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WICKSELL VERSUS TAYLOR: A QUEST FOR DETERMINANCY AND THE (IR)RELEVANCE OF THE TAYLOR PRINCIPLE[*]

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Abstract

In a new-Keynesian model we compare the determinacy regions of price-level targeting rules (called Wicksellian rules) and Taylor rules. We conclude that Wicksellian rules do not require the Taylor principle to be satisfied to induce determinacy. Moreover, the areas of determinacy are generally large under Wicksellian rules. Our results have two implications. First, estimating Taylor rules when the true rule is Wicksellian may lead to conclude that the equilibrium is indeterminate when this is not the case. Second, by following a Wicksellian rule, the frequency of occurrence of an active zero lower bound on the policy rate is greatly reduced.

Resumen

En un modelo neo Keynesiano, comparamos las regiones de determinación de reglas del nivel de precio (denominadas reglas Wickselianas) y reglas de Taylor. Concluimos que las reglas Wickselianas no requieren del principio de Taylor para inducir determinación. Adicionalmente, las áreas de determinación son generalmente más amplias bajo reglas Wickselianas. Nuestros resultados tienen dos implicancias. Primero, estimar reglas de Taylor cuando la regla verdadera es Wickseliana puede llevar a concluir que el equilibrio es indeterminado cuando este no es el caso. Segundo, siguiendo una regla Wickseliana, la ocurrencia de un límite cero de la tasa de interés se reduce de forma considerable.

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1 Introduction

From the seminal work of Taylor (1993), Taylor rules have received widespread attention in the empirical and theoretical modern literature of monetary policy. The reason behind this is that they are generally considered realistic yet sufficiently simple representations of how the monetary authority conducts policy. In a nutshell, Taylor rules relate in a linear fashion the policy rate to deviations of the inflation rate and a measure of the output gap. An alternative to Taylor rules are Wicksellian rules, in which the policy rate reacts to deviations of the *price level* instead of inflation. These rules have received much less attention in the literature, despite the fact that they may perform better in terms of welfare and are more robust to alternative shock processes than Taylor rules (Giannoni (2010)).

In this paper we study the areas of determinacy that arise when assuming that the central bank follows a Taylor rule or a Wicksellian rule. To this end, we consider the canonical three-equation New Keynesian model, and compute the model solution under the two alternative policy rules. We study several cases that vary in the set of information that the central bank takes into account when implementing each rule. These cases are rules with contemporaneous data, expectations of forward-looking variables, lagged data and a hybrid rule that comprises expectations of inflation and current output.

As it is well known in the literature, Taylor rules are able to induce determinacy [1]. Depending on whether the rule reacts to lagged, contemporaneous, or expected inflation and output, determinacy conditions may change across alternative rules. In general, however, determinacy requires a common constraint for all rules, which is the so-called *Taylor principle*. This principle states that the policy rate should react by more than the increase in inflation. In backward-looking models (Old Keynesian models) the Taylor principle guarantees that the real interest rate increases, dampening demand and inflation. As noted by Cochrane (2011), in New Keynesian models the Taylor principle works in a completely different way. In particular, by raising the policy rate in response to inflation, the central bank induces accelerating inflation or deflation unless inflation today jumps to a particular value. In this latter case, the Taylor principle rules out multiple equilibria and forces forward-looking solutions.

The main findings of this paper are as follows. First, in striking contrast with Taylor rules, Wicksellian rules do not require the Taylor principle to be satisfied in order to generate

[1]See Bullard and Mitra (2002) and Gali (2008) among others.

determinacy. Hence, under price level targeting rules the Taylor principle is irrelevant. This is true regardless of whether the rule reacts to lagged, contemporaneous, or expected inflation and output. Second, determinacy areas (in the policy coefficient space) are, in general, larger under Wicksellian rules. This implies that these rules present the monetary authority with a larger set of possible actions from which to choose in order to set its policy. Finally, under Wicksellian rules the price level, as well as inflation, are uniquely determined given the initial conditions. As a consequence, these rules avoid nonlocal nominal paths or nominal explosions as defined by Cochrane (2011).

The reason for the irrelevance of the Taylor principle has to do with the expectations of future movements in prices that Wicksellian rules entail. In particular, suppose that a cost-push shock creates inflationary pressures that drive the price level above its long-run equilibrium. Under a Wicksellian rule, agents know that inflation today will be counteracted by a deflation in future periods, in order for the price level to return to its target level. If the Taylor principle is not satisfied, the initial increase in the policy rate is of a lower magnitude than the increase in the inflation rate. However, expectations of a future deflation are responsible for driving the real interest rate up, and thus stabilizing the economy. In this case, it is expectations of future movements in prices, and not the reaction of the policy rate, what drives the real interest rate in the right direction. This is the reason why, in this case, the Taylor principle is not a necessary condition to achieve determinacy.

We emphasize the importance of our results by highlighting two situations in which the irrelevance of the Taylor principle for Wicksellian rules is crucial. In the first exercise, we draw attention to the problems that may arise when estimating Taylor rules to empirically characterize monetary policy, when the true underlying policy governing monetary policy is of the Wicksellian type. To this end, we simulate the economy characterized by a standard New Keynesian model with a simple Wicksellian rule. Then, we estimate a simple Taylor rule. We perform this exercise for several values of the Wicksellian rule such that the system is always determinate. We find that, when the coefficient governing the reaction of the policy rule to deviations of the price level is sufficiently small, the estimated Taylor rule displays a coefficient for inflation less than unity, thus violating the Taylor principle. This result rings a warning bell for empirical economists that use estimations of Taylor rules to extract conclusions about the nature of monetary policy in different time periods. In particular, our result suggest that, from Taylor rules estimates, it not possible to infer whether the central bank has moved from an "indeterminate" to a "determinate" regime, as suggested by Clarida et al. (2000).

In light of recent events, it is now clear that the problem of a binding zero lower bound is no longer a theoretical curiosity, but a relevant policy issue. The second situation we consider refers to the probability of occurrence of a binding zero lower bound on the policy rate associated to Taylor and Wicksellian rules. We argue that, given that the Taylor principle does not need to be satisfied under the Wicksellian rule, the policy response to deviations of the price level can be as small as desired. This gives great scope to the monetary authority to stabilize the economy without running the risk of hitting the lower bound of the policy rate. It is in this sense that a Wicksellian rule, by not requiring the Taylor principle to hold, reduces drastically the probability of hitting the zero lower bound[2].

There are a number of papers that study determinacy under different modelling assumptions. Bullard and Mitra (2002) study the determinacy and learnability properties that different specifications of a Taylor rule provide to a standard New Keynesian model. Bullard and Mitra (2007) extend the analysis to frameworks in which there is inertia in the policy rule. They find that monetary policy inertia improves learnability of a specified model. Ascari and Ropele (2009) analyze the case in which there is a positive trend inflation in steady state, instead of a zero steady state inflation as most models assume. Their results show that trend inflation shrinks the determinacy area in a New Keynesian model where policy is conducted through a contemporaneous rule and the Taylor principle is no longer a sufficient condition to ensure determinacy. In the spirit of Bullard and Mitra (2002), Llosa and Tuesta (2008) study the determinacy and learnability properties of a New Keynesian small open economy model. Dittmar and Gavin (2005) examine the areas of determinacy of flexible price RBC models with interest rate rules that target inflation and the price level. They find that, while inflation targeting often renders the system indeterminate, a price level target generally delivers determinacy. The exercise we perform in this paper is close in nature to the one of Bullard and Mitra (2002) since our specifications of the Taylor rule are inspired by theirs. We do not, however, address issues of learnability of different rules. We focus instead in the comparison between determinacy areas of Wicksellian and Taylor rules. We extend the results of Dittmar and Gavin (2005) and introduce nominal rigidities by analyzing a simple New Keynesian close economy model with zero steady state

[2] We are not taking into consideration in the analysis the issue of whether a rule that entails a low response of the policy rate to deviations of the price level is desirable when the economy is far from hitting the zero lower bound. This is a potentially very relevant question that exceeds the scope of the paper, so we leave it for future research.

inflation.

As we mentioned before, Wicksellian rules have not received broad attention in the literature, although there are some notable exceptions. Gorodnichenko and Shapiro (2007) show that a price level target can be understood as a commitment device to offset errors made by the central bank, and can therefore anchor inflation when the public and the central bank's optimism about the economy differ. Rules that target the price level are, consequently, superior to rules that target inflation. In a related contribution, Giannoni (2010) compares the performance of Taylor and Wicksellian rules in a simple New Keynesian model in terms of welfare and robustness to different specifications of shocks. He finds that, in general, optimal Wicksellian rules perform better than optimal Taylor rules in terms of welfare, and in addition are more robust to alternative shock processes. He only considers contemporaneous rules and, although he characterizes the determinacy area of Wicksellian contemporaneous rules, he does not relate his results to the irrelevance of the Taylor principle, as we do in this paper. Contrary to these papers, we do not seek to evaluate different rules in terms of their welfare implications since the scope of our analysis is mainly positive rather than normative.

The paper is organized as follows. In Section 2 we set up the model and characterize the determinacy regions of Taylor and Wicksellian rules that react to contemporaneous information, expectations of forward-looking variables or a combination of these. Section 3 describes, by means of an example, the mechanism behind the result that the Taylor principle is irrelevant when characterizing determinacy of systems with Wicksellian rules. In Section 4 we discuss two implications that arise from the irrelevance of the Taylor principle when policy is characterized by a Wicksellian rule. Section 5 concludes.

2 Determinacy regions in a New Keynesian model

The scope of this section is to determine the parameter regions for which a (linearized) standard monetary policy model has a unique solution, under a particular specification of the monetary policy rule followed by the central bank. We focus our attention in areas where a unique solution exists because areas where there are infinite solutions are areas in which some undesirable outcomes may arise, such as sunspots and equilibria in which fluctuations in inflation and the output gap are driven by self-fulfilling expectations. To this end, we study determinacy properties of different specifications of monetary policy rules in the standard workhorse New Keynesian model analyzed by, among many, Clarida et al.

(1999) and Woodford (2003).

The model economy can be represented by two equations, one intertemporal IS equation that is obtained from the intertemporal optimality condition of households, and a forward-looking New Keynesian Phillips curve that summarizes the optimal pricing decision of monopolistically competitive producers that cannot adjust prices every period.

The IS equation is

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^e), \tag{1}$$

where $x_t = (y_t - y_t^N)$ is the output gap defined as the difference between the level of output, y_t, and its flexible price level, y_t^N, π_t is inflation in period t, i_t is the nominal interest rate, r_t^e is the *natural rate of interest*, or the real interest rate of the flexible price economy, and σ is the inverse of the intertemporal elasticity of substitution. Each variable is represented as a percentage deviation from its steady state value.

The New-Keynesian Phillips curve (NKPC) reads

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t, \tag{2}$$

where κ is related to the frequency of price-adjustment by producers, β is the discount factor and u_t is a cost push shock that introduces a trade-off in the policy maker's decision problem[3].

