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ARE FORECAST COMBINATIONS EFFICIENT?

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Abstract

It is well known that weighted averages of two competing forecasts may reduce Mean Squared Prediction Errors (MSPE) and may also introduce certain inefficiencies. In this paper we take an in-depth view of one particular type of inefficiency stemming from simple combination schemes. We identify testable conditions under which every linear convex combination of two forecasts displays this type of inefficiency. In particular, we show that the process of taking averages of forecasts may induce inefficiencies in the combination, even when the individual forecasts are efficient. Furthermore, we show that the so-called "optimal weighted average" traditionally presented in the literature may indeed be sub-optimal. We propose a simple testable condition to detect if this traditional weighted factor is optimal in a broader sense. An optimal "recombination weight" is introduced. Finally, we illustrate our findings with simulations and an empirical application in the context of the combination of inflation forecasts.

Resumen

Es sabido en la literatura predictiva que el promedio ponderado de dos pronósticos puede reducir el Error Cuadrático Medio de Proyección (ECMP) y que también puede introducir algún tipo de ineficiencia. En este trabajo ahondamos en un tipo específico de ineficiencia que podría surgir a través del proceso de combinación de pronósticos. Identificamos condiciones testeables bajo las cuales todo promedio ponderado de dos pronósticos presenta el tipo de ineficiencia en cuestión. En particular, mostramos que la combinación de pronósticos puede ser ineficiente, incluso en el caso en que los dos pronósticos que conforman la combinación sean eficientes. Además, mostramos que la combinación que tradicionalmente es considerada óptima, podría ser subóptima en un sentido más amplio de la palabra. Proponemos también una simple condición testeable para evaluar esta situación e introducimos una recombinación óptima en caso que la optimalidad de la combinación tradicional sea rechazada. Finalmente, ilustramos nuestros hallazgos con ejemplos simulados y con una aplicación empírica en el contexto de la combinación de pronósticos de inflación.

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1 Introduction

It is common to find in practice situations in which decisionmakers are confronted with two or more forecasts for the same target variable. In this scenario, Elliot & Timmermann (2008) identify two possible different strategies: the search for the best possible single forecasting method, and the search for the best possible combination of the available forecasts. This last strategy has received a lot of attention in the literature since the seminal work of Bates & Granger (1969). In their article the authors combined two sets of forecasts coming from airline passenger data. They conclude that a composite forecast may display lower Mean-Squared-Prediction-Error (MSPE) than either of the single original projections[1].

Since 1969 a number of different papers have been written on topics directly or indirectly related to the combination of forecasts. During the next two decades a sample of influential work includes the articles by Newbold & Granger (1974), Granger & Ramanathan (1984), Clemen (1986) and Diebold (1988). More recent papers have also been published in the topic, including for instance, Batchelor & Dua (1995), Harvey, Leybourne & Newbold (1998), Stock & Watson (2004), Aiolfi & Timmermann (2006), Hansen (2008), Capistrán & Timmermann (2009), Clements & Harvey (2011), Poncela, Rodriguez, Sánchez-Mangas & Senra (2011), Kolassa (2011) and Costantini & Kunst (2011).

Despite the huge variety of combination methods available in the literature, two particular families of combination strategies have attracted special attention. Using the terms in Diebold (1988), these two families are known as the variance-covariance method of Bates & Granger (1969) and the regression method introduced by Granger & Ramanathan (1984). Broadly speaking, the first approach generates the combined forecast as a weighted average of the pool of single individual forecasts. Notice that this weighted average need not to be convex. The latter approach is one in which the combining weights are obtained as the coefficient estimates of a regression between the target variable and the set of available individual forecasts.

[1] Even when superior predictive ability of one forecast over another is suspected one could test for forecast encompassing. Granger & Newbold (1973, 1986) claim that the superior accuracy of one forecast over another does not necessarily mean that the inferior forecast is useless. It could be the case that the superior forecast could benefit from using some of the information contained in the outperformed forecast. Granger & Newbold consider the possibility that an average of the superior and inferior forecasts may yield a new and more accurate forecast. When this is not possible, it is said that the superior forecast is not only more accurate than the outperformed forecast, but also encompasses it (see Chong & Hendry, 1986 and Clements & Hendry, 1993).

In general terms, the combination of forecasts is reported as a successful strategy to improve forecast accuracy. Elliot & Timmermann show an interesting table in which the simple average of several methods outperforms either of the individual forecasts available for US inflation. This is just one example of a pattern that the literature has been exploring so far. More empirical examples of the good behavior of combination schemes are found, for instance, in the papers by Newbold & Granger (1974), Wright (2008) and Clements & Harvey (2011).

Different theoretical approaches aim at explaining the success of combination strategies. Given a set of forecasts and a loss function, the optimal combination could be found as the solution of an optimization problem looking for weights to minimize the expected loss. In many applications such an optimization problem is well defined and leads to non trivial optimal weights, ensuring reductions in the loss function, or in other words, ensuring combination gains. Timmermann (2006) provides an interesting summary of different environments in which combination gains are possible.

Despite these theoretical efforts, some questions are still unresolved. For instance, part of the relevant literature investigates what is known as the "Combination Puzzle ", which, in the version of Aiolfi, Capistrán & Timmermann refers to "*...the common finding that an equal-weighted forecast is surprisingly difficult to beat.*" Aiolfi, Capistrán & Timmermann (2011) page 2. More generally, as mentioned by Hansen (2008) it is still not entirely clear how to build the forecasts weights that will be used in a combination.

While it is clear that combination gains may exist in a number of applications, part of literature analyzes the efficiency of some combination strategies. For instance, Diebold (1988) indicates that the regression approach is a combination scheme that leaves room for improvement due to the autocorrelacion in the residuals that is inherent to this combination method. Diebold (1988) and Timmermann (2006) also mentions that the Bates and Granger approach is potentially inefficient due to the introduction of the constraint of the coefficients summing to unity. The extent to which these inefficiencies are indeed relevant requires a case by case analysis[2]. Nonetheless it is striking that in many applications in which a number of different forecasts are available, the combination of all of them seems to be the last step in the search of forecast accuracy, and no attempt to take advantage of potential inefficiencies stemming from the combination process is carried out.

[2]See, for instance, Clemen (1986).

In sharp contrast with this usual practice, in this article we explore a particular property of convex linear combination of forecasts. This property is called forecast auto-efficiency, and refers to the notion of efficiency analyzed by Mincer and Zarnowitz (1969). Under mild assumptions we show that linear convex forecast combinations are auto-inefficient with probability one, and therefore room for accuracy improvement is almost surely possible. This implies that greater reductions in MSPE are possible and has the further implication that the traditional optimal linear combination weights could be sub-optimal in a broader sense. Furthermore, this auto-inefficiency does not allow for a supplementary interpretation of MSPE as suggested by Patton & Timmermann (2012). We also show that certain symmetry condition is sufficient to ensure that the traditional combination scheme is optimal in this broader sense.

We focus on linear convex combinations because they are used by practitioners in many empirical applications. In particular Consensus Economics reports individual forecasts and their simple averages. Furthermore, many simple linear convex combinations are considered to be very accurate, which is consistent with the aforementioned combination puzzle. In addition, these combination strategies allow for an interpretation of the combination as a consensus forecast. Finally, many of these linear convex combinations do not require previous knowledge of the target variable to construct the combined forecast, which is a clear advantage of simple methods against the Granger & Ramanathan (1984) approach.

The rest of the paper is organized as follows. In Section 2 we set the econometric environment. Section 3 contains the main theoretical results. Section 4 displays illustrative examples of our findings as well as an empirical application. Finally Section 5 concludes and presents possible extensions for further research.

2 Econometric Environment

Let us consider $\{Y_t\}$ to be a stationary and ergodic time-series process. We will assume that at time t we want to forecast the random variable Y_{t+h} which is equivalent to say that we look for a h-step ahead forecast for our target variable. We will drop the t and h subindexes just for clarity of exposition. At time t we have two forecasts Y_1 and Y_2 for the target variable Y. We will further assume that the vector process

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y \end{pmatrix}_{3\times 1}$$

is weakly stationary, ergodic and has positive definite variance-covariance matrix $\mathbb{V}$.

