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OPTIMAL MONETARY POLICY RULES WHEN THE CURRENT ACCOUNT MATTERS

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Resumen

Este trabajo explora las implicancias sobre la regla de política monetaria óptima al incluir un objetivo de cuenta corriente (CC) entre las preferencias del banco central (BC). Utilizando un modelo macroeconómico simple pero realista de la economía chilena y métodos de programación dinámica estándar con variables *forward-looking*, este trabajo encuentra reglas óptimas bajos especificaciones alternativas para la función de pérdida cuadrática del BC. Los resultados muestran que las reacciones de política óptimas cambian substancialmente cuando hay un objetivo para la CC (adicional a la inflación). Más aun, una vez que la CC entra en la función objetivo del BC, la importancia relativa de la variabilidad producto *vis-à-vis* la variabilidad de la inflación es menos crucial en la determinación de la regla de política óptima. Asimismo, usando un modelo simple de 2 ecuaciones, este artículo investiga las implicancias de tener un objetivo asimétrico con respecto a la CC. Específicamente, se considera el caso en el cual las desviaciones negativas con relación al objetivo son consideradas relativamente más costosas. Los resultados indican que, en esta especificación no cuadrática, la política monetaria es claramente más agresiva contra shocks de inflación positivos que en el caso simétrico.

Abstract

This paper explores the implications for optimal monetary policy rules of including a target for the current account (CA) among central bank (CB) objectives. Using a simple but realistic macroeconomic model of the Chilean economy and standard dynamic programming with forward looking variables, the paper finds optimal rules under alternative specifications of a CB quadratic loss-function. The results show that optimal policy reactions change substantially when there is an objective for the CA (besides inflation). Furthermore, once the CA enters the CB objective function, the relative importance of output vis-à-vis inflation variability is less crucial in determining optimal policy rules. Using a simple 2-equation model, the paper then investigates the implications for monetary policy of having an asymmetric objective with respect to the CA. Specifically, it considers the case in which negative deviations from target are considered to be relatively more costly. The results indicate that, in this non-quadratic set-up, monetary policy is clearly more aggressive against positive inflation shocks than in the symmetric case.

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1 Introduction

Policymakers and the academic community have reached an increasing consensus during the last two decades: the primary objective of monetary policy should be to control inflation (see, e.g., King, 1999). A less obvious issue is the appropriate role of the central bank (CB) regarding other (say *secondary*) objectives. Some countries have explicitly included unemployment (or the output gap) within CB objectives, whereas others communicate monetary policy with an explicit reference to the output gap. For example, the FED has “to promote maximum employment” among its goals and the Reserve Bank of Australia has “the maintenance of full employment” as an explicit objective (besides price stability). The Sveriges Riksbank, the Bank of England, and even the ECB have expressed that output gap volatility is a reason to follow a gradualist approach. For example, the Bank of England has recently stated that there are alternative paths to achieve the inflation target and that, in choosing among them, the authority should be concerned about deviations of the level of output from capacity (see Svensson, 1999).

But the output gap and inflation are not the only relevant objectives from a practical point of view. According to its Constitutional Act, the Central Bank of Chile has a formal objective with regard to the stability and well functioning of the external payments system. Operationally, this objective has been interpreted as maintaining a sustainable current account (CA) deficit. Specifically, there is an operational objective of keeping this deficit between 4 and 5% measured at trend terms of trade.[1] Behind this CA objective there is the idea that an excessive CA deficit can easily jeopardize the normal functioning of the external payments system, including access to external financing.

Including a target for the CA deficit in the objective function of a CB potentially has important consequences for the formulation of monetary policy, particularly regarding the optimal reaction to alternative shocks. Both the magnitude and persistence of the appropriate movements in interest rates following a specific shock can change when this objective is considered. This paper investigates this issue deriving and comparing optimal policy rules using a simple macroeconomic model of the Chilean economy. It further analyzes what happens with optimal monetary policy when the CA objective is asymmetric, that is, when it is considered that positive deviations are relatively more

[1] See Massad (1998) for a description of the CB objectives and the way monetary policy is conducted in Chile based on an inflation target. One important distinction of the Chilean economy is its widespread indexation, including financial assets. It should also be mentioned that, before September 1999, the CB also used a target-zone framework as exchange rate policy.

undesirable than negative ones.[2]

The paper first studies the problem of a CB choosing interest rates in order to minimize the expected discount loss of a quadratic loss-function in an economy described by linear equations and in which there are some forward-looking variables. It makes use of standard dynamic programming and departs from other papers by explicitly incorporating the CA in the objective function and by considering a macroeconomic model with the appropriate structure and lags so as to realistically represent the Chilean economy.[3] The model structure is similar to a simple CB macroeconomic projection model. It includes equations for the output gap, the absorption-output gap, a Phillips curve, uncovered interest rate parity and a term structure. Using the model we investigate how optimal policy reactions change when the CA matters and the implications of incorporating the output gap among CB objectives.

The paper then studies in a considerably simpler framework the consequences of having an asymmetric objective for the CA. Simplicity, at that point, is a restriction for being able to solve the CB problem. The economy is described by a CB loss-function that depends on inflation and the absorption-output gap and two linear equations describing their evolution. Again, the CB has to choose interest rates in order to minimize the expected discount loss, this time of a non-quadratic loss-function. The solution method we use in this case is based on standard dynamic programming using a convenient discretization of the economy. We investigate how optimal policy reactions change both with the objective asymmetry and with uncertainty about future shocks.[4]

It is important to clarify at the outset that we refer to *optimal reactions* as the best possible policy rule in terms of maximizing the objective function in the context of the model under analysis. It is not necessarily the best policy for day-to-day policy-making because, by definition, any model is an incomplete description of reality. Actual policy-making might take into account these rules —they are indeed quite appealing as they give precise answers to a quite complex problem of combined lagged effects— but it should also evaluate the implication of developments that are not considered in the

[2] Of course, it is quite difficult to argue from first principles that the CA should be an objective in its own right. Ultimately, it is a short-cut to avoid a very complex model in which inflation (and output) is severely affected by highly non-linear events such as a BoP crisis.

[3] For example, Svensson (2000), Bharucha y Kent (1998), Ryan and Thompson (1999), and many others study optimal policy rules in inflation targeting frameworks for open economies. Among other issues, these papers analyze the usefulness of Monetary Conditions Indexes and the merits of targeting non-traded goods inflation.

[4] Because with asymmetric objectives the problem is not linear-quadratic, there is no certainty equivalence.

model. To be precise, the optimality of the rules we study are model-specific.

The main results of the paper yield relevant policy conclusions. For example, including the CA in the CB objectives has important consequences for optimal policy reactions. Interest rates react vigorously (and without much persistence) to shocks that affect the CA deficit whereas it is less responsive to output gap and inflation shocks. If the CA does not matter, optimal policy rules change considerably when the CB cares about the output gap. However, these changes are far less dramatic if the CB already has the CA among its objectives. An asymmetric CA objective does not change much optimal reactions to demand shocks, but makes monetary policy clearly more aggressive against positive inflation shocks. The change in this policy reaction is economically relevant. Finally, when there is an asymmetric CA objective, a higher volatility in CA shocks generates a more aggressive monetary policy.

The paper is organized as follows. Section 2 describes the structure of the economy as well as monetary policy objectives in a standard linear-quadratic framework. Section 3 presents the solution method for the CB problem and compares optimal policy rules for alternative CB preference parameters using a realistic model estimate. Section 4 studies policy rules in a simpler economy but one in which CA objectives are asymmetric. Finally, section 5 presents some concluding remarks.

2 The Economy

The economy is described by a series of state variables, some exogenous, some endogenous which are predetermined and subject to stochastic shocks, and some endogenous and forward-looking. These variables evolve according to a simple linear macroeconomic model in which monetary policy affects some variables instantaneously and some others with lags.[5] The exogenous variables represent the fundamentals for the economy and follow simple stochastic processes. The economy is endowed with a CB which has an explicit objective function. The frequency of the empirical counterpart of the model is quarterly and we assume that variables are observed contemporaneously.[6]

All variables in the model are measured as deviations from their (possibly stochastic)

[5]The model we consider is not immune to the Lucas' Critique, although expectations are completely rational and the public knows the CB objectives. It should be noticed, however, that Lucas' Critique is not that relevant when policy shocks are small (which is the case considered in this paper).

