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WELFARE COSTS OF INFLATION AND IMPERFECT COMPETITION IN A MONETARY SEARCH MODEL *

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Abstract

In this paper, I quantitatively measure the welfare costs of inflation. I build into standard money-search models, such as Rocheteau and Wright(2005) and Lagos and Wright(2005), by introducing endogenous imperfect competition based on free entry decisions that allow for the share of the transaction surplus going to firms to be determined endogenously. Under this framework, the welfare cost of inflation is amplified through a feedback loop, in which restricted money demand reduces the number of firms that the market can support. In turn, this reduction increases market concentration, reduces the consumer surplus, and further decreases the incentives to hold money. I find that, depending on the calibration, between 63 to 90 percent of the estimated welfare costs of inflation can be attributed to the interaction between money holdings and market concentration.

Resumen

En este trabajo se miden cuantitativamente los costos en bienestar de la inflación. Basándose en modelos estándar de búsqueda monetaria, tales como Rocheteau y Wright(2005) y Lagos y Wright(2005), se endogeniza el nivel de competencia, permitiendo con ello que la fracción del excedente que va a las firmas sea determinado endógenamente. Bajo esta estructura, el costo en bienestar de la inflación es amplificado por un mecanismo de retroalimentación, donde una menor demanda por dinero reduce el número de firmas que el mercado es capaz de sostener. Esto aumenta la concentración de mercado, reduce el excedente de los consumidores, y disminuye aún más los incentivos para demandar dinero. Se encuentra que, dependiendo de la calibración, entre el 63 y el 90 por ciento de los costos en bienestar de la inflación pueden atribuirse a la interacción entre demanda por dinero y concentración de mercado.

* I thank Carl Walsh, Kenneth Kletzer, and Grace Gu for their excellent comments and suggestions. The views in this document are those of the author and do not represent official positions of the Central Bank of Chile or its Board Members. Research Division, Economic Modeling Department, Central Bank of Chile. Address: Agustinas 1180, Santiago, Chile. Email: bgarcia@bcentral.cl.

1 Introduction

In the new monetarist literature, welfare costs of inflation can be derived from consumers restricting their monetary holdings, mainly because they pay the costs of holding money but receive only part of the welfare gains because part of the transaction surplus goes to the sellers. This surplus share is assumed to be constant and defined by an exogenous bargaining power parameter. In this paper, I endogenize the share of surplus going to consumers by introducing an explicit form of imperfect competition into a monetary search model. By introducing a Cournot type of imperfect competition with free entry, the share of the surplus going to firms is determined endogenously. Then, the welfare costs of imperfect competition and inflation are not independent but are jointly determined within the model. In this context, the welfare cost of a given inflation level is endogenous to the market structure of the economy. At the same time, the level of competition prevailing in the economy is influenced by the inflation level.

The model is based on Rocheteau and Wright, 2005(RW05) and Lagos and Wright, 2005(LW05). The very stylized framework of RW05 and LW05 allows for a tractable, albeit simplified, way to identify the welfare costs of inflation. Other extensions provide more general setups where, for example, there is no centralized markets and agents save money across periods¹. As discussed in Aruoba et al (2011), Williamson and Wright (2010) and Venkateswaran and Wright (2014), those models require keeping track of the distribution of money across agents as a state variable, a complicated affair even using numerical methods. These approaches provide complementary transmission channels at the expense of reduced tractability and increased complexity. The main insights and transmission mechanism of the baseline framework, however, remain present.

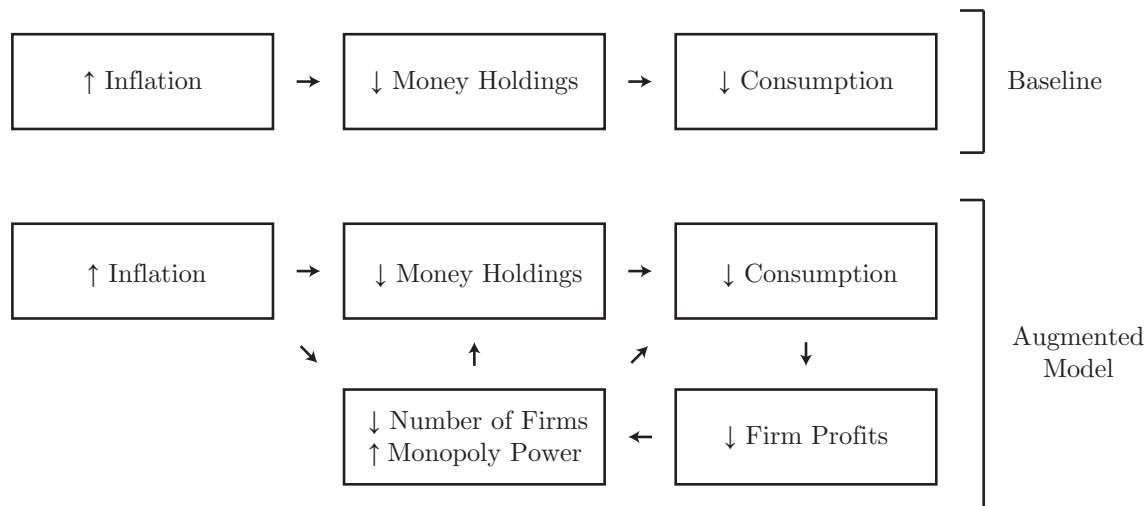
The main difference of this paper with respect to RW05 and LW05 is in the structure of the decentralized market, modified to allow each buyer to face more than one seller. The equilibrium

¹See Shi and Wang (2006), Molico (2006), Chiu and Molico (2010, 2011) and Dressler (2011).

prices and quantities are derived from a Cournot style competition between the sellers. As in LW05 and RW05, inflation is costly because it increases the cost of holding Money, which affects reducing the amount of money held by agents in the economy.

In this framework, the welfare loss from inflation is amplified by an "*imperfect competition multiplier*", with a mechanism similar to that of Jaimovich and Floetotto (2008). Reduced demand decreases firms' operational profits, which provides incentive for firms to leave the market, increasing market concentration and lowering both output and the share of surplus going to consumers. The reduction in consumer surplus resulting from higher monopoly power decreases the marginal benefit of carrying money, which in turn further reduces money holdings and amplifies the welfare loss. Figure 1 illustrates the mechanism.