2.1 The Combined Forecast

Consider the following combination of forecasts

$$\begin{aligned} Y^C &= \lambda Y_1 + (1-\lambda)Y_2 = \lambda(Y_1 - Y_2) + Y_2 \\ \lambda &\in [0,1] \end{aligned}$$

where Y^C denotes the combined forecast. The corresponding forecast errors are

$$\begin{aligned} u^C &= \lambda u_1 + (1-\lambda)u_2 \\ u^C &= \lambda \left[u_1 - u_2\right] + u_2 \end{aligned}$$

where u_1 and u_2 represent the errors associated to forecast Y_1 and Y_2 respectively:

$$\begin{aligned} u_1 &= Y - Y_1 \\ u_2 &= Y - Y_2 \end{aligned}$$

We will assume, without loss of generality, that the Mean Squared Prediction Error (MSPE) of forecast 2 is as good as that of forecast 1, that is to say:

$$MSPE_2 \equiv \mathbb{E}\left[u_2^2\right] \leq MSPE_1 \equiv \mathbb{E}\left[u_1^2\right]$$

When the combined forecast displays lower MSPE than forecasts Y_1 and Y_2 we will say that Combination Gains (CG) do exist[3]. Proposition 1 next shows conditions for this happen.

2.2 Combination Gains

Proposition 1 *If*

$$\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] < 0$$

then combination gains are possible for $\lambda \in (0,1)$.

[3] This is an expression used by Timmermann (2006).

Proof. Notice that

$$\begin{aligned}\mathbb{E}\left[u^C\right]^2 &= \mathbb{E}\left[\lambda\left[u_1-u_2\right]+u_2\right]^2 \\ &= \lambda^2\mathbb{E}\left[u_1-u_2\right]^2+\mathbb{E}\left[u_2\right]^2+2\lambda\mathbb{E}\left[u_1-u_2\right]\left[u_2\right]\end{aligned}$$

therefore

$$\mathbb{E}\left[u^C\right]^2-\mathbb{E}\left[u_2\right]^2=\lambda^2\mathbb{E}\left[u_1-u_2\right]^2+2\lambda\mathbb{E}\left[u_1-u_2\right]\left[u_2\right] \tag{1}$$

Suppose[4]

$$\mathbb{E}\left[u_1-u_2\right]^2=0$$

In this case

$$\mathbb{E}\left[u^C\right]^2-\mathbb{E}\left[u_2\right]^2=2\lambda\mathbb{E}\left[u_1-u_2\right]\left[u_2\right]<0 \text{ for all } \lambda>0$$

Let us suppose now that

$$\mathbb{E}\left[u_1-u_2\right]^2>0$$

then (1) is a strictly convex quadratic form with at most two different real roots. One of them is zero, which rules out the possibility of two complex roots. The other real root is

$$\lambda_2^u=\frac{-2\mathbb{E}\left[u_1-u_2\right]\left[u_2\right]}{\mathbb{E}\left[u_1-u_2\right]^2}>0$$

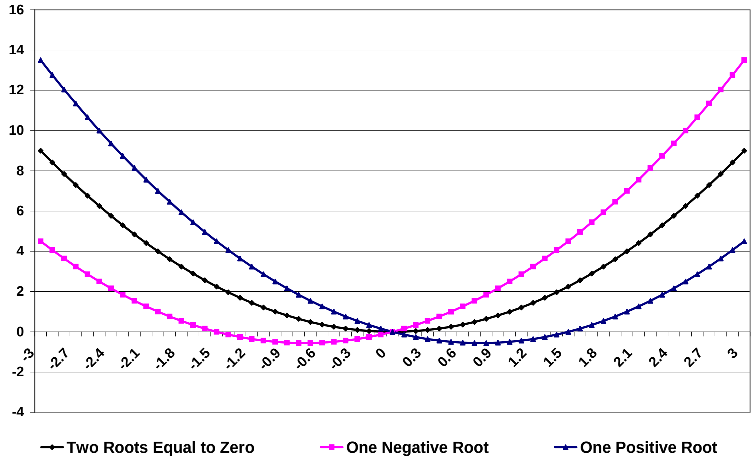
Univariate convex quadratic forms with a zero root must fall into one of the three cases depicted en Figure 1. Given that $\lambda_2^u>0$, we are in a situation like that depicted with the blue line. Therefore, combination gains are achieved in the interval

$$(0,\lambda_2^u)$$

which, in particular ensures combination gains in, at least, a subset of the open set (0,1). ■

[4] Notice that this condition is ruled out by the assumption of $\mathbb{V}$ being strictly definite-positive. We include this analysis to show that proposition 1 does not need this assumption to hold true.

Figure 1
Univariate Convex Quadratic Forms With a Zero Root



Well acquainted with this analysis, Harvey, Leybourne and Newbold (1998) propose testing the following null hypothesis of no encompassing

$$H_0 : \mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] = 0$$

Under this null hypothesis a combination of forecasts is not accuracy improving because

$$H_0 \Longrightarrow \mathbb{E}\left[u^C\right]^2 \geq \mathbb{E}\left[u_2\right]^2 \quad \text{for all } \lambda \in \mathbb{R}$$

and no combination gains are possible.

We will be also interested in analyzing a particular property of forecasts that we will name auto-efficiency:

2.3 Auto-Efficiency

Definition 2 *Consider a target variable Y and a forecast Y^f. We will say that the forecast Y^f is auto-efficient as long as*

$$\mathbb{C}ov(Y - Y^f, Y^f) = 0$$

which in the case of Y^f being unbiased could be expressed as

$$\mathbb{E}\left[Y - Y^f\right]\left[Y^f\right] = 0$$

This last expression indicates that forecast errors are orthogonal to the forecast itself. If this condition does not hold true, we will say that the forecast Y^f is auto-inefficient. In general, definitions in the same line are originally attributed to the early work of Mincer and Zarnowitz (1969) and have been called Minzer-Zarnowitz efficiency.

We notice that auto-efficiency is one of the conditions satisfied by optimal forecasts under quadratic loss. Furthermore, violations of auto-inefficiency are relevant for at least two reasons:

1. They allow for a supplementary interpretation of MSPE when forecasts are unbiased.
2. They allow for a simple modification of the forecast Y^f to produce a new revised forecast with lower MSPE than Y^f.

The first point above relies on a remark made in a recent work by Patton and Timmermann (2012). Notice that

$$Y = Y^f + u$$

therefore

$$\mathbb{E}\left[Y^2\right] = \mathbb{E}\left[Y^f\right]^2 + \mathbb{E}\left[u^2\right] + 2\mathbb{E}\left[Y^f u\right]$$

and

$$\begin{aligned} \mathbb{C}ov(Y,Y) &= \mathbb{C}ov(Y^f + u, Y^f + u) \\ \mathbb{V}(Y) &= \mathbb{C}ov(Y^f, Y^f) + 2\mathbb{C}ov(u, Y^f) + \mathbb{C}ov(u,u) \\ \mathbb{V}(Y) &= \mathbb{V}(Y^f) + 2\mathbb{C}ov(u, Y^f) + \mathbb{V}(u) \end{aligned}$$

When forecasts are unbiased we have that

$$\mathbb{E}\left[Y\right] = \mathbb{E}\left[Y^f\right]$$

so

$$\begin{aligned} \mathbb{E}(Y - Y^f) &= \mathbb{E}(u) = 0 \\ \mathbb{C}ov(Y^f, u) &= \mathbb{E}\left[Y^f u\right] \\ \mathbb{V}(u) &= \mathbb{E}\left[u^2\right] \end{aligned}$$

therefore

$$\mathbb{V}(Y) = \mathbb{V}\left(Y^f\right) + 2\mathbb{E}\left[Y^f u\right] + \mathbb{E}\left[u^2\right]$$

so, if auto-efficiency holds

$$\begin{aligned} \mathbb{V}(Y) &= \mathbb{V}\left(Y^f\right) + MSPE \\ MSPE &\equiv \mathbb{E}u^2 \end{aligned}$$

and there is an inverse relationship between MSPE and the "explained" variance of the model. Nevertheless, if auto-efficiency does not hold, we can have reductions in MSPE that are associated to reductions in the "explained" variance of the model as well, which is counterintuitive.