[6]One way of interpreting this assumption is to consider that monetary policy chooses end-of-period interest rates. An alternative is to consider that actual (off-model) short-term projections are very good (within a quarter).

long-run trend and consequently are stationary. For example, r_t represents deviations of the interest rate with respect to its long-run trend.[7] Appendix 1 shows the results of the estimation and calibration of the equations using Chilean quarterly data.

2.1 Monetary Policy Objectives

The CB has three objectives: (i) to keep inflation close to the target; (ii) to maintain the CA deficit close to a pre-announced value; and (iii) to keep the output gap close to zero. In addition, the CB dislikes large and sudden movements in interest rates.[8] These objectives are summarized in the following quadratic loss-function of period t:

$$\mathbf{l}_t = \mu_\pi \pi_t^2 + \mu_{cc} cc_t^2 + \mu_y y_t^2 + \mu_r \left(r_t - r_{t-1}\right)^2 \tag{1}$$

where π_t is the gap between actual and target inflation, cc_t is the gap between actual and target CA (as a proportion of GDP), and y_t is the gap between actual and potential output.[9] The term $(r_t - r_{t-1})^2$ captures the costs for the CB of frequent and sudden changes in monetary policy. Lastly, μ_π, μ_{cc}, μ_y and μ_r are non-negative constants that describe the CB type.

The CB is forward-looking and in each period t chooses the level of the real (indexed) interest rate in order to minimize the expected discount sum of future losses:

$$\mathbf{L}_t = E\left[\sum_{\tau=0}^{\infty} \delta^\tau \mathbf{l}_{t+\tau} \setminus \Omega_t\right] \tag{2}$$

where Ω_t represents the information set available in t. Moreover, because there are forward-looking variables, both the CB and the public know that the former can re-optimize in each period. Hence, the only possible solution is a discretional one.[10] Therefore, the optimal policy rule has the form $r_t = FX_t$, where X_t is the vector of predetermined variables and F is a vector of constants to be endogenously determined (see Backus and Driffill, 1986 and Söderlind, 1999a for further details).

[7] See footnote 13.

[8] See Woodford (1999a) for a derivation from first principles of this preferences for smooth movements in the policy instrument. Massad (1998) makes the case for instrument stability in Chile. The CB concern regarding the normal functioning of the domestic payments system gives another argument for such an objective.

[9] In this framework the authority recognizes that monetary policy can have only temporal effects on output. Moreover, there are no intertemporal inconsistency problems of the Barro-Gordon type. See footnote 13.

[10] A commitment solution would attain a better outcome (see Woodford, 1999b). However, like Svensson (1999), we assume that a commitment technology is not available. Thus, the only realist solution is discretional.

2.2 State Variables

There are three fundamental exogenous variables in this economy: the terms of trade (in logs), the country risk-premium, and the international real interest rate. We assume that the three variables are stationary (or alternatively, that they are detrended). Their law of motion is given by the following equations:

$$tot_t = \rho_{tot} tot_{t-1} + \rho_{tot}^{r} r_{t-1}^{*} + \xi_t^{tot}, \tag{3}$$

$$r_t^{*} = \rho_r r_{t-1}^{*} + \xi_t^{r}, \text{ and} \tag{4}$$

$$\varphi_t = \rho_{\varphi} \varphi_{t-1} + \xi_t^{\varphi}, \tag{5}$$

respectively, where $0 < \rho_j < 1$ and ξ_t^j are i.i.d. shocks ($j = tot$, r^*, and φ).[11]

Domestic inflation moves according to traded goods inflation, which, in turn, depends on international prices and the exchange rate, and according to no-traded goods inflation, which depends on lags of inflation (due to indexation), on previous periods inflation expectations, and on the output gap (due to a standard acceleracionist Phillips curve). In particular, the (annualized) inflation gap evolves according to:

$$\pi_t = (1-\omega)\pi_t^T + \omega \pi_t^N - \bar{\pi}, \tag{6}$$

where π_t^T is tradables inflation, π_t^N is non-tradables inflation, $\bar{\pi}$ is the inflation target, and ω a constant parameter. On the one hand, tradables inflation follows:

$$\pi_t^T = \alpha_T'(L)(\hat{e}_{t-1} + \pi_{t-1}^{*}) + \xi_t^T \tag{7}$$

where $\alpha_T'(L)$ is a polynomial in the lag operator L, $\hat{e}_t$ is the nominal devaluation, π_t^* is international inflation, and ξ_t^T is an i.i.d. shock. This specification assumes that the weak version of PPP obtains for tradables (controlling for changes in VAT and tariffs) and that there is no instantaneous pass-through. On the other hand, non-tradables inflation evolves according to:

[11] A more realistic approach would be to model the risk-premium as an endogenous variable depending on domestic variables such as the current account. Therefore, an increase in the current account deficit would cause a nominal devaluation following a rise in the risk-premium. Considering this possibility would only add an additional indirect CB concern regarding the current account without substantially changing the main results.

$$\pi_t^N = \alpha'_{\pi L}(L)\pi_{t-1} + E\left[\alpha'_{\pi F}(F)\pi_{t+1} \setminus \Omega_{t-1}\right] + \alpha'_y(L)y_t + \xi_t^N, \tag{8}$$

where $\alpha'_{\pi F}(F)$ is a polynomial in the forward (lead) operator. This means that $E\left[\pi_{t+1} \setminus \Omega_{t-1}\right]$ is a predetermined variable in period t.[12]

Let us define the real exchange rate (RER) as $q_t = e_t p_t^*/p_t$ and assume, without loss of generality, that $\bar{\pi} = 0$. Then the inflation gap equation can be written as:

$$\pi_t = \alpha_T(L)\hat{q}_{t-1} + \alpha_{\pi L}(L)\pi_{t-1} + E\left[\alpha_{\pi F}(F)\pi_{t+1} \setminus \Omega_{t-1}\right]) + \alpha_y(L)y_t + \xi_t^\pi, \tag{9}$$

where $\alpha_T(L) = (1-\omega)\alpha'_T(L)$, $\alpha_{\pi L}(L) = \omega\alpha'_{\pi L}(L) + (1-\omega)\alpha'_T(L)$, $\alpha_{\pi F}(F) = \omega\alpha'_{\pi F}(F)$, $\alpha_y(L) = \omega\alpha'_y(L)$ and $\xi_t^\pi = \omega\xi_t^N + (1-\omega)\xi_t^T$. Furthermore, in order to preserve homogeneity in prices we assume that $\omega\left[\alpha'_{\pi L}(1) + \alpha'_{\pi F}(1)\right] + (1-\omega)\alpha'_T(1) = 1$. Notice that in this framework monetary policy affects inflation through three different channels: the output gap, the exchange rate, and lagged expectations of future monetary policy (and hence, future inflation).

The RER is a forward-looking variable that is determined by the uncovered interest parity condition:

$$q_t = E\left[q_{t+1} \setminus \Omega_t\right] + 0.25(r_t^* - r_t + \varphi_t). \tag{10}$$

Besides the short-term real interest rate controlled by the CB, there is a long-term interest rate determined in a forward-looking variable bond market. According to the expectation hypothesis of the term-structure, the behavior of the long rate R_t can be approximated by:

$$R_t = \lambda r_t + (1-\lambda)E\left[R_{t+1} \setminus \Omega_t\right], \tag{11}$$

where λ is a constant that negatively depends on the bond duration.

The output gap has some persistence and reacts with lags to the interest rates, the terms of trade, and the RER:

$$y_t = \beta_y(L)y_{t-1} + \beta_R(L)R_{t-1} + \beta_r(L)r_{t-1} + \beta_{tot}(L)tot_{t-1} + \beta_q(L)q_{t-1} + \xi_t^y, \tag{12}$$

where ξ_t^y is an i.i.d. shock. This equation represents a standard dynamic IS.

[12] This structure is based on one of the projection models used at the CB of Chile and is similar to the price equation derived from first principles in Svensson (2000).

In order to construct the ratio CA (proxied by the trade balance) to GDP we consider a linear approximation that depends on the gap between absorption and output, denoted by y_t^d, the terms of trade, the output gap, and the RER. A first order (log) Taylor expansion of the trade balance yields the following approximation:

$$cc_t = -y_t^d - k_0 q_t + k_0 y_t + k_1 tot_t, \tag{13}$$

where k_0 is the trade balance to GDP ratio during the expansion year (2% in 1997) and k_1 is the exports to GDP ratio during the expansion year (30% in 1997). The CA deficit during the expansion year was 4.8% of GDP.