Figure 1: Consequences of an increase in inflation



2 A monetary search model with imperfect competition and free entry

The basic structure of the model is a modified version of the framework outlined in LW05 and RW05. In this version of the model, two types of agents exist: *buyers* and *sellers*. There is a continuum of buyers of mass 1. There is also an infinite number of potential sellers who are able to participate in the economy, from which N choose to do so.

Time is discrete, and each period is subdivided into the two subperiods Of day and night. For every type of agent, the discount factor is β_d between day and night, and β_n between night and the next period's day. $\beta_d, \beta_n \in [0, 1]$ and $\beta = \beta_d \beta_n$. During the day, all agents consume, but labor is only supplied by buyers. During the night, sellers supply labor, and only buyers are able to consume. During the day, a frictionless centralized market operates, whereas at night buyers search for any of S submarkets. Only if they are able to find one will they be able to consume. Each submarket is populated by $n = N/S$ sellers who compete among themselves in a Cournot fashion to serve the buyers. The number of submarkets is normalized to 1.

In general, for agents type i , preferences are $U_i(x_i, h_i, Q_i, H_i)$. Consumption and labor during the day and during the night are, respectively, (x_i, h_i) and (Q_i, H_i) , v_i and u_i are the utility derived from consumption, and ς_i is the disutility of labor:

$$U_i(x_i, h_i, Q_i, H_i) = v_i(x_i) - \varsigma_i(h_i) + \beta_d [u_i(Q_i) - \varsigma_i(H_i)] \quad (1)$$

For buyers, the utility of consumption is $v_b(x_b) = \omega \ln(x_b)$ during the day and $u_b(Q_b) = Q_b^{1-\eta_1} / (1 - \eta_1)$ during the night. Define $y = \int^1 x_b + \int^N x_s$ as the total amount of consumption goods produced by the buyers. During the day, production technology converts labor into consumption goods one to one, with a constant and unitary marginal cost of labor. Then, the cost of production for

buyers during the day is $c_b(y) = y$. Sellers' marginal utility of consumption during the day is linear and unitary: $v_s(x_s) = x_s$. During the night, sellers' labor disutility is $\varsigma_i(H_i) = H_i^{1+\eta_2}/(1+\eta_2)$. The amount of labor required to produce a unit of a night consumption good is 1. The cost function for night consumption goods can then be written as $c_s(q_s) = q_s^{1+\eta_2}/(1+\eta_2)$. In the night market, q_s is the production of each seller, where $\int^N q_s = \int^1 Q_b$.

Given these assumptions, the preferences for each type of agent can be characterized as

$$U_b = v_b(x_b) - y + \beta_d u_b(Q_b) \quad (2)$$

$$U_s = v_s(x_s) - \beta_d c_s(q_s) \quad (3)$$

To simplify the notation, the subscripts denoting the type of agent during the night market are dropped.

The real value of an amount of money m in the hands of an agent at date t is defined by $z_t = \phi_t m_t$. With a constant gross growth rate of money of γ , the evolution of the price of money should follow $\phi_{t+1}/\phi_t = 1/\gamma$ in the steady state. As in RW05, $\gamma \geq \beta$.

Let V_b and W_b be the value functions of a buyer in the night market and day market, respectively. In the centralized day market, the problem of a buyer j with z_j real money holdings is

$$\begin{aligned} W_b(z_j) = & \max_{z'_j, x_j, y_j} v_b(x_j) - y_j + \beta_d V_b(z'_j) \\ \text{st} \quad & z'_j + x_j = z_j + T_j + y_j \end{aligned} \quad (4)$$

where z'_j are the real money balances taken into the night market and T_j are real transfers attributable to changes in the aggregate money supply. By plugging the restriction into the maximization problem, the value function becomes $W_b(z_j) = z_j + T_j + \max_{z'_j, x_j} \{v_b(x_j) - x_j - z'_j + \beta_d V_b(z'_j)\}$. Two features of the value function become clear. First, W_b is linear in z_j . Second, the optimal values for z'_j and

x_j are independent from the amount of money brought to the day market.

In the night market, the value function of a buyer j who is carrying an amount of money z_j is

$$\begin{aligned} V_b(z_j) = & \max_{Q_j, d_j} \alpha_b(S) \left[u(Q_j) + \beta_n W_b\left(\frac{z_j - d_j}{\gamma}\right) \right] + (1 - \alpha_b(S)) \left[\beta_n W_b\left(\frac{z_j}{\gamma}\right) \right] \\ & st \quad pQ_j = d_j \\ & \quad z_j - d_j \geq 0 \end{aligned} \tag{5}$$

where z_j corresponds to the money brought to the day market, and d_j is the total monetary spending if a match is made. The probability that a buyer find a suitable match is $\alpha_b(S\lambda^{-1}) = S\lambda^{-1} (1 - e^{-S^{-1}\lambda})$, where $S\lambda^{-1}$ is the product between the possible successful matches (in this case, the number S of markets to be found) and λ^{-1} , a search efficiency parameter.

Under this specification, $\alpha'_b(S/\lambda) > 0$, $\alpha''_b(S/\lambda) < 0$, $\lim_{(S/\lambda) \rightarrow 0} \alpha_b(S/\lambda) = 0$ and $\lim_{(S/\lambda) \rightarrow \infty} \alpha_b(S/\lambda) = 1$. As the number of markets S is normalized to 1, the probability of a match will depend exclusively on the search efficiency parameter λ^{-1} .

As the value function in the night market is linear in money, the utility maximization problem can be expressed, conditional on a match, as one of quasilinear utility, where given money holdings z_j , buyers maximize utility considering that spending money on the night market means not being able to do so on the day market during the next period.

$$\begin{aligned} \max_{Q, d} \quad & U_j = u(Q_j) + a(z_j - d_j) \\ & st \quad pQ_j = d_j \\ & \quad z_j - d_j > 0 \end{aligned} \tag{6}$$

where $a = \beta_n/\gamma$ represents the utility of carrying money to the next period and, as in RW05, depends on the discount factor and the inflation rate.

If buyers carry enough money to make the non-negativity restriction on cash holdings not binding, the first-order conditions imply that

$$u'(Q_j) - ap = 0 \quad (7)$$

This equation provides the indirect demand function when money holdings do not restrict the buyer decision

$$p_U = \frac{u'(Q_j)}{a} \quad (8)$$

2.1 A decentralized market with Cournot competition

After searching, B_s buyers will find a market populated by n sellers, where $0 \leq B_s \leq 1$.

