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THE OUT-OF-SAMPLE PERFORMANCE OF AN EXACT MEDIAN-UNBIASED ESTIMATOR FOR THE NEAR-UNITY AR(1) MODEL*

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Abstract

We analyse the forecasting performance of several strategies when estimating the near-unity AR(1) model. We focus on the Andrews' (1993) exact median-unbiased estimator (BC), the OLS estimator and the driftless random walk (RW). We also explore two pairwise combinations between these strategies. We do this to investigate whether BC helps in reducing forecast errors. Via simulations, we find that BC forecasts typically outperform OLS forecasts. When BC is compared to the RW we obtain mixed results, favouring the latter while the persistence of the true process increases. Interestingly, we find that the combination of BC-RW performs well in a near-unity scheme.

Resumen

En este trabajo analizamos el rendimiento predictivo de algunas estrategias típicamente utilizadas para estimar un modelo AR(1) cercano a la raíz unitaria. Nos enfocamos en el estimador exacto insesgado en mediana de Andrews (1993) (BC), el estimador OLS y la caminata aleatoria sin constante (RW). También exploramos dos pares de combinaciones entre estas estrategias. Esto se realiza con el fin de investigar cuándo BC ayuda a reducir los errores de proyección. Vía simulaciones, encontramos que las proyecciones BC típicamente superan las OLS. Cuando BC es comparado con RW obtenemos resultados mixtos, favoreciendo RW cuando la persistencia del modelo verdadero se incrementa. Interesantemente, encontramos que la combinación BC-RW realiza un buen trabajo en un ambiente cercano a la raíz unitaria.

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1 Introduction

The fit of the AR(1) model $y_t = \widetilde{c} + \phi y_{t-1} + \varepsilon_t$, with $-1 \leq \phi < 1$, $\widetilde{c} = c(1-\phi)$, and $\varepsilon_t \sim iid\mathcal{N}(0, \sigma_\varepsilon^2)$, for a finite-sample autocorrelated variable has been fairly used in financial and applied economics for many purposes. If ϕ is not estimated and instead it is assumed equal to unity, the equation becomes the well-known random walk (RW) model. This particular specification has proved to be convenient for a number of economic time series. It is also common, especially within economics, that when the parameter ϕ is estimated, it is done through the OLS method, provided a biased estimator in finite samples (Marriott and Pope, 1954; Kendall, 1954).

According to the Frisch-Waugh-Lovell theorem, the OLS estimation is obtained also with $(y_t - \overline{y}) = \phi(y_{t-1} - \overline{y}) + \varepsilon_t$, where $\overline{y} = T^{-1}\sum_t^T y_t$. The downward bias comes from the correlation between $(y_{t-1} - \overline{y})$ and ε_t. There is no analytical expression for this bias, but it could be approximated with simulation methods (Hansen, 1999; Patterson, 2000).

Given this shortcoming, several bias-correction methods emerge.[1] One of the methods for the AR(1) case is the median-unbiased estimator of Andrews (1993). A number of applications are based on this estimator, covering from similar bias-correction procedures to several macroeconomic estimations.

Less attention has been devoted to the out-of-sample behaviour of these bias-correction procedures. However, Maekawa (1987), Gospodinov (2002), and Kim and Durmaz (2012) are some exceptions.

The aim of this article is to analyse the forecasting performance of several strategies for the near-unity AR(1) model. We focus on the Andrews' (1993) exact median-unbiased estimator (BC), the simple OLS estimator (OLS) and the artificial imposition of a unit root in the AR coefficient–*i.e.* a driftless RW. We also analyse two combinations: the equally weighted combination of BC and RW (labelled C1) and of OLS and BC (labelled C2). For completeness, we consider both Gaussian and fat-tails innovations. The latter is typically associated to financial time series. Notice that our investigation differs from the existing one by focusing both on short and long-horizon forecasts constructed using Andrews' (1993) methodology.

Via simulations, we find that BC typically outperforms OLS forecasts. When compared to the RW we obtain mixed results, favouring the latter while the persistence of the true process increases. Interestingly, we find that the combination of BC-RW performs well in a near-unity scheme, resembling Hansen's (2010) results.[2]

2 Econometric details

We focus on "Model 2" of Andrews' (1993): $y_t = \widetilde{c} + \phi y_{t-1} + \varepsilon_t$; with $-1 \leq \phi < 1$; $\widetilde{c} = c(1-\phi)$ and $\varepsilon_t \sim iid\mathcal{N}(0, \sigma_\varepsilon^2)$. We also consider the case in which $\varepsilon_t \sim t_8$ following the findings of Rachev *et al.* (2010) for financial variables. Mikusheva (2007) points out that Andrews' (1993) methodology could be also a valid procedure with fat-tails innovations.

