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Benjamín García

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Agustinas 1180, Santiago, Chile  
Teléfono: (56-2) 3882475; Fax: (56-2) 3882231

# **ZERO LOWER BOUND RISK AND LONG-TERM INFLATION IN A TIME VARYING ECONOMY\***

Benjamín García  
Banco Central de Chile

## **Abstract**

Using a Time Varying Parameters Vector Auto Regression framework, I construct an index, the Zero Probability Index (ZPI), based on the probability of the nominal interest rate hitting the zero lower bound (ZLB) within 10 quarters. I show how the probability of reaching the ZLB evolves over time and measure how a rise in the inflation target can reduce this probability. High ZPI episodes tend to occur during recessions and are c by a combination of the initial state of the variables and the estimated volatility of the shocks. However, not all episodes of a high ZPI share the same causes. In the US recessions of the 1980s, the probability was influenced significantly by an exceptionally volatile environment that overcame the dampening influence of the period's high nominal interest rates. On the other hand, the high ZPI for the 2001 and 2007 recessions were mainly defined by an initial state of low interest rates. Because of this difference, an increase in the inflation target was much more effective in reducing the estimated probability of the interest rate reaching the ZLB in the latter episodes.

## **Resumen**

Usando una metodología de vectores autoregresivos con parámetros variables, se construye un índice, el índice de probabilidad cero (ZPI), basado en la probabilidad de que las tasas nominales alcancen el límite de cero por ciento (ZLB) dentro de 10 trimestres. Se muestra cómo la probabilidad de alcanzar el ZLB evoluciona a través del tiempo y se cuantifica cómo un aumento en la meta de inflación puede reducir esta probabilidad. Episodios de alto ZPI tienden a ocurrir durante recesiones y están determinados por una combinación de estado inicial de las variables y probabilidad estimada de los shocks. Sin embargo, no todos los episodios de alto ZPI comparten las mismas causas. En las recesiones de EEUU de los 1980s, la probabilidad se vio fuertemente influenciada por un ambiente especialmente volátil que contrapesó la influencia de las altas tasas nominales del período. Por otro lado, los altos ZPI de las recesiones de 2001 y 2007 estuvieron determinados principalmente por un estado inicial de bajas tasas de interés. Debido a esta diferencia, un aumento en la meta de inflación hubiese sido mucho más efectivo en reducir la probabilidad estimada de alcanzar el ZLB en los episodios posteriores.

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# 1 Introduction

The so called *great moderation* – the period of low volatility in business cycle fluctuations that started in the mid-1980s – appeared to have ended after the last financial crisis. The empirical evidence from Keating and Valcarcel (2011, 2012) show an increase in the volatility of economic activity. Larger shocks require ever larger interest rates responses, and the zero rate interest bound is now an issue of practical importance. Blanchard, Dell’Ariccia, and Mauro (2010) argued that raising the inflation target is one way to give central banks more room for lowering interest rates without having to rely on alternative policies. Thus, the central bank will be able to respond to bigger shocks without reaching their policy limit.

In this paper I estimate a time-varying parameters vector auto regression (TVP-VAR), similar to Galí and Gambetti (2009), Primiceri (2005), and others. By modeling a time-varying economy where the variance of the shocks and the response of the variables to those shocks may change over time, I can compute the probability that interest rates will hit the zero lower bound (ZLB) in each period. I can also assess the sensitivity of this probability to changes in the inflation target, and how this sensitivity evolves over time.

Section 2 discusses the importance of the ZLB for the nominal interest rate and how the inflation target can influence the probability of reaching the ZLB. In Section 3, I describe the conceptual framework for the TVP VAR estimation and present the empirical results. The paper concludes in Section 4.

## 2 Volatility, inflation target, and monetary policy

### 2.1 The end of the great moderation and the zero bound in nominal rates

Clark (2009) shows an increase in the volatility of shocks after the recession that started in 2007. Changes in volatility also affect the likelihood that nominal interest rates reach the ZLB. If the economy is entering a phase of increased volatility, it will also imply a period where the zero bound will constrain monetary policy more frequently, reducing the effectiveness of monetary policy to manage economic fluctuations.

However, there is no conclusive evidence for how ineffective monetary policy may become at the zero bound. Chung, Laforge, Reifschneider, and Williams (2012) suggest that while the Federal Reserve's quantitative easing policy improved macroeconomic conditions, it did not prevent the ZLB from having first-order adverse consequences. In contrast, Eggertsson and Woodford (2003) argue that while the zero bound restricts possibilities for stabilization it only does so modestly. But one fact is clear: interest rates, when available, are the preferred instrument of monetary policy. Heterodox policies rarely occur when the interest rate is far from the zero bound. Nonetheless, while potentially effective, alternative policies can be costly. Bernanke (2012) emphasizes that non-traditional policies involve costs beyond those generally associated with more standard policies. Hamilton and Wu (2012) estimate that at the ZLB, buying \$400 billion in long-term maturities is required to reduce the 10-year rate by 13 basis points.

Blinder (2000) advises, *"don't go there, prevention is far better than the cure"*. Bernanke, Reinhart, and Sack (2004) find that despite some evidence that nonstandard policies might be effective, policy makers should remain cautious while their effects remain quantitatively uncertain. After 4 years of QE policies, Bernanke (2012) concludes that the *"estimates of the effects of nontraditional policies on economic activity and inflation are uncertain"*.

Jung, Teranishi, and Watanabe (2005) emphasize the critical impact of CB credibility during ZLB episodes. Adam and Billi (2006) agree that a lack of credibility could severely affect monetary policy effectiveness adversely. This is because the current inability to change interest rates means that it is crucially important to be able to credibly affect future expectations. It would logically follow then, that a CB that is not completely sure of its credibility or its credibility in the event of a crisis, would find that a policy that avoids the zero bound is a good policy.