Each seller maximizes profits subject to an inverse demand function $p(Q/B_s)$, a cost function $c(q_i)$, and a fixed cost of entry τ_f .

It is assumed that all buyers search together, and all of them either find the night market with probability α_b or fail to find anything. An alternative specification could be a completely independent search, through which every buyer with infinitely small mass has a random chance of having a successful search. In this case, the market will receive with certainty a mass α_b of buyers. The night market will never receive more and will never receive less.

These two specifications can be thought of as the two polar cases of a binomial distribution characterization of the probability that a market will receive customers. Assume that buyers organize their search by grouping into G clusters, where $G \in \mathbb{N}_{\geq 1}$. For a given G , the set containing all possible clusters of buyers that the market can receive is denoted by $\psi = g/G$, where $g = \{\mathbb{N}_0^G\}$. The chance that each cluster of buyers finds the market is α_b . The probability that the market receives a mass ψ of buyers is then $pr(B_s = \psi) = \binom{G}{g} (\alpha_b)^g (1 - \alpha_b)^{G-g}$, whereas $pr(B_s = \theta \mid \theta \notin \psi) = 0$. For example, assume $\alpha_b = 0.5$, and $G = 2$, meaning that there is a 50% chance that a buyer can

find the market, and buyers are grouped into two clusters. Then $g = \{0, 1, 2\}$, $\psi = \{0, 0.5, 1\}$, and $pr(B_s = \psi) = \{0.25, 0.5, 0.25\}$. The probability that the market will receive no customers is 25%, that it will receive a mass 0.5 of buyers is 50%, and that it will receive a mass 1 of buyers is 25%. The assumption that all buyers search together corresponds to the case of $G = 1$, whereas the alternative of a completely independent search corresponds to $G \rightarrow \infty$. The main difference between these parameterizations is the volatility of the expected customers received. When $G = 1$, uncertainty is maximized, and when $G \rightarrow \infty$, uncertainty goes to 0. In particular, the volatility of expected customers can be expressed as $\sigma_{B_s}^2 = \alpha_b(1 - \alpha_b)/G$. Although G could be calibrated to match an empirical volatility moment, a unitary value is assumed because it greatly simplifies the mathematical derivation and solving the model. Under this assumption, if a match is made, $B_s = 1$ and $Q_j = Q$.

When maximizing profits, sellers take into account that, although they have to pay the cost of producing overnight, they can only use the obtained revenue at the next period day market. Therefore, the relevant revenue value is not pq_i , but is apq_i , where $a = \beta_n/\gamma \leq 1$.

$$\begin{aligned} \max_{q_i} \quad & Profit_i = aq_i p(Q) - c(q_i) - \tau_f \\ st \quad & p = \frac{u'(Q)}{a} \\ & Q = \sum_{i=1}^N q_i \end{aligned} \tag{9}$$

The first-order condition with respect to q_i simply shows that the optimal firm behavior requires the discounted marginal revenue to be equal to the marginal cost of the last unit sold.

$$\underbrace{u'(Q) + q_i u''(Q)}_{Mg \text{ revenue}_u} = \underbrace{c'(q_i)}_{Mg \text{ cost}} \tag{10}$$

The first term on the left-hand side (LHS) corresponds to the quantity effect on revenue. The second term corresponds to the price effect of an extra unit put on the market. Assuming positive and

marginally decreasing utility of consumption, the first term is expected to be positive and the second is expected to be negative.

If every seller in the market has the same preferences and cost functions, $q_i = Q/n$, the first-order condition becomes

$$\underbrace{u'(Q)}_{ap} + \frac{Q}{n} u''(Q) = c' \left(\frac{Q}{n} \right) \quad (11)$$

As the number n of firms increases, the influence of each firm's output on prices decreases. At the limit, if $n \rightarrow \infty$, then the second term disappears and the result converges to the perfect competition solution for which the discounted price ap equals the marginal cost.

2.2 Restricted money holdings

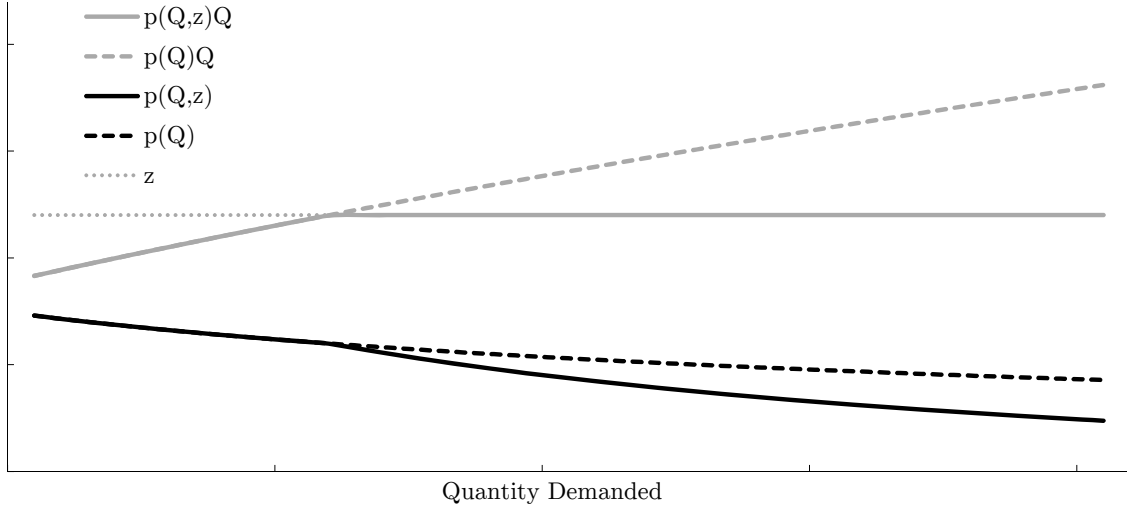
The inverse demand function from equation (8) is only valid when buyers carry enough money to pay for that quantity of goods. If a point in the demand function is not feasible because of a lack of sufficient money holdings, the only way to make the buyer willing to absorb that quantity is to lower the price enough to ensure that she is able to buy it.

