

DOCUMENTOS DE TRABAJO

The Impact of the Minimum Wage on Capital Accumulation and Employment in a Large-Firm Framework

Sofia Bauducco
Alexandre Janiak

N.º 755 Abril 2015

BANCO CENTRAL DE CHILE



DOCUMENTOS DE TRABAJO

The Impact of the Minimum Wage on Capital Accumulation and Employment in a Large-Firm Framework

Sofia Bauducco
Alexandre Janiak

N.º 755 Abril 2015

BANCO CENTRAL DE CHILE





BANCO CENTRAL DE CHILE

CENTRAL BANK OF CHILE

La serie Documentos de Trabajo es una publicación del Banco Central de Chile que divulga los trabajos de investigación económica realizados por profesionales de esta institución o encargados por ella a terceros. El objetivo de la serie es aportar al debate temas relevantes y presentar nuevos enfoques en el análisis de los mismos. La difusión de los Documentos de Trabajo sólo intenta facilitar el intercambio de ideas y dar a conocer investigaciones, con carácter preliminar, para su discusión y comentarios.

La publicación de los Documentos de Trabajo no está sujeta a la aprobación previa de los miembros del Consejo del Banco Central de Chile. Tanto el contenido de los Documentos de Trabajo como también los análisis y conclusiones que de ellos se deriven, son de exclusiva responsabilidad de su o sus autores y no reflejan necesariamente la opinión del Banco Central de Chile o de sus Consejeros.

The Working Papers series of the Central Bank of Chile disseminates economic research conducted by Central Bank staff or third parties under the sponsorship of the Bank. The purpose of the series is to contribute to the discussion of relevant issues and develop new analytical or empirical approaches in their analyses. The only aim of the Working Papers is to disseminate preliminary research for its discussion and comments.

Publication of Working Papers is not subject to previous approval by the members of the Board of the Central Bank. The views and conclusions presented in the papers are exclusively those of the author(s) and do not necessarily reflect the position of the Central Bank of Chile or of the Board members.

Documentos de Trabajo del Banco Central de Chile
Working Papers of the Central Bank of Chile
Agustinas 1180, Santiago, Chile
Teléfono: (56-2) 3882475; Fax: (56-2) 3882231

THE IMPACT OF THE MINIMUM WAGE ON CAPITAL ACCUMULATION AND EMPLOYMENT IN A LARGE-FIRM FRAMEWORK*

Sofía Bauducco
Banco Central de Chile

Alexandre Janiak
Universidad de Chile

Abstract

We study the effect of a binding minimum wage on labor market outcomes, the accumulation of capital and welfare. We consider a large firm that invests in physical capital and hires several types of workers. Labor markets are characterized by search and matching frictions, while incomplete wage contracts allow workers to appropriate part of the return on each factor. Absent a minimum wage, the model in general gives rise to inefficient levels of capital and employment. We show that, when labor types are substitutes, the introduction of a binding minimum wage has positive effects on capital and positive and small effects on employment, and these effects depend on the ability of the minimum wage to deter rent appropriation by workers.

Resumen

En este artículo estudiamos el efecto de un salario mínimo activo en los resultados del mercado laboral, la acumulación de capital y el bienestar. Consideramos el caso de una empresa grande que invierte en capital físico y emplea distintos tipos de trabajadores. Los mercados de trabajo se caracterizan por fricciones de búsqueda y de emparejamiento, mientras que los contratos salariales incompletos permiten que los trabajadores se apropien de parte del retorno de cada factor. En ausencia del salario mínimo, el modelo en general arroja niveles ineficientes de capital y trabajo. Mostramos que, cuando los tipos de trabajadores son sustitutos entre sí, la introducción de un salario mínimo activo tiene efectos positivos sobre el capital y efectos positivos pero pequeños sobre el empleo, y estos efectos dependen de la habilidad del salario mínimo de evitar la apropiación de renta de los trabajadores.

* An earlier version of this paper circulated under the title “Minimum wages strike: the effects on capital and labor demands in a large-firm framework”. We are grateful to Felipe Balmaceda, Wouter den Haan and Etienne Wasmer for useful comments, as well as participants at the Winter Econometric Society Meeting in Los Angeles, the European Economic Association Meeting in Toulouse, the CEF conference in Vancouver, the Econometric Society conference in Sao Paulo and EALE conference in Bonn, and seminar participants at Banque de France, Central Bank of Chile, UDP, USACH, PUC-Chile and CEA. We acknowledge funding from the Anillo in Social Sciences and Humanities (project SOC 1402 on “Search models: implications for markets, social interactions and public policy”). Alexandre Janiak also thanks Fondecyt (project no 1151053) and the Millennium Institute for Research in Market Imperfections and Public Policy. All errors are our own. Email: sbauducco@bcentral.cl y ajaniak@dii.uchile.cl.

1 Introduction

The available empirical evidence on the effects of minimum wages on employment has challenged the traditional models of the labor market and highlighted the need for alternative views of how this market works. Two reasons support this statement. First, although common wisdom suggests that a minimum wage should have a negative effect over employment, there is an extensive empirical literature on this topic that suggests a weak—or even a positive—relation between the two variables. [Card and Krueger \(1994\)](#) is the seminal contribution that led to a rapid expansion of this literature, which mainly challenges the neoclassical view that increases in the regulatory minimum level necessarily lead to a decrease in employment ([Stigler, 1946](#)).¹ Second, as [Teulings \(2000\)](#) points out, the weak response of employment to an increase in the minimum wage could be reconciled with standard neoclassical models if substitutability between types of labor (that differ, for example in skills) is low. This is at odds, however, with direct evidence that suggests that substitutability is actually quite high. [Teulings \(2000\)](#) refers to this puzzle as the *minimum wage paradox*.

In this paper, we show that the presence of a binding minimum wage can be consistent with a response of employment in line with the empirical literature mentioned before when different labor types are substitutes in production. To do so, we introduce a binding minimum wage into a “large-firm” framework in which there are two types of labor used in the production of final goods and labor markets are characterized by search frictions. Our model is able to generate situations where a minimum wage has a small positive impact on the employment level of workers earning the minimum. Moreover, this positive impact on employment is stronger for higher degrees of substitutability between labor types.²

Our choice of the model setup is motivated by a growing literature that has shown

¹[Card and Krueger \(1994\)](#) exploit a difference-in-difference approach to study the impact of an increase in the New Jersey’s minimum wage on the employment level of fast-food restaurants by using establishments of the same industry in Pennsylvania as a control group. Their conclusion is that the rise in the minimum wage did not seem to reduce employment. Similar conclusions have been reached in other studies for the United States, such as [Katz and Krueger \(1992\)](#), [Card \(1992b\)](#) and [Card \(1992a\)](#), while [Machin and Manning \(1994\)](#), [Machin and Manning \(1996\)](#), [Stewart \(2004b\)](#) and [Stewart \(2004a\)](#) obtain similar results for the United Kingdom, and [Dolado et al. \(1996\)](#) for France, the Netherlands and Spain. Negative effects over employment may sometimes be found by some authors, especially in the case of young workers (e.g. [Linneman \(1982\)](#), [Currie and Fallick \(1996\)](#), [Dolado et al. \(1996\)](#), [Burkhauser et al. \(2000\)](#) and [Portugal and Cardoso \(2006\)](#)) or vulnerable sectors (e.g. [Machin et al. \(2003\)](#) and [Machin and Wilson \(2004\)](#)).

²The literature has stressed the importance of minimum wages for wage inequality. See for instance [Lee \(1999\)](#), [Teulings \(2000\)](#), [Manning \(2003\)](#) and [Autor et al. \(2010\)](#), among others. In particular, this literature emphasizes the presence of spill-over effects on the wage of workers not directly affected by minimum wages. Our model is able to generate these effects through an increase in the demand for high-wage workers. See Section 5.

that the quality of labor relations is a determinant of labor market outcomes.³ The large-firm framework is a natural way to embed labor tensions into a standard search and matching model: although it is a simple extension of [Pissarides \(1985\)](#) matching model to a multi-factor framework, it brings out a rich set of strategic interactions between factors in wage bargaining, allowing for the possibility of appropriation between different labor types, capital and the firm. The setup relies on contract incompleteness and wage renegotiation. [Stole and Zwiebel \(1996b\)](#) and [Stole and Zwiebel \(1996a\)](#) have shown that, in this context, firms *ex ante* choose a specific organizational structure in order to influence the outcome of the bargaining process *ex post*. This may occur for instance because the firm aims at limiting the effect of appropriation by workers as in [Grout \(1984\)](#)—the so called “holdup” problem. We model these inefficiencies in a framework with search and matching frictions as in [Bertola and Caballero \(1994\)](#), [Smith \(1999\)](#), [Bertola and Garibaldi \(2001\)](#) and [Cahuc et al. \(2008\)](#), among others, and ask how labor and capital demands react to the introduction of a binding minimum wage in this setup.

In our setup, the introduction of a minimum wage may imply an increase in employment, and even an increase in the demand for workers earning the minimum wage. Two mechanisms explain this result. First, our model economy is characterized by a standard holdup problem, yielding lower incentives to invest in physical capital when capital and labor are complements.⁴ When the minimum wage is binding, part of the disincentives to accumulate capital are eliminated, since the firm no longer underinvests as a way to reduce the wage of workers earning the minimum regulatory level. This increase in capital demand in turn tends to increase labor demand through the complementarity between the two factors. We call this mechanism the *capital demand effect*.

A second mechanism, which we call the *rent appropriation effect*, may also imply an

³[Blanchard and Philippon \(2006\)](#) document a robust negative cross-country correlation between an index of the quality of labor relations and unemployment. They argue that lack of trust produces bargaining failures, which results in higher unemployment. Similarly, [David et al. \(2010\)](#) suggest that depressed labor markets make individuals invest in non-professional forms of social capital, which amplifies unemployment issues. These channels in turn determine labor-market regulation. Indeed, according to [Aghion et al. \(2010\)](#), the minimum wage is higher in countries where cooperation between workers and employers is weak because parties do not trust each other in wage negotiation. [Algan and Cahuc \(2009\)](#) argue that trustful relationships lead an economy to provide labor-market insurance through unemployment benefits, while economies with lack of civic virtue rely more on employment protection.

⁴The assumption that capital and labor are complements is in line with US data. [Krusell et al. \(2000\)](#) estimate a production function for the US that includes two types of capital, structures and equipment, and two types of labor, skilled and unskilled. They find that, although the elasticity of substitution is higher between capital equipment and unskilled labor than between capital equipment and skilled labor, capital equipment is a complement of both types of labor. This is also true for capital structures.

increase in employment when the different types of labor considered in the production function are substitutes. [Cahuc et al. \(2008\)](#) have shown that, absent a minimum wage, the firm chooses to overemploy one type of labor in order to reduce the wage of the other type: because the equilibrium wage depends on the marginal product of labor under Nash bargaining, overemployment allows to reduce the other type's wage by decreasing its marginal product. Unfortunately for the firm, this leads to an additional appropriation problem as the overemployed workers claim part of the decrease in the other type's wage rate when they negotiate with the firm. In a context where the wage of one type of workers is fixed by regulation, rent appropriation by those workers is no longer possible. Thus, overemploying these workers becomes a more attractive option for the firm to affect the wage of the other type.

In light of our results, minimum wages may serve as a potential explanation for the higher capital-output ratios observed in Continental Europe as compared to the US.⁵ The suggestion that labor market institutions may be responsible for the higher capital-output ratios observed in Europe is not new in the literature. Standard explanations rely on capital-labor substitution: an increase in the relative cost of labor forces firms to substitute away from labor and towards capital, as argued in [Caballero and Hammour \(1998\)](#). However, a mechanism based on minimum wages can hardly account for an increase in capital if it has to generate a small or positive impact on employment simultaneously. Our paper shows that both employment and capital may increase following the introduction of a minimum wage.⁶

The literature on the economic effects of minimum wages is vast. The lack of clear-cut evidence supporting a negative effect of minimum wages on employment has favored the popularity of theoretical models that predict a positive effect as an alternative to the neo-classical framework. An example is the monopsony model, where employment is increased through an increase in labor supply without labor demand being necessarily

⁵Data from [Caselli and Feyrer \(2007\)](#) shows that the capital-output ratio is 36% larger in France than in the US, 29% in Spain and 22% in the Netherlands. See also [Hall and Jones \(1999\)](#). At the same time, those countries are also characterized by higher minimum wages. Data from the OECD reveals that the ratio of minimum to median wage of full-time workers is 36% in the US, while it is 56% in France, 43% in Spain and 51% in the Netherlands. In other countries such as Germany and Italy, there is no minimum wage regulation for most sectors of the economy. Instead, wages are determined through collective bargaining. In our model, as long as collective bargaining imply that individual firms take the stream of future wages as given, it should generate similar results on employment and capital accumulation as a minimum wage. According to the OECD, the proportion of workers covered by collective agreements is 92% in Germany, 82% in Italy and 18% in the US.

⁶Because a minimum wage helps deter rent appropriation by workers, it may also generate an increase in measures of aggregate welfare. Our paper is not the first to show welfare-improving properties of minimum wages. See [Flinn \(2006\)](#) and the references therein.

affected.⁷ In our paper, employment increases through an increase in labor demand. This increase in labor demand even occurs at the *microeconomic* level, in the sense that this result does not rely on any general-equilibrium mechanism, and may include the labor type for which the minimum wage is binding.

Our analysis is closely related to the work of [Smith \(1999\)](#). This paper also studies the effect of a minimum wage in the context of a large-firm search model with firm entry and exit.⁸ In his model, firms hire one type of labor and the production function displays decreasing returns to scale. Those assumptions generate two effects. First, firms tend to overemploy workers in order to decrease wages by decreasing their marginal product. Second, overemployment creates a negative externality that forces some firms out of the industry by increasing the labor-market search cost. As a result, there are too few firms in the economy and they are inefficiently too large. By introducing a minimum wage, it is possible to eliminate this inefficiency: firms stop overemploying, which increases firm entry and aggregate employment. However, the seminal study of [Card and Krueger \(1994\)](#) shows a positive effect of the minimum wage on firm size and no significant effect on the number of firms. Our results are consistent with Card and Krueger’s findings.