The second point above indicates that, when auto-inefficiency holds true, the forecast itself contains information that could be used to predict its own forecast errors. We could then build a linear model for the forecast errors as follows

$$\begin{aligned} u &\equiv Y - Y^f = \alpha + \beta Y^f + u^* \\ \mathbb{E}u^* &= \mathbb{E}Y^f u^* = 0 \end{aligned}$$

which defines the following coefficients

$$\beta \equiv \frac{\mathbb{C}ov(Y^f, u)}{\mathbb{V}\left(Y^f\right)}; \alpha \equiv \mathbb{E}u - \beta\mathbb{E}Y^f$$

We could build a revised forecast Y^{fr} as follows

$$\begin{aligned} Y^{fr} &= \alpha + (1+\beta)Y^f \\ &= \mathbb{E}u - \beta\mathbb{E}Y^f + (1+\beta)Y^f \\ &= Y^f + \mathbb{E}u + \beta(Y^f - \mathbb{E}Y^f) \\ &= Y^f + \mathbb{E}Y - \mathbb{E}Y^f + \beta(Y^f - \mathbb{E}Y^f) \\ &= (1+\beta)(Y^f - \mathbb{E}Y^f) + \mathbb{E}Y \end{aligned}$$

Notice that

$$Y^f = (Y^f - \mathbb{E}Y^f) + \mathbb{E}Y^f$$

when forecast are unbiased we have

$$Y^f = (Y^f - \mathbb{E}Y^f) + \mathbb{E}Y$$

therefore

$$\beta \equiv \frac{\mathbb{C}ov(Y^f, u)}{\mathbb{V}\left(Y^f\right)}$$

provides information regarding the need of a shrinkage or an upscale adjustment in the term $(Y^f - \mathbb{E}Y^f)$.

The new forecast error is

$$\begin{aligned} Y - Y^{fr} &= Y - \alpha - (1+\beta)Y^f \\ &= Y - Y^f - \alpha - \beta Y^f \\ &= u - (\alpha + \beta Y^f) \\ &= u^* \end{aligned}$$

Interestingly

$$\mathbb{E}\left[u^*\right]^2 < \mathbb{E}\left[u\right]^2$$

this is so as long as $\alpha \neq 0$ or $\beta \neq 0$. In fact

$$\begin{aligned} \mathbb{E}\left[u^*\right]^2 &= \mathbb{E}\left[u - (\alpha + \beta Y^f)\right]^2 \\ &= \mathbb{E}\left[u\right]^2 + \mathbb{E}\left[\alpha + \beta Y^f\right]^2 - 2\mathbb{E}\left[u(\alpha + \beta Y^f)\right] \\ &= \mathbb{E}\left[u\right]^2 + \mathbb{E}\left[\alpha + \beta Y^f\right]^2 - 2\mathbb{E}\left[(\alpha + \beta Y^f + u^*)(\alpha + \beta Y^f)\right] \\ &= \mathbb{E}\left[u\right]^2 + \mathbb{E}\left[\alpha + \beta Y^f\right]^2 - 2\mathbb{E}\left[\alpha + \beta Y^f\right]^2 - \mathbb{E}[u^*(\alpha + \beta Y^f)\right] \\ &= \mathbb{E}\left[u\right]^2 - \mathbb{E}\left[\alpha + \beta Y^f\right]^2 \end{aligned} \tag{2}$$

Notice that even if the original forecast is unbiased we will have $\alpha \neq 0$. This is so because

$$\alpha \equiv \mathbb{E}u - \beta \mathbb{E}Y^f$$

so in the unbiased case

$$\alpha \equiv -\beta \mathbb{E}Y$$

and α may still be different from zero, due to the auto-inefficiency of Y^f, as long as

$$\mathbb{E}Y \neq 0$$

2.4 Assumptions

In summary we will be interested in an environment characterized by

1. One target variable Y.
2. Two forecasts Y_1 and Y_2 such that

$$\begin{aligned} MSPE_2 &\equiv \mathbb{E}\left[u_2^2\right] \leq MSPE_1 \equiv \mathbb{E}\left[u_1^2\right] && (3) \\ \mathbb{E}\left[u_1\right] &= \mathbb{E}\left[u_2\right] = 0 && (4) \end{aligned}$$

3. Combination Gains do exist in some region of the open set (0,1). In other words

$$\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] < 0 \tag{5}$$

We will also make use of the following assumption:

4. The vector

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y \end{pmatrix}_{3\times 1}$$

is weakly stationary, ergodic and with positive definite variance-covariance matrix $\mathbb{V}$.

We have displayed the basic econometric framework with which we will be working in this paper. In the next section we show the main results of the article.

3 Main Theoretical Results

One of the main points of this paper is to show that traditional weighted averages of forecasts are auto-inefficient almost surely. In fact, the next proposition shows that the majority of forecast combinations with $\lambda \in (0,1)$ are auto-inefficient. Previous to that, notice that with straightforward algebra the auto-efficiency of the combined forecast could be expressed in the following way:

$$\mathbb{E}[Y^C u^C] = -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \tag{6}$$

It would be also useful to express (6) as follows

$$\mathbb{E}[Y^C u^C] = \lambda^2 \mathbb{E}\left[Y_1 u_1\right] + (1-\lambda)^2 \mathbb{E}\left[Y_2 u_2\right] + \lambda(1-\lambda)\mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right] \tag{7}$$

The proof of these expressions are in the appendix.

3.1 Auto-Inefficiency of Forecast Combinations

Proposition 3 *Let Y denote a target variable and Y_1, Y_2 two forecasts for Y such that assumptions 1-4 hold true. Then there will be at most two different combinations $\lambda_1, \lambda_2 \in (0,1)$ for which the combined forecast is auto-efficient.*

Proof. Let us consider the expected value of the combined forecast times its forecast error:

$$\mathbb{E}[Y^C u^C] = \mathbb{E}\left[\lambda\left[Y_1 - Y_2\right] + Y_2\right]\left[\lambda\left[u_1 - u_2\right] + u_2\right]$$

this expression defines the following quadratic form

$$\mathbb{E}[Y^C u^C] = -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2 u_2\right]$$

We notice that

$$\mathbb{E}\left[Y_1 - Y_2\right]^2 > 0 \tag{8}$$

otherwise

$$0 = \mathbb{E}\left[Y_1 - Y_2\right]^2 = \mathbb{V}(Y_1 - Y_2) + \left[\mathbb{E}\left[Y_1 - Y_2\right]\right]^2 \tag{9}$$

and

$$\mathbb{V}(Y_1 - Y_2) = 0$$

which means that

$$\mathbb{V}(Y_1) + \mathbb{V}(Y_2) = 2\mathbb{C}ov(Y_1, Y_2)$$

Let us consider V_{12} to be defined as the variance covariance matrix of the sub-vector

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}_{2\times 1}$$

then it has to be the case that V_{12} is a positive definite matrix as well, as all the leading principal minors of $\mathbb{V}$ are also strictly positive. Nevertheless

$$\begin{aligned} \begin{bmatrix} 1 & -1 \end{bmatrix} V_{12} \begin{bmatrix} 1 \\ -1 \end{bmatrix} &= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \mathbb{V}(Y_1) - \mathbb{C}ov(Y_1, Y_2) \\ \mathbb{C}ov(Y_1, Y_2) - \mathbb{V}(Y_2) \end{bmatrix} \\ &= \mathbb{V}(Y_1) - \mathbb{C}ov(Y_1, Y_2) - \mathbb{C}ov(Y_1, Y_2) + \mathbb{V}(Y_2) = 0 \end{aligned}$$

which is a contradiction with the fact that V_{12} is positive definite. With

$$\mathbb{E}\left[Y_1 - Y_2\right]^2 > 0$$

the expression $\mathbb{E}[Y^C u^C]$ is a strictly concave quadratic form and of course is different from the zero function. As a consequence, $\mathbb{E}[Y^C u^C]$ will have at most two real roots which may or may not lie within the $(0,1)$ interval, so it may be the case that every single combination is auto-inefficient. In any case, at most two combinations are auto-efficient. ■

