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INTRODUCING LIQUIDITY RISK IN THE CONTINGENT-CLAIM ANALYSIS FOR THE BANKS[*]

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Abstract

Traditional approaches to assess the default risk of a bank fail to recognize their basic operations, granting loans and receiving deposits from the public. The contingent claims approach (CCA) is an extension of the Black-Scholes (1973) and Merton (1974) models that calculate the credit risk of a company by characterizing the company's equity as a call option on its assets. Although a bank can be understood as a firm, banks receive deposits and unlike a common debt, these have to be renewed before the maturity of the assets.

This paper proposes a new approach to measure the default risk of a bank based on the CCA, but it includes a stochastic distress barrier in order to capture the funding volatility of the bank. This new framework provides a method to calculate the implied volatility the bank's assets and their corresponding distance to distress (DD) and the correlation between assets and deposits returns.

This methodology is applied to the Chilean banking system in the period of 2003-2012. The results show that there was an important decrease of the DD that coincides with the increment in the funding volatility during the recent crisis.

Resumen

Las metodologías tradicionales para evaluar el riesgo de incumplimiento de los bancos fracasan en reconocer su negocio principal, otorgar créditos y captar depósitos del público. La metodología de derechos contingentes (CCA) es una extensión del modelo de Black-Scholes (1973) y Merton (1974) que calcula el riesgo de crédito de una firma al caracterizar el capital como una opción de compra sobre los activos. Aunque un banco puede ser entendido como una firma, los bancos reciben depósitos. A diferencia de una deuda común, los depósitos tienen que ser renovados antes de la madurez de los activos.

Este documento propone un nuevo enfoque para medir el riesgo de incumplimiento de un banco basado en CCA. Propone una forma de adaptar el modelo a la estructura bancaria al incluir una barrera estocástica para capturar la volatilidad de los fondos del banco. Esta nueva aproximación proporciona un método para calcular la volatilidad implícita de los activos del banco y su distancia a la insolvencia (DD).

Esta metodología se aplica al sistema bancario chileno en el período del 2003-2012. Los resultados muestran que la DD disminuyó considerablemente, lo que coincide con el incremento en la volatilidad del financiamiento durante la reciente crisis.

[*] I am grateful to Ander Perez and Elisa Alòs for their helpful comments on this paper. E-mail: dodaze@bcentral.cl.

I. Introduction

Among the many techniques to identify fragilities in the banking sector, a common approach is to use Contingent Claim Analysis (CCA) to determine the Probability of Default (PoD) or the Distance-to-Distress (DD) of an institution (Gray, Merton, and Bodie, 2005). The CCA is an extension of the Black-Scholes (1973) and Merton (1974) models, based on the insight that a shareholder has an implicit call option on the value of the assets of the firm. This method is helpful in identifying corporate sector vulnerabilities, according to Gapen et al. (2004). Indeed, Moody's KMV provide a commercial version of PoD, the Expected Default Frequency (EDF), which is based on the DD measure obtained from Merton's model (Dwyer and Qu, 2007). The EDF's are widely used to measure and monitoring credit risk of firms. This indicator has a good performance to predict default of firms over other alternatives such as agency ratings, Altman's Z-Scores, and a simpler version of the Merton model (Korablev and Dwyer, 2007). Moreover, it has been used to analyse credit risk in the recent financial crisis[1]/.

The original Merton model is based on a set of assumptions such as (i) default can occur only at the maturity date of the debt; (ii) there is a fixed distress barrier; (iii) there is a constant risk-free rate; and (iv) asset volatility is constant.

In order to relax the assumption (i), Black and Cox (1976) introduced a "first passage time" model, where default can occur prior to the maturity of the debt when the asset falls below the barrier for the first time. Geske (1977) modelled defaultable coupon debt as a compound option on the firm's value. Related to the assumption (iii), Shimko *et al.* (1993) include a Vasicek interest rate term structure model for the risk-free rate (the discount factor). Longstaff and Schwart (1995) combined the Black and Cox model with stochastic interest rates. Ericsson and Reneby (1998) and Briys and de Varenne (1997) modelled the option to default as a barrier option.

Heston (1993) dealt with the assumption (iv) by introducing a stochastic volatility into the model. The stochastic asset volatility creates the existence of "fat tails" in the asset distribution. On the other hand, Zhou (1997) included periodic jumps in the assets price in the Merton model.

Despite of the numerous extension of the original Merton model, none of them relaxed the assumption of a fixed barrier taking it as a stochastic process[2]/. Additionally, the main objective of these extensions is to value options, and its introduction in the CCA approach is focused on corporate firms.

Nonetheless, there are important differences in the structure of a financial institution. For instance, banks have deposits and a high level of leverage. On the one hand, deposits' interest rates contain relevant market information about bank risks. In addition, funding constraints and bank runs are important sources of risk. On the other hand, high levels of leverage distort the calculation of the implied asset volatility, because the amount of equity is low compared to that of assets[3]/.

Is it correct to directly apply CCA to banks? Do we have to incorporate the volatility of bank's deposits and other liabilities into the model? How can this be done?

[1]/ Dwyer and Woo (2007) analyzed the subprime market fallout using EDF credit measures. Another example is the Fannie Mae and Freddie Mac case of study from Moody's KMV.
[2]/ As we will see later, Margrabe (1978) can be understood as Merton model with a stochastic barrier, even though it is applied to an option to exchange one risky asset for another risky asset at maturity.
[3]/ For example, equity represents in average 7% of the assets of the Chilean bank system.

This paper examines the differences in the structure of a regular non-financial firm with banks, and evaluates if those differences generate distortions in the calculation of the probability of default. If there is any relevant distortion, then it is necessary to adjust CCA for banks. Empirically, it seems that there are some differences. In particular, Moody's KMV compute EDF for banks considering some adjustments (Sellers and Arora, 2004), however, those are justified in empirical facts more than theoretical grounds.

The main idea is to incorporate the information contained in the interest rate of the deposits to calculate the probability of default of a bank. In particular, we include the volatility of the funding cost (approximated by the interest rate on deposits) in the CCA methodology. Although the derivative pricing model in which the model is based was developed by Margrabe (1978), it has never been used before in the CCA. This paper goes further and calculates the implied volatility of the assets when the distress barrier is stochastic.

In addition to the technical discussion, we observe during the recent financial crisis that many central banks and governments applied financial programs in order to mitigate its effects on the real economy and on the stability of the financial system. Some of these programs have had a direct impact on the liabilities of banks. That is the case of deposit insurance programs, bank capitalisation with public funds, and the extension of emergency liquidity facilities. What has been the impact of these policies on financial stability? Are they reflected on the default probability of banks? The inclusion of the volatility of liabilities into the CCA model might incorporate this effect, since these policies should tend to reduce this volatility. If that is true, how well can an adjusted CCA model capture the risk of a bank?

In section II, I develop the pricing equation of corporate equity and the standard contingent-claim methodology. In section III, I disentangle the differences of the corporate and bank liabilities. I also discuss the incorporation of a stochastic distress barrier. In the section IV I solve the mathematical problem. Section V shows how so solve the model. The section VI presents an application of the methodology for the Chilean banking system in 2003-2012. Section VIII mentions some futures steps in this line of research. In the last section, I summarize the findings.

II. Contingent-Claim analysis

Default risk can be defined as the uncertainty surrounding a firm's ability to service its debts and obligations. As a default event cannot be observed until it occurs, the best we can do is to make probabilistic assessments of its likelihood. Nonetheless, due to the low frequency of defaults[4]/, modeling a probabilistic function is a difficult task.

The contingent claim analysis (CCA) is a method that uses the option-pricing theory, pioneered by Black-Scholes (1973) and Merton (1973), to calculate the likelihood of default. This method takes advantage of high frequency data, such as equity market information (forward looking), and combines it with balance sheet data of a firm (backward looking).