Our paper is also related to the literature that discusses the effect of a minimum wage on capital accumulation. In a context with search frictions, [Acemoglu and Shimer \(1999\)](#) show that the holdup problem is avoided if the wage rate (as a function of the capital stock) is constant in the neighborhood of the efficient capital stock. In our model, introducing a minimum wage allows to fulfill this necessary condition. [Acemoglu \(2001\)](#) builds a model where firms open too few capital-intensive jobs because workers appropriate part of the return on capital. The introduction of a binding minimum wage helps correct for this externality and enhances the creation of capital-intensive jobs. However, an increase in the minimum wage always results in an increase in unemployment in his model, while the effect is marginally negative in the context

⁷In the monopsony model, the firm has some market power over workers, which allows it to fix wages below the competitive level. Because the slope of the labor supply curve is positive, the introduction of a binding minimum wage increases employment by enhancing the incentives to supply more labor. The marginal impact on employment remains positive as long as the minimum regulatory level does not go beyond the competitive wage. The consequences of a minimum wage are similar in the oligopoly extension of the basic monopsony model and in its version with search frictions. See e.g. [Boal and Ransom \(1997\)](#), [Burdett and Mortensen \(1998\)](#), and [Manning \(2003\)](#).

⁸[Acemoglu and Hawkins \(2014\)](#) recently extended the analysis in [Smith \(1999\)](#) by considering convex vacancy costs. This extension allows to generate the realistic feature of slow employment growth at the firm level. See also [Ebell and Haefke \(2009\)](#), [Felbermayr and Prat \(2011\)](#) and [Janiak \(2013\)](#). In a model without firm entry, [Elsby and Michaels \(2013\)](#) also match key properties of the cross-sectional distributions of employment and employment growth. However, none of these papers consider the impact of a minimum wage.

of our model. Moreover, our model considers a richer set of strategic interactions between workers, the firm and capital. Similarly, [Kaas and Madden \(2008\)](#) illustrate the beneficial effects of a minimum wage for capital investment in the context of an oligopsonistic model, but they do not obtain a positive effect on employment.

The rest of the paper is organized as follows. In [Section 2](#) we describe the model. [Section 3](#) characterizes the equilibrium. The implications of a minimum wage for welfare are analyzed in [Section 4](#). In [Section 5](#), we illustrate the quantitative impact of a minimum wage over factor demands, wages, profits and welfare. [Section 6](#) concludes.

2 The model

We consider an economy in steady state, where time is continuous and discounted at a rate r and agents are risk neutral. For notational simplicity, we suppress the time indices t when describing the economy and analyzing the equilibrium, while we denote by primes variables evaluated at time $(t + dt)$, where dt is an arbitrarily small interval of time.

2.1 A representative firm

Output is produced by a representative firm. The firm hires two types of workers in quantities n_h and n_l and owns capital in quantity k . The constant-returns-to-scale production function f is increasing and concave in each argument.⁹ Standard Inada conditions are assumed such that an equilibrium exists on all markets for inputs, independently of the flow value of being unemployed.

Without loss of generality, we assume that the first type of labor is the most productive one, in the sense that it supplies more efficiency units than the other group. We refer hereafter to the two types of labor as “high productivity” and “low productivity” workers respectively, hence the subscripts h and l . We interpret these types as corresponding both to unskilled labor but with varying job characteristics.

2.2 Labor

There are two labor markets corresponding to the two types of labor the firm hires. The mass of each type of workers is equal to ς_i with $i \in \{l, h\}$. A high-productivity worker cannot hold a low-productivity job and *vice versa*.

⁹We consider constant returns to scale because, under decreasing returns to scale, new firms would have incentives to enter since profits would be positive. The comparative statics of [Section 2.7](#) are still valid under decreasing returns to scale.

Workers on each market can be either employed or unemployed. The presence of search and matching frictions explains the existence of unemployment on the two markets (in quantities u_h and u_l respectively). Firms post vacancies at a flow cost c in order to hire workers. We denote by v_i , $i \in \{h, l\}$, the mass of posted vacancies by the representative firm on each labor market, while V_i , $i \in \{h, l\}$, is the aggregate mass of vacancies in the economy. In equilibrium, $v_i = V_i$, but the representative firm takes V_i as given while v_i is a control variable. Vacancies on market i , $i \in \{h, l\}$, are filled at a rate $q(\theta_i)$ that depends negatively on the labor market tightness $\theta_i \equiv \frac{V_i}{u_i}$, i.e. the vacancy-unemployment ratio. This rate is derived from a matching function $m(u_i, V_i)$ with constant returns to scale, increasing in both arguments, concave and satisfying the property $m(u_i, 0) = m(0, V_i) = 0$, implying that $q(\theta_i) = \frac{m(u_i, V_i)}{V_i} = m(\theta_i^{-1}, 1)$. Separations occur at an exogenous rate s .¹⁰

2.3 Prices

Workers choose to earn either the minimum wage \bar{w} or negotiate *à la* Nash with the firm. There is continuous wage renegotiation.¹¹ If they choose to bargain, they obtain the negotiated wage $\check{w}^i(n_h, n_l, k)$. We denote by χ_i the fraction of i -type workers who negotiate their wage with the firm: because workers may be indifferent between earning the minimum wage or bargaining the wage with the firm in equilibrium, they randomize according to a mixed strategy, where the probability they bargain with the firm is χ_i .¹² The probability χ_i is endogenous and determined through arbitrage.¹³ We also denote by $\bar{n}_j = (1 - \chi_j)n_j$ and $\check{n}_j = \chi_j n_j$ the mass of workers who choose to earn the minimum wage and negotiate their wage with the firm respectively.

Hence, the expected wage $w^i(n_h, n_l, k)$ paid to a worker of type i is

$$w^i(n_h, n_l, k) = \chi_i \check{w}^i(n_h, n_l, k) + (1 - \chi_i) \bar{w}. \quad (1)$$

Notice that our notation for wages explicitly emphasizes their dependence on the

¹⁰For notational simplicity we assume that the parameters s , b , c and the function m is common across labor groups. However, all the results presented in the next sections go through when those parameters and functions are allowed to differ across groups.

¹¹See [Hawkins \(2015\)](#) for a model that considers commitment over wages.

¹²For some values of the minimum wage, it may happen that no equilibrium in pure strategy exists. The intuition is the following: consider the case where a minimum wage is introduced, this minimum wage is marginally binding and it causes an increase in labor demand. Then, a contradiction would arise since workers would renegotiate higher wages because of the increase in labor demand, but this requires the minimum wage to be binding at the same time. Hence, no equilibrium in pure strategy would exist in this case, but an equilibrium in mixed strategy does.

¹³Section 3 gives further details on the determination of χ_i in equilibrium.

employment levels n_h and n_l and the capital stock k : the firm may choose a particular level of employment or capital before wages are negotiated in order to influence the outcome of the bargaining process *ex post*.¹⁴ For example, [Smith \(1999\)](#) shows that when the production function is concave in each factor, the firm may choose to overemploy in order to reduce wages through a reduction in the marginal product of labor¹⁵. [Cahuc and Wasmer \(2001a\)](#) and [Cahuc et al. \(2008\)](#) show that the complementarity (substitutability) between different types of labor may induce the firm to underemploy (overemploy) one type of labor in order to reduce the wage of other workers.

Finally, the purchase of a unit of capital is priced one unit of final good and capital depreciates at a rate δ .

2.4 Value functions

The present-discounted value of profits of the representative firm is

$$\begin{aligned} \Pi(n_h, n_l, k) = \max_{\{v_h, v_l, a\}} & \quad (2) \\ \frac{1}{1+r} \frac{d}{dt} \left(\left[f(n_h, n_l, k) - \sum_{j=\{h,l\}} [w^j(n_h, n_l, k)n_j + v_j c] - a \right] dt + \Pi(n'_h, n'_l, k') \right), \end{aligned}$$

subject to the constraints

$$\dot{n}_i = q(\theta_i)v_i - sn_i, \quad \forall i \in \{h, l\} \quad (3)$$

and

$$\dot{k} = a - \delta k, \quad (4)$$

where a denotes investment in physical capital and dt is an arbitrarily small interval of time. We specifically consider the case where dt tends to zero.

The value of being unemployed for an i -type worker follows a standard formulation and reads in steady state as

¹⁴We assume that the firm acquires capital *before* wages are bargained, and that the capital stock of the firm cannot be adjusted while wage negotiation occurs. If capital could be freely adjusted, the holdup problem would not be present, see [Cahuc and Wasmer \(2001a\)](#) for a discussion.

¹⁵We refer to a situation of *overemployment* when (in partial equilibrium) the firm hires a quantity of labor larger than the level that would prevail under a situation where the firm takes the stream of future wages as given. Similarly, when employment is below that level, we refer to a situation of *underemployment*.

$$rU_i = b + \theta_i q(\theta_i) [W_i - U_i], \quad (5)$$

with b the flow utility of being unemployed, while the value of being employed follows

$$W_i = \max\{\check{W}_i, \bar{W}_i\}, \quad (6)$$

where \check{W}_i is the value if a worker chooses to negotiate,

$$r\check{W}_i = \check{w}^i(n_h, n_l, k) + s[U_i - W_i], \quad (7)$$

and \bar{W}_i is the value when a worker chooses to earn the minimum wage,

$$r\bar{W}_i = \bar{w} + s[U_i - W_i]. \quad (8)$$

2.5 Intra-firm Nash bargaining

In a context with mixed strategies, a worker randomizes between negotiating the wage with the firm with probability χ_i and earning the minimum wage with probability $(1 - \chi_i)$. Consequently, for the purpose of the negotiation, only the mass of workers participating in the negotiation process $\check{n}_i = \chi_i n_i$ is relevant. The remaining mass $\bar{n}_i = (1 - \chi_i)n_i$ is taken as a given input in the production function, since their wage is exogenously determined and given by the minimum wage \bar{w} .

Denote by $f_x(x, y)$ the partial derivative of a function f with respect to argument x , i.e., $f_x = \frac{\partial f(x, y)}{\partial x}$. The wage rates under Nash bargaining $\check{w}_i(\cdot)$ are determined following a standard Nash bargaining rule and solve:

$$\beta \Pi_{\check{n}_i} = (1 - \beta) [\check{W}_i - U_i], \quad \forall i \in \{h, l\}, \quad (9)$$

where \check{W}_i is defined in (7) and the firm's surplus $\Pi_{\check{n}_i}$ is calculated by applying the envelope theorem to (2)¹⁶:

$$\Pi_{\check{n}_i} = \frac{f_{n_i} - \check{w}^i - \sum_{j \in \{h, l\}} \check{w}_{\check{n}_i}^j \check{n}_j}{r + s}, \quad \forall i \in \{h, l\}.$$

The equation above describes the marginal value a negotiating worker brings to the firm. It is equal to the discounted sum of marginal profits, taking into account that hiring this marginal worker affects the wage of all negotiating workers (the last term

¹⁶For ease of notation, we have avoided specifying the arguments of functions Π , f and \check{w}^i . Notice that, since $n_i \equiv \check{n}_i + \bar{n}_i$, $f_{\check{n}_i} = f_{n_i}$.

in the equation above). Notice that, because a fraction $(1 - \chi_j)$ of workers earn the minimum wage, the marginal worker only affects the wage of a share χ_j of workers. Hence, the lower χ_j , the more limited the ability of the firm to act strategically.

The Nash solution for the negotiated wage reads as

$$\check{w}^i(n_h, n_l, k) = \beta f_{\check{n}_i} + (1 - \beta)rU_i - \beta \sum_{j \in \{h, l\}} \check{w}_{\check{n}_i}^j \check{n}_j, \quad \forall i \in \{h, l\}. \quad (10)$$

The wage equation (10) differs from the standard equation from [Pissarides \(1985\)](#) through the last term. Under Nash bargaining, workers can appropriate part of the decrease in wages of the other workers. This explains the difference between the wage equation (10) and the standard one. Notice that the lower χ_j is, the lower is the size of this additional term, because the set of wages that can be affected through intra-firm bargaining is smaller. This suggests that the minimum wage may affect the strategic behavior of the firm through a *rent-appropriation* effect.

Expression (10) states a system of nonlinear differential equations in \check{w}^i . The following proposition characterizes the solution to this system.

Proposition 1. *The negotiated wage of an i -type worker is*

$$\check{w}^i(n_h, n_l, k) = \beta \check{\Omega}_i f_{n_i} + (1 - \beta)rU_i, \quad \forall i \in \{h, l\} \quad (11)$$

where

$$\begin{aligned} \check{\Omega}_i &= \frac{\int_0^1 f_{n_i \zeta_i^z}(n_h \zeta_h^z, n_l \zeta_l^z, k) \check{\varphi}(z) dz}{f_{n_i}(n_h, n_l, k)}, \\ \zeta_i^z &= \chi_i z + 1 - \chi_i, \\ \check{\varphi}(z) &= \frac{1}{\beta} z^{\frac{1-\beta}{\beta}}. \end{aligned} \quad (12)$$

Proof. See [Appendix A.1](#). □

As in [Cahuc et al. \(2008\)](#), the wage equation (11) differs from the one-worker-per-firm wage equation by the presence of $\check{\Omega}_i$. We provide intuition for this term in [Section 2.7](#).

2.6 First-order conditions of the firm

The first-order conditions for vacancy posting and capital investment are, respectively,

$$\frac{c}{q(\theta_i)} = \frac{f_{n_i}(n_h, n_l, k) - w^i(n_h, n_l, k) - \sum_{j \in \{h, l\}} w_{n_i}^j(n_h, n_l, k) n_j}{r + s}, \quad \forall i \in \{h, l\}, \quad (13)$$

and

$$r + \delta = f_k(n_h, n_l, k) - \sum_{i \in \{h, l\}} w_k^i(n_h, n_l, k) n_i \quad (14)$$

The vacancy-posting conditions (13) equate the expected search cost of hiring a worker of type i to the discounted sum of profits that the marginal worker brings to the firm after being hired. They differ from the condition of the standard model with one worker per firm (Pissarides, 1985) through two strategic effects. First, the employment level of group i may affect the wage of that group. Incentives to overemploy may appear when the production function is concave in n_i (Smith, 1999). Second, the employment level of group i may affect the wage of the other group $j \neq i$. Incentives to overemploy may appear when factors are substitutes and underemployment may result from complementarity between factors (Cahuc et al., 2008).

