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A NOTE ON YIELD SPREAD AND OUTPUT GROWTH*

Rodrigo Alfaro
Gerencia de Estabilidad Financiera
Banco Central de Chile

Damián Romero
Gerencia de Investigación Económica
Banco Central de Chile

Abstract

In this paper we compute the impact of the yield spread on output growth, based on a standard DSGE model. As it is supported by empirical literature, we found that yield spread can be used only to forecast output growth for short-term horizons (less than 2 years). Moreover, the size of that impact obtained from calibration of the model is consistent with previous empirical results.

Resumen

Calculamos el impacto del diferencial de tasas sobre el crecimiento del producto, basándonos en un modelo DSGE estándar. Apoyados en la literatura empírica, encontramos que el diferencial de tasas se puede utilizar solo para predecir el crecimiento del producto en horizontes cortos (menos de dos años). En tanto, la magnitud del impacto obtenido de la calibración del modelo es coherente con resultados empíricos anteriores.

* Emails: ralfaro@bcentral.cl y dromero@bcentral.cl, respectivamente.

1 Introduction

There is an extensive literature on empirical evidence of the predictive power of the yield spread in economic activity (Estrella & Hardouvelis 1989, Furlong 1989, Estrella & Hardouvelis 1990 Fama 1990, Estrella & Hardouvelis 1991, Funke 1997, Estrella 1997, Hamilton & Kim 2002, Wheelock & Wohar 2009, among others), however, few papers study why there is such as predicting power and how that is rooted in standard macroeconomic models. An early work on this path is the work of Smets and Tsatsaronis (1997), in that paper authors use a SVAR to identify the relative importance of demand, supply and monetary shocks on the determination of nominal and real components of the yield spread. Imposing both short and long run restrictions, they conclude that both demand and supply shocks are relevant to explain the predictive power of the yield spread. Hardouvelis & Malliaropulos (2003) use a CAPM model with price stickiness to analyze the predictive power of spread, finding that productivity and money supply shocks are behind this phenomena. An exception of previous analysis, is Estrella (2005). The author uses a 3-equation DSGE model, concluding that the predictive power of the yield spread can be rooted into this framework.

In this paper, we extend Estrella (2005)'s approach by using an hybrid DSGE model, similar to the one presented in Fuhrer (2009). We calibrate its parameters consistently with the empirical findings of Bekaert et al. (2010) for the US economy. Defining yield spread as the difference between the 10 years and 3 the month interest rates, we study the predictive power of that over output growth in the next 16 quarters. Following the work of Fuhrer (2009) in inflation persistence, we explore the implications of changing structural parameters of the DSGE model to analyze the relevance of backward and forward components in the model.

In our numerical exercises we found no predictive power on economic activity for horizons greater than 2 years. This is in line with both common knowledge that yield spread can be used only for short-term forecast, and empirical papers on the area including Estrella & Hardouvelis (1991) for which we are also able to replicate their empirical results using our calibrated DSGE model.

The following section presents the structural model of Fuhrer (2009) and it develops the relation between yield spread and future output growth. Next, we present our numerical results on this relationship based on a baseline calibration and some alternative calibrations. A final section concludes.

2 Structural Model and Yield Spread

2.1 Structural Model

Our structural model follows Fuhrer (2009) and it comprises an hybrid inflation specification or Aggregate Supply (equation 1), an IS or Aggregate Demand which characterizes output gap (equation 2), and a Taylor rule or Monetary Policy Reaction Function (equation 3):

$$\pi_t = \mu\pi_{t-1} + (1-\mu)E_t(\pi_{t+1}) + \gamma x_t + u_t \tag{1}$$

$$x_t = \delta x_{t-1} + (1-\delta)E_t(x_{t+1}) - \lambda[i_t - E_t(\pi_{t+1})] + v_t \tag{2}$$

$$i_t = \rho i_{t-1} + (1-\rho)[\varpi\pi_t + \theta x_t] \tag{3}$$

where π_t is the annualized quarter-over-quarter inflation in deviations form its mean, x_t is the output gap and i_t is the (nominal) monetary policy rate or short term interest rate, also in deviations from its mean. Also, we assume that both supply (u_t) and demand (v_t) shocks are independent and normally distributed with zero mean and variances σ_u^2 and σ_v^2, respectively. The system (1)-(3) can be written in matrix form as:

