DOCUMENTOS DE TRABAJO

Bailouts and Prudential Policies - A Delicate Interaction

Ernesto Pastén

N.º 743 Diciembre 2014

BANCO CENTRAL DE CHILE

DOCUMENTOS DE TRABAJO

Bailouts and Prudential Policies - A Delicate Interaction

Ernesto Pastén

N.º 743 Diciembre 2014

BANCO CENTRAL DE CHILE

BANCO CENTRAL DE CHILE

CENTRAL BANK OF CHILE

Documentos de Trabajo del Banco Central de Chile
Working Papers of the Central Bank of Chile
Agustinas 1180, Santiago, Chile
Teléfono: (56-2) 3882475; Fax: (56-2) 3882231

Documento de Trabajo
N° 743

Working Paper
N° 743

BAILOUTS AND PRUDENTIAL POLICIES—A DELICATE INTERACTION*

Ernesto Pastén
Banco Central de Chile

Abstract

Could prudential policies backfire by making the lack of commitment problem of bailouts worse? This commitment problem refers to the excessive risk taken by banks and financial institutions in expectations of bailouts if crises occur, which in turn increase financial fragility and the severity of crises. Ex-ante policies, such as prudential policies, have a variety of effects on the various components of the ex-post incentives of an authority to implementing a bailout. Thus, the interaction between prudential policies and bailouts is delicate: In different conditions, a given prudential policy may backfire or increase its effectiveness by worsening or alleviating the lack of commitment problem of bailouts. Liquidity requirements and prudential taxes are examples of prudential policies that may backfire. Public debt is an example of an ex-ante policy usually with no prudential motivation that may play such a role.

Resumen

¿Es posible que políticas prudenciales terminen siendo perjudiciales al agravar el problema de commitment de una política de salvatajes financieros? Este problema de commitment se refiere al hecho de que expectativas de bancos e instituciones financieras de recibir un salvataje en caso de una crisis financiera los estimula a tomar riesgo excesivamente aumentando la fragilidad de la economía y la profundidad de las crisis financieras. Este artículo muestra que políticas pre-crisis, como las políticas prudenciales, cambian de forma no trivial el comportamiento de los bancos e instituciones financieras, lo que a su vez cambia de forma no trivial los incentivos de la autoridad de implementar un salvataje durante una crisis. Estos efectos no triviales implican que la relación entre las políticas prudenciales y los salvatajes financieros es delicada: En diferentes condiciones, una cierta política prudencial puede tener efectos indirectos positivos o negativos al respectivamente aliviar o agravar el problema de commitment de una política de salvatajes financieros. Regulaciones que ponen piso a la liquidez que deben mantener los bancos y una política de impuestos a la toma de riesgo son ejemplos de políticas pre-crisis que pueden terminar siendo perjudiciales. El manejo de la deuda pública es un ejemplo de políticas pre-crisis que usualmente no tienen un rol prudencial que pueden cumplir dicho rol.

* This paper is an extensive revision of a paper that has previously circulated under the title "Time-Consistent Bailouts" (Central Bank of Chile Working Paper #635-2012.) I especially thank Jean Tirole and Bob King for comments and discussions. I also thank feedback from Alvaro Aguirre, Elias Albagli, Bruno Biais, Itay Goldstein, Francisco Gonzalez, Christian Hellwig, Anton Korinek, Bart Lipman, Per Krusell, Enrique Mendoza, Guillaume Plantin, Jean-Charles Rochet and seminar participants at the Central Bank of Chile, IIES, LUISS, Maryland, PUC-Chile, Toulouse, UChile-CEA, USACH, Yale, and the Institute of Banking and Finance at the University of Zurich. The views expressed herein are those of the author and do not necessarily represent the position of the Central Bank of Chile. Errors and omissions are my own. Email: epasten@bcentral.cl.

1 Introduction

After the 2007-2009 financial crisis there has been a surge of "prudential" regulatory and policy proposals. A common premise behind these proposals is that financial institutions (in short, banks) misbehave by taking too much risk which leads to financial crises; prudential policies are called to correct such a misbehavior. Two sources are usually blamed for banks' misbehavior: an externality arising when banks do not internalize the systemic effects of their risk taking decisions, and expectations of bailouts due to a lack of commitment problem.[1] Policy proposals focusing on correcting the externality usually abstract from bailouts.[2] Proposals focusing on bailouts usually postulate prudential policies that solve the lack of commitment problem.[3] However, to solve this problem, strong assumptions must be made regarding the information available for the authority and the enforceability of prudential regulations and policies in financial markets.

Thus, from a positive perspective, the following questions arise: When the lack of commitment problem of bailouts is inevitable, how do bailouts and prudential policies interact? Is it possible that prudential policies may backfire by fueling expectations of even larger bailouts?

This paper addresses these questions in an infinitely repeated version of the model of Farhi and Tirole (2009, 2012) where banks jointly decide risk taking and liquidity holding while the authority must balance the trade-offs and reputation concerns involved in a bailout.[4] The authority's incentives of bailing out are endogenous, so there is indeed an interaction between bailouts and prudential policies. Further, this interaction is *delicate*: Under different conditions, a given prudential policy may backfire or increase its effectiveness by worsening or alleviating the lack of commitment problem of bailouts. I illustrate this point by separatedly studying liquidity requirements and prudential taxes.[5] Besides, the endogeneity of bailouts also shows up when ex-ante policies with no prudential motivation may play such a role by their own interplay with the commitment problem of bailouts. I illustrate this point by introducing public debt into the model. This paper also makes a methodological contribution by studying the lack of commitment problem of policy in a repeated game where such a problem yields fragility instead of the standard time-inconsistency.

In the model the economy is populated by an infinite sequence of non-overlapping generations of a

[1] For instance, Brunnermeier (2009), Diamond and Rajan (2009) and Kelly, Lustig and Van Nieuwerburgh (2012).

[2] Examples are Lorenzoni (2008), Bianchi (2011), Bianchi and Mendoza (2010, 2013) and Jeanne and Korinek (2010).

[3] Examples are Farhi and Tirole (2009, 2012), Kocherlakota (2010), Chari and Kehoe (2013) and Keister (2014).

[4] Section 2 discusses the importance of reputational concerns in the analysis.

[5] Liquidity requirements is an example of the many regulations that try to impose bounds on banks' choices. Prudential taxes is a popular proposal that seeks either to force banks to pay for their own bailouts or to discourage banks' risk taking.

continuum of entrepreneurs (interpreted as banks) and households who live one period. There is also a benevolent authority who lives infinite periods. Each period is exactly the same as any other and it is broken into three stages. In the initial stage entrepreneurs borrow from households subject to a credit constraint to make risky and riskless investment that mature in the last stage. Riskless investment is interpretable as liquidity since it is useful to raise further funds from households if there is "distress" in the interim stage. Distress is a metaphor for a liquidity shock in which risky investment needs refinancing to continue; otherwise it is lost. The authority may help refinancing risky investment by implementing a systemic "bailout" –a reduction in the cost of funding of all entrepreneurs at the time of distress. A bailout harms households since it distorts their consumption schemes and involves implicit transfers to entrepreneurs.

An authority with commitment chooses no bailouts. This is because expectations of no bailouts gives entrepreneurs incentives to fully hedge against distress risk which yields the constraint-efficient allocation.

In contrast, if the authority has no commitment, there are multiple equilibrium bailout policies. To see this, assume that the authority has no reputation concerns. A no-bailouts policy is indeed an equilibrium policy. This is because, given expectations of no bailouts, entrepreneurs hold enough liquidity so no risky investment is lost if there is distress, thus there is no need of a bailout. But some positive bailouts are also equilibrium policies. If entrepreneurs choose less liquidity in expectations of a bailout of a given size, the authority implements such a bailout if its cost for households is smaller than its benefit for entrepreneurs. The marginal cost of a bailout for households is increasing in its size since their utility is concave while the marginal benefit of bailouts on avoiding the lost of risky investment is more or less constant. Therefore any bailout from size zero to a given cap are equilibrium bailout policies.

This equilibrium multiplicity leads to a *fragility* problem: Good allocations are possible in which banks well-behave in expectations of no bailouts, but bad allocations are also possible in which banks do misbehave in expectations of a bailout. This fragility problem is conceptually different than the *time-inconsistency* problem usually arising when policy lacks commitment. For instance, for capital taxation or inflationary policy, if agents behave consistently with the commitment policy by investing a lot or adjusting prices by little respectively in expectations of low taxes or low inflation, the authority deviates by setting high taxes or high inflation. For bailouts it is only when banks deviate from the behavior consistent with the commitment policy that the authority deviates too to bail them out. Entrepreneurs prefer larger bailouts, but they are assumed too small (in the limit, atomistic) to have strategic power over the authority or other entrepreneurs.[6]

The paper introduces reputation concerns by applying the notion of Sustainable Plans (Chari and Kehoe, 1990). This is the standard approach to study lack of commitment of policy in a repeated game when private agents are atomistic. A given policy is a "sustainable plan" if the authority does not deviate from it given that private agents act consistently with the policy plan and impose a "penalty" to a deviation of the authority from the plan –in short, a reputation cost. This penalty is the difference in the discounted sum of future generations' welfare delivered by the sustainable plan and the worst equilibrium policy –in the bailouts context, the largest bailout in the static game. I reach two main results. First, the fragility problem still applies. This is because the well-known result that every equilibrium in a static game is also an equilibrium in the repetition of the game. Second, the standard approach of focusing on the best sustainable plan in terms of welfare is uninformative as a criterion to evaluate the severity of the lack of commitment problem when such a problem leads to fragility instead of time-inconsistency. This is because the best sustainable bailout plan is simply no bailouts. Importantly, the best sustainable bailout plan is invariant to changes in the authority's discounting and, for the seek of this paper, to changes in regulations and prudential policies.

I subsequently propose a refinement to the set of sustainable bailout plans that is useful for the policy analysis. I call the bailout plans that pass this refinement "resistant bailouts." Specifically, I ask: What sustainable bailout plans does not the authority deviate from even if all entrepreneurs collectively take so much risk that they need a higher-than-planned bailout if there is distress? In other words, what bailout policies are resistant to the fragility problem? In the paper I discuss the connection between this refinement and the literature on refinements involving collective deviations. The main finding is that the size of the smallest resistant bailout (the best in terms of welfare) is decreasing in the authority's discount factor: If the authority fully disregards future generations, only the largest equilibrium bailout is resistant; if it weighs high future generations, even the commitment no-bailouts policy is resistant.

I use this refinement to study the interplay of bailouts with regulations and prudential policies when the authority has moderate concern about future generations, so the best resistant bailout is in the middle ground between the largest equilibrium bailout and the commitment bailout policy. I start by introducing liquidity requirements. One case in which this regulation backfires is when its effectiveness is so limited that it is binding under the worst equilibrium but it is not binding in the equilibrium path of the best resistant bailout.

[6]This assumption is discussed in Section 2.

This is because liquidity requirements increases welfare only in the worst equilibrium, so the penalty to deviations from bailout plans becomes smaller. Thus, the best resistant bailout increases and entrepreneurs increase risk taking. This result is informative because liquidity requirement is one of the many regulations that impose bounds on banks's choices. Historically banks have shown to be quite skilled to round regulations. This result highlights that rounded regulations may have a pervasive effect by exacerbating the fragility problem due to lack of commitment of bailout policy.

I study next two variations of prudential taxes. In both variations entrepreneurs are taxed and rebated; only the timing of the rebates differs. In the first variation the rebate is in the interim stage, so it provides funds to entrepreneurs if there is distress. This variation is motivated by the proposal of using taxes to force banks to pay ex-ante for the bailouts they get ex-post. I find that this prudential policy may exacerbate the lack of commitment problem of bailouts. This is because if the needs of funding of entrepreneurs during distress can be met with a smaller burden on households, then the size of the best resistant bailout increases. However, this result is ambiguous by the action of two offsetting forces at play. First, the tax reduces banks' risky investment given expectations of a bailout. Second, the penalty to a policy deviation increases when the tax makes the worst static equilibrium even worse. This ambiguity in the result highlights the delicacy of the interplay of prudential policies and bailouts.