The CCA methodology is widely used by financial market participants to measure the default probability of a firm (commonly non-financial). One of the most notable applications of CCA has

[4]/ According to Moodys KMV, a typical firm has a default rate of around 2% in any year. This rate is lower, the better rating the firm has. Then, the default for banks is probably very rare, but an important event, especially for large banks.

come from Moody's KMV. KMV uses an extended model of Black-Scholes-Merton, known as the Vasicek-Kealhofer (VK) model, to calculate the probability of default of a firm[5]/.

The CCA method is based on three principles:

1) The economic value of a firm's equity and liabilities is derived from the economic value of its assets.
2) Equity and liabilities have different seniorities and, therefore, different levels of credit risk.
3) There is a stochastic component in the temporal evolution of the asset value, therefore, in the value of the liabilities and equity.

This method links the option pricing models as follows:

1) The underlying financial (contingent) guarantee can be treated as a put option, and
2) The residual (contingent) claim as a call option, to price equity.

The main idea of this methodology is that the market price of the assets is equal to the sum of the market price of the contingent (or implicit options) of the owners of equity and creditors, respectively, on the underlying price of their assets, which has a stochastic component. In that sense, the market price of the assets is equal to the market price of their liabilities and equity, which incorporate the price of the underlying options.

The market price represents the collective forecasts of many investors. On this line, contingent claim analysis is prospective. Although the perception of the investors is based on historical data, that is their best estimate of future behaviour using current available information. Nonetheless, this kind of analyses contains more forward looking information in contrast to that based on historical financial reports.

Incorporating market information, which is constantly adjusted, into the analysis have two main advantages. First, the speed of change in economic and financial conditions is much greater than the frequency of the available historical information from financial statements. Market indicators are forward looking and they are available at a high frequency. Secondly, the contingent claims approach explicitly considers assets volatility in monitoring and evaluating default risk.

The volatility of assets is essential in this method. Two entities with similar capital structure (equity and liabilities) can have different probabilities of default if the underling volatility of their assets is different. Therefore, the methodology used to capture this volatility has a big impact on the results of the analysis. Thus, we have to be careful in the selection of the method, since there are many ways of calculating a price volatility[6]/.

Contingent claim analysis also has some limitations, which are mainly related to the markets' capacity to correctly assess risks[7]/. Market prices can reflect changes in conditions that might not be related to financial stability. For example, market price rises that are reflected in a higher distance-to-distress (lower default risk) could be due to abundant liquidity, market overreactions to good news, herd behavior, or a different risk assessment than that of the authorities (due to the opaqueness of banks), more than to improvements in fundamentals.

[5]/ The product is called Expected Default Frequency TM (EDF TM)
[6]/ For a survey, see Alfaro and Silva (2008).
[7]/ Persson, M. and M. Blavarg (2003).

Another disadvantage of CCA is that measures of distance-to-distress and the default probability do not adequately capture very short-term financial risks, since they do not allow discontinuities in asset prices, and an agent's debt level is assumed to be constant[8]/.

On the other hand, although contingent claims analysis is a conceptually robust tool for monitoring credit risk, its data requirements are high. In addition, it is computationally complex to implement, mainly due to the difficulties of integrating diverse data from multiple sources.

Nevertheless, indicators based on the behavior of market prices have proved to be good predictors of financial stress, risk ratings[9]/, and several credit risk indicators[10]/.

There is a line of research that applied CCA to the banking sector. Gambacorta (2009) use the EDF as an indicator of bank's risk-taking and link it to monetary policy. Gilchrist *et al.* (2009) uses the EDF to construct an EDF-based bond portfolios and comparing them to standard credit spread indexes.

III. Corporate debt against Bank debt

In the CCA, equity is understood as a subordinate claim whose value is derived from the residual value of the firm once all priority claims (or debt) have been met. Thus an entity's equity owners implicitly have a *call option* on the residual value of the entity's total assets. The economic value of this *call option* fluctuates with the market prices and volatility of the entity's equity.

Debt (or liabilities) is a priority (senior) claim over the asset value, but it is risky because the entity's asset value may be insufficient to cover promised payments. Therefore, the economic value of the debt is thus equal to its risk-free value (or the present value of promised payments) less the expected loss due to the event that the asset value falls below the value of promised payments or debt, which is called the distress barrier. This is because the shareholders have an incentive to declare bankruptcy and turn over the remaining assets to the creditors for liquidation.

Creditors are to be paid the full value of debt but are exposed to the expected loss, which can be modeled as an implicit *put option.* In the event the assets fall below the distress barrier, the creditors would pay out the value of the *put option* to the equity holders and receive the assets of the defaulted entity. Therefore the net position of the creditors is to receive the default-free value of the debt but have losses equal to the implicit *put option*. The value of this implicit guarantee or expected loss is equivalent to the value of an implicit *put option* on the debtor's assets, whose strike price is given by the default barrier and whose value fluctuates according to the market value and volatility of the asset value[11]/.

Hence, as the underlying asset value moves closer to the default barrier, the value of the creditors' implicit *put option* (implicit guarantee) increases, and the value of the risky debt falls. At the same time, the value of the equity owners' implicit *call option* (contingent claim) on the assets also falls. The result is that the market value of the assets approaches the default.

[8]/ The elimination of this assumption is the main idea of this research.
[9]/ Tudela and Young (2003) as well as Gropp, Vesala y Bulpes (2002) find that the distance-to-default measure anticipates changes in the risk ratings of banks in Europe.
[10]/ Chan-Lau and Gravelle (2005); Chan-Lau *et.al.* (2004); Dionne *et al.* (2006).
[11]/ Merton (1974); Chacko *et.al.* (2006).

a. Pricing of corporate equity

In order to develop the contingent-claim pricing model, we make the following assumptions.

i. Frictionless markets. There are no transaction costs, taxes or problems with the indivisibilities of assets.
ii. Borrowing and short-selling are allowed without restrictions.
iii. There exists an exchange market for borrowing and lending at the same rate of interest.
iv. Trading takes place continuously in time.
v. Riskless asset (*r*). There is a riskless asset whose rate of return per unit time is known and the term structure is flat.

The dynamics for the value of the firm (*A*) through time can be described by a diffusion-type stochastic process with stochastic differential equation (as Brennan-Schwartz, 1980)

$$dA = (c + \alpha A)dt + \sigma A dz \tag{1}$$

Where α is the instantaneous expected rate of return on the firm per unit time; c is the total payouts by the firm per unit of time if positive, and it is the net dollar received by the firm if negative; σ^2 is the instantaneous variance of the return on the firm per unit time; and dz is a standard Gauss-Wiener process.