[1] Some examples–for the AR(p) also–are the methods of Orcutt and Winokur (1969), Stine and Shaman (1989), Andrews and Chen (1994), Hansen (1999), So and Shin (1999), Roy and Fuller (2001), Kim (2003) and Withers and Nadarajah (2011) among others.

[2] We use equal weights following the "combination puzzle" (Bates and Granger, 1969; Stock and Watson, 2004). This states that complex estimations of the weights in a composite forecast do not guarantee a superior out-of-sample performance.

While OLS is widely known, we provide some details of BC. Suppose that $\widehat{\phi}$ is an estimator with a median function given by $m(\phi)$, that is uniquely defined and strictly increasing on the finite parameter space $-1 \leq \phi < 1$. Thus, the function $m(\phi)$ is the unique median of $\widehat{\phi}$ when ϕ is the true parameter. Then, the following estimator $\widehat{\phi}_{BC}$:

$$\widehat{\phi}_{BC} = \begin{cases} 1 & if\ \widehat{\phi} > m(1) \\ m^{-1}(\widehat{\phi}) & if\ m(-1) < \widehat{\phi} \leq m(1) \\ -1 & if\ \widehat{\phi} \leq m(-1), \end{cases} \tag{1}$$

corresponds to the median-unbiased estimator of ϕ. Notice that $m(-1) = \lim_{\phi \to -1} m(\phi)$, and the function $m^{-1} : (m(-1), m(1)] \to (-1, 1]$ acts as the inverse function of $m(\cdot)$.

Aside from these two methods (OLS and BC) we consider the case when ϕ is exactly equal to 1– consequently, $\widetilde{c} = 0$. Forecasts from this misspecified model have been analysed by Clements and Hendry (2001) and Pincheira and Medel (2015), among others. When the true model is near unity, RW forecasts perform relatively well; a result supported in many articles. In particular, Pincheira and Medel (2015) notice that forecasts from the–incorrectly specified–RW model have the appealing property of unbiasedness. Iterating forward the AR(1) we obtain:

$$y_{t+h} = \widetilde{c}\left[\frac{1-\phi^h}{1-\phi}\right] + \phi^h y_t + \sum_{i=0}^{h-1} \phi^i \varepsilon_{t+h-i}.$$

If y_t were a driftless RW, then the optimal forecast would simply be y_t at any forecasting horizon. Accordingly, the expected value of the RW associated forecast error h-step-ahead forecast, $\mathbb{E}\left[\varepsilon_t^{RW}(h)\right] = \mathbb{E}\left[y_{t+h} - y_t^{RW}(h)\right]$, would satisfy:

$$\begin{aligned} \mathsf{Bias}(h) &\equiv \mathbb{E}\left[\varepsilon_t^{RW}(h)\right] = \mathbb{E}\left[c\left[\frac{1-\phi^h}{1-\phi}\right] - (1-\phi^h)y_t + \sum_{i=0}^{h-1}\phi^i \varepsilon_{t+h-i}\right], \\ &= c\left[\frac{1-\phi^h}{1-\phi}\right] - (1-\phi^h)\mathbb{E}\left[y_t\right] = 0, \end{aligned} \tag{2}$$

as $\mathbb{E}\left[y_t\right] = c/(1-\phi)$.

The following proposition, extracted from Pincheira and Medel (2015), generalize to the case of any stationary process.

Proposition 1 | ***Driftless RW Forecast Unbiasedness***[3] *Let y_t be a covariance stationary process: $\mathbb{E}[y_t] = \mu$, $\mathbb{V}\left[y_t\right] = \gamma_0$, $\mathbb{C}\left[y_t, y_{t-s}\right] = \gamma_s$, $\forall t, s$ (not dependent on t). Then, driftless RW-based forecasts are unbiased and display a bounded Mean Squared Forecast Error (MSFE) as the forecasting horizon goes to infinity.*