## **2.2 Costs and benefits of inflation**

There is consensus that policymakers should avoid high inflation because it is bad for the economy. Walsh (2003) provides a good summary of the costs of inflation in a New Keynesian environment, emphasizing the welfare loss from the deviation from the optimal consumption basket. In the presence of sticky prices, during which firms do not adjust prices simultaneously, inflation results in

an inefficient dispersion of relative prices and induces consumers to consume more of the cheaper goods and less of the most expensive ones. The gains from consuming more of the cheaper goods are less than the loss from consuming less of the more expensive goods due to the diminishing marginal utility. We can estimate the welfare costs under this framework under a zero inflation policy.

Why do we not observe zero inflation targets? The reasons are various but generally correlated. One explanation is the theory of inflation, "greasing the wheels of the labor market" as in Tobin (1972) or Akerlof, Dickens, Perry, Gordon, and Mankiw (1996) where the downward nominal rigidity of wages would make some inflation desirable in order to reduce real wages to fall in case of adverse shocks. However, most argue for reducing the risk of deflation and liquidity traps that, as noted by Svensson (2003) and many others, can have severe negative consequences: as the real value of debt increases, commercial banks' balance sheets deteriorate and unemployment rises, magnified by the downward nominal rigidity that can further deteriorate aggregate demand. Hence, inflation below a certain level increases the risk that a shock will cause deflation. Moreover if we consider Bernanke, Laubach, Mishkin, and Posen (2001) argument that because consumers tend to replace goods that become more expensive with less expensive options, the measured inflation likely contains an upward bias.

Higher inflation provides another benefit in terms of avoiding the zero bound limit. Low levels of inflation induce low levels of nominal interest rates, leaving the Central Bank with little room to lower interest rates in the event of a recession. Blanchard, Dell'Ariccia, and Mauro (2010) argue that higher inflation before a crisis, and thus higher interest rates to begin with, would have allowed the Fed to cut interest rates more and thus probably reduce the drop in output and deteriorating fiscal positions. Blinder (2000) suggest setting a  $p^*$  sufficiently high to make the probability of encountering  $r = 0$  extremely small. Billi (2011) and Williams (2009), among others, show in simulations how a higher inflation target can reduce the probability of reaching the zero bound.

In this paper, I estimate a *time varying* sensitivity of the likelihood of reaching the ZLB to a rise of the inflation target. This is done by allowing for both the volatility and the rest of the structure of the economy to evolve over time.

### 3 Empirical Approach

I estimate a TVP VAR to quantitatively assess the impact of an increase in the inflation target on the probability of hitting the ZLB. This allows for a time-varying structure of the economy, and therefore, the time-varying risk that the interest rate will reach the ZLB.

I then use the estimation results to compute the variance in the shocks and the impulse response functions for the economic variables in each period. Additionally, multiple trajectories are then simulated to compute the probability of reaching the ZLB in each period within a certain horizon. I estimate counter-factual scenarios to calculate the effects of a higher inflation target on the probability of reaching the ZLB and the required inflationary increase for each period to keep this probability below an arbitrary threshold.

#### 3.1 Estimation and Results

This section presents the framework that allows me to compute a time-varying probability of reaching the ZLB. I construct a *ZLB probability index* (ZPI), which – for every period  $t$  – is defined as the likelihood of reaching the ZLB within the next 10 quarters.

Following the methodology from Galí and Gambetti (2009), and similar to Primiceri (2005), Cogley and Sargent (2005), and Cogley and Sbordone (2008), I estimate a Bayesian VAR with  $n$  variables and  $p$  lags using:

$$x_t = A_{0,t} + A_{1,t}x_{t-1} + \dots + A_{p,t}x_{t-p} + u_t \quad (1)$$

Where  $x_t$  is a vector of endogenous variables,  $A_{0,t}$  is a vector of time-varying coefficients, and  $A_{i,t}, i = 1, \dots, p$  are matrices of time-varying coefficients. The residuals  $u_t$  are normally distributed with mean zero and var-cov matrix  $\Sigma_t$ . Let  $A_t = [A_{0,t}, A_{1,t}, \dots, A_{p,t}]$  and  $\theta_t = \text{vec}(A_t')$  represent a vector that stacks all elements of  $A_t$ . Assume that the parameters from  $\theta_t$  evolve as random walks subject to reflecting barriers that impose stability, ruling out explosive behaviors for the variables. The residuals  $u_t$  are normally distributed with mean zero and a variance-covariance matrix of  $\Sigma_t$  that can also change over time<sup>1</sup>.

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<sup>1</sup>The appendix, section A, presents a full description of the estimation procedure.

As in Primiceri (2005), the VAR has 2 lags and 3 endogenous variables that intend to replicate a small reduced form new Keynesian model: inflation, unemployment rate, and a short-term nominal interest rate. The sample size covers 1953 Q3 to 2008 Q3<sup>2</sup>. The training period uses the first 40 quarters to initialize the priors. The model is estimated using data from the US starting from 1964 Q3.

Inflation is measured by the annual growth of the CPI. Unemployment is measured as the civilian unemployment of all workers over 16. The nominal interest rate is the effective federal funds rate<sup>3</sup>.

Figure 1 summarizes the data.