In the second variation of prudential taxes the rebate is given to entrepreneurs in the last stage of each period. This variation responds to the motivation of using taxes to discourage risk taking. This policy turns out to be innocuous. This is because the rebate to entrepreneurs is a perfect substitute to riskless assets, so entrepreneurs anticipate the rebate by choosing a portfolio more tilt to risky investment. But at the same time the tax increases the actual cost of risky investment for entrepreneurs without changing the incentives of the authority to implementing a bailout. The overall effect is that both the scale of risky investment in the economy and the severity of the lack of commitment problem of bailouts are invariant to the tax; however the level of riskless investment chosen by entrepreneurs is smaller.

Then I turn to study public debt as an example of a policy usually without a prudential motivation that may play such a role. A key feature of public debt is that it is predetermined when a bailout must be implemented, so it works as a substitute of commitment. One way in which public debt mitigates the fragility problem is to increasing the cost of bailouts by increasing households' marginal utility in distressed times –for instance, if taxes are raised from households to serve the debt.[7] However, similarly to above, this result is delicate since it depends on the multiple effects of public debt on the static and inter-temporal trade-offs involved in the authority's problem of bailing out. Depending on the sign of its effect, a high or a low stock of public debt may be desirable from a prudential perspective.

The rest of the paper is organized as follows. Section 2 discusses the contribution of this paper relative to different strands of literature. Section 3 presents the static game to motivate the fragility problem. Section 4 repeats this game to introduce Sustainable Plans and the refinement proposed in this paper. Section 5 studies the interplay of bailouts with liquidity requirements, prudential taxes, and public debt. Section 6 concludes. An appendix displays some proofs omitted from the main text.

2 Discussion and contribution relative to the literature

The most natural connection of this paper is with Farhi and Tirole (2009, 2012).[8] In these papers the lack of commitment of bailouts gives banks incentives to correlate their risk exposures, so any source of risk is aggregate risk. This point is also in Acharya and Yorulmazer (2007). This is a type of banks' misbehavior I take as granted, so I use a model with only aggregate risk. In this model I study the interplay of bailouts with regulations and prudential policies. I thus see the analysis in this paper as complementary to theirs.

In the best of my knowledge, all papers studying the lack of commitment problem of bailouts propose prudential policies solving this problem, e.g., Farhi and Tirole (2009, 2012) with liquidity requirements, Chari and Kehoe (2013) with loan-to-value limits, and Keister (2014) with taxes on banks' liabilities. In contrast, this paper focuses on a positive instead of normative analysis to point out that the interaction of the lack of commitment problem of bailouts and prudential policies is delicate. This paper also speaks to the large bulk of literature proposing prudential regulations and policies without considering this interaction.

A couple of specific results deserve special mention. First, prudential taxes have emerged as a popular proposal to control banks' risk taking.[9] In my results prudential taxes that respond to this motivation are

[7] An alternative mechanism not expored in the paper is to decreasing the benefit of a bailout, for instance, by crowding risky investment out so a smaller amount of resources in the economy are exposed to distress risk.

[8] Farhi and Tirole (2009) is the working paper version of Farhi and Tirole (2012). The main difference between both versions is the modelling approach for liquidity. In the former there are long-run riskless assets that may be used to raise funds from financial markets once funds are needed. In the latter there are one-period riskless assets that can be directly used once funds are needed. In what matters for this paper both approaches are equivalent.

[9] See Lorenzoni (2008), Bianchi and Mendoza (2010), Kocherlakota (2010) and Jeanne and Korinek (2011).

innocuous. This is because in the model the rebate of prudential taxes is a perfect substitute of liquidity. In contrast, in many macro-finance models there is no role for liquidity. Second, in this paper a prudential motive arises for the management of public debt. This result complements those of Woodford (1990) who sees a prudential role for public debt by creating supply of liquid assets.

From a different angle, this paper highlights the importance of reputation concerns in the bailout decision for policy analysis. First, in the paper the authority's static and inter-temporal concerns may be affected in opposite directions by prudential policies. For instance, in the example of liquidity requirements, the worst static equilibrium improves yet the fragility problem becomes worse. Second, the welfare cost of bailouts is convex in the bailout size, so exactly when the authority's discount factor is small (i.e., little reputation concerns), variations in the severity of the fragility problem due to prudential policies are more important. Third, all results in this paper apply when the authority's reputation concerns are small. Regarding literature, most papers studying bailouts without commitment focus on static environments. Examples are Acharya and Yorulmazer (2007), Farhi and Tirole (2009, 2012), and Nosal and Ordonez (2014).[10] The only exception is Chari and Kehoe (2013). They study lack of commitment of bailouts in a repeated game where bankruptcy is efficient ex-ante to encourage firms' managers to exert high effort, but the authority has incentives to prevent bankruptcies ex-post. Their focus is quite different than mine, as mentioned above, besides in their model there is no role for liquidity which is quite important in my results.

This paper builds on an economy where lack of commitment of bailouts creates equilibrium multiplicity – a fragility problem. This source of multiplicity is mentioned in Kydland and Prescott (1977)'s seminal paper on time-inconsistency[11] and it is prevalent in many financial applications.[12] Although the fragility and time-inconsistency problems share a common source, they are different conceptually. In fact, in the bailouts context, the commitment policy is time-consistent and yet there is a fragility problem. Besides, this paper shows that the standard approach of looking at the best sustainable plan (Chari and Kehoe, 1990) in a repeated game is uninformative for policy analysis. Instead, this paper proposes a refinement that recovers the usefulness of studying repeated games for policy analysis. In this refinement the role of the authority's concern about the future is not to allow for better equilibrium allocations, as in Sustainable Plans, but to rule

[10] They shows that the fragility problem is mitigated when the authority is uncertain about shocks being aggregate or idiosyncratic.

[11] They use an example on flood control: If agents expect no dams (no bailouts) to be built (implemented) on a flood plain (distressed economy), no houses are constructed there (banks keep enough liquidity), so dams (bailouts) are not necessary. But if houses are built on the flood plain (banks do not keep enough liquidity), the authority will be forced to build the dams (to bail out).

[12] Examples are Schneider and Tornell (2004), Diamond and Rajan (2009), and Ennis and Keister (2009).

the worst equilibrium allocations out by limiting the fragility problem.

This fragility problem crucially depends on the assumption of an atomistic private sector. In fact, it can be shown that, if the authority fully disregards future generations, all equilibria but the worst are ruled out by a subgame perfection argument when there is a single entrepreneur in each generation instead of many. There are a few comments to make in this regard. First, the results about Sustainable Plans (Chari and Kehoe, 1990) and the "resistance" refinement are general contributions to the literature of repeated games with focus on policy analysis. Second, I argue that atomistic entrepreneurs is a sensible assumption given the large number of banks operating in financial markets. Although a few banks are large, it is unclear that they can be modelled as a single bank. Third, the point of this paper regarding bailouts and prudential policies also applies under the standard time-inconsistency problem. I choose an environment where the fragility problem is prevalent because liquidity and leverage play meaningful roles in the analysis. Besides, if the time-inconsistency problem applies, an endogenous bailout size requires that the authority is willing to implement larger-than-needed bailouts in certain situations. I consider this requirement implausible.

In game theoretical terms, the "resistance" refinement follows a long tradition of refinements in cooperative games, but it has its differences. Two examples of this kind in static games are "resilient equilibrium" (Aumann, 1959) and "strong-perfect equilibrium" (Rubinstein, 1980). These refinements are so strong in terms of what deviations are admissible and who could form a coalition that usually rule out all equilibria. In contrast, the "resistance" refinement is designed for a repeated game and it is weaker than those above, which allows for meaningful results. Refinements of this kind in repeated games focus on ruling some equilibria out for requiring penalties that are not self-enforcing. Two examples are "coalition-proof equilibrium" (Berheim, Peleg and Whinston, 1987) and "dynamically consistent equilibrium" (Berheim and Ray, 1989). The resistance refinement also relies on self-enforcing penalties, but its focuses on checking for equilibria "resistant" to the fragility problem within the set of equilibrium bailout policies in the static game.

Finally, Bianchi (2013) and Keister (2014) argue in different frameworks that the insurance role of bailouts justifies positive bailouts even under commitment. In this paper the desirability of a no-bailouts policy under commitment is not fundamental. It is only important that the commitment bailout policy is contained in the set of equilibrium bailout policies without commitment.

3 A static model

This section displays a game between households, entrepreneurs and an authority where lack of commitment of bailouts creates a fragility problem. This game is based on Farhi and Tirole (2009) and it is static in the sense that the authority has a single opportunity to implementing a bailout.[13] The next section infinitely repeats this game to allow for reputation concerns in the authority's bailout decision.

3.1 Setup

Consider an economy with three stages, $s = 0, 1, 2$.[14] There are two types of atomistic agents each with total mass one, *households* and (*banking) entrepreneurs*, and an *authority* to be introduced below.

Households have exogenous endowments e_0 and e_1 in stages $s = 0, 1$ and utility

$$V = c_0^h + \log(c_1^h) + c_2^h, \tag{1}$$

where c_s^h denotes households' consumption in stages $s = 0, 1, 2$.

Entrepreneurs have exogenous endowment A only in the initial stage $s = 0$ and utility

$$U = c_0^{ent} + c_1^{ent} + c_2^{ent}. \tag{2}$$

where c_s^{ent} denotes entrepreneurs' consumption in stages $s = 0, 1, 2$.

Households can make riskless investment while entrepreneurs can make riskless and risky investments. Riskless investment simply transfers consumption from $s = 0$ or $s = 1$ to $s = 2$. Risky investment is made in $s = 0$ and pays in $s = 2$ with gross return $\rho_1 > 1$ if there is "no distress" in $s = 1$. Distress has probability $1 - \alpha$ and implies that risky investment needs refinancing in $s = 1$; otherwise it is lost. The part of risky investment that survives distress pays a gross return ρ_1 in $s = 2$. Entrepreneurs are financially constrained since they can only pledge up to a fraction ρ_0 of the surviving scale of their risky investment.[15]

[13] In the modelling I follow Farhi and Tirole (2009) instead Farhi and Tirole (2012) since both models are equivalent for this paper but the former explicitly solves for the objective function of the authority –which plays a central role in my analysis.

[14] Notation t is reserved for each repetition of this game in Section 4.

[15] Limited pledgeability is assumed exogenous, but it may be justified by an optimal contract between households and entrepreneurs that induce the latter to exert high effort (Holmstrom and Tirole, 1998).

The *authority* (for instance, a central bank) can change the return of riskless investment in $s = 1$ by levying a contingent tax on riskless investment which is rebated via lump-sum transfers in $s = 2$. The after-tax return of riskless assets in $s = 2$ is thus $R \leq 1$. A policy $R < 1$ is interpreted as a "bailout" of size $1 - R$. The authority's objective is

$$V + \beta U \tag{3}$$

where β represents the relative weight of entrepreneurs' welfare on the authority's objective.

Timing. *Initial stage,* $s = 0$: Households receive e_0 and decide c_0^h, their riskless investment and loans to entrepreneurs. Entrepreneurs receive A and decide c_0^{ent}, their risky and riskless investments, i and xi.

Interim stage, $s = 1$: Households receive e_1 and the state is revealed: "no distress" (with probability α) or "distress" (with probability $1 - \alpha$). In the distress state households decide their riskless investment and their new loans to entrepreneurs; entrepreneurs receive no endowment and decide the surviving scale j of their risky investment. In the no distress state there is no need of reinvestment, so households only invest in riskless assets. In either state, taxes on riskless assets are collected, and private agents choose c_1^h and c_1^{ent}.

Last stage, $s = 2$: Investment pays, taxes are rebated, and private agents respectively consume c_2^h and c_2^{ent}.