Proof. Suppose that we forecast y_{t+h} assuming a driftless RW as true model, delivering the forecast $y_t^{RW}(h) = y_t$, with forecasting errors, $\varepsilon_t^{RW}(h) = y_{t+h} - y_t$. Because of the stationarity of y_{t+h} we have that:

$$\mathbb{E}\left[y_{t+h} - y_t^{RW}(h)\right] = \mathbb{E}\left[y_{t+h} - y_t\right] = \mathbb{E}\left[y_{t+h}\right] - \mathbb{E}\left[y_t\right] = 0, \tag{P1}$$

[3] We denote the expected value, variance, covariance, and autocovariance with $\mathbb{E}\left[\cdot\right]$, $\mathbb{V}\left[\cdot\right]$, $\mathbb{C}\left[\cdot,\cdot\right]$, and γ_t, respectively.

and therefore, *driftless RW-based forecasts* are unbiased. The MSFE is given by:

$$\begin{aligned}\mathsf{MSFE}(h) &= \mathbb{E}\left[y_{t+h} - y_t^{RW}(h)\right]^2 \\ &= \mathbb{E}\left[y_{t+h} - y_t\right]^2 \\ &= 2\mathbb{V}\left[y_t\right] - 2\gamma_h.\end{aligned} \tag{P2}$$

So,

$$\begin{aligned}\mathsf{MSFE}(h) &= \left|2\mathbb{V}\left[y_t\right] - 2\gamma_h\right| \leq 2\mathbb{V}\left[y_t\right] + 2|\gamma_h| \\ \mathsf{MSFE}(h) &\leq 2\mathbb{V}\left[y_t\right] + 2\sqrt{\mathbb{V}\left[y_{t+h}\right]\mathbb{V}\left[y_t\right]} \\ \mathsf{MSFE}(h) &\leq 4\mathbb{V}\left[y_t\right],\end{aligned} \tag{P2'}$$

and

$$\lim_{h\to\infty} \mathsf{MSFE}(h) \leq 4\mathbb{V}\left[y_t\right] < \infty, \tag{P3}$$

and then MSFE(h) is a bounded sequence. ■

3 Simulations

We explore the out-of-sample performance of the five strategies considered: BC, OLS, RW, C1 and C2, via Monte Carlo simulations.