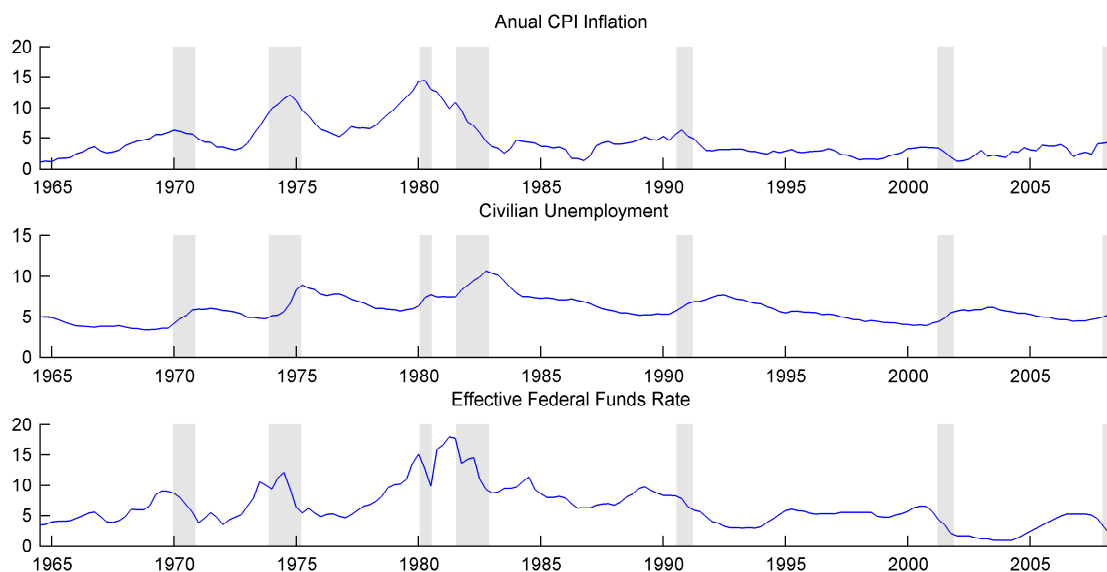


Figure 1: Sample period inflation, unemployment, and interest rate

Williams (2014) identifies two key factors that affect the simulated probability of hitting the ZLB: the size and the duration of the economic shocks. In the context of this VAR estimation, the former will be represented by the estimated standard deviation of the residuals. A measure of how long a shock can influence a variable is obtained by summing up the lag coefficients for each equation. Figure 2 provide the estimation for both measures.

The estimated persistence of the processes show no significant changes over the sample period. For the estimated volatility of the shocks, the sample can be separated into three distinct periods:

<sup>2</sup>The end of the sample is restricted to exclude periods when the ZLB was binding

<sup>3</sup>All data is taken from the FRED Database of the Saint Louis Federal Reserve.



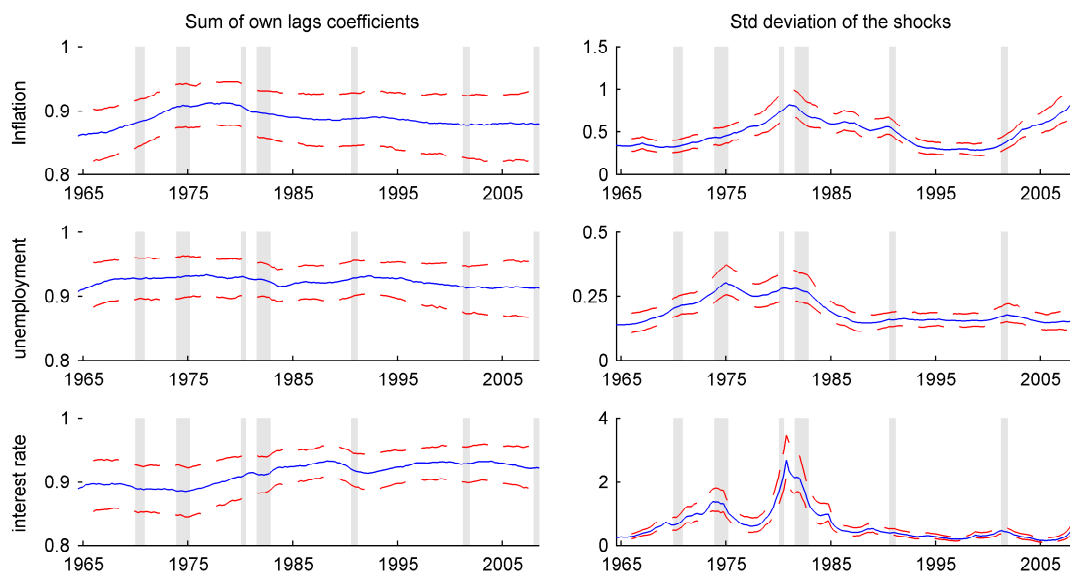


Figure 2: Estimated persistence of processes and the volatility of shocks.

One of rising volatility starting in the first half of the 1970s until the mid-1980s, followed by a period of markedly low volatility interrupted after the 2001 recession with a moderate increase in volatility that increases strongly again with the financial crisis. The price equation shows the biggest volatility during the financial crisis. The unemployment volatility shows comparable peaks during the recessions of 1974, 1980, 1982, and 2008. The interest rate equation dynamics are dominated by the volatility peak during the 1980 recession.

It is worth noting that the increase in the interest rate volatility during the 1980 recession is not an isolated incident. During most recessions, the estimated volatility of the non-systematic part of monetary policy – the shocks to the interest rate equation– tends to increase. This is in line with Calani, Cowan, and García (2011) in the sense that the linearity assumptions that permit equivalence between simple policy rules and more complex optimal rules break down in the presence of large shocks, and therefore the VAR identifies the changes in interest rates as a non-systematic response to other variables.

Regarding the systematic part of monetary policy – i.e the response of the interest rate to the variables observed in the model – a measure of the strength of the response to a unitary shock is constructed by summing over the corresponding impulse-response functions. Figures 3 and 4 show

both the monetary policy response to shocks in inflation and unemployment for different periods and the evolution of the measure of aggregate response over time.