$$AZ_t = BE_tZ_{t+1} + CZ_{t-1} + \varepsilon_t \tag{4}$$

where $Z_t = (\pi_t, x_t, i_t)'$, $\varepsilon_t = (u_t, v_t, 0)'$, and

$$A = \begin{pmatrix} 1 & -\gamma & 0 \\ 0 & 1 & \lambda \\ -\varpi(1-\rho) & -\theta(1-\rho) & 1 \end{pmatrix}; B = \begin{pmatrix} 1-\mu & 0 & 0 \\ \lambda & 1-\delta & 0 \\ 0 & 0 & 0 \end{pmatrix}; C = \begin{pmatrix} \mu & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \rho \end{pmatrix};$$

In contrast to Estrella (2005) the system does not have a close-form solution, however a stable solution of the model is a VAR(1) i.e $Z_t = MZ_{t-1} + N\varepsilon_t$ (De Jong & Dave 2011), where all the roots of M are inside of the unit circle.

2.2 Yield Spread

The VAR(1) representation can be used to compute the long-term interest rate (R), from the n quarters ahead forecast of the short rate, by using the Expectation Hypothesis. Indeed, the long-term interest rate (R_{nt}) minus its mean ($\overline{R}$) can be computed as follows:

$$\begin{aligned} r_{nt} &\equiv R_{nt} - \overline{R} = \left[\frac{1}{n}\sum_{j=0}^{n-1} E_t(i_{t+j} + \bar{i})\right] - \overline{R} = d_3' \frac{1}{n}\sum_{j=0}^{n-1} E_t(Z_{t+j}) \\ &= d_3' \left(\frac{1}{n}\sum_{j=0}^{n-1} M^j\right) Z_t = d_3'(I - M)^{-1}(I - M^n)\frac{Z_t}{n} \end{aligned}$$

where d_i is a 3×1 zero vectors with one in position i. The expression above is valid since in steady state we have: $\bar{i} = \overline{R}$.[1]

Taking s_{nt} to be the yield spread, is clear that:

$$s_{nt} \equiv r_{nt} - i_t = d_3'(I - M)^{-1}(I - M^n)\frac{Z_t}{n} - d_3' Z_t = a_{1n}' Z_t \qquad (5)$$

where $a_{1n} \equiv [(I - M)^{-1}(I - M^n)(1/n) - I]' d_3$.

Also, the VAR(1) framework provides forecasts for output gap as follow:

$$E_t(x_{t+k}) = E_t(d_2' Z_{t+k}) = d_2' E_t(Z_{t+k}) = d_2' M^k Z_t$$

meanwhile for output gap growth we have:

$$E_t(\Delta x_{t+k}) = d_2'(M^k - M^{k-1})Z_t = a_{2k}' Z_t$$

where $a_{2k} = (M^k - M^{k-1})' d_2$.

[1] We note that an intercept is needed in the long-term interest rate in order to accommodate Jensen inequality term, however such term is time-invariant in this case because disturbances are normally distributed. Therefore is irrelevant for the purpose of the next section (β parameter).

2.3 Output Growth and Yield Spread

Note that empirical exercises relate output growth with yield spread, meanwhile the model presented above is based on output gap. Output gap is obtained as residual, and two choices are typically used: (i) Stochastic Trend (e.g. Hodrick-Prescott or Baxter-King) and (ii) Deterministic Trend (a polynomial of time). In this paper we follow Bekaert et al. (2010) which consider a deterministic trend. Thus, the log of real GDP (y_t) can be decomposed into output gap (x_t) and a trend component (p_t) which is a function of t. Thus, the (quarter-over-quarter) change in output is

$$g_{t+k} \equiv y_{t+k} - y_{t+k-1} = \Delta x_{t+k} + \Delta p_{t+k} \tag{6}$$

Usually, the predictive power of yield spread on GDP growth is evaluated in OLS regression:

$$g_{t+k} = \alpha + \beta s_{nt} + w_{t+k},$$

where w_{t+k} is an error term. By conditional expectation we have:

$$g_{t+k} = E_t(g_{t+k}) + e_{t+k}$$

and by using previous definitions we have:

$$E_t(g_{t+k}) = E_t(\Delta x_{t+k}) + p_{t+k} - p_{t+k-1}$$

then we can write output growth as follows:

$$g_{t+k} = E_t(\Delta x_{t+k}) + \Delta p_{t+k} + e_{t+k} = a'_{2k} Z_t + \Delta p_{t+k} + e_{t+k}$$

