% and 9%. In the absence of a robust estimate of the discount rate, I use a conservative value of 3% in the simulations.

The coefficient of risk aversion r, in turn, takes the values 0.5, 1 and 5. Gourinchas and Parker (1997) estimate a value for r of 0.5. This estimation is lower than what is generally considered a "reasonable range," which is between 1 and 5.

Income Process

The income profiles are taken from Hubbard, Skinner and Zeldes (1994) (HSZ henceforth). HSZ estimate income profiles over the life cycle with PSID data for 1983-1987 for population groups with three levels of educational attainment: Non-High School (NHS), High School Degree (HS) and College Degree (COL). Figure 1 shows the income profiles used. This information is summarized in the model using the level of permanent income at the start of active life, P_{25}, and a vector of growth factors G_t for t= 26, …, L, where L is the last year of active life.

HSZ estimate the variance of income and decompose it on the variance of permanent and transitory shocks. The model for income that they use is similar to that of this paper (equations (5) and

(7)), except that they allow for a coefficient of the persistence of permanent income to be other than one[9]. The parameters they obtain are summarized in Table 1:

Table 1: Variance of Permanent and Income Shocks by Educational Attainment
Percentages

Education	Variance of Permanent Shock	Variance of Transitory Shock
Non-High School	0.033	0.04
High School	0.025	0.021
College Degree	0.016	0.014

Source: HSZ, 1994.

Population Composition

The share of workers at each level of educational attainment is obtained from Census data for 1990. Educational Attainment for the population over 25 years of age in 1990 was: 22.4% Non-High School, 56.3% High-School Degree and 21.3% College Degree (Bureau of the Census, 1992).

Age of Retirement and Expected length of life

The assumed ages of retirement are: 61 years for NHS, 63 years for HS and 65 years for COL (Laibson *et al*., 1998). Length of life in the model is set at the population expected length of life at retirement. These are calculated based on the tables of mortality by educational attainment in Rogot *et al* (1992), applying the methodology described in *Vital Statistics for the U.S. 1994*. This calculation gives us an expected age of death of 81 years for NHS, 82 years for HS and 83 years for COL.

Interest Rate

The risk-free interest rate is assumed to be 3%.

[9] In log form, equation (7) would be $p_t = g_t + gp_{t-1} + n_t$. The coefficient g is assumed to be 1 in this paper, while HSZ estimate it to be between 0.945 and 0.955. Results change slightly when these numbers are considered. A value of

3.3 Pensions systems

The initial DB system

The initial scenario closely resembles the current pay-as-you-go system in the U.S. Benefits are defined over the average lifetime covered wages. The replacement rate (the ratio of the level of the retirement benefit over the average of covered wages) is a decreasing function of average covered wages. This means that the replacement rate the system will give a worker with a history of low wages will be higher than that given to a worker with a history of higher wages. Therefore, the benefit formula is progressive.

The replacement rate in the model is calculated according to the same formula of benefits as in the U.S. Social Security system. In terms of equation (10), the replacement rate, y, is a function of the average of taxable wages over active life:

- 90% of the average annual income up to $5112
- An additional 32% of the average annual income in excess of $5112 but below $30,804
- An additional 15% of the average annual income in excess of $30,804[10]

There is a maximum taxable wage of $61,200 (Social Security Administration, 1997). The contribution rate is set at 10.6%. Figure 2 shows the benefit formula.

The Internal Rate of Return (IRR) in the DB system

Given the benefit formula described above and the assumptions on the distribution of population, I then calculate the IRR of the system. This calculation will tell us whether the whole set of demographic and systemic assumptions provides a good description of the current situation of the pensions system in the U.S.

To calculate the IRR, I calculate the expected income profile for each type of worker. I run 5,000 histories for each type of worker with their respective income process as described in the previous section. Using the population weights, I obtain the profile of contributions paid and pensions received over the lifetime of an average worker. With these payment flows, I can calculate the implicit IRR in the system, which is the rate of return at which it would be necessary to invest contributions in order to finance the observed pension benefits. I compare the IRR that I calculate with other calculations in the literature. Also, the IRR that I calculated must be close to growth of the population and labor productivity ($n+x$) during the same years. This is because the IRR is ultimately the return that an unfunded system can

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1 is used in this paper for computational simplicity.

[10] The cutoff points in the formula ("bend points") are determined by the Social Security Administration in terms of monthly earnings. In monthly terms the cutoff points are $426 and $2567.

pay. If the IRR that I calculate is too high in relationship to (n+x) –say above 2.5%-- it would mean the parameterization that I chose is unrealistic, and that the system is paying benefits that are too high. On the other side, if the IRR is too low –say, below 0.5%-- it would mean that I have modeled a system that gives too little benefits.

The IRR that I estimate is 1.72%. Leimer (1994), in a model with a lot more demographic detail, estimates the IRR to be 1.73% for the cohort born in 1995 and 1.71% for the cohort born in 2050, with the IRR falling slowly for the cohorts in between. This means that the chosen parameterization provides a description consistent with reality. With respect to the growth of population and productivity, figures from the Board of Trustees (1996) show that the geometric average for (n+x) in the period 1990-95 was 1.41%. Considering the cyclical factors that are affecting these realizations, projections for the following years from the Board of Trustees start at a level of 2.2%. In the intermediate scenario, this figure goes down steadily, driven basically by decreases in the growth rate of the labor force. The IRR would reach a level of 1.3% in year 2020. Therefore, a number between 1.3% and 2.2% is a reasonable average forecast for the medium run.

The DC system

The case that I analyze in this paper is one in which the DB system is replaced by a DC system that has the same expected IRR for a worker that has not yet started her active life. In the DC system workers will contribute to an individual account. The balance accumulated in this account will earn an interest equal to the ex-post average (across population) return paid by the DB system. This implies that before starting active life a worker has the same expected benefit in both systems, but the variance of the benefit will differ.

This case is relevant for two reasons. The first is that it provides the appropriate structure for comparing the effect of the differences in uncertainty with respect to retirement income implied by different systems, which is the objective of this paper. If we were to assume a higher IRR in the DC system then the results would be due to both differential uncertainty and level of returns.

The second reason is that this is the scenario that would arise if the reform were fair, in the sense that all generations would be paid what they were promised, and no generation is offered a lower IRR than what they would have had in the old system. To understand this, it should be noted that the IRR to contributions that a system can offer is related to the growth in the asset that backs the promise of future payment. In a funded system, the return that the system can offer is that of the assets in which funds are invested. In unfunded systems, the promise of future payments is backed by the ability to collect contributions from future generations in order to finance the payment of benefits. The growth in that “asset” is the growth in covered wages, which can be decomposed into the growth of the labor force (n)

and growth of real wages due to increases in productivity (x). This is the rate of return that an unfunded system can offer (Samuelson, 1958).

Therefore, when an unfunded system is reformed, there is an unfunded liability that must be dealt with. This is basically the same as public debt, with the only differences being that it normally does not appear in fiscal accounts and that the government has more discretion to change the return that it pays on it. The question is how to pay for this liability. If it is paid by raising taxes, then generations living when the taxes were raised for this purpose will be harmed. This is equivalent to making some generations pay for retiring public debt. If it is paid by issuing public debt, then this debt will have to pay market interest rates (r), higher than ($n+x$) in an economy without an excess of capital. If this is the case, the government will have to raise some tax in order to pay for the return that its debt pays in excess of the return on the "asset" it possesses ($r > n+x$). If this is the case, then some agent in the economy will be harmed by the rise in taxes and, therefore, the reform will not be neutral.

One way to obtain a neutral reform is the following. Workers in the new system should buy the new public debt. If this promises market interest rates then a tax should be raised in order for the debt not to grow indefinitely over time. The tax can be a tax on the returns to this new debt, such that the return after tax equals ($n+x$). Thus, the debt is not growing indefinitely over time and workers are receiving a return on their contributions equal to what they received before. Alternatively, the new public debt can simply be issued with the lower return, with the workers mandated to buy it. If buying it were not mandatory, workers would prefer buying assets with higher returns and the government would have to raise the return it offers but increase some tax with the consequences described earlier. This equivalence result is proven in the appendix. Valdés-Prieto (1997) and Rangel (1998) provide alternative proofs of the same result.

3.4 Solution Method

The problem is solved using backwards induction. The control variable of this problem is consumption at each age of life. The state variables are $\{t, X_t, P_t, B_t\}$ where t (age) goes from 25 to (H-1). P_t (permanent income) disappears as state variable for t=H to D, the retirement period and the last period of active life. P_t is not relevant here because there is no more labor income coming in the next periods. The value function that corresponds to the problem described by (2) can be written as:

$$V_t(X_t, P_t, B_t) = \max_{C_t}\left[u(C_t) + bE_t V_{t+1}(X_{t+1}, P_{t+1}, B_{t+1})\right] \qquad (13)$$

while a similar equation without P_t as a state variable holds for t=65 to 82, the retirement period. We want to solve the policy functions for the control variable as a function of the state variables ($C = C(t, X_t, P_t, B_t)$). Given that I have assumed the absence of bequest motives, the policy function for $t=82$, the last period, is straightforward: utility is maximized when all available resources are consumed ($C = X$). With this, I know the form of the value function at t=82 ($V_{82}(.) = u(X)$). With this information the maximization in (13) can be done, obtaining V_{81} and the policy function for consumption at that age. Knowing V_{81} allows me to examine the previous period and maximize (13) again, obtaining the policy and value function for that period. The procedure is repeated through the first period of active life.