Corollary 4 *Under assumptions 1-4, if we further assume that both individual forecasts are auto-efficient, then any weighted average of the forecasts will display positive auto-inefficiency.*

Proof. We already saw that

$$0 < \mathbb{E}\left[Y_1 - Y_2\right]^2$$

Notice that

$$\begin{aligned} 0 &< \mathbb{E}\left[Y_1 - Y_2\right]^2 = \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - u_1\right] \\ 0 &< \mathbb{E}\left[Y_1 - Y_2\right]^2 = \mathbb{E}\left[Y_1 u_2 - Y_1 u_1 - Y_2 u_2 + Y_2 u_1\right] \\ 0 &< \mathbb{E}\left[Y_1 - Y_2\right]^2 = \mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right] \end{aligned}$$

Using that

$$\mathbb{E}[Y^C u^C] = \lambda^2 \mathbb{E}\left[Y_1 u_1\right] + (1-\lambda)^2 \mathbb{E}\left[Y_2 u_2\right] + (1-\lambda)\lambda \mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right]$$

Because the two individual forecasts are auto-efficient we have

$$\mathbb{E}\left[Y_1 u_1\right] = \mathbb{E}\left[Y_2 u_2\right] = 0$$

therefore we conclude that

$$\mathbb{E}[Y^C u^C] = (1-\lambda)\lambda \mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right] > 0 \text{ for all } \lambda \in (0,1)$$

■

Proposition 3 showed that most of the possible forecast combinations are auto-inefficient. The previous corollary showed a particular case ensuring that all possible combinations within (0,1) are auto-inefficient. As a consequence, tests based upon aggregated information from surveys might incorrectly reject the null hypothesis of rational agents.

The next proposition provides more general conditions under which every single combination in (0,1) is auto-inefficient.

Proposition 5 *For Y, Y_1 and Y_2 as in proposition 3, let us assume that*

$$\mathbb{E}\left[Y_1 u_1\right] \geq 0 \quad (10)$$

$$\mathbb{E}\left[Y_2 u_2\right] \geq 0 \quad (11)$$

then for every single combination $\lambda \in (0,1)$ the combined forecast displays auto-inefficiency.

Proof. Using that

$$\begin{aligned} 0 &< \mathbb{E}\left[Y_1 - Y_2\right]^2 = \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - u_1\right] \\ 0 &< \mathbb{E}\left[Y_1 - Y_2\right]^2 = \mathbb{E}\left[Y_1 u_2 - Y_1 u_1 - Y_2 u_2 + Y_2 u_1\right] \end{aligned}$$

we obtain

$$0 < \mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right] - \mathbb{E}\left[Y_1 u_1 + Y_2 u_2\right]$$

therefore

$$\mathbb{E}\left[Y_1 u_1 + Y_2 u_2\right] < \mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right]$$

but, by assumption the LHS is greater or equal than zero

$$0 \leq \mathbb{E}\left[Y_1 u_1 + Y_2 u_2\right] < \mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right]$$

therefore

$$0 < \mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right]$$

Using again that

$$\mathbb{E}[Y^C u^C] = \lambda^2 \mathbb{E}\left[Y_1 u_1\right] + (1-\lambda)^2 \mathbb{E}\left[Y_2 u_2\right] + (1-\lambda)\lambda \mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right]$$

we conclude that

$$\mathbb{E}[Y^C u^C] = \lambda^2 \mathbb{E}\left[Y_1 u_1\right] + (1-\lambda)^2 \mathbb{E}\left[Y_2 u_2\right] + (1-\lambda)\lambda \mathbb{E}\left[Y_1 u_2 + Y_2 u_1\right] > 0 \text{ for all } \lambda \in (0,1)$$

It is important to remark that conditions (10) and (11) may be tested using a strategy based on the “reality check” of White (2000) which has also been used by Pincheira (2012). ■

Remark 6 *Notice that*

$$0 < \mathbb{E}\left[Y_1 - Y_2\right]^2$$

is also a necessary condition for combination gains to hold true. In fact, if instead we had

$$0 = \mathbb{E}\left[Y_1 - Y_2\right]^2$$

then

$$0 = \mathbb{E}\left[Y_1 - Y_2\right]^2 = \mathbb{E}\left[u_2 - u_1\right]^2 = \mathbb{E}\left[u_2\right]^2 + \mathbb{E}\left[u_1\right]^2 - 2\mathbb{E}\left[u_2 u_1\right]$$

in other words we would have

$$2\mathbb{E}\left[u_2 u_1\right] = \mathbb{E}\left[u_2\right]^2 + \mathbb{E}\left[u_1\right]^2$$

Let us analyze the sign of following expression having in mind that we will make use of assumption (3):

$$\begin{aligned} 2\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] &= 2\mathbb{E}\left[u_2 u_1\right] - 2\mathbb{E}\left[u_2\right]^2 = \mathbb{E}\left[u_2\right]^2 + \mathbb{E}\left[u_1\right]^2 - 2\mathbb{E}\left[u_2\right]^2 \\ &= \mathbb{E}\left[u_1\right]^2 - \mathbb{E}\left[u_2\right]^2 \geq 0 \end{aligned}$$

and no combination gains would be possible.

Proposition 7 *For Y, Y_1 and Y_2 as in proposition 3, then we can find a unique $\lambda^* \in (0,1)$ such that*

$$\lambda^* = \underset{\lambda \in (0,1)}{\arg\min} \mathbb{E}\left[\lambda u_1 + (1-\lambda) u_2\right]^2$$

For this λ^* we also have

$$\mathbb{E}[Y^C(\lambda^*) u^C(\lambda^*)] = \lambda^* \mathbb{E}\left[Y_2 u_1\right] + (1-\lambda^*)\mathbb{E}\left[Y_2 u_2\right]$$

Proof.

$$\mathbb{E}\left[\lambda u_1 + (1-\lambda) u_2\right]^2 = \lambda^2 \mathbb{E}\left[u_1 - u_2\right]^2 + \mathbb{E}\left[u_2\right]^2 + 2\lambda \mathbb{E}\left[u_1 - u_2\right]\left[u_2\right]$$

this is a strictly convex quadratic function which admits a unique global minimum defined by the following first order conditions

$$2\lambda \mathbb{E}\left[u_1 - u_2\right]^2 + 2\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] = 0$$

this equation is solved by

$$\lambda^* = \frac{-\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right]}{\mathbb{E}\left[u_1 - u_2\right]^2}$$

now, let us recall that by assumption

$$0 \leq \mathbb{E}\left[u_1^2 - u_2^2\right] = \mathbb{E}\left[u_1 - u_2\right]\left[u_1 + u_2\right]$$

therefore

$$0 \leq \mathbb{E}\left[u_1^2 - u_2^2\right] = \mathbb{E}\left[u_1 - u_2\right]\left[u_1 - u_2 + 2u_2\right] = \mathbb{E}\left[u_1 - u_2\right]^2 + 2\mathbb{E} u_2 \left[u_1 - u_2\right]$$

so, dividing by $\mathbb{E}\left[u_1 - u_2\right]^2$ we have

$$\frac{\mathbb{E}\left[u_1 - u_2\right]^2 + 2\mathbb{E} u_2 \left[u_1 - u_2\right]}{\mathbb{E}\left[u_1 - u_2\right]^2} \geq 0$$

or

$$\begin{aligned}
1 + \frac{2\mathbb{E}\left[u_1 - u_2\right]u_2}{\mathbb{E}\left[u_1 - u_2\right]^2} &\geq 0 \\
-1 - \frac{2\mathbb{E}\left[u_1 - u_2\right]u_2}{\mathbb{E}\left[u_1 - u_2\right]^2} &\leq 0 \\
-\frac{2\mathbb{E}\left[u_1 - u_2\right]u_2}{\mathbb{E}\left[u_1 - u_2\right]^2} &\leq 1
\end{aligned}$$