Thus, the population OLS estimator of β is defined as:

$$\begin{aligned}\hat{\beta} &= \frac{\text{cov}(g_{t+k}, s_{nt})}{\text{var}(s_{nt})} = \frac{\text{cov}[a'_{2k}Z_t + \Delta p_{t+k} + e_{t+k}, a'_{1n}Z_t]}{\text{var}(a'_{1n}Z_t)} \\ &= \frac{\text{cov}[a'_{2k}Z_t, a'_{1n}Z_t]}{\text{var}(a'_{1n}Z_t)} = \frac{a'_{2k}\Gamma a_{1n}}{a'_{1n}\Gamma a_{1n}},\end{aligned}$$

where $\Gamma \equiv E_t(Z_t, Z'_t)$ is the variance-covariance matrix of endogenous variables.

3 Numerical Results

In the previous section we provide a close-form relationship between yield spread and expected output growth under the assumption that economy can be characterized by a standard DSGE model. In this section, we describe the calibration of the model, which is based on Bekaert et al. (2010), the empirical results of Estrella & Hardouvelis (1991), and the numerical results obtained from the calibrated model.

3.1 Calibration of the Model

We calibrate our model based on Bekaert et al. (2010). That paper uses data from 1961Q1 to 2003Q4 for US economy and it studies the implications to include the term structure into a standard DSGE model. It uses both CPI inflation and GDP deflator as price measures, output deviations from a linear trend and quadratically detrended output as output gap, 3-months T-bill rate as short-term interest rate, and 3 and 5 years rates as long rates. The structural model is estimated via GMM, and implied parameters are reported in Table 1, along with Fuhrer (2009), and our baseline calibration. As we can see, our baseline calibration is a modified version of both Fuhrer and Bekaert, Cho and Moreno's calibration. Some interesting features of those parameters are: (i) the effects of output gap on inflation and real interest rate on activity are low, but significant, (ii) output gap has no effect on monetary policy rate, which is consistent with other empirical literature of United States (Clarida et al. 1999).

Table 1: DSGE calibration

	μ	γ	δ	λ	ρ	ϖ	θ	σ_u	σ_v
Bekaert et al. (2010)	0.39	0.06	0.58	0.13	0.72	1.53	0.00	0.71	0.71
Fuhrer (2009)	0.50	0.10	0.50	0.10	0.80	1.50	0.50	0.50	0.50
Baseline	0.40	0.05	0.60	0.10	0.80	1.50	0.00	0.71	0.71

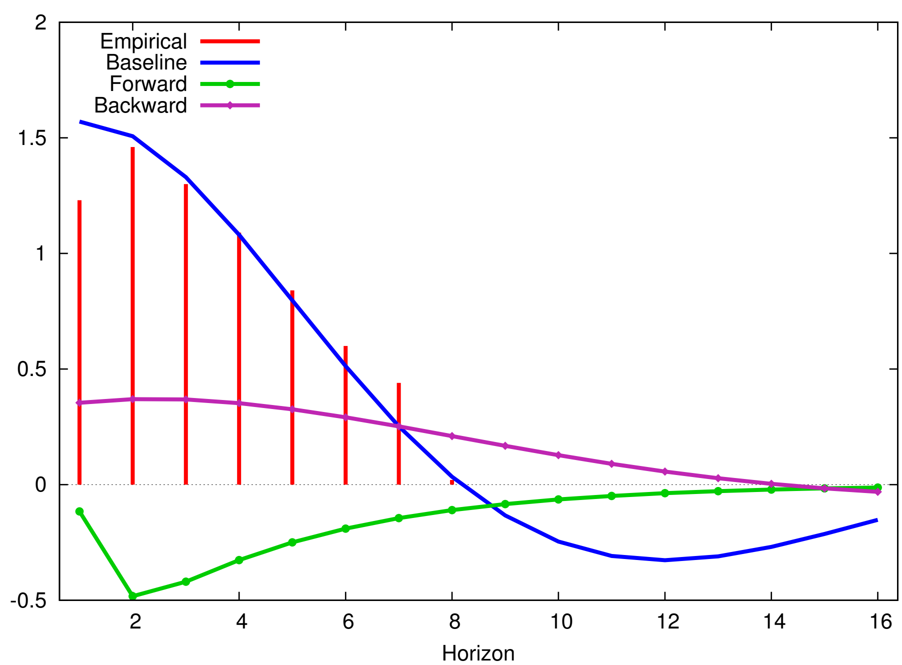
As sensitive analysis we consider: (i) "forward" calibration, setting $\mu = \delta = 0.10$, which is close to forward model in Estrella (2005), and (ii) "backward" calibration, setting $\delta = 0.90$.