The capital investment condition (14) equates the opportunity cost of capital to the marginal income of capital. The latter differs from its neoclassical counterpart through the effect on wages: depending on the complementarity/substitutability of capital with labor, the representative firm may choose to underinvest/overinvest in order to reduce wages.

2.7 Front-load factors

Conditions (13) and (14) depend on the negotiated wages \check{w}^h and \check{w}^l through equation (1). The following proposition characterizes the vacancy-posting conditions and the capital investment condition considering the solution for negotiated wages given by expression (11).

Proposition 2. *The vacancy-posting conditions and the capital investment condition read respectively as*

$$(r + s) \frac{c}{q(\theta_i)} = \Omega_i f_{n_i} - w^i(n_h, n_l, k), \quad \forall i \in \{h, l\} \quad (15)$$

and

$$r + \delta = \Omega_k f_k, \quad (16)$$

with

$$\Omega_i = \chi_i \check{\Omega}_i + (1 - \chi_i) \bar{\Omega}_i, \quad \forall i \in \{h, l\} \quad (17)$$

where $\check{\Omega}_i$ is defined as in (12), and $\bar{\Omega}_i$ and Ω_k are

$$\bar{\Omega}_i = \frac{\int_0^1 f_{n_i \zeta_i^z} (n_h \zeta_h^z, n_l \zeta_l^z, k) \bar{\varphi}(z) dz}{f_{n_i}}, \quad (18)$$

$$\Omega_k = \frac{\int_0^1 f_k (n_h \zeta_h^z, n_l \zeta_l^z, k) \bar{\varphi}(z) dz}{f_k}, \quad (19)$$

$$\bar{\varphi}(z) = \frac{1 - \beta}{\beta} z^{\frac{1-2\beta}{\beta}}.$$

Proof. See Appendix A.2. □

Notice the presence of the *front-load factors* $\Omega_i > 0$ and $\Omega_k > 0$ in equations (15) and (16). Their presence is the outcome of the strategic interactions between workers and the representative firm in bargaining. When Ω_i for any $i \in \{h, l\}$ takes a value larger than one, we refer to this situation as a situation of *overemployment*, in the sense that the firm employs a quantity of i -type workers larger than in the case where the firm considers future wages as given. *Underemployment* of factor i appears when the respective factor is lower than one. Similarly, the values of Ω_k illustrate how investment by the representative firm responds to contract incompleteness: we refer to *overinvestment* when $\Omega_k > 1$ and *underinvestment* when $\Omega_k < 1$.

To provide intuition for the values of Ω_i , Ω_k , $\check{\Omega}_i$ and $\bar{\Omega}_i$, we will analyze some special cases. In particular, we will study the case in which the minimum wage is not binding for any labor group (i.e. $\chi_l = \chi_h = 1$), and the case in which there is only one labor type.

2.7.1 Special case: no minimum wage earner

First, notice that, when $\chi_l = \chi_h = 1$, Ω_i , $\check{\Omega}_i$ and Ω_k take the same form as in Cahuc et al. (2008) as $\zeta_l^z = \zeta_h^z = z$:

$$\Omega_i = \check{\Omega}_i = \frac{\int_0^1 f_{n_i z} (n_h z, n_l z, k) \check{\varphi}(z) dz}{f_{n_i}},$$

$$\Omega_k = \frac{\int_0^1 f_k (n_h z, n_l z, k) \bar{\varphi}(z) dz}{f_k},$$

while, from (17), the value of $\bar{\Omega}_i$ becomes irrelevant.

Ω_i is the ratio of two elements: its denominator is the marginal product of labor, while its numerator is a weighted average of the infra-marginal products, where the weights in the integral are given by the density $\check{\varphi}(z)$, with $\int_0^1 \check{\varphi}(z)dz = 1$. Notice that $\Omega_i = 1$ when $\check{\varphi}(z)$ has a mass point around $z = 1$ and $\check{\varphi}(z) = 0$ for all $z < 1$, since the numerator is equal to the denominator in this case. Ω_i is also equal to one when the marginal product of i -type labor is independent of n_i and n_j . For other values of $\check{\varphi}(z)$ and with a non-linear production function, Ω_i may differ from one.

Three effects may drive the value of the front-load factors away from one.¹⁷ First, the concavity in n_i of the production function tends to increase their value: the more concave the production function is, the larger are the incentives for the firm to overemploy i -type workers in order to reduce their wage. Second, the substitutability (complementarity) with j -type workers tends to increase (decrease) the value of Ω_i : overemployment (underemployment) allows to decrease the wage of j -type workers by decreasing their marginal product. Third, the shape of the density $\check{\varphi}(z)$ also affects the values of Ω_i by weighting the different infra-marginal products of labor at a different intensity. Specifically, when the bargaining power of workers is large, the representative firm has more incentives to reduce wages.

Moreover, under Nash bargaining the negotiated wage is a function of the capital stock. Because workers do not share in the cost of *ex ante* investments in the absence of binding wage contracts, this leads to underinvestment (overinvestment) when capital is complementary (substitutable) to labor: the representative firm anticipates that investing more (less) in physical capital amounts to bargaining to a higher wage.

2.7.2 Special case: one labor type

Consider the case where χ_i may differ from 1, but we only have one labor type. In this case, a first difference with respect to Cahuc et al. (2008) appears: the front-load factor Ω_i becomes an average of the front-load factor for negotiating workers $\check{\Omega}_i$ and the front-load factor for minimum wage workers $\bar{\Omega}_i$, as shown in equation (17).

With the change of variable $x = \zeta^z$, these front-load factors can be interpreted more easily:

$$\Omega_k = \frac{\int_0^1 f_k(nx, k) \bar{\psi}(x) dx}{f_k},$$

$$\check{\Omega} = \frac{\int_0^1 f_{nx}(nx, k) \check{\psi}(x) dx}{f_n} \quad \text{and} \quad \bar{\Omega} = \frac{\int_0^1 f_{nx}(nx, k) \bar{\psi}(x) dx}{f_n},$$

¹⁷See Cahuc et al. (2008) and Cahuc and Wasmer (2001a) for more details.

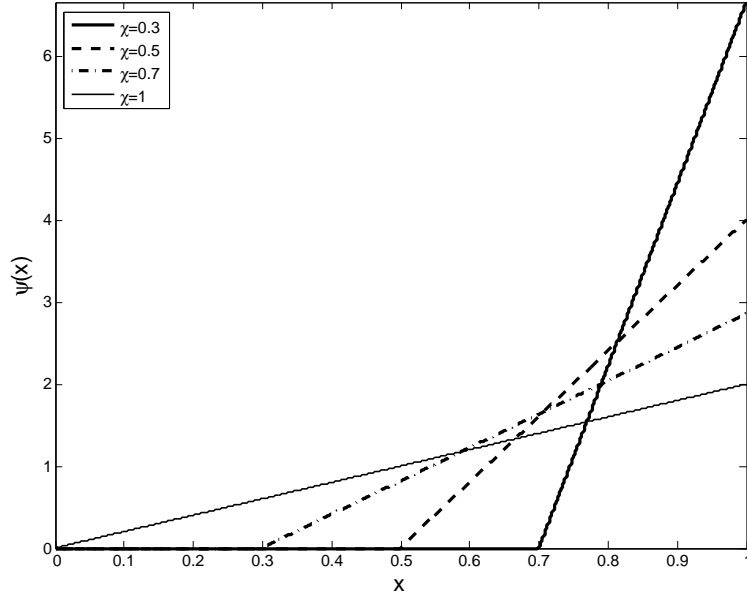


Figure 1: The effect of χ on the $\check{\psi}(x)$ density, examples with $\beta = 1/2$

with

$$\check{\psi}(x) = \begin{cases} 0 & \text{if } x < 1 - \chi \\ \frac{(x-1+\chi)^{\frac{1-\beta}{\beta}}}{\beta\chi^{1/\beta}} & \text{if } x \geq 1 - \chi \end{cases}$$

and

$$\bar{\psi}(x) = \begin{cases} 0 & \text{if } x < 1 - \chi \\ \frac{(1-\beta)(x-1+\chi)^{\frac{1-\beta}{\beta}}}{\beta\chi^{\frac{1-\beta}{\beta}}} & \text{if } x \geq 1 - \chi \end{cases}$$

where the subscripts i are not considered since we only have one labor type. Notice that the densities $\check{\psi}(x)$ and $\bar{\psi}(x)$ fulfill the property $\int_0^1 \check{\psi}(x)dx = \int_0^1 \bar{\psi}(x)dx = 1$.

The front-load factors have a structure similar to the one in [Cahuc et al. \(2008\)](#) with the difference that the share of negotiating workers χ has an effect on the density that appears in the integral. In particular, the lower the value of χ is, the more concentrated are the $\check{\psi}(x)$ and $\bar{\psi}(x)$ distributions around $x = 1$. Figure 1 depicts examples of the $\check{\psi}(x)$ density for $\beta = \frac{1}{2}$ and several values of χ .

The effect of χ on the densities suggests that the firm's strategic behavior gets

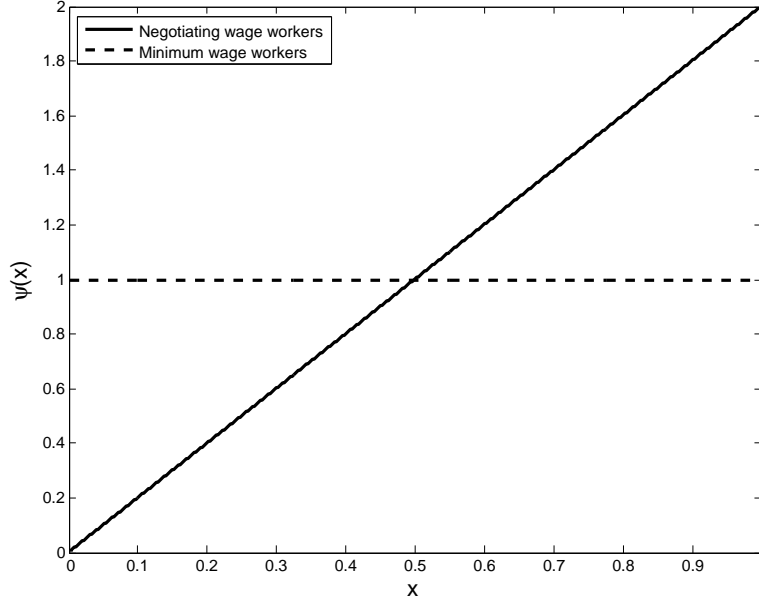


Figure 2: Comparing the $\check{\psi}(x)$ and $\bar{\psi}(x)$ densities, example with $\beta = 1/2$ and $\chi = 1$

more limited as the share of negotiating workers decreases. For example, when capital and labor are complements, the firm chooses to underinvest (i.e. $\Omega_k < 1$); but, as χ decreases and the distribution $\bar{\psi}(x)$ gets more concentrated around $x = 1$, underinvestment becomes weaker, generating an increase in the demand for capital: the lower the fraction of workers that negotiate their wage with the firm is, the lower are the effects that the wage negotiation exerts over the investment decision. This implies that $\Omega_k \rightarrow 1$ when $\chi_i \rightarrow 0$ for $i \in \{h, l\}$.¹⁸

There may also be situations where the firm's strategic behavior actually gets exacerbated. To understand when this may happen, first notice that the $\check{\psi}(x)$ density is more concentrated around $x = 1$ than $\bar{\psi}(x)$. Figure 2 illustrates this fact with an example: it compares the $\check{\psi}(x)$ density (solid line) with $\bar{\psi}(x)$ (dashed line) in the case where $\beta = \frac{1}{2}$. This suggests that, *ceteris paribus*, overemployment (underemployment) should be stronger in the case of minimum wage workers than in the case of negotiating workers. The intuition for this is *rent appropriation*: negotiating workers claim part of the change in the wage of other workers resulting from intra-firm bargaining, while minimum wage workers do not. As a consequence, overemployment (or underemployment) may become stronger as χ decreases because rent appropriation by workers is more limited. This can be observed in equation (17), where Ω depends more on $\bar{\Omega}$ than

¹⁸Notice also that, if $\chi_i \rightarrow 0 \forall i \in \{h, l\}$, $\check{\Omega}_i \rightarrow 1$ and $\bar{\Omega}_i \rightarrow 1 \forall i \in \{h, l\}$, as in this case the firm cannot strategically influence any wage rate.

$\check{\Omega}$ as χ decreases.

2.7.3 General case

In the general case with two labor types and χ_i that may differ from one, the front-load factors are written as in (12), (17), (18) and (19). All the comparative statics and interpretations in Sections 2.7.1 and 2.7.2 still hold with the additional ingredient that the share of negotiating workers may interact with the fact that the two labor types are substitutes or complements.

For example, when labor types are substitutes, the firm has incentives to overemploy them. If χ_l decreases, the firm would have less incentives to overemploy h -type workers because it would influence the wage of a lower fraction of l -type workers. In the case of overemployment of l -type workers, the effect is ambiguous. On the one hand, overemployment may be more limited as in the case of h -type workers. On the other hand, lower rent appropriation by l -type workers may actually enhance the firm's strategic behavior, leading to higher overemployment.