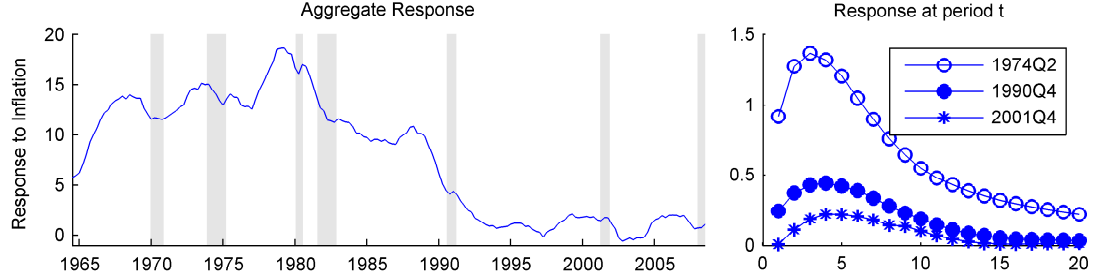


Figure 3: Estimated response of the federal funds rate to a unitary shock in inflation

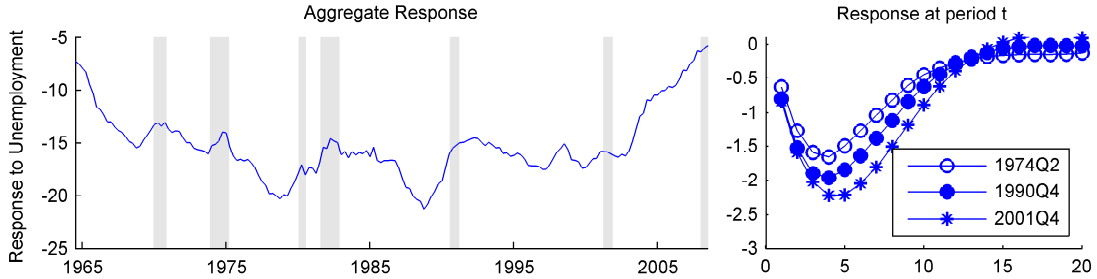


Figure 4: Estimated response of the federal funds rate to a unitary shock in unemployment

This calculations allow me to compute the ZPI index for every period in the sample. The index computed based on the forecasted trajectory of the interest rate starting at period  $t^4$ . Let equation (1) be expressed in companion form:  $x_t = \mu_t + A_t x_{t-1} + u_t$ , where  $x_t \equiv [x'_t, x'_{t-1}, \dots, x'_{t-p+1}]'$ ,  $u_t \equiv [u'_t, 0, \dots, 0]'$ ,  $\mu_t \equiv [A'_{0,t}, 0, \dots, 0]'$ , and  $A_t$  is the corresponding companion matrix. Since  $\mu_t$  and  $A_t$  evolve as random walks,  $E_t(\mu_{t+j}) = \mu_t$  and  $E_t(A_{t+j}) = A_t$ , and the forecast for  $j$  periods ahead can be recursively computed as  $E_t(x_{t+j}) = \mu_t + A_t E_t(x_{t+j-1})$ . Note that this forecast will not necessarily converge monotonically towards the trend value as in Cogley and Sbordone (2008) definition as the level at which the variable is expected to settle after the short-run fluctuations die out,  $\bar{x}_t = \lim_{j \rightarrow \infty} E_t(x_{t+j})$ . For example, Figure 5 shows how by the end of the 2001 recession, the interest rate, while already below the trend, is expected to keep dropping for the next 3 quarters due to the combination of a relatively high unemployment rate (5.7%) and low inflation (1.2%).

<sup>4</sup>Given the random walk nature of the time-varying estimates, at each period  $t$ , I expect all parameters to remain constant for the foreseeable future. Therefore, for every period, the simulated trajectories assume a constant parameterization of the economic structure.

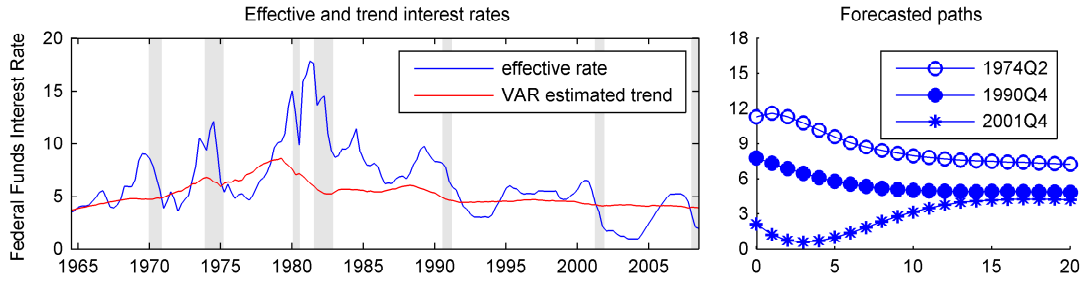


Figure 5: Estimated federal funds rate trend and convergence paths

In addition to the expected path for the interest rate, I simulate 25,000 alternative trajectories for each period based on the estimated impulse response functions and a series of random shocks drawn from the period's VAR-COV matrix.

Figures 6 and 7 show some interesting results. First, high ZPI events tend to occur during NBER defined recessions. Special cases are the 1969-1970 recession, where the peak is reached afterwards, and the 1990-1991 recession, the only incident without a relevant spike in the index.