In order to apply this procedure I use standard numerical techniques (Judd, 1998). I discretize the state-space of the problem, with a grid that is finer for lower values of the variables. At a given age, for each possible combination of the discretized state-space, I calculate the expected utility for that combination. To do this, I approximate the continuous density function of the shocks that affect the state variables using ten points equally spaced over a range of six times the standard deviation around the mean[11]. The utility associated with each possible point is obtained by interpolating the value function for the next period when points do not coincide exactly with points on the grid. This gives the continuation pay off, or the second term in equation (13). The next step is to calculate for each possible point in the state-space the utility associated with each possible value of the control variable, and then to pick the maximum. When a maximum is picked, I do a second search around it with a finer (interpolated) grid, in order to obtain a more precise estimate of the policy function.

3.5 Simulation results

Figure 3 shows the estimated consumption function for the case {d=3%, r=0.5, r=3%} at different ages. The horizontal axis measures cash-on-hand[12]. Different levels of B_t are represented by different lines, with the lowest representing a zero pension and the highest a pension equal to the income at the start of active life. Graphs are derived for one particular level of permanent income, which is that of a worker with a high-school degree at the start of life. Consumption functions start as a 45^o line. This indicates that for low levels of cash-on-hand the worker chooses to consume all his income, i.e., liquidity constraints are binding. At higher levels of cash-on-hand, the worker starts to save, which is indicated by a 'kink' in the consumption function. Notice that the graph is derived for a worker with an expected income of one unit at age 25 who expects income to rise (see age-income profiles in figure 1). Despite

[11] This method is suggested in Deaton (1992) and used also by Carroll (1992, 1997).
[12] All figures indicating levels of money are divided by the income of a worker at 25 years of age ($28,000 aprox.)

this, she starts saving when her effective income is slightly above 0.5, due to the precautionary saving motive. This is maintained well into active life, as the graph for the 50-year old worker shows.

The extent to which consumers behave differently according to the amount of pension they already know they will receive at retirement varies with age. Even at the start of life, workers behave differently according to their expected pension, distinguishing between the cases of zero and higher pensions. At ages 40 and 50 distinction is higher. Low levels of pension reduce consumption, while differentiation in behavior at higher benefit levels is still low. When all uncertainty in retirement income is resolved, at age 65, consumption responds 1 to 1 to differences in pensions.

Figure 5 shows a different case. Here the consumption profiles seem to show two 'kinks'. The reason for the second kink is that in the case depicted, permanent income is low. For high levels of B_t, the worker would prefer to consume part of her retirement income in advance. The second kink, therefore, appears because of the impossibility of borrowing against retirement income.

Figures 6 and 7 show how consumption changes with different levels of the coefficient of risk aversion. The values considered for CRRA are 0.5, 1 and 5. Figure 6 considers the case of a worker without a high school degree, with permanent income equal to that expected at the start of working life and a pension equal to half of that. With the highest value of Relative Risk Aversion consumption is much lower than in the other cases. This considerable difference subsists even with high values of cash-on-hand. High risk aversion leads to low consumption even late in life. The second graph in figure 6 shows low consumption when Relative Risk Aversion is high for low values of cash-on-hand. For higher values of cash-on-hand, consumption approaches consumption levels obtained with lower values of CRRA.

The difference in behavior persists even when the income profile is increasing. Figure 7 shows the case of a worker with a college degree. In this case income is expected to grow more during the lifetime, as can be seen in figure 1. Despite this at age 25, workers with high risk aversion still save a lot more than workers with low risk aversion. At age 50, results change in a similar way as in the case of a worker with no high school degree. Consumption is markedly lower for low levels of cash-on-hand, while it tends to converge to the consumption of workers with lower risk aversion.

In order to generate expected consumption profiles, I simulate 5000 histories with independent transitory and permanent shocks to individual income. Those histories are averaged in order to obtain an aggregate profile. Figure 8 shows one such history. The continuous line represents permanent income, the dashed line indicates consumption, and the dotted line indicates the level of pension that the worker is certain about receiving at retirement. In this realization, permanent shocks to income are very bad in the early forties. The worker adjusts her consumption sharply and reaches retirement with virtually zero assets.

It should be stressed that results are partial equilibrium. The level of wages and the interest rate are given. Endogenizing these variables would require the addition of a production sector. Although this is feasible, the iterative procedure that searches for the equilibrium factor prices may take 10 or 20 times the CPU time of a partial equilibrium estimation, which is already high. Partial equilibrium analysis with comparative static can provide a reasonable approximation at a much lower computational cost.

4. Redistribution under current formula in U.S. PAYG system

This section studies the implications of the current redistributive formula in the PAYG system in the U.S. Here I analyze the extent of redistribution implied by the formula, the impact on welfare of eliminating redistribution and the impact on savings that such a reform would have. By eliminating redistribution, I mean that the benefit formula is defined in such a way that before starting life every worker has the same expected IRR from her contributions to the system. This IRR is the one that the system can afford given the demographic assumptions. This formulation is the extent to which a defined benefit system can go to eliminate redistribution. An alternative formulation, such as a flat replacement rate for all types of workers, would redistribute income from those with a shorter expected retirement, a longer active life and an income path with a higher growth rate towards those with a longer expected retirement, shorter active life and a lower income growth rate path. These attributes are observable demographic characteristics. A system that intends to have a minimal degree of redistribution, as is the case in this simulation, should use such characteristics in its design.

a. The Extent of Redistribution

There are several ways to build a picture of the extent of redistribution. Here I look at four of them. The first is the expected IRR by type of worker before starting active life. To obtain this, I simulate the income process for each type of worker and for each type, I obtain the average contribution by age and the average pension implied by the formula. With the average flow of contributions by age and the average pension at retirement, I calculate the implied IRR for each type of worker. Table 2 presents the results.

Table 2: Expected IRR by type of worker before starting active life.
Percentages

Type of Worker	Expected IRR		Expected Replacement Rate	
Education	with Redistribution	without Redistribution	with Redistribution	without Redistribution
Non-High School	2.56%	1.72%	41.4%	32.3%
High School	1.79%	1.72%	37.2%	36.6%
College	1.01%	1.72%	32.8%	40.7%

Source: Author's calculations.

Table 2 shows that workers with a lower expected income receive a higher expected return on their contributions than workers with a higher expected income. In expected terms, both NHS and HS workers benefit from redistribution at the expense of COL workers. The third and fourth columns indicate the replacement rate that each type of worker will receive in the scenario with and without redistribution, respectively. As mentioned before, the replacement rate differs across groups because it is adjusted according to observable demographic characteristics of worker groups in order to avoid redistribution among them.

The previous calculations stress the differences in the expected value of the benefits. The redistributive formula also affects the variance of expected benefits. Figure 9 shows the variance of effective pensions paid by the system with and without redistribution. These are the distributions of expected pensions that a worker faces before starting active life[13]. In all cases, benefits have lower variance under the system with redistribution.

A different view of redistribution looks at the effective IRR received by workers. The history of income realizations of a worker is summarized by the present value of labor income. Figure 10 shows that the redistributive formula allows workers with the lowest lifetime income to obtain the highest return on their contributions.

b. Valuation of Redistribution

[13] During active life the distributions change according to the particular realizations of income for a worker.

Change in welfare

A primary measure of the value of redistribution is the compensatory variation. This is defined as the amount of resources that workers should be given (taken away) in order for them to be as equally well off after the reform as before. A positive number indicates that the reform harmed them because they need to be compensated to be equally well off. The change in resources is measured as the fraction of permanent income during each period of the workers' working life. Therefore, the change upwards or downwards is measured over the entire profile of permanent income. A common alternative is to measure compensatory variations as the change in resources in the first period of active life. Given that I am assuming borrowing constraints, measuring compensatory variation as resources given in the first period of active life would bias the measure of the effects of the policy. With this measure, part of the gains would come from reducing the desire to anticipate consumption. By changing the entire income profile, the extent to which borrowing constraints reduce welfare does not change.

The procedure to measure the change in welfare is straightforward. Since the value function in the first period summarizes utility for the remaining lifetime, I can calculate the expected utility to be achieved under a certain regime. Then I take the value function in the first period obtained under the new regime. With this, I calculate the additional income that should be given to or taken away from a worker so that she will reach the same level of welfare as in the original scenario.

A final specification we must make refers to the information set that the agent is assumed to have at the moment her welfare is calculated. There are two alternative assumptions. The first is to assume that the agent does not know her final level of educational attainment, and that the chances to achieve any one level are given by the current average educational attainment of the population. The second is to measure the welfare change of the worker once their educational attainment level is known.

Table 3 shows the compensatory variation required to be indifferent to the elimination of redistribution in the DB system. The first line presents an agent that does not know what level of education she will attain, although she knows what preferences she will have. The second section of the table considers workers that already know their educational attainment and therefore, their expected income profile.

Table 3: Compensatory Variation Required for Acceptance of the Elimination of Redistribution in the DB System.
Percentage change in permanent income

Education	CRRA=0.5	CRRA=1	CRRA=5
Ex-ante	3.0%	3.8%	0.3%
Ex-post			
Non-High School	7.2%	8.4%	0.6%
High School	2.3%	2.6%	0.0%
College Degree	0.2%	0.6%	-0.1%

Source: Author's calculations.