3 Equilibrium

In general equilibrium, the labor market tightness θ_i and the present-discounted value of being unemployed U_i are endogenous. The former is obtained in a standard way by equating the flows in and out of employment, leading to the Beveridge relations

$$n_h = \varsigma_h \frac{\theta_h q(\theta_h)}{s + \theta_h q(\theta_h)} \quad \text{and} \quad n_l = \varsigma_l \frac{\theta_l q(\theta_l)}{s + \theta_l q(\theta_l)}, \quad (20)$$

while, by combining equations (5)-(8), one can show that the value of unemployment can be written as

$$rU_i = \frac{(r + s)b + \theta_i q(\theta_i)w^i}{r + s + \theta_i q(\theta_i)}. \quad (21)$$

To close the model, we also need to determine the equilibrium values for χ_l and χ_h . These have to be consistent with the optimizing behavior of workers: the choice by workers between negotiating with the firm or earning the minimum wage must be the most attractive one. Arbitrage thus yields the following equilibrium condition:

Equilibrium condition 1. *Workers' wage strategy is optimal:*

- The fraction $\chi_i = 1$, $i = \{h, l\}$, is an equilibrium if $\check{w}_i > \bar{w}$.

- The fraction $\chi_i = 0$, $i = \{h, l\}$, is an equilibrium if $\check{w}_i < \bar{w}$.
- The fraction $\chi_i \in (0, 1)$, $i = \{h, l\}$, is an equilibrium if $\check{w}_i = \bar{w}$.

This yields the following definition of equilibrium:

Definition 1. A steady-state general equilibrium is a set of employment levels n_h, n_l , a capital stock k , a set of fractions χ_l and χ_h of workers who earn the negotiated wage, negotiated wage rates \check{w}_h and \check{w}_l and labor market tightness θ_h and θ_l such that the first-order conditions (13) and (14), the wage equations (11), the value of unemployment (21), the Beveridge relations (20) and Equilibrium condition 1 are satisfied, given a minimum wage \bar{w} .

Equilibrium condition 1 suggests the possibility of multiple equilibria. This is confirmed by Figure 3, which illustrates several examples of the determination of χ with one type of labor. Each panel in the figure compares the value of the minimum wage with the value of the negotiated wage as a function of χ to pin down possible equilibria. The upper left panel illustrates the case with one labor type, no capital and a linear production function. In this case, because the minimum wage is higher than the negotiated wage, nobody negotiates with the firm and the equilibrium value of χ is zero. The two upper right panels illustrate the situation with decreasing returns to scale and no capital. The production function is $f(n) = n^\alpha$, with $\alpha \in (0, 1)$. The first example is characterized by multiple equilibria (two equilibria are in pure strategy and one in mixed strategy), while the second example shows one equilibrium in pure strategy.¹⁹ Finally, the bottom panels show the determination of χ for three values of the minimum wage when the production function is $f(n, k) = n^\alpha k^{1-\alpha}$. There is uniqueness in these cases. When the minimum wage is too low, χ equals one. When it starts to bind, the unique equilibrium is characterized by mixed strategies. For higher values of the minimum wage, χ is equal to zero.

In the numerical examples of Section 5, we focus only on situations where the equilibrium is unique.

¹⁹Intuitively, multiple equilibria arise in this example due to a general-equilibrium effect. When agents coordinate to a higher χ and a larger proportion of agents negotiate, the representative firm chooses to overhire to reduce their wage because there are decreasing returns to labor. However, the firm does not consider the effect on labor-market tightness in general equilibrium: overemployment pushes the tightness upwards, which actually increases wages above the minimum wage. When the agents coordinate to a lower value of χ , the opposite happens.

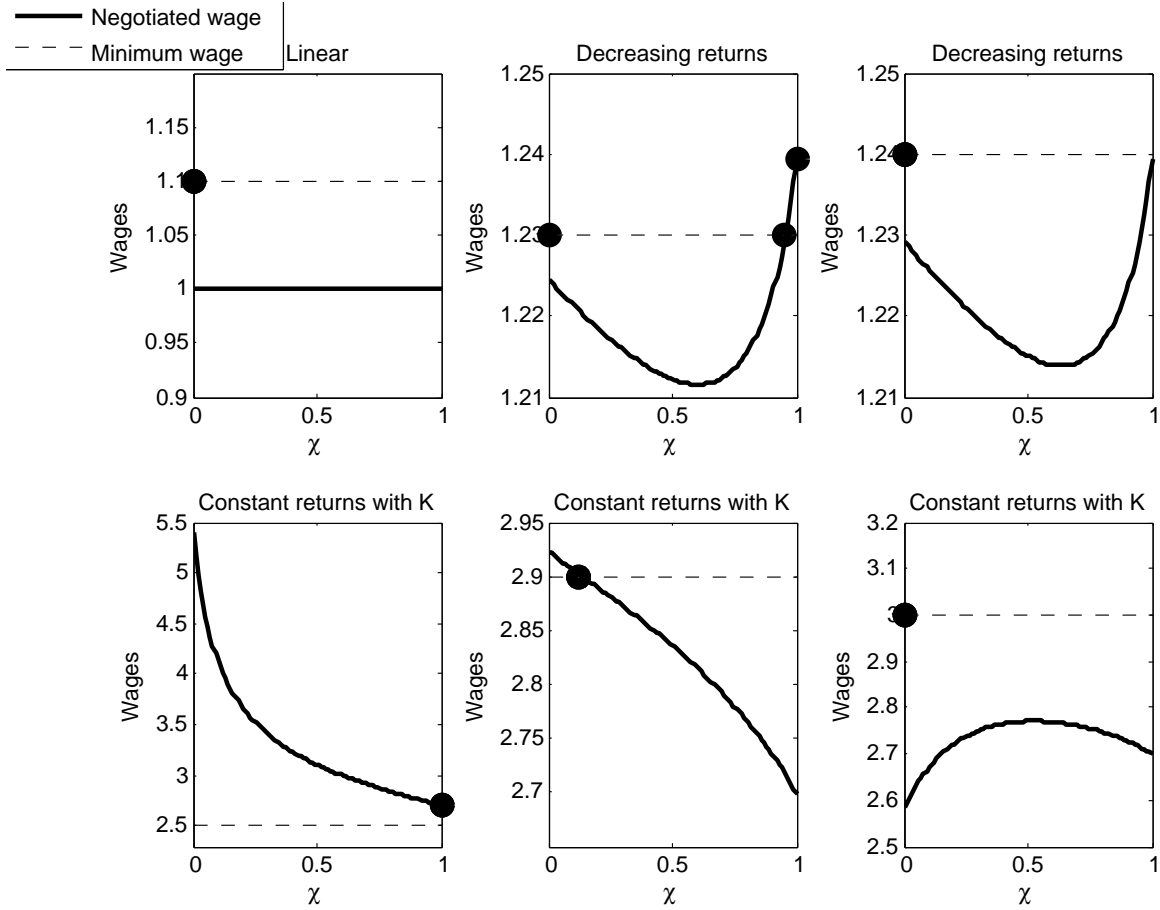


Figure 3: Determination of χ : several examples

Notes: the upper left panel illustrates the case with one labor type, no capital and a linear production function. The production function for the two upper right panels is $f(n) = n^\alpha$, with $\alpha \in (0, 1)$. The bottom panel show the determination of χ for three values of the minimum wage when the production function is $f(n, k) = n^\alpha k^{1-\alpha}$.

4 Welfare

Our model is characterized by two types of inefficiencies. First, congestion externalities are not necessarily internalized by the Nash bargaining rule as in [Hosios \(1990\)](#). Second, appropriability distorts employment and capital decisions, as in [Grout \(1984\)](#) and [Cahuc et al. \(2008\)](#) among others. Both inefficiencies may partly compensate each other; for instance, the social losses of a large bargaining power, which leads to too few vacancies in the standard model with one worker per firm, may be reduced by an overemploying representative firm.

We now illustrate these ideas in the context of the constrained social planner problem.²⁰ The value function characterizing the social planner's solving problem is

$$\mathcal{V}(n_h, n_l, k) = \max_{\{v_h, v_l, a\}} \frac{1}{1+r} \frac{d}{dt} \left(\left[f(n_h, n_l, k) + b \left(\sum_{j=\{h,l\}} \varsigma_j - n_h - n_l \right) - \sum_{j=\{h,l\}} v_j c - a \right] dt + \mathcal{V}(n'_h, n'_l, k') \right), \quad (22)$$

subject to the constraints (3), (4), $\theta_h = \frac{v_h}{\varsigma_h - n_h}$ and $\theta_l = \frac{v_l}{\varsigma_l - n_l}$.

We show in [Appendix A.3.1](#) that the first-order conditions for a maximum are, in steady state,

$$\frac{c}{q(\theta_i)} = \frac{(f_{n_i} - b)(1 - \eta(\theta_i))}{r + s - \theta_i^2 q_{\theta_i}(\theta_i)}, \quad \forall i \in \{h, l\}, \quad (23)$$

and

$$r + \delta = f_k, \quad (24)$$

where $\eta(\theta_i) \equiv -\frac{\theta_i q_{\theta_i}(\theta_i)}{q(\theta_i)}$.

These optimality conditions can be compared to the vacancy-posting and capital-investment conditions of the representative firm in the context of the steady-state equilibrium given in [Definition 1](#). This allows us to establish the following result on the efficiency of the equilibrium:

Proposition 3. *The constrained-efficient allocations are a set of employment levels n_h and n_l , a capital stock k and labor-market tightness θ_h and θ_l such that the optimality conditions (23) and (24) and the Beveridge relations (20) are satisfied.*

Hence, a steady-state equilibrium is efficient if, $\forall i \in \{h, l\}$,

²⁰See also [Smith \(1999\)](#) and [Cahuc and Wasmer \(2001b\)](#).

$$\beta = \frac{\check{\Omega}_i f_{n_i} - b - \Delta_i (f_{n_i} - b)(r + s)}{\check{\Omega}_i f_{n_i} - b + \Delta_i (f_{n_i} - b)\theta_i q(\theta_i)} \quad \text{for } 0 < \chi_i \leq 1, \quad (25)$$

$$\bar{w} = \bar{\Omega}_i f_{n_i} - \Delta_i (r + s)(f_{n_i} - b) \quad \text{for } 0 \leq \chi_i < 1, \quad (26)$$

$$\Omega_k = 1 \quad \text{for } 0 < \chi_i < 1, \quad (27)$$

$$\text{where } \Delta_i = \frac{1 - \eta(\theta_i)}{r + s + \theta_i q(\theta_i) \eta(\theta_i)}.$$

Proof. See Appendix A.3.2. □

Condition (27) is a standard condition for an efficient capital allocation, while condition (25) is an augmented Hosios-Pissarides condition, with (26) being its counterpart in presence of a binding minimum wage. It is easy to show that both reduce to the standard condition of a model with one worker per firm when $\check{\Omega}_i = \bar{\Omega}_i = 1$, with $\beta = \eta(\theta_i)$ when $\chi_i > 0$ and $\bar{w} = f_{n_i} - \Delta_i (r + s)(f_{n_i} - b)$ when $\chi_i = 0$.²¹ When $\check{\Omega}_i > 1$, a value for β larger than $\eta(\theta_i)$ is required in order to compensate for overemployment by the representative firm. The opposite occurs when $\check{\Omega}_i < 1$.

A minimum wage can fill one of the efficiency conditions given in Proposition 3. For example, condition (26) may be satisfied when the decentralized equilibrium wage is inefficiently too low absent a minimum wage legislation. This happens when the representative firm has incentives to overemploy ($\check{\Omega}_i > 1$) or when the share it obtains under wage bargaining is too high, generating congestion on the vacancy side. Similarly, condition (27) is satisfied when the minimum wage is binding for both labor groups. In this case, intrafirm bargaining cannot take place, which alleviates the holdup problem.

However, these efficiency conditions are rarely satisfied together. Moreover, the fulfilment of a subset of them does not necessarily produce an improvement in welfare. It may be the case that reaching optimality on one market leads to augmented inefficiencies on another market. For example, implementing (26) in the case of low-wage workers may induce the front-load factor for high-wage workers to deviate even more from its social optimum. Similarly, implementing (27) on the capital market or (26) on the market for high-wage workers may be too expensive if it requires increasing the wage cost of l -type workers.

²¹It can be shown that, if $\bar{\Omega}_i = 1$ and $\chi_i = 0$, then

$$\bar{w} = \eta(\theta_i) f_{n_i} + (1 - \eta(\theta_i))b + \eta(\theta_i)\theta_i c.$$

5 Quantitative analysis

The discussion in Section 2.7 emphasizes how the introduction of a binding minimum wage affects the incentives to overemploy or underemploy by analyzing how the front-load factors change with the policy. It is important to notice that, throughout the analysis of the variation in the Ω_i 's, all the levels of n_h , n_l and k are held constant. It is tempting to extrapolate these results to the whole incentives to open up vacancies and believe that, if a front-load factor increases (holding constant n_h , n_l and k), then the marginal income of the respective factor increases too. However, we know that this may not be true because of the possible complementarities or substitutabilities between factors in the production process.

We now quantitatively assess how the factor utilization levels n_h , n_l and k vary for different values of the exogenous minimum wage \bar{w} . We start with the case of one labor type in Section 5.1. This section shows that the *capital demand* effect alone is enough for employment and capital to rise following an increase in the minimum wage. Section 5.2 then extends this benchmark to two labor types to emphasize the *rent appropriation* effect.

5.1 One labor type

The one-labor-type case is a special case of the more general model of Section 2 where $f(n_h, n_l, k)$ is assumed not to vary with n_h and ς_l and ς_h are respectively set equal to one and zero. Without loss of generality, we assume that the labor type hired by the representative firm is l . We first calibrate the economy without a minimum wage to the US. We then build a grid for different values of the minimum wage over the interval $(0, \bar{w}_{\max}]$, where \bar{w}_{\max} is the maximum value of the minimum wage such that $n_l > 0$. For each point of this grid, we compute relevant statistics such as the employment and capital levels for the economy in steady state or a measure of welfare. The integrals in the expression for the front-load factors in Proposition 1 and Proposition 2 are approximated using a Gauss-Legendre quadrature.²²

5.1.1 Calibration

When there is only one labor type, it turns out that the steady state of the economy described in Section 2 is exactly the same as in Pissarides (2009) when $\Omega_l f_{n_l} = 1$, even though Pissarides (2009) focuses on the standard one-worker-per-firm matching model. For this reason, we take most of the parameter values from his calibration,

²²See Appendix B for a description of the algorithm used to solve the model.