To close the model, we need to specify a monetary policy rule to provide a specification for the nominal interest rate. In what follows we consider simple rules, that is, monetary policy rules that depend on variables observed by the central bank and that, consequently, should be feasible to adopt. We compare two different types of rules: Taylor rules and Wicksellian rules. Taylor rules have been widely analyzed in both the theoretical and empirical literature of monetary policy. Under this type of rules, the nominal interest rate reacts to deviations of the output gap and the inflation rate from specified target values. Wicksellian rules, on the other hand, have received much less attention in the literature of simple rules[4]. A Wicksellian rule implies that the nominal interest rate reacts to deviations of the output gap

[3] The shock u_t represents, for example, changes is distortionary taxes or exogenous variations in the degree of market power of firms. More generally, this shock may be interpreted as the difference between the efficient level of output, y_t^E, and its natural level, y_t^N. The inclusion of such shocks does not alter the results on determinacy of different monetary policy rules.

[4] Some exceptions are Giannoni (2010) and Dittmar and Gavin (2005).

and the *price level* from target values.

In the next sections, we study the determinacy properties of different specifications for Taylor and Wicksellian rules. In particular, we follow Bullard and Mitra (2002) and consider interest rate equations that react to forward expectations and contemporaneous data. We incorporate in the analysis an additional rule, which we call a *hybrid* rule, that reacts to expected inflation and contemporaneous data. This rule has not received too much attention in the theoretical literature despite the fact that it has been used extensively in empirical research[5]. In the Appendix we study the empirically less relevant case of rules with lagged data.

When possible, we analytically characterize the regions of determinacy of different rules. In all cases we illustrate our results by means of numerical examples. We follow the calibration by Woodford (1999) and Bullard and Mitra (2002) and set $\beta = 0.99$, $\sigma = 0.157$ and $\kappa = 0.024$.

2.1 Contemporaneous rules

Contemporaneous rules are the most common type of rules considered in the literature due to their simplicity, both for theoretical and empirical purposes. From the seminal contribution of Taylor (1993), papers such as Adolfson (2007), Lubik and Schorfheide (2007) and Aizenman et al. (2011) have estimated this type of rules, and many papers and some books, of which Gali (2008) and Gali and Monacelli (2005) are notable examples, have assessed the behavior of such rules in theoretical frameworks.

The contemporaneous Taylor rule we consider is

$$i_t = \varphi_\pi \pi_t + \varphi_x x_t \tag{3}$$

where $\varphi_\pi > 0$ and $\varphi_x > 0$ represent the reaction of the nominal interest rate to deviations of the output gap and the inflation rate from target values[6] [7].

[5]See Clarida et al. (1998), Clarida (2001), Engel and West (2006) and Adolfson et al. (2011) among others.

[6]These coefficients can be determined so as to minimize the loss function of the central bank. In this case, we say that the simple rule is optimal, in the sense that it is the best possible rule given the restriction imposed by the specific functional form of the rule we are assuming. This "optimal simple rule", however, will in general not implement the first best allocation.

[7]For simplicity, we are assuming that the target values for inflation and the output gap are zero. Considering any other constant target would not change the results on determinacy.

The system of equations (1), (2) and (3) can be rewritten as

$$y_t = BE_t y_{t+1} + Cz_t,$$

where $y_t = [x_t; \pi_t]$, $z_t = [r_t^e; u_t]$ and

$$B = \frac{1}{\sigma + \varphi_\pi \kappa + \varphi_x} \begin{pmatrix} \sigma & 1 - \beta\varphi_\pi \\ \kappa\sigma & \kappa + \beta(\sigma + \varphi_x) \end{pmatrix}.$$

This system has two endogenous non-predermined variables. Thus, following Blanchard and Kahn (1980), for the equilibrium to be determinate, that is, for the system to have a unique solution, it is required that the the matrix B has two eigenvalues inside the unitary circle. If B has less than two eigenvalues inside the unitary circle, the system is said to be indeterminate, and there exist an infinity of solutions[8].

The determinacy properties of this rule have already been studied in Bullard and Mitra (2002). We reproduce their results for comparison purposes.

Proposition 1. *Under contemporaneous Taylor rules as (3) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that*

$$\kappa(\varphi_\pi - 1) + (1 - \beta)\varphi_x > 0. \tag{4}$$

Proof. See Bullard and Mitra (2002). □

The determinacy conditions under a contemporaneous Taylor rule require that the *Taylor principle* is satisfied. Following Bullard and Mitra (2002), to interpret this condition, let us rewrite equation (4) as

$$\varphi_\pi + \frac{1 - \beta}{\kappa}\varphi_x > 1.$$

Notice that, from the NKPC (2), a one percent permanent change in inflation implies that the output gap increases by $(1 - \beta)/\kappa$, under the assumption that $u_t = 0$. Then, condition (4) states that the increase in the nominal interest rate given a permanent one percent increase in inflation should be higher than one percent. In other words, the nominal interest rate should react by more than the increase in permanent inflation. This guarantees

[8]If the system is indeterminate, one cannot exclude the presence of solutions that encompass bubbles and history dependence.

that the *real interest rate* increases as well, therefore dampening aggregate demand through the IS equation (1) and, consequently, reducing the output gap and inflation.

The contemporaneous Wicksellian rule is

$$i_t = \psi_p p_t + \psi_x x_t, \tag{5}$$

where $\psi_p > 0$ and $\psi_x > 0$ represent the magnitude of the reaction of the nominal interest rate to deviations of the output gap and the price level [9] [10].

The system of equations (1), (2) and (11) can be written as [11]

$$y_t = BE_t y_{t+1} + Cz_t,$$

where $y_t = [x_t; \pi_t; p_{t-1}]$, $z_t = [r_t^e; u_t]$ and

$$B = \begin{pmatrix} \frac{\sigma\beta+\beta\psi_x+\kappa}{\sigma\beta} & \frac{\beta\psi_p-1}{\sigma\beta} & \frac{\psi_p}{\sigma} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{pmatrix}^{-1}. \tag{6}$$

This system has two endogenous non-predermined variables, x_t and π_t, and one endogenous predetermined variable, p_{t-1}. Thus, following Blanchard and Kahn (1980), for the equilibrium to be determinate, that is, for the system to have a unique solution, it is required that the matrix B has two eigenvalues inside the unitary circle and one outside it. If B has less than two eigenvalues inside the unitary circle, the system is said to be indeterminate. If, on the other hand, B has more than two eigenvalues inside the unitary circle, the system is said to be explosive.

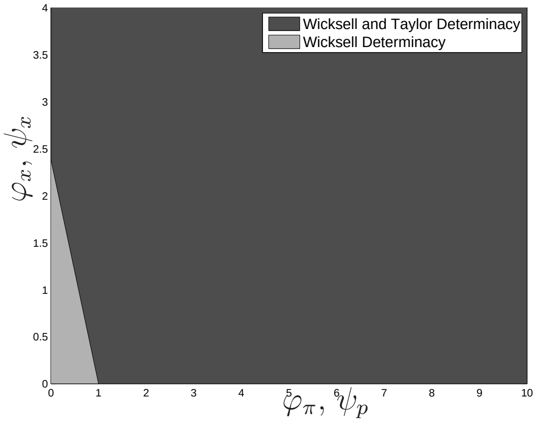
The following proposition states that the necessary and sufficient conditions for determinacy when the rule (5) is implemented are that the central bank increases the nominal interest rate when inflation or the output gap are positive. This proposition is already

[9] Notice that, as we are working with variables expressed in deviations from the steady state, the variable p_t can be interpreted as the deviation of the price level from a predetermined target.

[10] We are implicitly assuming that the steady state of this economy is an undistorted one, i.e., inflation and the output gap are zero. If we considered a model with a different steady state, we would need to define the Wicksellian rule in terms of the deviation of the price level with respect to some deterministic trend for the price level. See Giannoni (2010) for an example.

[11] If the system had a different steady state where $\pi^{ss} \neq 0$ and/or $x^{ss} \neq 0$, we could still write it in terms of deviations from the steady state. The matrices $\tilde{B}$ and $\tilde{C}$ would be unchanged and the results that follow would still hold.

Figure 1: Determinacy areas of contemporaneous rules



Note: The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The lighter area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate.

established in Giannoni (2010).

Proposition 2. *Under contemporaneous Wicksellian rules such as* (5) *all possible values of* $\psi_p > 0$ *and* $\psi_x > 0$ *yield a rational expectations equilibrium that is unique (determinate).*

Proof. See Appendix. □

The following corollary summarizes the results for Taylor and Wicksellian rules.

Corollary 1. *Under contemporaneous Wicksellian rules the coefficient* ψ_p *need not satisfy the Taylor principle, as it must be the case for* φ_π *in the Taylor rule. Moreover, the area of determinacy is larger under contemporaneous Wicksellian rules than under contemporaneous Taylor rules.*

Proof. Compare Propositions 1 and 2. □

Notice that a comparison between the areas of determinacy in the space of coefficients $\{\varphi_x, \varphi_\pi\}$ and $\{\psi_x, \psi_p\}$ is not immediate, because φ_π and ψ_p are coefficients accompanying different variables. They are, however, comparable in the following sense: consider shocks that cause inflation and the output gap to react on impact in the same magnitude under both types of rules, then $\varphi_\pi = \psi_p$ implies the same reaction of the nominal interest rate *on impact* to such shocks. Then, a larger determinacy area for combinations of the coefficients of the Wicksellian rule translates into a larger set of actions that the monetary authority can take following a shock that causes a given initial reaction of inflation and the output gap, compared to the case in which the monetary authority follows a Taylor rule.

We illustrate the analytical results previously depicted by means of a numerical example. Figure 1 shows the determinacy and indeterminacy areas of contemporaneous Taylor and Wicksellian rules for different combinations of parameters $\{\varphi_x, \varphi_\pi\}$ and $\{\psi_x, \psi_p\}$. The darker area in the figure corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which both systems are determinate. The lighter area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate.

The conclusions highlighted in Corollary 1 are clearly illustrated in Figure 1. The lighter shaded area corresponds to the area where the Taylor principle is not satisfied by the Taylor rule, thus the system is indeterminate. However, the Wicksellian rule yields determinacy in this area. Moreover, the area of determinacy for the Wicksellian rule is undoubtedly larger than the one of the Taylor rule.

2.2 Forward-looking rules

In line with Clarida et al. (2000), Woodford (2000) and Svensson and Woodford (2004) we specify purely forward-looking rules in which the monetary authority's actions (policy rate) at any time are conditioned solely upon forecasts of its target variables. In particular, we consider a forward-looking Taylor rule in which the policy rate i_t reacts to expected inflation, $E_t\pi_{t+1}$, and expected output gap, $E_t x_{t+1}$:

$$i_t = \varphi_\pi E_t\pi_{t+1} + \varphi_x E_t x_{t+1}, \tag{7}$$

where $\varphi_\pi > 0$ and $\varphi_x > 0$ represent the reaction of the nominal interest rate to deviations of expectations of the output gap and the inflation rate from target values.