3.2 Competitive allocations given (R, R^e)

I start the analysis by solving for the allocation resulting from the competitive interaction between households and entrepreneurs taking actual and expected policy R and R^e as exogenous variables. This is an artifact that allows to represent allocations in a way that facilitate the subsequent policy analysis and at the same time respect sequential rationality of private agents in the game.

The total loan that entrepreneurs receive in stage $s = 0$ is $i + xi - A$ if $1 + (1 - \alpha) < \rho_1$. This assumption ensures that entrepreneurs set $c_0^{ent} = 0$. Households' endowment e_0 is assumed large enough, so there is no shortage of supply of funds and risky investment i is pinned down by households' break-even condition in $s = 0$:

$$i + xi - A = \alpha (\rho_0 + x) i.$$

In words, households are willing to lend to entrepreneurs up to the expected return of the loan, $\alpha (\rho_0 + x) i$. The lending contract prescribes that entrepreneurs only repay to households if there is no distress (which has probability α). If there is no distress in $s = 1$ households receive the pledgeable part ρ_0 of entrepreneurs' risky investment i and all proceedings of entrepreneurs' riskless investment xi. This expression solves

$$i(x) = \frac{A}{1 + (1 - \alpha)x - \alpha\rho_0}. \tag{4}$$

If there is distress, entrepreneurs must raise new funds in $s = 1$ to preserve a scale j of their risky investment. Only households receive new endowment in $s = 1$. The break-even condition for households in $s = 1$ now is

$$Rj = \rho_0 j + xi$$

after also assuming that households' endowment e_1 in $s = 1$ is high enough such that there is no shortage of supply of funding. Once the "distress" state is realized there is no more uncertainty, so households' income for these new loans is the pledgeable part ρ_0 of entrepreneurs' surviving scale j of their risky investment and all proceedings of entrepreneurs' riskless investment xi. Opportunity cost for these loans is R, which is the policy instrument. This break-even condition solves:

$$j(x, R, i) = \min\left\{\frac{x}{R - \rho_0}, 1\right\} i \tag{5}$$

where the upper bound on j is justified by the implicit assumption that $j \leq i$.

Finally I solve for entrepreneurs' choice in $s = 0$ for their riskless to risky investment ratio x. Risky investment $i(x)$ in (4) is decreasing in x while its surviving scale $j(x, R, i)$ after distress in (5) is increasing in x. Since entrepreneurs' utility is linear, it can be shown that if $\alpha < 1$, entrepreneurs choose:

$$x(R^e) = R^e - \rho_0. \tag{6}$$

which implies that risky investment i and its surviving scale after distress j solve:

$$i(R^e) = \frac{A}{1 + (1 - \alpha)R^e - \rho_0}, \tag{7}$$

$$j(R, R^e) = \min\left\{\frac{R^e - \rho_0}{R - \rho_0}, 1\right\} i(R^e). \tag{8}$$

Equations (6), (7) and (8) characterize allocations only as functions of actual and expected bailout policy R and R^e. These expressions are obtained after imposing competitive and sequentially rational behavior of households and entrepreneurs. A critical result in this section is that part of entrepreneurs' risky investment is lost in the distress state if $R^e < R$ or, equivalently, given a bailout $1-R$, entrepreneurs' riskless investment is not enough to raise the funds to avoid the loss of some risky investment.

3.3 Bailouts with and without commitment

I now turn to solve for equilibrium bailout policy with and without commitment. With commitment, households and entrepreneurs observe the policy R before taking their decisions in $s = 0$ or, equivalently, $R^e = R$ is internalized by the authority. Without commitment, the authority chooses R in $s = 1$ taking households and entrepreneurs' decisions in $s = 0$ as given or, equivalently, taking R^e as exogenous and requiring $R^e = R$ only as an equilibrium condition.

Since the focus of this paper is bailouts when the authority lacks commitment and this section follows closely the analysis in Farhi and Tirole (2009), I state the equilibrium bailout policy under commitment in the next proposition and relegate its proof to the appendix.

Proposition 1 *There are no bailouts in equilibrium under commitment, $R_c^* = 1$, if*

$$\beta\left(\rho_1 - \rho_0\right) \leq (1-\alpha) + (1-\rho_0). \tag{9}$$

For the rest of this paper I assume that (9) holds. Proposition 1 states that there are no bailouts in equilibrium under commitment. This is because there is no risk of inefficient loss of risky investment under distress in $s = 1$ for any policy R. Hence, implementing a policy $R < 1$ has no benefit for entrepreneurs and only costs for households, so the authority chooses $R = 1$. The trade-off between households' and entrepreneurs' welfare is at the core of the policy analysis without commitment that comes next.

To solve for bailout policy without commitment, I compute social welfare after the distress state is realized for an arbitrary policy R. From (1), households' welfare is given by

$$V = cons + \log\left(e_1 - S^d\right) + RS^d + (1-R)\left[S^d - j\left(R, R^e\right)\right].$$

Households' welfare V sums the (log) utility for consumption in $s = 1$, which depends on endowment e_1 and savings S^d, and the (linear) utility for consumption in $s = 2$. Households' consumption in $s = 2$ is composed by the return of their savings, RS^d, and the rebate for taxation on riskless return. Note that there is no rebate for the portion of households' savings that are lent to entrepreneurs, so a bailout $R < 1$ involves implicit transfers from households to entrepreneurs.

From (2), entrepreneurs' welfare is simply given by $U = (\rho_1 - \rho_0)\, j\,(R, R^e)$: the non-pledgeable part of the return of their surviving scale of risky investment in the distress state. This is because entrepreneurs' linear utility implies that they only consume in $s = 2$.

Households' savings satisfy $S^d(R) = e_1 - \frac{1}{R}$, so from (3) and equations (6), (7) and (8) the authority's objective in $s = 1$ is

$$W(R, R^e) = cons - \log(R) - \frac{1}{R} + [\beta(\rho_1 - \rho_0) - (1 - R)] \min\left\{\frac{R^e - \rho_0}{R - \rho_0}, 1\right\} i(R^e). \qquad (10)$$

This objective shows that the authority only has incentives to set $R < 1$ if $R^e < 1$ or, in words, when in the distress state and given the cost of funding entrepreneurs do not have enough riskless investment to raise enough funds to ensure the continuation of risky investment at full scale. In this situation the authority faces a trade-off: It can avoid the (inefficient) downsizing of risky investment by reducing R but at a cost of distorting households' consumption. Since $j \leq i$, the authority has no incentives to set $R < R^e$. The authority has no incentives either to set $R > 1$. In equilibrium it must hold that $R^e = R$. The next proposition states the set of bailout policies in equilibrium.

Proposition 2 *A policy $R \in \Re_d = [\underline{R}, 1]$, the set of equilibrium bailouts without commitment in the static game, if*

$$W(R, R^e = R) \geq W\left(\widetilde{R}, R^e = R\right) \quad \forall \widetilde{R} \in [R, 1], \textit{or}$$

$$\log\left(\frac{R}{\widetilde{R}}\right) + \left(\frac{1}{R} - \frac{1}{\widetilde{R}}\right) \leq \omega \frac{\widetilde{R} - R}{\widetilde{R} - \rho_0} i(R) \quad \forall \widetilde{R} \in [R, 1]. \qquad (11)$$

with $\omega = \beta(\rho_1 - \rho_0) - (1 - \rho_0) > 0$. The largest bailout in equilibrium satisfies $\underline{R} \geq \rho_0$.

Proof. The first equation is simply the definition of an equilibrium. The second equation states its closed form solution. Any policy $\widetilde{R} > 1$ is suboptimal when $R^e = R \leq 1$ because $\widetilde{R} > R^e$ implies downsizing risky investment, $j < i$, and a distortion of households savings because $\widetilde{R} > 1$. Besides, any policy $\widetilde{R} < R^e = R$ is suboptimal because $\widetilde{R} < R$ implies more distortion of households savings than R but not more reinvestment since $j(R, R) = i(R)$. Hence, candidate policy deviations are $\widetilde{R} \in [R, 1]$. If $R^e = R < 1$ but it is close to one, then R is an equilibrium bailout without commitment. This is because households' utility is concave, so the welfare cost of the distortion of households' savings is small relative to avoiding $j < i$. The cost of the distortion for households is increasing and convex as R decreases. Hence for the condition in (11) is met by all $R^e \in [\underline{R}, 1]$ and it is violated by any $R^e \notin [\underline{R}, 1]$. ■

3.4 Discussion on the lack of commitment problem

Although the lack of commitment problem is common to many policy contexts, in bailouts it takes a special form. Proposition 2 implies that, by construction, any policy $R \in \Re_d$ is time-consistent—including the optimal policy under commitment, $R_c^* = 1$. This is because a promise in $s = 0$ of a policy $R_c^* = 1$ is implemented at $s = 1$ when households and entrepreneurs take decisions in $s = 0$ that are consistent with such a policy, i.e., when $R^e = R_c^*$. Nevertheless there still is a lack of commitment problem since any $R \in [\underline{R}, 1]$ may be realized in equilibrium. Among them, only R_c^* is -efficient ex-ante.

In contrast, in the standard policy contexts such as capital taxation or inflation policy, the optimal policy under commitment is time-inconsistent. This is technically captured by equilibrium sets with and without commitment that do not intersect; usually equilibrium policy in both cases is unique.[16] For bailouts the equilibrium sets with and without commitment do intersect; in particular, there are multiple equilibrium policies without commitment. Hence, instead of a time-inconsistency problem, for bailout there is a fragility problem—good allocations are possible in equilibrium, but bad allocations are equally possible when entrepreneurs take too much risk ex-ante and large distortionary bailouts are implemented ex-post.

The key characteristic behind this difference lies on the momentary incentives of the authority to deviate from its ex-ante optimal policy. In the cases of capital taxation or inflation policy, the authority has incentives to set high capital taxes or high inflation when private agents act consistently with expectations of low taxes or low inflation (which are respectively the ex-ante optimal policies). In contrast, in the case of bailouts, the authority has no incentives to set a bailout when private agents act consistently with expectations of no

[16] In monetary models where the authority lacks commitment usually there is implementation multiplicity since many interest rate policies are consistent with a single inflation policy. However, the optimal inflation policy is unique.

bailout. This is because bailouts are the authority's response to entrepreneurs' misbehavior due to moral hazard; hence, if there is no misbehavior, there is no need for bailouts. In a more complicated game where entrepreneurs cannot fully protect themselves against liquidity risk, a no bailout policy would not be optimal under commitment. However the same result holds in the sense that the optimal bailout with commitment is also one of the equilibrium policies without commitment (when there is no misbehavior of entrepreneurs).

It is worthwhile to note that the set of equilibrium bailouts without commitment also collapses to a single policy different than the optimal policy with commitment if there is a single entrepreneur in the game (besides the single authority and the continuum of households). Following a subgame perfection argument, the entrepreneur may strategically exploit the timing of the game by choosing low riskless investment to force the authority to implement $\underline{R}$ (the largest bailout without commitment) under distress. However, such an argument could not be applied when entrepreneurs are many and very small (in the limit, atomistic) so they do not have strategic power over the authority, households or other entrepreneurs. This is the situation in which this paper focuses. Nevertheless, the notion of "resistance" to be introduced in the next section shares its flavor with subgame perfection in the static game with a single entrepreneur.

4 Infinite policy horizon

I now extend the game to allow for an infinite sequence of non-overlapping generations that preserves the three-stages structure of the static game but introduces an infinitely lived authority that internalizes the future effects of its actions. I apply in this context the notion of sustainable plans (Chari and Kehoe, 1990) to study the way in which the history of bailout policies affect the set of equilibrium bailouts without commitment. I find that sustainable plans are uninformative as a criterion to define the severity of the lack of commitment problem of bailouts – which is critical for the analysis regarding prudential policies. I then propose "resistant bailouts" to define such a criterion.