The first noteworthy result is that for CRRA values of 0.5 and 1, all compensatory variations are positive, meaning that all types of workers positively value the income redistribution of the current formula in the U.S. DB system. The interesting fact about this is that it is true even for college graduates, who have a higher expected pension in the system without redistribution. They value redistribution because they also run the risk of having bad income realizations, in which case the system with redistribution would give them a better pension.

A second interesting result is that the valuation of redistribution is high with low coefficients of risk aversion. This is because when CRRA is very high, workers save much on their own to prepare for adverse shocks during their active life. These savings will also provide income at retirement. The incremental security that the redistributive formula gives to them is minimal when compared to the insurance they provide for themselves through savings. Therefore, as CRRA increases, there is a trade-off between two forces. A higher CRRA implies higher distaste for risk, and thus, compensatory variation should be higher. However, at the same time, self-insurance through voluntary savings increases, resulting in a reduction of the valuation of the redistributive formula. The table shows that for CRRA values between 0.5 and 1 the first effect dominates, while between 1 and 5 the second dominates. In the case of the worker with a college degree, the second effect is high enough to counter the first, and she experiences a positive welfare change when redistribution is abandoned.

Variance of Income

A second contribution of income redistribution to welfare comes from its impact in reducing the dispersion of consumption in society. The reduction in the disparity between consumption would be positively valued by a society that prefers a more egalitarian distribution of resources. Dispersion of consumption increases as individuals in a certain cohort age. Deaton and Paxson (1994) document this fact with data for Taiwan, the U.K. and the U.S., and it also arises in the model of this paper. An interesting result is that when there is redistribution in the pensions system, the variance of consumption decreases during active life. Those workers with lower income realizations can afford to save less for retirement because they can count on a higher retirement income. The opposite is true for workers with higher income realizations.

c. Effects on Savings.

Table 4 presents the change in aggregate savings offered to the economy by each type of worker when redistribution is eliminated in the DB system. The change in aggregate savings is calculated by assuming that the economy is populated by overlapping generations of each type of worker. Each generation has the same profile of expected income, but each starts with an expected income x% greater than the previous, where x is the productivity growth of cohorts. Therefore the aggregate supply of capital by each type of worker can be obtained by totaling the supply of savings of the representative agent at each age, adjusted by the size and productivity level of the cohort that it represents. Formally, let $\{S_t\}^{D}_{t=25}$ be the sequence of average savings offered by a class of workers during their lifetimes, where D is the age of death. Aggregate savings (S^T) in one period will be:

$$S^T = \sum_{t=25}^{D} \frac{S_t}{(1+n+x)^{t-1}} \qquad (14)$$

According to the arguments given in section 3.3, I use (n+x) = 1.72%. The results are virtually unchanged when any number between 1% and 2% is used. The stock of savings is calculated under both scenarios (DB and DC) and its change is reported in Table 4.

Table 4: Change in aggregate savings by type of worker and total population when redistribution is eliminated in a DB system
Percentages

Education	CRRA=0.5	CRRA=1	CRRA=5
Non-High School	15.3%	14.2%	7.5%
High School	4.5%	4.3%	3.4%
College Degree	-2.6%	-2.1%	-1.0%
Total Population	4.2%	4.1%	2.7%

Source: Author's calculations.

Results show that NHS workers are those who increase savings the most when redistribution is eliminated. These are the most affected given that their expected pension is lower and their income process has the highest variance. On the other extreme, COL workers reduce their savings when redistribution is eliminated. In this case, the negative effect on savings implied by a higher expected pension is larger than the higher savings implied by the higher variance in pensions.

When the CRRA is higher, the increase in aggregate savings is lower for NHS and HS workers, while the savings of workers with a college degree increase marginally. The explanation for this lies in the fact that aggregate savings in the initial scenario are higher with a higher CRRA, given that workers have a stronger precautionary motive. Since there is substitution, the existing buffer stock can be used to cover for the increased uncertainty.

Finally, a change in aggregate savings is calculated by measuring the total aggregate savings offered in the initial and final scenarios. These are determined as the weighted averages of aggregate savings of each type of worker. Results indicate that the change in aggregate savings can be considerable: between 2.7% and 4.2% depending on the CRRA that prevails in the economy.

5. Change in the benefit formula: From DB to DC

This section considers the effects of changing the pension formula from a DB to a DC scheme. The change is done in such a way that the expected pension before starting active life is the same in both systems for each type of worker.

a. Information Effects

The main difference between a DB system and a DC system can be summarized by Figure 11. In section 2 we saw that most pensions formulas –in particular the DB and DC formulas as defined here- can be represented as a weighted average of the realizations of labor income during each period of life. A DB system without redistribution implies that the weights of realizations of income at different years are the same. In a DC system, realizations early in life receive a higher weight than those later in life. This has two implications.

First, information about the pension level at retirement is disclosed earlier under the DC system. This reduces the level of uncertainty faced during lifetime, reducing precautionary savings. The second effect comes from the fact that variance of income increases with age. In the model, this comes from the fact that the shock to the log of income follows an AR1 process with a coefficient equal to one. This means that log income is a random walk (with a drift), and therefore the variance increases in each period. Beyond the model, however, several empirical studies provide evidence in support of this representation of the income process (HSZ, 1994; MaCurdy, 1982). Also, Deaton and Paxson (1994) provide evidence using household data for Taiwan, the U.K. and the U.S. that the variance of income increases with age.

Thus, a DC system appropriately weights the labor income realizations to reduce the uncertainty about pensions. The consequences of this can be seen in Figure 12, which shows the variance of pensions under both systems for each type of agent. In all cases, the mean is the same while the variance of pensions under the DC system is lower; that is, distribution of pensions under a DC system second order stochastically dominates the distribution under DB system.

Additional insight into this situation is given by the relationship between IRRs and the present value of permanent income. Figure 13 shows that the IRR is slightly lower for those with a low present value of income, while the opposite is true when the present value of income is high. Therefore, *ex-post* income redistribution in the DB system without *ex-ante* redistribution results in the opposite direction than desired.

b. Welfare

Table 5 shows the compensatory variations required for this reform. The welfare impact of changing the benefit formula is positive in all cases, i.e. the required compensatory variation is negative. This means that workers will be willing to pay in order to have the reform done. Note that in this case the expected pension at the start of life is the same under both scenarios; therefore, the welfare change is the result only of a change in variance of the benefits, and not a change in the mean benefit. Welfare loss is higher for NHS workers. This is due to the fact that the variance in income shocks faced is higher.

Therefore, they more highly value the DC formula, which weights more heavily the income realizations in the early part of life, when income variance is lower.

Table 5: Compensatory variation required by workers to accept a change from a DB formula to a DC formula with the same level of expected benefits. Percentage change in permanent income

Education	CRRA=0.5	CRRA=1	CRRA=5
Ex-ante	-1.0%	-1.5%	-0.4%
Ex-post			
Non-High School	-3.5%	-4.3%	-0.4%
High School	-0.3%	-0.4%	-0.3%
College Degree	-0.2%	-0.4%	-1.0%

Source: Author's calculations.

For higher values of CRRA, a result similar to the previous section arises. Higher values of CRRA make workers put a higher value on mechanisms that reduce uncertainty, as is the case here in the shift towards a DC formula. But at the same time, workers save more on their own to cover for other risks, which they can use to cover for pension uncertainty too. So the reduction in uncertainty that they obtain from the DC formula is less valued.

c. Aggregate Savings.

The change in aggregate savings by type of worker and by total population is calculated as described in the previous section and presented in table 6. The fact that the DC system has a lower variance of savings and reveals information earlier in life allows workers to reduce savings. The reduction in savings is higher for those who have the higher gains from the shift, i.e. the NHS workers. A result analogous to that in the previous section is obtained; the change in savings is lower for higher values of the coefficient of relative risk aversion.

Looking at total population, the results indicate that a change from a DB to a DC formula reduces total savings between 0.9% and 1.4%.

Table 6: Change in aggregate savings by type of worker and total population when pensions system changes from a DB formula to a DC one with the same expected level of benefits.
Percentages

Education	CRRA=0.5	CRRA=1	CRRA=5
Non-High School	-4.2%	-4.0%	-0.9%
High School	-0.8%	-0.9%	-0.9%
College Degree	-0.6%	-0.8%	-0.9%
Total Population	-1.4%	-1.4%	-0.9%

Source: Author's calculations.

6. Conclusions

The combined effects of the two reforms are presented herein. Table 7 indicates the welfare impact of going from a DB system with redistributive formula, as in the U.S., to a DC system without redistribution. The results indicate that the combined effect of the reform is harmful to workers when values of CRRA are 0.5 and 1. Within this range, the higher the risk aversion, the larger the welfare loss from the combined reform. When CRRA=5, welfare impact of the reform becomes less relevant, because workers provide insurance to themselves through voluntary savings.

Table 7: Compensatory variation required by workers to accept a change from a DB system with redistribution to a DC system without redistribution.
Percentage change in permanent income.

Education	CRRA=0.5	CRRA=1	CRRA=5
Ex-ante	2.0%	2.3%	-0.1%
Ex-post			
Non-High School	3.5%	3.7%	0.2%
High School	2.0%	2.2%	-0.3%
College Degree	-0.0%	0.2%	-1.2%

Source: Author's calculations.

Results in Table 7 confirm the previous results from tables 3 and 5, that for r = 0.5 and 1 the welfare loss from the losing the redistribution in the DB system is larger than the gain from changing the formula from DB to DC.