by the assumption of combination gains we have

$$\mathbb{E}\left[u_1 - u_2\right]u_2 < 0$$

therefore

$$0 < -\frac{2\mathbb{E}\left[u_1 - u_2\right]u_2}{\mathbb{E}\left[u_1 - u_2\right]^2} \leq 1$$

or

$$0 < \lambda^* = -\frac{\mathbb{E}\left[u_1 - u_2\right]u_2}{\mathbb{E}\left[u_1 - u_2\right]^2} \leq \frac{1}{2}$$

therefore

$$0 < \lambda^* < 1$$

Now, let us find an expression for $\mathbb{E}[Y^C(\lambda^*)u^C(\lambda^*)]$:

$$\begin{aligned}
\mathbb{E}[Y^C(\lambda^*)u^C(\lambda^*)] &= -\left[\lambda^*\right]^2\mathbb{E}\left[Y_1 - Y_2\right]^2 + \left[\lambda^*\right]\mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2u_2\right] \\
&= -\left[\lambda^*\right]^2\mathbb{E}\left[u_2 - u_1\right]^2 - \left[\lambda^*\right]\mathbb{E}\left[u_1 - u_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2u_2\right] \\
&= -\lambda^*\left[\lambda^*\mathbb{E}\left[u_2 - u_1\right]^2\right] - \left[\lambda^*\right]\mathbb{E}\left[u_1 - u_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2u_2\right] \\
&= \lambda^*\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] - \lambda^*\mathbb{E}\left[u_1 - u_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2u_2\right] \\
&= \lambda^*\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] - \lambda^*\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] + \lambda^*\mathbb{E}\left[u_1 - u_2\right]\left[Y_2\right] + \mathbb{E}\left[Y_2u_2\right] \\
&= \lambda^*\mathbb{E}\left[u_1 - u_2\right]\left[Y_2\right] + \mathbb{E}\left[Y_2u_2\right] \\
&= \lambda^*\mathbb{E}\left[Y_2u_1\right] + (1 - \lambda^*)\mathbb{E}\left[Y_2u_2\right]
\end{aligned}$$

■

This last expression indicates that the optimal combination may be auto-inefficient as well. For instance, if

$$\mathbb{E}\left[Y_2u_1\right] \text{ and } \mathbb{E}\left[Y_2u_2\right]$$

share the same sign, then there is no way for the optimal combination to display auto-efficiency.

3.2 Size of the Auto-Inefficiency in the Forecast Combinations

Let us consider the following notation:

$$\begin{aligned} f(\lambda) &\equiv \mathbb{E}\left(u^C\right)^2 = \lambda^2 \mathbb{E}\left[u_1 - u_2\right]^2 + 2\lambda \mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] + \mathbb{E}\left(u_2\right)^2 \\ g(\lambda) &\equiv \mathbb{E}\left[Y^C u^C\right] = -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\ \mathbb{S} &= \mathbb{E}\left[Y_1 - Y_2\right] Y = -\mathbb{E}\left[u_1 - u_2\right] Y \end{aligned}$$

We notice that

$$\begin{aligned} f(\lambda) + g(\lambda) &= \mathbb{E}\left(u_2\right)^2 + \mathbb{E}\left[Y_2 u_2\right] + 2\lambda \mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] \\ &= \mathbb{E}\left(u_2\right)^2 + \mathbb{E}\left[Y_2 u_2\right] + 2\lambda \mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] - \lambda \mathbb{E}\left[u_1 - u_2\right] Y + 2\lambda \mathbb{E}\left[u_1 - u_2\right]\left[Y_2\right] \\ &= \mathbb{E}\left(u_2\right)^2 + \mathbb{E}\left[Y_2 u_2\right] + 2\lambda \mathbb{E}\left[u_1 - u_2\right] Y - \lambda \mathbb{E}\left[u_1 - u_2\right] Y \\ &= \mathbb{E}\left(u_2\right)^2 + \mathbb{E}\left[Y_2 u_2\right] + \lambda \mathbb{E}\left[u_1 - u_2\right] Y \end{aligned}$$

This las expression is equivalent to write:

$$\left[\mathbb{E}\left(u^C\right)^2 - \mathbb{E}\left(u_2\right)^2\right] = -\left[\mathbb{E}\left(Y^C u^C\right) - \mathbb{E}\left(Y_2 u_2\right)\right] + \lambda \mathbb{E}\left[u_1 - u_2\right] Y$$

or

$$\left[\mathbb{E}\left(u^C\right)^2 - \mathbb{E}\left(u_2\right)^2\right] + \left[\mathbb{E}\left(Y^C u^C\right) - \mathbb{E}\left(Y_2 u_2\right)\right] = -\lambda \mathbb{S}$$

which could be written as

$$CG(\lambda) + AEG(\lambda) = -\lambda \mathbb{S}$$

where AEG stands for Auto-Efficiency Gains (or losses) relative to the auto-efficient status of forecast 2. The first term in the LHS is a strictly convex quadratic form with a zero root. The second term in the LHS is a strictly concave quadratic form with a zero root. If the symmetry condition holds true ($\mathbb{S}=0$) then the two quadratic forms would be the same but with opposite sign. This implies that movements along the quadratic forms are totally compensated. When $\mathbb{S} \neq 0$ this is not the case, and movements along the quadratic forms are not totally compensated. They are only partially compensated. The size of the symmetry conditon $\mathbb{S}$ is indeed key for combination gains to be totally or just partially compensated by changes in auto-efficiency. We will see this with examples in subsection 3.4.

3.3 Improving on Forecast Combinations

At this stage we know that the majority of forecast averages are auto-inefficient. We also know that, for every auto-inefficient $\lambda \in (0,1)$ we could build a revised forecast using an OLS adjustment. A natural question to ask is whether the optimal combination λ^* will remain optimal after the OLS adjustment. If the answer is no, then we would want to find the optimal $\lambda \in (0,1)$ for which the combination, adjusted by OLS and correcting by auto-inefficiency, provides the lowest MSPE. We will denote this optimal recombination by λ^{**}. It is convenient now to recall expression (2) which provides the magnitude of recombination gains:

$$\mathbb{E}\left[u^*\right]^2 = \mathbb{E}\left[u\right]^2 - \mathbb{E}\left[\alpha + \beta Y^f\right]^2 \qquad (12)$$

$$\beta \equiv \frac{\mathbb{C}ov(Y^f, u)}{\mathbb{V}\left(Y^f\right)}; \alpha \equiv \mathbb{E}u - \beta \mathbb{E}Y^f \qquad (13)$$

From (12) we see that recombination gains are equal to:

$$\mathbb{E}\left[\alpha + \beta Y^f\right]^2$$

When the forecast Y^f coincides with the combined forecast then recombination gains are:

$$\mathbb{E}\left[\alpha_C^\lambda + \beta_C^\lambda Y^C\right]^2$$

$$\beta_C^\lambda \equiv \frac{\mathbb{C}ov(Y^C, u^C)}{\mathbb{V}\left(Y^C\right)}; \alpha_C^\lambda \equiv \mathbb{E}u^C - \beta_C^\lambda \mathbb{E}Y^C$$

We could build a revised forecast Y^{Cr} as follows

$$\begin{aligned} Y^{Cr} &= \alpha_C^\lambda + (1+\beta_C^\lambda)Y^C \\ &= \mathbb{E}u^C - \beta_C^\lambda \mathbb{E}Y^C + (1+\beta_C^\lambda)Y^C \\ &= Y^C + \beta_C^\lambda\left[Y^C - \mathbb{E}Y^C\right] \\ &= Y^C + \frac{\mathbb{C}ov(Y^C, u^C)}{\mathbb{V}\left(Y^C\right)}\left[Y^C - \mathbb{E}Y^C\right] \end{aligned}$$

the new forecast error is

$$\begin{aligned} u^{**} &\equiv Y - Y^{Cr} = Y - \alpha_C^\lambda - (1+\beta_C^\lambda)Y^C \\ &= u^C - (\alpha_C^\lambda + \beta_C^\lambda Y^C) \end{aligned}$$

with

$$\mathbb{E}\left[u^{**}\right]^2 < \mathbb{E}\left[u^C\right]^2$$

as long as $\alpha_C^\lambda \neq 0$ or $\beta_C^\lambda \neq 0$. In fact, as shown before,

$$\mathbb{E}\left[u^{**}\right]^2 = \mathbb{E}\left[u^C\right]^2 - \mathbb{E}\left[\alpha_C + \beta_C^\lambda Y^C\right]^2$$