The system formed by equations (1), (2) and (7) can be rewritten as

$$y_t = BE_t y_{t+1} + Cz_t,$$

where $y_t = [x_t; \pi_t]$, $z_t = [r_t^e; u_t]$ and

$$B = \begin{pmatrix} \frac{\sigma-\varphi_x}{\sigma} & \frac{1-\varphi_\pi}{\sigma} \\ \kappa\frac{\sigma-\varphi_x}{\sigma} & \kappa\frac{1-\varphi_\pi}{\sigma}+\beta \end{pmatrix}.$$

This system has two endogenous non-predetermined variables. Hence, for the equilibrium to be determinate, that is, for the system to have a unique solution, it is required that the the matrix B has two eigenvalues inside the unitary circle. As before, if B has less than two eigenvalues inside the unitary circle, the system is said to be indeterminate, and there exist an infinity of solutions. [12]

The determinacy properties of this rule have already been studied in Bullard and Mitra (2002). We reproduce their result here in order to compare the determinacy conditions of Taylor and Wicksellian rules.

Proposition 3. *Under forward-looking Taylor rules such as (7) the necessary and sufficient conditions for a rational expectations equilibrium to be unique (determinate) are that* [13]

$$\kappa(\varphi_\pi - 1) + (1+\beta)\varphi_x < 2\sigma(1+\beta), \tag{9}$$

and

$$\kappa(\varphi_\pi - 1) + (1-\beta)\varphi_x > 0. \tag{10}$$

Proof. See Bullard and Mitra (2002). □

Notice that this rule requires the *Taylor principle*, equation (10), to be satisfied. This principle implies that the equilibrium, under rule (7), will be unique if φ_π and φ_z are large enough to guarantee that the real rate eventually rises in the face of an increase in inflation.

Now, a forward-looking Wicksellian rule can be specified as follows

$$i_t = \psi_p E_t p_{t+1} + \psi_x E_t x_{t+1}, \tag{11}$$

where $\psi_p > 0$ and $\psi_x > 0$ represent the magnitude of the reaction of the nominal interest rate to deviations of expectations of the price level and to expectations of the output gap,

[12] See Blanchard and Kahn (1980)

[13] Bullard and Mitra (2002) list as an additional condition

$$\varphi_x < \sigma(1+\beta^{-1}). \tag{8}$$

It can be easily shown that (9) and (10) imply (8), so we do not include it as a necessary and sufficient condition.

respectively [14].

The system formed by equations (1), (2) and (11) can be written as

$$y_t = BE_t y_{t+1} + Cz_t,$$

where $y_t = [x_t; \pi_t; p_{t-1}]$, $z_t = [r_t^e; u_t]$ and

$$B = \begin{pmatrix} \frac{\sigma\beta+\kappa(1-\psi_p)}{(\sigma-\psi_x)\beta} & -\frac{1-\psi_p(1+\beta)}{(\sigma-\psi_x)\beta} & \frac{\psi_p}{\sigma-\psi_x} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{pmatrix}^{-1}. \tag{12}$$

This system has two endogenous non-predetermined variables, x_t and π_t, and one endogenous predetermined variable, p_{t-1}. As before, for the equilibrium to be determinate, that is, for the system to have a unique solution, it is required that the the matrix B has two eigenvalues inside the unitary circle and one outside it.

The following proposition characterizes the necessary and sufficient conditions for determinacy when rule (11) is implemented.

Proposition 4. *Under forward-looking Wicksellian rules such as (11) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that*

$$\kappa(\psi_p - 2) + 2(1+\beta)\psi_x < 4\sigma(1+\beta). \tag{13}$$

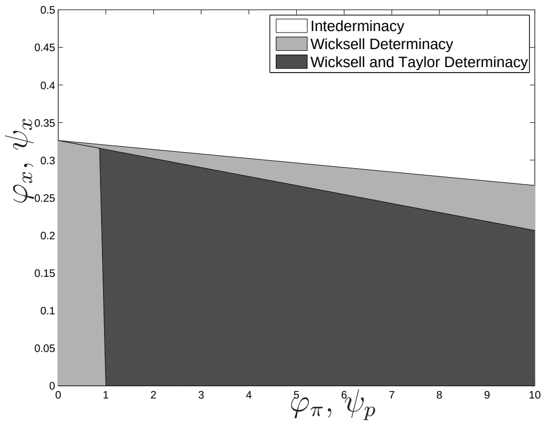
Proof. See Appendix A.3 . □

A natural corollary of the previous proposition is that the Taylor principle is not a condition for determinacy and that the area of determinacy is larger under forward-looking Wicksellian rules than under forward-looking Taylor rules:

Corollary 2. *Under forward-looking Wicksellian rules the coefficient ψ_p need not satisfy the Taylor principle, as it must be the case for φ_π in the Taylor rule. Moreover, the area of indeterminacy is larger under forward-looking Wicksellian rules than under forward-looking Taylor rules.*

[14]Once again, we explicitly include a target price level $\bar{p}$ in the specification of the rule. The results of this section do not depend on the value assumed for this parameter.

Figure 2: Determinacy areas of forward-looking rules



Note: The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate. The non-shaded area corresponds to corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which both systems are indeterminate.

Proof. Compare conditions (9) and (10) to condition (13). Notice that condition (9) can be written as

$$\varphi_\pi < \frac{2\sigma(1+\beta) - (1+\beta)\varphi_x}{\kappa} + 1,$$

while condition (13) can be written as

$$\psi_p < 2\left(\frac{2\sigma(1+\beta) - (1+\beta)\psi_x}{\kappa} + 1\right).$$

□

The Wicksellian rule does not need to satisfy the Taylor principle, condition (10), as in rule (7). In fact, as can be seen from (13), relatively "weak" responses to both price and output deviations are able to induce determinacy. In particular, in the limiting case in which $\psi_x = 0$, determinacy is achieved by an arbitrarily small and positive ψ_p coefficient.

Figure 2 shows the determinacy and indeterminacy areas of forward-looking Taylor and Wicksellian rules for different combinations of parameters $\{\varphi_x, \varphi_\pi\}$ and $\{\psi_x, \psi_p\}$. The darker area in the figure corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which both systems are determinate. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate. The non-shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which both systems are indeterminate.

It is clear from Figure 2 that the region of the Taylor principle (the light shaded region to the right of the dark shaded one) implies determinacy for the Wicksellian rule but indeterminacy for the Taylor one. Also, the larger region of determinacy for a Wicksellian rule in comparison to a Taylor rule translates into a larger area from which the monetary authority can choose its response to inflation and output gap, given a shock.

2.3 Hybrid rules

In many empirical exercises[15] Taylor rules are estimated using a specification in which the policy instrument reacts to expected inflation and contemporaneous output (and some other external variables in the case of open economies). This type of rules can be expressed as:

$$i_t = \varphi_\pi E_t \pi_{t+1} + \varphi_x x_t, \tag{14}$$

where $\varphi_\pi > 0$ and $\varphi_x > 0$.

The rationale for this specification is that expected inflation, rather than the contemporaneous level of it, could be influenced by monetary policy. This type of rule is usually refered in the literature as a forward-looking Taylor rule. In order to distinguish this specification from the pure forward-looking one considered in the previous section, we call it a *hybrid* Taylor rule.

The system formed by equations (1), (2) and (14) can be rewritten as

$$y_t = B E_t y_{t+1} + C z_t,$$

where $y_t = [x_t; \pi_t]$, $z_t = [r_t^e; u_t]$ and

$$B = \frac{1}{\sigma + \varphi_x} \begin{pmatrix} \sigma & 1 - \varphi_\pi \\ \kappa\sigma & \beta(\sigma + \varphi_x) + \kappa(1 - \varphi_\pi) \end{pmatrix}. \tag{15}$$

This system has two endogenous non-predetermined variables. Thus, for the equilibrium

[15] See Clarida et al. (1998), Clarida (2001), Engel and West (2006) and Adolfson et al. (2011) among others.

to be determinate, it is required that the the matrix B has two eigenvalues inside the unitary circle.

To our knowledge, this is the first paper to derive the determinacy properties of this rule, which are summarized in the following proposition.

Proposition 5. *Under hybrid Taylor rules such as (14) the necessary and sufficient conditions for a rational expectations equilibrium to be unique (determinate) are that*

$$\kappa(\varphi_\pi - 1) - (1+\beta)\varphi_x < 2\sigma(1+\beta) \tag{16}$$

and

$$\kappa(\varphi_\pi - 1) + (1-\beta)\varphi_x > 0. \tag{17}$$

Proof. See Appendix A.4. □

Notice that in the case of the hybrid Taylor rule, determinacy also requires the Taylor principle to be satisfied (conditions (10) and (17) are identical). However, comparing (9) and (16), it is easy to see that the previous conditions imply a larger area of determinacy for the hybrid rule than for the forward-looking one, but a smaller one than for the contemporaneous rule. This is to be expected, as this rule uses a combination of contemporaneous data and expectations of future variables.

A hybrid Wicksellian rule can be defined as:

$$i_t = \psi_p E_t p_{t+1} + \psi_x x_t, \tag{18}$$

where $\psi_p > 0$ and $\psi_x > 0$.

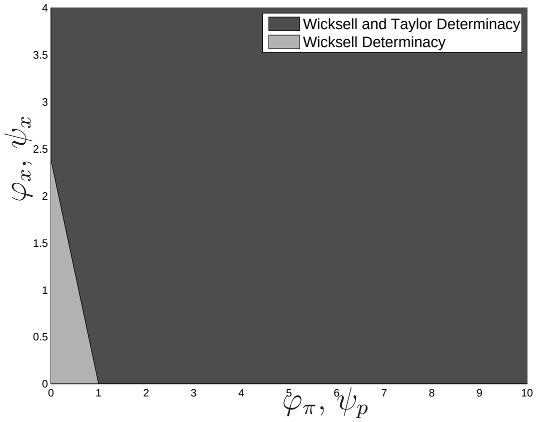
As before, the system composed by equations (1), (2) and (18) can be cast in the form:

$$y_t = BE_t y_{t+1} + Cz_t,$$

where $y_t = [x_t; \pi_t; p_{t-1}]$, $z_t = [r_t^e; u_t]$. In this particular case, the $\tilde{B}$ matrix takes de form:

$$B = \begin{pmatrix} \frac{\sigma\beta+\kappa(1-\psi_p)+\beta\psi_x}{\sigma\beta} & -\frac{1-\psi_p(1+\beta)}{\sigma\beta} & \frac{\psi_p}{\sigma} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{pmatrix}^{-1}. \tag{19}$$

Figure 3: Determinacy areas of hybrid rules



Note: The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate.

This system has two endogenous non-predetermined variables, x_t and π_t, and one endogenous predetermined variable, p_{t-1}. Thus, as in the case of the pure forward-looking Wickesellian rule analyzed previously, for the equilibrium to be determinate, that is, for the system to have a unique solution, it is required that the the matrix B has two eigenvalues inside the unitary circle and one outside it.