4.1 A repeated game

Consider an economy populated by generations of households and entrepreneurs. Each generation lives for only one period $t = 0, 1, ..., \infty$. Each period is broken into three stages $s = 0, 1, 2$ which are identical to those in the static game in Section 2. Endowments remain exogenous and there are no inter-generational transfers or state variables, so there is no interaction among generations.

Allocations given actual and expected policy at t, (R_t, R_t^e), may be represented as in Section 2 by only adding subindex t:

$$x\left(R_t^e\right) = R_t^e - \rho_0, \tag{12}$$

$$i\left(R_t^e\right) = \frac{A}{1 + (1-\alpha) R_t^e - \rho_0}, \tag{13}$$

$$j\left(R_t, R_t^e\right) = \min\left\{\frac{R_t^e - \rho_0}{R_t - \rho_0}, 1\right\} i\left(R_t^e\right). \tag{14}$$

Equation (12) is generation-t entrepreneurs' optimal ratio $x\left(R_t^e\right)$ of riskless to risky investment. This ratio depends on entrepreneurs' expectations R_t^e at the initial stage $s = 0$ at t about the bailout to be implemented in the interim stage $s = 1$ if there is distress at t. Equation (13) describes generation-t entrepreneurs' risky investment $i\left(R_t^e\right)$ which depends on their endowment A, the limit of pledgeability ρ_0, and their optimal choice of riskless to risky assets ratio $x\left(R_t^e\right)$. Finally, $j\left(R_t, R_t^e\right)$ in (14) is entrepreneurs' surviving scale of risky investment if there is distress in t which depends on R_t and R_t^e.

The only twist of this economy with respect to Section 2 is that the authority, unlike households and entrepreneurs, live infinite periods. Its objective in t is

$$\mathbb{E}_t\left\{\sum_{k=0}^{\infty} \delta^k W_{t+k}\right\}. \tag{15}$$

where W_{t+k} is welfare, as defined in Section 2, of generation $t+k$ and δ is the discount factor. If $\delta = 0$, optimal bailouts with and without commitment are respectively identical to those in Propositions 1 and 2.

Overlapping generations. If generations are overlapped, a bailout at t will affect the reinvestment scale j_t of the 'old' generation of entrepreneurs and the risky investment i_t of the 'young' generation of entrepreneurs. This paper abstracts from this effect since it is not central for results.

4.2 Sustainable bailout plans

I now turn to solve for sustainable bailout plans. The authority chooses R_t in stage 1 at every period t. A sustainable plan is a sequence of policies in a Perfect Bayesian Equilibrium (PBE) such that any deviation from this sequence implies that private agents behave as in the worst equilibrium of the stage game for all subsequent periods. In short, there is a fixed cost for policy deviations.

I focus on pure and symmetric strategies for all players. A pure strategy for the authority is an infinite sequence $\sigma = \{\sigma_t\}_{t=0}^{\infty}$ where the authority's strategy at any given period t depends on the history $h_{t-1} = (R_0, R_1, ..., R_{t-1})$ of bailouts, i.e., $\sigma_t(h_{t-1})$. A symmetric strategy for generation-t households $f_t = (f_{0t}, f_{1t})$ is composed by $f_{0t}(h_{t-1})$ for their decisions in the initial stage $s = 0$ at t, which depend on the history h_{t-1} of policies up to $t-1$, and $f_{1t}(h_t)$ for their decisions in the interim stage $s = 1$ at t, which depend on the history of bailouts up to t, $h_t = (h_{t-1}, \sigma_t)$. Similarly, a strategy for generation-t entrepreneurs is $g_t = (g_{0t}, g_{0t})$ where $g_{0t}(h_{t-1})$ and $g_{1t}(h_t)$ account for their decisions in stages $s = 0, 1$ at t. Sequences $f = \{f_t\}_{t=0}^{\infty}$ and $g = \{g_t\}_{t=0}^{\infty}$ denote strategies of all generations of households and entrepreneurs.

Definition 1 *A sustainable equilibrium is a triple (σ, f, g) of sequences that satisfies:*

(i) Given σ, f and g solve for households' and entrepreneurs' problem of all generations in a competitive fashion and satisfying sequential rationality;

(ii) Given f and g, the continuation of σ solves the authority's problem in (15) for every history of past bailouts h_{t-1}.

(iii) The financial market clears in $t = 0$ and $t = 1$.

There are few comments to make about this definition. First, a key feature of a repeated policy game is that it gives a repeated opportunity to the authority to implement its policy, which in turn allows for history-dependent strategies of private agents –in this paper, households and entrepreneurs. This gives room to new equilibria that do not exist in the static game. This is captured in Definition 1.

Second, the key feature of sustainable equilibria is that assumes that private agents act competitively: They do not have strategic power over the authority or other private agents, so the history of their actions can be ignored. In contrast, the authority recognizes the effect of its policies on histories and thus it does not behave competitively. As shown by Chari and Kehoe (1990), the competitive private agents assumption greatly simplifies the analysis in game theoretic terms but, as I show below, it is the reason why sustainable plans are uninformative to study the lack of commitment problem of bailouts.

Third, the requirement of sequential rationality in condition (i) imposes that households and entrepreneurs act optimally according to the intra-period timing in the stage game. Besides, condition (iii) imposes that the competitive interaction between households and entrepreneurs is such that the interest rates paid by entrepreneurs for households' loans clear the market. In the simple economy studied in this paper this interest rate equals the effective return of riskless investment: 1 in $s = 0$ and $s = 1$ if there is no distress in t, and R_t in $s = 1$ if there is distress in t.

Fourth, the sequence of policy strategies in a sustainable equilibrium is a *sustainable plan*. Characterizing the whole set of sustainable plans is typically difficult. But Chari and Kehoe (1990), building on Abreu (1988), provide a simple test to check whether a given policy plan is sustainable. The authority should not find a profitable deviation from a sustainable plan when private agents' strategies prescribe a penalty to an authority's deviation. This penalty refers to all generations of private agents taking their decisions consistently with the worst equilibrium if they observe a policy deviation from the plan in the past.

Finally, fifth, in the literature of sustainable plans sometimes it is difficult to find the overall worst equilibrium, so the penalty is usually defined as the worst equilibrium in the stage game. In this paper the worst equilibrium in the stage game is the one with the largest bailout. However, as this subsection discusses below, results are robust to choosing a more elaborated worst equilibrium to get a higher penalty.

Applications of sustainable plans typically focus on the best sustainable plan: the policy in the sustainable equilibrium set yielding the highest ex-ante welfare. I now follow this approach.

To establish the conditions under which a bailout plan is sustainable, I first compute the *penalty* –the difference of discounted sum of future generations' welfare if the authority deviates from the plan at t versus if it does not deviate from it. Then I check whether there exist a profitable deviation for the authority from the plan given its momentary incentives to do so and the penalty.

To compute the penalty I find convenient to use the competitive allocations at period t as functions of actual and expected policy R_t and R_t^e from Section 3.1: the ratio of entrepreneurs' riskless to risky investment $x(R_t^e)$ in (12), entrepreneurs' risky investment $i\left(R_t^e\right)$ in (13), and the surviving scale $j\left(R_t, R_t^e\right)$ of entrepreneurs' risky investment in case of distress in (14). These functions satisfy competitive behavior and sequential rationality of private agents. The ex-ante expected welfare of generation-t households for an arbitrary pair (R_t, R_t^e) thus is

$$\begin{aligned} V^{ex-ante}\left(R_t, R_t^e\right) &= \left\{e_0 - i\left(R_t^e\right) - x\left(R_t^e\right) i\left(R_t^e\right) + A\right\} \\ &\quad + \alpha\left\{u\left(e_1 - S_t^{nd}\right) + S_t^{nd} + \left(\rho_0 + x\left(R_t^e\right)\right) i\left(R_t^e\right)\right\} \\ &\quad + (1-\alpha)\left\{u\left(e_1 - S_t^d\right) + R_t S_t^d + (1 - R_t)\left[S_t^d - j\left(R_t, R_t^e\right)\right]\right\} \end{aligned}$$

where the first, second and third lines on the right-hand side are respectively households' expected consumption in stage $s = 0, 1, 2$ at t considering the probability of distress in $s = 1$ and the fact that there is a bailout ($R_t < 1$) only under distress. Using the break-even condition for households in $s = 0$ and $s = 1$ that clears the financial market and the optimal choice of savings S_t^{nd}, I obtain

$$V^{ex-ante}\left(R_t, R_t^e\right) = cons + (1-\alpha)\left[-\log\left(R_t\right) - \frac{1}{R_t} - (1 - R_t)\, j\left(R, R^e\right)\right].$$

Generation-t entrepreneurs receive $\rho_1 - \rho_0$ of their surviving risky investment at the end of period t—which is $i\left(R_t^e\right)$ if there is no distress and $j(R_t, R_t^e)$ if there is distress. Hence, after imposing the equilibrium condition that $R_t = R_t^e$, generation-t welfare is

$$W^{ex-ante}\left(R_t\right) = cons - (1-\alpha)\left(\log\left(R_t\right) + \frac{1}{R_t}\right) + \left[\beta\left(\rho_1 - \rho_0\right) - (1-\alpha)\left(1 - R_t\right)\right] i\left(R_t\right). \quad (16)$$

In this very simple economy the only link between periods is the history of realized bailout policies; I thus focus on time-invariant bailouts plans. Consider an arbitrary policy plan $R_t = R\ \forall t$. The penalty of deviating from this plan is:

$$\wp\left(R, \underline{R}\right) = \frac{\delta}{1-\delta}\left[W^{ex-ante}(R) - W^{ex-ante}(\underline{R})\right].$$

where $\underline{R}$ is defined in Proposition 2.

The momentary incentives of the authority to follow R is given by welfare of one generation after distress is realized in that period from (10):

$$W\left(R, R_t^e\right) = cons - \log\left(R\right) - \frac{1}{R} + \left[\beta\left(\rho_1 - \rho_0\right) - (1 - R)\right] \min\left\{\frac{R_t^e - \rho_0}{R - \rho_0}, 1\right\} i\left(R_t^e\right).$$

The next proposition states the condition for a time-invariant bailout plan R to be sustainable.

Proposition 3 *A bailout plan $R_s \in \Re_s$, the set of sustainable bailout plans, if*

$$W(R_s, R_t^e = R_s) \geq W(\widetilde{R}, R_t^e = R_s) - \wp\,(R_s, \underline{R}) \quad \forall \widetilde{R} \in [R_s, 1] \text{ and } \forall t, \tag{17}$$

or equivalently

$$\log\left(\frac{R_s}{\widetilde{R}}\right) + \left(\frac{1}{R_s} - \frac{1}{\widetilde{R}}\right) \leq \omega \frac{\widetilde{R} - R_s}{\widetilde{R} - \rho_0} i\,(R) - \wp\,(R_s, \underline{R}) \quad \forall \widetilde{R} \in [R_s, 1] \text{ and } \forall t \tag{18}$$

with ω defined as in Proposition 1.

Besides, $\Re_s$ satisfies $\Re_d \subseteq \Re_s$ and $\inf(\Re_d) = \inf\,(\Re_s)$.

Proof. The first equation is a standard condition that sustainable plans must satisfy and the second equation states its closed form. In words, the current benefit of a deviation from the plan $R_t = R_s\ \forall t$ must be weakly smaller than implementing the plan including the deviation penalty $\wp\,(R_s, \underline{R})$. Since $\wp\,(R_s, \underline{R}) \geq 0$ for any $R_s \in \Re_d$, then $\Re_d \subseteq \Re_s$. That $\inf(\Re_s) = \inf(\Re_d)$ follows from $\wp\,(R_s, \underline{R}) < 0$ for any $R_s < \underline{R}$. ■

A standard result is that an equilibrium in the stage game is also an equilibrium in the repeated game, so $\Re_d \subseteq \Re_s$. There is no sustainable plan $R_s < \underline{R}$ since the authority has no momentary incentives to implement such a policy (i.e., it is not part of an equilibrium in the stage game) and a deviation from it implies a negative penalty (i.e., $\wp\,(R_s, \underline{R}) < 0$). Some bailout plans with $R_s > 1$ and $\wp\,(R_s, \underline{R}) > 0$ are sustainable. The same results holds if a policy deviation implies a worse outcome than in the worst stage game. The only difference would be that $\Re_s$ would expand to lower and higher interest rate policies.