Table 8 shows the total effect of the combined reform on aggregate savings. NHS and HS workers increase their supply of savings, while COL workers reduce it. Overall, aggregate savings in the population increases by between 1.8% and 2.8%.

Table 8: Change in aggregate savings by type of worker and total population when pension system is changed from a DB with redistribution to a DC without redistribution.
Percentages

Education	CRRA=0.5	CRRA=1	CRRA=5
Non-High School	10.4%	9.7%	6.6%
High School	3.6%	3.4%	2.5%
College Degree	-3.2%	-2.8%	-1.9%
Total Population	2.8%	2.7%	1.8%

Source: Author's calculations.

Results in this section indicate that the impact on aggregate savings is not very sensitive to the assumption about r. The situation is different for the welfare evaluation. When r is changed from 1 to 5, welfare evaluation changes radically. This paper has shown that when r takes a value of 5 consumption behavior changes dramatically (see figures 6 and 7) for reasonable values of uncertainty. Clearly, r = 5 should be taken as an upper bound, because the amounts of savings it generates early in life are not consistent with what is observed in the data. Thus, a better estimate of the welfare effects of the reform are given by r =1.

The main findings of this chapter can be summarized as follows:

- Redistribution in the current DB system in the U.S. is valued positively by all types of workers for a relevant range of preference parameters, even by those who have an expected welfare loss from the redistributive formula. This is because they positively value the insurance provided by the redistributive formula.

- DB and DC systems have important differences in terms of the uncertainty about retirement benefits that they have associated. Given the empirical fact that the variance of income increases during lifetime, a DC-type formula is preferred for pension benefits as it reduces the variance of the benefits received.

- Pension reform can affect aggregate savings even when additional burdens are not imposed upon transition generations. This increased savings comes at the cost of increased uncertainty over lifetime income and therefore, reduced welfare.

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Appendix

This appendix shows that it is possible to change from a system based on PAYG financing to a system with individual accounts without imposing any cost on public finances and allowing workers to earn benefits at least as good as the ones they had under the PAYG system. This implies that such a reform is feasible without harming any generation. The down side is that the move towards individual accounts will not allow workers to earn better returns on their contributions than what they obtained in the previous system, even if the funds accumulated in the individual accounts earn market interest rates. This is true under the constraint that transition costs are not imposed on any particular generation.

Consider the following reform. At year *t*, contributions to Social Security start to be directed towards individual accounts. Funds accumulated in these accounts are invested in financial instruments. Let us assume first that they are invested in bonds specially issued by the government. Current retirees will continue receiving their pensions. Workers that contributed to the old system are guaranteed that they will receive a retirement benefit that is at least equal to what they would have obtained under the old system. This means that the government will supply the difference between their old pension and what they will obtain under the new system. If the pension in the new system is higher than the previous, then the government does nothing. Workers that did not contribute to the old system are not covered by the guarantee.

Before starting, I will introduce some useful notation:

$$C(N,m) = \sum_{i=1}^{N} (1+m)^{i-1} = 1 + (1+m) + (1+m)^2 + ... + (1+m)^{N-1}$$

$$A(T,m) = \frac{(1+m)^T}{C(T,m)}$$

The annuity factor $A(T,m)$ gives us the size of a constant payment (annuity) that we can get from a stock S at time t, from year $t+1$ to year $t+T$, when the net interest rate at which the funds can be invested is m. The size of the annuity will be $S \times A(T,m)$.

The Model

Consider an overlapping generations model where workers have *R* years of active life (ages 1 to *R*). After *R* they live in retirement until they die at age *D* with certainty[14]. Retirees receive a real annuity from ages *R+1* to *D*. Population grows at rate *n*. Assume for simplicity that there is no productivity growth. The production function shows constant returns to scale, so in the steady state total output grows at rate *n*.

The level of pensions in a balanced system

We assume that the initial system is balanced, that is, income equals expenditures in every period. Given a contribution rate *s* and a wage profile $\{w_{age}\}^{R}_{age=1}$, total Social Security contributions and expenditures in year *t* are given by:

$$s\sum_{age=1}^{R}\frac{w_{age,t}}{(1+n)^{age-1}} = \sum_{age=R+1}^{D}\frac{p}{(1+n)^{age-1}} \qquad (1)$$

where *p* is the level of pension, which is the same for every cohort[15]. It is interesting to note that the formula is the same as the present value of contributions by a worker with an *n*% discount rate. Solving for *p*:

$$p = A(D-R,n)\,(1+n)^{R-1}\,s\sum_{age=1}^{R}\frac{w_{age,t}}{(1+n)^{age-1}}$$

Assuming constant wages throughout the life cycle, the previous expression can be written as:

$$p = swA(D-R,n)\,(1+n)^{R-1}\,\frac{C(R,n)}{(1+n)^{R-1}} = sw\,C(R,n)\,A(D-R,n) \qquad (2)$$

Note that this does not depend on the formula that determines the worker's pension as a function of her wage history. This formula can take any form, but the value of the pension given must be equal to (2) for the system to be in equilibrium.

[14] This can be interpreted as the life expectancy of the cohort. Note that by assuming that retirement benefits are annuities and focusing on the cohort as the unit of analysis, the assumption of uncertainty in life expectancy becomes irrelevant.

The Reform

Consider the case of R=3 and D=5. The flow budget constraint for the government is:

$$B_t - B_{t-1} = \Delta B_t = G_t + r^b B_{t-1} \tag{3}$$

where B_t is the stock of public debt, G_t is the expenditure in pensions of the old system and r^b is the interest rate that the bonds related to the reform will pay. Our constraint is that other expenditures, taxes and public debt are not affected by the reform, so we can omit them from the analysis.

For the private sector (workers), the flow budget constraint is given by:

$$A_t - A_{t-1} = \Delta A_t = C_t - X_t + r^b A_{t-1} \tag{4}$$

where A_t is the assets held by the private sector on their individual accounts, C_t is the flow of contributions in period t and X_t is the reduction in assets for retirement purposes (i.e., pensions in the new system).

My claim is that the reform is feasible without raising taxes. Therefore, the deficit in the public sector (change in debt) has to be equal to the accumulation of assets in individual accounts plus other possible accumulation of assets in the private sector. Formally,

$$\Delta B_t = \Delta A_t + \boldsymbol{b}_t \tag{5}$$

where $\boldsymbol{b}_t$ is the demand for public debt that is necessary to cover cash needs of the government. Substituting (3) and (4) into (5),

$$G_t + X_t + r^b \left(B_{t-1} - A_{t-1}\right) = C_t + \boldsymbol{b}_t \tag{6}$$

We also need to impose a transversality condition to avoid Ponzi-game type financing. This requires the growth rate of B_t (call it b) to converge to n, the growth rate of output.

[15] This is consistent with our assumption of no productivity growth. Productivity growth can be considered in the model without changing the conclusions.

$$\lim_{t \to \infty} [b_t - n] = 0 \qquad (7)$$

Below we revise the conditions under which (6) and (7) hold true. We normalize the size of the cohort that is in its last year of active life at the time of the reform to 1. In the year of the reform (year 1), two cohorts are due payments of pensions, and no withdrawal of funds from individual accounts occurs yet (X_t=0). Flows in the period are:

$$G_1 = p \left(\frac{1}{(1+n)} + \frac{1}{(1+n)^2} \right) = sw\, C(3,n)\ A(2,n) \left(\frac{1}{(1+n)} + \frac{1}{(1+n)^2} \right) = sw\, C(3,n) \quad (8)$$

$$C_1 = sw \left((1+n)^2 + (1+n) + 1 \right) = sw\, C(3,n) \qquad (9)$$

The equality among the flows is not surprising, given that the system was in balance before the reform and the reform does not alter the size of the contributions nor the level of benefits in that year.

Consider now the situation of the system when it has matured, in the sense that no living cohort has ever contributed to the old system. This occurs in year 5 according to the parameters chosen in this exercise. I assume that in the new system workers receive an annuity from the funds accumulated in their individual accounts. G_t = 0, and the other flows are:

$$X_5 = sw\, C(3, r^b)\ A(2, r^b) \left((1+n)^2 + (1+n)^3 \right) \qquad (8')$$

$$C_5 = sw\, C(3,n)\, (1+n)^4 \qquad (9')$$

Assume first that during the transition, $b = 0$ in all periods. Therefore, $B_t = A_t$ for $t=1,\ldots,4$. In this case it is easy to verify that when $r^b = n$, $X_5 = C_5$ and therefore, $b_5 = 0$. We argue that this is the only solution of (6) that satisfies (7). If $r^b > n$, payment of bonds will exceed workers' contributions ((8') > (9')) and therefore, $b > 0$. Issuing bonds in excess of what mandatory contributions can buy, however, will make public debt grow without limit.

This can be confirmed by noting that in year 5 and beyond, total outstanding public debt is equal to:

$$B_t = sw \left[C(3, r^b) + C(2, r^b)(1+n) + (1+n)^2 \right] (1+n)^{t-1} + \ldots$$

$$+ sw\, C(3, r^b)\left(\frac{1+r^b}{2+r^b}\right)(1+n)^{t-2} + a_t \tag{10}$$

where,

$$a_t = a_{t-1}(1+r^b) + b_t$$

The first term of the sum represents the aggregate balances in the individual accounts of active workers. In other words, this is the amount of outstanding bonds issued in exchange for active workers' regular contributions. The second term is the remaining balance in the cohort that is on its first period of retirement; the other cohort has zero remaining assets at the end of the period. Note that these two elements grow at a rate *n* in a steady state. The term a is the accumulated value of $b > 0$ in previous periods.