Notice that when working with unbiased forecasts we have

$$\alpha_C^\lambda \equiv \mathbb{E}u^C - \beta_C^\lambda \mathbb{E}Y^C = -\beta_C^\lambda \mathbb{E}Y$$

and gains from OLS recombination would be given by

$$\begin{aligned}
\mathbb{E}\left[\alpha_C^\lambda + \beta_C^\lambda Y^C\right]^2 &= \mathbb{E}\left[\beta_C^\lambda Y^C - \beta_C^\lambda \mathbb{E}Y\right]^2 \\
&= \mathbb{E}\left[\beta_C^\lambda Y^C - \mathbb{E}\left[\beta_C^\lambda Y^C\right]\right]^2 \\
&= \mathbb{V}\left[\beta_C^\lambda Y^C\right] \\
&= \left[\beta_C^\lambda\right]^2 \mathbb{V}\left[Y^C\right]
\end{aligned}$$

On the other hand

$$\beta_C^\lambda \equiv \frac{\mathbb{C}ov(Y^C, u^C)}{\mathbb{V}\left(Y^C\right)} = \frac{\mathbb{E}\left[Y^C u^C\right]}{\mathbb{V}\left(Y^C\right)}$$

therefore, the total gain from recombining is

$$\mathbb{E}\left[\alpha_C^\lambda + \beta_C^\lambda Y^C\right]^2 = \frac{\left(\mathbb{E}\left[Y^C u^C\right]\right)^2}{\mathbb{V}\left(Y^C\right)} = \frac{\left(\mathbb{E}\left[Y^C u^C\right]\right)^2}{\mathbb{E}\left(Y^C\right)^2 - \left[\mathbb{E}Y\right]^2} \tag{14}$$

In the next two propositions we will try to find $\lambda^{**} \in (0,1)$ for which (14) reaches a local minimum.

Proposition 8 *Let Y, Y_1 and Y_2 be as in proposition 3. Let $\lambda^* \in (0,1)$ represents the optimal combination. Furthermore, let us assume that the following symmetry condition is met*

$$\mathbb{S} \equiv \mathbb{E}\left[Y(Y_1 - Y_2)\right] = 0 \tag{15}$$

then λ^* will be a critical point of the combined MSPE function corrected by OLS. Furthermore, λ^* corresponds to a local minimum of the optimal OLS corrected combination. In other words

$$\lambda^* = \underset{B(\lambda^*)}{\arg\min} \mathbb{E}\left[u^{**}\right]^2 = \mathbb{E}\left[u^C\right]^2 - \mathbb{E}\left[\alpha_C + \beta_C Y^C\right]^2$$

Proof. The total gain from the OLS recombination is given by

$$\mathbb{E}\left[\alpha_C + \beta_C Y^C\right]^2 = \frac{\left(\mathbb{E}\left[Y^C u^C\right]\right)^2}{\mathbb{V}\left(Y^C\right)} = \frac{\left(\mathbb{E}\left[Y^C u^C\right]\right)^2}{\mathbb{E}\left(Y^C\right)^2 - \left[\mathbb{E}Y\right]^2}$$

we will use the following notation:

$$\begin{aligned} f(\lambda) &\equiv \mathbb{E}\left(u^C\right)^2 \\ g(\lambda) &\equiv \mathbb{E}\left[Y^C u^C\right] \\ h(\lambda) &\equiv \mathbb{V}\left(Y^C\right) \end{aligned}$$

The critical points of $\mathbb{E}\left[u^{**}\right]^2$ for $\lambda \in (0,1)$ must satisfy:

$$\frac{\partial \mathbb{E}\left[u^{**}\right]^2}{\partial \lambda} = 0$$

which can be written as

$$\frac{\partial\left(f(\lambda) - g^2(\lambda)/h(\lambda)\right)}{\partial \lambda} = 0$$

so we have

$$f' - \left[\frac{2gg'h - g^2h'}{h^2}\right] = 0$$

Notice that under (15) we have

$$\left[\mathbb{E}\left(u^C\right)^2 - \mathbb{E}\left(u_2\right)^2\right] + \left[\mathbb{E}\left(Y^C u^C\right) - \mathbb{E}\left(Y_2 u_2\right)\right] = \lambda \mathbb{E}\left[u_1 - u_2\right] Y = 0$$

which implies

$$\frac{\partial \mathbb{E}\left(u^C\right)^2}{\partial \lambda} = -\frac{\partial \mathbb{E}\left(Y^C u^C\right)}{\partial \lambda}$$

Besides, (15) also implies that

$$\begin{aligned} \mathbb{E}\left[Y^C\right]^2 &= \mathbb{E}\left[\lambda\left(Y_1 - Y_2\right) + Y_2\right]^2 = \lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \mathbb{E}\left[Y_2\right]^2 + 2\lambda \mathbb{E}\left[\left(Y_1 - Y_2\right) Y_2\right] \\ &= \lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \mathbb{E}\left[Y_2\right]^2 + 2\lambda \mathbb{E}\left[\left(Y_1 - Y_2\right)\left(Y - u_2\right)\right] \\ &= \lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \mathbb{E}\left[Y_2\right]^2 + 2\lambda \mathbb{E}\left[\left(Y_1 - Y_2\right) Y\right] - 2\lambda \mathbb{E}\left[\left(Y_1 - Y_2\right) u_2\right] \\ &= \lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \mathbb{E}\left[Y_2\right]^2 - 2\lambda \mathbb{E}\left[\left(Y_1 - Y_2\right) u_2\right] \\ &= \lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \mathbb{E}\left[Y_2\right]^2 + 2\lambda \mathbb{E}\left[\left(u_1 - u_2\right) u_2\right] \\ &= \lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \mathbb{E}\left[Y_2\right]^2 - 2\lambda \mathbb{E}\left[\left(Y_1 - Y_2\right)\left(Y - Y_2\right)\right] \\ &= \lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \mathbb{E}\left[Y_2\right]^2 + 2\lambda \mathbb{E}\left[\left(Y_1 - Y_2\right) Y_2\right] \end{aligned}$$

from

$$\begin{aligned}\mathbb{E}[Y^C u^C] &= -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\ &= -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[Y - 2Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\ &= -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 - 2\lambda \mathbb{E}\left[Y_1 - Y_2\right] Y_2 + \mathbb{E}\left[Y_2 u_2\right]\end{aligned}$$

we conclude that

$$\begin{aligned}\mathbb{E}\left[Y^C\right]^2 - \mathbb{E}\left[Y_2\right]^2 &= \lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + 2\lambda \mathbb{E}\left[\left(Y_1 - Y_2\right) Y_2\right] \\ \mathbb{E}\left[Y^C\right]^2 - \mathbb{E}\left[Y_2\right]^2 &= -\left[\mathbb{E}[Y^C u^C] - \mathbb{E}\left[Y_2 u_2\right]\right] \qquad (16)\end{aligned}$$

therefore, under (15) we have

$$\left[\mathbb{E}\left(u^C\right)^2 - \mathbb{E}\left(u_2\right)^2\right] = -\left[\mathbb{E}\left(Y^C u^C\right) - \mathbb{E}\left(Y_2 u_2\right)\right] = \left[\mathbb{E}\left[Y^C\right]^2 - \mathbb{E}\left[Y_2\right]^2\right] = \mathbb{V}\left[Y^C\right] - \mathbb{V}\left[Y_2\right]$$

but also

$$\begin{aligned}\mathbb{V}\left[Y^C\right] &= \mathbb{E}\left[Y^C\right]^2 - \left[\mathbb{E}Y^C\right]^2 \\ \mathbb{V}\left[Y^C\right] &= \mathbb{E}\left[Y^C\right]^2 - \left[\mathbb{E}Y\right]^2\end{aligned}$$