The following proposition characterizes the necessary and sufficient conditions for determinacy when the rule (18) is implemented.

Proposition 6. *Under hybrid Wicksellian rules such as (18) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that*

$$\kappa(\psi_p - 2) - 2(1+\beta)\psi_x < 4\sigma(1+\beta). \tag{20}$$

Proof. See Appendix A.5 . □

Again, a natural corollary of the previous proposition is that the Wicksellian rule does not require the Taylor principle to be satisfied, and that the area of determinacy is larger under hybrid Wicksellian rules than under hybrid Taylor rules:

Corollary 3. *Under hybrid Wicksellian rules the coefficient ψ_p need not satisfy the Taylor principle, as it must be the case for φ_π in the Taylor rule. Moreover, the area of indeterminacy is larger under hybrid Wicksellian rules than under hybrid Taylor rules.*

Proof. Compare conditions (16) and (17) to condition (20). Notice that condition (16) can be written as

$$\varphi_\pi < \frac{2\sigma(1+\beta) + (1+\beta)\varphi_x}{\kappa} + 1,$$

while condition (20) can be written as

$$\psi_p < 2\left(\frac{2\sigma(1+\beta) + (1+\beta)\psi_x}{\kappa} + 1\right).$$

□

As in the case of forward-looking rules, the area of determinacy for ψ_p, under the hybrid Wicksellian rule (second inequality) is larger than the area of determinacy for φ_π under the hybrid Taylor rule (first inequality).

Figure 3 shows the determinacy areas of hybrid Taylor and Wicksellian rules. The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate. Notice that for the configuration of parameters that we are considering, Figure 3 is identical to Figure 1.

The last type of rules we consider are backward-looking Taylor and Wicksellian rules. Since these rules have received less attention in the empirical and theoretical monetary policy literature, we defer the analysis to Appendix A.6.

2.4 Comments on the solution

The New Keynesian model with a Taylor rule is such that the dynamics of the endogenous variables, $y_t = [x_t; \pi_t]$, is a function of the exogenous shocks, r_t^e and u_t

$$y_t = ar_t^e + bu_t$$

Under this solution there is no mechanism to pin down inflation or the price level. Furthermore, the fact that the Taylor principle holds ensures that inflation jumps to the unique value that leads to a locally bounded equilibrium path. As noted by Cochrane (2011), in this context there is no economic reason why the economy should pick this path.

Now, in the case of the model with a Wicksellian rule, there is an additional endogenous predetermined variable, p_{t-1}, so the system has a solution of the form

$$y_t = ar_t^e + bu_t + cp_{t-1}$$

where $y_t = [x_t; \pi_t; p_t]$. In the case of the Wicksellian rule, the element of vector c that corresponds to the price level is smaller than one, that is

$$p_t = a_p r_t^e + b_p u_t + c_p p_{t-1}$$

Then, the price level is uniquely determined by this equation given that the initial condition for the price level and the law of motion for prices determines a non-explosive path. As a consequence, this Wicksellian rules avoid nonlocal nominal paths or nominal explosions as defined by Cochrane (2011).

3 The Irrelevance of the Taylor Principle: mechanism

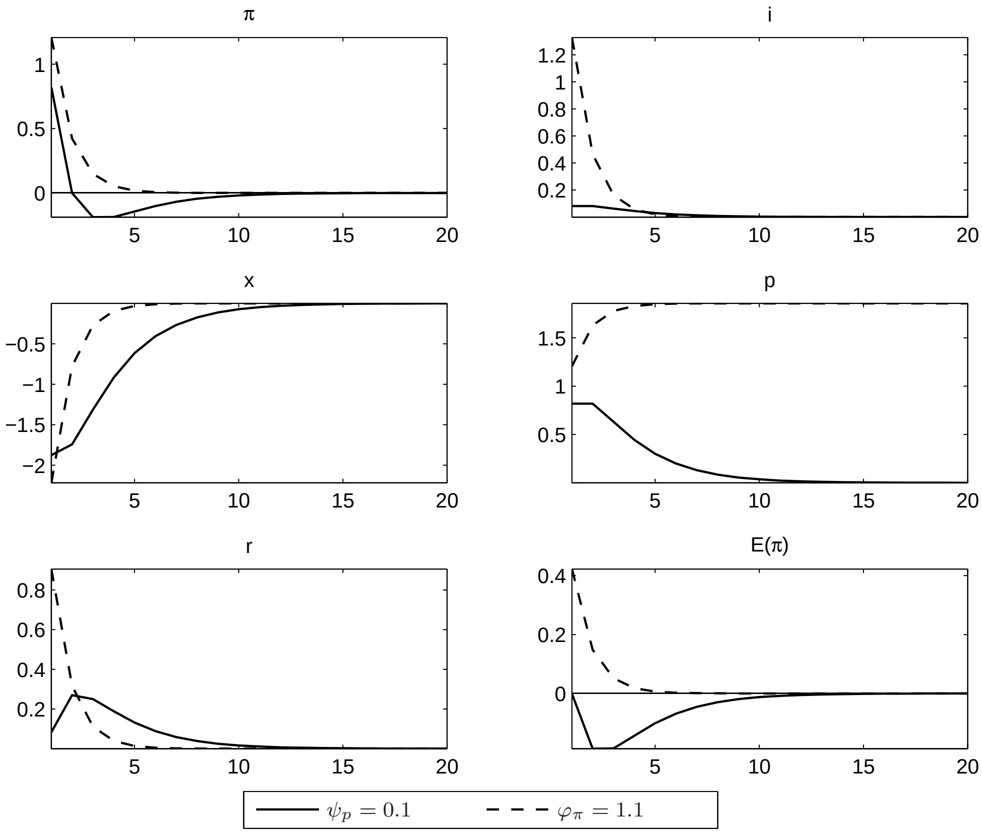
As shown in the previous section, Wicksellian rules do not require the Taylor principle to be satisfied. To understand the mechanism through which determinacy is achieved, even without satisfying the Taylor principle, we perform a simple exercise: we simulate the dynamic response of the economy, characterized by (1) and (2), in the face of a cost push shock, under two alternative instrument rules. The first one is a simple Wicksellian rule of the form:

$$i_t = \psi_p p_t \tag{21}$$

where we have excluded, for simplicity, a response to the output gap (hence $\psi_x = 0$). We impose a small policy response to the price level, $\psi_p = 0.1$, which ensures determinacy but implies, on impact, that the Taylor principle is not satisfied. In fact, in this case the nominal interest rate moves less than one to one with inflation.

The second rule we consider is a simple Taylor rule which only reacts to contemporaneous inflation. This rule takes the form:

Figure 4: Responses to a 1% Cost Push Shock (in percentage deviation from steady state)



$$i_t = \varphi_\pi \pi_t \tag{22}$$

where the policy response to inflation is marginally greater than one, $\varphi_\pi = 1.1$, so determinacy conditions (and the Taylor principle) are satisfied.

The response of the economy to a cost push shock is presented in Figure 4. In the case of the simple Taylor rule, as expected, the interest rate increases, on impact, more than inflation (contemporaneous and expected). As a consequence, the real ex-ante interest rate increases, pushing down the output gap. The decline in the output gap reduces marginal costs, inducing a decline in inflation over time. This is the standard transmission mechanism in basic New Keynesian models when there is a cost-push shock: the central bank moves the policy rate in order to induce movements in the real rate and in the output gap, that in the end stabilize the inflation rate. In this case, if a Taylor rule is followed, the only way in which the required increase in the real rate (and decline in output) could be implemented is through an increase in the nominal rate above inflation.

In the case of the Wicksellian rule in (21) the transmission mechanism is quite different. The cost push shock generates, on impact, an increase in the price level and in inflation of the same magnitude. Given the fact that $\psi_p = 0.1$, it follows that the increase in the nominal rate is well below the increase in inflation (see top row of Figure 4). Hence, on impact, the Taylor principle is not satisfied. In subsequent periods, the policy rate remains above its steady state level as long as the price level does not adjust to its initial value. To induce this adjustment, inflation should decline which, in turn, requires a contraction in the output gap. This contraction is only possible if the real rate increases. Now, in this particular case, the increase in the real rate is driven, mostly, by expected future deflation. As a consequence, determinacy is achieved despite the fact that the Taylor principle is not satisfied [16].

4 The irrelevance of the Taylor principle: practical implications

In previous sections we have shown that the Taylor principle is irrelevant to characterize areas of determinacy when monetary policy is conducted following a Wicksellian rule. This is important because it endows the monetary authority with a larger set of policy actions, compared to a situation in which the central bank follows a Taylor rule.

In this section, we discuss two practical implications related to the irrelevance of the Taylor principle for Wicksellian rules. The first refers to problems that may arise when estimating Taylor rules to empirically characterize monetary policy, when the true underlying policy governing monetary policy is of the Wicksellian type. Our main objective is to determine the extent to which misleading conclusions about the nature of monetary policy may be drawn in this context. In particular, we show that, if the coefficient ψ_p associated to deviations of the price level in a Wicksellian rule is sufficiently small, the researcher can mistakenly conclude that the estimated Taylor rule implies that the equilibrium is indeterminate, when in fact this is not the case. This will also happen in hybrid rules in which the policy coefficients, ψ_p and ψ_p are different from zero.

[16] After the initial period, the policy rate is slightly above its steady state level, whereas inflation is systematically below its long-run value. As a consequence, there is a negative relationship between the interest rate and inflation after the initial period, so the Taylor principle is not satisfied in subsequent periods.

Second, we describe the importance of Wicksellian rules in relation to the problem of a zero lower bound on the policy rate. In addition to the well-known benefits of adopting price-level targeting rules when the bound is active [17], we assess the relative advantage of Wicksellian rules to prevent the occurrence of such bound. More specifically, given that the Taylor principle need not hold in order to achieve determinacy with a Wicksellian rule, the response of the policy rate can be as small as desired when the economy is hit by an inflationary (deflationary) shock: ψ_p can be arbitrarily small, as long as it is positive. Thus, for a small value of ψ_p, the frequency of occurrence of an active zero lower bound may be greatly diminished with respect to a case in which the Taylor rule is in place.

4.1 Pitfalls of estimated Taylor rules

There are several studies that characterize the nature of monetary policy through the estimation of simple Taylor rules. Taylor (1999) estimates Taylor rules for different periods of the US monetary policy history. He concludes that in the 1960s and 1970s the US experienced excessive monetary policy ease that lead to high levels of inflation around that period. For a set of industrialized economies, Clarida et al. (1998) estimate hybrid Taylor rules where, besides reacting to expected inflation and output, the central bank also responds to foreign variables (exchange rates and foreign interest rate). In a subsequent study, Clarida et al. (2000) estimate forward-looking Taylor rules for the USA before and after the Volcker-Greenspan era. They conclude that the policy response to expected inflation in the Volcker-Greenspan era appears to have been more much sensitive to changes in expected inflation than in the pre-Volcker period. Furthermore, they find that the point estimates for the inflation and output gap coefficients during the pre-Volcker era fall within the indeterminacy region of a canonical model they use to characterize the USA economy. As a consequence, they conclude that the Volcker-Greenspan rule is stabilizing when compared to rules in the previous period. Lubik and Schorfheide (2004) use bayesian techniques to show how the likelihood-based estimation of dynamic stochastic general equilibrium models can be extended to allow for indeterminacies and sunspot fluctuations. They apply this methodology to estimate a New Keynesian model of the US economy, and conclude, as Clarida et al. (2000), that monetary policy in the pre-Volker era was not consistent with the model being determinate. Boivin (2006) estimates forward-looking Taylor rules with drifting coefficients

[17] See Evans (2012).

for the US using real-time data and accounting for the presence of heteroskedasticity in the policy shock. Again, he finds that the Fed's response to the real-time forecast of inflation was weak in the second half of the 1970's, which implies that it is possible that the Taylor principle was not satisfied during this period.