Table 1: Calibration: parameter values

| Parameter | Value | Description | Target/Source |
|------------------|--------|-----------------------------|--|
| One labor type: | | | |
| η | 0.5 | Matching func. elasticity | Standard/Pissarides (2009) |
| β | 0.5 | Bargaining power | Standard/Pissarides (2009) |
| m_0 | 0.7 | Matching function scale | Job finding rate/Pissarides (2009) |
| r | 0.004 | Discount rate | Interest rate/Pissarides (2009) |
| s | 0.036 | Job separation rate | Job separation rate/Pissarides (2009) |
| b | 0.71 | Flow value of unemp. | Pissarides (2009) |
| c | 0.356 | Vacancy cost | Tightness/Pissarides (2009) |
| δ | 0.0087 | Capital depreciation | 10% annual deprec./Gomme and Rupert (2007) |
| α | 0.425 | Production func. elasticity | Labor share/Gomme and Rupert (2007) |
| A | 0.327 | TFP | $\Omega_l f_{n_l} = 1$ |
| s_l | 1 | l population size | Assumption |
| s_h | 0 | h population size | Assumption |
| Two labor types: | | | |
| η | 0.5 | Matching func. elasticity | One labor-type case |
| β | 0.5 | Bargaining power | One labor-type case |
| m_0 | 0.7 | Matching function scale | One labor-type case |
| r | 0.004 | Discount rate | One labor-type case |
| s | 0.036 | Job separation rate | One labor-type case |
| b | 0.71 | Flow value of unemp. | One labor-type case |
| c | 0.356 | Vacancy cost | One labor-type case |
| δ | 0.0087 | Capital depreciation | One labor-type case |
| α | 0.425 | Production func. elasticity | One labor-type case |
| A | 0.297 | TFP | 5.7% unemployment rate |
| π | 2.02 | Labor services for h | Non-college earnings quartiles/BLS |
| ν | 1 | CES exponent | Perfect substitutes |
| s_l | 1 | l population size | Assumption |
| s_h | 1 | h population size | Assumption |

which targets the US economy at a monthly frequency. In particular, we choose a Cobb-Douglas structure for the matching function:

$$m(s_l - n_l, V_l) = m_0(s_l - n_l)^\eta V_l^{1-\eta}, \quad (28)$$

Regarding the parameters that are as in [Pissarides \(2009\)](#), we fix the discount rate $r = 0.004$, the value of leisure $b = 0.71$, the vacancy cost $c = 0.356$, the matching function scale parameter $m_0 = 0.7$ and the elasticity of the matching function η and the bargaining power β equal to 0.5.

We also consider a Cobb-Douglas form for the production function:

$$f(n_l, k) = A n_l^{1-\alpha} k^\alpha. \quad (29)$$

Because we want our calibration to be consistent with [Pissarides \(2009\)](#), we fix the parameter A such that $\Omega f_{n_l} = 1$. This produces a value for $A = 0.327$. The parameter α is calibrated to obtain a realistic labor share. We rely on [Gomme and Rupert \(2007\)](#), who document a labor share of 71.7%. The resulting value for α is 0.425, a bit above the value considered by [Cooley and Prescott \(1995\)](#). Finally, we target a 10% capital depreciation rate, which is consistent with evidence in [Gomme and Rupert \(2007\)](#), implying $\delta = 0.0087$.

The values of α and δ generate an investment share in aggregate output of 18.5%, which is close to its data counterpart as documented in the NIPA tables. Other labor market moments are of course consistent with [Pissarides \(2009\)](#): a 5.7% unemployment rate, labor-market tightness equal to 0.72 and a 59.4% job finding rate.

These parameter values are reported in [Table 1](#).

5.1.2 Results

[Figure 4](#) shows how the equilibrium values of employment and the capital stock change with the minimum wage. It also shows the values of the front-load factors for capital and labor and a measure of welfare, which is the flow value displayed in [equation \(22\)](#). The vertical dashed line on each panel of the figure refers to the level of the minimum wage beyond which it is binding. To the left of this line, the minimum wage does not bind and the allocations correspond to the case in which $\chi_i = 1$ for $i = h, l$. To the right of this line, $\chi_l < 1$ ²³.

Absent a minimum wage, the representative firm overemploys and underinvests ($\Omega_l = 1.27 > 1$ and $\Omega_k = 0.63 < 1$). Overemployment occurs because the production function is concave in labor: by increasing employment the firm can reduce the marginal

²³In [Appendix C.1](#) we show the evolution of χ_l for different values of the minimum wage.

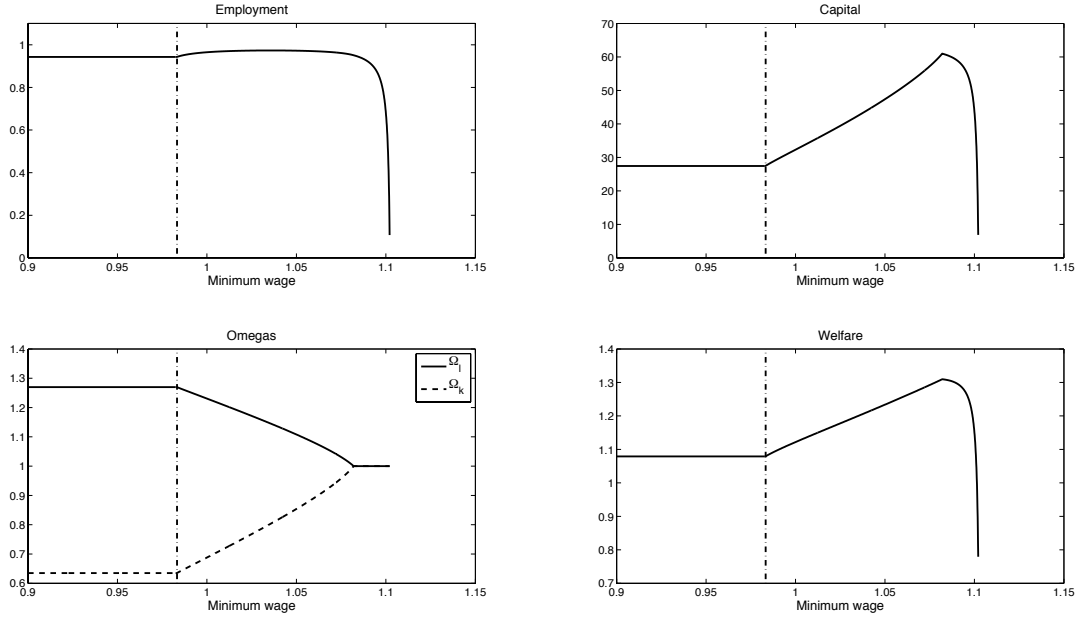


Figure 4: One labor type case: factor utilization, front-load factors and welfare

Notes: The vertical dashed line refers to the level of the minimum wage beyond which it is binding

product of labor and the negotiated wage. Underinvestment occurs because of the complementarity between labor and capital: the firm decreases the marginal product of labor by owning a lower capital stock. When the minimum wage starts binding, the share χ_l starts decreasing and both front-load factors converge towards one as the value of the minimum wage increases, lowering the importance of overemployment and underinvestment and improving welfare. This occurs because it becomes harder for the firm to influence the value of the wage when χ_l decreases.

As a result, the firm invests more and the capital stock increases. Note that the impact on capital is quite strong in this case: comparing the stock of capital of an economy without a minimum wage with the stock of an economy with a binding minimum wage, the latter may be up to 120% higher than the former.

The impact on employment is much weaker. When the minimum wage starts binding, we observe an increase in employment that may amount to 3%. Employment then remains relatively unchanged for most values of the minimum wage until it abruptly falls for a value of the minimum wage that is too high. Notice that the increase in employment occurs even though the front-load for labor decreases with the minimum wage. The reason for this is due to the increase in the capital stock and the fact that labor and capital are complements in the production function: as capital increases, the marginal product of labor increases, providing incentives for the firm to hire more

workers. This is the *capital demand* effect.

Finally, if the minimum wage is too high, employment eventually decreases since labor costs become too stringent, implying a decrease in labor demand. This negatively impacts investment in capital too, because of the complementarity between capital and labor.

5.2 Two labor types

5.2.1 Parametrization

We now consider the case with two labor types and reproduce a similar exercise in this context. This will allow us to illustrate the *rent appropriation* effect of the minimum wage. We consider a CES specification for the production function:

$$f(n_h, n_l, k) = [(\pi n_h)^\nu + n_l^\nu]^{\frac{1-\alpha}{\nu}} k^\alpha, \quad (30)$$

where $\pi > 1$ in order to generate higher wages for h -type workers. The parameter π can be interpreted as the amount of labor services provided by h -type workers, while ν influences the elasticity of substitution between the two labor types.

Most of the parameter values we consider are as in the one-labor-type case. We only recalibrate the TFP parameter so that the unemployment rate is the same as in the previous exercise (5.7%).²⁴ Moreover, we need to find appropriate values for the amount of labor services provided by h -type workers and the parameter ν .

We interpret both labor types as coming from the same skill category. We fix a value for $\nu = 1$, implying that both labor types are substitutes (in the sense that the cross-derivative in the production function is negative) and that the elasticity of substitution is infinite²⁵. We consider the first and the last quartile of usual weekly earnings of full-time non-college workers provided by the BLS to calibrate the value of π . In 2014, it is shown that high-school graduates on the last quartile earn 98% more than graduates on the first quartile. In the case of workers in the category “Some college or associate degree”, this difference is higher (113%), while it is lower in the case of workers with less than a high-school diploma (81%), but the sample size for this latter group is much smaller. Hence, we choose a value for π so that the wage of h -type workers (interpreted as being on the third quartile) is twice the wage of l -type workers (interpreted as being on the first quartile). This implies a value for $\pi = 2.02$. Because of the quartile interpretation, we also fix $\varsigma_l = \varsigma_h = 1$.

All the parameter values are reported on Table 1.

²⁴The labor income share and the investment-output ratio are also the same as in the previous exercise.

²⁵Section 5.2.4 shows results for other values of ν .

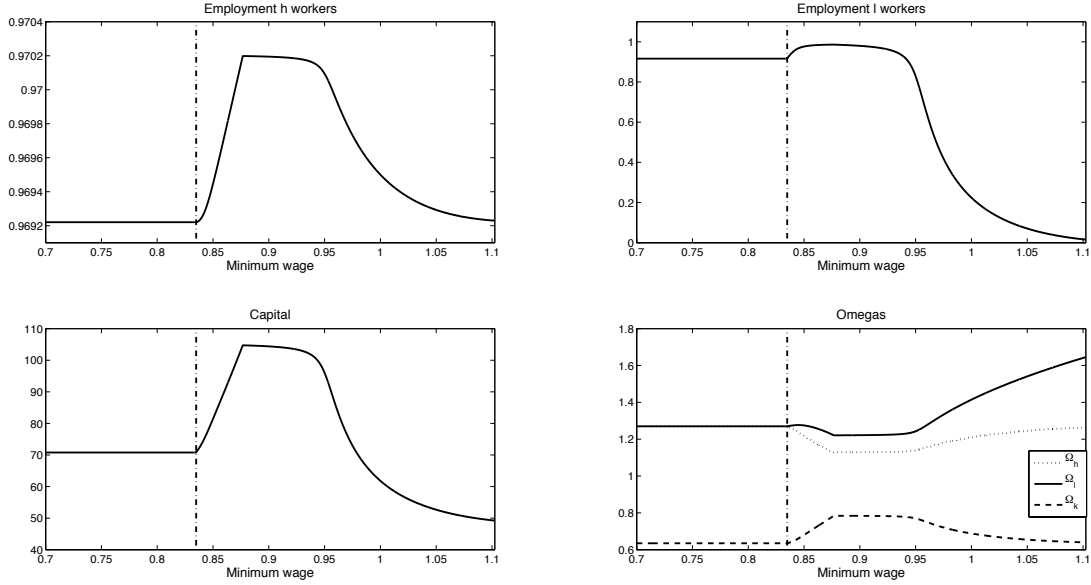


Figure 5: Two labor types case: factor utilization, front-load factors and welfare

Notes: The vertical dashed line refers to the level of the minimum wage beyond which it is binding

5.2.2 Results

Figure 5 shows how the equilibrium values of labor and capital utilization and the front-load factors change for different values of the minimum wage in the two-labor-type case.

The graphs on Figure 5 share many characteristics with the one-labor-type case. Absent a binding minimum wage, the representative firm underinvests and overemploys, with the same values for Ω_l and Ω_k as in the one-labor-type case. When the minimum wage becomes binding for l -type workers, the holdup problem on capital investment is alleviated, which encourages the firm to invest more. Consequently, Ω_k increases. Moreover, since capital is complementary to both labor inputs, the increase in capital leads to an increase in the marginal productivities of h -type workers and l -type workers and, as a consequence, to an increase in n_h and n_l . This is the same *capital demand* effect described above in the one-labor-type case. Notice, however, that the quantitative impact of the minimum wage on capital accumulation is lower than in the one-labor-type case though it is still strong: the difference between the size of the capital stock in an economy without a binding minimum wage and an economy with a binding minimum wage may reach up to 48% versus 120% in the one-labor-type case. The reason for this quantitative difference is due to the fact that the share of negotiating workers in the entire population of workers is *a fortiori* higher because of

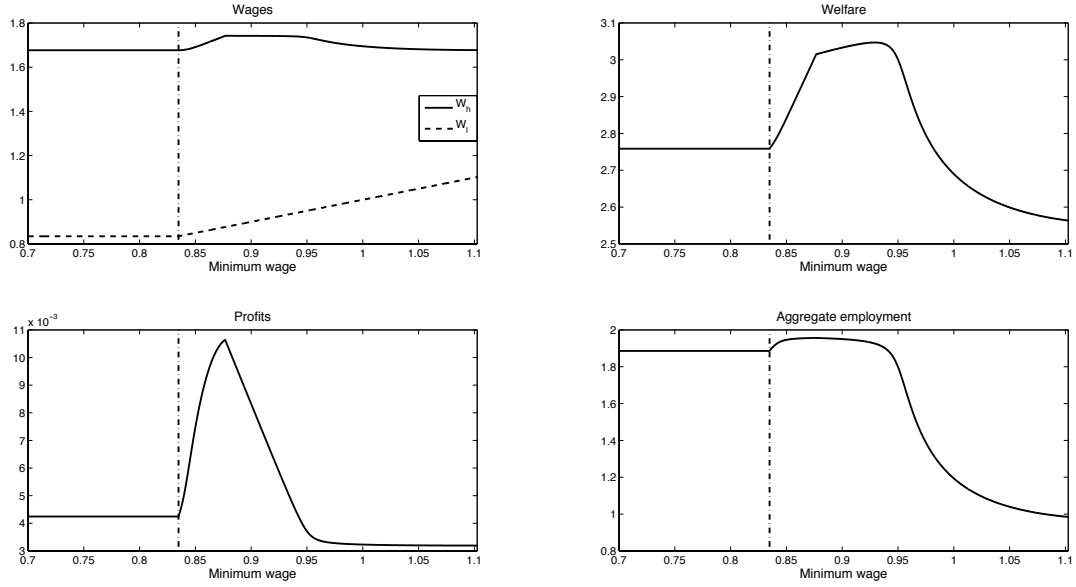


Figure 6: Wages, profits and welfare

Notes: The vertical dashed line refers to the level of the minimum wage beyond which it is binding

the existence of h -type workers now.