The best sustainable bailout plan is simply $R = 1$, the optimal bailout policy under commitment. This is also the best equilibrium bailout policy without commitment in the static game. Hence, the standard exercise of seeking for the best sustainable plan is uninformative to study the lack of commitment problem in a repeated policy game of bailouts. Such an exercise is useful in policy contexts in which lack of commitment implies a time-inconsistency problem, but it is not useful in contexts in which it implies a fragility problem –for bailouts: good allocations are possible in equilibrium but if entrepreneurs take excessive risk there is nothing that prevents bad allocations where large distortionary bailouts are implemented.

4.3 Resistant bailout plans

This section proposes a rather natural approach to study policy in a repeated game when lack of commitment of policy materializes in the fragility problem. In particular, I ask: Which sustainable bailout plans would be carried out even if entrepreneurs take too much risk? In other words, is there a profitable collective deviation of entrepreneurs in a given period to choose less riskless investment than what is optimal for them given the bailout plan? A sustainable bailout plan in which there is no such a profitable deviation is a "resistant bailout plan", or, put differently, a sustainable bailout plan resistant to the fragility problem.

Resistant bailout plans recover the logic of subgame perfection in a static game when there is a single entrepreneur but in a repeated game with a continuum of entrepreneurs. In a static game with a single entrepreneur many equilibria are possible, but in some of these equilibria there is no room for profitable deviations of the entrepreneur: The entrepreneur can choose low riskless investment to force the authority to implement a large bailout if there is distress. Similarly, in a repeated game with a continuum of entrepreneurs, in some equilibria there is no room for collective deviation of entrepreneurs. However, the set of resistant equilibria in a static game is in general different than in a repeated game –it is a subset. The reason is that private agents' strategies are history dependent in a repeated game. Hence, the authority must take into account the future effect of abandoning a bailout plan.

The most attractive feature of this approach is that the best resistant sustainable plan is not invariant to characteristics of the game such as the authority's discount factor or other policies affecting households or entrepreneurs—the best sustainable plan, $R_t = 1\ \forall t$, is indeed invariant to such characteristics. The interplay between the best resistant bailout plan and other policies is the focus of next section.

I now define resistant sustainable bailout plans:

Definition 2 *A resistant bailout plan is a sustainable bailout plan $R \in \Re_s$ that satisfies*

$$W\left(R, R_t^e\right) \geq W\left(R_t^e, R_t^e\right) - \wp\left(R, \underline{R}\right) \quad \forall R_t^e \in \left[\underline{R}, R\right] \textit{ and } \forall t. \tag{19}$$

or equivalently

$$\log\left(\frac{R_t^e}{R_r}\right) + \left(\frac{1}{R_t^e} - \frac{1}{R_r}\right) \geq \omega \frac{R_r - R_t^e}{R_r - \rho_0} i\left(R_t^e\right) - \wp\left(R_r, \underline{R}\right) \quad \forall R_t^e \in \left[\underline{R}, R_r\right] \textit{ and } \forall t. \tag{20}$$

That is, a resistant plan satisfies two conditions. It is sustainable, so it is an equilibrium plan in the repeated policy game. In addition, the plan is not abandoned if private agents in a given period expect a larger-than-planned bailout ($R_t^e < R$) provided that abandoning the plan implies entrepreneurs in the future to behave as in the worst equilibrium in the stage game (when policy is $\underline{R}$). I restrict attention to $R_t^e \geq \underline{R}$ to ensure that if the authority abandons the plan the outcome is self-enforcing.

Compare the condition in (17) for sustainable plans and the condition in (19) for resistance. In (17) it is a deviation of the authority what must be unprofitable given the penalty. In contrast, in (19) it is a collective deviation of entrepreneurs what must be unprofitable given the penalty. Such a deviation is modelled as expectations of a larger-than-planned bailout. Equation (20) is simply the closed-form solution of (19).

Note that the "resistance" requirement may not be necessarily satisfied by all sustainable equilibria; in game theoretical language, it is a refinement. I use the label "resistance" to distinguish it from other refinements in the literature that use collective deviations. Examples are "resilient equilibria" (Aumann, 1959), "strong-perfect equilibria" (Rubinstein, 1980), "coalition-proof equilibria" (Berheim, Peleg and Whinston, 1987), "dynamically consistent equilibria" (Berheim and Ray, 1989) among others. These refinements are variations of the requirements that on a given equilibrium path no subset of players have either incentives to coordinate their strategies or to deviate from the prescribed penalty at the time of implementing it.

A "resistant" sustainable equilibrium also satisfies these requirements in the simple game studied in this paper. There is no collective action of one generations of entrepreneurs that could force a higher-than-planned resistant bailout. Besides, the penalty to a policy deviation is that all future generations of entrepreneurs behave as in the worst equilibrium in the stage game, i.e., when the bailout is the highest, no subset of entrepreneurs have incentives to deviate from the penalty. Nevertheless, if entrepreneurs live many (or infinite) periods, they may design complicated coordinated strategies involving low riskless investment and low penalties to policy deviations to induce large bailouts every period. Such coordination is not possible for non-overlapping generations of private agents that live a single period under the assumption that one generation cannot coordinate with generations which have already died or are not yet born. If coordination among generations is possible, for instance, because private agents live many periods and/or generations are overlapped, the "resistance" refinement is weaker than the other refinements relying on coordinated strategies cited above. This is actually a desirable property since many of these refinements are so strong that often rule out all or the most appealing equilibria.

I now turn to study some properties of the best "resistant" sustainable plan R_r^*.

Proposition 4 *(i) The best resistant sustainable plan is* $R_r^* = \min\{\sup(\Re_R), R_c^*\}$ *.*

(ii) R_r^* *is increasing in* $\delta \leq \overline{\delta}$ *such that* $R_r^* \to \underline{R}$ *as* $\delta \to 0$ *and* $R_r^* \to R_c^*$ *as* $\delta \to \overline{\delta}$*.*

(iii) The social cost of bailouts is convex in the bailout size.

Proof. (i) It follows from $W^{ex-ante}(R_t)$ being increasing in $R_t < 1$ with a maximum at $R_c^* = 1$.

(ii) If $\delta = 0$, $\wp(R_r, \underline{R}) = 0$ for any R_r. Then the equilibrium condition in (17) and the resistance condition (19) hold simultaneously only for $R_r = \underline{R}$. To see this, take a plan $R_s = \underline{R}$. (17) holds for any $\widetilde{R} \in [\underline{R}, 1]$. Now assume that some $R_r > \underline{R}$ is sustainable. (20) must hold for any $R_t^e \in [\underline{R}, R_r]$. This is a contradiction for $R_t^e = \underline{R}$. It is not a contradiction for $R_r = \underline{R}$.

If $\delta > 0$, $\wp(R_r, \underline{R}) \geq 0$ for $R_r \in \Re_s$. For $R_r = \underline{R}$ the left hand side of (20) is smaller than the right hand side for $\wp(R_r, \underline{R}) = 0$ by the argument for $\delta = 0$. Since $\underline{R}$ is the smallest equilibrium policy, (17) must hold with equality. All functions are monotone in R_t and R_t^e for $R_t^e \leq R_r$, so for any $\wp(R_r, \underline{R}) \geq 0$ for $R_r \in \Re_s$ there exist policies $R_r > \underline{R}$ such that (20) holds.

However, in general not all sustainable policies are resistant. To see this, note that the left hand side of (20) decreases in $R_r \leq 1$ if $R_r - R_t^e$ is hold constant. This is because the concavity of households' utility. Hence, for a given penalty function $\wp(R_r, \underline{R})$ there exist a $R_r^* \leq 1$ which increases as δ increases. As $\delta \to 1$ the penalty $\wp(R_r, \underline{R}) \to \infty$, so there exist a $\overline{\delta}$ such that $R_r^* = 1$.

(iii) It follows from $W^{ex-ante}(R_t)$ being concavewith a maximum at $R_t = 1$ and $R_r^* \leq 1$. ■

Proposition 4 states useful properties of the resistant sustainable bailout plans. Result (i) simply pins the best resistant sustainable bailout down as the one that yields the highest ex-ante welfare.

Result (ii) establishes that, if the authority is myopic $(\delta = 0)$, the only resistant sustainable plan is $\underline{R}$, the largest equilibrium policy and the worst equilibrium allocation from an ex-ante perspective. This is the only equilibrium policy for which a collective deviation of entrepreneurs is not profitable. In contrast, if the authority is forward-looking $(\delta > 0)$, some equilibrium policies $R_s > \underline{R}$ become resistant. As the discount factor of the authority δ increases, the lack of commitment problem gets alleviated since smaller bailouts become resistant ($R_r^* \leq 1$ increases). Good and bad equilibria may exist, but on the path of a resistant bailout plan all worse allocations are ruled out even if all entrepreneurs could coordinate their actions. This is the counterpart of the standard result for the best sustainable plan when there is a time-inconsistency problem. However this result is fundamentally different since the standard context an increase in δ allows to sustain better allocations as an equilibrium; here it is that more bad equilibria can be ruled out.

Result (iii) has important policy implications. Because of a concave utility of households, larger distortions on households' decisions due to a bailout has an increasing detrimental effect on welfare. Thus, small variations in the severity of the lack of commitment problem of bailouts have relative large effects on welfare. This result justifies the focus of this paper of studying the effects of regulations and prudential policies on the severity of the lack of commitment problem of bailouts.

5 Resistant bailouts and prudential policies

The last section establishes a criterion to evaluate the severity of the lack of commitment problem of bailout policy: The size of the best resistant sustainable bailout plan. In this section I restrict attention to the case in which the authority's discount factor is $\delta \in \left(0, \overline{\delta}\right)$, so the best resistant sustainable plan is in the middle ground between no bailouts and the largest bailout in the stage game. I study in this context the interplay of such a plan and ex-ante policies – so called prudential policies. The basic point is that many ex-ante policies affect the incentives of the authority to implementing bailouts under distress. Hence, a side effect of such ex-ante policies is to alleviating or exacerbating the lack of commitment problem of bailouts. The sign of these sides effects is usually ambiguous which gives rise to a delicate interaction between prudential policies and bailouts.

For concreteness, I study three examples: liquidity requirements, prudential taxes, and public debt. For liquidity requirements and prudential taxes I focus on the case in which these policies backfire by increasing instead of decreasing entrepreneurs' risk taking. For public debt I focus on the converse case to show that a prudential role of the management of public debt arises.

5.1 Liquidity requirements

I see liquidity requirements as an example of the variety of regulations proposed after the 2007-2009 financial crisis that takes the form of bounds on banks' choices (in this paper, entrepreneurs).

A liquidity requirement implements the first best if it can effective impose the constraint on entrepreneurs that the ratio of riskless to risky investment must be $x_t \geq 1 - \rho_0$. I instead study a different case: when the effectiveness of regulation is limited, so it cannot implement the first best. For liquidity requirements, entrepreneurs' ratio of riskless to risky investment must be $x_t \geq \underline{x} < 1 - \rho_0$. The motivation for this analysis relies on the fact that historically many financial regulations have only had limited success on restricting misbehavior of participants in financial markets. I show in this section that liquidity requirements in this case may have a detrimental effect on welfare by exacerbating the fragility problem or, in the terminology used in this paper, by increasing the size of the best resistant bailout plan (by decreasing R_r^*).