These results seem to suggest that high inflation and output volatility in the pre-Volker era are due, to a large extent, to bad policy, in the sense that the monetary authority followed an accommodative policy that failed to increase the real interest rate in the presence of shocks leading to inflationary pressures. This timid reaction of the monetary authority to inflation implied that the economy transited a region of indeterminacy that exposed the economy to sunspots and undesired volatile paths for inflation, output and employment.

Several authors have challenged this view by suggesting that the conclusions extracted from the estimation of Taylor rules before 1980 may be misleading. Orphanides, in a number of papers in the early 2000s[18], argues that, in order to obtain an accurate characterization of monetary policy, it is necessary to take into account the information set that the authority had available at each point in time when conducting policy. This implies that estimations of Taylor-type rules should use real-time data instead of ex-post revised data, as it is usually common practice. Taking this into consideration, Orphanides (2004) estimates forward-looking monetary policy reaction functions before and after the Volker era, using real time data available to the FOMC. He concludes that the reaction functions before and after 1979 are broadly similar, and in both time periods monetary policy can be characterized by its forward-looking nature and by its strong reaction to expected inflation, thus contradicting previous findings. In a related work, Primiceri (2006) suggests that the high inflation of the 1960s and 1970s and rapid disinflation of the 1980s may be due to incorrect beliefs of the monetary authority about the natural rate of unemployment and the persistence of inflation in the Phillips curve. He estimates a model under these premises and finds that the model can account reasonably well for the evolution of beliefs, stabilization policy and the postwar behavior of inflation and unemployment in the US. Once more, this study challenges the view that the macroeconomic performance of the pre-Volker period is a result of bad policy, as the policymaker is always rational and optimizing within the information set it possesses. Finally, Davig and Leeper (2007) propose a generalization of the Taylor principle when there are changes in the policy regime. They challenge the results from

[18]See Orphanides (2000), Orphanides (2001). Orphanides (2002), Orphanides (2003) and Orphanides (2004).

Lubik and Schorfheide (2004) by showing that, depending on whether the "passive" regime of the 1960s and 1970s was expected to last for a long period or not, the economy may have been in an indeterminate equilibrium or not, and fluctuations in inflation and output may be the consequence of sunspots or not.

We contribute to the previous debate by claiming that the estimation of Taylor-type rules when the true rule governing the monetary authority's actions is of the Wicksellian type may lead to wrong conclusions about the determinacy of the equilibrium. This casts doubts on the validity of arguments of "bad policy" based on estimations of coefficients that imply indeterminacy.

To illustrate the potential problems that may emerge when estimating Taylor rules in practice, we simulate the economy characterized by the output gap in equation (1), the New Keynesian Phillips curve in (2), and the simple Wicksellian rule in (21) that reacts only to contemporaneous inflation. We allow for demand and supply shocks, i.e. disturbances to the IS and New Keynesian Phillip curve. We do not consider stochastic shocks to the Wicksellian rule in (21) [19]. We perform this simulation for several positive values of ψ_p, so that the system is always determine. Then, we empirically estimate, using OLS, the policy coefficient in a simple Taylor rule like

$$i_t = \varphi_\pi \pi_t + \epsilon_t \tag{23}$$

The above rule incorporates a disturbance term, ϵ_t, that in this particular case correspond to the lagged interest rate, i_{t-1} [20]. As a consequence, the OLS estimation of φ_π is going to be biased and inconsistent. In particular, it can be proved that in the case of (23) the OLS estimator of φ_π is going to be a biased estimator of ψ_p. This bias is always negative and it is such that $\hat{\varphi_\pi} = \frac{\psi_p}{2}$. Estimations presented in Table (1) show this fact and confirm one of the conjectures discussed before: for data sets generated through a Wicksellian rule for which ψ_p is small, the (incorrectly) estimated Taylor rule does not satisfy the Taylor principle. Furthermore, for values of ψ_p between 0.1 and 2.0, the estimated Taylor rule coefficient, $\hat{\varphi}_\pi$, falls within the indeterminacy region. The implication of this result is that, based on Taylor rule estimates, one may conclude -erroneously- that the central bank is following an accommodative monetary policy that is potentially destabilizing, when in fact

[19] We simulate the dynamic response of the economy to a sequence of 1.000 i.i.d shocks to r_t^e and u_t, with mean zero and variances σ_{r^e}=1 and σ_u=0.05

[20] The rule $i_t = \psi_p p_t$ is equivalent to $i_t = i_{t-1} + \psi_p \pi_t$, hence the disturbance term in (23) is, in fact, i_{t-1}.

Table 1: Contemporaneous Taylor Rule Estimates (OLS)

Estimates	(1) $\psi_p = 0.1$	(2) $\psi_p = 0.5$	(3) $\psi_p = 1$	(4) $\psi_p = 1.5$	(5) $\psi_p = 2$	(6) $\psi_p = 3$	(7) $\psi_p = 5$	(8) $\psi_p = 10$
$\hat{\varphi}_\pi$	0.050***	0.250***	0.500***	0.750***	1.000***	1.500***	2.500***	5.000***
	(0.005)	(0.023)	(0.043)	(0.061)	(0.078)	(0.111)	(0.172)	(0.315)
Observations	1,001	1,001	1,001	1,001	1,001	1,001	1,001	1,001
R-squared	0.084	0.106	0.122	0.133	0.142	0.156	0.174	0.201

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

the Wicksellian rule in place induces determinacy. The indeterminacy result is overcome if the rule in (23) is extended by including i_{t-1} as regressor. In particular, a partial adjustment specification of the form:

$$i_t = \rho i_{t-1} + (1-\rho)\varphi_\pi \pi_t + \epsilon_t \tag{24}$$

which is widely used in the empirical literature [21], solves the indeterminacy problem. This specification generates an estimate of ρ which is close to one at the cost, however, of generating an estimated value of φ_π which is way above the value of ψ_p [22].

Now, using the same sequence of stochastic shocks, we simulate the economy under a hybrid Wicksellian rule [23]. This rule takes the form:

$$i_t = \psi_p E_t p_{t+1} + \psi_y y_t, \tag{25}$$

As before, we consider alternative positive values for the ψ_p coefficient and set ψ_y=2.0 [24]. In each case, we estimate, following Clarida et al. (1998) and Clarida et al. (2000), a partial adjustment hybrid Taylor rule of the form:

$$i_t = \rho i_{t-1} + (1-\rho)\left[\varphi_\pi E_t \pi_{t+1} + \varphi_y y_t\right] + \varepsilon_t \tag{26}$$

[21] See Clarida et al. (1998) and Clarida et al. (2000), among others.
[22] For brevity, we do not present here the results of estimating (24). Results are available upon request.
[23] We assume the central bank cannot observe the natural level of output accurately, but can observe the level of output. Our results are, in any case, robust to the inclusion of the output gap.
[24] The results that follow are robust to alternative values of the ψ_y coefficient.

Table 2: Hybrid Taylor Rule Estimates (2-Step GMM-IV)

Estimates	(1) $\psi_p = 0.1$	(2) $\psi_p = 0.5$	(3) $\psi_p = 1$	(4) $\psi_p = 1.5$	(5) $\psi_p = 2$	(6) $\psi_p = 3$	(7) $\psi_p = 5$	(8) $\psi_p = 10$
$\hat{\rho}$	0.220***	0.266***	0.277***	0.282***	0.286***	0.296***	0.313*	0.476
	(0.018)	(0.043)	(0.055)	(0.035)	(0.078)	(0.102)	(0.177)	(1.125)
$\hat{\varphi}_\pi$	0.168***	0.202**	0.203**	0.201***	0.200*	0.201*	0.226**	0.283
	(0.065)	(0.091)	(0.098)	(0.048)	(0.108)	(0.111)	(0.102)	(0.263)
$\hat{\varphi}_y$	0.737***	0.304***	0.172***	0.106***	0.064	0.010***	-0.054	-0.065
	(0.050)	(0.065)	(0.052)	(0.018)	(0.044)	(0.040)	(0.040)	(0.197)
Observations	996	993	994	995	995	990	991	991
Hansen J-test (p-value)	0.989	0.997	0.967	0.998	0.955	0.968	0.967	0.948

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

In this particular case it can be shown that the error term, ε_t, contains an endogenous variable (lagged value of output) that is correlated to expected inflation, the output level and the lagged interest rate. This may generate biased and inconsistent estimates, independently of the sample size T. To correct the bias generated by the correlation between the error term and the explanatory variables, we estimate the Taylor rule in (26) using an instrumental variable approach as in Clarida et al. (1998) and Clarida et al. (2000). In particular, we remove the unobserved expected inflation deviation by rewriting the policy rule (26) in terms of realized variables as follows:

$$i_t = \rho i_{t-1} + (1-\rho)\left[\varphi_\pi \pi_{t+1} + \varphi_y y_t\right] + \nu_t \tag{27}$$

where the error term, ν_t, is a combination of the forecast errors of inflation and the disturbance ε_t. We define a vector of variables $\mathbf{u}_t$ within each central bank's information set, at the time each one chooses the interest rate, that is orthogonal to ν_t. Hence, $E\left[\nu_t|\mathbf{u}_t\right] = 0$. In order to estimate the parameters of interest, we use the generalized method of moments (GMM), instrumenting expected inflation, output and the lagged interest rate. The set of instruments $\mathbf{u}_t$ we use includes lagged values of inflation. We check the validity of instruments with the Hansen test (which we report) and the orthogonality of each instrument with the Eichenbaum, Hansen and Singleton test (which we do not report, but are available upon request).

The results from estimating equation (26) are presented in Table (2). As the policy response to the level of prices in the Wicksellian rule increases, the estimated degree of persistence, $\hat{\rho}$, is larger. At the same time, for larger values of ψ_p, the importance of the policy response to inflation in the Taylor rule, the φ_π coefficient, increases relative to the policy response to output, φ_y. Despite this fact the Taylor rules' estimated coefficients fall within the indeterminacy area of the model for every value of ψ_p.

The results previously depicted illustrate the extent to which misleading conclusions may be drawn if the nature of monetary policy is to be interpreted based on estimated Taylor rules. In particular, one may conclude that the Taylor principle is violated even in cases in which the system is determinate (results in Table 1 and Table 2).