Additionally, we assume that the firm cannot issue any new senior claims on the firm nor can it pay cash dividends or do share repurchase prior to the maturity of the debt. Therefore $c = 0$ in our case and the model becomes (as Black and Scholes, 1973)

$$\frac{dA}{A} = \alpha dt + \sigma dz \tag{2}$$

The partial differential equation for the value of the equity (*E*) is

$$\frac{1}{2}\sigma^2 A^2 E_{AA} + rAE_A - rE + E_t = 0 \tag{3}$$

Solving this PDE, subject to the appropriate terminal condition; we obtain the price for the equity. The firm agrees to pay *B* dollars to debt-holders at date *T*, and the excess correspond to the equity-holders. As in the case examined by Black and Scholes (1973), this corresponds to a common-stock call option with and exercise price of *B* dollars and an expiration date of *T*. Define $\tau \equiv T - t$, the boundary conditions can be written as

$$E/_{A} \leq 1 \tag{4}$$

$$E(0, \tau) = 0 \tag{5}$$

$$E(A, 0) = max(0, A - B) \tag{6}$$

The value of the equity (*E*) is

$$E(A, \tau) = A\Phi(d_1) - Bexp(-r\tau)\Phi(d_2) \tag{7}$$

Where

$$\Phi(x) \equiv \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{x} exp\left(-\frac{1}{2}z^2\right) dz \quad (8)$$

$$d_1 \equiv \frac{\left[log\left(\frac{A}{B}\right)+\left(r+\frac{1}{2}\sigma^2\right)\tau\right]}{\sigma\tau^{1/2}} \quad (9)$$

$$d_2 \equiv d_1 - \sigma\tau^{1/2} \quad (10)$$

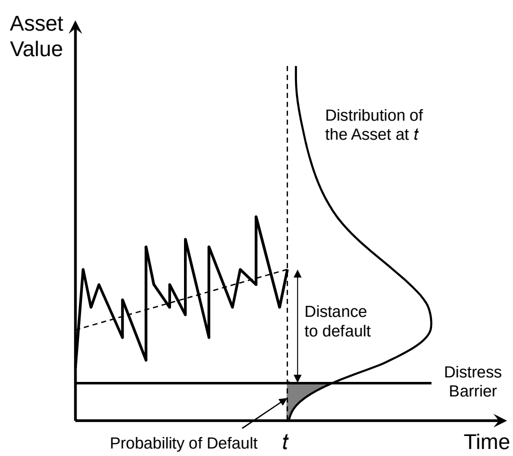
Using the option *delta*, the relationship between volatility of firm assets and volatility of equity is given by

$$E(A,\tau) = \frac{\sigma}{s} A\Phi(d_1) \quad (11)$$

The value of the Equity (*E*) is observed in the market; the volatility of the equity (*s*) can be calculated empirically using market data; the distress barrier (*B*) can be approximated by the firm's debt plus the promised interests; and expiration time can be set arbitrarily, it is usually fixed at one year. The asset value (*A*) and asset volatility (*σ*) can be obtained by solving the system of equations (7) and (11).

Intuitively, we can understand the default probability as it is showed in the next figure:

Figure 1



The distance-to-distress (*DD*) is defined as the difference between the implicit market value of assets and the default barrier, divided by the standard deviation of the implicit market value of assets. It is interpreted as the number of standard deviations at which the implicit asset level, at market value, is away from the default barrier, given the value and volatility of equity, the default barrier, the risk free interest rate, and the horizon period. The *DD* then combines, in a single indicator, the difference between the market value of assets (*A*) and the distress barrier (*B*), with the volatility of the market value of assets (*σ*):

$$DD = \frac{A-B}{A\sigma} \quad (12)$$

This formula can also be used to estimate the default probability. This involves establishing the relationship between the calculated *DD* and the associated probability under the assumption that the *DD* has a standard normal distribution.

b. Pricing of bank equity

As we can notice, the distress barrier (*B*) for the firm is assumed fixed. However, in the case of banks, the distress barrier includes deposits which can change instantly. If the asset turns more risky, it directly affects the funding rate. Funding rate increases to compensate the risk, and, if the level of risk is too high, a bank-run can occur. Intuitively, the depositors set the payoff (*B*) for its deposits (*D*) in the contract in such a way that it is at least as good as their best alternative use.

On the other hand, we can assume that there exists a demand for money. This affects directly the quantity of the deposits and its volatility, and the interest rate demanded by the depositors in order to hold their deposits.

This kind of risk could be interpreted as a Liquidity risk. Funding liquidity risk is a type of liquidity risk that arises when the necessary liquidity to fund illiquid assets positions over a time horizon cannot be obtained in the expected terms and with immediacy. Holmström and Tirole (1998) affirm that liquidity risk arises because revenues and outlays are not synchronised.

Drehmann and Nikolaou (2009) show that funding liquidity risk has two components: i) future (random) flows of money and future (random) prices of obtaining funding liquidity from different sources.

Additionally, information about funding is of high frequency. Moreover, data from deposits interest rates or interbank interest rate can be used[12]/.

Consider a bank that operates for one period, from t to *T*. The bank held a total amount $D(t)$ of deposits and $E(t)$ dollars of equity at the beginning of the period. The bank invests the total amount of funds $D(t) + E(t)$ in risky assets. Suppose that the bank bought a combination of loans and financial instruments with a value of $A(t) = D(t) + E(t)$.

The bank also promised to repay to their depositors the amount of $D(T)$ at the end of the period. The yield to maturity of the deposits r_D is

$$D(T) = D(t)\exp\,(r_D\tau) \qquad (13.a)$$

$$\frac{D(t)}{D(T)} = \exp\,(-r_D\tau) \qquad (13.b)$$

Suppose a three-period economy, where $t < t^* < T$, in which a continuum of agents (depositors) wants to consume at dates t^* and T. The agents (depositors) are subject to liquidity shocks.

At time t the market is in equilibrium and the depositors and bank agree a repayment of $D^*(T)$ at the end of the period. The depositors and the equity holders observe the leverage of the bank at time t, and both agree in the volatility of the assets and deposits (σ_A, σ_D). In other words, given the leverage of the firm the investors know the risk level of the assets.

[12]/ Suppose that the bank is able to face any deposit withdrawal with a loan in the interbank market.

Suppose that at time t^* a fraction $\pi(t^*) \geq 0$ of the deposits are retired. If the depositors liquidate prematurely its deposits, they receive a lower repayment ($D(t) \leq D^*(t^*) \leq D^*(T)$). The bank can react in two ways:

1. The bank liquidates part of their assets to repay the depositors. The bank has to sell an amount $\pi(t^*)D^*(t^*)$ of assets at time t^*.

   Then, the final repayment at time T corresponds to

$$D(T) = \big(1 - \pi(t^*)\big)D^*(T) \leq D^*(T) \tag{14.a}$$

$$\frac{A(t^*) - \pi(t^*)D^*(t^*)}{A(t^*)} A(T) \leq A(T) \tag{14.b}$$

   Therefore, the distress barrier $D(T)$ do not correspond necessarily to the amount $D^*(T)$ set in the contract at time t, and the correlation between assets and deposits is not null[13]/.

2. The bank roll over their debt at a different interest rate. Usually the bank has to choose an expensive funding.

$$D(T) = \big(1 - \pi(t^*)\big)D^*(T) + \pi(t^*)\widetilde{D}(T) \lesseqgtr D^*(T) \tag{15}$$

   In this case, assets and deposits are not correlated if and only if they do not have a common factor. In other words, roll over the debt do not imply assets liquidation. However, other factors, such unemployment, can affect both assets returns and deposits interest rate.

When there is a strong liquidity contraction and the bank is forced to liquidate part of their assets, it enters in fire sales and it has to sell their assets at a price lower than $A(t^*)$. This lowers the final return of their assets, amplify its volatility and, therefore, increment their probability of a bankrupt.

The second case is related to the maturity transformation function of the bank. The risk here is based on the ability of the bank to roll over their deposits. A stable funding reduces the interest demanded by the depositors and its volatility. In this sense, we expect that the lower the mean and variance of the deposits, the lower the risk of default.

In either case, the effective repayment for the debt at time T is not necessarily $D^*(T)$ which was the initial contract. Therefore, the typically distress barrier (*B*) is not fixed anymore, and then we have to introduce a stochastic distress barrier $D(.)$.

However, in what follows, we will assume that the bank roll over their entire debt ($\pi(t^*) = 1$) and, therefore, we will focus in the volatility of the deposits interest rates. In other words, banks do not liquidate assets in order to pay short-term obligations. Instead, they renew their short-term debt at a new interest rate[14]/. Nonetheless, we do not impose a model for the movements of the deposits interest rates but assume that they follow a stochastic process.

[13]/ Notice that a similar conclusion is reach if we consider an increment of the deposits at time t^* taking $\pi(t^*) \leq 0$.