Equation (10) confirms that $r^b = n$ has to hold for an equilibrium to exist. If not, b would be positive and debt would be growing at a rate higher than *n*. The same argument implies that a has to be zero; that is, during the transition $b = 0$ is true in all periods. Let's see what this implies for the evolution of r^b during the transition.

Year 2 of the transition, the older cohort of retirees receives their pension from the government, while the younger receives a transfer and sells part of their bonds. The transfer is equal to the difference between the pension they received in the old system and what they obtain under the new system. Contributions to the new system earn a return r^b. Total expenses of the government are pension payments and the repayment of bonds, and equal:

$$G_2 + X_2 = p\frac{1}{(1+n)} + \left[p - s\,A(2, r^b)\right] + s\,A(2, r^b) =$$

$$= \begin{cases} swC(3,n)\,A(2,n)\left(\frac{(2+n)}{(1+n)}\right) = swC(3,n)(1+n) & \text{if } \left[p - s\,A(2, r^b)\right] > 0 \\ p\frac{1}{(1+n)} + s\,A(2, r^b) > swC(3,n)(1+n) & \text{otherwise} \end{cases} \tag{8''}$$

The term in square brackets is the necessary supplement to the pensions paid by the new system in order to reach the level of pensions prior to the reform.

Contributions to the system are given by:

$$C_2 = sw\left((1+n)^3 + (1+n)^2 + (1+n)\right) = sw\, C(3,n)\,(1+n) \qquad (9'')$$

which equals expenditures in the period only if the term in square brackets is positive.

Equations (8’’) and (9’’) show that (6) will not hold only if the term in brackets is negative, which occurs only at a very high level of interest rate r^b [16]. This means that, for the reform that we analyze here, it seems possible for the government to offer in the first year a return on its bonds higher than *n*. This would be sustainable in this year because the higher payments that this would imply would be matched by reductions in the payment of pensions of the old system. Public debt would grow faster in this period.

The problem with this strategy would arise in the future when cohorts that have been promised rates of return higher than *n* start to retire. Equations (8’) and (9’) show that the only way that income and expenditures match is when $r^b = n$. Therefore, this argument shows that during the transition, it is also necessary that $r^b = n$ holds.

Generalization

The argument can be generalized to a case in which *R* and *D* take any value, provided that $D > R$. The equations for the flows of a mature system are:

$$X = sw\, C(R,r^b)\; A(D-R,r^b)\; \sum_{i=1}^{D-R} (1+n)^{i-1} \qquad (8''')$$

$$C = sw\, C(R,n)\; (1+n)^{D-R} \qquad (9''')$$

The evolution of B_t in the new steady state is given by,

$$B_t = sw\left[\sum_{j=1}^{R} C(j,r^b)\,(1+n)^{R-j}\right](1+n)^t + \ldots$$

$$+\, sw\, C(R,r^b)\left[\sum_{k=1}^{D-R}\left((1+r^b)^k - A(D-R,r^b)\sum_{i}^{k}(1+r^b)^{k-i}\right)\right](1+n)^t$$

which grows at rate *n* when the system is mature.

[16] Above 100% in the context of this model.

Interpreting the results

Equations equivalent to those above would hold under a slightly different scenario. Suppose that the government issues bonds at market interest rate and assume that this rate is above *n*. Equations (8') and (9') show that inflows to the system can not sustain a return to contributions of more than *n*. Therefore a tax should be imposed on some agent in the economy such that these higher payments can be met. The natural agent to tax is precisely that which is gaining from the reform, i.e. the workers that have access to a higher rate of return. In brief, we can think of returns to bonds being taxed in order to finance the higher return that these bonds obtain. In practice, bonds will be earning an after tax rate of return equal to *n*, and the situation of public finances will be exactly as described in the previous equations.

Although obvious, the example above is useful to illustrate another step. Instead of government bonds paying market interest rates, individual contributions can be used to buy private securities. Government will issue bonds to the general public at market interest rates. The way to make this sustainable in steady state is through a tax on the returns of the pension system. This tax will, again, bring returns down to rate *n*. Note that this type of reform would allow workers to change the portfolio composition of their pension wealth, moving towards a more desired point on the risk-return frontier. The impact of this change goes beyond the scope of this paper.

Uncertainty

The analysis in this appendix is not affected when considering idiosyncratic uncertainty, as in the model presented in this paper. The reason is that at the cohort level individual shocks average out, and the flows pertaining to a cohort remain unaltered. This is true even in the case of a redistributive formula in the PAYG. The only requirement is that aggregate flows of contributions and benefits remain unaltered when uncertainty is considered.

These flows will be altered in the presence of aggregate shocks to income. In this context the design of the PAYG system would differ, allowing for a stock to buffer aggregate fluctuations. The rules governing the system and the transition towards a system with individual accounts would differ from the one presented here. The particulars of that do not seem to prevent us from applying the same logic as in the case here to obtain the same equivalence result.

Figure 1: Labor income profiles by Educational Attainment
U.S. Dollars of 1990. (Hubbard, Skinner and Zeldes, 1994)

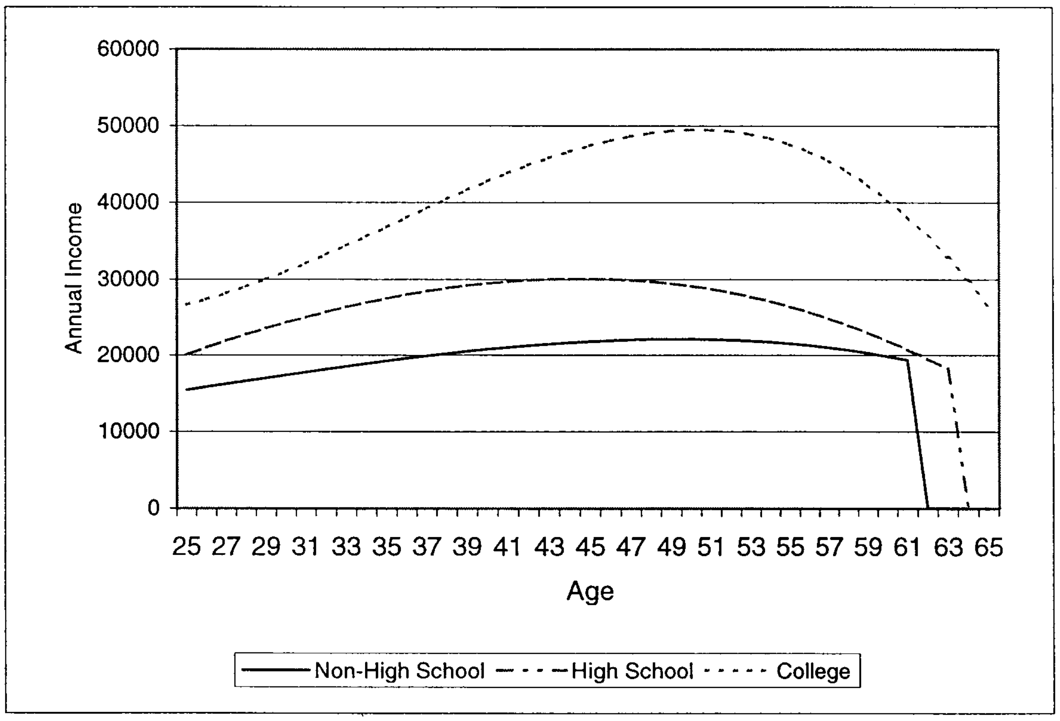


Figure 2: Benefit Formula. U.S. Social Security System
U.S. Dollars of 1990 (Social Security Administration, 1997)

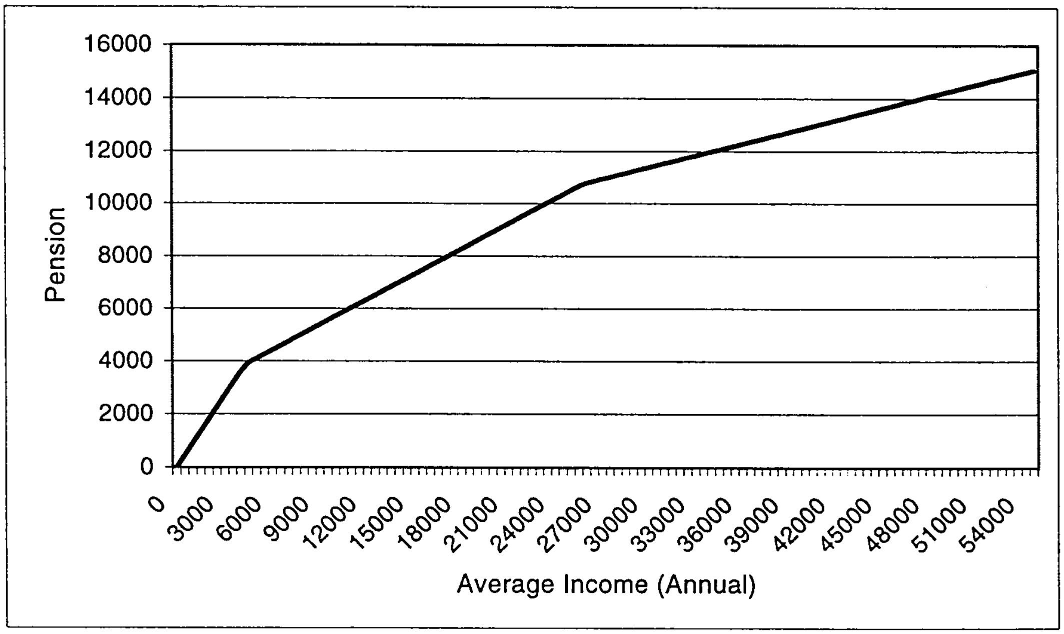


Figure 3: Consumption Functions by Age

Different lines indicate different levels of pensions at retirement. Graphs are derived for a worker with a high school degree. Permanent income is fixed. Preferences parameters are δ=3%, ρ=0.5. Interest rate is 3%.