A difference between Figure 4 and Figure 5 is that, while Ω_l monotonically decreases as the minimum wage increases on Figure 4, both Ω_l and Ω_h eventually increase beyond a certain value for \bar{w} on Figure 5. Ω_l even overshoots to become higher than its original value in the economy without a minimum wage. Three effects explain the behavior of Ω_l . Two of these effects are responsible for the initial decrease in Ω_l : i) the fact that the production function is concave in n_l gives incentives for the firm to overemploy l -type workers in the economy without a minimum wage ($\Omega_l > 1$); ii) the substitutability between labor types also leads to overemployment. When the minimum wage starts binding, these incentives start disappearing and the firm overemploys less. As the minimum wage increases, a third effect eventually dominates the first two—a *rent appropriation* effect. Because minimum wage workers do not claim part of the change in the wage of other workers, the firm can benefit more easily from the substitutability between labor types and choose to overemploy l -type workers by more in order to decrease the wage of h -type workers.

The decrease in Ω_h when the minimum wage starts binding is due to the fact that the firm exploits less the substitutability between factors to decrease the wage of l -type workers. However, this mechanism is not strong enough to counteract the increase in labor demand coming from an increase in the capital stock, as shown by the small

increase in n_h on the graph. Interestingly, when the minimum wage is binding, Ω_h is always lower than Ω_l . This is a consequence of the *rent appropriation* effect, which influences the hiring decision of l -type workers and does not play a role for hiring h -type workers. Moreover, the impact on employment of h -type workers is also lower than the impact on employment of l -type workers: the difference in employment between an economy without a binding minimum wage and an economy with a binding minimum wage may reach up to 0.1% in the case of h -type workers, while this difference is up to 7.6% in the case of l -type workers. Notice also that the difference between Ω_h and Ω_l increases as \bar{w} increases, meaning that the rent appropriation effect becomes stronger for higher values of \bar{w} .

Eventually, when the minimum wage is too high, the increase in the costs of production and the consequent reduction in profits imply that firms reduce their demand for all productive factors, thus offsetting the effects due to the alleviation of the holdup problem. For a high value of \bar{w} , the levels of k and n_l are lower than they would be were the minimum wage absent.

Figure 6 shows the equilibrium wages of h -type and l -type workers, and the profiles of welfare and firm profits for different values of the minimum wage. For a marginally binding minimum wage for l -type workers, the wage of h -type workers increases. This is due to the complementarity between capital and labor: the increase in the demand for capital causes an increase in wages through an increase in the marginal product of labor. Notice that, despite the increase in wages, profits of the firm increase as well. The effect on the flow of aggregate welfare, computed as the flow in equation (22), is also positive when the minimum wage becomes binding. Two reasons explain the positive impact on profits and welfare. First, the minimum wage, when not increased by too much, partially corrects for the holdup problem. Second, the negative externalities brought by congestion in the labor markets may be weaker.

5.2.3 A decomposition

In order to understand the importance of the capital demand and rent appropriation effects over the allocations, we perform the following exercise inspired by Chari et al. (2007): we assume that Ω_h , Ω_l and Ω_k are wedges that firms take as given when deciding how many vacancies to open and how much to invest.²⁶ For every $\bar{w} \in (0, \bar{w}_{max}]$ we set Ω_h and Ω_l equal to their corresponding values in the benchmark specification when the minimum wage is not binding. Conversely, Ω_k is set equal to the series generated in Section 5.2.2 for every $\bar{w} \in (0, \bar{w}_{max}]$. In other words, we shut down the effects of the

²⁶Notice that, if we take the series of Ω_h , Ω_l and Ω_k generated in Section 5.2.2 and plug it in equations (15), (16) and (20), we obtain the allocations for n_h , n_l and k depicted in Figure 5.

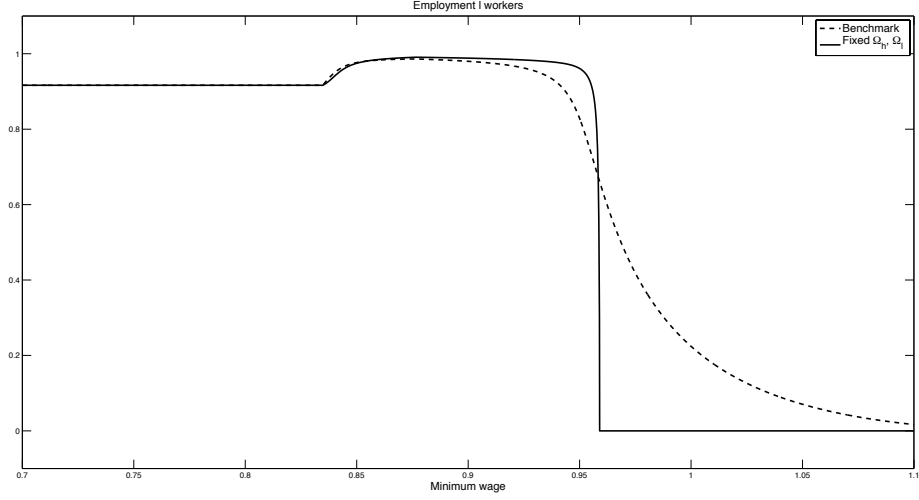


Figure 7: Capital demand effect

change in Ω_h and Ω_l over the allocations when the minimum wage becomes binding, and retain exclusively the effect of the change in Ω_k , which we have called the *capital demand effect*.

Figure 7 shows employment levels of l -type workers in the benchmark case (solid line) and in the case in which Ω_h and Ω_l are kept constant at their pre-minimum wage values (dashed line). For values of \bar{w} for which the minimum wage is marginally binding, χ_l is close to one²⁷ and the changes in Ω_h and Ω_l are small. Thus, in this region, the capital demand effect rules the response of n_l to the introduction of the minimum wage. For higher values of \bar{w} , the decrease of Ω_h and Ω_l actually exerts a negative effect over n_l : keeping Ω_h and Ω_l constant results in a higher level of n_l (the dashed line is over the solid line). As \bar{w} becomes sufficiently large, the capital demand effect quickly dies out and the rent appropriation effect kicks in, causing the solid line to be above the dashed one. This confirms the claim in Section 5.2.2 that the rent appropriation effect becomes stronger for higher values of \bar{w} .

It is clear from this exercise that both the capital demand and the rent appropriation effect exert a positive effect over n_l . The capital demand effect is the relevant one for lower values of \bar{w} , whereas the rent appropriation effect is responsible for the attenuated effect of a high minimum wage over n_l .

²⁷See Appendix C.2.

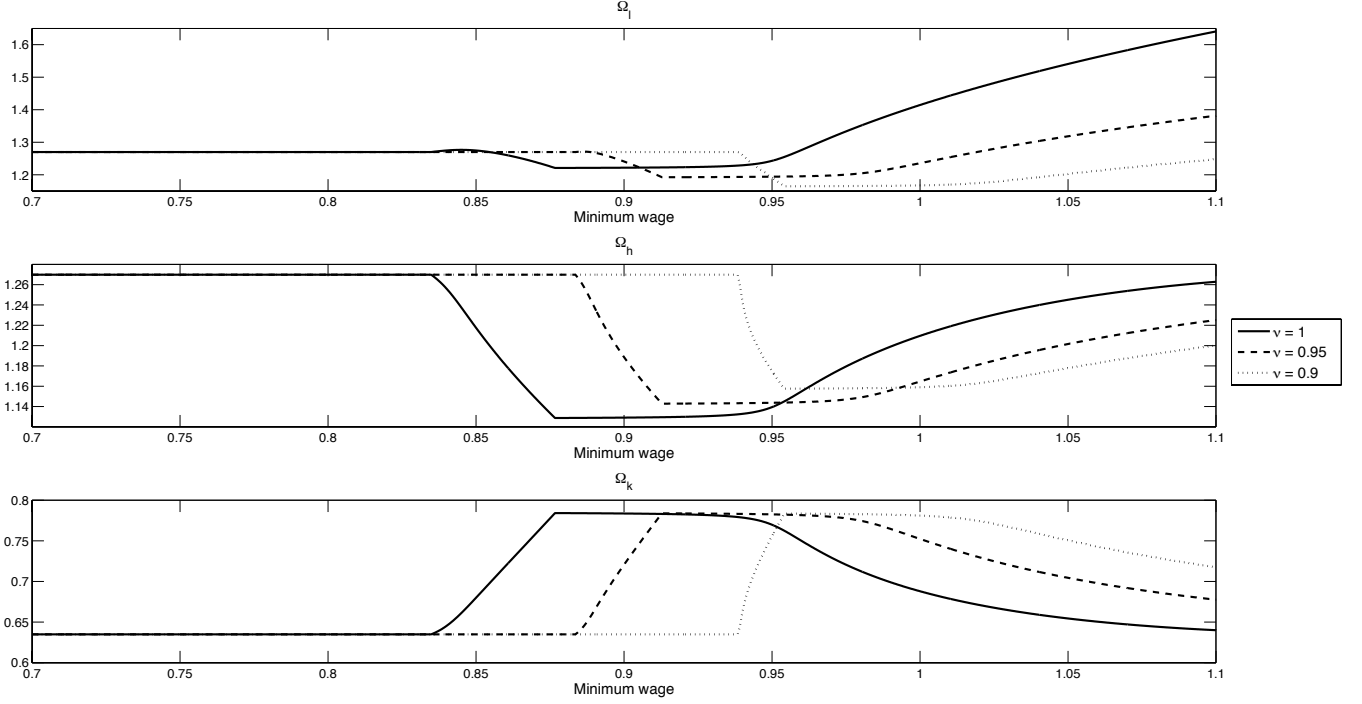


Figure 8: Effect of substitutability on Ω_h , Ω_l and Ω_k

5.2.4 Degree of substitutability between labor types

In this section, we argue that the rent appropriation effect is larger, the higher the degree of substitutability between labor types measured by the parameter ν , while the capital demand effect is not significantly affected by this parameter. This implies that, for higher values of ν , the positive effect of the minimum wage over n_l is larger. This finding allows us to reconcile the empirical evidence on the effect of the minimum wage over employment with the estimates of the elasticity of substitution between labor types; consequently, we provide a possible explanation to the minimum wage paradox.

In the benchmark parameterization considered in Section 5.1.1, we set $\nu = 1$. This is the highest possible value such that the production function is concave in n_l and n_h . Figure 8 shows how Ω_l , Ω_h and Ω_k change when considering $\nu = 0.95$ and $\nu = 0.9$.²⁸

The top panel of Figure 8 shows Ω_l for different values of ν . Notice that for lower values of ν , which imply lower substitutability between labor types, the initial decrease in Ω_l is more pronounced and the subsequent increase is less pronounced than in the benchmark case of $\nu = 1$. This implies that the rent appropriation effect is stronger

²⁸We do not show results for the case in which labor types are complements, i.e. $\nu < 1 - \alpha$, because for those parameterizations we obtain multiple equilibria, as described in Section 3.

for higher degrees of substitutability. The intuition behind this result is simple: the more substitutable labor types are, the higher the incentives for the firm are to over hire l -type workers in order to lower the negotiated wage of h -type workers. Similarly, the second panel shows that for lower values of ν the decrease in Ω_h becomes less pronounced, as the incentives to overhire h -type workers to decrease the negotiated wage of l -type workers are less important in this case. Finally, the lower panel of Figure 8 shows that the behavior of Ω_k is not substantially modified by ν , as the capital demand effect does not rest on the degree of substitutability between labor types.

6 Conclusions

Common wisdom among economists suggests that a minimum wage should have a negative effect over employment. The rationale for this is simple: a minimum wage acts as a price floor that reduces the demand for labor by firms because of the increase in its cost. However, a large body of literature that has tried to assess this effect in the data has found either that the introduction (or increase) of a minimum wage has an insignificant effect over employment, or even that this effect is positive.

This empirical finding of a small response of employment to the introduction of a minimum wage seems to suggest that substitutability between labor types is small. This is at odds with the high degree of substitutability usually found in the empirical literature. This gives rise to what has been dubbed as the minimum wage paradox.

We reconcile these two sets of empirical findings by studying the effects of a minimum wage on labor and capital demand in a standard large firm model with search frictions in the labor market. We are able to generate a slightly positive effect on employment through an increase in the demand for labor of firms in a context in which the firm hires two types of labor that are substitutes in production. The impact of the minimum wage over the demand for labor can be explained by two effects: first, the minimum wage fosters the demand for capital by alleviating a holdup problem that leads to underinvestment. We call this a *capital demand* effect. Second, since labor types are substitutes, the firm strategically overemploys one type to exert a downward pressure over wages of the other type. If the minimum wage is binding for one type of worker, the firm does not share the benefits of over employment with that type, and the incentives to overemploy exacerbate. We call this a *rent appropriation* effect.