From the point of view of the seller, the relevant inverse demand function—defined as the price at which the buyer is willing (and able) to acquire any specific amount of goods—is the minimum between the unrestricted demand and the maximum units that the money holdings are able to buy.

$$p = \min \left\{ \underbrace{u'(Q)/a}_{p_u}, \underbrace{z/Q}_{p_r} \right\} \quad (12)$$

Figure 2 illustrates this concept. When money holdings are not enough to buy a particular set (p, Q) , the demand function is depressed to comply with the constraint of non-negativity of money holdings.

Figure 2: Demand in the decentralized market with restricted money holdings



Let $d_u = p_u(Q)Q$ be the spending that would occur if money holdings were not binding. If $z \geq d_u$, then the effective spending is the same as in the unrestricted case, and the equilibrium price and quantity is derived from equations (8) and (11). If $z < d_u$, then the market equilibrium will be determined by firms maximizing profits subject to $p(Q, z) = z/Q$.

$$\begin{aligned}
 \max_{q_i} \quad & Profit_i = apq_i - c(q_i) - \tau \\
 st \quad & p = \frac{z}{Q} \\
 & Q = \sum_{i=1}^N q_i \\
 & \tau = \tau_f + \tau_m
 \end{aligned} \tag{13}$$

Again, the first-order condition with respect to q_i shows that the optimal firm behavior requires the discounted marginal revenue to be equal to the marginal cost of the last unit sold.

$$\underbrace{(az/Q)(1 - q/Q)}_{Mg \text{ revenue}_R} = \underbrace{c'(q_i)}_{Mg \text{ cost}} \tag{14}$$

In equilibrium, $Q = nq$ and the discounted price is closer to the marginal cost as the number of

firms increases.

$$\underbrace{\frac{az}{Q}}_{ap} \left(1 - \frac{1}{n}\right) = c'(q_i) \quad (15)$$

In this case, the markups—defined as the ratio of prices and marginal costs—can be expressed as an explicit function of the discount factor, inflation level, and the equilibrium number of firms:

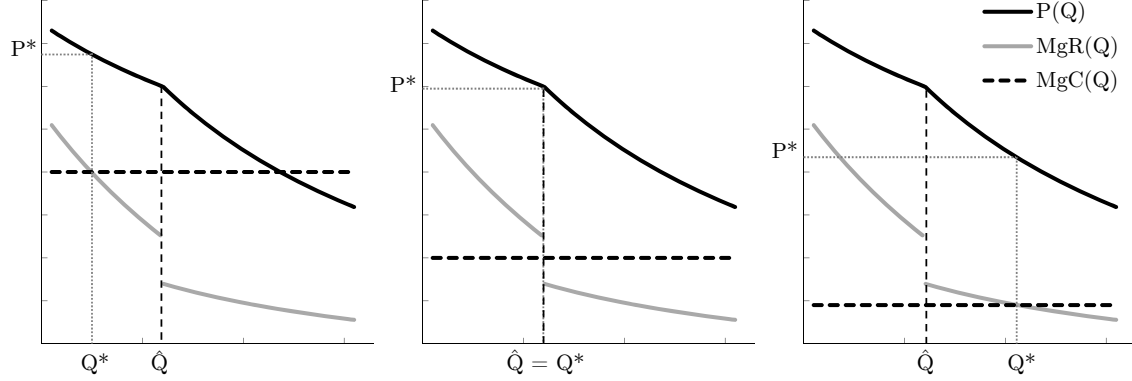
$$Mkup = \left(\frac{n}{n-1}\right) \frac{\gamma}{\beta_n}$$

Depending on the level of money holdings, the equilibrium quantities sold in the night market is given by equations (11) or (15). Define \hat{Q} as the quantity at which money holdings became restrictive, the quantity such that $u'(\hat{Q})/a = z/\hat{Q}$. Figure 3 shows that at \hat{Q} , the slope of the inverse demand function changes suddenly, going from the unrestricted case from the left to the restricted case to the right. This change introduces a discontinuity in the marginal revenue function and three possible equilibria, depending on the relationship between the marginal cost and revenue evaluated at \hat{Q} :

$$\begin{aligned} \text{if } C'(\hat{Q}) > MgR_U(\hat{Q}) & \quad \begin{cases} MgR_U(Q^*) = C'(Q^*) \\ P^* = P_U(Q^*) \end{cases} \\ \text{if } MgR_R(\hat{Q}) < C'(\hat{Q}) < MgR_U(\hat{Q}) & \quad \begin{cases} Q^* = \hat{Q} \\ P^* = P_U(Q^*) = P_R(Q^*) \end{cases} \\ \text{if } C'(\hat{Q}) < MgR_R(\hat{Q}) & \quad \begin{cases} MgR_R(Q^*) = C'(Q^*) \\ P^* = P_R(Q^*) \end{cases} \end{aligned} \quad (16)$$

Given these equations, the equilibrium price and quantity for the night market can be pinned down for any z and N .

Figure 3: Possible equilibria in the decentralized market with restricted money holdings



2.3 Equilibrium money holdings

The money holdings that a buyer brings to the night market are derived from maximizing the value function. Assume that each individual buyer is small enough that she is incapable of modifying the equilibrium prices². From equation (4), the FOC of $W^b(z_b)$ with respect to z'_b is

$$\beta_d V_{z'_b}^b(z'_b) - 1 = 0 \quad (17)$$

From equation (5), if $z_b > d_u$ then $\beta_d V_{z'_b}^b = \frac{\beta_n}{\gamma}$. With $\gamma \geq \beta > 0$ and $\beta = \beta_d \beta_n$, it follows that $\frac{\beta_n}{\gamma} \leq 1$. Clearly, money holdings larger than d_u will never be optimal. Because holding money is costly, holding money that is not going to be spent implies a welfare loss.