4.2 The Taylor principle and the zero lower bound

In light of the financial crisis of 2008, in recent years there has been a renewed interest in studying issues of monetary policy and the zero lower bound. As Blinder (2000) points out, there are two crucial matters that deserve attention from the profession. The first is the preemptive issue on how to avoid reaching the zero lower bound (ZLB). The second refers to the policy actions that should be taken once the ZLB has been reached.

The latter issue has been the subject of many studies in the last decade[25]. The seminal contribution of Eggertsson and Woodford (2003) proposes a price-level targeting rule in the spirit of the Wicksellian rules analyzed in this paper as a near-optimal rule when the ZLB is binding. The usefulness of this rule lies in the fact that, given a deflationary process, a Wicksellian rule induces expectations of future inflation that are necessary to drive real interest rates down, a result that cannot be achieved by simply lowering the nominal interest rate because of the ZLB.

In the current analysis, we complement the result of Eggertsson and Woodford (2003) about the usefulness of a Wicksellian rule in the ZLB. In particular, we argue that this type of rule is also very effective in tackling the preemptive issue raised by Blinder (2000)[26].

[25]Many of the recent contributions have focused in analyzing the efficacy of unconventional monetary policy instruments that have been put to practice in the last years or that may be feasible alternatives to those: see, for example, Bernanke et al. (2004), Fernandez-Villaverde et al. (2011), Curdia and Woodford (2011), Gertler and Karadi (2011), del Negro et al. (2011), Hamilton and Wu (2012) and Gambacorta et al. (2012).

[26]Several papers have addressed this topic (see Adam and Billi (2007) and de Fiore and Tristani (2012)), but not many of them have acknowledged the benefits that the adoption of a price level target may bring in this respect. Some exceptions are Blinder (2000) and Gaspar et al. (2007). Blinder (2000) discusses some of the measures that may be taken in order to avoid a situation in which the ZLB is active, among which

As it is clear from the discussion of previous sections, a Wicksellian rule does not require the Taylor principle to hold in order to render the equilibrium of the economy determinate. This result is independent from the set of information that the central bank takes into account when implementing the rule. In general terms, this implies that the nominal interest rate may respond less than one-for-one with inflation when there is an inflationary shock. Moreover, in order to attain determinacy, for low values of the response of the rule to output gap ψ_x, the only condition for determinacy that a Wicksellian rule imposes is that the response of the nominal interest rate to the price level is positive, $\psi_p > 0$.

The fact that the Taylor principle need not be satisfied when implementing a Wicksellian rule gives the monetary authority significant freedom to choose the response parameters of such rule. In particular, the central bank may choose a very mild (but positive) response of the nominal interest rate to deviations of the price level. As an example, Figure 4 shows the impulse responses of the variables of interest to a given cost-push shock when $\psi_x = 0$ and $\psi_p = 0.1$. As it is clear from this example, the nominal interest rate barely reacts when inflation spikes. Due to expectations of future deflation, however, this minimal reaction of the nominal interest rate suffices to drive the economy back to its steady state and to prevent the economy to be subject to sunspots.

A straightforward conclusion that can be drawn from the previous example is that, in the presence of deflationary pressures, when the central bank follows a Wicksellian rule with a low ψ_p, the ZLB is hit much less frequently than when the policy regime is governed by a Taylor rule. This is the case because, for a given shock, the nominal interest rate is much less responsive under a Wicksellian rule than under a Taylor rule. In other words, the adoption of a Wicksellian rule may shield the economy from reaching the ZLB much more effectively than with a Taylor rule, due to the fact that in this case the Taylor principle does not govern determinacy and, consequently, the response of the nominal interest rate to a deflation may be less than one-for-one.

he lists a price level target for the Central Bank. The reasons for listing a price level target, however, are more related to the effectiveness of this policy regime to exit a situation of ZLB once this has taken place. Gaspar et al. (2007) point to the fact that, under price level targeting, nominal interest rates need to adjust less than under inflation targeting and as a result the frequency of hitting the zero lower bound is diminished. Nevertheless, neither of these papers relate the frequency of occurrence of a ZLB episode to the absence of a Taylor principle for Wicksellian rules, as we do here.

5 Conclusions and future research

In a standard New Keynesian model, we compare the areas of determinacy of two alternative instrument rules. The first one is a simple Taylor rule in which the policy rate reacts to movements in inflation and output. The second one is a Wicksellian rule in which the interest rate reacts to movements in the price level and output. Our main findings are as follows. First, the area of determinacy for the policy coefficients is, in general, larger for Wicksellian rules. Second, Wicksellian rules do not need to satisfy the Taylor principle in order to induce determinacy. In particular, such rules are able to generate determinacy even in cases in which the nominal rate reacts less than one to one with inflation. In those cases the main mechanism ensuring determinacy, or an increase in the real ex-ante interest rate in the face of supply shocks, is the expected future deflation which a Wicksellian rule generates. Finally, under Wicksellian rules the price level, as well as inflation, are uniquely determined given the initial conditions. As a consequence, this rule avoids nonlocal nominal paths or nominal explosions as defined by Cochrane (2011).

There are two practical implications related to the irrelevance of the Taylor principle for Wicksellian rules. The first is that estimating a Taylor rule when the true underlying rule is Wicksellian, may lead to conclude -erroneously- that the equilibrium is indeterminate even in the case in which the price level rules ensures determinacy. The second is that, when the central bank follows a Wicksellian rule, the frequency of occurrence of an active zero lower bound on the policy rate is greatly reduced.

Based on the findings of this paper, there are theoretical issues that could be addressed in future research. First, in open economies some forms of managed exchange rate rules may alleviate problems of indeterminacy. In other words, an augmented Taylor rule that, besides reacting to inflation and output, also moves in the face of changes in the exchange rate, increases the determinacy area of the policy coefficients (see Llosa and Tuesta (2008)). Second, the existence of trend inflation in a standard New Keynesian model modifies substantially the determinacy properties of simple Taylor rules (see Ascari and Ropele (2009)). In particular, when trend inflation is considered, neither the Taylor principle nor the generalized Taylor principle, which requires the nominal interest rate to be raised by more than the increase in inflation in the long run, is a sufficient condition for local determinacy of equilibrium. In this respect, it should be interesting to check the extent to which our main results for the Wicksellian rules hold in an open economy environment and in the presence of trend inflation.

From a practical perspective, we have shown that empirical results from Taylor rules estimates may lead to misleading conclusions about the nature of monetary policy. In this context, to see the extent to which, under the pre-Volcker period, the policy followed by the Fed was indeed stabilizing, we could estimate Wicksellian rules during that period in order to assess the extent to which determinacy conditions under such rules were satisfied. Finally, recent papers (Giannoni (2010)) have shown that Wicksellian rules have a better performance in terms of welfare than simple Taylor rules. This analysis does not incorporate the advantages of Wicksellian rules under the ZLB. In this respect, it would be useful to assess the extent to which the existence of a ZLB is an element that increases even further the welfare gains of price level targeting rules.

A Appendix: proofs

A.1 Necessary and sufficient conditions for determinacy

In order to prove Proposition 2, Proposition 4 and Proposition 6, we need to determine if matrix $\tilde{B} = B^{-1}$ has exactly two eigenvalues outside the unitary circle. To this end, we follow Proposition C.2 in Woodford (2003) that lists a set of necessary and sufficient conditions for this to be the case. For ease of exposition, we reproduce this proposition here.

Let the characteristic polynomial of the 3x3 matrix $\tilde{B}$, be defined as

$$\mathcal{P}(\lambda) = \lambda^3 + \mathcal{A}_2\lambda^2 + \mathcal{A}_1\lambda + \mathcal{A}_0,$$

where λ are the eigenvalues of $\tilde{B}$. This equation has one root inside the unit circle and two roots outside if and only if:

- **Case I**

  $$\mathcal{P}(1) = 1 + \mathcal{A}_2 + \mathcal{A}_1 + \mathcal{A}_0 < 0, \tag{28}$$

  and

  $$\mathcal{P}(-1) = 1 + \mathcal{A}_2 - \mathcal{A}_1 + \mathcal{A}_0 > 0; \tag{29}$$

  or

- **Case II**

  $$\mathcal{P}(1) = 1 + \mathcal{A}_2 + \mathcal{A}_1 + \mathcal{A}_0 > 0, \tag{30}$$

  $$\mathcal{P}(-1) = 1 + \mathcal{A}_2 - \mathcal{A}_1 + \mathcal{A}_0 < 0, \tag{31}$$

  and

  $$\mathcal{A}_4 = \mathcal{A}_0^2 - \mathcal{A}_0\mathcal{A}_2 + \mathcal{A}_1 - 1 > 0, \tag{32}$$

  or

- **Case III**

  Conditions (30) and (31) hold in addition to

  $$\mathcal{A}_4 = \mathcal{A}_0^2 - \mathcal{A}_0\mathcal{A}_2 + \mathcal{A}_1 - 1 < 0 \tag{33}$$

and

$$|\mathcal{A}_2| > 3. \tag{34}$$

A.2 Determinacy of contemporaneous Wicksellian rules

This section reproduces the proof in Giannoni (2010).

In order to prove Proposition 2, we need to determine if matrix $\tilde{B} = B^{-1}$, where matrix B is defined in (6), has exactly two eigenvalues outside the unitary circle.

The characteristic polynomial of matrix $\tilde{B}$ is

$$\mathcal{P}(\lambda) = \lambda^3 + \mathcal{A}_2\lambda^2 + \mathcal{A}_1\lambda + \mathcal{A}_0,$$

where λ are the eigenvalues of $\tilde{B}$ and

$$\mathcal{A}_2 = -\frac{2\sigma\beta + \beta\psi_x + \kappa + \sigma}{\sigma\beta},$$

$$\mathcal{A}_1 = \frac{\psi_p\kappa + \psi_x(1+\beta) + \sigma(\beta+2) + \kappa}{\sigma\beta},$$

$$\mathcal{A}_0 = -\frac{\sigma + \psi_x}{\sigma\beta}.$$

Case III of Woodford (2003) states that a sufficient condition for determinacy is that[27]

$$\mathcal{P}(1) > 0, \quad \mathcal{P}(-1) < 0 \quad \text{and} \quad |A_2| > 3.$$

Next, we show that, as long as ψ_p and ψ_x are positive, all values satisfy the conditions previously stated and, thus, yield determinacy. Notice that

$$\mathcal{P}(1) = \frac{\psi_p\kappa}{\sigma\beta} > 0 \quad \text{if } \psi_p > 0,$$

$$\mathcal{P}(-1) = -\frac{4\sigma\beta + 4\sigma + 2\psi_x + 2\beta\psi_x + 2\kappa + \psi_p\kappa}{\sigma\beta} < 0 \quad \text{if } \psi_p > 0, \psi_x > 0,$$

$$|A_2| = 2 + \frac{1}{\beta} + \frac{\beta\psi_x + \kappa}{\sigma\beta} > 3 \quad \text{since } \beta < 1 \text{ and } \psi_x > 0.$$

This completes the proof.