[14]/ The bank can always raise their deposits interest rate to be able to maintain their level of deposits. A strong liquidity constraint, therefore, is viewed as a sharp increment in the promised payment.

The bank's assets at the end of the period correspond to $A(T)$ which use to pay their debt and equity. Two things can happen at the end of the period:

1. If $A(T) \geq D(T)$, the bank is solvent, the depositors gets $D(T)$, and the equity holders gets $A(T) - D(T)$.
2. If $A(T) \leq D(T)$, the assets of the bank are liquidated, and the depositors gets only $A(T)$. Given the limited responsibility, the equity holders get nothing. In such a case $A(T) = D(T)$.

Thus, the terminal payoff for the depositors and equity holders are

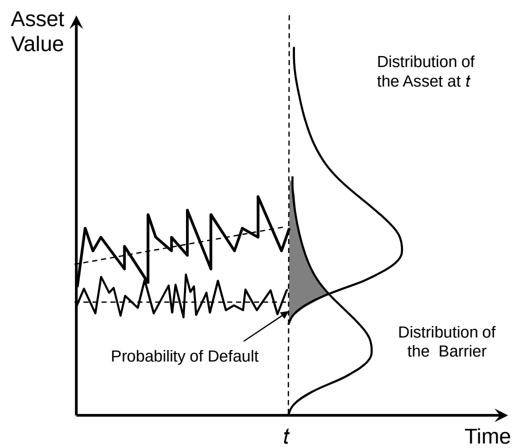
$$D(T) = min\big(D(T), A(T)\big) \leq A(T) \tag{16}$$

$$E(T) = max\big(0, A(T) - D(T)\big) \tag{17}$$

From equations (16) and (17) we can obtain a desirable property. If the bank is fully funded by deposits ($A(t) = D(t)$) then the depositors will get the entire payoff of the assets. In this case, the value of the equity ($E(t)$) must collapse to zero.

The effect on the CCA of the volatility of liabilities can be recognized as follow:

Figure 2



We can use funding interest rate to capture the volatility of (*B*). However, the volatility of *B* does not only affect directly the distress barrier, it also have to be incorporated in the calculation of the assets (*A*) and assets volatility (σ). A simple relationship between those two factors is as follows:

Figure 3

Corporation

Corporate Assets	**Debt** (default-free value minus put option)
	Equity (call option on corporate assets)

Financial Institution

Loans and other Assets (including loans to corporate and public sector)	**Liabilities / Deposits** (default-free value minus put option)
Financial Guarantees (modeled as put option)	**Equity** (call option on financial sector total assets)

IV. The mathematical problem and solution

We keep the same assumptions of the corporate contingent-claim model. The capital market is perfect, and all the returns come from the capital gains. The rate of return of assets (*A*) and deposits (*D*) are given by

$$dA = \alpha_A A dt + \sigma_A A dz_A \tag{18.a}$$
$$dD = \alpha_D D dt + \sigma_D D dz_D \tag{18.b}$$

Where dz_A and dz_D are Wiener processes. The correlation between dz_A and dz_D is ρ_{AD}. We want to value the equity (E) which has the initial condition

$$E(A, D, 0) = max(0, A - D) \tag{19}$$

The equity (*E*) worth at least zero and not more than the assets (*A*), if assets (*A*) and deposits (*D*) are worth at least zero

$$0 \leq E(A, D, \tau) \leq A \tag{20}$$

Following Margrabe (1978), the equity holder can hedge his position by selling $E_A \equiv {}^{\partial E}/_{\partial A}$ units of assets and buying $-E_D \equiv -{}^{\partial E}/_{\partial D}$ units of deposits. Thus, by Euler's theorem, the hedger's investment will be

$$E - E_A A - E_D D = 0 \tag{21}$$

Applying the Itô's lemma, we get

$$dE(.) = E_A dA + E_D dD + E_t dt + \frac{1}{2}[\sigma_A^2 A^2 E_{AA} + 2E_{AD}\sigma_A\sigma_D\rho_{AD}AD + \sigma_D^2 D^2 E_{DD}]dt \tag{22}$$

Taking (21) and (22) we have

$$E_t + \frac{1}{2}[\sigma_A^2 A^2 E_{AA} + 2E_{AD}\sigma_A\sigma_D\rho_{AD}AD + \sigma_D^2 D^2 E_{DD}] = 0 \tag{23}$$

The solution to this differential equation subject to (27) and (28) is

$$E(A, D, \tau) = A\Phi(d_1) - D\Phi(d_2) \tag{24}$$

$$d_1 \equiv \frac{\left[log\left(\frac{A}{D}\right) + \frac{1}{2}\sigma^2\tau\right]}{\sigma\tau^{1/2}} \tag{25}$$

$$d_2 \equiv d_1 - \sigma\tau^{1/2} \tag{26}$$

Where $\Phi(x)$ is the cumulative standard normal density function (8) and $\sigma^2 = \sigma_A^2 - 2\sigma_A\sigma_D\rho_{AD} + \sigma_D^2$ is the variance of $(A/D)^{-1}d(A/D)$.

From the equation (22) we have that

$$\sigma_E^2(A,D,\tau) = \frac{1}{E^2}[\sigma_A^2 A^2 (E_A)^2 + 2E_A E_D \sigma_A \sigma_D \rho_{AD} AD + \sigma_D^2 D^2 (E_D)^2] \tag{27.a}$$

$$E(A,D,\tau) = \frac{1}{\sigma_E}\left[\sigma_A^2 A^2 \left(\Phi(d_1)\right)^2 - 2\Phi(d_1)\Phi(d_2)\sigma_A \sigma_D \rho_{AD} AD + \sigma_D^2 D^2 \left(\Phi(d_2)\right)^2\right]^{1/2} \tag{27.b}$$

The Black and Scholes formula is a special case of (24), where $D = Bexp(-r\tau)$, and $\sigma_D^2 = 0$. Also, if we believe that is indeed the interest rate stochastic, we can obtain the Merton (1973) model taking $D = BP(\tau)$, where $P(\tau)$ is the stochastic value of a default-free discount bond and $P(0) = 1$.

In the Black and Scholes model, the volatility of the deposits comes only from the assets, since they are a put option on the assets. Notice that in this model the volatility of the deposits is due to more of one factor other than the assets volatility.

a. Model I: Return on assets and deposits are uncorrelated

If assets and deposits are uncorrelated, then $\rho_{AD} = 0$. Under this assumption we impose that the bank do not liquidate its assets till maturity, efficiently rollover their deposits, and there is not a factor that affects both assets and deposits.

The information available is the actual price of the equity of the bank (*E*), the variance of the changes in the stock prices (σ_E), the amount of the deposits (*D*), and the variance of the passive interest rates (σ_D). The value of the riskless interest rate (*r*) is no longer necessary.

Therefore, in order to calculate the probability of default we need to find two unknowns, the value of the bank assets (*A*) that is not directly observable, and the variance of the assets returns (σ_A). These values are obtained by solving the following system of equations.

$$E = A\Phi(d_1) - D\Phi(d_2) \tag{s1.1}$$

$$\sigma_E^2 E^2 = \sigma_A^2 A^2 \left(\Phi(d_1)\right)^2 + \sigma_D^2 D^2 \left(\Phi(d_2)\right)^2 \tag{s1.2}$$

$$d_1 \equiv \frac{\left[log\left(\frac{A}{D}\right) + \frac{1}{2}\sigma^2 \tau\right]}{\sigma \tau^{1/2}}$$

$$d_2 \equiv d_1 - \sigma \tau^{1/2}$$

$$\sigma^2 \equiv \sigma_A^2 + \sigma_D^2$$

b. Model II: Return on assets and deposits are correlated but unknown

If we cannot set a fixed value for the correlation, we need additional information to calculate it. Thus, we use the equation of the correlation between equity returns and deposits interest rate to complete the system of equations.