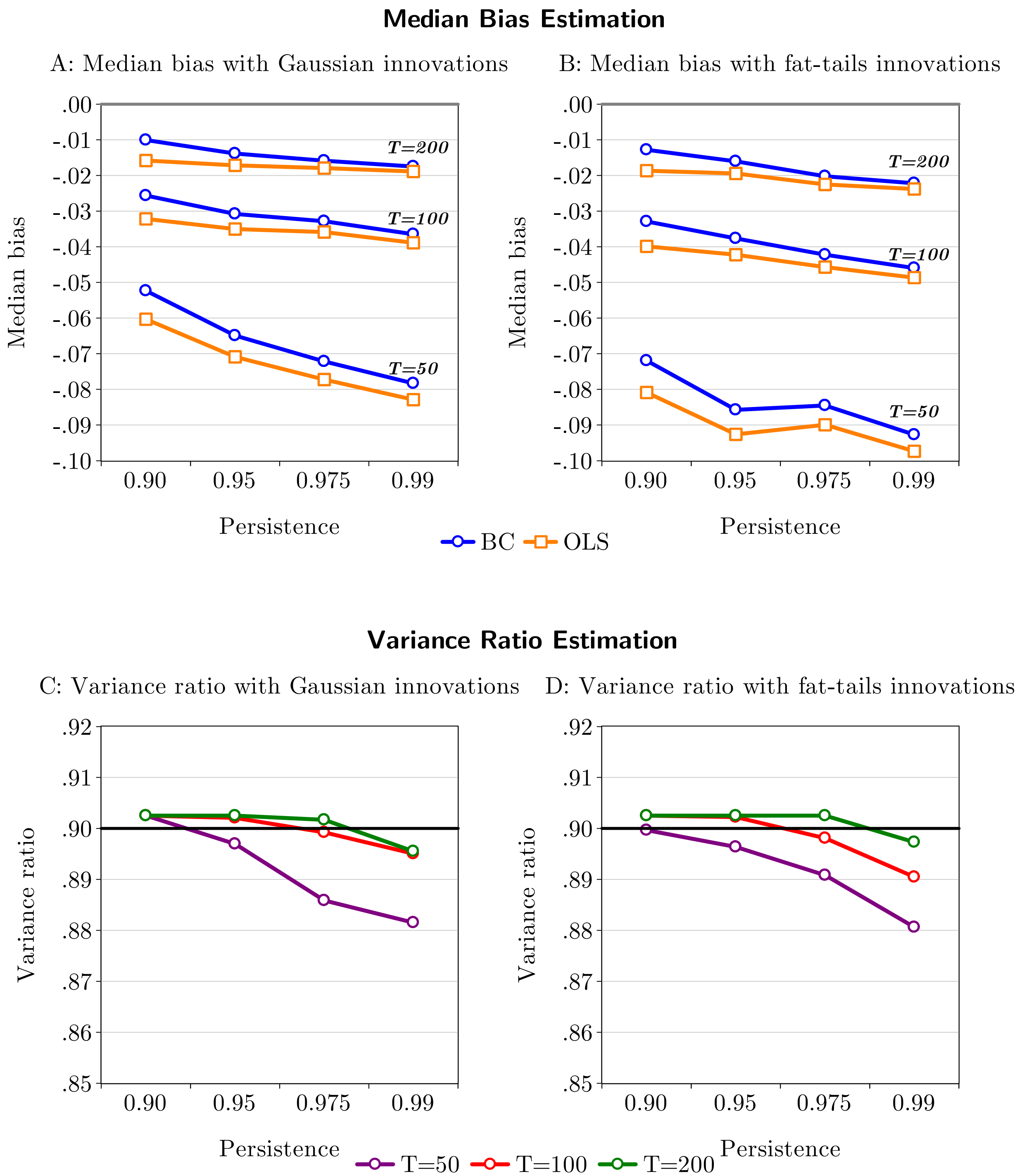
3.1 Simulation set-up

First, we generate observations from a stationary AR(1) model. Second, we pick a rolling estimation window to estimate the model (when needed). Third, we generate forecasts at several horizons. Finally, we compute and compare the sample Root MSFE (RMSFE) between the procedures.

We obtain the median unbiased estimator of ϕ as follows. Given a value of ϕ, we generate a grid of AR(1) models with persistence parameter $\phi - \delta_i$, where $\delta_i = 0.01 \times i$, $i \in [1, 20]$. For each of these models we generate 1000 simulations. Then, for each of the simulated series, we estimate an AR(1) with OLS using T observations. We also estimate by OLS an AR(1) model with the series coming from the true model. This last estimation–that of the true series–delivers the value $\widehat{\phi}^{OLS}$. We save the median of the 1000 simulations for each of the models with parameter $(\phi - \delta_i)$. Finally, we find the median-unbiased estimator $\widehat{\phi}^{BC}$ as $\phi - \delta_j$ such that this latter term delivers the minimum value of $\left|m(\phi - \delta_i) - \widehat{\phi}^{OLS}\right|$.

In Figure 1A and 1B, we show the estimation of the median bias for different choices of T. With a small T, bias in absolute value is higher, and the curve across persistence is steeper than with a greater T. In both innovation cases, for any given T, the bias is directly related to persistence. In Figure 1A and 1B the lowest (absolute) bias is 0.01, achieved with the combination $(\min\{\phi\}, \max\{T\})$. The lowest value is obtained with the BC estimator when the process is generated with Gaussian innovations. The highest (absolute) bias is 0.097, as reported in Figure 1A and 1B (obtained with OLS/fat-tails). Interestingly, this result is obtained with the combination $(\max\{\phi\}, \min\{T\})$.

Figure 1: Median bias and relative variance of the estimators (*)



(*) A-B: Median bias is defined as: $\widehat{\phi}^{i} - \phi$, with i={BC;OLS} for different sample sizes.
C-D: Variance ratio is defined as $V(\widehat{\phi}^{BC})/V(\widehat{\phi}^{OLS})$. Number of replications: 10000.
Source: Authors' elaboration.