Finally, we assume that the absorption-output gap has some persistence and depends on interest rates, the terms of trade, and the RER. In particular, this gap is determined by the following equation:

$$y_t^d = \gamma_d(L) y_{t-1}^d + \gamma_R(L) R_{t-1} + \gamma_r(L) r_{t-1} + \gamma_{tot}(L) tot_{t-1} + \gamma_q(L) q_{t-1} + \xi_t^d, \tag{14}$$

where ξ_t^d is an i.i.d. shock. In order to be consistent with our prior definition of y_t, we consider y_t^d as the gap between (i) the difference between actual and trend absorption and (ii) the output gap.[13] Of course, because both equations include output in the LHS and both represent cyclical movements of the economy, equations (14) and (12) are closely related. Thus, the shocks ξ_t^y and ξ_t^d not need to be orthogonal.[14] In the exercises we present below we consider both independent and common shocks to both equations.

In sum, the economy is described by equations (3) to (5) and (9) to (14). Appendix 1 presents the estimation results of these equations using quarterly Chilean data. We choose a lag structure in each equation so as to maximize the realism and fit of the model, despite this strategy generates several state variables.

3 Solution

The solution of the model requires us to re-write the model so as to represent it in the state-space form. Once it is represented in this way one can directly apply the

[13] In this framework, therefore, we study the effects of monetary policy on the CA without considering secular trends in the expenditure/output gap. This is consistent with the idea that the CB is only capable of choosing (temporal) deviations of the real interest rate with respect to its long-run trend. The latter is actually endogenous and responds to the aforementioned secular trend.

[14] The estimation presented in appendix 1 shows a correlation of 0.25.

algorithms described in Backus y Driffill (1986), Svensson (1994) and Söderlind (1999a). As mentioned before, we consider a discretional solution.

3.1 State-Space Representation

Following the same notation as Svensson (2000), let X_t be the (column) vector of predetermined variables, Y_t the (column) vector of variables that enter as arguments in the CB loss-function, x_t the (column) vector of forward-looking variables, and ξ_t the (column) vector of shocks. Considering the lag structure embedded in the model estimation presented in appendix 1, one has:

$$X_t = \left(tot_t, tot_{t-1}, tot_{t-2}, r_t^*, \varphi_t, q_{t-1}, q_{t-2}, q_{t-3}, \pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_t, y_{t-1}, r_{t-1}, R_{t-1}, y_t^d\right)',$$

$$Y_t = (\pi_t, cc_t, y_t, r_t - r_{t-1})',$$

$$x_t = (R_t, q_t, E\left[\pi_{t+1} \setminus \Omega_{t-1}\right], E\left[\pi_{t+2} \setminus \Omega_{t-1}\right])', \text{ and}$$

$$\xi_t = \left(\xi_t^{tot}, 0, 0, \xi_t^r, \xi_t^\varphi, 0, 0, 0, \xi_t^\pi, 0, 0, 0, \xi_t^y, 0, 0, 0, \xi_t^d\right)'.$$

Let n_1, n_2, n_3, and $n = n_1 + n_2$ be the dimensions of X_t, x_t, Y_t, and Z_t, respectively. In the particular model we consider one has $n_1 = 17$, $n_2 = 4$, and $n_3 = 4$. Let $Z_t = (X_t', x_t')'$ be the vector that describes the state of the economy. Using these definitions the model can be written in the following way (in the state-space):

$$\begin{bmatrix} X_{t+1} \\ E\left[x_{t+1} \setminus \Omega_t\right] \end{bmatrix} = AZ_t + Br_t + \begin{bmatrix} \xi_{t+1} \\ 0 \end{bmatrix}, \tag{15}$$

$$Y_t = C_Z Z_t + C_r r_t, \text{ and} \tag{16}$$

$$\mathbf{l}_t = Y_t' K Y_t, \tag{17}$$

where A is a $n \times n$ matrix, B is a $n \times 1$ column vector, C_Z is a $n_3 \times n$ matrix, C_r is a $n_3 \times 1$ column vector, and K is a $n_3 \times n_3$ diagonal matrix with $(\mu_\pi, \mu_{cc}, \mu_y, \mu_r)$ in the diagonal. Appendix 2 describes in detail the construction of these matrices.

The solution to the CB problem is characterized by a policy function of the following form:

$$r_t = FX_t, \tag{18}$$

where F is a $1 \times n_1$ row vector to be endogenously determined.

The solution also characterizes the evolution of the forward-looking variables according to the following linear function:

$$x_t = HX_t, \tag{19}$$

where H is a $n_2 \times n_1$ matrix to be endogenously determined.

Hence, the dynamics of the economy is given by (18), (19), and the following pair of equations:

$$X_{t+1} = M_{11}X_t + \xi_{t+1}, \text{ and} \tag{20}$$

$$Y_t = \left(C_{z1} + C_{z2}H + C_r F\right) X_t, \tag{21}$$

where M_{11}, C_{z1}, and C_{z2} are $n_1 \times n_1$, $n_3 \times n_1$ and $n_3 \times n_2$ matrices, respectively, corresponding to the partitions of:

$$M \equiv A + B \begin{bmatrix} F \\ 0 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \text{ and}$$

$$C_Z = \begin{bmatrix} C_{Z1} & C_{Z2} \end{bmatrix}$$

according to X_t and x_t:

In order to find the matrices F and H we use the algorithm mentioned above.

3.2 Optimal Reaction Functions

The optimal policy rule depends both on the structure of the economy and the preferences of the CB. Preferences, in turn, are described by a discount rate and by a set of relative weights for the loss-function (1). We consider a discount factor of 0.99 and four sets of weights for (1) and thus define four types of CBs. These are: “Hawk”, which has inflation as its almost unique objective; “Dove”, whose objectives include both inflation and the output gap; “Strict Condor”, whose objectives are inflation and the CA deficit; and “Flexible Condor”, whose objectives include inflation, the CA deficit, and the output gap. We assume that the four CBs dislike large changes in interest rates and therefore have $\mu_r > 0$. Table 1 presents the weights (the μs) of the loss-function of each of these CBs.[15]

Table 1: Four Central Bank Types

	Preference Parameters			
	μ_π	μ_{cc}	μ_y	μ_r
Hawk (H)	0.95	0.00	0.05	0.10
Dove (D)	0.50	0.00	0.50	0.10
Strict Condor (SC)	0.65	0.30	0.05	0.10
Flexible Condor (FC)	0.35	0.30	0.35	0.10

Table 2 presents the optimal reaction functions for each of the four CB types. Each column corresponds to the vector F', which, multiplied by the vector X_t yields the optimal interest rate deviation for period (quarter) t. For example, if the CB is Hawk-type, then it would increase the (indexed) interest rate by 0.45% after a 1% shock in inflation.[16]

The dynamic reaction of the interest rate to alternative shocks is another way of analyzing optimal policy rules. These reactions show the path of interest rates that would prevail if the rule is followed and no other shocks occur, and thereby include reactions to expected future movements in other endogenous variables.

Figure 1 presents the path of interest rates between quarter zero (on impact) and quarter 7 following different exogenous shocks. Besides the 6 basic (structural) shocks of the model we consider a combined shock of output and absorption-output gap, which could be thought of as the case of an over-heated economy. The figure presents the reactions of the four CBs. The LHS panels show the reaction of H and P, while the RHS ones show the reaction of SC and FC.

The results of table 2 and figure 1 have relevant implications in at least three dimensions (besides the numerical results that may be useful as a benchmark for policy-making). First, once the CA is part of the CB objectives, optimal policy reactions change

[15]The CB preferences are "deep" parameters unaffected by the structure of the economy (e.g., the degree of indexation). The structure of the economy modifies the optimal reaction functions, not preferences.

[16]Notice that interest rate persistence depends on both the parameter for r_{t-1} and R_{t-1}.

Table 2: Optimal Policy Functions

	Central Bank Type			
	Hawk	Dove	Strict Condor	Flexible Condor
tot_t	-0.02	-0.01	-0.11	-0.10
tot_{t-1}	0.02	0.03	0.05	0.05
tot_{t-2}	0.01	0.02	0.01	0.01
r_t^*	0.87	0.43	0.52	0.34
φ_t	0.72	0.32	0.43	0.27
q_{t-1}	-0.13	-0.06	-0.09	-0.06
q_{t-2}	-0.09	-0.04	-0.07	-0.04
q_{t-3}	-0.11	-0.05	-0.06	-0.04
π_t	0.45	0.28	0.24	0.17
π_{t-1}	0.32	0.19	0.17	0.11
π_{t-2}	0.19	0.11	0.10	0.07
π_{t-3}	0.09	0.05	0.04	0.03
y_t	0.57	0.60	0.31	0.33
y_{t-1}	0.12	0.07	0.06	0.04
r_{t-1}	0.28	0.12	-0.04	-0.07
R_{t-1}	-0.27	-0.37	-0.15	-0.20
y_t^d	0.00	0.00	0.13	0.12

in many respects, sometimes substantially. As expected, SC and CF respond to shocks in the absorption-output gap and in terms of trade in a very different way from what H and D do. These two shocks have direct effects in the CA, for which the two latter CBs do not care. Interestingly, SC and CF are less responsive to international interest rate and risk-premium shocks (vis-à-vis H and D, respectively). More importantly, when there is a target for the CA, monetary policy is clearly less aggressive in responding to shocks to inflation (compare H to SC) and to the output gap. All these differences are economically relevant.