We perform a numerical exercise and find that the introduction of a minimum wage causes the demand for capital and the two types of labor to increase. Both the capital demand effect and the rent appropriation effect work in this direction. Moreover, the

rent appropriation effect is larger when the degree of substitutability between labor types is higher.

Given the correspondence between the empirical findings mentioned before and the results of our model, we believe this type of model proves useful to study issues of redistribution between types of workers and efficiency in the allocation of production factors. We leave this exercise for future research.

A Appendix: proofs

A.1 Proof of Proposition 1

Define $\check{f}(\check{n}_h, \check{n}_l; \bar{n}_h, \bar{n}_l, k) \equiv f(\check{n}_h + \bar{n}_h, \check{n}_l + \bar{n}_l, k)$. Given that $f_{\check{n}_h} = \check{f}_{\check{n}_h}$, the system of differential equations to be solved can be rewritten as

$$\begin{cases} \check{w}^h = \beta \check{f}_{\check{n}_h} + (1 - \beta)rU_h - \beta \sum_{j \in \{h, l\}} \check{w}_{\check{n}_h}^j \check{n}_j \\ \check{w}^l = \beta \check{f}_{\check{n}_l} + (1 - \beta)rU_h - \beta \sum_{j \in \{h, l\}} \check{w}_{\check{n}_l}^j \check{n}_j. \end{cases}$$

The solution to this system is given in [Cahuc et al. \(2008\)](#), who use spherical coordinates to solve for it. This leads to the wage expression

$$\check{w}^i = \beta \check{\Omega}_i \check{f}_{\check{n}_i} + (1 - \beta)rU_i, \quad (31)$$

where

$$\check{\Omega}_i = \frac{\int_0^1 \frac{1}{\beta} z^{\frac{1-\beta}{\beta}} \check{f}_{z\check{n}_i}(z\check{n}_h, z\check{n}_l; \bar{n}_h, \bar{n}_l, k) dz}{\check{f}_{\check{n}_i}(\check{n}_h, \check{n}_l; \bar{n}_h, \bar{n}_l, k)}. \quad (32)$$

Given the definition of \check{f} we introduced above, the front-load factor in equation (32) can be rewritten as

$$\check{\Omega}_i = \frac{\int_0^1 \frac{1}{\beta} z^{\frac{1-\beta}{\beta}} f_{z\check{n}_i}(z\check{n}_h + \bar{n}_h, z\check{n}_l + \bar{n}_l, k) dz}{f_{n_i}(n_h, n_l, k)},$$

implying the more compact formulation in equation (12).

A.2 Proof of Proposition 2

A.2.1 Capital front-load factor

To obtain equation (16), first notice that, by using equation (1), the fact that $\bar{w}_k^i = 0$ and the definition of \check{f} in Appendix A.2, equation (14) can be rewritten as

$$r + \delta = \check{f}_k - \sum_{j \in \{h, l\}} \check{w}_k^j \check{n}_j \quad (33)$$

Then, by deriving (31) with respect to k , we have that

$$\check{w}_k^j = \int_0^1 z^{\frac{1-\beta}{\beta}} \check{f}_{z\check{n}_j, k}(z\check{n}_h, z\check{n}_l; \bar{n}_h, \bar{n}_l, k) dz, \quad \forall j \in \{h, l\}.$$

We integrate by parts $\sum_{j \in \{h, l\}} \check{w}_k^j \check{n}_j$ as

$$\sum_{j \in \{h, l\}} \check{w}_k^j \check{n}_j = \check{f}_k - \frac{1-\beta}{\beta} \int_0^1 z^{\frac{1-2\beta}{\beta}} \check{f}_k(z \check{n}_h, z \check{n}_l; \bar{n}_h, \bar{n}_l, k) dz.$$

Plugging this expression in condition (33) yields

$$r + \delta = \int_0^1 \frac{1-\beta}{\beta} z^{\frac{1-2\beta}{\beta}} \check{f}_k(z \check{n}_h, z \check{n}_l; \bar{n}_h, \bar{n}_l, k) dz.$$

Finally, by using the definition of \check{f} in Appendix A.1, the equation above can be rewritten as equation (16) with the respective expression for Ω_k .

A.2.2 Labor front-load factors

To obtain equation (15), notice first that condition (13) can be rewritten as

$$\frac{c}{q(\theta_i)} = \frac{f_{n_i} - w^i - \sum_{j \in \{h, l\}} \check{w}_{n_i}^j \check{n}_j}{r + s}, \quad \forall i \in \{h, l\}, \quad (34)$$

given equation (1) and the fact that $\bar{w}_{n_i}^i = 0$.

Then, derive (11) with respect to n_i :

$$\check{w}_{n_i}^j = \int_0^1 \zeta_i^z z^{\frac{1-\beta}{\beta}} f_{n_j \zeta_j^z, n_i \zeta_i^z}(n_h \zeta_h^z, n_l \zeta_l^z, k) dz$$

and calculate $\sum_{j \in \{h, l\}} \check{w}_{n_i}^j \check{n}_j$:

$$\sum_{j \in \{h, l\}} \check{w}_{n_i}^j \check{n}_j = \sum_{j \in \{h, l\}} \int_0^1 (\chi_i z + 1 - \chi_i) z^{\frac{1}{\beta}-1} f_{n_j \zeta_j^z, n_i \zeta_i^z}(n_h \zeta_h^z, n_l \zeta_l^z, k) \chi_j n_j dz$$

$$\begin{aligned} \sum_{j \in \{h, l\}} \check{w}_{n_i}^j \check{n}_j &= \\ &\chi_i \int_0^1 z^{\frac{1}{\beta}} \sum_{j \in \{h, l\}} f_{n_j \zeta_j^z, n_i \zeta_i^z}(n_h \zeta_h^z, n_l \zeta_l^z, k) \chi_j n_j dz \\ &+ (1 - \chi_i) \int_0^1 z^{\frac{1}{\beta}-1} \sum_{j \in \{h, l\}} f_{n_j \zeta_j^z, n_i \zeta_i^z}(n_h \zeta_h^z, n_l \zeta_l^z, k) \chi_j n_j dz \end{aligned}$$

By integrating by parts the two integrals in the equation above, we obtain

$$\begin{aligned}
\sum_{j \in \{h, l\}} \check{w}_{n_i}^j \check{n}_j = & \\
& \chi_i f_{n_i} - \chi_i \int_0^1 \frac{1}{\beta} z^{\frac{1}{\beta}-1} f_{n_i \zeta_i^z} (n_h \zeta_h^z, n_l \zeta_l^z, k) dz \\
& + (1 - \chi_i) f_{n_i} - (1 - \chi_i) \int_0^1 \frac{1 - \beta}{\beta} z^{\frac{1}{\beta}-2} f_{n_i \zeta_i^z} (n_h \zeta_h^z, n_l \zeta_l^z, k) dz
\end{aligned}$$

By plugging this expression in condition (34), we obtain (15) and the respective expressions for Ω_i and $\bar{\Omega}_i$. This completes the proof.

A.3 Proof of Proposition 3

A.3.1 Constrained-efficient allocations

We show first how to obtain equations (23) and (24). Define $p(\theta) = \theta q(\theta)$. The first-order conditions of the program in (22) are

$$c = \mathcal{V}_{n_i}(n_h, n_l, k) p'(\theta_i), \quad \forall i \in \{h, l\},$$

and

$$\mathcal{V}_k(n_h, n_l, k) = 1.$$

By applying the envelope theorem, we get

$$(r + s - \theta_i^2 q'(\theta_i)) \mathcal{V}_{n_i}(n_h, n_l, k) = f_{n_i} - b, \quad \forall i \in \{h, l\},$$

and

$$(r + \delta) \mathcal{V}_k(n_h, n_l, k) = f_k.$$

Plugging these two equations in the first-order conditions yields equation (24) and

$$\frac{c}{p'(\theta_i)} = \frac{f_{n_i} - b}{r + s - \theta_i^2 q'(\theta_i)}, \quad \forall i \in \{h, l\}.$$

Notice that $p'(\theta_i) = q(\theta_i)(1 - \eta(\theta_i))$. Hence,

$$c (r + s - \theta_i^2 q'(\theta_i)) = (f_{n_i} - b) q(\theta_i) (1 - \eta(\theta_i)), \quad \forall i \in \{h, l\}.$$

By rearranging this equation, one can obtain equation (23).

A.3.2 Efficiency of the decentralized equilibrium

Equations (25)-(27) are obtained by comparing (23) and (24) with the first-order conditions in the case of the decentralized equilibrium. From equations (1), (11), (15) and (17), the vacancy creation condition can be rewritten as

$$(r + s) \frac{c}{q(\theta_i)} = \chi_i(1 - \beta) \left(\check{\Omega}_i f_{n_i} - rU_i \right) + (1 - \chi_i) \left(\bar{\Omega}_i f_{n_i} - \bar{w} \right), \forall i \in \{h, l\} \quad (35)$$

while, as a reminder, the capital investment condition is

$$r + \delta = \Omega_k f_k. \quad (36)$$

Notice that, from equations (1), (11) and (21), and imposing the equilibrium condition by which $\check{w}^i = \bar{w}$ when $0 < \chi_i < 1$, we can write

$$rU_i = \frac{(r + s)b + \beta\theta_i q(\theta_i) \check{\Omega}_i f_{n_i}}{r + s + \beta\theta_i q(\theta_i)} \quad \forall 0 < \chi_i \leq 1. \quad (37)$$

Plugging equation (37) in (35), we obtain

$$\frac{c}{q(\theta_i)} = \chi_i(1 - \beta) \frac{\check{\Omega}_i f_{n_i} - b}{r + s + \beta\theta_i q(\theta_i)} + (1 - \chi_i) \frac{\bar{\Omega}_i f_{n_i} - \bar{w}}{r + s}. \quad (38)$$

Equation (23) can be rewritten as

$$\frac{c}{q(\theta_i)} = \chi_i(1 - \eta(\theta_i)) \frac{f_{n_i} - b}{r + s + \theta_i q(\theta_i) \eta(\theta_i)} + (1 - \chi_i)(1 - \eta(\theta_i)) \frac{f_{n_i} - b}{r + s + \theta_i q(\theta_i) \eta(\theta_i)}. \quad (39)$$

Comparing (38) to (39) we obtain conditions (25) and (26). Similarly, by comparing (36) with (24), we obtain condition (27). This completes the proof.

B Appendix: numerical algorithm

We briefly describe here the numerical algorithms to solve the examples depicted in Section 5.

1. Specify the parameter values and functional forms according to Table 1 and expressions (29) and (30). Set $\chi_h = 1$.
2. Construct a grid for the minimum wage such that $\bar{w} \in [\bar{w}_{min}; \bar{w}_{max}]$.
3. For every $\bar{w} \in [\bar{w}_{min}; \bar{w}_{max}]$:
 - (a) Construct a fine grid for $\chi_l \in [0, 1]$.
 - (b) For every possible value of χ_l , obtain the equilibrium values of $\theta_h, \theta_l, n_h, n_l$ and k such that equations (1), (11), (15), (16) and (20) are satisfied. In order to compute the integrals in $\Omega_i, i = l, h, k$, use a Gauss-Legendre quadrature, suited to compute the area under a curve.²⁹ Compute \check{w}_i in each case.
 - (c) Given Definition 1:
 - i. if $\check{w}_l > \bar{w}$ for $\chi_l = 1$, then set $\chi_l = 1$
 - ii. if $\check{w}_l < \bar{w}$ for $\chi_l = 0$, then set $\chi_l = 0$
 - iii. if $\check{w}_l < \bar{w}$ for $\chi_l = 1$ and $\check{w}_l > \bar{w}$ for $\chi_l = 0$, then set χ_l such that $\check{w}_l = \bar{w}$
 - (d) Check that the equilibrium is unique, i.e., only one of the previous situations arises.

²⁹To compute the integral, we use 100 quadrature nodes.