In contrast, if $z_b < d_u$, then $z_b = d$, and all money holdings are going to be spent if a match occurs, thus $Qp = z$. In this case, the FOC with respect to money holdings can be expressed as

$$\beta_d V_z^b(z) - 1 = \beta_d \alpha_b \left[\frac{\partial u(Q)}{\partial Q} \frac{\partial Q}{\partial z} \right] + \frac{\beta}{\gamma} (1 - \alpha_b) - 1 = 0 \quad (18)$$

²This assumption can also potentially generate suboptimal money holdings because individual buyers will not consider the possible benefits that their money holdings could bring to other buyers and sellers

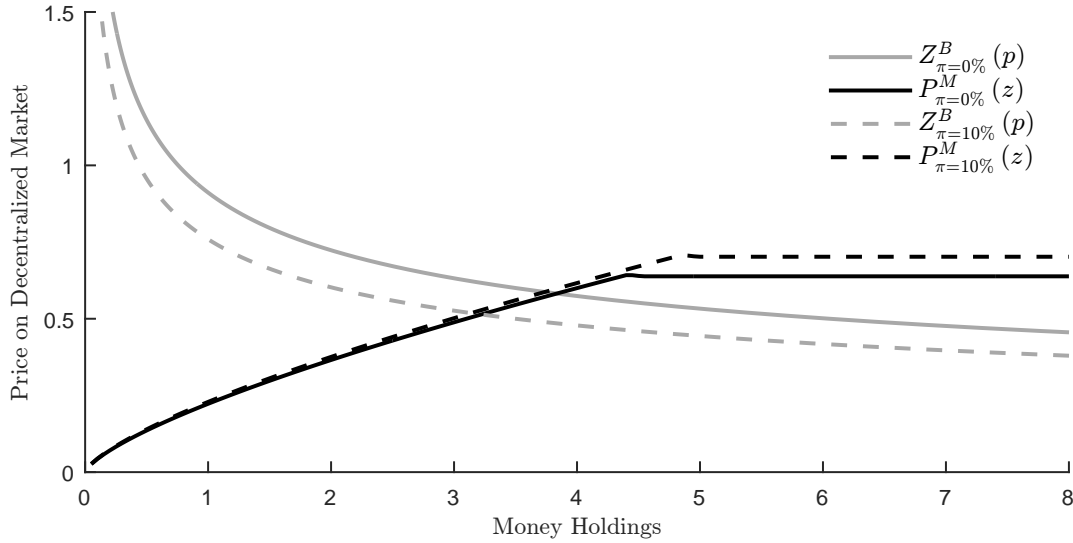
For any price that is expected to clear the market in the case of a match, the amount of money that buyers are willing to hold is denoted by the function $Z^B(p)$, which corresponds to the money holdings such that

$$u(p/Z^B)' / Z^B = (\beta_d \alpha_b)^{-1} - a(\alpha_b^{-1} - 1) \quad (19)$$

The equilibrium can be thought of as the intersection of two best response functions. On the one hand, to buyers bringing an amount of money z , the market will respond with a clearing price of $P^M(z)$. On the other hand, to each price p , buyers will respond with money holdings equal to $Z^B(p)$.

Figure 4 shows how, given a number of firms n , both P^M and Z^B respond to an increase in the inflation level $\pi = \gamma - 1$.

Figure 4: Inflation and equilibrium money holdings with a fixed number of sellers



Regarding P^M , higher inflation make sellers demand higher compensation for their labor because the value of money will be lower by the time they will be able to spend their monetary payments. This phenomenon causes an upward shift in the $P^M(z)$ function, implying that for any z , the market

clears with a higher price. The best response function $P^M(z)$ appears flat to the right because, after money holdings stop being binding, a buyer carrying extra money changes neither the equilibrium prices nor the quantities.

The second and far more influential manner in which inflation affects equilibrium money holdings is by shifting the best response function $Z^B(p)$. Higher inflation implies that when no match is made, the loss of value of the buyer's money holdings are also higher. Consequently, for any price p , buyers are willing to bring less money to the night market, shifting $Z^B(p)$ to the left.

For a given number of firms and assuming an equilibrium with restricted money holdings (it will be for all chosen parameterizations), an analytic expression can be found for the equilibrium $z^*(n), p^*(n)$ and $Q^*(n)$:³

$$z^*(n) = A_1 A_2^{A_3 + A_4} \quad (20)$$

$$p^*(n) = A_1 A_2^{A_4} \quad (21)$$

$$Q^*(n) = A_2^{A_3} \quad (22)$$

where $A_1 = \frac{\gamma \beta_d \alpha_b}{\gamma + (\alpha_b - 1)\beta}$, $A_2 = \left(\frac{A_1 \beta_n (n-1)}{\gamma n^{1-\eta_2}} \right)^{\frac{1}{1+\eta_2}}$, $A_3 = \frac{1+\eta_2}{\eta_1 + \eta_2}$, and $A_4 = -A_3 \eta_1$.

2.4 Free entry and number of sellers

The equilibrium number of firms is determined by the free entry condition, where firms enter until their expected profits equal zero.

$$E[Profit_i] = \alpha_k \cdot (ap^* q_i^* - c(q_i^*)) - \tau = 0 \quad (23)$$

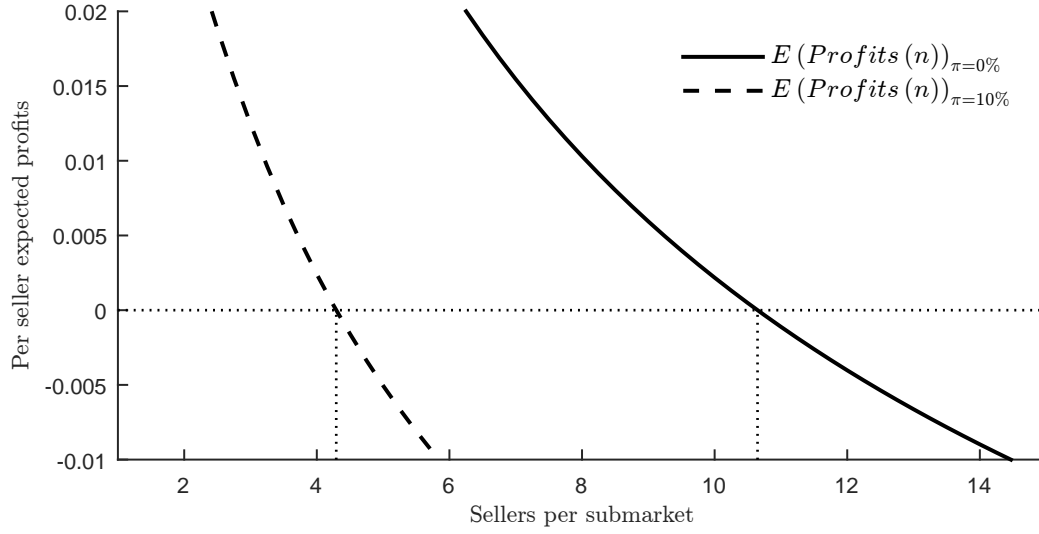
³If the equilibrium does not involve restricted money holdings, $z^*(n), p^*(n)$, and $Q^*(n)$ can be found using numerical methods.