Second, the results show that, in general, reactions are less persistent for shocks that directly and indirectly affect the CA when the CA matters. This is probably due to the relative lower persistence in the absorption-output gap in comparison to other variables. However, the reaction to inflation and output shocks are more persistent in SC and FC than in H and D.

Finally, the differences in optimal rules that arise when the CB incorporates the output gap among its objectives are far less dramatic when the CA is already part of its objectives. Indeed, the differences between H and D reactions are considerably more important than the difference between SC and FC reactions. One case in which this difference appears most clearly is in the dynamic CB reaction to an inflation shock — probably the key issue in monetary policy-making. In this case SC is only marginally more *hawkish* than FC. Thus, it is possible to conclude that incorporating the output gap into CB objectives does not appear to be crucial when the CA already matters. The intuition for this result is simple: the CA also presents a trade-off for monetary policy as more aggressive inflation shock responses generate larger CA deviations. However, for this result to hold, it is necessary to have a symmetric CA objective. The next section analyzes optimal policy reactions when this objective is asymmetric.

4 Optimal Monetary Policy with an Asymmetric CA Objective

Ultimately, behind the objective of keeping the CA under control is the notion that an "excessive" deficit usually brings about a serious BoP crisis, in which voluntary financing disappears and the economy enters a deep recession. The converse, however, is not true. Too small a deficit is not considered to be a threat to external stability. But the loss function described by (1) is symmetric, weighting positive and negative deviations as equally undesirable. Therefore, the rules we have derived are, by construction, symmetric. This section investigates the implications for monetary policy of having an asymmetric loss-function for the CA.

We analyze two issues that arise with asymmetric objectives. First, we investigate how much optimal reactions change, both against price and demand (CA) shocks. Second, we analyze whether optimal monetary policy changes when shock volatility increases. We evaluate both how policy changes and the economic relevance of these changes.

Departing from the linear-quadratic framework (i.e., from a symmetric CA objective) has the benefit of allowing for a more realistic loss-function. However, at the same time, it seriously complicates the problem. In the linear-quadratic problem we were able to find closed-form solutions of the form $r_t = FX_t$, whereas in this case we have to resort to numerical simulations. To be able to solve the CB problem we have to simplify the model of the economy substantially. In particular, we calibrate a simple stochastic two-equation backward-looking economy with two state variables, namely inflation and the absorption-output gap, and directly associate the latter with the CA deficit. For simplicity, we assume that a positive absorption-output gap has inflationary consequences.[17] Moreover, we do not consider any preferences over interest rate variability, nor bounds for possible interest rate values. Thus, the rules we derive will in general prescribe a more aggressive monetary policy than the one a CB would normally follow. Accordingly, rather than taking the quantitative results we derive at face value, they should be analyzed relative to the baseline-case scenario of rules for a symmetric CA objective (derived from a standard quadratic loss-function).

4.1 A Simpler Economy

The economy is described by the following two equations:

$$\pi_t = \pi_{t-1} + \alpha_y y^d_{t-1} + \varepsilon^\pi_t \tag{22}$$

and

$$y^d_t = \beta_y y^d_{t-1} + \beta_r r_{t-1} + \varepsilon^y_t \tag{23}$$

[17]It should be stressed that this is a simplification. There is no reason for this gap to have direct inflationary consequences. What happens is that it is usually highly correlated to the output gap, which does affect inflation.

where, as before, π_t is the inflation gap in period t, y_t^d is the absorption-output gap, r_t is the real (or indexed) interest rate (again measured as deviation with respect to its long run trend), and ε_t^π and ε_t^y are serially uncorrelated mean-zero stochastic shocks. α_y, α_r, and β_y are constant parameters.

As before, the CB problem at time t is to choose a sequence of interest rates $\{r_{t+\tau}\}_{\tau=0}^{\infty}$ so as to minimize the intertemporal loss function (2). Let x_t be the vector with the state variables, $x_t = (\pi_t, y_t^d)'$. We seek to characterize the interest rate sequence through a time-invariant (probably non-linear) policy function for a loss-function of the following type:

$$\mathbf{l}(x_t) = \begin{cases} a\pi_t^2 + b_L(y_t^d)^2 & \text{for} \quad y_t^d \in (-\infty, 0) \\ a\pi_t^2 + b_H(y_t^d)^2 & \text{for} \quad y_t^d \in (0, \infty), \end{cases} \tag{24}$$

where a, b_L, and b_H are positive constants. The asymmetric CA objective is described by $b_L < b_H$.

Therefore, the CB problem is to set interest rates in order to minimize (2) with (24) subject to (22) and (23). We consider three alternative CB "types": Quadratic, Asymmetric, and One-sided. Table 3 presents the preference parameters of each of these CBs. The quadratic CB has the standard symmetric loss-function, the asymmetric CB cares considerably more about positive CA deviations from target (vis-à-vis negative ones), whereas the one-sided CB dislikes positive CA only (besides inflation). In all cases, on average, there is a 2 : 1 weight relation between inflation and CA gap deviations.

Table 3: Loss-Functions of Alternative Central Banks

	Quadratic	Asymmetric	One-sided
a	1	1	1
b_L	0.5	0.2	0.0
b_H	0.5	0.8	1.0
δ	0.95	0.95	0.95

We calibrate the model using the following parameter values: $\alpha_y = 0.5$, $\beta_y = 0.6$, $\beta_r = -0.5$, $S.D.(\varepsilon_t^\pi) = 0.8\%$, $S.D.(\varepsilon_t^y) = 0.8\%$, and zero covariance between the shocks. These parameters are approximately in line with the estimation of equations (22) and (23) with Chilean data —with the caveat that we use the output gap in the former equation (see footnote 17).[18] The frequency of the real-world counterpart of the model can be thought of as being either quarterly or semi-annual.

4.2 Solution

In order to solve the problem we discretize the economy along the lines followed by Medina and Valdés (2000) and apply standard dynamic programming techniques. Appendix 3 briefly reviews the procedure. It should be mentioned that this approximation yields quite accurate solutions for the linear-quadratic case.

Figure 2 shows the optimal policy functions for the CB problem considering alternative combinations of inflation and absorption-output gap. Each panel shows the optimal reaction function of the three types of CBs. The x-axis shows a gap deviation and the y-axis shows the optimal policy reaction (given by the interest rate deviation).

The results show two interesting features. First, and more importantly, the asymmetry generates a more aggressive respond to positive inflation shocks. The intuition is straightforward: there is no trade-off involved. The magnitudes of the differences are quite large. For example, if the CA gap is zero while the inflation gap is 3%, a one-sided CB will increase interest rates almost twice as much as the symmetric one. The asymmetric CB will increase them by one-third more than the symmetric one. In the case of negative inflation shocks, the asymmetry generates less aggressive monetary policy (higher interest rates). Hence, the two effects together imply that a CB with an asymmetric CA objective can end up having inflation, on average, below its target.[19]

Second, at least for the case in which there is a negative or a zero inflation gap, the asymmetric CA objective does not affect much the optimal policy response against demand (CA) shocks. The reason for this result is that even if an expenditure gap does not matter directly, it can indirectly matter through its future impact on inflation. Because inflation has a unit root the more so. This is not the case if there is a positive absorption-output gap and a positive inflation gap. In this case monetary policy is more aggressive because the no-trade-off argument applies.

Figure 3 presents what happens to optimal policy reactions when the standard deviation of CA shocks increases. As one could have expected, when the standard deviation

[18] The estimation of (22) yields a considerably higher inflation volatility. However, as shown by Magendzo (1998), in Chile there is close relation between the level and volatility of inflation. The estimation of the same equation using the difference between actual and long-run trend inflation yields $S.D.(\varepsilon_t^{\pi}) = 0.2\%$.