We consider the levels of persistence $\phi = \{0.90; 0.95; 0,975; 0.99\}$. When we compute the OLS and the BC estimators, we use rolling windows of a fixed size T. We explore the robustness of our results across different choices of $T = \{50; 100; 200\}$. The forecasting horizons are $h = \{1; 12; 24; 36; 48; 96; 120\}$, referring to monthly frequency. We set the total number of h-step-ahead forecasts to 2000, made in an indirect-iterative manner. The constant term in our true DGP is set to unity, the variance of the stochastic term is also set to $\sigma_\varepsilon^2 = 1$, and the total number of simulations is 10000. We construct forecast combinations as follows:

$$\begin{aligned} \mathsf{C1} &: \; y_t^{C1}(h) = \lambda_1 \cdot y_t^{RW}(h) + (1-\lambda_1) \cdot y_t^{BC}(h), \\ \mathsf{C2} &: \; y_t^{C2}(h) = \lambda_2 \cdot y_t^{BC}(h) + (1-\lambda_2) \cdot y_t^{OLS}(h), \end{aligned} \tag{3}$$

where $\lambda_1 = \lambda_2 = 0.5$, following the suggestion of Stock and Watson (2004), and $y_t^M(h)$ denotes an h-step-ahead forecast constructed at time t using method $M \in \{BC; OLS; RW; C1; C2\}$. The predictive comparison is based on the RMSFE statistic:

$$RMSFE(h) = \left[\frac{1}{T} \sum_{t=1}^{T} (y_{t+h} - y_t^M(h))^2 \right]^{\frac{1}{2}}, \tag{4}$$

Specifically, we focus on the RMSFE ratio between the BC acting as a pivot and the competing forecast $M' \in \{OLS; RW; C1; C2\}$: $RMSFE^{BC}/RMSFE^{M'}$, for any given horizon. Hence, values above unity imply a worse performance of BC-based forecasts.

In Figure 1C and 1D we show the variance ratio defined as $\mathbb{V}(\widehat{\phi}^{BC})/\mathbb{V}(\widehat{\phi}^{OLS})$. Notice that a similar profile is found with both kinds of innovations. $\mathbb{V}(\widehat{\phi}^{BC})$ is always lower than $\mathbb{V}(\widehat{\phi}^{OLS})$, in a ratio ranging around 0.880–0.905. The variance of the OLS estimator is the highest with ϕ=0.99, suggesting better relative predictive results for BC.

3.2 Simulation results

The RMSFE results are presented in Table 1.

3.2.1 BC versus OLS

BC performance is outstanding compared to OLS, as 95% of the corresponding ratios are below unity. These predictive gains tend to increase with both the persistence and the forecasting horizon, but tend to decrease as the sample size increases–especially at long horizons. We notice that OLS seem an inappropriate alternative for h>36 and T=50, with high persistence levels. In these cases, Table 1 shows ratios that are negligible. These results are robust to the source of innovations. To have a quantitative flavour notice that since 24-periods-ahead there are obtained remarkable predictive gains; for instance, BC achieves the half of OLS RMSFE (49%; T=50, ϕ=0.975). Also remarkable is that in the least favourable case for BC, OLS display a small gain of only 1%.

3.2.2 BC versus RW

In most cases, BC outperforms RW performance: more than two-thirds of the corresponding ratios (Table 1) are below unity (71%). These gains of BC show a tendency to increase with both the forecasting horizon and the sample size, but to decrease with the persistence level. These results are robust to the distribution of innovations. Differing from the previous comparison with OLS, BC forecasts are outperformed by RW when the level of persistence is high. For instance, in the least favourable case to BC, RW forecasts display a gain of 22% in RMSFE. Therefore, RW results in a valid benchmark at high levels of persistence, indicating that in this set-up, misspecification is less harmful than parameter uncertainty.

3.2.3 Combinations performance

Roughly speaking, C1 dominates C2. This result, plus the fact that BC quasi-dominates OLS forecasts, opens the question about the best forecasting strategy between BC, RW and C1. Table 1 helps to produce a rule-of-thumb-type answer. For levels of persistence between 0.90 and 0.95, a good strategy is the construction of forecasts using BC (outperforming RW and C1 in 89% of our simulations). Similarly, as the persistence increases, C1 provides a good and relatively robust performance (outperforming RW and BC in 52% of our simulations). This result is similar to that of Hansen (2010).

Note from Table 1 that BC and C1 do not *always* provide the most accurate forecasts. Nevertheless, when they are outperformed by either one of the other two best forecasting strategies it is often by just a small margin.

4 Concluding remarks

We analyse the forecasting performance of several strategies for the near-unity AR(1) model. We focus on the BC, OLS and the driftless RW. We explore also the forecasting performance of two combinations: the equally weighted combination of BC and RW, and of OLS and BC. For completeness, we analyse the behaviour of these forecasts with both Gaussian and fat-tails innovations.