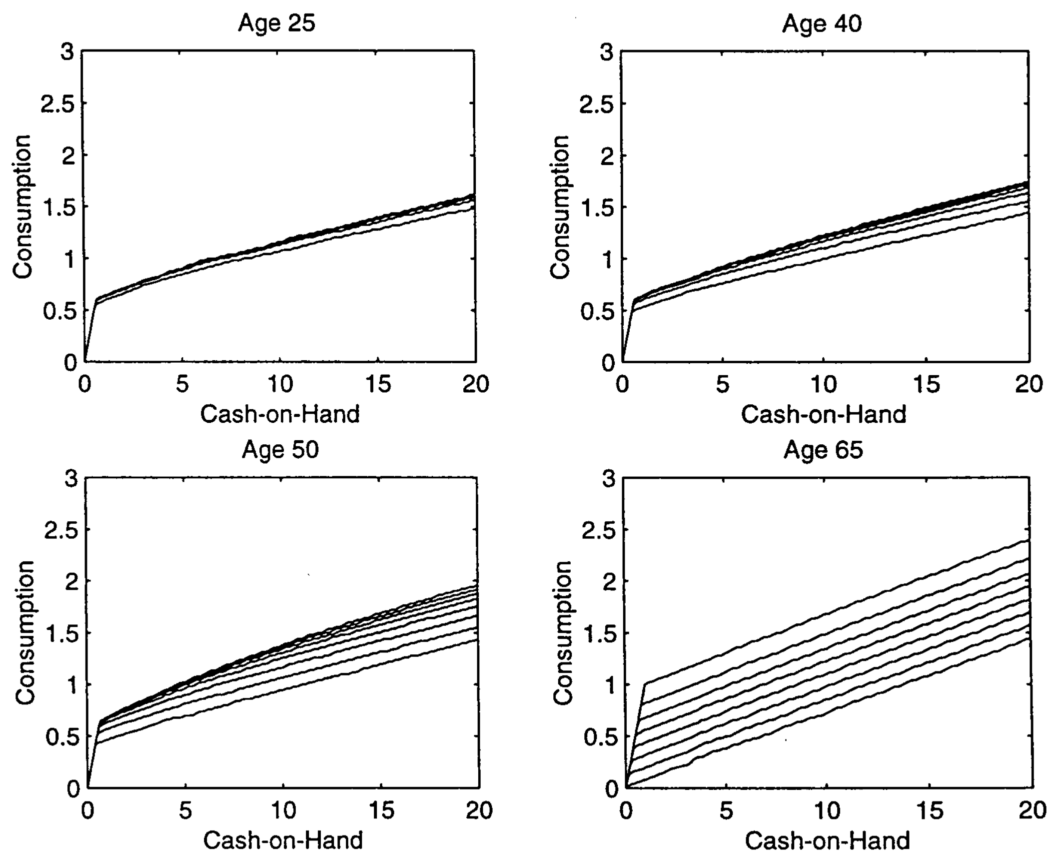


Figure 4: Value Functions by Age

Different lines indicate different levels of pensions at retirement. Graphs are derived for a worker with high school degree. Permanent income is fixed. Preferences parameters are δ=3%, ρ=0.5. Interest rate is 3%.

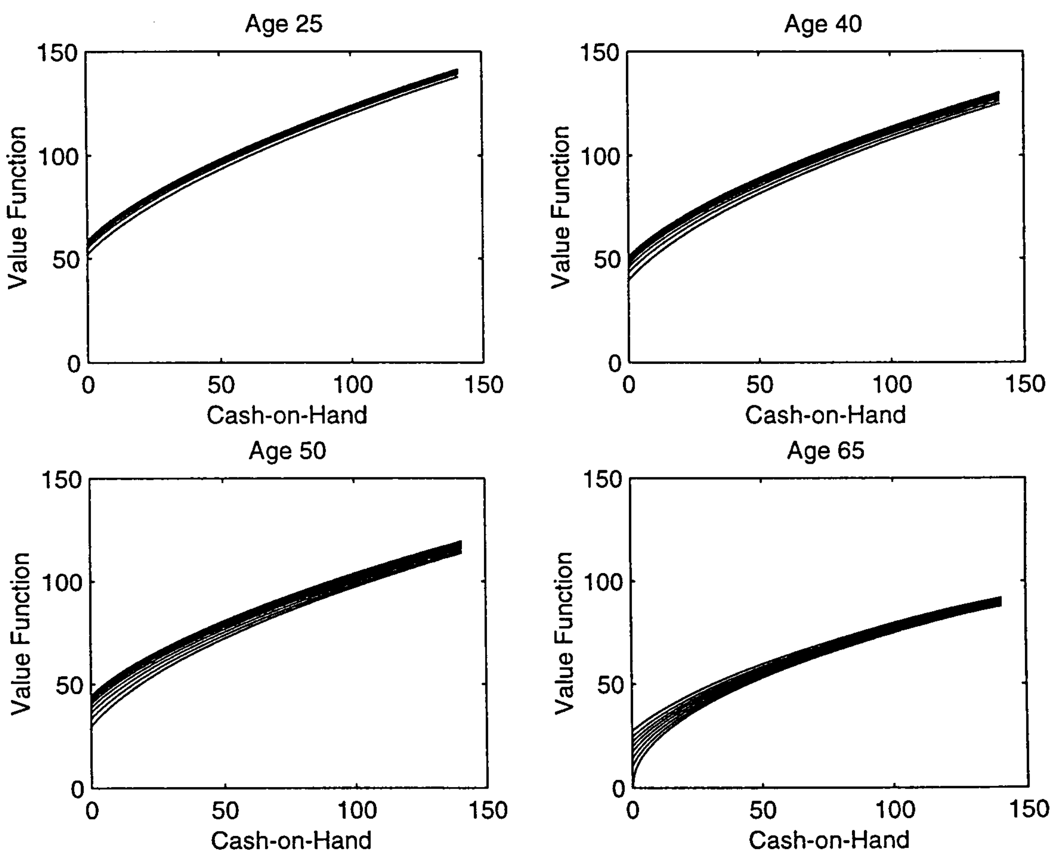


Figure 5: Consumption Function

Different lines indicate different levels of pensions at retirement. Graphs are derived for a worker with high school degree with 60 years of age. Permanent income is fixed. Preferences parameters are δ=3%, ρ=0.5. Interest rate is 3%.

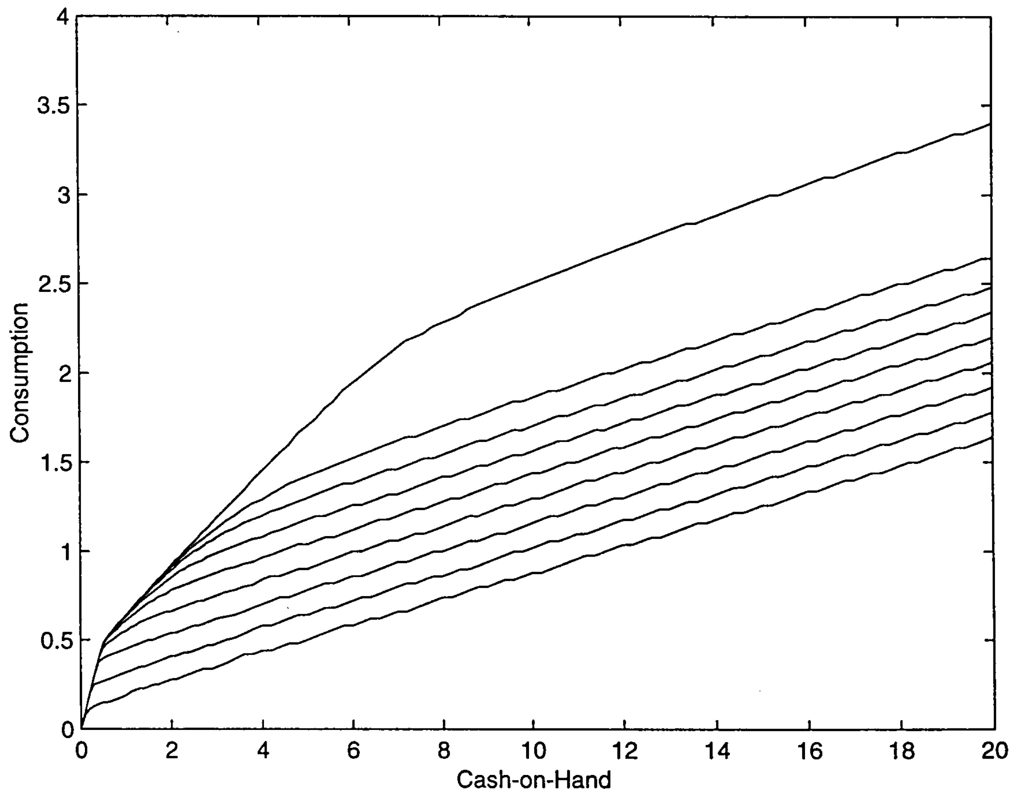


Figure 6: Consumption Function, varying CRRA

Worker with Non-High School degree. δ=3%, Interest rate is 3%. CRRA (ρ)= 0.5 (solid line), 1 (dashed line) and 5 (dotted line).