[19] Of course, one can argue that the bias is for (not against) inflation if the counterfactual is a CB with no CA objective at all.

rises from 0.8% (the baseline case) to 1.4%, the CB reactions are more aggressive against inflation. However, the economic relevance of these changes are not important. Only when the standard deviation is 2% the differences start to be more relevant, although they still are less important than the differences created by the asymmetry itself. In the side of negative inflation shocks, we find that a higher shock volatility makes monetary policy also more aggressive. One possible explanation for such a finding is that when the inflation gap is very negative a higher demand volatility risks a movement towards an even more negative gap (through it Phillips curve effect). Preemptively, when volatility is higher, monetary policy tries to return inflation to target faster. Again, the differences are not that relevant from an economic perspective.

5 Concluding Remarks

This paper has presented two different models in order to analyze the implications for monetary policy of having a target for the CA deficit among CB objectives. The first model is a simple linear-quadratic model estimated (calibrated) for the Chilean economy with several state variables, some of which are forward-looking. The second model is a simple 2-equation model but in which the authority has an asymmetric objective for the CA.

The analysis of the realistic linear-quadratic economy indicates that including the CA in the CB objectives has important consequences for optimal policy reactions. Interest rates are less reactive to inflation and the output gap shocks and, as expected, more reactive to shocks that directly affect the CA. For example, the optimal reaction to a terms of trade shocks is completely different once the CB cares about the CA.

Furthermore, the results show that optimal monetary rules change much less when the CB incorporates the output gap among its objectives if it already has a target for the CA among them. Therefore, while the discussion about incorporating unemployment in the CB objectives is very relevant in Chile, it is considerably less so than in a country in which there are no CA objectives. Of course, this result hinges from having a symmetric objective for something that offers a trade-off to a rapid inflation control.

The asymmetric nature of the CA objective has important consequences for optimal policy reactions against inflation shocks. Compared to the case of a symmetric CA objective, monetary policy is considerably tighter when the objective is asymmetric. When the CA shock volatility increases and the CB has an asymmetric CA objective, monetary policy becomes more aggressive (more reactive). This last change, however, is quantitatively less important from an economic perspective.

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Appendix 1. Model Estimation/Calibration

This appendix describes the estimation results and calibration for the Chilean economy of the model presented in the main text. The data has quarterly frequency and the sample size varies according to data availability.

For the risk-premium equation, we assume an AR(1) parameter of 0.90, equal to the one estimated for the international interest rate. We calibrate the CA gap equation with a log-Taylor expansion in 1997 (see main text for further details). We estimate the inflation equation using a specification in levels with international inflation (which includes nominal depreciation, foreign inflation, and VAT and tariff changes) as part of the explanatory variables and impose homogeneity in prices. We then modify the equation incorporating the inflation target and the RER. In the output equation we assume elasticities with respect to the RER and estimate the rest of the parameters. In this and the next appendix we denote the conditional expectation $E\left[x_{t+\tau} \setminus \Omega_t\right]$ as $x_{t+\tau|t}$.

- **Terms of Trade**

$$tot_t = \underset{(21.4)}{\underset{0.86}{\rho_{tot}}} \; tot_{t-1} + \underset{(-3.7)}{\underset{-0.53}{\rho_{tot}^{r}}} \; r^*_{t-1} + \xi_t^{tot}$$

OLS, Sample: 77:2–98:2, Robust Newey-West t-tests
$\bar{R}^2 = 0.85$, F-stat = 237.4, Durbin-h = 1.19

- **International Real Interest Rate**

$$r^*_t = \underset{(18.8)}{\underset{0.90}{\rho_r}} \; r^*_{t-1} + \xi_t^r$$

OLS, Sample: 77:2–98:2, Robust Newey-West t-tests
$\bar{R}^2 = 0.84$, F-stat = 443.0, Durbin-h = 2.30

- **Country Risk-premium (calibration)**

$$\varphi_t = \underset{0.90}{\rho_\varphi} \; \varphi_{t-1} + \xi_t^\varphi$$

- **Inflation (levels)**

$$\pi_t - \pi_{t-1} = \underset{\substack{0.41 \\ (3.4)}}{\psi_0} \left[\left(\pi_{t-2} + \pi_{t-3} + \pi_{t-4}\right)/3 - \pi_{t-1}\right] + \underset{\substack{0.29 \\ (2.0)}}{\psi_1} \left[\left(\pi_{t+1|t-1} + \pi_{t+2|t-1}\right)/2 - \pi_{t-1}\right]$$

$$+ \underset{\substack{0.15 \\ (2.3)}}{\psi_2} \left[\left(p^*_{t-1} + e_{t-1} - p^*_{t-4} - e_{t-4}\right)/3 - \pi_{t-1}\right]$$

$$+ \underset{\substack{0.39 \\ (2.1)}}{\psi_3} \left(y_{t-1} + y_{t-2}\right)/2 + \underset{\substack{0.08 \\ (6.2)}}{\psi_4} D912 + \underset{\substack{1.97 \\ (2.1)}}{\psi_5} DTax + \xi_t^{\pi}$$

TSLS, Sample: 88:3–98:4, Robust Newey-West t-tests
$\bar{R}^2 = 0.59$, F-stat = 11.52
Instruments: inflation target, wages, R_{t-1}, and lags.

- **Inflation Gap (transformation)**

$$\pi_t = \underset{0.20}{\alpha^0_{\pi L}} \pi_{t-1} + \underset{0.37}{\alpha^1_{\pi L}} \left(\pi_{t-2} + \pi_{t-3}\right)/2 + \underset{0.14}{\alpha^2_{\pi L}} \pi_{t-4} + \underset{0.29}{\alpha_{\pi F}} \left(\pi_{t+1|t-1} + \pi_{t+2|t-1}\right)/2$$

$$+ \underset{0.60}{\alpha_q} \left(q_{t-1} - q_{t-4}\right)/3 + \underset{0.39}{\alpha_y} \left(y_{t-1} + y_{t-2}\right)/2 + \xi_t^{\pi}$$

- **Long-term Real Interest Rate**

$$R_t = \underset{\substack{0.13 \\ (2.1)}}{\lambda} r_t + \underset{\substack{0.87 \\ (18.8)}}{(1-\lambda)} R_{t+1|t}$$

TSLS, Sample: 87:3–98:4, Robust Newey-West t-tests
$\bar{R}^2 = 0.74$, F-stat = 110.1, D-W = 1.64
Instruments: π_t, y_t^d, tot_t, and lags.

- **Output Gap**

$$y_t = \underset{(11.1)}{\underset{0.61}{\beta_y}} y_{t-1} + \underset{(-3.3)}{\underset{-0.97}{\beta_R}} \left(R_{t-1} + R_{t-2}\right)/2 + \underset{(-1.7)}{\underset{-0.28}{\beta_r}} r_{t-2}$$
$$+ \underset{(4.2)}{\underset{0.04}{\beta_{tot}}} \left(tot_{t-2} + tot_{t-3}\right)/2 + +0.005q_{t-1} + 0.010q_{t-2} + 0.015q_{t-3} + 0.020q_{t-4} + \xi_t^y$$

OLS, Sample: 87:3–98:4, Robust Newey-West t-tests
$\bar{R}^2 = 0.57$, F-stat = 16.1, Durbin-h = −0.72

- **Absorption-Output Gap**

$$y_t^d = \underset{(2.4)}{\underset{0.29}{\gamma_d}} y_{t-1}^d + \underset{(-3.3)}{\underset{-0.91}{\gamma_r}} (r_{t-1} + r_{t-2})/2 + \underset{(-0.5)}{\underset{-0.02}{\gamma_{q1}}} q_{t-1}$$
$$+ \underset{(-0.8)}{\underset{-0.02}{\gamma_{q2}}} q_{t-2} + \underset{(-2.0)}{\underset{-0.03}{\gamma_{q3}}} q_{t-4} + \underset{(-0.3)}{\underset{-0.02}{\gamma_{q4}}} q_{t-4} + \underset{(4.2)}{\underset{0.08}{\gamma_{tot}}} tot_{t-2} + \xi_t^d$$

OLS-PDL for q, Sample: 86:2–98:4, Robust Newey-West t-tests
$\bar{R}^2 = 0.49$, F-stat = 11.1, Durbin-h = 0.85

- **Current Account Gap (calibration)**

$$cc_t = -y_t^d + \underset{0.02}{k_0} q_t - \underset{0.02}{k_0} y_t + \underset{0.30}{k_1} tot_t$$

Calibration based on a log-Taylor expansion with 1997 as the base year.