If all firms behave the same, $q_i = Q/n$, the equilibrium number of firms is the n such that

$$n = \frac{\left((1 + \eta_2) a p^*(n) - (Q^*(n)/n)^{\eta_2} \right) \alpha_k Q^*(n)}{\tau (1 + \eta_2)} \quad (24)$$

The effect of inflation on the number of firms comes from its effect on profits. First, inflation directly affects the discount factor a and, therefore, the discounted income. Additionally, higher inflation restricts buyers' money holdings and, therefore, total spending pq . Because less competition increases the per-seller expected profits, lower income requires the market to clear with fewer firms to maintain zero profits at reduced demand. Figure 5 shows the effect of inflation on expected profits and how it induces a smaller number of sellers, increasing market concentration.

Figure 5: Inflation and the equilibrium number of sellers under free entry



By solving equations (20), (21), (22), and (24), the equilibrium with buyers bringing an amount z^* of money to a market with n^* sellers, and buying Q^* goods at a price p^* , can be fully characterized for any set of parameters.

3 Calibration and results

The parameters in the baseline model are calibrated as follows. The labor disutility parameter η_2 is set at 2.503, as in Smets and Wouters (2003). The discount factor β is set at an annual rate of 0.96, with $\beta_d = 1$ and $\beta_n = 0.96$. This assumes no discount between subperiods. The fixed cost of participation for the seller τ_f is calibrated such that, with an inflation rate of 2%, the markups in the decentralized market equal 20%, as in Craig and Rocheteau 2008 (CR08). In a similar vein as LW05 and CR08, parameters ω , λ , and η_1 are chosen to match an empirical money demand.

Following Lucas (2000), the ratio of real money holdings to income $L = M/PY$ is defined as a function of interest rates i , where the ratio between real balances and income depends on the cost of holding cash. In this model specification, M is analogous to z . Nominal GDP is constructed by multiplying the output in each subperiod by its price. During the night, prices and output are given by equations (21) and (22). In the centralized frictionless day market, prices are equal to marginal costs and, therefore, buyers equalize marginal utility with the unitary marginal cost. Then, buyers' optimal consumption x_b equal ω . In contrast, sellers have money holdings from the previous night market, although its value will have diminished because of inflation. Given the assumption of $\beta \leq \gamma$, it's optimal for sellers to spend all of their cash, which allows them—in case they were matched last period—to buy z/γ units of the day's good. Then, similar to LW05 and CR08, the model's counterpart to Lucas money demand $L(i) = M/PY$ is defined as

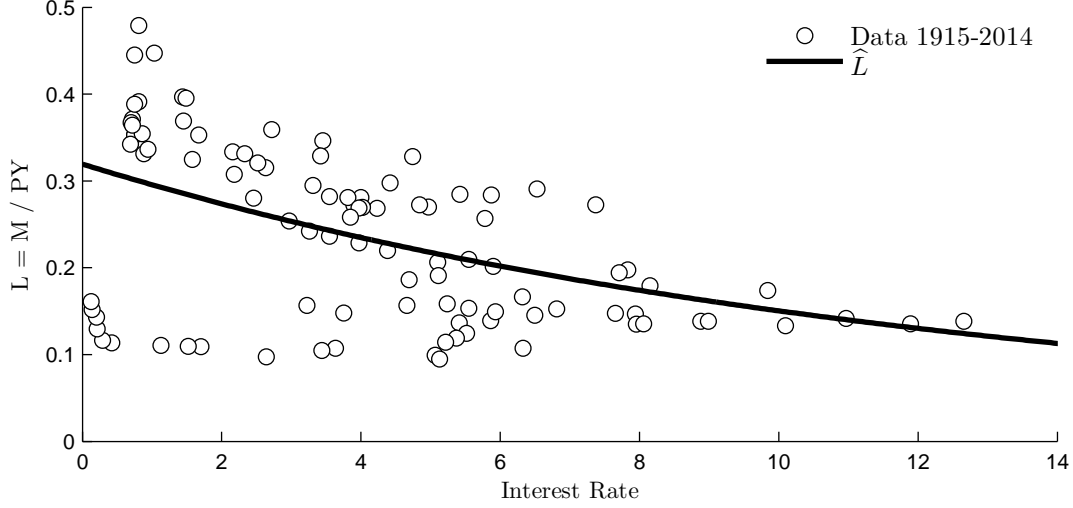
$$L(\gamma) = \frac{M}{PY} = \frac{z^*(\gamma)}{x_b + x_s + \alpha_b p Q} = \frac{z^*(\gamma)}{\omega + z^*(\gamma) \cdot \alpha_b \cdot (1 + \gamma^{-1})} \quad (25)$$

Figure 6 shows how the model is able to fit the downward slope of the empirical money demand for the 1915–2014 sample⁴.

⁴As in Lucas(2000), the interest rate is the short rate for commercial paper. For 1915 to 1975, this rate is taken from *Historical Statistics of the United States, Earliest Times to the Present: Millennial Edition*(HSUS), compiled from Friedman and Schwartz(1982). For 1976 to 2014, the rate is from the Federal Reserve Bank of St. Louis FRED Database.

The Money Supply is M1 in billion of dollars. For 1915 to 1958, M1 is from HSUS, compiled from Friedman and

Figure 6: Money demand and model fit.



This baseline parameterization allows for an analysis of the consequences of an increase in inflation. The focus is on the impact of inflation going from 0% to 10% on four variables: number of sellers participating in the economy n , money holdings z , consumer surplus share during the night market, and welfare losses from inflation.

As in LW05 and CR08, let $(1 - \Delta_0^\pi)$ be the welfare loss, defined as the percentage of consumption that agents would be willing to give up to go from an inflation rate of π to zero⁵. We define the long-term total utility of buyers and sellers given an inflation rate of π as

$$U_\pi = v_b(x_{b,\pi}) + v_s(x_{s,\pi}) - (x_{b,\pi} + x_{s,\pi}) + \alpha_b \beta_d [u(Q_\pi) - n_\pi c(q_\pi)] - N_\pi \tau_f \quad (26)$$

Schwartz(1982) and Rasche(1987). For 1959 to 2014, M1 is from the FRED database.

GDP from 1915 to 1946 is from HSUS's *GDP Millennial Edition Series*, compiled from varied sources. For 1947 to 2014, GDP is from the FRED Database.