hence

$$\frac{\partial \mathbb{E}\left(u^C\right)^2}{\partial \lambda} = -\frac{\partial \mathbb{E}\left(Y^C u^C\right)}{\partial \lambda} = \frac{\partial \mathbb{V}\left(Y^C\right)}{\partial \lambda} = \frac{\partial \mathbb{E}\left[Y^C\right]^2}{\partial \lambda}$$

or

$$f' = h' = -g'$$

therefore we have

$$\begin{aligned}f' - \left[\frac{2gg'h - g^2 h'}{h^2}\right] &= 0 \\ f' - \left[\frac{-2gf'h - g^2 f'}{h^2}\right] &= 0 \\ f'\left[\frac{h^2}{h^2}\right] - \left[\frac{-2gf'h - g^2 f'}{h^2}\right] &= 0 \\ -\left[\frac{-f'h^2 - 2gf'h - g^2 f'}{h^2}\right] &= 0 \\ f'\left[\frac{h^2 + 2gh + g^2}{h^2}\right] &= 0 \\ \frac{f'}{h^2}\left[h + g\right]^2 &= 0\end{aligned}$$

but from (16) we have

$$\begin{aligned}
\mathbb{E}\left[Y^C\right]^2-\mathbb{E}\left[Y_2\right]^2 &= -\left[\mathbb{E}[Y^C u^C]-\mathbb{E}\left[Y_2 u_2\right]\right] \\
\mathbb{E}\left[Y^C\right]^2-\left(\mathbb{E}\left[Y^C\right]\right)^2-\mathbb{E}\left[Y_2\right]^2+\left(\mathbb{E}\left[Y^C\right]\right)^2 &= -\left[\mathbb{E}[Y^C u^C]-\mathbb{E}\left[Y_2 u_2\right]\right] \\
\mathbb{V}\left(Y^C\right)-\mathbb{V}\left(Y_2\right) &= -\left[\mathbb{E}[Y^C u^C]-\mathbb{E}\left[Y_2 u_2\right]\right] \\
h-\mathbb{V}\left(Y_2\right) &= -\left[g-\mathbb{E}\left[Y_2 u_2\right]\right]
\end{aligned}$$

or

$$\begin{aligned}
h+g &= \mathbb{V}\left(Y_2\right)+\mathbb{E}\left[Y_2 u_2\right]=\mathbb{E}\left[Y_2^2\right]-\left(\mathbb{E}\left[Y_2\right]\right)^2+\mathbb{E}\left[Y_2 u_2\right] \\
&= \mathbb{E}\left[Y_2 Y\right]-\left(\mathbb{E}\left[Y_2\right]\right)^2=\mathbb{E}\left[Y_2 Y\right]-\mathbb{E}\left[Y_2\right]\mathbb{E}\left[Y\right] \\
&= \mathbb{C}ov(Y_2,Y) \text{ for all } \lambda
\end{aligned}$$

therefore the critical points of

$$\mathbb{E}\left[u^{**}\right]^2=\mathbb{E}\left[u^C\right]^2-\mathbb{E}\left[\alpha_C+\beta_C Y^C\right]^2$$

satisfy

$$\frac{f'}{h^2}\left[\mathbb{C}ov(Y_2,Y)\right]^2=0$$

When $\mathbb{C}ov(Y_2,Y)=0$, then our objective function is flat and every single $\lambda\in\mathbb{R}$ will be a critical point and a global solution of our optimization problem, in particular $\lambda=\lambda^*$ will solve the problem. When $\mathbb{C}ov(Y_2,Y)\neq 0$ then the unique critical point corresponds to $\lambda=\lambda^*$. Let us explore the behavior of the second derivative of our objective function evaluated at $\lambda=\lambda^*$. We want:

$$\begin{aligned}
\frac{\partial^2\left(f(\lambda)-g^2(\lambda)/h(\lambda)\right)}{\partial\lambda^2} &= \frac{\partial}{\partial\lambda}\left[\frac{\partial\left(f(\lambda)-g^2(\lambda)/h(\lambda)\right)}{\partial\lambda}\right] \\
&= \frac{\partial}{\partial\lambda}\left[\frac{f'}{h^2}\left[h+g\right]^2\right] \\
&= \left[\frac{f''h^2-2f'hh'}{h^4}\right]\left[h+g\right]^2+2\frac{f'}{h^2}\left[h+g\right]\left[h'+g'\right]
\end{aligned}$$

therefore

$$\left.\frac{\partial \mathbb{E}\left[u^{**}\right]^2}{\partial \lambda}\right|_{\lambda=\lambda^*} = \left[\left(\frac{f''h^2-2f'hh'}{h^4}\right)\left[h+g\right]^2\right]_{|\lambda=\lambda^*} + \left[2\frac{f'}{h^2}\left[h+g\right]\left[h'+g'\right]\right]_{|\lambda=\lambda^*}$$

$$\left.\frac{\partial \mathbb{E}\left[u^{**}\right]^2}{\partial \lambda}\right|_{\lambda=\lambda^*} = \left[\left(\frac{f''}{h^2}\right)\left[h+g\right]^2\right]_{|\lambda=\lambda^*}$$

$$\left.\frac{\partial \mathbb{E}\left[u^{**}\right]^2}{\partial \lambda}\right|_{\lambda=\lambda^*} = \left[\left(\frac{f''(\lambda^*)}{\left[V(Y^C(\lambda^*))\right]^2}\right)\left[\mathbb{C}ov(Y_2,Y)\right]^2\right] > 0 \text{ if } \mathbb{C}ov(Y_2,Y)\neq 0$$

and in this way λ^* is a local minimum of the OLS-adjusted objective function because it is a global minimum of $\mathbb{E}\left(u^C\right)^2$ and therefore $f''(\lambda^*) > 0$. ■

When the symmetry condition (15) is not met, the optimal combination with and without the OLS adjustment may differ. In fact it may not even exist. The following proposition may be useful in this scenario.

Proposition 9 *Let Y, Y_1 and Y_2 be as in proposition 3. Let us assume that the following conditions are met*

$$\mathbb{S} \equiv \mathbb{E}\left[Y(Y_1-Y_2)\right]\neq 0 \quad (17)$$

$$\mathbb{V}(Y^C) > 0 \text{ for all } \lambda\in\mathbb{R} \quad (18)$$

$$\mathbb{C}ov\mathbb{E}\left[u_1-u_2\right]^2-\mathbb{S}\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right] \neq 0 \quad (19)$$

$$\mathbb{S}^2\left[\mathbb{E}\left[Y_2u_2\right]-\mathbb{C}ov\right]+2\mathbb{S}\mathbb{C}ov\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right]-\left[\mathbb{C}ov\right]^2\mathbb{E}\left[u_1-u_2\right]^2 \neq 0 \quad (20)$$

where

$$\mathbb{C}ov\equiv\mathbb{C}ov\left(Y,Y_2\right)$$

then

$$\lambda^{**} = -\frac{\mathbb{C}ov(Y_2,Y)\mathbb{E}\left[u_1-u_2\right]\left[u_2\right]+\mathbb{S}\mathbb{E}\left[Y_2u_2\right]}{\mathbb{C}ov(Y_2,Y)\mathbb{E}\left[u_1-u_2\right]^2-\mathbb{S}\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right]}$$

will be a critical point of the combined MSPE function corrected by OLS for $\lambda\in\mathbb{R}$. Furthermore, λ^{**} corresponds to a local minimum of the optimal OLS corrected combination.

Proof. See the Appendix. ■

Remark 10 *The symmetry condition can be easily tested using a t-type test similar to that proposed by Diebold & Mariano (1995) and West (1996).*

In the next section we will see illustrative examples of some of our results. In particular we will show a situation in which $\lambda^{**}\in(0,1)$ is a global minimum of $\mathbb{E}\left[u^{**}\right]^2$ in the $(0,1)$ interval.