- **Uncovered Interest Rate Parity**

$$q_t = q_{t+1|t} + 0.25(r_t^* + \varphi_t - r_t)$$

Appendix 2. State-space Representation

This appendix shows how to write the model in the state-space form. As noted by Svensson (2000), given rational expectations, the inflation gap has the following behavior:

$$\pi_{t+1} = \pi_{t+1|t} + \xi_{t+1}^{\pi}.$$

Moreover, shifting one period forward the inflation equation and taking expectations with information up to time t, one has an equation that can be solved for $\pi_{t+3|t}$ as a function of known variables and of $\pi_{t+2|t}$ and $\pi_{t+1|t}$. With these considerations one can write the matrix A of equation (15) in the following way:

$$A = \begin{bmatrix} \rho_{tot} E_1 + \rho_{tot}^{r} E_4 \\ E_1 \\ E_2 \\ \rho_r E_4 \\ \rho_{\varphi} E_5 \\ E_{19} \\ E_6 \\ E_7 \\ E_{20} \\ E_9 \\ E_{10} \\ E_{11} \\ A_{13\cdot} \\ E_{13} \\ 0 \\ E_{18} \\ A_{17\cdot} \\ \left[1/\left(1-\lambda\right)\right] E_{18} \\ E_{19} - 0.25 E_4 - 0.25 E_5 \\ E_{21} \\ A_{21\cdot} \end{bmatrix}$$

where E_j represents a $1 \times n$ row vector with a one in position j and zeros otherwise, and $A_{j\cdot}$ is row j of matrix A. Furthermore,

$$A_{13\cdot} = \beta_y E_{13} + \beta_R \left(E_{16} + E_{18}\right)/2 + \beta_r E_{15} + \beta_{tot} \left(E_2 + E_3\right)/2 + \beta_{q1} E_{19} + \beta_{q2} E_6 + \beta_{q3} E_7 + \beta_{q4} E_8,$$

$$A_{17\cdot} = \gamma_d E_{17} + \gamma_r E_{15}/2 + \gamma_q \left(E_6 + E_{19}\right)/2 + \gamma_{tot} E_2 + \gamma_{q1} E_{19} + \gamma_{q2} E_6 + \gamma_{q3} E_7 + \gamma_{q4} E_8,$$

and

$$A_{21\cdot} = \frac{2}{\alpha_{\pi F}} \left\{ \begin{array}{c} E_{20} - \alpha_{\pi L}^{0} E_9 - \alpha_{\pi L}^{1} \left(E_{10} + E_{11}\right)/2 - \alpha_{\pi L}^{2} E_{12} \\ -\alpha_{\pi F} E_{21}/2 - \alpha_q \left(E_{19} - E_8\right)/3 - \alpha_y \left(E_{13} + E_{14}\right)/2 \end{array} \right\}.$$

The B vector, in turn, is given by:

$$B = \left(0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,\gamma_r/2,-\lambda/\left(1-\lambda\right),0.25,0,0\right)'.$$

The C_Z matrix and the C_r vector of (16) are given by:

$$C_Z = \begin{bmatrix} E_9 \\ -E_{17} - k_0 E_{19} + k_0 E_{13} + k_1 E_1 \\ E_{13} \\ -E_{15} \end{bmatrix} \text{ y } C_r = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Finally, the K matrix of (17) is given by:

$$K = \begin{bmatrix} \mu_\pi & 0 & 0 & 0 \\ 0 & \mu_{cc} & 0 & 0 \\ 0 & 0 & \mu_y & 0 \\ 0 & 0 & 0 & \mu_r \end{bmatrix}.$$

Appendix 3. Discretization of the Economy and Dynamic Programming

This appendix describes the discretization of the economy presented in section 4. It follows Medina and Valdés (2000).

In order to transform the continuous economy described by equations (22) and (23) and the control variable r_t into a discrete-state space economy we proceed as follows. First assume that the economy can be in m possible states collected in a set $\Gamma = \left\{(\pi_1, y_1^d), (\pi_2, y_2^d), \ldots, (\pi_m, y_m^d)\right\}$. Further, assume that the interest rate r_t can take n different values arranged in a set $\Psi = \{r_1, r_2, \ldots, r_n\}$.

To preserve the dynamic and stochastic properties embedded in equations (22) and (23) we calculate transition probabilities between states that depend on the interest rate. Assume that the shocks ε_t^π and ε_t^y follow a bivariate normal distribution with variance-covariance matrix Σ. It is then straightforward to calculate the conditional distribution of the vector $x_t = (\pi_t, y_t^d)'$ conditional on $(\pi_{t-1}, y_{t-1}^d)'$ and r_{t-1} as the bivariate normal distribution.

To construct the transition matrix associated to each interest rate in Ψ we divide a rectangle $[a_\pi, b_\pi] \times [a_y, b_y] \subseteq \Re^2$ into m equal-sized rectangles $\theta_1, \theta_2, \ldots, \theta_m$ and consider that the center of each of these rectangles form the set Γ. For each feasible interest rate $r_k \in \Psi$ we can define the transition probability p_{ij}^k as the likelihood of a movement from state $\left(\pi_i, y_i^d\right)$ in $t-1$ to state $\left(\pi_j, y_j^d\right)$ one period ahead given that $r_{t-1} = r_k$. This probability is given by:

$$p_{ij}^k = \Pr\left[\left(\pi_t, y_t^d\right)' \in \theta_j \setminus \left(\pi_{t-1}, y_{t-1}^d\right) = \left(\pi_i, y_i^d\right), r_{t-1} = r_k\right]$$

$$= \left(\int_{\theta_j} \frac{1}{2\pi|\Sigma|^{1/2}} \exp\left[-\frac{\left(x - \bar{x}_i^k\right)' \Sigma^{-1} \left(x - \bar{x}_i^k\right)}{2}\right] dx\right) \div$$

$$\int_{[a_\pi, b_\pi]\times[a_y, b_y]} \frac{1}{2\pi|\Sigma|^{1/2}} \exp\left[-\frac{\left(x - \bar{x}_i^k\right)' \Sigma^{-1} \left(x - \bar{x}_i^k\right)}{2}\right] dx,$$

where

$$\bar{x}_i^k = E\left(\begin{bmatrix} \pi_t \\ y_t^d \end{bmatrix} \setminus \pi_{t-1} = \pi_i, y_{t-1}^d = y_i^d, r_{t-1} = r_k\right).$$

Because we consider a subset of the possible states of the economy, this discretization is only an approximation. In the neighborhood of the borders of the rectangle $[a_\pi, b_\pi] \times [a_y, b_y]$ the approximation is far from accurate. When we calculate optimal policy rules we use the complete rectangle, but analyze them using the neighborhood of its center only.

With the discretiziced economy, the original CB problem becomes:

$$\min_{\{r_{t+\tau}\}_{\tau=0}^{\infty}} E \sum_{\tau=0}^{\infty} \delta^\tau l(\pi_{t+\tau}, y_{t+\tau}^d)$$

$$\begin{aligned} s.t. \quad & \left(\pi_{t+1}, y_{t+1}^d\right) \in \Gamma = \left\{\left(\pi_1, y_1^d\right), \ldots, \left(\pi_m, y_m^d\right)\right\} \\ & r_t \in \Psi = \{r_1, \ldots, r_k\} \\ & \Pr\left[\left(\pi_{t+1}, y_{t+1}^d\right)' \in \theta_j \setminus \left(\pi_t, y_t^d\right) = \left(\pi_i, y_i^d\right), r_t = r_k\right] = p_{ij}^k, \end{aligned}$$

where $\sum_{j=1}^m p_{ij}^k = 1$, for all $i = 1, \ldots, m$ and for all $k = 1, \ldots, n$.

Given the transition probabilities, the associated Bellman equation is:

$$V\left(\pi_i, y_i^d\right) = \min_{r_k \in \Psi} \left\{ l(\pi_i, y_i^d) + \delta \sum_{j=1}^m p_{ij}^k V\left(\pi_j, y_j^d\right) \right\}, \tag{25}$$

for all $i = 1, \ldots, m$ and for all $k = 1, \ldots, n$, where $V\left(\pi_i, y_i^d\right)$ is the optimal value of the objective function starting from an inflation π_i and a gap y_i^d in t.

The solution of the CB problem is characterized by a value function $V(.,.)$ that satisfies (25) and an associated policy function $r' = h(\pi, y^d)$ mapping the state of the economy (π, y^d) into an optimal choice of interest rate.