From (18) and (22) we can obtain.

$$\sigma_{DE} = \frac{E_A}{E}\sigma_A \sigma_D \rho_{AD} A + \frac{E_D}{E}\sigma_D^2 D \tag{28.a}$$

$$\sigma_{DE} = \frac{\Phi(d_1)}{E}\sigma_A\sigma_D\rho_{AD}A - \frac{\Phi(d_2)}{E}\sigma_D^2 D \quad (28.b)$$

The covariance of equity returns and deposits interest rate is obtained from the empirical data. Therefore, we can solve the following system of three equations.

$$E = A\Phi(d_1) - D\Phi(d_2) \quad (s2.1)$$

$$\sigma_E^2 E^2 = \sigma_A^2 A^2 \big(\Phi(d_1)\big)^2 - 2\Phi(d_1)\Phi(d_2)\sigma_A\sigma_D\rho_{AD}AD + \sigma_D^2 D^2 \big(\Phi(d_2)\big)^2 \quad (s2.2)$$

$$\sigma_{DE}E = \Phi(d_1)\sigma_A\sigma_D\rho_{AD}A - \Phi(d_2)\sigma_D^2 D \quad (s2.3)$$

$$d_1 \equiv \frac{\left[log\left(\frac{A}{D}\right)+\frac{1}{2}\sigma^2\tau\right]}{\sigma\tau^{1/2}}$$

$$d_2 \equiv d_1 - \sigma\tau^{1/2}$$

$$\sigma^2 \equiv \sigma_A^2 - 2\sigma_A\sigma_D\rho_{AD} + \sigma_D^2$$

Comparing the value of ρ_{AD} obtained with this system and the model I we can test if the value is statistical equal to zero. The volatility comes from the estimation of the covariance matrix of *E* and *D*. Given the non-linearity of the equations, the confidence intervals must be calculated numerically.

The IMF[15]/ demonstrates that the two-dimensional Newton method used to solve the nonlinear system fails to get a good numerical solution when the equity value is too small. They suggest a transformation of the system and equations for the initial values. An idea is to incorporate such adjustment[16]/.

V. System solution

The Newton method can be used to solve the nonlinear system proposed in the Model II. Notice that $0 \le \Phi(d_1) \le 1$ and $0 \le \Phi(d_1) \le 1$, then we can construct the following intervals using the system of equations assuming that $\rho_{AD} = 0$.

$$\sigma_E^2 E^2 = \sigma_A^2 A^2 \big(\Phi(d_1)\big)^2 + \sigma_D^2 D^2 \big(\Phi(d_2)\big)^2$$

$$\sigma_E^2 E^2 = \sigma_A^2 A^2 \left(\frac{E + D\Phi(d_2)}{A}\right)^2 + \sigma_D^2 D^2 \big(\Phi(d_2)\big)^2$$

$$\sigma_A = \left[\frac{\sigma_E^2 E^2 - \sigma_D^2 D^2 \big(\Phi(d_2)\big)^2}{\big(E + D\Phi(d_2)\big)^2}\right]^{1/2}$$

Then

$$\left[\frac{\sigma_E^2 E^2 - \sigma_D^2 D^2}{(E+D)^2}\right]^{1/2} \le \sigma_A \le \sigma_E \quad (29)$$

[15]/ The method is documented in the Balance Sheet Risk Analysis (BSRA) used at the IMF, and implemented in the IMFLibrary.dll.
[16]/ This adjustment is applied in section V.

And

$$\sigma_E^2 E^2 = \sigma_A^2 A^2 \big(\Phi(d_1)\big)^2 + \sigma_D^2 D^2 \big(\Phi(d_2)\big)^2$$

$$\sigma_E^2 E^2 = \sigma_A^2 A^2 \big(\Phi(d_1)\big)^2 + \sigma_D^2 D^2 \left(\frac{A\Phi(d_1) - E}{D}\right)^2$$

$$\big(\Phi(d_1)\big)^2 (\sigma_A^2 + \sigma_D^2) A^2 - 2\sigma_D^2 \Phi(d_1) E A + (\sigma_D^2 - \sigma_E^2) E^2 = 0$$

Then

$$A \geq \frac{2\sigma_D^2 E \pm E\sqrt{4\sigma_D^4 - (\sigma_A^2 + \sigma_D^2)(\sigma_D^2 - \sigma_E^2)}}{2(\sigma_A^2 + \sigma_D^2)} \qquad (30)$$

Then we can set the initial value of σ_A as the middle of the interval; the initial value for *A* as his lower bound corresponding for σ_A^0; and ρ_{AD} as the lower correlation possible.

$$\sigma_A^0 = \frac{1}{2}\left(\left[\frac{\sigma_E^2 E^2 - \sigma_D^2 D^2}{(E+D)^2}\right]^{1/2} + \sigma_E\right)$$

$$A^0 = \frac{2\sigma_D^2 E \pm E\sqrt{4\sigma_D^4 - (\sigma_A^{02} + \sigma_D^2)(\sigma_D^2 - \sigma_E^2)}}{2(\sigma_A^{02} + \sigma_D^2)}$$

$$\rho_{AD}^0 = 0$$

The system of equations has three equations and three unknowns. We can reduce the system to two unknowns[17]/. From equation (24) we have

$$A\Phi(d_1) = E + D\Phi(d_2) \qquad \text{(s2.1)}$$

Replacing (s2.1) into equation (s2.3) we have that

$$\sigma_{DE} E = \sigma_A \sigma_D \rho_{AD} \big(E + D\Phi(d_2)\big) - \sigma_D^2 D \Phi(d_2) \qquad (31)$$

Then

$$\Phi(d_2) = \frac{E(\sigma_{DE} - \sigma_A \sigma_D \rho_{AD})}{D(\sigma_A \sigma_D \rho_{AD} - \sigma_D^2)}$$

$$d_2 = \Phi^{-1}\left(\frac{E(\sigma_{DE} - \sigma_A \sigma_D \rho_{AD})}{D(\sigma_A \sigma_D \rho_{AD} - \sigma_D^2)}\right) = f_2(\sigma_A^2, \rho_{AD}) \qquad (32)$$

Additionally, we know that

$$d_1 = d_2 + \sigma\tau^{1/2} = f_1(\sigma_A^2, \rho_{AD}) \qquad (33)$$

[17]/ Using a similar procedure we can reduce the Black and Scholes problem to a single equation. Then we can solve the one-dimensional problem using the bisectional method.

On the other hand, by the definition of d_2 we have that

$$d_2\sigma\sqrt{\tau} = log(A) - log(D) - \frac{\tau}{2}\sigma^2$$

$$A = Dexp\left(d_2\sigma\sqrt{\tau} + \frac{\tau}{2}\sigma^2\right) = \tilde{g}(\sigma_A^2, \rho_{AD}, d_2) = g(\sigma_A^2, \rho_{AD}) \quad (34)$$

Then we can substitute (32), (33), and (34) into equations (s2.2) and (s2.3) and the system becomes

$$\sigma_E^2 E^2 = \sigma_A^2\big(g(\sigma_A^2, \rho_{AD})\big)^2\big(\Phi(d_1)\big)^2 - 2\Phi(d_1)\Phi(d_2)\sigma_A\sigma_D\rho_{AD}g(\sigma_A^2, \rho_{AD})D + \sigma_D^2 D^2\big(\Phi(d_2)\big)^2 \quad (s3.1)$$

$$\sigma_{DE}E = \Phi(d_1)\sigma_A\sigma_D\rho_{AD}g(\sigma_A^2, \rho_{AD}) - \Phi(d_2)\sigma_D^2 D \quad (s3.2)$$

Where σ_A^2 and ρ_{AD} are the two unknowns. Then we can solve the system using a two-dimensional Newton's method. However, the Newton method family algorithms can present some numerical problems in the calculation of the derivatives[18]/. Therefore, I use the Nelder-Mead Method[19]/ or downhill simplex method in order to avoid numerical problems that affects derivative-based optimization algorithms.