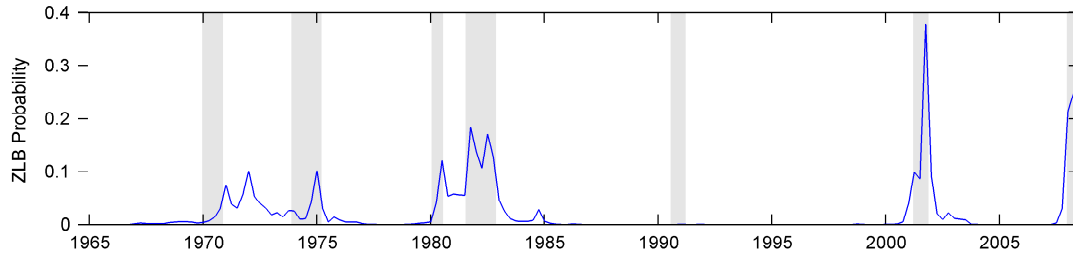


Figure 6: Estimated probabilities of reaching the ZLB within 10 quarters: ZPI index

The three main high risk episodes, during which the ZPI passed the 10% mark, occurred during the recessions of the early 1980s, 2001, and after the financial crisis. As Figure 7 shows, the 1980s episode has a comparable starting point and expected convergence path compared to 1970 Q2, while the 2001 and 2008 episodes have similar dispersion on the simulated paths. However, all three episodes have considerably higher estimated probabilities. I hypothesize that the high estimated ZPI in the 1980s episode is mainly due to high economic uncertainty, while in 2001 and 2008 it is due to low initial state.

I test the validity of this hypothesis by simulating two counterfactuals. To isolate the impact of varying starting points, I simulate the ZPI with the assumption that at each period, the initial state

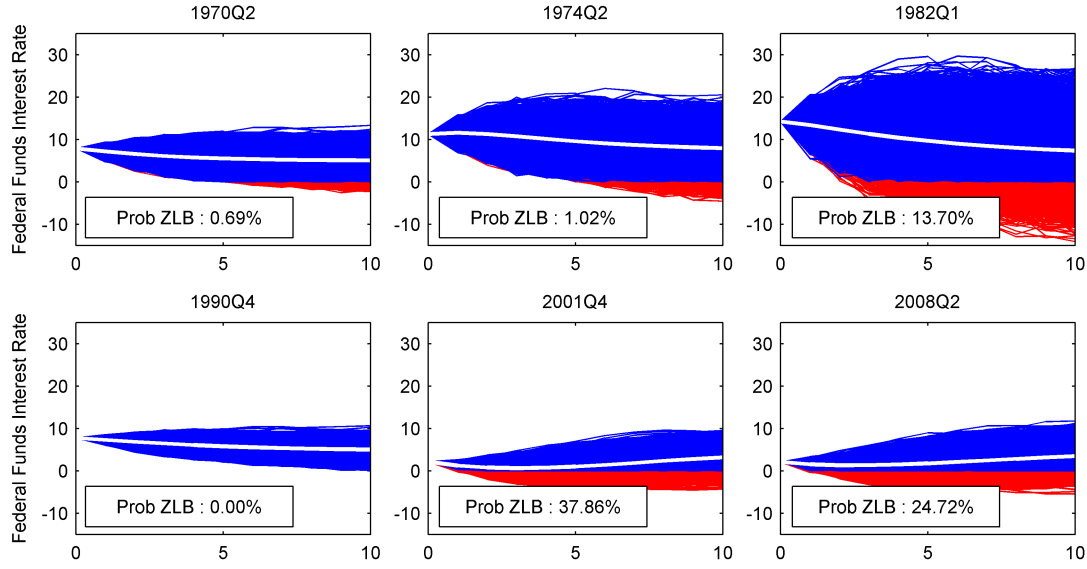


Figure 7: Simulated interest rate trajectories during NBER-defined recessions. Red trajectories cross the ZLB at some point; the white trajectory has no shocks

is the long-term trend. To deal with the implications of changing volatility, I compute the ZPI while maintaining a constant level of uncertainty equal to the average from 2001.

The results in Figures 8 and 9 seem to validate the previous hypothesis. Choosing a starting point equal to the trend values significantly reduces the estimated ZPI for the 2001 and 2007-2008 episodes, while it simultaneously increases the ZPI for the 1980s recessions, where the effective interest rates were above the estimated trends. When computing the ZPI under the assumption of a constant variance equal to the 2001 average, the ZPI is completely wiped out for the 1980s recessions. Interestingly, this counterfactual also increases the ZPI for the period following the 2001 recession, a period characterized by very low interest rates.