C Appendix: numerical results

C.1 One labor-type case: χ_l

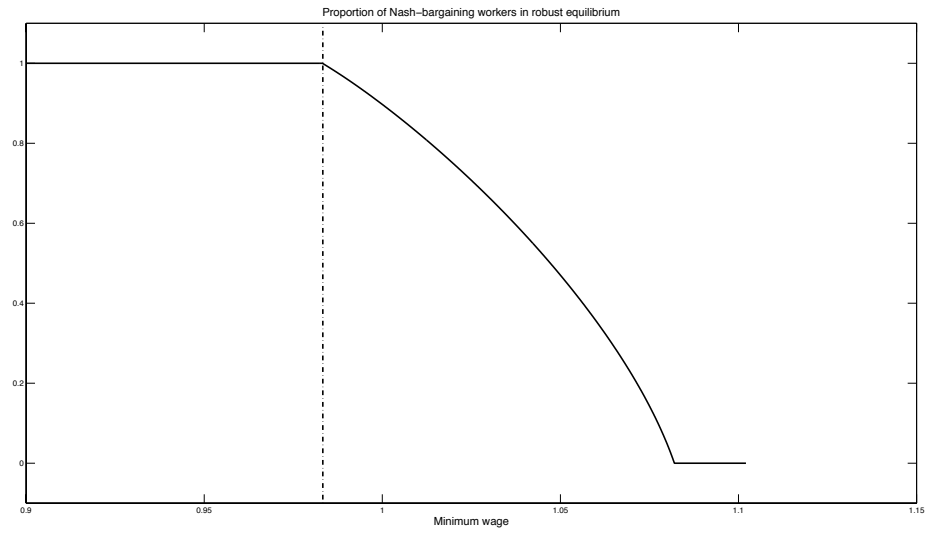


Figure 9: Evolution of χ_l

C.2 Two labor-type case: χ_l

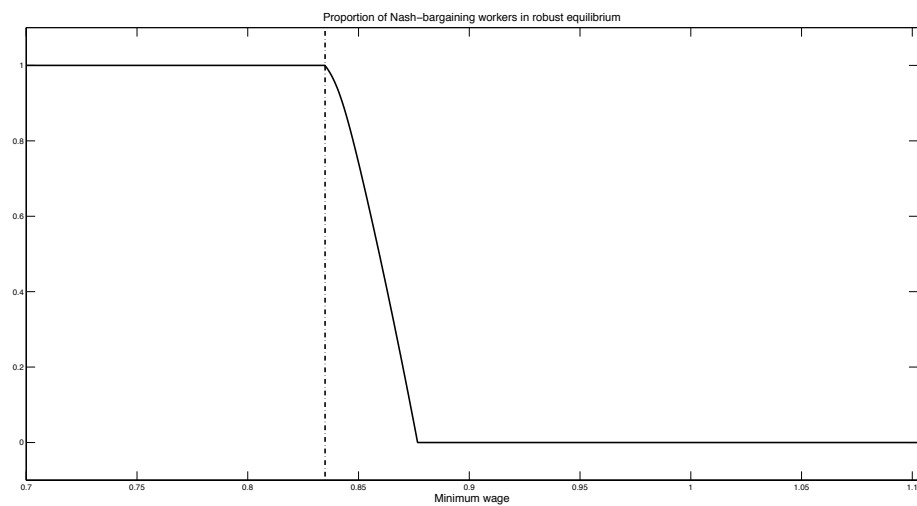


Figure 10: Evolution of χ_l

References

- Acemoglu, Daron**, “Good Jobs versus Bad Jobs,” *Journal of Labor Economics*, 2001, 19 (1), pp. 1–21.
- **and Robert Shimer**, “Holdups and Efficiency with Search Frictions,” *International Economic Review*, 1999, 40 (4), pp. 827–849.
- **and William Hawkins**, “Search with multiworker firms,” *Theoretical Economics*, 2014, 9, pp. 583–628.
- Aghion, Philippe, Yann Algan, and Pierre Cahuc**, “Civil society and the State: the interplay between cooperation and minimum wage regulation,” *Journal of the European Economic Association*, 2010, 9 (1), pp. 3–42.
- Algan, Yann and Pierre Cahuc**, “Civic Virtue and Labor Market Institutions,” *American Economic Journal: Macroeconomics*, 2009, 1 (1), pp. 111–145.
- Autor, David H., Alan Manning, and Christopher L. Smith**, “The Contribution of the Minimum Wage to U.S. Wage Inequality over Three Decades: A Re-assessment,” LSE working paper 2010.
- Bertola, Giuseppe and Pietro Garibaldi**, “Wages and the Size of Firms in Dynamic Matching Models,” *Review of Economic Dynamics*, 2001, 4, pp. 335–368.
- **and Ricardo J. Caballero**, “Cross-Sectional Efficiency and Labour Hoarding in a Matching Model of Unemployment,” *Review of Economic Studies*, 1994, 61 (3), pp. 435–456.
- Blanchard, Olivier Jean and Thomas Philippon**, “The quality of labor relations and unemployment,” Technical Report 2006.
- Boal, William M. and Michael R. Ransom**, “Monopsony in the Labor Market,” *Journal of Economic Literature*, 1997, 35 (1), pp. 86–112.
- Burdett, Kenneth and Dale T. Mortensen**, “Wage differentials, employer size, and unemployment,” *International Economic Review*, 1998, 39 (2), pp. 257–273.
- Burkhauser, Richard V., Kenneth A. Couch, and David C. Wittenburg**, “A reassessment of the new economics of the minimum wage literature with monthly data from the Current Population Survey,” *Journal of Labor Economics*, 2000, 18, pp. 653–701.
- Caballero, Ricardo J. and Mohamad L. Hammour**, “Jobless growth: appropriability, factor substitution, and unemployment,” *Carnegie-Rochester Conference Series on Public Policy*, 1998, 48, pp. 51–94.

- Cahuc, Pierre and Etienne Wasmer**, “Does intrafirm bargaining matter in the large firm’s matching model?,” *Macroeconomic Dynamics*, 2001, 5, pp. 742–747.
- **and —**, “Labor Market Efficiency, Wages and Employment when Search Frictions Interact with Intrafirm Bargaining,” IZA Discussion Paper 2001.
- **, François Marque, and Etienne Wasmer**, “A theory of wages and labor demand with intrafirm bargaining and matching frictions,” *International Economic Review*, 2008, 48 (3), pp. 943–72.
- Card, David**, “Do Minimum Wages Reduce Employment? A Case Study of California, 1987-89,” *Industrial and Labor Relations Review*, 1992, 46 (1), pp. 38–54.
- **,** “Using Regional Variation in Wages to Measure the Effects of the Federal Minimum Wage,” *Industrial and Labor Relations Review*, 1992, 46 (1), pp. 22–37.
- **and Alan B. Krueger**, “Minimum wages and employment: a case-study of the fast-food industry in New Jersey and Pennsylvania,” *American Economic Review*, 1994, 84 (4), pp. 772–793.
- Caselli, Francesco and James Feyrer**, “The marginal product of capital,” *Quarterly Journal of Economics*, 2007, CXXII (2), pp. 535–568.
- Chari, V.V, Patrick J. Kehoe, and Ellen R. McGrattan**, “Business cycle accounting,” *Econometrica*, 2007, (3).
- Cooley, Thomas F. and Edward C. Prescott**, *Frontiers of Business Cycle Research*, Princeton University Press,
- Currie, Janet and Bruce C. Fallick**, “The Minimum Wage and the Employment of Youth Evidence from the NLSY,” *Journal of Human Resources*, 1996, 31 (2), pp. 404–428.
- David, Quentin, Alexandre Janiak, and Etienne Wasmer**, “Local social capital and geographical mobility,” *Journal of Urban of Economics*, 2010, 6 (2), pp. 191–204.
- Dolado, Juan José, Francis Kramarz, Stephen Machin, Alan Manning, David Margolis, and Coen Teulings**, “Minimum wages: the European experience,” *Economic Policy*, 1996, 11 (23), pp. 318–372.
- Ebell, Monique and Christian Haefke**, “Product market deregulation and the US employment miracle,” *Review of Economic Dynamics*, 2009, 12, pp. 479–504.
- Elsby, Michael and Ryan Michaels**, “Marginal Jobs, Heterogeneous Firms, and Unemployment Flows,” *American Economic Journals: Macroeconomics*, 2013, 5 (1), pp. 1–48.

- Felbermayr, Gabriel and Julien Prat**, “Product Market Regulation, Firm Selection and Unemployment,” *Journal of the European Economic Association*, 2011, 9 (2), pp. 278–317.
- Flinn, Christopher J.**, “Minimum wage effects on labor market outcomes under search, matching, and endogenous contact rates,” *Econometrica*, 2006, 74 (4), pp. 1013–1062.
- Gomme, Paul and Peter Rupert**, “Theory, measurement and calibration of macroeconomic models,” *Journal of Monetary Economics*, 2007, 54, pp. 460–497.
- Grout, Paul**, “Investment and Wages in the Absence of Binding Contracts,” *Econometrica*, 1984, 52, pp. 449–460.
- Hall, Robert E. and Charles I. Jones**, “Why Do Some Countries Produce so Much More Output per Worker than Others?,” *Quarterly Journal of Economics*, 1999, 64, pp. 83–116.
- Hawkins, William**, “Bargaining with commitment between workers and large firms,” *Review of Economic Dynamics*, 2015.
- Hosios, Arthur J.**, “On the efficiency of matching and related models of search and unemployment,” *Review of Economic Studies*, 1990, 57, pp. 279–298.
- Janiak, Alexandre**, “Structural unemployment and the costs of firm entry and exit,” *Labour Economics*, 2013, 23, 1–19.
- Kaas, Leo and Paul Madden**, “Holdup in oligopsonistic labour markets - a new role for the minimum wage,” *Labour Economics*, 2008, 15, pp. 334–349.
- Katz, Lawrence F. and Alan B. Krueger**, “The Effect of the Minimum Wage on the Fast-Food Industry,” *Industrial and Labor Relations Review*, 1992, 46 (1), pp. 6–21.
- Krusell, Per, Lee E. Ohanian, José-Víctor Ríos-Rull, and Giovanni L. Violante**, “Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis,” *Econometrica*, 2000, 68 (5), pp. 1029–1053.
- Lee, David S.**, “Wage inequality in the United States during the 1980s: rising dispersion or falling minimum wage?,” *Quarterly Journal of Economics*, 1999, 114 (3), pp. 977–1023.
- Linneman, Peter**, “The Economic Impacts of Minimum Wage Laws: A New Look at an Old Question,” *Journal of Political Economy*, 1982, 90 (3), pp. 443–469.
- Machin, Stephen and Alan Manning**, “The effects of minimum wages on wage dispersion and employment: evidence from the U.K. wages councils,” *Industrial and Labor Relations Review*, 1994, 47 (2), pp. 319–329.

- **and** — , “Employment and the Introduction of a Minimum Wage in Britain,” *Economic Journal*, 1996, *106* (436), pp. 667–676.
- **and Joan Wilson**, “Minimum wages in a low-wage labour market: care homes in the UK,” *Economic Journal*, 2004, *114*, pp. C102–C109.
- , **Lupin Rahman, and Alan Manning**, “Where the minimum wage bites hard: introduction of minimum wages to a low wage sector,” *Journal of the European Economic Association*, 2003, *1*, pp. 154–180.
- Manning, Alan**, *Monopsony in Motion: Imperfect Competition in Labor Markets*, Princeton University Press, 2003.
- Pissarides, Christopher A.**, “Short-run equilibrium dynamics of unemployment, vacancies, and real wages,” *American Economic Review*, 1985, *75*, pp. 676–690.
- , “The unemployment volatility puzzle: is wage stickiness the answer?,” *Econometrica*, 2009, *77*, pp. 1339–1369.
- Portugal, Pedro and Ana Rute Cardoso**, “Disentangling the Minimum Wage Puzzle: An Analysis of Job Accessions and Separations from a Longitudinal Matched Employer-Employee Data Set,” *Journal of the European Economic Association*, 2006, *4* (5), pp. 988–1013.
- Smith, Eric**, “Search, concave production, and optimal firm size,” *Review of Economic Dynamics*, 1999, *2* (2), pp. 456–471.
- Stewart, Mark**, “The Employment Effects of the National Minimum Wage,” *Economic Journal*, 2004, *114* (494), pp. C110–116.
- , “The Impact of the Introduction of the UK Minimum Wage on the Employment Probabilities of Low Wage Workers,” *Journal of the European Economic Association*, 2004, *2* (1), pp. 67–97.
- Stigler, George J.**, “The Economics of Minimum Wage Legislation,” *American Economic Review*, 1946, *36* (3), pp. 358–365.
- Stole, Lars A. and Jeffrey Zwiebel**, “Intra-Firm Bargaining under Non-Binding Contracts,” *Review of Economic Studies*, 1996, *63* (3), pp. 375–410.
- **and** — , “Organizational Design and Technology Choice under Intrafirm Bargaining,” *American Economic Review*, 1996, *86* (1), pp. 195–222.
- Teulings, Coen N.**, “Aggregation bias in elasticities of substitution and the minimum wage paradox,” *International Economic Review*, 2000, *41* (2), pp. 359–398.

| Documentos de Trabajo Banco Central de Chile | Working Papers Central Bank of Chile |
|--|--|
| NÚMEROS ANTERIORES | PAST ISSUES |
| <p>La serie de Documentos de Trabajo en versión PDF puede obtenerse gratis en la dirección electrónica:</p> <p>www.bcentral.cl/esp/estpub/estudios/dtbc.</p> <p>Existe la posibilidad de solicitar una copia impresa con un costo de Ch\$500 si es dentro de Chile y US\$12 si es fuera de Chile. Las solicitudes se pueden hacer por fax: +56 2 26702231 o a través del correo electrónico: bcch@bcentral.cl.</p> | <p>Working Papers in PDF format can be downloaded free of charge from:</p> <p>www.bcentral.cl/eng/stdpub/studies/workingpaper.</p> <p>Printed versions can be ordered individually for US\$12 per copy (for order inside Chile the charge is Ch\$500.) Orders can be placed by fax: +56 2 26702231 or by email: bcch@bcentral.cl.</p> |

DTBC – 754

Identification of Earnings Dynamics using Rotating Samples over Short Periods: The Case of Chile

Carlos Madeira

DTBC – 753

The Impact of Commodity Price Shocks in a Major Producing Economy. The Case of Copper and Chile

Michael Pedersen

DTBC – 752

Nominal Term Structure and Term Premia: Evidence from Chile

Luis Ceballos, Alberto Naudon y Damián Romero

DTBC – 751

The Labor Wedge: New Facts Based on US Microdata

David Coble

DTBC – 750

El Rol de las Asimetrías en el Pass-Through: Evidencia para Chile

Lucas Bertinatto y Diego Saravia

DTBC – 749

Dissent in FOMC Meetings and the Announcement Drift

Carlos Madeira y Joao Madeira

DTBC – 748

Post-crisis Financiera y Expansión de las Exportaciones: Micro-Evidencia para Chile

Roberto Álvarez y Camila Sáez

DTBC – 747

Exchange Rate Pass-Through to Prices: VAR Evidence for Chile

Santiago Justel y Andrés Sansone

DTBC – 746

A New Liquidity Risk Measure for the Chilean Banking Sector

Sebastián Becerra, Gregory Claeys y Juan Francisco Martínez

DTBC – 745

Sensibilidad de las Exportaciones al TCR: Un Análisis Sectorial y por Destino

Samuel Carrasco, Diego Gianelli y Carolina Godoy

DTBC – 744

Agrupación de Instituciones Bancarias a Partir del Análisis de Cluster: Una Aplicación al Caso de Chile

Alejandro Jara y Daniel Oda

DTBC – 743

Bailouts and Prudential Policies-A delicate Interaction

Ernesto Pasten

DTBC – 742

Facing Volatile Capital Flows: The Role of Exchange Rate Flexibility and Foreign Assets

Rodrigo Cifuentes and Alejandro Jara

DTBC – 741

Risk Matters: The Impact of Nominal Uncertainty in Chile

Luis Ceballos y Damián Romero

DTBC – 740

Credit Stabilization through Public Banks: The Case of BancoEstado

Luis Felipe Lagos y Matías Tapia



BANCO CENTRAL
DE CHILE