VI. Application

a. Data

I calculate the Distant-to-distress for the Chilean banking system for the period of 2003-2012 using information from three different sources. First, I use the Balance Sheet of banks which is compiled by the Superintendency of Banks and Financial Institutions (SBIF). The data is presented in monthly basis and is available since 1998. This dataset includes information about the liabilities of the banks including deposits.

Second, the *Information System* of the SBIF registers in the file D31 the interest rates of banks transactions daily. This information is available since 2001. I use this file to calculate the average interest rate of the deposits for each bank.

Third, I obtain the number and the price of the stocks from *Bloomberg*. There are approximately 25 banks operating in Chile between 2001 and 2012. However, only five of those banks are registered in the stock exchange. Some of them have a low amount of transactions and the price movements are flat, or its price history is too short. Therefore, I chose to work only with three banks. Even so, these three banks represent about 50% of the total banking assets.

Combining these three sources, I have a daily dataset from January 2003 to October 2012, constituting 2568 observations. Notice, however, that the monthly information from the balance sheets is repeated for each day in the month.

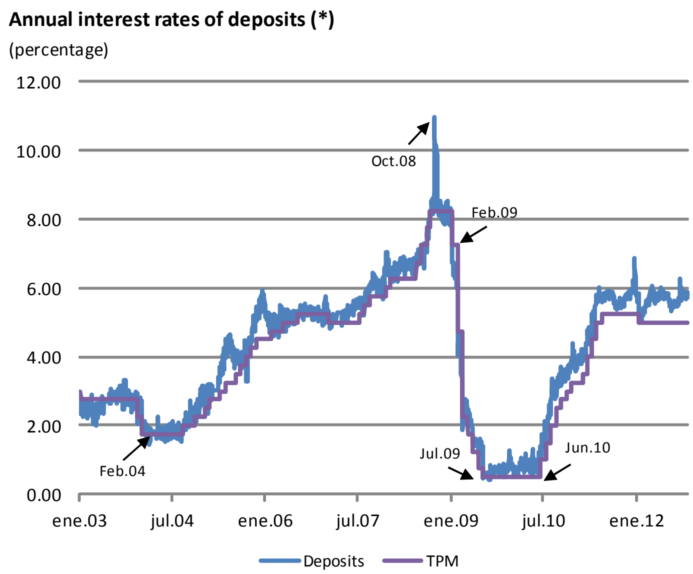
[18]/ Newton's method can run into difficulties. The Jacobian may be singular, and so the Newton steps are not even defined. Also, the exact Newton steps may be expensive to compute. In addition, Newton's method may not converge if the starting point is far from the solution.

[19]/ The Nelder-Mead (1965) method is one of the best known algorithms for multidimensional unconstrained optimization without derivatives.

b. Equity and Interest rates volatility

Using the data from the D31, I calculated the average interest rate for deposits in Chilean Pesos (CLP) per day. Deposits in CLP are the main source of fund for the banks. Thus, I take the nominal interest rate as a representative rate for the liabilities[20]/. The interest rates of the deposits are presented in the Graph 1. We consider the monetary policy rate (*Tasa de Política Monetaria* – TPM) as the risk-free interest rate. Subsequently, we take the spread between the interest rate of deposits and the TPM (risk premium) to clear the movements of the riskless asset.

Graph 1



(*) Weigthed average for the three bigest banks of Chile.

Then, I use a GARCH(1,1) model to estimate the volatilities of the equity returns and deposits interest rate spread. Another common practice is calculating the volatilities using the exponentially weighted moving average (EWMA) model. The EWMA model is a special case of the GARCH model where $\alpha + \beta = 1$. However, this hypothesis is not accepted for equity, but not rejected for the deposits interest rates (Annex 1). The estimated volatilities are showed in the Graph 2. The covariance between deposits and equity returns is estimated by a varying conditional correlation multivariate GARCH, Tse and Tsui (2002).

The period of high volatility in 2004 is explained by the increment in the rate of return on government bond in the U.S. In mid-March, the rate of return on 10-year bonds raised 50 basis points. Stock indices for the most developed economies were more volatile in the second quarter of 2004, mainly associated with uncertainty about how quickly interest rates would return to normality.

In 2008, the complex financial situation of many of the international banks in the U.S. and Europe caused a deep uncertainty in the capital markets in developed countries. These increased the perception of financial risk in the banking industry and raised the precautionary demand for liquidity, driving up deposits rates. This is reflected in the increase of the volatility of the banks' equity shares and deposits interest rates between late September and early October. The volatility of the deposits

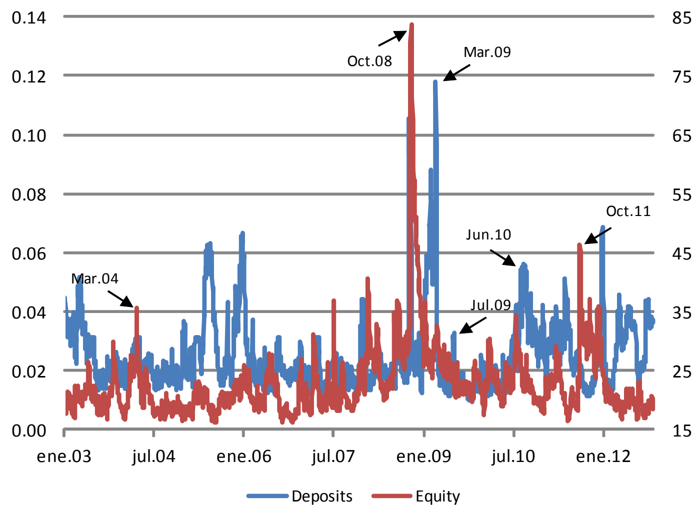
[20]/ The other sources of funding are constituted in USD and UF (*Unidades de Fomento*).

interest rates reached its peak in the first week of October, just after the Lehman Brothers filed for bankruptcy, while the volatility of equity increased till October 15th.

Graph 2

Volatility of Deposits and Equity Returns (*)

(percentage)



(*) Weigthed average for the three bigest banks of Chile.

In this context, on October 10th, the Central Bank of Chile proceeded to temporally accept banks deposits as collateral for seven-day repo operations to be in effect, initially, for a period of six months. On December 10th, the period was extended through the end of 2009 and, additionally, a complementary mechanism was introduced to provide longer-term liquidity, based on a line of credit that accepts General Treasury bonds, among others, as effective collateral.

The volatility of the deposits' interest rates dropped until the first quarter of 2009. This was due to the expectations of a monetary policy loosening and the aggressive reductions in the policy rates[21]/. The following rebound of the volatility is probably a response to a reduction in inflation expectations, a faster-than-expected economic recovery, and an increase in external long-term interest rates[22]/.

In July 9th, the Central Bank of Chile cut the monetary policy rate to a historical low of 0.5% in annual terms. Additionally, it introduced the Term Liquidity Facility (FLAP) for banking institutions, through the Bank provides liquidity at 90 and 180 days at the prevailing monetary policy rate. With the introduction of the FLAP, the funding interest rate sharply decreased after its introduction. This dramatically change in the interest rate curve increased the volatility of interest rates. This effect lasted until the FLAP closes in May 2010. After that period, the volatility returned to a level similar to the pre financial crisis period.

The jump in the equity volatility in October 2011 raised in a context of uncertainty about the European Union, and especially in a loss of confidence in the Spanish banking sector, which affects direct or indirectly the Chilean banking system.