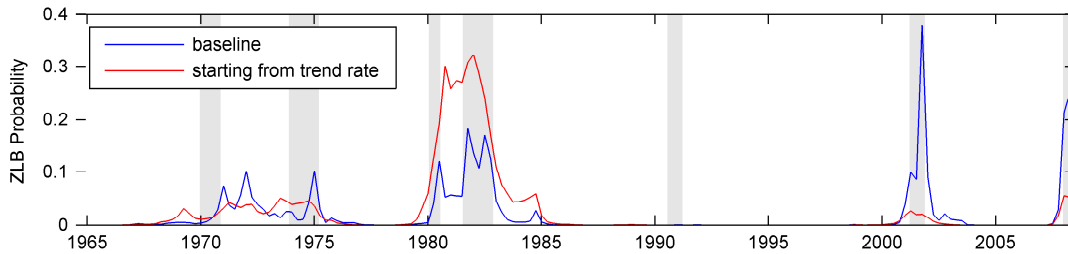


Figure 8: Effect of the initial state on the ZPI estimate

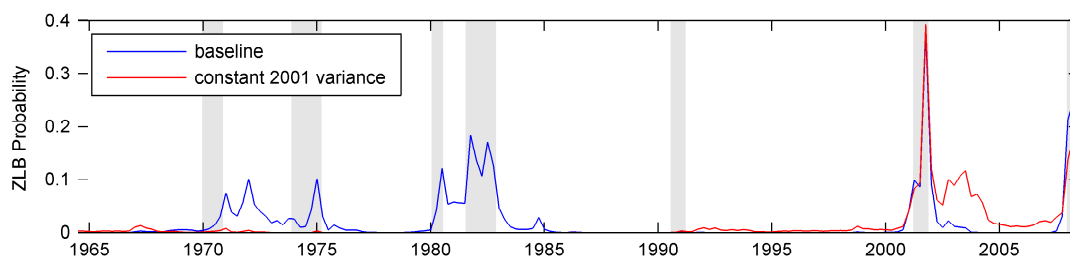


Figure 9: Effect of volatility on the ZPI estimate

### 3.2 Changes in the inflation target counter-factual

Blanchard, Dell’Ariccia, and Mauro (2010), among others, argue that a rise in the inflation target could help reduce the likelihood of reaching the ZLB. Assuming the super-neutrality of money – that is, a change in the trend inflation level should not affect the real variables – I can assess the effects of a change in the inflation target on the probability of hitting the ZLB in a straightforward manner. This is because the change would simply imply a shift in the nominal interest rate of the same magnitude.

Figure 10 presents the counter-factual following the suggestion from Blanchard, Dell’Ariccia, and Mauro (2010) by using an inflation target of 200 basis points above the baseline.

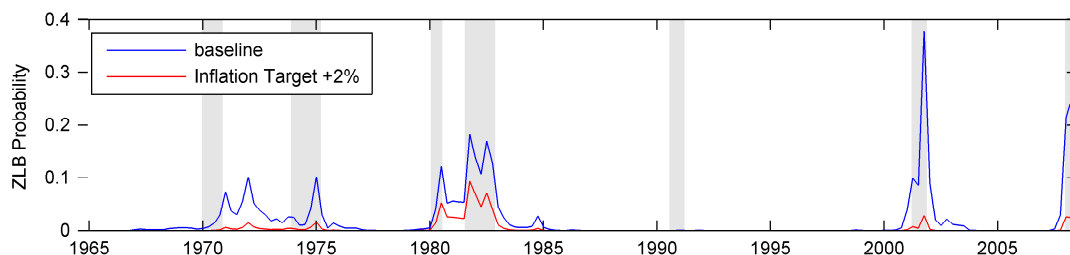


Figure 10: Effect of an increase in the inflation target on the ZPI estimate

I find a considerable decrease in the estimated probability of reaching the ZLB in all relevant episodes. When the inflation target increases by 2%, the estimated ZPI almost vanishes for all episodes but the 1980s recessions. The ZPI in the 1980s is still considerably reduced, dropping to approximately half the estimate in the base scenario. This is consistent with the results in the previous section of the higher relative impact of volatility on the estimated ZPI for the 1980s episodes. On the other hand, the higher impact of a higher inflation target in the latter episodes is consistent with the

diagnosis of a ZPI greatly affected by a low initial interest rate.

I also look at an alternative approach to assess the effect of an increased inflation trend on the estimated ZPI. I specifically assess the required inflation increase for each period to maintain a ZPI below some threshold level. Figure 11 shows that the increase in inflation required to keep the ZPI below 5% and 1% for the entire sample period is 270 and 650 basis points, respectively.

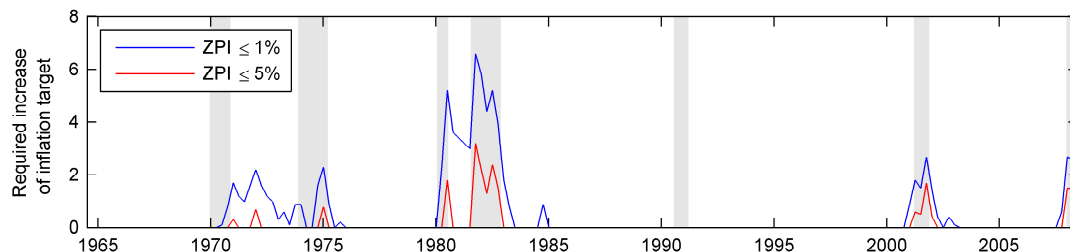


Figure 11: Inflation target increase required to attain a lower ZPI

Even if the estimated ZPI is lower for the 1980s recessions than the 21st century ones, reducing the probability requires a higher increase in the inflation level. This due to the higher estimated volatility in the period compared to other recessive periods.

## 4 Conclusions

Using a TVP VAR framework, I construct an index, the Zero Probability Index (ZPI), based on the probability that the nominal interest rate will reach the ZLB within 10 quarters.

I show empirically how the probability of reaching the ZLB evolves over time, and how an increase in the inflation target can help reduce this probability. In particular, raising the inflation target by 200 basis points, as suggested by Blanchard et al (2010), could significantly reduce this probability, in many cases by more than an order of magnitude.

I also find that high ZPI episodes tend to occur during recessions and are determined by a combination of the initial state of the variables and the estimated volatility of the shocks.

However, each high ZPI episode does not have the same causes. In the 1980s recessions, the high ZPI is a consequence of an exceptionally volatile environment that eventually overcame the dampening

influence of the period's high nominal interest rates. On the other hand, the high ZPI estimates for the 2001 and 2007 recessions are mainly determined by an initial state of low interest rates. Thus, an increase in the inflation target is a much more effective means to reduce the estimated probability that the interest rate will reach the ZLB in later episodes.