[27] Notice that we do not need to compute the sign of $\mathcal{A}_4$ since, in case it is positive, we are back to Case II, and if it is negative, we fall into Case III. In either case, the conditions stated here are sufficient for determinacy.

A.3 Determinacy of forward-looking Wicksellian rules

In order to prove Proposition 4, we need to determine if matrix B defined in (12) has exactly two eigenvalues inside the unitary circle. To simplify the algebra, we define $\tilde{B} = B^{-1}$, so to achieve determinacy, $\tilde{B}$ needs to have two eigenvalues outside the unit circle.

The characteristic polynomial of matrix $\tilde{B}$ is

$$\mathcal{P}(\lambda) = \lambda^3 + \mathcal{A}_2\lambda^2 + \mathcal{A}_1\lambda + \mathcal{A}_0,$$

where λ are the eigenvalues of $\tilde{B}$ and

$$\mathcal{A}_2 = -\frac{(\sigma - \psi_x)(1+\beta) + \sigma\beta + \kappa(1-\psi_p)}{(\sigma - \psi_x)\beta},$$

$$\mathcal{A}_1 = \frac{\sigma(2+\beta) + \kappa - \psi_x}{(\sigma - \psi_x)\beta},$$

$$\mathcal{A}_0 = -\frac{\sigma}{(\sigma - \psi_x)\beta}.$$

As a result, we can compute $\mathcal{P}(1)$ and $\mathcal{P}(-1)$ to obtain

$$\mathcal{P}(1) = \frac{\kappa\psi_p}{(\sigma - \psi_x)\beta}, \tag{35}$$

$$\mathcal{P}(-1) = \frac{2(1+\beta)(\psi_x - 2\sigma) - \kappa(2-\psi_p)}{(\sigma - \psi_x)\beta}. \tag{36}$$

We restrict our attention to values of the coefficients of the Wicksellian rule larger or equal than zero, that is, $\psi_p \geq 0$ and $\psi_x \geq 0$. In this case, the sign of $\mathcal{P}(1)$ and $\mathcal{P}(-1)$ depends on whether $(\sigma - \psi_x)$ is positive or negative. We consider each case separately.

A.3.1 Case I: $\sigma < \psi_x$

Notice that, in this case, $\mathcal{P}(1) < 0$ and $\mathcal{P}(-1) > 0$ if the following condition is satisfied:

$$\kappa(\psi_p - 2) + 2(1+\beta)\psi_x < 4\sigma(1+\beta). \tag{37}$$

Hence, if $\sigma < \psi_x$ the system is determinate if condition (37) is satisfied.

A.3.2 Case II and III: $\sigma > \psi_x$

Notice that, in this case, $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$ hold if condition (37) is satisfied. To have determinacy, however, it is requiered in addition that $\mathcal{A}_4 > 0$ (Case II) or $\mathcal{A}_4 < 0$ and $|\mathcal{A}_2| > 3$ (Case III).

Now, notice that:

$$\mathcal{A}_4 = \frac{\psi_x\beta(\sigma-\psi_x)(\beta-1)+(\sigma-\psi_x)\beta\kappa+\psi_x\sigma(1-\beta)+\sigma\kappa(\psi_p-1)}{(\sigma-\psi_x)^2\beta^2}. \tag{38}$$

For (38) to be positive, since the denominator is positive, the numerator should be positive as well. This is the case if:

$$\kappa(\psi_p-1) > -\psi_x(1-\beta)^2 - \frac{\psi_x^2(1-\beta)\beta}{\sigma} - \frac{(\sigma-\psi_x)\beta\kappa}{\sigma}. \tag{39}$$

If conditions (37) and (39) are satisfied, then the system is determinate (Case II).

Now, if condition (39) is not satisfied, so that $\mathcal{A}_4 < 0$, we require that $|\mathcal{A}_2| > 3$ (Case III). Notice that

$$|\mathcal{A}_2| = \left| -\frac{(\sigma-\psi_x)(1+\beta)+\sigma\beta+\kappa(1-\psi_p)}{(\sigma-\psi_x)\beta} \right|. \tag{40}$$

Assume ψ_p is not very large, so $\mathcal{A}_2 < 0$. Then, condition $|\mathcal{A}_2| > 3$ requires that [28]

$$-\frac{(\sigma-\psi_x)(1+\beta)+\sigma\beta+\kappa(1-\psi_p)}{(\sigma-\psi_x)\beta} < -3.$$

or

$$\kappa(\psi_p-1) < \sigma(1-\beta)+\psi_x(2\beta-1). \tag{41}$$

We show next that if condition (39) is not satisfied, then condition (41) is automatically satisfied. To this end, we need to check if, when

$$\kappa(\psi_p-1) < -\psi_x(1-\beta)^2 - \frac{\psi_x^2(1-\beta)\beta}{\sigma} - \frac{(\sigma-\psi_x)\beta\kappa}{\sigma},$$

it is always the case that condition (41) holds. This is the case if the right hand side of equation (41) is larger than the right hand side of equation (39). We prove this by contradiction. Suppose

$$-\psi_x(1-\beta)^2 - \frac{\psi_x^2(1-\beta)\beta}{\sigma} - \frac{(\sigma-\psi_x)\beta\kappa}{\sigma} > \sigma(1-\beta)+\psi_x(2\beta-1).$$

Rearranging,

$$-\left(\psi_x(1-\beta)^2 + \frac{\psi_x^2(1-\beta)\beta}{\sigma} + \frac{(\sigma-\psi_x)\beta\kappa}{\sigma} + (\sigma-\psi_x)(1-\beta)+\psi_x\beta\right) > 0,$$

which is clearly a contradiction because all terms inside the brackets in the left hand side of the previous expression are positive. We can conclude that all cases that satisfy $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$, either satisfy condition (39) or condition (41). This completes the proof.

[28] If ψ_p is larger than one, condition (39) is always satisfied, so Case III becomes irrelevant.

A.4 Determinacy of hybrid Taylor rules

In order to prove Proposition 5, we need to determine if matrix B defined in (15) has exactly two eigenvalues inside the unitary circle.

The characteristic polynomial of B is given by

$$\mathcal{P}(\lambda) = \lambda^2 + \mathcal{A}_1\lambda + \mathcal{A}_0,$$

where

$$\mathcal{A}_0 = \frac{\beta\sigma}{\sigma + \varphi_x}$$

and

$$\mathcal{A}_1 = -\frac{1}{\sigma + \varphi_x}(\sigma(1+\beta) + \varphi_x\beta + \kappa(1-\varphi_\pi)).$$

Both eigenvalues are inside the unitary circle if and only if [29]

$$|\mathcal{A}_0| < 1, \quad |\mathcal{A}_1| < 1 + \mathcal{A}_0.$$

Condition $|\mathcal{A}_0| < 1$ implies that $\varphi_x > (\beta - 1)\sigma$. This condition is satisfied for all positive values of φ_x. On the other hand, condition $|\mathcal{A}_1| < 1 + \mathcal{A}_0$ implies that:

$$\kappa(\varphi_\pi - 1) - \varphi_x(1+\beta) < 2(1+\beta)\sigma, \tag{42}$$

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) > 0. \tag{43}$$

Hence, the hybrid Taylor rule induces determinacy if conditions (42) and (43) are satisfied. This completes the proof.

A.5 Determinacy of hybrid Wicksellian rules

In order to prove Proposition 6, we need to determine if matrix B defined in (19) has exactly two eigenvalues inside the unitary circle. To simplify the algebra, we define $\tilde{B} = B^{-1}$, so to achieve determinacy, $\tilde{B}$ needs to have two eigenvalues outside the unit circle.

The characteristic polynomial of matrix $\tilde{B}$ is

$$\mathcal{P}(\lambda) = \lambda^3 + \mathcal{A}_2\lambda^2 + \mathcal{A}_1\lambda + \mathcal{A}_0,$$

where λ are the eigenvalues of $\tilde{B}$ and

[29] See LaSalle (1986).

$$\mathcal{A}_2 = \frac{\kappa(\psi_p - 1) - \psi_x\beta - \sigma}{\sigma\beta} - 2,$$

$$\mathcal{A}_1 = \frac{\beta(\sigma + \psi_x) + \kappa + 2\sigma + \psi_x}{\sigma\beta},$$

$$\mathcal{A}_0 = -\frac{\sigma + \psi_x}{\sigma\beta}.$$

As a result, we can compute $\mathcal{P}(1)$ and $\mathcal{P}(-1)$ to obtain

$$\mathcal{P}(1) = \frac{\kappa\psi_p}{\sigma\beta} > 0, \tag{44}$$

$$\mathcal{P}(-1) = \frac{\kappa(\psi_p - 2) - 4\sigma(1+\beta) - 2\psi_x(1+\beta)}{\sigma\beta}. \tag{45}$$

Since $\mathcal{P}(1) > 0$ for all values of the parameters, as long as $\psi_p > 0$, we can already discard Case I as a relevant case to establish areas of determinacy. Instead, we consider Case II and Case III.

A.5.1 Case II and III:

These conditions require that $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$. The first condition is automatically satisfied. To satisfy the second condition, given that the denominator is always positive, the numerator must be negative, that is

$$\kappa(\psi_p - 2) - 2(1+\beta)\psi_x < 4\sigma(1+\beta). \tag{46}$$

Case II requires the following additional condition to hold

$$\mathcal{A}_4 = \frac{\psi_x^2(1-\beta) + \kappa\sigma(\psi_p + \beta - 1) - \psi_x(\kappa(\psi_p - 1) + \sigma(1-\beta)^2)}{\sigma^2\beta^2} > 0. \tag{47}$$

For (47) to be positive, since the denominator is positive, the numerator should be positive as well. This is the case if

$$\psi_p > \frac{-\psi_x^2(1-\beta) + \kappa\sigma(1-\beta) + \psi_x\kappa - \psi_x\sigma(1-\beta)^2}{\kappa(\sigma + \psi_x)}. \tag{48}$$

If conditions (46) and (48) are satisfied, then the system is determinate (Case II). If condition (48) is not satisfied, so that $\mathcal{A}_4 < 0$, we require $|\mathcal{A}_2| > 3$ (Case III). Notice that

$$|\mathcal{A}_2| = \left| \frac{\kappa(\psi_p - 1) + \psi_x\beta - \sigma - 2\sigma\beta}{\sigma\beta} \right|.$$

Assume ψ_p is not very large, so $\mathcal{A}_2 < 0$. Then, condition $|\mathcal{A}_2| > 3$ requires that [30]

$$\frac{\kappa(\psi_p - 1) - \psi_x\beta - \sigma - 2\sigma\beta}{\sigma\beta} < -3,$$

or

$$\psi_p < \frac{\sigma(1-\beta) + \psi_x\beta}{\kappa} + 1. \tag{49}$$

We show next that if condition (48) is not satisfied, then condition (49) is automatically satisfied. To this end, we need to check if, when

$$\psi_p < \frac{-\psi_x^2(1-\beta) + \kappa\sigma(1-\beta) + \psi_x\kappa - \psi_x\sigma(1-\beta)^2}{\kappa(\sigma + \psi_x)},$$

it is always the case that condition (49) holds. This is the case if the right hand side of equation (49) is larger than the right hand side of equation (48). We prove this by contradiction. Suppose

$$\frac{-\psi_x^2(1-\beta) + \kappa\sigma(1-\beta) + \psi_x\kappa - \psi_x\sigma(1-\beta)^2}{\kappa(\sigma + \psi_x)} > \frac{\sigma(1-\beta) + \psi_x\beta}{\kappa} + 1.$$

Rearranging,

$$\sigma^2(1-\beta) + \sigma\psi_x\beta + \psi_x\sigma(1-\beta) + \psi_x^2 + \sigma\kappa\beta + \psi_x\sigma(1-\beta)^2 < 0,$$

which is clearly a contradiction because all terms in the left hand side are positive. We can conclude that all cases that satisfy $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$, either satisfy condition (48) or condition (49). Then, the only relevant condition for determinacy is condition (46). This completes the proof.