[21]/ Financial Stability Report First Half 2009, Central Bank of Chile.
[22]/ Financial Stability Report First Half 2009, Central Bank of Chile.

c. Distress Barrier

The promised payment to the liabilities represents a barrier that triggers default. In the Merton model, the default takes place when the value of assets is less than the promised payments due on the debt. In the real world, default occurs at higher assets values. This is because of a material violation of a debt covenant, or because assets cannot be sold to meet the payments ("inadequate liquidity"), or because a strategic default. Gray and Malone (2008) pointed out, to capture these real-world conditions for default in the model, we have to specify a market value of total assets at which the sovereign will default, which is called the "distress barrier".

Usually, the barrier level is set equal to the sum of the book value of the short-term debt, promised interest payments for the next 12 months, and half of long-term debt[23]/. In order to calculate the Merton model, I define the distress barrier as[24]/:

Demand deposits + 0.65 x (Time deposits + Bonds)

In the model presented in this paper, the bank maintains $D(t)$ deposits with the condition to pay $D(t)\exp(r_D\tau)$ at the maturity. Following this model, the distress barrier corresponds to the full debt plus the promised interest payments. I consider that the bank needs to roll-over the entire debt at time T. Therefore, we do not need to define a distress barrier at time *T*, but the value of the debt, giving that we know the distribution of r_D. The value of the debt is defined as:

Demand deposits + Time deposits + Bonds

This paper centers the discussion in the inclusion of a stochastic distress barrier. As we saw in the previous section the interest rate is not fixed. In addition, the volatility of the interest rates changes over time and it is different from zero. Despite of its low magnitude, it becomes important because the bank leverage is *per se* high. In the case of Chile, the liabilities are close to the 80% of the assets.

d. Implied volatilities and Distant-to-Distress

The implied volatility of assets is calculated for both the Merton and the model presented is this paper. By convention, the time to maturity is fixed to one year and I considered the TPM as the risk-free return. The results are presented in the Graph 3. As the volatility of the stock returns, the assets volatility has three peaks, in February 2004, October 2008, and October 2011. The implied volatility calculated using the Merton model is higher. However, the movements of the two series are practically the same.

This is actually an expected result. When we introduce an additional risk in the contract, say the inclusion of the deposits volatility[25]/, the value of the equity (the call-option price) is lower. But, when we go backwards in order to calculate the implied volatility, both models starts with the same equity value. Therefore, the implied asset volatility is lower when we introduce more risks.

[23]/ See Crouchy *et al.* (2000)

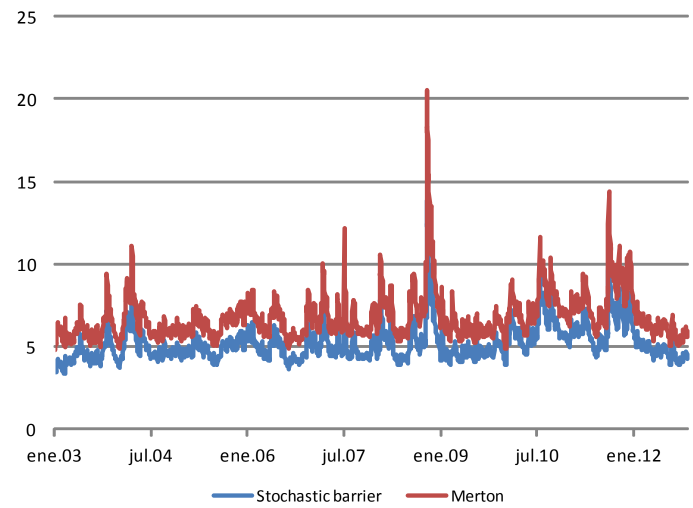
[24]/ We take 0.65 of the long-term as a conservative barrier. The results of the Merton model using the full debt plus the interest payments do not change the conclusions qualitatively.

[25]/ Black and Cox (1975) considered a safety covenant in the senior debt contract. While this increase the value of the senior debt (deposits in our case), the value of the equity decreases. Since debt-holders can force the bank into bankruptcy before the maturity if the value of the assets falls to the distress barrier, the expected payoffs for the equity holders reduces.

Graph 3

Implied Volatility of Assets by type of distress barrier (*)

(percentage)



(*) Weigthed average for the three bigest banks of Chile.

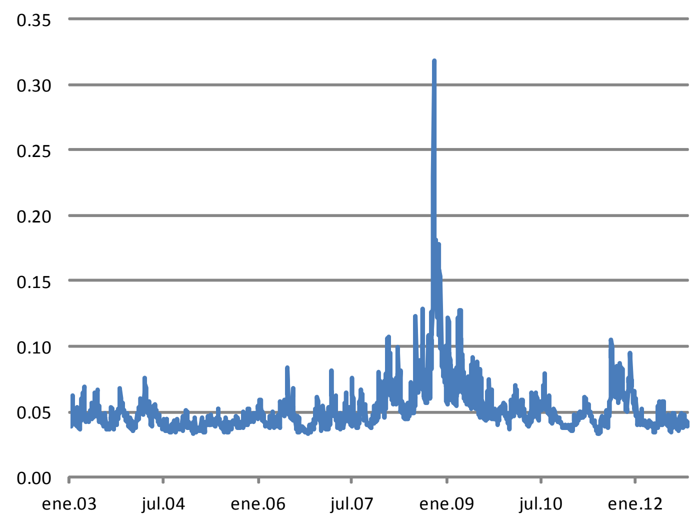
The CCA models suppose that the agents evaluate the equity as a call-option. However, in practice, we cannot verify which kind of contract really is, but we observe a market price. I assume, in my model, that the agents incorporate the information of the deposits volatility in their assessment.

On the other hand, as we can see in the Graph 4, the implied correlation between assets and deposits returns is positive and generally low. The correlation increased in the period of the recent crisis reached a maximum of 0.32 on October 17^{th} 2008.

Graph 4

Implied Assets and Deposits correlation (*)

(units)



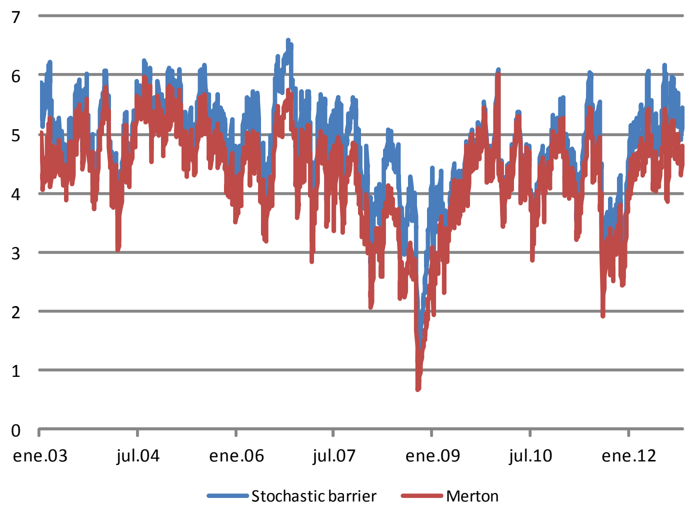
(*) Weigthed average for the three bigest banks of Chile.

The distant-to-distress indicator shows the same behavior for both models (Graph 5). The Merton model indicates a lower distant or higher probability of default. Notice that in normal periods, the distant-to-distress is over 2 standards deviations, indicating an almost zero probability of default. Despite that no bankruptcy events occurred in the Chilean banking system during the recent crisis, the probability of default increased. The latter can be explained in part by the sharp increment in the funding cost and the loss of confidence in the banking activity due to the turbulence in the U.S. market (2008-2009) and in Europe (2011-2012).