To see why, note that with the liquidity requirement $\underline{x}$ entrepreneurs' riskless investment is

$$x\left(R_t^e; \underline{x}\right) = \max\left\{R_t^e - \rho_0, \underline{x}\right\}$$

so the regulation is binding only for $R_t^e \leq \underline{x} + \rho_0$. Thus, risky investment $i\left(R_t^e; \underline{x}\right)$ is strictly smaller than $i\left(R_t^e\right)$ in (13) for $R_t^e \leq \underline{x} + \rho_0$ and otherwise both expressions are identical. Similarly, $j\left(R_t, R_t^e; \underline{x}\right)$ is strictly larger than $j\left(R_t, R_t^e\right)$ in (14) for $R_t^e \leq \underline{x} + \rho_0$ and otherwise both expressions are identical.

Proposition 5 *A liquidity requirement $\underline{x}$ increases entrepreneurs' risk taking by exhacerbating the lack of commitment of bailouts if such the liquidity requirement is binding in the worst static equilibrium ($\underline{R} < \underline{x} + \rho_0$) but it is not binding on the equilibrium path of the best resistant bailout plan ($R_r^* > \underline{x} + \rho_0$).*

Proof. In the stage game, with the liquidity requirement the authority has no incentives to set $R_t < \underline{x} + \rho_0$ since doing so increases the distortion in households saving decisions but it has no benefit on avoiding the inefficient downscale of risky investment for any expected policy $R_t^e \in [\rho_0, \underline{x} + \rho_0]$. Hence, if the liquidity requirement is binding, then $\underline{R} = \underline{x} + \rho_0$, so $\underline{R}$ is increasing in $\underline{x}$.

In the repeated game, if without the liquidity requirement the best resistant sustainable bailout is $R_r^* > \underline{x} + \rho_0$ but the worst equilibrium bailout policy in the stage game is $\underline{R} < \underline{x} + \rho_0$, then the only effect of $\underline{x}$ on the condition (20) is by reducing the penalty $\wp\left(R_r, \underline{R}\right)$. Therefore, R_r^* is decreasing in $\underline{x}$. ■

This proposition highlights a case in which the side effect of a liquidity requirement makes the fragility problem of bailout policy without commitment worse. This result applies when the restriction imposed by the liquidity requirement is binding on the worst equilibrium in the stage game but it is not binding on the equilibrium path in which the best resistant bailout policy is implemented, i.e., when $\underline{R} < \underline{x} + \rho_0 < R_r^*$.

This is because in this situation the liquidity requirement $\underline{x}$ acts as a restriction that enhances ex-ante welfare of the worst equilibrium allocation in the stage game. But at the same time it increases the fragility problem of bailout policy by decreasing the incentives of an infinite horizon authority to carry a bailout plan out: The "penalty" that the authority suffers if it deviates from a bailout plan decreases when the worst possible equilibrium allocation improves. This is the only effect that such a regulation has on the authority's problem in (20). Thus, the regulation $x_t \geq \underline{x}$ is not binding on entrepreneurs' decisions when their expected policy R_t^e equals the best resistant bailout R_r^*.

The current analysis may be extrapolated that any regulation that enters into the game as a bound on entrepreneurs' decisions. Other examples are capital requirements or limits on leverage. Besides, the proposition 5 highlights the different notions of the severity of lack of commitment of policy in static and dynamic setups: The allocation in the worst equilibrium in the static game improves after the liquidity requirement is imposed, the best sustainable bailout is invariant, yet the allocation in the path of the best resistant sustainable plan becomes worse, all in terms of ex-ante welfare.

5.2 Taxes on borrowing

Prudential taxes is another popular policy proposal that has emerged as a response to the 2007-2009 financial crisis. It has two alternative motivations. One is the informal argument that banks should pay ex-ante for the bailouts they get ex-post. The other, more formally analyzed, proposes a Pigouvian correction of an externality that leads to overborrowing (e.g., Jeane and Korinek, 2010; Bianchi and Mendoza, 2010). This externality arises because one borrower hitting its borrowing constraint tightens other borrowers' constraints in general equilibrium. The economy studied in this paper is too simple to capture this externality, but it suffices to show that a potential drawback of prudential taxes is that it may induce entrepreneurs to increase their risk exposure. The mechanism varies according to the treatment of the tax rebate.

Consider a proportional tax τ on risky investment i_t to be collected at stage $s = 0$ in every period t. I explore two variations of this policy according to what is done with the tax revenue: (i) lump-sum rebates T_t to entrepreneurs in $s = 1$, so it could help finance reinvestment j_t if there is distress; and (ii) lump-sum rebates T_t to entrepreneurs in $s = 2$, so the policy has the standard interpretation of Pigouvian taxation.

(i) The tax is rebated to entrepreneurs in $s = 1$. This variation captures the motive of taxing banking entrepreneurs ex-ante to finance bailouts during distress. Households' break-even condition in $s = 0$ is now

$$(1 + \tau) i_t + x_t i_t - A = \alpha \left[(\rho_0 + x_t) i_t + T_t \right],$$

implying

$$i(x_t, \tau, T_t) = \frac{A + \alpha T_t}{1 - \alpha \rho_0 + (1 - \alpha) x_t + \tau}.$$

In words, generation-t entrepreneurs must raise $(1 + \tau) i_t$ in stage $s = 0$ to obtain a scale of risky investment i_t. The tax rebate T_t if there is no distress works in $s = 0$ exactly as riskless investment $x_t i_t$. This is because T_t is invested in riskless assets in $s = 1$ if there is no distress, i.e., it is fully pledgeable in $s = 0$ after adjusting by the probability α of no distress. Therefore, the scale of entrepreneurs' risky investment is decreasing in the tax τ and increasing in the rebate T_t.

In turn, households' break-even condition in $s = 1$ is now

$$R_t (j_t - T_t) = \rho_0 j_t + x_t i_t,$$

implying

$$j(x_t, T_t) = \min \left\{ \frac{x_t i_t + R_t T_t}{R_t - \rho_0}, i_t \right\}.$$

In words, if there is distress in stage $s = 1$, the rebate T_t to generation-t entrepreneurs works exactly as endowment. The amount of loans needed by entrepreneurs is only $j_t - T_t$ to ensure that a scale j_t of risky investment survives distress. Thus, ceteris paribus, j_t is increasing in T_t.

Taking into account that entrepreneurs in this economy choose their riskless investment x_t to continue their risky investment at full scale and the equilibrium condition $T_t = \tau i_t$, I obtain

$$\begin{aligned} x(R_t^e; \tau) &= (1 - \tau) R_t^e - \rho_0, \\ i(R_t^e; \tau) &= \frac{A}{1 + (1 - \alpha) \left[R_t^e + (1 - R_t^e) \tau \right] - \rho_0}, \\ j(R_t, R_t^e; \tau) &= \min \left\{ \frac{R_t^e - \rho_0}{R_t - \rho_0}, 1 \right\} i(R_t^e; \tau). \end{aligned}$$

These expressions must be compared to (12), (13), and (14) when $\tau = 0$. Generation-t entrepreneurs anticipate the rebate, so their choice of the ratio x_t of riskless to risky investment in stage $s = 0$ is decreasing in the tax τ given an expected bailout policy R_t^e. As a result, conditioning on R_t and R_t^e, the ratio j_t/i_t between the scale of risky investment after and before distress is unaffected by τ. However, the scale of risky investment i_t is decreasing in τ if $R_t^e < 1$ –which is the relevant case. The reason is that the effect of the rebate on entrepreneurs' financing capacity during distress is also affected by the bailout policy $R < 1$.

Proposition 6 *A prudential tax designed to finance transfers to entrepreneurs during distress may increase entrepreneurs" risk taking by exacerbating the lack of commitment of bailout policy.*

Proof. See the appendix ■

This proposition states that a possible side effect of prudential taxes to finance bailouts is to worsen the lack of commitment problem of bailouts. In technical words, the best resistant sustainable plan R_r^* may be decreasing in τ. The proof of Proposition 6 in the appendix states the conditions in which this possibility takes place. To build intuition, the condition for a bailout plan R_r to be resistant given the modality of prudential taxes studied here is

$$\log\left(\frac{R_t^e}{R_r}\right) + \left(\frac{1}{R_t^e} - \frac{1}{R_r}\right) \geq \omega \frac{R_r - R_t^e}{R_r - \rho_0} i\left(R_t^e;\tau\right) + \left(R_r - R_t^e\right)\tau i\left(R_t^e;\tau\right) - \wp\left(R_r, \underline{R}\left(\tau\right);\tau\right)$$

$\forall R_t^e \in \left[\underline{R}\left(\tau\right), R_r\right]$ and $\forall t$.

The prudential tax τ enters into this problem by several components on the right hand side of this condition. First, the rebate to entrepreneurs' implies that, given the bailout plan R_r, the transfer of resources from households to entrepreneurs decreases in the tax τ. Thus, the burden on households due to the bailout is decreasing in the rebate (and thus in τ). As a result, the detrimental effect of a bailout on households' welfare is decreasing in τ. This decreases the "static" cost for the authority in terms welfare of deviating from the bailout plan R_r, which in turn increases the fragility problem due to lack of commitment. This effect is captured in the term $\left(R_r - R_t^e\right)\tau i\left(R_t^e;\tau\right)$.

However, there is an off-setting force at play: The scale of entrepreneurs' risky investment $i\left(R_t^e;\tau\right)$ is decreasing in τ. By this force, less resources are exposed in the economy to distress risk such that the static benefit for the authority of deviating from the bailout plan R_r is smaller. These two forces interact in the momentary incentives of the authority to implementing a bailout as well as in the penalty $\wp\left(R_r, \underline{R}\left(\tau\right);\tau\right)$ by two channels: The difference in ex-ante welfare varies in the tax τ given the bailout plan R_r and the worst static equilibrium policy $\underline{R}$, and because $\underline{R}$ is also affected by the tax.

The proof in the appendix shows that the tax τ decreases the monetary incentives of the authority to carry out a bailout plan when the bailout plan $R_r - \rho_0$ is above a certain endogenous level. Similarly, the worst equilibrium bailout policy $\underline{R}\left(\tau\right)$ is decreasing in τ if $\underline{R}\left(\tau\right) - \rho_0$ is also above this level. Thus, it is possible that the tax τ decreases the momentary incentives of the authority to carry out a bailout plan for a plan $R_r > \underline{R}$ but these incentives are increasing in τ in the equilibrium path in which $\underline{R}$ is implemented.

Regarding the penalty term, the effect of the tax τ is ambiguous. It depends on the relative forces explained above and the effect of the tax on the worst equilibrium policy $\underline{R}\left(\tau\right)$. In any case, if the reputation concerns of the authority are small (i.e., small discount factor δ), then the effect of the tax on the momentary incentives of the authority to carrying out a bailout plan dominate.

(ii) The tax is rebated to entrepreneurs in $s = 2$. This variation captures the motive of using taxes to reduce entrepreneurs' risk taking, for instance, because of a systemic externality (Bianchi, 2011) or because of the expectations of bailouts (Kocherlakota, 2010). This paper suggests that this tax policy is ineffective on reducing entrepreneurs' risk taking and ends up only reducing entrepreneurs' liquidity holding.

To see this, the break-even condition of generation-t households in stage $s = 0$ is

$$\left(1+\tau\right)i_t + x_t i_t - A = \alpha\left[\left(\rho_0 + x_t\right)i_t + T_t\right],$$

which is identical to the variation (i) for prudential taxes studied above in this section. This is because the rebate in stage $s = 2$ still works as a perfect substitute of riskless investment from the standpoint of $s = 0$. Both the rebate and riskless investment are certain inflow of resources to entrepreneurs in $s = 2$, so both are fully pledgeable in $s = 0$ up to the extent of the probability α of no distress. The closed form of risky investment i_t as a function of the rebate T_t is thus identical to the one derived in (i) above.