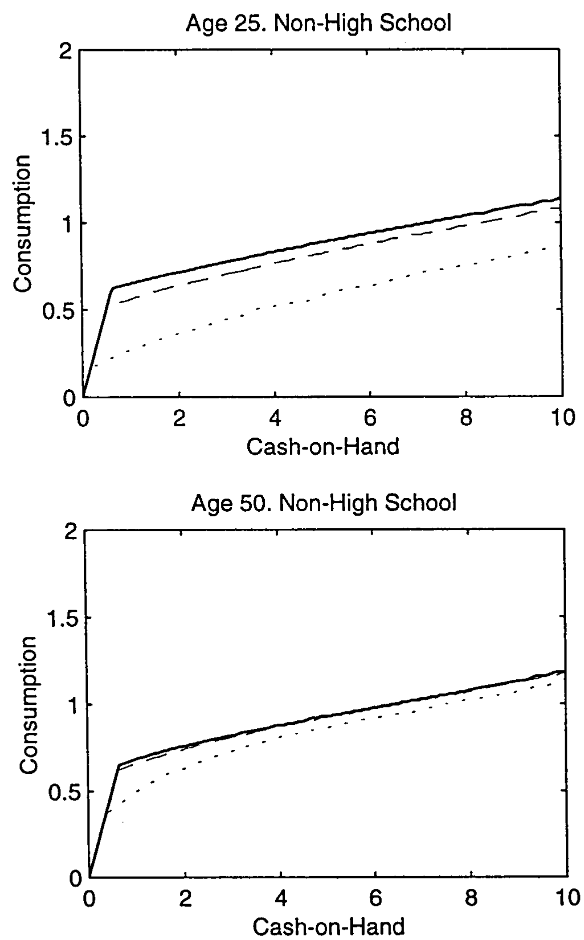


Figure 7: Consumption Function, varying CRRA
Worker with College degree. δ=3%, Interest rate is 3%. CRRA (ρ)= 0.5 (solid line), 1 (dashed line) and 5 (dotted line).

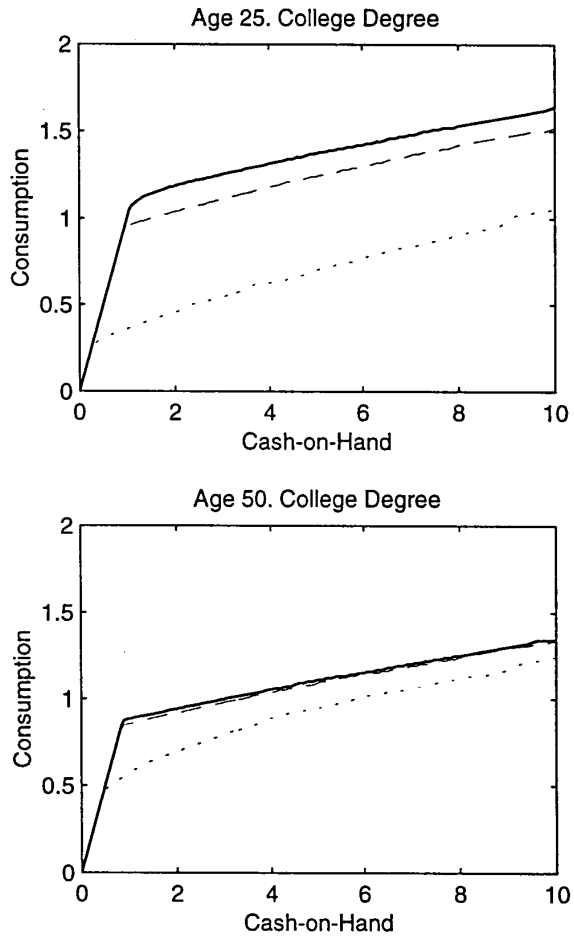


Figure 8: A Life-cycle history: Income Realizations and Consumption
Permanent Income (solid line), Consumption (dashed line) and Retirement Benefit (dotted line).

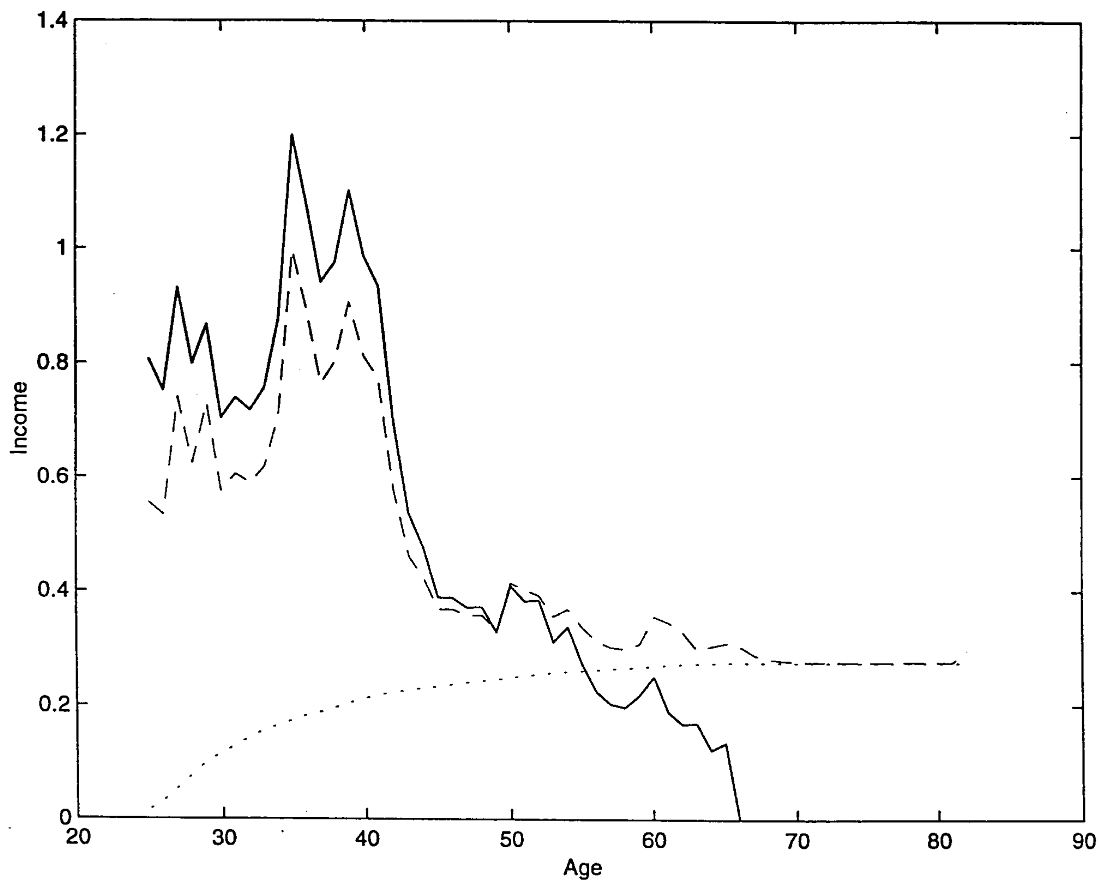


Figure 9: Distribution of Retirement Benefits under a DB system with redistribution (dots) and without redistribution (crosses)

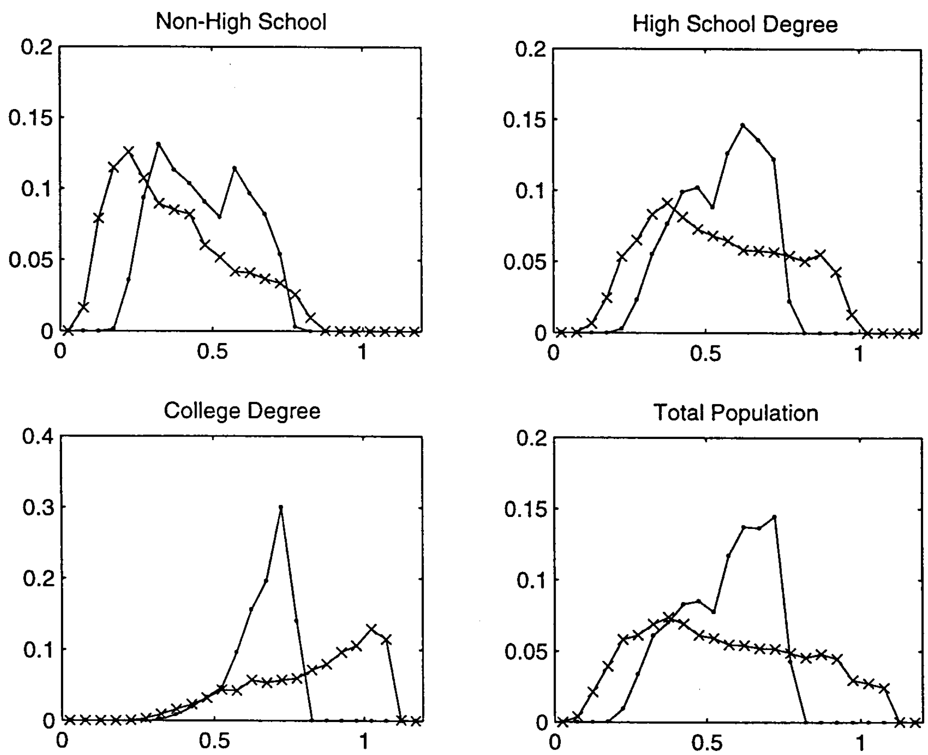


Figure 10: Internal Rate of Return versus Present Value of Permanent Income by type of worker. DB system with redistribution