Graph 5

Distant-to-Distress by type of distress barrier (*)

(number of standard deviations)

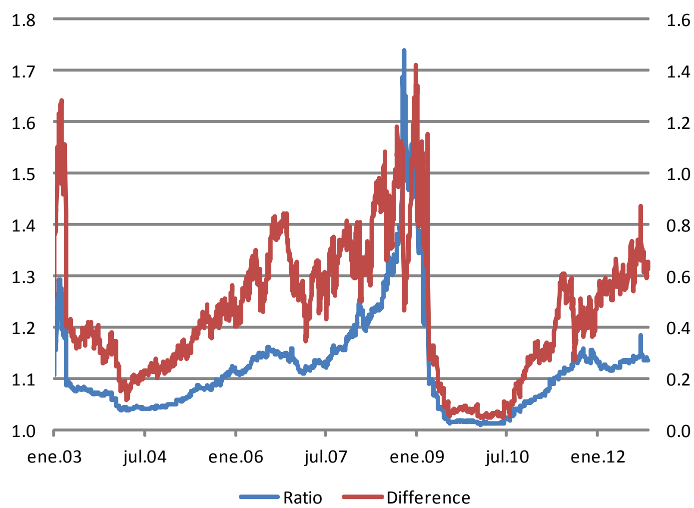


(*) Weigthed average for the three bigest banks of Chile.

Graph 6

Discrepancies between the stochastic and fixed distress barrier indicator (*)

(times)



(*) Weigthed average for the three bigest banks of Chile.

Nonetheless, there is not a fixed relation between the Merton model and our method. As we can appreciate in the Graph 6, neither the ratio nor the difference between both indicators is constant over time. The bigger divergence between both indicators is also presented during the recent crisis in October 2008.

We know that our model encompasses the Merton model as a special case. Considering a sufficiently low risk-free interest rate, if the volatility of the deposits converges to zero, both models will give us the same magnitude of default probability and distant-to-distress. The bigger the volatility of the deposits is, the bigger the discrepancy between the models.

One advantage of this model is that we have an impact of the funding volatility in the probability of distress of a bank. In that sense, efforts to reduce that volatility can be observed in our indicators. In other words, funding stability reduces the stress of the banks.

VII. Other applications and extensions

We can use this framework to assess the vulnerabilities of a particular banking system. Given the information available, we can calculate the distant-to-distress for each bank and looks how it behave during a crisis. Some evidence from the aggregate results are presented in this paper. Nonetheless, the aggregation of the probabilities of default or the construction of an indicator for the whole banking system is not trivial. The interbank market is an important factor in the funding stability of the system. Therefore, a particular or isolated bank funding distress can be solved through this channel, but this is not necessary true when the problem systemic.

Also, we can use this model to simulate the effect of a policy that affects directly the deposits stability (deposits volatility forecasting), such insurances, liquidity easing, and so on. We can obtain the parameters using real data for a given point in time. Then, we use those parameters to simulate the probabilities of default in scenarios of stress.

The difference between the stock prices of a model with and without funding volatility can be interpreted as a funding volatility premium. The stock price is directly related with the Distant-to-distress. In that sense, the discrepancies of DD can be informative about the ability of the bank to roll-over their deposits. A liquidity shock can be introduced here as a jump in the funding volatility that limit the capacity to rollover the debt.

There are some extensions of this paper. We can construct a formal method to test the value of the correlation between assets and deposits. In this line, we can investigate if this correlation comes from common factors, say macro-variables, or from the bank management.

Additionally, we can include a “safety covenant” in a same fashion as Black and Cox (1976) for the deposits. If we believe that the depositor has the right to force bankruptcy before the maturity (as a bank run), we have to consider the first time that the bank falls below the barrier. There is a probability that the assets value be greater that the promised payment at the maturity date, even when the assets value crossed the distress barrier before. Such an event can be viewed as a probability of a bank run following the idea of Diamond and Dybvig (1983).

The objective of CCA is to capture the implied assets value and asset volatility (and implied probability of default) from the market data. In other words, we are trying to find which parameters the investors are using. But in this task, we are assuming that the investors value the equity in a given way, not necessarily correct, but following a certain model. Find the used model is probably unfeasible, and take the more realistic one is the best we can do.

Despite of that, there still one parameter that we are not adjusting to the investors behaviour, which is the time to maturity. Time to maturity is usually fixed to one year, but it must correspond to the investment horizon of the equity holders. The election of the time to maturity is not trivial because it has a direct impact on the equity value (Theta is different from zero). One next step in this line of research is to evaluate the sensitivity of the results to changes in the time to maturity and how the results match to the observed default frequency.

On the other hand, we can use the CCA framework to the deposits which are modelled as a put-option. In the case of Chile, only few banks are registered in the stock market, but the information about their deposits are available for all of them. Another advantage of that is that we have information regarding the term structure of the debt, and, therefore, we can set the time to maturity. If Modigliani-Miller still holds, the results must be the same as the analysis using the equity.

VIII. Conclusions

A bank can be defined as *"an institution whose current operations consist in granting loans and receiving deposits from the public"*. Banks finance a large fraction of their assets through deposits, and this is one of the main factors of the fragility of the banks. Unlike a common debt, the nature of the deposits is not necessarily the one specified in the initial contract. Early withdrawals, changes in the interest rates or variations in the economy activity may affect the deposits volume and their demanded return. Even if we assume that the deposit volume is constant, the maturity mismatch between assets and deposits makes the banks to rollover their debt at a different interest rate. Therefore, we can understand the deposits as a stochastic barrier when we modeled the equity as a call option on the assets.

In this new problem, there are three non-observable variables, the value of the bank's assets, the volatility of the assets, and the correlation between assets and deposits. The new framework encompasses the Black and Scholes model as a special case.

The application of this framework to the Chilean banking system showed some interesting results. First, we observe that the interest rate volatility is not null and is not constant. This confirm the need of include this information in the model. Second, the implied correlation of assets and deposits is also not null and is higher during the crisis.

Distant-to-distress reduced sharply after Lehman failed. The evidence suggests that this was an effect of the loss in confidence in the financial market and the increment in the financing cost. After that episode, a set of policies was implemented in Chile to, among other objectives, relief the liquidity requirements of the financial institutions.

The model presented in this paper shows plausible results during the financial crisis. One advantage of the model is that incorporates the effect of the funding volatility in a comprehensive framework. As we saw, discrepancies between include or not the funding volatility are important in the calculation of distress indicators. This is a good starting point to develop more realistic models for the banking system based on market information.

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Annex 1
Volatility model: GARCH(1,1)

Stock returns

	Bank 1	Bank 2	Bank 3
Level			
Constant	0.008	0.017	0.013
	(0.028)	(0.025)	(0.022)
ARCH			
ARCH(1)	0.059	0.097	0.077
	(0.005)	(0.007)	(0.006)
GARCH(1)	0.921	0.810	0.910
	(0.006)	(0.016)	(0.006)
Constant	0.052	0.171	0.030
	(0.009)	(0.021)	(0.005)
Observations	2595	2595	2595
Ho: $\alpha + \beta = 1$			
$Chi^2(1)$	24.7	63.4	13.0
Prob > Chi^2	0.000	0.000	0.000

** Standard errors in parenthesis.*

Deposist's interest rate

	Bank 1	Bank 2	Bank 3
Level			
Constant	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
ARCH			
ARCH(1)	0.314	0.421	0.253
	(0.025)	(0.030)	(0.024)
GARCH(1)	0.648	0.558	0.753
	(0.025)	(0.024)	(0.020)
Constant	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
Observations	2762	2759	2864
Ho: $\alpha + \beta = 1$			
$Chi^2(1)$	5.420	1.600	0.390
Prob > Chi^2	0.020	0.207	0.534

** Standard errors in parenthesis.*

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