⁵Lucas(2000) computes the welfare cost in a slightly different but primarily equivalent manner. He asks, given inflation of π , the amount of extra consumption that should be given to agents to make them indifferent to a zero inflation equilibrium

The welfare cost of inflation is the value of $(1 - \Delta_0^\pi)$ such that

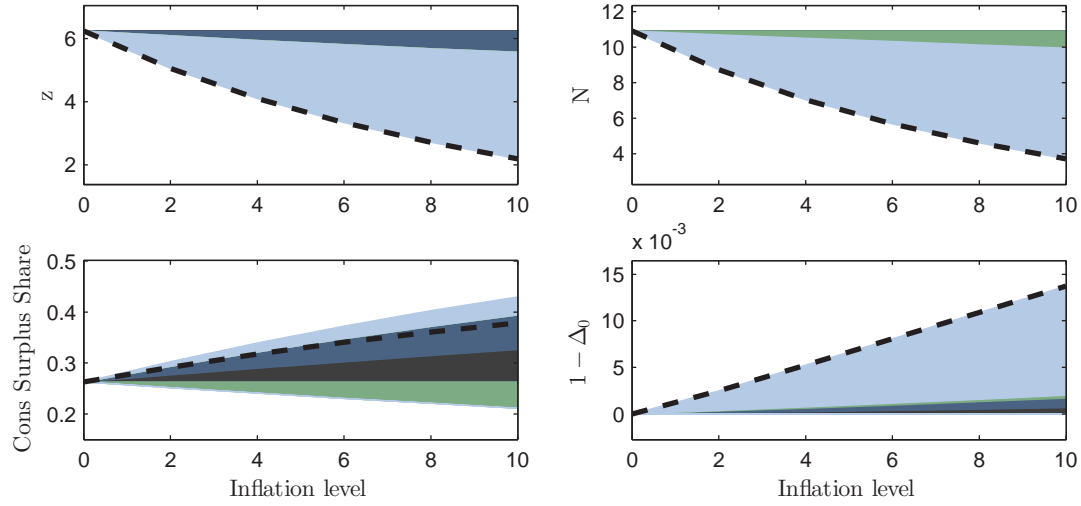
$$U_\pi = v_b(x_{b,0} \cdot \Delta_0^\pi) + v_s(x_{s,0} \cdot \Delta_0^\pi) - (x_{b,0} + x_{s,0}) + \alpha_b \beta_d [u(Q_0 \cdot \Delta_0^\pi) - n_0 c(q_0)] - N_0 \tau_f \quad (27)$$

The effect of inflation on the analyzed variables is decomposed into four parts. The first part is the direct effect of inflation on prices and quantities sold, maintaining both z and n at their zero inflation values. The second part is the additional effect attributable to the role that inflation plays in determining the equilibrium number of sellers. The share is obtained by computing the model with a level π of inflation, but keeping money holdings fixed at their zero inflation levels: $z_\pi = z_0$. To avoid double counting, the direct effect of inflation on prices and quantities is subtracted. The direct effect of inflation on equilibrium money holdings are computed in the same manner as the previous one, but fixing the number of firms such that $n_\pi = n_0$. Finally, the additional effect of the feedback loop between reduced money holdings and fewer participating firms are computed by subtracting all three contributions previously obtained from the total impact of inflation on the model variables. The decomposition previously described is presented in Figure 7.

As expected, higher inflation increases the cost of money and causes a decrease in the desired holdings. However, less than one-fifth of the effect can be attributed directly to the increase in inflation. The second-round effects have the most impact, where the decline in money holdings diminishes the number of sellers, further reducing the desired holdings. For the number of sellers willing to participate in the economy, the proportion of the variation attributable to second-round effects has a similar magnitude.

The share of the total surplus going to the buyers during the night is a key variable because it contributes to the suboptimality of money holdings. Buyers only consider the private benefits of money. Because the benefits to the sellers are not taken into account when making the decision, less money is brought than what is optimal for the economy as a whole.

Figure 7: Consequences of an increase in the inflation level, different contributions.^b[1]



^aThe dark gray area represents the direct effect on prices. The blue area represents the direct effect of inflation on money holdings. Green represents the direct effect on the number of sellers willing to participate. The light blue area corresponds to the feedback loop between money holdings and number of sellers. The black discontinuous line represents the total effect.

The direct impact of inflation on prices and quantities increases the buyer's share of the surplus because this effect mainly affects the sellers. With higher inflation, the monetary payments that sellers receive for their labor are worth less in terms of next subperiod purchasing power. This phenomenon reduces the sellers' surplus given the night market's transactions.

The effect of inflation on the desired money holdings increases the buyer's share because the reduction in demand decreases sellers' profits.

The reduction in the number of participating sellers attributable to inflation tends to decrease the buyers' surplus share because higher monopoly power allows sellers to extract more surplus from the buyers.

The feedback loop between fewer firms and reduced money holdings does not have a clear predicted

consequence on the buyer's surplus share because it is a combination between the positive effect of the reduction in the money holdings and the negative effect attributable to fewer firms participating. For the baseline calibration, the overall effect is an increase in the buyers' surplus share. This increase helps dampen the welfare costs of inflation because a higher proportion of the benefits from holding money will be internalized given a higher consumer surplus, reducing the inefficiency in the equilibrium money holdings.

Regarding welfare costs, the overall cost of a 10% level of inflation is 1.37%, meaning that agents would be willing to give up 1.37% of consumption to have zero instead of 10% inflation⁶. Again, the majority of the effect is from the interaction between changes in money holdings and number of firms, which accounts for 85% of the total cost.