# Appendix

## A Time varying parameter VAR estimation procedure

Following Galí and Gambetti (2009), and similar to Primiceri (2005), Cogley and Sargent (2005), and Cogley and Sbordone (2008), I estimate a Bayesian VAR with  $n$  variables and  $p$  lags with the following specification:

$$x_t = A_{0,t} + A_{1,t}x_{t-1} + \dots + A_{p,t}x_{t-p} + u_t \quad (2)$$

Where  $x_t$  is a vector of endogenous variables,  $A_{0,t}$  is a vector of time-varying coefficients, and  $A_{i,t}, i = 1, \dots, p$  are matrices of time-varying coefficients. The residuals  $u_t$  are normally distributed with mean zero and a variance-covariance matrix  $\Sigma_t$ . Let  $A_t = [A_{0,t}, A_{1,t}, \dots, A_{p,t}]$  and  $\theta_t = \text{vec}(A_t')$  be a vector that stacks all elements of  $A_t$ . Assume that the parameters from  $\theta_t$  evolve as random walks subject to reflecting barriers that impose stability, and thus ruling out explosive behaviors for the variables. Then, apart from the reflecting barrier,  $\theta_t$  evolves as

$$\theta_t = \theta_{t-1} + \omega_t \quad (3)$$

Where  $\omega_t \sim N(0, \Omega)$ . The variance-covariance  $\Sigma_t$  also changes over time. Let  $\Sigma_t = F_t D_t F_t'$ , where  $F_t^{-1}$  is the lower triangular matrix

$$F_t^{-1} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \gamma_{2,1,t} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \gamma_{n,1,t} & \dots & \gamma_{n,n,t} & 1 \end{bmatrix} \quad (4)$$



and  $D_t$  is the diagonal matrix

$$D_t = \begin{bmatrix} \sigma_{1,t} & 0 & \dots & 0 \\ 0 & \sigma_{2,t} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sigma_{n,t} \end{bmatrix} \quad (5)$$

The evolution of  $\Sigma_t$  is determined by the evolution of  $\gamma_t$  and  $\sigma_t$ , where the first is a vector of the non-zero and non-one elements of  $F_t^{-1}$ , and  $\sigma_t$  is the vector of the diagonal elements of  $D_t$ .

$$\gamma_t = \gamma_{t-1} + \zeta_t \quad (6)$$

$$\ln(\sigma_t) = \ln(\sigma_{t-1}) + \xi_t \quad (7)$$

Where  $\zeta_t \sim N(0, \Psi)$  and  $\xi_t \sim N(0, \Xi)$ . Let  $\theta^T, \gamma^T$ , and  $\sigma^T$  be the sequence of the corresponding variables to time  $T$ . The conditional prior density is assumed to be

$$p(\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega) \propto I(\theta^T) f(\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega) \quad (8)$$

where

$$I(\theta^T) = \prod_{t=0}^T I(\theta_t) \quad (9)$$

$$f(\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega) = f(\theta_0) f(\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega) \quad (10)$$

Additionally,  $f(\theta^T | \gamma^T, \sigma^T, \Psi, \Xi, \Omega)$  are consistent with (3). The index function  $I(\theta_t)$  equals one if the absolute value of every root from the associated VAR polynomial is greater than one, and zero otherwise. It ensures that the estimated system will not have explosive behavior by setting the likelihood of those parameters equal to zero. Let  $\widehat{z}_{OLS}$  be the estimated parameter  $z$  from a time invariant VAR using a training sample with  $T_0$  observations. As in Benati and Mumtaz (2007) and Benati and Mumtaz (2007), the prior densities and parameters take the form of

$$p(\theta_0) \propto I(\theta_0) N\left(\hat{\theta}_{OLS}, \sigma_{\hat{\theta}_{OLS}}^2\right) \quad (11)$$

$$p(\log \sigma_0) = N(\log \hat{\sigma}_{OLS}, 10 \times I) \quad (12)$$

$$p(\gamma_0) = N(\hat{\gamma}_{OLS}, |\hat{\gamma}_{OLS}|) \quad (13)$$

$$p(\Omega) = IW\left(\frac{1}{0.005 \times \sigma_{\hat{\theta}_{OLS}}^2}, T_0\right) \quad (14)$$

$$p(\Psi) = IW\left(\frac{1}{0.001 \times |\hat{\gamma}_{OLS}|}, 2\right) \quad (15)$$

$$p(\Xi_{i,i}) = IG\left(\frac{0.0001}{2}, \frac{1}{2}\right) \quad (16)$$

The realizations from the posterior density are drawn using a Markov chain Monte Carlo (MCMC) algorithm – the Gibbs sampler – which works iteratively. Each iteration has four steps, within which the realizations of a subset of the parameters are drawn conditional on a particular realization of the remaining coefficients. Each subsequent step draws another subset of parameters conditional on the draws from the previous step. Under regularity conditions, the iteration of these four steps produces draws from the joint density.

For each iteration  $i$  of the Gibbs sampler, the first step draws realizations for  $\theta_i^T$  conditional on  $x^T, \gamma_{i-1}^T, \sigma_{i-1}^T, \Psi_{i-1}, \Xi_{i-1}$  and  $\Omega_{i-1}$  using the Carter and Kohn (1994) algorithm. In the second step, using the same procedure described in Primiceri (2005), the draws of  $\gamma^T$  are obtained conditional on  $x^T, \theta_i^T, \sigma_{i-1}^T, \Psi_{i-1}, \Xi_{i-1}$ , and  $\Omega_{i-1}$ . The third step draws from  $\sigma_i^T$  conditional on  $x^T, \theta_i^T, \gamma_i^T, \Psi_{i-1}, \Xi_{i-1}$ , and  $\Omega_{i-1}$  using Jacquier, Polson, and Rossi (2002) algorithm. Finally, as in Gelman, Carlin, Stern, and Rubin (1995), the draws in the fourth step from  $\Psi_i, \Xi_i$ , and  $\Omega_i$  are conditional on  $x^T, \theta_i^T, \gamma_i^T$ , and  $\sigma_i^T$ . The parameters  $\gamma_0^T, \sigma_0^T, \Psi_0, \Xi_0$ , and  $\Omega_0$  are initialized using the correspondent parameters of the training sample estimated VAR.