A.6 Backward-looking rules

Under backward-looking rules, the policy maker adjusts the policy t rate in response to deviations of the lagged values of the variables of interest with respect to their target values. Rules that react to lagged inflation and output gap are not too used in the theoretical literature of monetary policy, with the exception of Bullard and Mitra (2002), that study the determinacy and E-stability properties of such rules. However, these rules have been used in studies that attempt to estimate the actual rules used by central banks, such as Rotemberg and Woodford (1999) and Giannoni and Woodford (2004).

We consider a backward-looking Taylor rule in which the policy rate reacts to lagged inflation, π_{t-1} and lagged output gap, x_{t-1}:

[30] If ψ_p is larger than one, condition (48) is always satisfied, so Case III becomes irrelevant.

$$i_t = \varphi_\pi \pi_{t-1} + \varphi_x x_{t-1}, \tag{50}$$

where φ_x and φ_π represent the reaction of the policy rate to deviations of the output gap and the inflation rate from target values.

The system formed by equations (1), (2) and (50) can be rewritten as

$$E_t y_{t+1} = \tilde{B} y_t + \tilde{C} z_t,$$

where $y_t = [x_t; \pi_t; i_t]$, $z_t = [r_t^e; u_t]$ and

$$\tilde{B} = \begin{pmatrix} \frac{\beta\sigma+\kappa}{\beta\sigma} & -\frac{1}{\beta\sigma} & \frac{1}{\sigma} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \varphi_x & \varphi_\pi & 0 \end{pmatrix}. \tag{51}$$

This system has two endogenous non-predetermined variables, x_t and π_t, and one endogenous predetermined variable, i_t. The following proposition characterizes the necessary and sufficient conditions for determinacy when the rule (50) is implemented [31].

Proposition 7. *Under backward-looking Taylor rules such as (50) the necessary and sufficient conditions for a rational expectations to be unique (determinate) are either that*

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) < 0 \quad \textit{and}$$
$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) > 2\sigma(\beta+1),$$

or

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) > 0 \quad \textit{and}$$
$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) < 2\sigma(\beta+1),$$

Proof. In order to prove the proposition, we need to determine if matrix $\tilde{B}$ defined in (51) has exactly two eigenvalues outside the unitary circle.

The characteristic polynomial of $\tilde{B}$ is given by

$$\mathcal{P}(\lambda) = \lambda^3 + \mathcal{A}_2\lambda^2 + \mathcal{A}_1\lambda + \mathcal{A}_0,$$

where λ are the eigenvalues of $\tilde{B}$ and

$$\mathcal{A}_2 = -\frac{\beta\sigma + \kappa + \sigma}{\beta\sigma},$$
$$\mathcal{A}_1 = -\frac{\beta\varphi_x - \sigma}{\beta\sigma},$$

[31] Bullard and Mitra (2002) also study the determinacy areas of these type of rules, but only determine analytically sufficient conditions for determinacy, while we establish necessary and sufficient conditions.

$$\mathcal{A}_0 = \frac{\varphi_\pi \kappa + \varphi_x}{\beta\sigma}.$$

We can compute $\mathcal{P}(1)$ and $\mathcal{P}(-1)$ to obtain

$$\mathcal{P}(1) = \frac{\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta)}{\beta\sigma},$$

$$\mathcal{P}(1) = \frac{-2\beta\sigma - \kappa(1-\varphi_\pi) - 2\sigma + \varphi_x(1+\beta)}{\beta\sigma}.$$

We consider cases I and II separately.

A.6.1 Case I:

Notice that, in this case, $\mathcal{P}(1) < 0$ and $\mathcal{P}(-1) > 0$ if the following conditions are satisfied:

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) < 0,$$
$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) > 2\sigma(1+\beta).$$

A.6.2 Case II:

Notice that, in this case, $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$ if the following conditions are satisfied:

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) > 0,$$
$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) < 2\sigma(1+\beta).$$

Moreover, we need that

$$\mathcal{A}_4 = \frac{(\varphi_\pi\kappa + \varphi_x)(\varphi_\pi\kappa + \varphi_x + \beta\sigma + \kappa + 1) - \beta^2\sigma\varphi_x + \sigma^2\beta - \beta^2\sigma^2}{\beta^2\sigma^2} > 0.$$

For this expression to be positive, it is required that the numerator is positive. Rearranging the numerator, it can be expressed as

$$\varphi_\pi\kappa(\varphi_\pi\kappa + \varphi_x + \beta\sigma + \kappa + \sigma) + \varphi_x(\varphi_\pi\kappa + \varphi_x + \kappa + \sigma) + \varphi_x\beta\sigma(1-\beta) - \beta\sigma^2(\beta - 1),$$

which is clearly positive because $\beta < 1$ and, thus, all terms in the previous expression are positive. This completes the proof.

□

In the case of backward-looking Taylor rules, determinacy conditions may or may not require the Taylor principle to be satisfied. This is the case because, although inflation is still a non-predetermined endogenous variable, now the current rule for the policy rate is a predetermined variable, and it includes past inflation as a determinant. Then, determinacy can be achieved even for relatively small values of φ_π (that do not satisfy the Taylor principle), as long as the response of the policy rate to the output gap is sufficiently large. That is, the increase in the nominal interest rate given a permanent one percent increase in inflation need not be higher than one percent, so long as the policy rate responds strongly enough to the output gap associated to the permanent increase in inflation [32].

A backward-looking Wicksellian rule reads:

$$i_t = \psi_p(p_{t-1} - \bar{p}) + \psi_x x_{t-1}, \tag{53}$$

where $\psi_p > 0$ and $\psi_x > 0$.

As before, the system composed by equations (1), (2) and (53) can be cast in the form:

$$E_t y_{t+1} = \tilde{B} y_t + \tilde{C} z_t + \tilde{D}\bar{p},$$

where $y_t = [x_t; \pi_t; i_t; p_{t-1}]$, $z_t = [r_t^e; u_t]$. In this particular case, the $\tilde{B}$ matrix takes de form:

$$\tilde{B} = \begin{pmatrix} \frac{\beta\sigma+\kappa}{\beta\sigma} & -\frac{1}{\beta\sigma} & \frac{1}{\sigma} & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 \\ \psi_x & \psi_p & 0 & \psi_p \\ 0 & 1 & 0 & 1 \end{pmatrix}. \tag{54}$$

Notice that this system has two endogenous non-predetermined variables, x_t and π_t, and two endogenous predetermined variables, i_t and p_{t-1}. In this case, for the equilibrium to be determinate it is required that the matrix $\tilde{B}$ has two eigenvalues outside the unitary circle and two inside.

Given that matrix $\tilde{B}$ is a 4×4 matrix, we cannot characterize analytically the regions of determinacy, indeterminacy and explosive solutions [33]. Instead, we claim that the region of

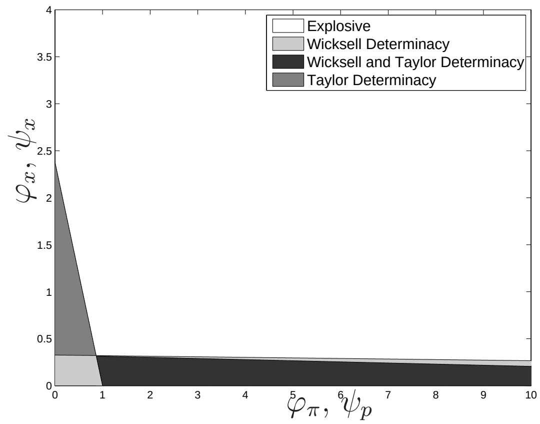
[32] Notice that $\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta)$ can be written as:

$$\varphi_\pi + \frac{1+\beta}{\kappa}\varphi_x > \frac{2\sigma(\beta+1)}{\kappa} + 1. \tag{52}$$

The left hand side of expression (52) is the Taylor rule that would be implemented if inflation had increased one percent last period and were to decrease one percent this period. Similarly, the right hand side corresponds to what the policy rate should be if the inflation rate decreased one percent this period and increased one percent next period, and this cycle continued for ever. Then, determinacy requires that, in the presence of such a shock, the policy rate increases by more than what an oscillating behavior as previously described requires. That is, determinacy requires that, in the event of a positive inflation shock, either the Taylor principle is satisfied, or the policy rule reacts strongly enough to counteract any oscillating pattern of inflation and the output gap.

[33] To our knowledge, in the case of a 4×4 matrix there are no theorems that characterize regions where the eigenvalues are inside or outside the unitary circle, as is the case with 3×3 matrices (see Woodford (2003)).

Figure 5: Determinacy areas of backward-looking rules



Note: The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The semi-dark shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is determinate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is explosive. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate. Finally, the non-shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are explosive.

determinacy is the same as the one for forward-looking and hybrid Wicksellian rules.

Claim 1. *Under backward-looking Wicksellian rules such as (53) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that*

$$\kappa(\psi_p - 2) + 2(1+\beta)\psi_x < 4\sigma(1+\beta).$$

Notice that, if Claim 1 is correct, backward-looking Wicksellian rules do not need to satisfy the Taylor principle. In this case, however, we cannot ascertain that a Wicksellian rule leads to a larger area of determinacy than a Taylor rule, because there is an additional area of determinacy in the Taylor rule for which the Taylor principle is not satisfied.

We assess the validity of Claim 1 through a numerical example in the spirit of the ones conducted before. The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The semi-dark shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is determinate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is explosive. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate. Finally, the non-shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are explosive.

As it is clear from Figure 5, the region of determinacy of a Wicksellian backward-looking rule is identical to the region of determinacy under a forward-looking rule. Then, Claim 1 is confirmed (in this particular example). From the numerical example, it is not possible to conclude that one area of determinacy is larger than the other. It is still the case, however, that the Wicksellian rule does not need to satisfy the Taylor principle. The Taylor rule, on the contrary, has to satisfy this principle for some combinations of parameters $\{\varphi_x, \varphi_\pi\}$ in order to attain determinacy.

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