If there is distress in $s = 1$, households' break-even condition is

$$R_t j_t = \rho_0 j_t + x_t i_t + T_t,$$

which is now different than the one obtained in (i) above. This is because the rebate is still a substitute for riskless investment in $s = 1$. In (i), the rebate is substitute of endowment in $s = 1$ and thus its financing capacity is affected by the bailout policy R_t. This in not the case in the current variation of prudential taxes. As a result, entrepreneurs' optimal choice of riskless assets in $s = 0$ once the equilibrium condition $T_t = \tau i_t$ is imposed is

$$x\left(R_t^e; \tau\right) = R_t^e - \rho_0 - \tau.$$

In words, since the rebate is a perfect substitute of riskless investment both in $s = 0$ and $s = 1$, entrepreneurs adjust their choice of x_t such that the tax has no effect on entrepreneurs choice of risky investment i_t, the surviving scale j_t of this risky investment after distress, or the amount of transfers from households to entrepreneurs implicit in a bailout. These three variables are the sole determinants of welfare in this economy, so prudential taxes have no effect on the authority's problem to implement a bailout under distress.

The next proposition summarizes this result, so it needs no proof.

Proposition 7 *A prudential tax designed to reduce entrepreneurs' risk taking is innocuous in reducing entrepreneurs' exposition to distress risk and instead it only reduces entrepreneurs' liquidity.*

The prudential tax can only correct the externality that arises in general equilibrium if it can effectively reduce entrepreneurs' risk taking. In this economy this is not the case. This is because the rebate to which the tax is attached to relaxes the borrowing constraint of entrepreneurs during distress, exactly as riskless investment does. This effect is not present in the literature studying prudential taxation with a Pigouvian motivation (e.g., Jeanne and Korinek, 2010; Bianchi and Mendoza, 2013) because in this literature the borrowing constraint of investors (in this paper, entrepreneurs) is assumed to take an ad hoc form in which there is no role for liquidity. Specifically, the literature usually assumes that the borrowing constraint is defined as a fixed proportion of the value of capital. In contrast, this paper follows Holmstrom and Tirole (1998) and Farhi and Tirole (2009, 2012) by modelling entrepreneurs' borrowing constraint in a way that may be derived from optimal contracts and liquidity plays a meaningful role. Limited pledgeability of risky investment can be understood as a requirement imposed in an optimal contract when entrepreneurs' effort is correlated to the return of risky investment but such an effort is entrepreneurs' private information. Liquidity (in this paper, riskless investment) plays a meaningful role under this contract because it increases the ability of entrepreneurs to raise funds in the distress state.

5.3 Public debt as a prudential policy

This section shows that a prudential role of public debt emerges when the lack of commitment problem of bailouts yields misbehavior of financial institutions. The key is that public debt enters as a state variable in the authority's problem of bailouts that changes its incentives of implementing a bailout at the time of distress. The delicate interaction between bailouts and prudential policies shows up here because in some parameter subspace the best resistant bailout R_r^* is increasing as public debt increases (i.e., the fragility problem is alleviated) while in other subspace R_r^* is decreasing as public debt increases. In either case, the appropriate management of public debt decreases less risk-taking by entrepreneurs.[17]

A simple mechanism to generate such a prudential role is by assuming that the service of public debt must be financed by distortionary taxes to households in a way that increases marginal utility of consumption at the time of distress. For concreteness, assume that a constant tax rate ζ is levied from the return of households' investment in the interim subperiod (regardless on whether it is riskless investment or loans to entrepreneurs). For simplicity the stock of public debt is assumed to have no other effect on generation-t than the need of raising funds from households to pay its service. Thus welfare of generation-t households relevant for the authority if there is distress at t is

$$V = cons + \log\left(e_1 - S_t^D\right) + R_t S_t^D + (1 - R_t)\left(S_t^D - j_t\right) - \zeta R_t S_t$$

so the public debt service equals $\zeta R_t S_t$. Note that households savings in the distress state S_t^D now solve

$$S_t^D = e_1 - \frac{1}{(1-\zeta)\, R_t}.$$

In words, this way to finance the public debt service has no effect on the rebate that generation-t households receive from the authority, $(1 - R_t)\left(S_t^D - j_t\right)$, but it distorts their consumption schemes. This distortion amplifies the welfare cost on households of a bailout at t of size $1 - R_t$.

An important implicit assumption in this setup is that the authority cannot repudiate the service of its debt. Hence, the tax rate ζ on households is not in the control of the authority at the time of distress. One may rationalize this assumption by introducing a cost on the authority for defaulting its debt. Then results

[17] The use of public debt as a commitment device for the time-inconsistency problem of monetary and fiscal policy has been studied respectively by Persson, Persson and Svensson (2006) and Dominguez (2007).

in this section apply in a parameter subspace in which there is a lack of commitment problem of bailouts but the authority does not repudiate the service of its debt.

The main result in this section is the following:

Proposition 8 *Public debt management affects the welfare cost of bailouts and thus decreases entrepreneurs' risk taking by alleviating the lack of commitment problem of bailouts.*

Proof. See the appendix. ■

The proof in the appendix establishes that the best resistant bailout increases with the tax ζ when the reputation concerns of the authority are weak, i.e., when the authority heavily discounts welfare of future generations. This is because a higher cost of a bailout to the current generation increases the authority's incentives of carrying out its bailouts plans. However, the opposite result holds when these reputation concerns are strong. This is because the penalty term is decreasing in ζ. Therefore, more or less public debt may be desirable from a prudential standing point depending on the sign of this effect.

6 Concluding remarks

This paper calls attention to the delicate interaction between ex-ante policies, such as financial regulations and prudential policies, and the lack of commitment problem of bailouts. This interaction is delicate because there are conditions in which financial regulation and prudential policies may exacerbate or alleviate this lack of commitment problem. This paper focuses on liquidity requirements and prudential taxes are examples of well-intended prudential regulations and policies that may backfire. Besides, this paper also studies public debt management as an example of policies usually not motivated by prudential concerns that may play such a role by its side effect on the lack of commitment problem of bailouts.

I make these points in a model where leverage and liquidity play an important role and where the authority has a long-run horizon. In this environment the lack of commitment of bailouts creates a fragility problem instead of the standard time-inconsistency problem. I argue that in this context the standard approach of Sustainable Plans (Chari and Kehoe, 1990) is uninformative for policy analysis. I propose a refinement that recovers the usefulness of studying repeated games to consider the reputation concerns of an authority at the time of implementing a bailout.

7 References

Abreu, D. (1988). "On the Theory of Infinitely Repeated Games with Discounting," Econometrica 56(2):383-96.

Acharya, V. and T. Yorulmazer (2007). "Too Many to Fail – An Analysis of Time-Consistency in Bank Closure Policies," Journal of Financial Intermediation 16(1):1-31.

Aumann, R. (1959). "Acceptable Points in General Cooperative n-Person Games," Contributions to the Theory of Games, Annals of Mathematical Studies IV:287–324

Berheim D., B. Peleg and M. Whinston (1987). "Coalition Proof Nash Equilibria I: Concepts," Journal of Economic Theory 42(1):1-12.

Berheim D. and D. Ray (1989). "Collective Dynamic Consistency in Repeated Games," Games and Economic Behavior 1(4):295-326.

Bianchi, J. (2011). "Overborrowing and Systemic Externalities in the Business Cycle," American Economic Review 101(7):3400-26.

Bianchi, J. (2013). "Efficient Bailouts?," mimeo, Wisconsin.

Bianchi, J. and E. G. Mendoza (2010). "Overborrowing, Financial Crises and 'Macro-Prudential' Taxes," NBER Working Paper #16091, June.

Brunnermeier, M (2009). "Deciphering the Liquidity and Credit Crunch 2007-08," Journal of Economic Perspectives 23(1):77-100.

Chari, V. and P. Kehoe (1990), "Sustainable Plans," Journal of Political Economy 98(4): 783-802.

Chari, V. and P. Kehoe (2013). "Bailouts, Time Inconsistency and Optimal Regulation," NBER Working Paper #19192, June.

Diamond, D. and R. Rajan (2009). "The Credit Crisis: Conjectures about Causes and Remedies," American Economic Review 99(2):606-10.

Diamond, D. and R. Rajan (2012). "Illiquid Banks, Financial Stability, and Interest Rate Policy," Journal of Political Economy 120(3):552-91.

Dominguez, B. (2007). "Public Debt and Optimal Taxes without Commitment," Journal of Economic Theory 135(1):159-70.

Farhi, E. and J. Tirole (2009). "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts," NBER Working Paper #15138, July.

Farhi, E. and J. Tirole (2012). "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts," American Economic Review 102(1):60-93.

Holmstrom, B. and J. Tirole (1998). "Private and Public Supply of Liquidity," Journal of Political Economy 106(1):1-40.

Jeanne, O. and A. Korinek (2011). "Managing Credit Booms and Busts: A Pigouvian Taxation Approach," NBER Working Paper #16377, September.

Keister, T. (2014). "Bailouts and Financial Fragility," mimeo, Rutgers University, January.

Kelly, B., H. Lustig and S. Van Nieuwerburgh (2012). "Too-Systemic-To-Fail: What Option Markets Imply About Sector-Wide Government Guarantees," Chicago Booth Research Paper 11-12, March.

Kocherlakota, N. (2010). "Taxing Risk and the Optimal Regulation of Financial Institutions," Federal Reserve Bank of Minneapolis, Economic Policy Paper 10-3, May.

Kydland, F. and E. C. Prescott (1977). "Rules Rather than Discretion: The Inconsistency of Optimal Plans," Journal of Political Economy 85(3):473-91.

Lorenzoni, G. (2008). "Inefficient Credit Booms," Review of Economic Studies 75(3):809-33.

Mailath, G. and L. Mester (1994). "A Positive Analysis of Bank Closure," Journal of Financial Intermediation 3(3):272-99.

Nosal, J. and G. Ordonez (2014). "Uncertainty as Commitment," mimeo, Columbia, Penn, January.

Persson, M. T. Persson and L.E.O Svensson (2006). "Time Consistency of Fiscal and Monetary Policy: A Solution", Econometrica, 74(1): 193-212.

Rubinstein (1980). "Strong Perfect Equilibria," International Journal of Game Theory 9(1):1-12.

Schneider, M. and A. Tornell (2004). "Balance Sheet Effects, Bailout Guarantees, and Financial Crises," Review of Economic Studies 71(3):883-913.

Woodford, M. (1990). "Public Debt as Private Liquidity," American Economic Review 80(2):382-88.

Appendix

Proof of Proposition 1. Under the condition $\beta(\rho_1 - \rho_0) \leq (1 - \rho_0) + (1 - \alpha)$ it holds that $\frac{\partial W^{ex-ante}(R)}{\partial R} > 0$ where $W^{ex-ante}(R)$ is identical to (16) after removing the time subindex.

Proof of Proposition 6. The condition for a policy R_r to be a resistant bailout plan in the economy with prudential taxes and rebates to generation-t entrepreneurs in $s = 1$ is

$$\log\left(\frac{R_t^e}{R_r}\right) + \left(\frac{1}{R_t^e} - \frac{1}{R_r}\right) \geq \omega \frac{R_r - R_t^e}{R_r - \rho_0} i\left(R_t^e; \tau\right) + \left(R_r - R_t^e\right) \tau i\left(R_t^e; \tau\right) - \wp\left(R_r, \underline{R}(\tau); \tau\right)$$

$\forall R_t^e \in [\underline{R}(\tau), R_r]$ and $\forall t$.

The prudential tax τ enters into this condition by two forces that affect both its static and inter-temporal components. The first force is that the tax reduces transfers from households to entrepreneurs that are implicit in the bailout. This is because, given the surviving scale j_t of generation-t's risky investment after distress, the funds that entrepreneurs must raise from households is decreasing in the tax rebate (and thus in the tax rate). This is the main driving force of the result stated in the proposition. The second force is that the scale of risky investment is decreasing in the tax τ. This force is captured by the term $i\left(R_t^e; \tau\right)$ which is derived in section 4.2 and operates in opposite direction to the former one.