## References

- ADAM, K., AND R. M. BILLI (2006): “Optimal monetary policy under commitment with a zero bound on nominal interest rates,” *Journal of Money, Credit, and Banking*, 38(7), 1877–1905.
- AKERLOF, G. A., W. T. DICKENS, G. L. PERRY, R. J. GORDON, AND N. G. MANKIW (1996): “The macroeconomics of low inflation,” *Brookings papers on economic activity*, 1996(1), 1–76.
- BENATI, L., AND H. MUMTAZ (2007): “U.S. evolving macroeconomic dynamics: a structural investigation,” Working Paper Series 0746, European Central Bank.
- BERNANKE, B. (2012): “Monetary Policy since the Onset of the Crisis,” in *Federal Reserve Bank of Kansas City Economic Symposium, Jackson Hole, Wyoming*, vol. 31.
- BERNANKE, B., V. REINHART, AND B. SACK (2004): “Monetary policy alternatives at the zero bound: An empirical assessment,” *Brookings papers on economic activity*, 2004(2), 1–100.
- BERNANKE, B. S., T. LAUBACH, F. S. MISHKIN, AND A. S. POSEN (2001): *Inflation targeting: lessons from the international experience*. Princeton University Press.
- BILLI, R. M. (2011): “Optimal inflation for the US economy,” *American Economic Journal: Macroeconomics*, 3(3), 29–52.
- BLANCHARD, O., G. DELL’ARICCIA, AND P. MAURO (2010): “Rethinking macroeconomic policy,” *Journal of Money, Credit and Banking*, 42, 199–215.
- BLINDER, A. S. (2000): “Monetary policy at the zero lower bound: balancing the risks,” *Journal of Money, Credit and Banking*, 32(4), 1093–1099.
- CALANI, M., K. COWAN, AND P. GARCÍA (2011): “Inflation Targeting in Financially Stable Economies: Has it Been Flexible Enough?,” *Central Banking, Analysis, and Economic Policies Book Series*, 16, 283–368.
- CARTER, C. K., AND R. KOHN (1994): “On Gibbs sampling for state space models,” *Biometrika*, 81(3), 541–553.

- CHUNG, H., J.-P. LAFORTE, D. REIFSCHNEIDER, AND J. C. WILLIAMS (2012): “Have we underestimated the likelihood and severity of zero lower bound events?,” *Journal of Money, Credit and Banking*, 44(s1), 47–82.
- CLARK, T. E. (2009): “Is the Great Moderation over? An empirical analysis,” *Economic Review*, 2009, Q4–5.
- COGLEY, T., AND T. J. SARGENT (2005): “Drifts and volatilities: monetary policies and outcomes in the post WWII US,” *Review of Economic dynamics*, 8(2), 262–302.
- COGLEY, T., AND A. M. SBORDONE (2008): “Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve,” *The American Economic Review*, 98(5), 2101–2126.
- EGGERTSSON, G. B., AND M. WOODFORD (2003): “Zero bound on interest rates and optimal monetary policy,” *Brookings Papers on Economic Activity*, 2003(1), 139–233.
- GALÍ, J., AND L. GAMBETTI (2009): “On the Sources of the Great Moderation,” *American Economic Journal: Macroeconomics*, pp. 26–57.
- GELMAN, A., J. B. CARLIN, H. S. STERN, AND D. B. RUBIN (1995): *Bayesian data analysis*. Chapman and Hall.
- HAMILTON, J. D., AND J. C. WU (2012): “The effectiveness of alternative monetary policy tools in a zero lower bound environment,” *Journal of Money, Credit and Banking*, 44(s1), 3–46.
- JACQUIER, E., N. G. POLSON, AND P. E. ROSSI (2002): “Bayesian analysis of stochastic volatility models,” *Journal of Business & Economic Statistics*, 20(1), 69–87.
- JUNG, T., Y. TERANISHI, AND T. WATANABE (2005): “Optimal monetary policy at the zero-interest-rate bound,” *Journal of Money, Credit, and Banking*, 37(5), 813–835.
- KEATING, J., AND V. VALCARCEL (2011): “Greater moderations,” *Economics Letters*.
- KEATING, J. W., AND V. J. VALCARCEL (2012): “What’s so Great about the Great Moderation? A Multi-Country Investigation of Time-Varying Volatilities of Output Growth and Inflation,” Working Papers Series in Theoretical and Applied Economics 201204, University of Kansas, Department of Economics.

- PRIMICERI, G. E. (2005): “Time varying structural vector autoregressions and monetary policy,” *The Review of Economic Studies*, 72(3), 821–852.
- SVENSSON, L. E. (2003): “Escaping from a liquidity trap and deflation: The foolproof way and others,” Discussion paper, National Bureau of Economic Research.
- TOBIN, J. (1972): “Inflation and Unemployment,” *The American Economic Review*, 62(1/2), pp. 1–18.
- WALSH, C. E. (2003): *Monetary theory and policy*. MIT press.
- WILLIAMS, J. C. (2009): “Heeding daedalus: Optimal inflation and the zero lower bound,” *Brookings Papers on Economic Activity*, 2009(2), 1–37.
- (2014): “Monetary Policy at the Zero Lower Bound,” *Hutchins Center on Fiscal & Monetary Policy at Brookings*.

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