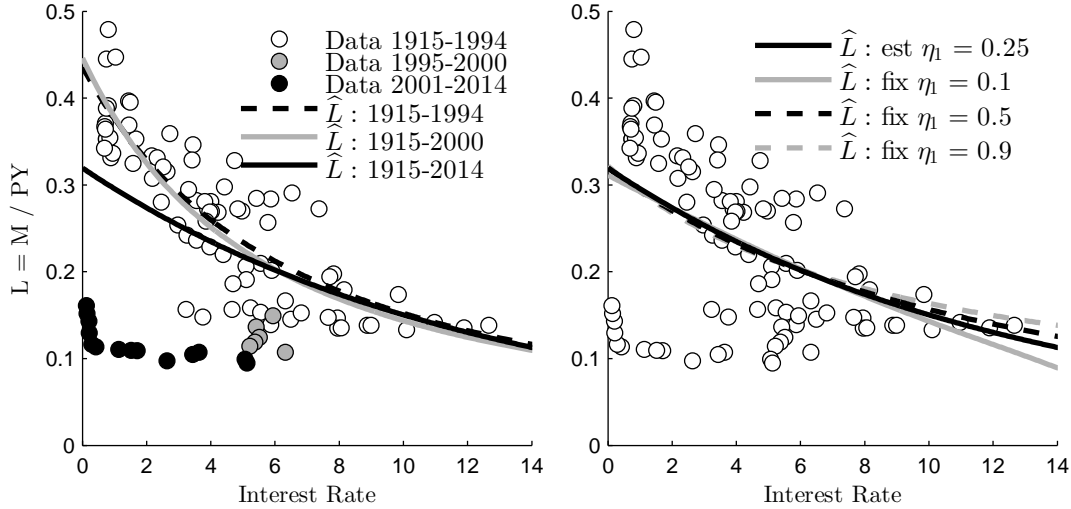
As a robustness check, several alternative parameterizations are chosen. Regarding the money demand empirical match, the parameters ω , λ , and η_1 are re-estimated to fit more restricted samples to obtain a better comparison with Lucas and LW05, who end their sample in the years 1994 and 2000, respectively. Additionally, in alternative specifications, the elasticity parameter η_1 , instead of estimated to fit the data, is fixed at 0.1, 0.5, and 0.9 because different values just barely affect the fit at moderate inflation levels. Figure 8 show the fit of the model with these different parameterizations and sample sizes.

Other specifications involved alternative balances for the day and night discount factors, and different values for the labor disutility parameter.

Table 1 presents the estimated welfare costs of a 10% rate of inflation across different specifications. The estimated cost, ranging from 0.9% to 1.7%, sits on the lower part of the range of 1.2%–4.6% found by LW05 and 0.8%–10.4% found by CR08, above the 0.62% of Chiu and Molico (2010), and

⁶The baseline measure for the cost of inflation is defined as the cost from going from 0% inflation. As a benchmark for inflation costs, it is also possible to compute $(1 - \Delta_f^T)$, the cost of deviating from the Friedman rule for optimal inflation, which sets nominal interest rates at 0%. Using this measure, the cost of a 10% inflation rate increases to 1.44%

Figure 8: Money demand and model fit, different samples, and parameterizations



closer to Lucas' estimated range of 0.5%–1.5%.

Across specifications, the total welfare cost is mainly derived through interacting effects between changes in money holdings and number of firms. The feedback loop accounts for between 63% and 90% of the total cost of inflation.

4 Conclusions

By augmenting a monetary search model with imperfect competition and free entry, I show that the cost of inflation can be derived not only from a reduction in money holdings but also from the effect of inflation on the equilibrium number of firms that the market can support. Moreover, these two effects reinforce each other. On the one hand, a decrease in money holdings reduces the number of firms that a market can support. On the other hand, a lower number of firms— from an increase in monopoly power—shrinks the incentives to carry money. Under this framework, the vast majority

Table 1: Estimated welfare cost of a 10% rate of inflation.^b[1]

	Sample: 1915-2014		Sample: 1915-2014		Sample: 1915-2014		Sample: 1915-2014		Sample: 1915-2014	
	$\beta_d=1.00$	$\eta_1=0.15$	$\beta_d=\mathbf{0.98}$	$\eta_1=0.17$	$\beta_d=\mathbf{0.96}$	$\eta_1=0.19$	$\beta_d=1.00$	$\eta_1=0.17$	$\beta_d=1.00$	$\eta_1=0.19$
	$\omega=19.8$	$\eta_2=2.50$	$\omega=16.6$	$\eta_2=2.50$	$\omega=14.2$	$\eta_2=2.50$	$\omega=14.3$	$\eta_2=\mathbf{1.50}$	$\omega=14.3$	$\eta_2=\mathbf{1.00}$
	$\lambda=1.29$	$\tau_f=0.23$	$\lambda=1.51$	$\tau_f=0.20$	$\lambda=1.73$	$\tau_f=0.18$	$\lambda=1.53$	$\tau_f=0.15$	$\lambda=1.71$	$\tau_f=0.10$
	(1)		(2)		(3)		(4)		(5)	
$1-\Delta_0$	0.014		0.014		0.014		0.014		0.014	
% $\pi \rightarrow q$	0.049		0.049		0.049		0.062		0.072	
% $\pi \rightarrow z$	0.076		0.086		0.095		0.138		0.206	
% $\pi \rightarrow n$	0.023		0.023		0.023		0.010		0.000	
% $z \leftrightarrow n$	0.853		0.843		0.833		0.791		0.722	
	Sample: 1915-2014		Sample: 1915-2014		Sample: 1915-2014		Sample: 1915- 2000		Sample: 1915- 1994	
	$\beta_d=1.00$	$\eta_1=\mathbf{0.10}$	$\beta_d=1.00$	$\eta_1=\mathbf{0.50}$	$\beta_d=1.00$	$\eta_1=\mathbf{0.90}$	$\beta_d=1.00$	$\eta_1=0.67$	$\beta_d=1.00$	$\eta_1=0.62$
	$\omega=21.0$	$\eta_2=2.50$	$\omega=8.53$	$\eta_2=2.50$	$\omega=3.55$	$\eta_2=2.50$	$\omega=4.63$	$\eta_2=2.50$	$\omega=5.21$	$\eta_2=2.50$
	$\lambda=0.58$	$\tau_f=0.37$	$\lambda=5.45$	$\tau_f=0.03$	$\lambda=10.63$	$\tau_f=0.006$	$\lambda=13.4$	$\tau_f=0.006$	$\lambda=11.1$	$\tau_f=0.008$
	(6)		(7)		(8)		(9)		(10)	
$1-\Delta_0$	0.017		0.010		0.009		0.012		0.012	
% $\pi \rightarrow q$	0.048		0.039		0.038		0.028		0.029	
% $\pi \rightarrow z$	0.045		0.193		0.260		0.228		0.221	
% $\pi \rightarrow n$	0.011		0.056		0.073		0.055		0.055	
% $z \leftrightarrow n$	0.896		0.713		0.630		0.689		0.696	

^aFor each specification, the parameters that differ from the baseline are highlighted

of the welfare costs of inflation can be traced back to the feedback loop between the reduction of money holdings and the increase in market concentration.

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