Via simulations, we find that BC typically outperforms OLS forecasts. When compared to the RW we obtain mixed results, favouring the latter while the persistence of the true process increases. Interestingly, we find that the combination of BC-RW performs well in a near-unity scheme, resembling Hansen's (2010) results. As a rule of thumb, we propose the use of BC forecasts when the level of persistence is medium-high, and the use of an equally weighted combination between RW and BC when the persistence is near unity.

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Table 1: RMSFE-ratio between BC forecast and candidates, $M' \in \{OLS;RW;C1;C2\}$ (*)

		Gaussian innovations							*Fat-tails innovations*						
								T=50							
	h:	*1*	*12*	*24*	*36*	*48*	*96*	*120*	*1*	*12*	*24*	*36*	*48*	*96*	*120*
ϕ=0.90	OLS	1.00	0.96	0.94	0.91	0.86	0.62	0.53	0.99	0.95	0.90	0.82	0.64	0.08	0.02
	RW	0.99	0.84	0.74	0.74	0.72	0.65	0.69	0.99	0.79	0.68	0.71	0.65	0.69	0.68
	C1	1.02	1.01	0.96	0.96	0.97	0.90	0.94	1.02	0.98	0.92	0.94	0.89	0.95	0.94
	C2	1.02	0.97	0.90	0.90	0.87	0.72	0.72	1.01	0.93	0.86	0.84	0.74	0.23	0.06
ϕ=0.95	OLS	0.99	0.93	0.79	0.61	0.35	0.01	0.00	1.00	0.91	0.71	0.49	0.27	0.00	0.00
	RW	1.04	0.97	0.82	0.78	0.76	0.77	0.80	1.06	0.96	0.87	0.79	0.76	0.80	0.79
	C1	1.04	1.07	0.99	0.95	0.94	0.96	0.98	1.05	1.06	1.01	0.97	0.95	0.97	0.97
	C2	1.04	1.01	0.87	0.77	0.60	0.02	0.00	1.05	0.99	0.85	0.70	0.53	0.02	0.00
ϕ=0.975	OLS	1.00	0.86	0.49	0.12	0.02	0.00	0.00	1.00	0.92	0.76	0.57	0.34	0.00	0.00
	RW	1.04	1.14	1.04	0.96	0.94	0.86	0.86	1.08	1.12	1.04	0.97	0.90	0.87	0.86
	C1	1.04	1.14	1.09	1.04	1.03	0.98	0.98	1.05	1.13	1.10	1.05	1.01	0.99	0.98
	C2	1.03	1.06	0.81	0.35	0.06	0.00	0.00	1.05	1.07	0.95	0.81	0.63	0.02	0.00
ϕ=0.99	OLS	0.98	0.90	0.71	0.46	0.23	0.00	0.00	1.00	0.84	0.42	0.09	0.01	0.00	0.00
	RW	1.06	1.22	1.14	1.14	1.10	1.04	1.02	1.06	1.20	1.14	1.10	1.06	1.00	1.00
	C1	1.05	1.16	1.13	1.12	1.10	1.05	1.05	1.04	1.15	1.12	1.10	1.08	1.04	1.03
	C2	1.03	1.10	0.97	0.80	0.53	0.01	0.00	1.04	1.08	0.80	0.31	0.04	0.00	0.00
								T=100							
ϕ=0.90	OLS	1.00	0.99	0.98	0.98	0.98	0.99	0.98	1.00	0.99	0.98	0.99	0.97	0.97	0.99
	RW	1.01	0.73	0.65	0.60	0.60	0.60	0.58	0.99	0.76	0.68	0.65	0.62	0.58	0.60
	C1	1.03	0.94	0.90	0.87	0.87	0.88	0.86	1.01	0.96	0.92	0.90	0.89	0.87	0.88
	C2	1.02	0.92	0.88	0.84	0.84	0.85	0.83	1.01	0.95	0.90	0.88	0.86	0.83	0.85
ϕ=0.95	OLS	1.00	0.97	0.93	0.83	0.75	0.74	0.48	1.00	0.99	0.96	0.95	0.94	0.90	0.90
	RW	0.99	0.89	0.81	0.74	0.71	0.70	0.69	1.01	0.90	0.77	0.73	0.69	0.64	0.64
	C1	1.01	1.02	0.99	0.95	0.94	0.95	0.94	1.02	1.03	0.97	0.95	0.93	0.91	0.91
	C2	1.00	0.99	0.94	0.86	0.82	0.77	0.50	1.02	1.01	0.93	0.90	0.88	0.83	0.82
ϕ=0.975	OLS	1.00	0.99	0.96	0.93	0.90	0.82	0.80	1.00	0.96	0.91	0.85	0.78	0.53	0.38
	RW	1.02	1.09	1.01	0.94	0.89	0.82	0.82	1.02	1.00	0.94	0.90	0.84	0.78	0.76
	C1	1.02	1.10	1.08	1.04	1.02	0.98	0.98	1.02	1.06	1.05	1.03	0.99	0.96	0.94
	C2	1.02	1.08	1.04	0.98	0.94	0.86	0.85	1.02	1.03	0.98	0.93	0.86	0.72	0.61
ϕ=0.99	OLS	1.00	0.98	0.97	0.92	0.89	0.63	0.48	1.00	0.94	0.86	0.76	0.67	0.31	0.16
	RW	1.03	1.17	1.18	1.13	1.13	1.01	0.97	1.03	1.14	1.08	1.03	1.01	0.99	0.96
	C1	1.03	1.14	1.15	1.13	1.13	1.07	1.04	1.03	1.12	1.10	1.08	1.07	1.05	1.03
	C2	1.02	1.11	1.12	1.07	1.06	0.87	0.76	1.02	1.07	1.01	0.94	0.88	0.60	0.41