To solve the problem we define v to be a vector in $\Re^m$ and $T[.]$ to be an operator that maps a vector v into a new vector $T[v] = (tv_1, tv_2, \ldots, tv_m)'$ in which each element tv_i is given by:

$$tv_i = l\left(\pi_i, y_i^d\right) + \min\left\{ \sum_{j=1}^m p_{ij}^1 v_j, \sum_{j=1}^m p_{ij}^2 v_j, \ldots, \sum_{j=1}^m p_{ij}^k v_j \right\}. \tag{26}$$

Thus, the Bellman equation can be represented by

$$v = T[v],$$

which can be solved by iterating until convergence the following recursion:

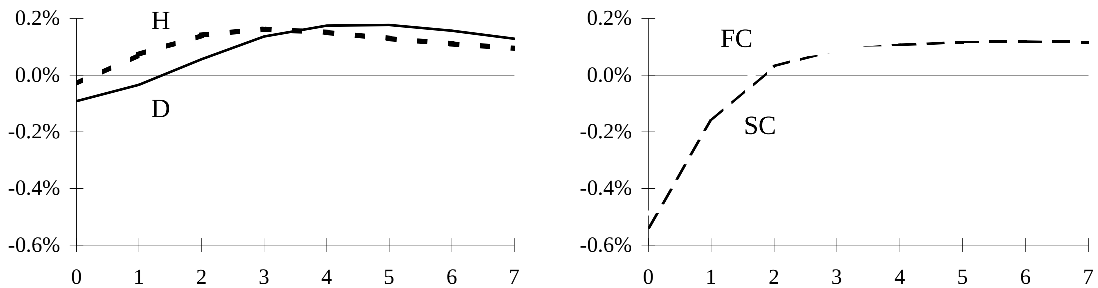
$$(v)_{s+1} = T[(v)_s].$$

Given the solution v^*, the optimal policy function h satisfies:

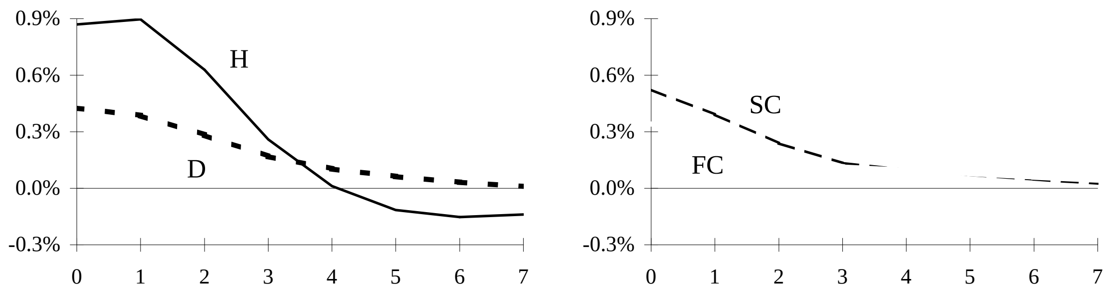
$$r' = h(\pi, y^d) \in \arg\min_{r_k \in \Psi} \left\{ \sum_{j=1}^m p_{ij}^k v_j^* \right\}.$$

Figure 1: Reaction Functions of Four Central Banks

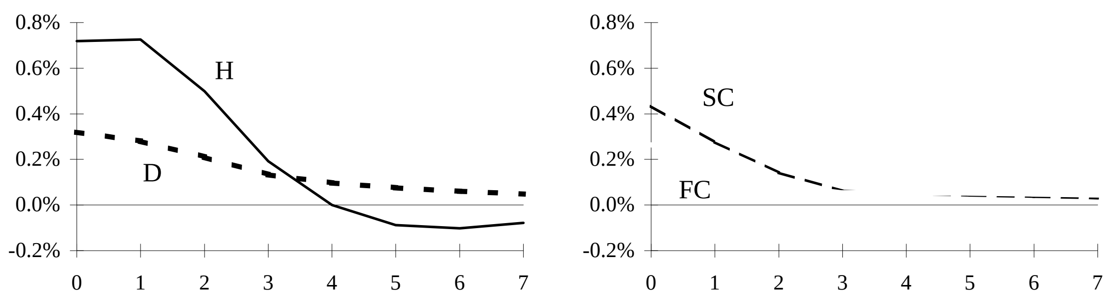
5% Shock in Terms of Trade



1% Shock in Real International Interest Rate



1% Shock in Country Risk-Premium



1% Shock in Inflation Gap

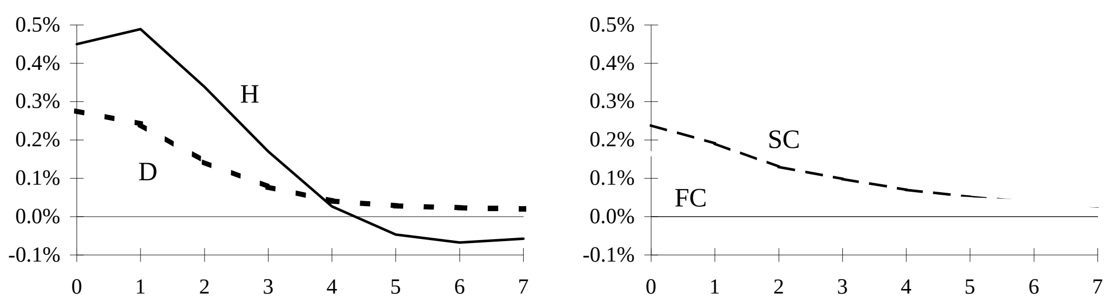
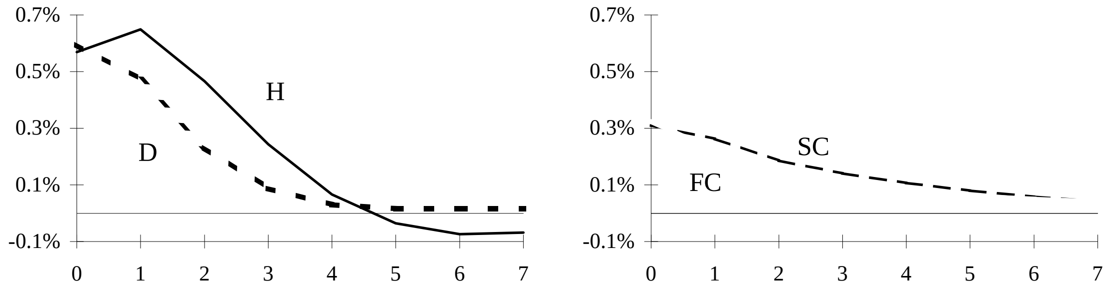
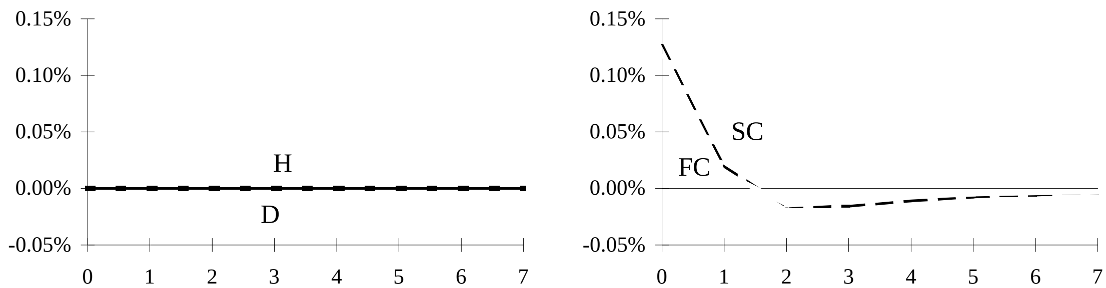


Figure 1: Reaction Functions of Four Central Banks (cont'd.)