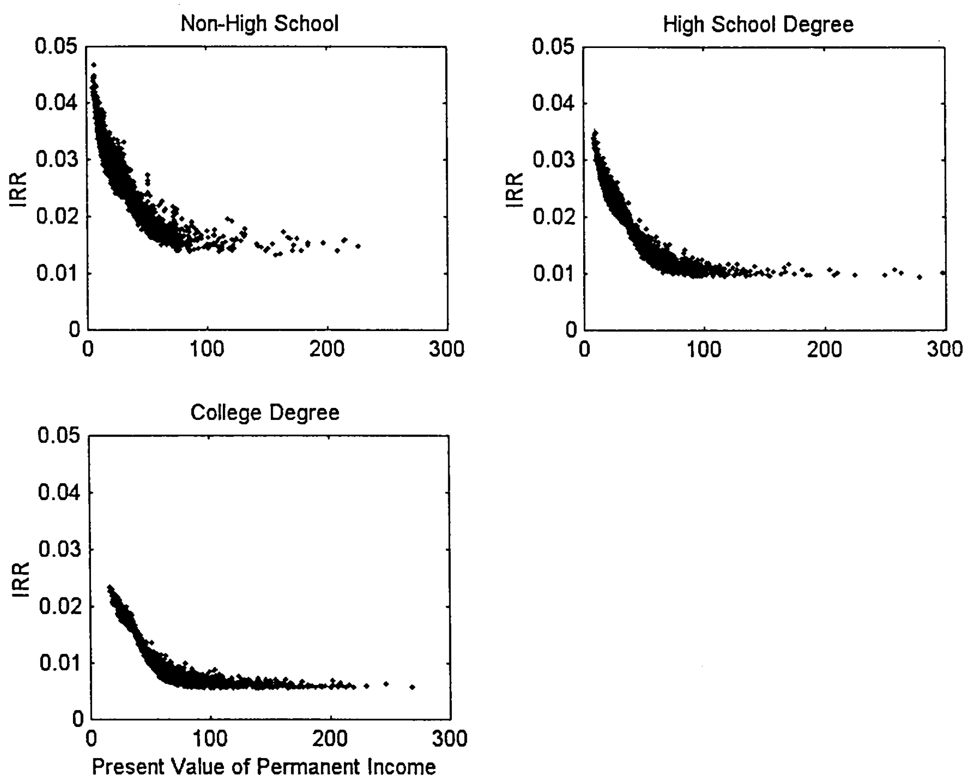


Figure 11: Fraction of the wage that goes into pension in a DB system without redistribution (dots) and a DC system without redistribution (crosses)

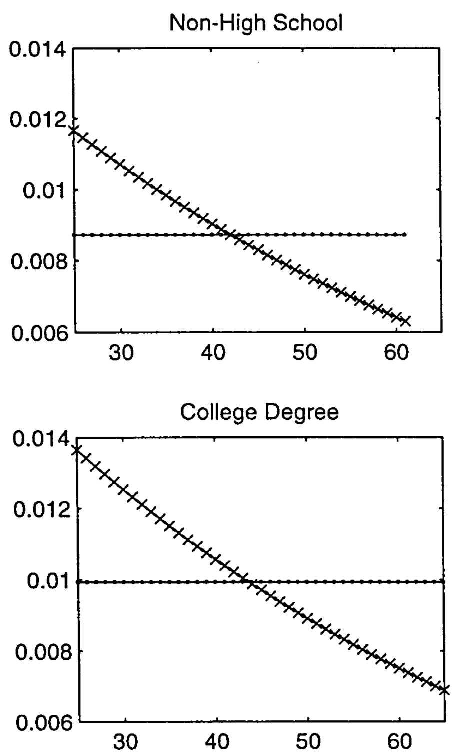


Figure 12: Distribution of Retirement Benefits under a DB system without redistribution (dots) and DC without redistribution (crosses)

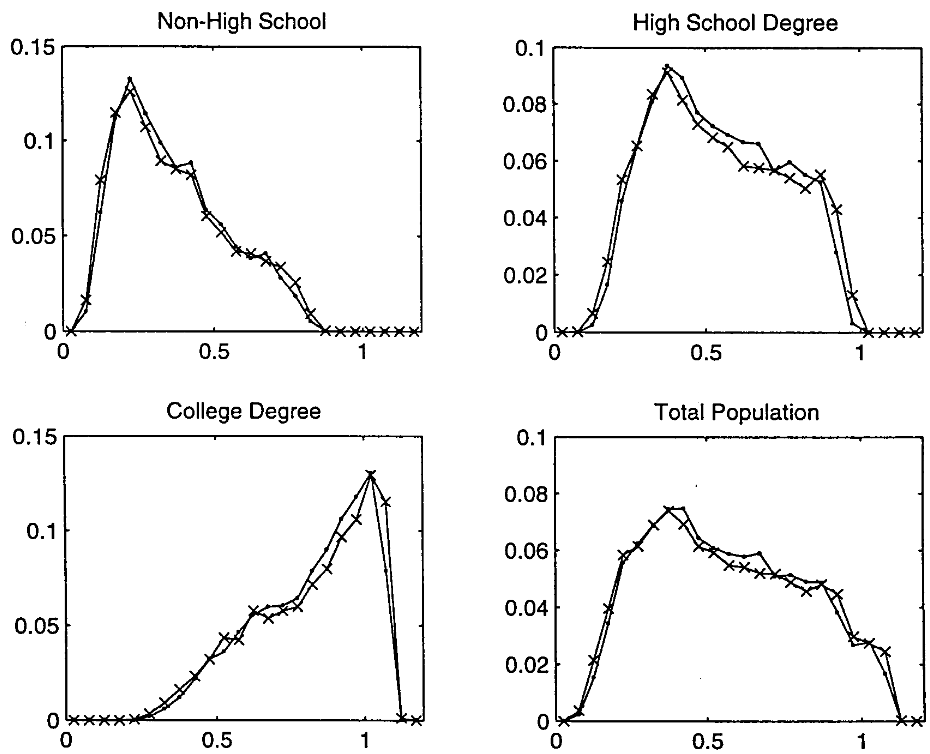
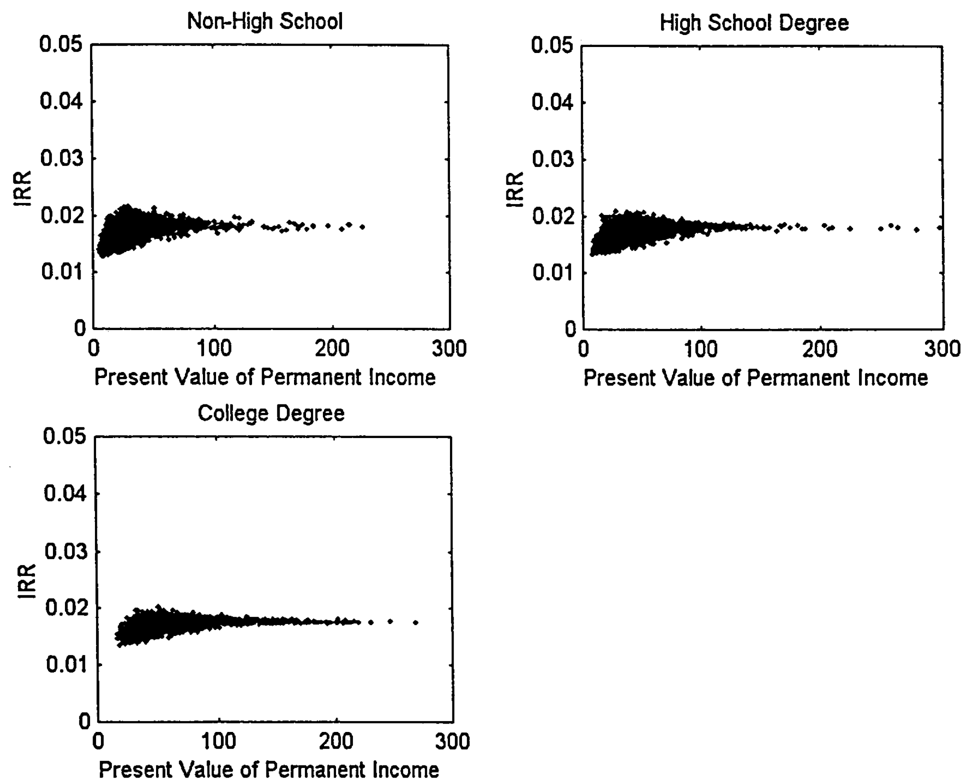


Figure 13: Internal Rate of Return versus Present Value of Permanent Income by type of worker. DB system without redistribution



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