(*) The values below 1 imply a better performance of the BC forecast, highlighted in shaded cells.

Source: Authors' elaboration.

Table 1: RMSFE-ratio between BC forecast and candidates, $M' \in \{OLS;RW;C1;C2\}$ (*) (...continued)

		Gaussian innovations							*Fat-tails innovations*						
		T=200													
	h:	*1*	*12*	*24*	*36*	*48*	*96*	*120*	*1*	*12*	*24*	*36*	*48*	*96*	*120*
ϕ=0.90	OLS	1.00	1.00	0.99	0.99	0.99	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.01	1.00
	RW	0.95	0.67	0.59	0.53	0.56	0.54	0.54	0.95	0.69	0.59	0.56	0.54	0.57	0.54
	C1	0.99	0.89	0.87	0.82	0.84	0.83	0.84	0.99	0.92	0.87	0.85	0.83	0.87	0.83
	C2	0.99	0.89	0.86	0.80	0.82	0.82	0.82	0.99	0.91	0.86	0.84	0.82	0.86	0.82
ϕ=0.95	OLS	1.00	0.99	1.00	1.00	0.99	0.99	0.98	1.00	0.99	0.99	0.99	0.98	0.98	0.99
	RW	0.96	0.87	0.76	0.69	0.65	0.60	0.62	0.98	0.85	0.71	0.69	0.63	0.60	0.63
	C1	0.99	1.00	0.95	0.92	0.90	0.88	0.89	1.00	0.99	0.93	0.93	0.89	0.88	0.91
	C2	0.99	0.99	0.94	0.90	0.88	0.85	0.86	1.00	0.98	0.91	0.91	0.87	0.85	0.88
ϕ=0.975	OLS	1.00	0.99	0.97	0.96	0.95	0.93	0.90	1.00	0.99	0.98	0.97	0.96	0.92	0.92
	RW	1.01	0.97	0.89	0.82	0.78	0.72	0.68	1.00	0.90	0.82	0.80	0.76	0.63	0.61
	C1	1.01	1.03	1.01	0.98	0.96	0.94	0.92	1.01	1.00	0.98	0.97	0.96	0.87	0.86
	C2	1.01	1.02	0.98	0.94	0.92	0.88	0.85	1.01	0.99	0.95	0.94	0.92	0.81	0.80
ϕ=0.99	OLS	1.00	0.99	0.96	0.94	0.92	0.83	0.77	1.00	0.99	0.99	0.97	0.94	0.82	0.74
	RW	1.02	1.09	1.09	1.05	1.01	0.94	0.92	1.02	1.09	1.07	1.05	1.03	0.98	0.91
	C1	1.02	1.08	1.10	1.09	1.08	1.04	1.03	1.02	1.09	1.10	1.09	1.09	1.07	1.03
	C2	1.01	1.07	1.07	1.04	1.01	0.93	0.89	1.01	1.07	1.08	1.06	1.04	0.95	0.87

(*) The values below 1 imply a better performance of the BC forecast, highlighted in shaded cells.

Source: Authors' elaboration.

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