Turning to the static components, the first force –less transfers from households to entrepreneurs given j_t– decreases the social cost of a bailout for the current generation of households as τ increases. This effect is captured in the second term on the right hand side of the resistance condition above, $\left(R_r - R_t^e\right) \tau i\left(R_t^e; \tau\right)$. The second force –$i\left(R_t^e; \tau\right)$ decreasing in τ— mitigates this effect by reducing the base of taxation.

That $i\left(R_t^e; \tau\right)$ is decreasing in τ also implies that the amount of resources exposed to distress risk is decreasing in the tax. This effect is captured in the first term on the right hand side of the resistance condition, $\omega \frac{R_r - R_t^e}{R_r - \rho_0} i\left(R_t^e; \tau\right)$.

Overall, the static component of this condition

$$\omega \frac{R_r - R_t^e}{R_r - \rho_0} i\left(R_t^e; \tau\right) + \left(R_r - R_t^e\right) \tau i\left(R_t^e; \tau\right)$$

is increasing in τ if

$$R_r > \rho_0 + \frac{\omega B(R^e)}{1 - \tau B(R^e)} > \rho_0 \tag{*}$$

where $B(R^e) = \frac{(1-\alpha)(1-R^e)}{1+(1-\alpha)(R^e+(1-R^e)\tau)-\rho_0}$.

This force implies that the momentary incentives of the authority to carrying out a bailout plan R_r are decreasing in τ if (*) is satisfied. It also implies that the largest bailout policy in the stage game $\underline{R}(\tau)$ is decreasing in τ if this condition is satisfied for $R_r = \underline{R}(\tau)$. Note that $B'(R^e) < 0$. Hence, the higher $\underline{R}(\tau)$ or $R_r^*(\tau)$, the weaker the condition in (*) is.

I now turn to the dynamic components of this condition which is captured in the penalty $\wp(R_r, \underline{R}(\tau); \tau)$

$$\wp(R_r, \underline{R}(\tau); \tau) = \frac{\delta}{1-\delta}\left[W^{ex-ante}(R_r; \tau) - W^{ex-ante}(\underline{R}(\tau); \tau)\right]$$

where

$$W^{ex-ante}(R; \tau) = cons - (1-\alpha)\left(\log(R) + \frac{1}{R}\right) + \left[\beta(\rho_1 - \rho_0) - (1-\alpha)(1-\tau)(1-R)\right] i(R; \tau).$$

for $R = \{\underline{R}(\tau), R_r\}$.

The first force –less transfers from households to entrepreneurs given j_t– implies that welfare of future generations if the bailout plan is carried out, $W^{ex-ante}(R_r; \tau)$, is increasing in τ and the second force – $i(R_t^e; \tau)$ decreasing in τ– goes in opposite direction. The same is true for the welfare of future generations if the authority deviates from the bailout plan, $W^{ex-ante}(\underline{R}(\tau); \tau)$. However, in this latter component there is an extra effect of τ: The worst equilibrium policy in the static game $\underline{R}(\tau)$ is decreasing in τ if the condition in (*) is satisfied as established above. The term $\underline{R}(\tau)$ enters in the penalty by a variety of effects on households and entrepreneurs.

Overall,

$$\begin{aligned}\frac{1-\delta}{\delta}\frac{\partial}{\partial\tau}\wp\left(R_r,\underline{R}\left(\tau\right);\tau\right) &= \left[\beta\left(\rho_1-\rho_0\right)-\left(1-\alpha\right)\left(1-\tau\right)\right]\left(\frac{\partial i\left(R_r;\tau\right)}{\partial\tau}-\frac{\partial i\left(\underline{R}\left(\tau\right);\tau\right)}{\partial\tau}\right)+ \\ &+\left(1-\alpha\right)\left(1-\tau\right)\left[R_r\frac{\partial i\left(R_r;\tau\right)}{\partial\tau}-\underline{R}\left(\tau\right)\frac{\partial i\left(\underline{R}\left(\tau\right);\tau\right)}{\partial\tau}\right] \\ &+\left(1-\alpha\right)\left[\left(1-R_r\right)i\left(R_r;\tau\right)-\left(1-\underline{R}\left(\tau\right)\right)i\left(\underline{R}\left(\tau\right);\tau\right)\right] \\ &-\frac{\partial W^{ex-ante}\left(\underline{R}\left(\tau\right);\tau\right)}{\partial\underline{R}\left(\tau\right)}\frac{\partial\underline{R}\left(\tau\right)}{\partial\tau}\end{aligned}$$

The first term in parenthesis on the right hand side is weakly positive since $1\geq R_r\geq\underline{R}\left(\tau\right)$ and $\frac{\partial i(\underline{R}(\tau);\tau)}{\partial\tau}\leq\frac{\partial i(R_r;\tau)}{\partial\tau}<0$. The second term in parenthesis is negative if $\underline{R}\left(\tau\right)\geq\frac{1}{2}$. The third term is negative and the last term is positive if the condition (*) is satisfied for $R_r=\underline{R}\left(\tau\right)$.

Overall, if the discount factor δ is small, then the result depends on the condition (*). Otherwise, the result is ambiguous.

Proof of Proposition 7. The condition for a bailout plan R_r to be resistant when there is a tax ζ on households to serve the public debt is

$$\log\left(\frac{R_t^e}{R_r}\right)+\frac{1}{1-\zeta}\left(\frac{1}{R_t^e}-\frac{1}{R_r}\right)+\tau e_1\left(R_t^e-R_r\right)\geq\omega\frac{R_r-R_t^e}{R_r-\rho_0}i\left(R_t^e\right)-\wp\left(R_r,\underline{R}\left(\zeta\right);\zeta\right)$$

$\forall R_t^e\in\left[\underline{R}\left(\zeta\right),R_r\right]$ and $\forall t$.

The tax ζ enters into this problem by a single force: a higher ζ increases the welfare cost on households of a bailout policy R. This in turn affects several components of this condition. The first is on the left hand side of the resistance condition by increasing the cost on households of deviating from R_r to implementing $R_t^e\in\left[\underline{R}\left(\zeta\right),R_r\right]$. The second is on the largest bailout policy in the static game $\underline{R}\left(\zeta\right)$ which enters in the penalty term $\wp\left(\cdot\right)$. The third is the direct effect of the tax on ex-ante welfare, which is implicit in the definition of the penalty:

$$\wp\left(R_r,\underline{R}\left(\zeta\right);\zeta\right)=\frac{\delta}{1-\delta}\left[W^{ex-ante}\left(R_r;\zeta\right)-W^{ex-ante}\left(\underline{R}\left(\zeta\right);\zeta\right)\right]$$

where

$$W^{ex-ante}\left(R;\zeta\right) = cons-\left(1-\alpha\right)\left(\log\left(R\right)+\frac{1}{\left(1-\zeta\right)R}+\tau e_1 R\right)+\left[\beta\left(\rho_1-\rho_0\right)-\left(1-\alpha\right)\left(1-R\right)\right]i\left(R\right).$$

The static component of the resistance condition

$$\log\left(\frac{R_t^e}{R_r}\right)+\frac{1}{1-\zeta}\left(\frac{1}{R_t^e}-\frac{1}{R_r}\right)+\tau e_1\left(R_t^e-R_r\right)$$

is unambiguously decreasing in the tax ζ since $R_t^e \in \left[\underline{R}\left(\zeta\right), R_r\right]$. By the same force, $\underline{R}\left(\zeta\right)$ is increasing in ζ. In turn, $W^{ex-ante}\left(R;\zeta\right)$ is decreasing in ζ. Therefore, the effect of ζ on the penalty to a deviation from R_r is

$$\frac{\partial}{\partial\tau}\wp\left(R_r,\underline{R}\left(\tau\right);\tau\right)=\frac{\delta\left(1-\alpha\right)}{1-\delta}\left\{\left(\frac{1}{\underline{R}\left(\zeta\right)}-\frac{1}{R_r}\right)-\left(R_r-\underline{R}\left(\zeta\right)\right)e_1\right\}+\frac{\delta}{1-\delta}\frac{\partial W^{ex\ ante}\left(\underline{R};\zeta\right)}{\partial\underline{R}}\frac{\partial\underline{R}\left(\zeta\right)}{\partial\zeta}$$

which is ambiguous but negative if households endowment e_1 is large (which is assumed in the paper) and given that $\frac{\partial W^{ex\ ante}(\underline{R};\zeta)}{\partial\underline{R}} > 0$ under the conditions in Proposition 1 and $\frac{\partial\underline{R}(\zeta)}{\partial\zeta} < 0$ as established above. Therefore, the tax ζ reduces the inter-temporal incentives of the authority of carrying out its bailouts plans.

Overall, the best resistant bailout plan R_r^* is increasing in the tax ζ to serve the public debt if reputation concerns by the authority are small (δ is small) and otherwise R_r^* is decreasing in the tax ζ Depending on the sign of this effect is that more or less public debt decreases entrepreneurs' risk taking.

Documentos de Trabajo **Banco Central de Chile** NÚMEROS ANTERIORES La serie de Documentos de Trabajo en versión PDF puede obtenerse gratis en la dirección electrónica: www.bcentral.cl/esp/estpub/estudios/dtbc. Existe la posibilidad de solicitar una copia impresa con un costo de Ch$500 si es dentro de Chile y US$12 si es fuera de Chile. Las solicitudes se pueden hacer por fax: +56 2 26702231 o a través del correo electrónico: bcch@bcentral.cl.	**Working Papers** **Central Bank of Chile** PAST ISSUES Working Papers in PDF format can be downloaded free of charge from: www.bcentral.cl/eng/stdpub/studies/workingpaper. Printed versions can be ordered individually for US$12 per copy (for order inside Chile the charge is Ch$500.) Orders can be placed by fax: +56 2 26702231 or by email: bcch@bcentral.cl.

DTBC – 742
Facing Volatile Capital Flows: The Role of Exchange Rate Flexibility and Foreign Assets
Rodrigo Cifuentes and Alejandro Jara

DTBC – 741
Risk Matters: The Impact of Nominal Uncertainty in Chile
Luis Ceballos y Damián Romero

DTBC – 740
Credit Stabilization through Public Banks: The Case of BancoEstado
Luis Felipe Lagos y Matías Tapia

DTBC – 739
Trade with Asymmetric Information
Robert Hall

DTBC – 738
Commodity Price Cycles and Financial Stability
Carola Moreno, Carlos Saavedra y Bárbara Ulloa

DTBC – 737
Uncovering our Self-imposed Limits: Changes in Loan-to-Value and the Mortgage Market
Daniel Oda y Fernando Sepúlveda

DTBC – 736

The Long-term Divergence Between your CPI an Mine, The Case of Chile
Andrea Bentancor y Pablo Pincheira

DTBC – 735

Probabilidad Clásica de Sobreajuste con Criterios de Información: Estimaciones con Series Macroeconómicas Chilenas
Carlos Medel

DTBC – 734

Vulnerability to Changes in External Financing due to Global Factors
Gabriela Contreras y Francisco Pinto

DTBC – 733

Carry Trade y Turbulencias Cambiarias en el Peso Chileno
José Carreño y Paulo Cox

DTBC – 732

The Yield Curve Information under Unconventional Monetary Policies
Luis Ceballos y Damián Romero

DTBC – 731

Análisis de Flujos en el Mercado Laboral Chileno
Gonzalo Castex y Roberto Gillmore

DTBC – 730

The Changing Nature of Real Exchange Rate Fluctuations. New Evidence for Inflation Targeting Countries
Rodrigo Caputo, Gustavo Leyva y Michael Pedersen

DTBC – 729

Predicción del Empleo Sectorial y Total en Base a Indicadores de Confianza Empresarial
Pablo Pincheira

DTBC – 728

Caracterización del Mercado Laboral en Chile y su Evolución en los Últimos 25 años
Gonzalo Castex y Fabián Sepúlveda

BANCO CENTRAL
DE CHILE