3.2 Main Results

Estrella & Hardouvelis (1991) uses the difference between the 10-year government bonds rate and 3-month T-bill rates as yield spread. They study the predictability of both annualized cumulative percentage change in the seasonally adjusted real GNP and the annualized marginal percentage change of the same variable, k-periods ahead for 1955Q2-1988Q4. Using both measures of activity, they find that the yield spread has predictive power in a range of one to eight quarters, even controlling for other variables.

Figure 1 presents the results for our numerical exercises. For comparison, we include empirical results of Estrella & Hardouvelis (1991) for the US economy. We consider $n = 28$ as it is in line with typical duration of 10-year maturity bond, which is 7 years. Our baseline calibration accurately replicates empirical results for horizons two to eight and overestimates the effect of slope on output growth for the first quarter. For alternative parameter settings results are misaligned: (i) Forward calibration has a poorly performance, without being able to replicate the sign of empirical results, and (ii) Backward calibration is able to replicate the sign of predictions but with a lower impact.

Figure 1: DSGE predictions on output growth



Predictions of model (1)-(3) on output growth. Baseline defined in Table 1. Forward uses $\mu = \delta = 0.05$. Backward use $\delta = 0.90$. Empirical corresponds to Estrella & Hardouvelis (1991).

4 Conclusions

Following the large empirical tradition that studies the importance of the yield spread as a leading indicator of output growth, we analyze the structural determinants behind this phenomenon. Indeed, by using an standard hybrid DSGE model calibrated for the US economy, we able replicate the empirical results presented in Estrella & Hardouvelis (1991).

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A Reduced form matrices

In this section we show the exact form of matrices M, N, and the eigenvalues of M for our different parameter settings. We obtain M by iterative methods (DeJong and Dave, 2011) and $N = (A - BM)^{-1}$.

Baseline model

$$M = \begin{pmatrix} 0.481 & 0.433 & -0.427 \\ -0.068 & 1.102 & -0.424 \\ 0.144 & 0.130 & 0.672 \end{pmatrix} ; N = \begin{pmatrix} 1.202 & 0.722 & -0.533 \\ -0.169 & 1.837 & -0.530 \\ 0.361 & 0.217 & 0.840 \end{pmatrix} ; \mathrm{eig}(M) = \begin{pmatrix} 0.594 \\ 0.83 + 0.252i \\ 0.83 - 0.252i \end{pmatrix}$$

Forward model

$$M = \begin{pmatrix} 0.106 & 0.007 & -0.106 \\ -0.017 & 0.110 & -0.404 \\ 0.032 & 0.002 & 0.768 \end{pmatrix} ; N = \begin{pmatrix} 1.061 & 0.066 & -0.133 \\ -0.174 & 1.100 & -0.505 \\ 0.318 & 0.020 & 0.960 \end{pmatrix} ; \mathrm{eig}(M) = \begin{pmatrix} 0.762 \\ 0.115 \\ 0.107 \end{pmatrix}$$

Backward model

$$M = \begin{pmatrix} 0.533 & 1.070 & -0.419 \\ 0.004 & 1.168 & -0.167 \\ 0.160 & 0.321 & 0.674 \end{pmatrix} ; N = \begin{pmatrix} 1.334 & 1.189 & -0.524 \\ 0.011 & 1.298 & -0.209 \\ 0.400 & 0.357 & 0.843 \end{pmatrix} ; \mathrm{eig}(M) = \begin{pmatrix} 0.622 \\ 0.877 + 0.163i \\ 0.877 - 0.163i \end{pmatrix}$$

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