1% Shock in Output Gap



1% Shock in Absorption-Output Gap



1% Simultaneous Shock in Output Gap and Absorption-Output Gap

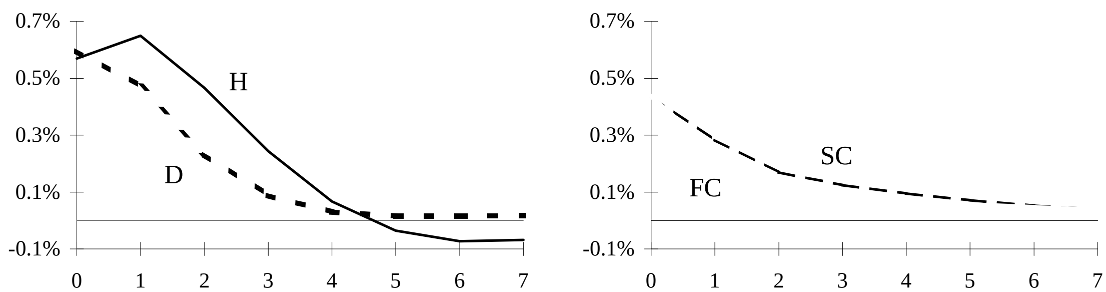


Figure 2: Optimal Policy Functions with an Asymmetric CA Objective

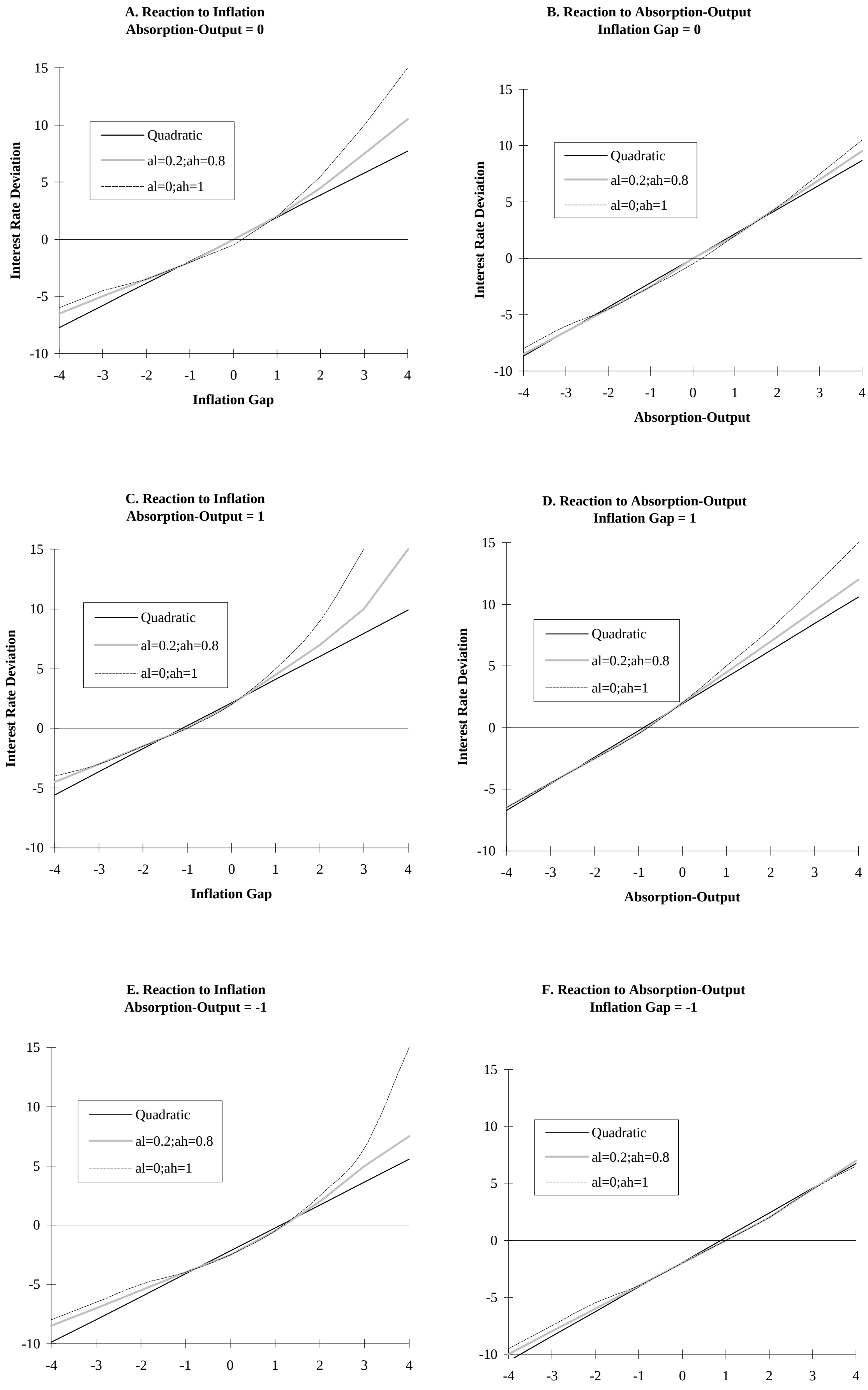
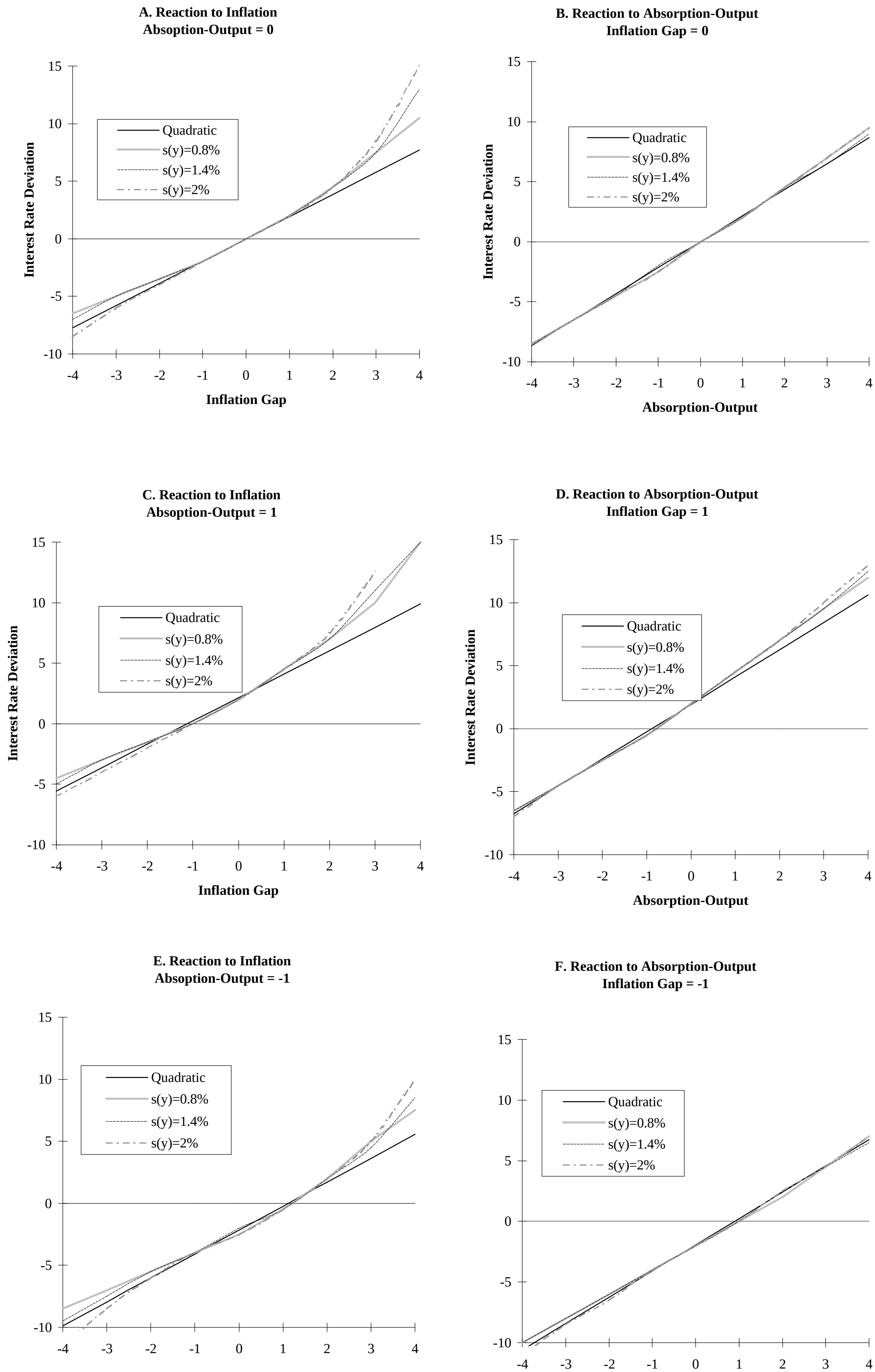


Figure 3: Effects of Volatility on Optimal Policy Functions

(Case: ah = 0.8; al = 0.2)



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