3.4 Illustrative Examples

In this subsection we will illustrate with simple examples the main results showed in previous sections. In all these cases we will impose the existence of combination gains and will explore if auto-efficiency is satisfied or not. Let us consider the three-dimensional vector

$$W_t = \begin{pmatrix} Y_t \\ X_{t-1} \\ Z_{t-1} \end{pmatrix}_{3\times 1} \rightsquigarrow N(0, \Omega)$$

We will consider the following four cases for the matrix Ω:

$$\Omega(1) = \begin{pmatrix} 1.600 & 0.600 & 0.750 \\ 0.600 & 0.700 & 0.250 \\ 0.750 & 0.250 & 0.900 \end{pmatrix}; \Omega(2) = \begin{pmatrix} 1.600 & 0.600 & 0.750 \\ 0.600 & 0.600 & 0.250 \\ 0.750 & 0.250 & 0.750 \end{pmatrix}$$

$$\Omega(3) = \begin{pmatrix} 2.500 & 1.125 & 1.250 \\ 1.125 & 2.000 & 0.250 \\ 1.250 & 0.250 & 2.250 \end{pmatrix}; \Omega(4) = \begin{pmatrix} 1.750 & 0.600 & 1.500 \\ 0.600 & 0.600 & 0.300 \\ 1.500 & 0.300 & 1.500 \end{pmatrix}$$

All these matrices are symmetric and definite positive as their eigenvalues are $(2.33, 0.33, 0.54)$; $(2.28, 0.25, 0.42)$; $(4.13, 0.76, 1.87)$ and $(3.29, 0.04, 0.53)$ respectively. We will use two different forecasts for Y_t :

$$\begin{aligned} Y_1^f &\equiv X_{t-1} \\ Y_2^f &\equiv Z_{t-1} \end{aligned}$$

Let us analyze the case in which we have $\Omega(1)$. The respective MSPE and Mean Squared Forecasts (MSF) are

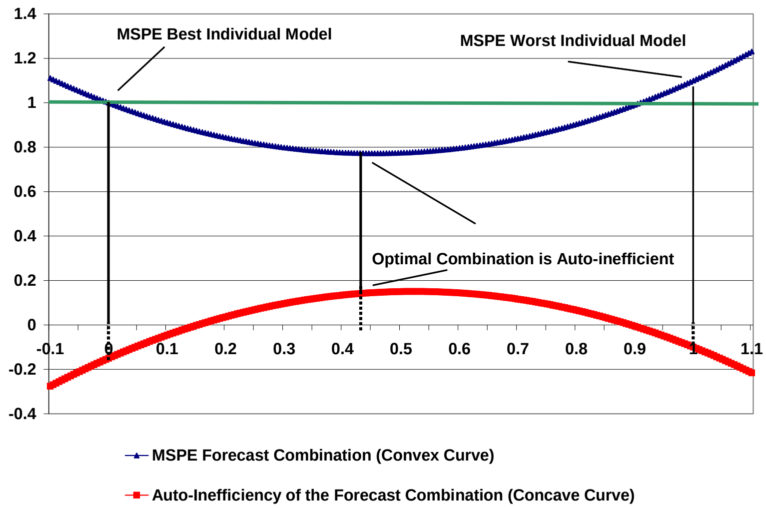
$$\begin{aligned} MSPE(Y_1^f) &\equiv \mathbb{E}(Y_t - X_{t-1})^2 = \mathbb{E}Y_t^2 + \mathbb{E}X_{t-1}^2 - 2\mathbb{E}Y_t X_{t-1} = 1.6 + 0.7 - 1.2 = 1.1 \\ MSPE(Y_2^f) &\equiv \mathbb{E}(Y_t - Z_{t-1})^2 = \mathbb{E}Y_t^2 + \mathbb{E}Z_{t-1}^2 - 2\mathbb{E}Y_t Z_{t-1} = 1.6 + 0.9 - 1.5 = 1 \\ MSF(Y_1^f) &\equiv \mathbb{E}(X_{t-1})^2 = 0.7 \\ MSF(Y_2^f) &\equiv \mathbb{E}(Z_{t-1})^2 = 0.9 \end{aligned}$$

So clearly forecast 2 is more accurate than forecast 1 in terms of MSPE and displays higher MSF. Nevertheless, forecast 1 is not encompassed by forecast 2 and combination gains are possible. This is so because

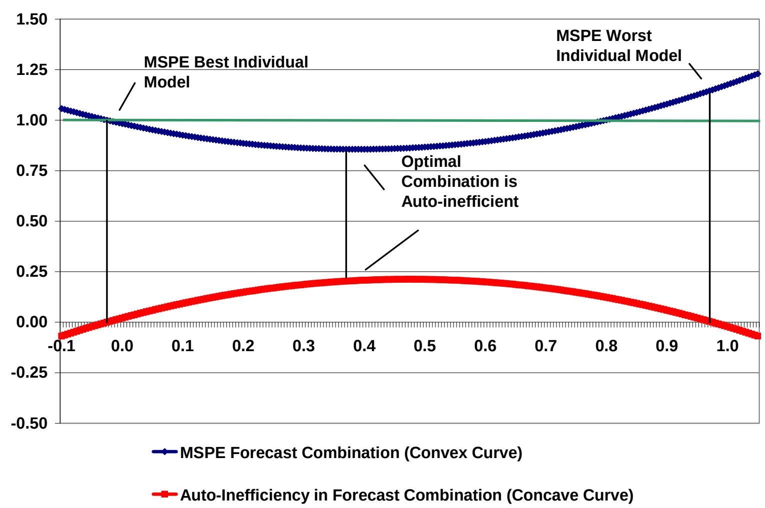
$$\begin{aligned} \mathbb{E}[u_1 - u_2][u_2] &= \mathbb{E}[Y_t - X_{t-1} - (Y_t - Z_{t-1})][Y_t - Z_{t-1}] \\ &= \mathbb{E}[Z_{t-1} - X_{t-1}][Y_t - Z_{t-1}] \\ &= \mathbb{E}[Z_{t-1}Y_t] - \mathbb{E}\left[Z_{t-1}^2\right] - \mathbb{E}[X_{t-1}Y_t] + \mathbb{E}[Z_{t-1}X_{t-1}] \\ &= 0.75 - 0.9 - 0.6 + 0.25 = -0.5 < 0 \end{aligned}$$

From Picture 2 we show that most combinations in (0,1) display reductions in MSPE compared to the best performing individual forecast. At the same time, most of these combinations are auto-inefficient.

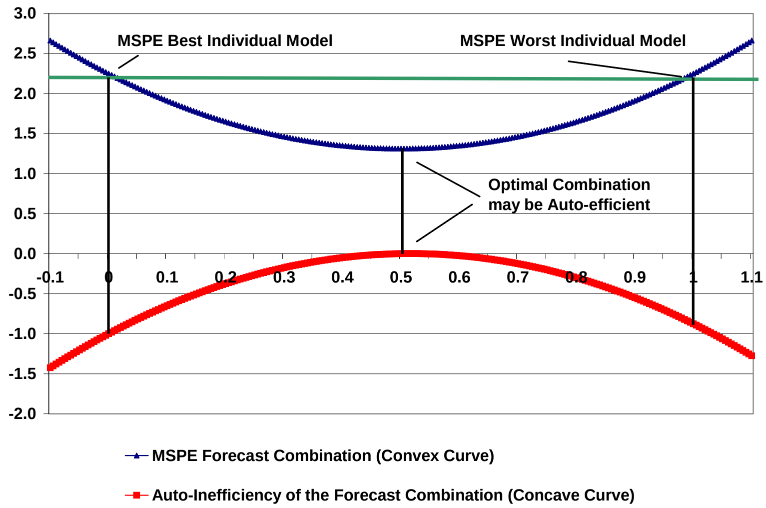
Picture 2
Auto-inefficiency of Most Forecast Combinations



Picture 3
Auto-inefficiency of Forecast Combinations When Individual Forecasts are Auto-efficient



Picture 4
The Optimal Combination May Display Auto-Efficiency



Picture 3 is associated to $\Omega(2)$. This picture depicts a situation in which all combinations in $(0,1)$ are auto-inefficient. Picture 4 comes from $\Omega(3)$. This picture provides an example in which the optimal combination is auto-efficient.

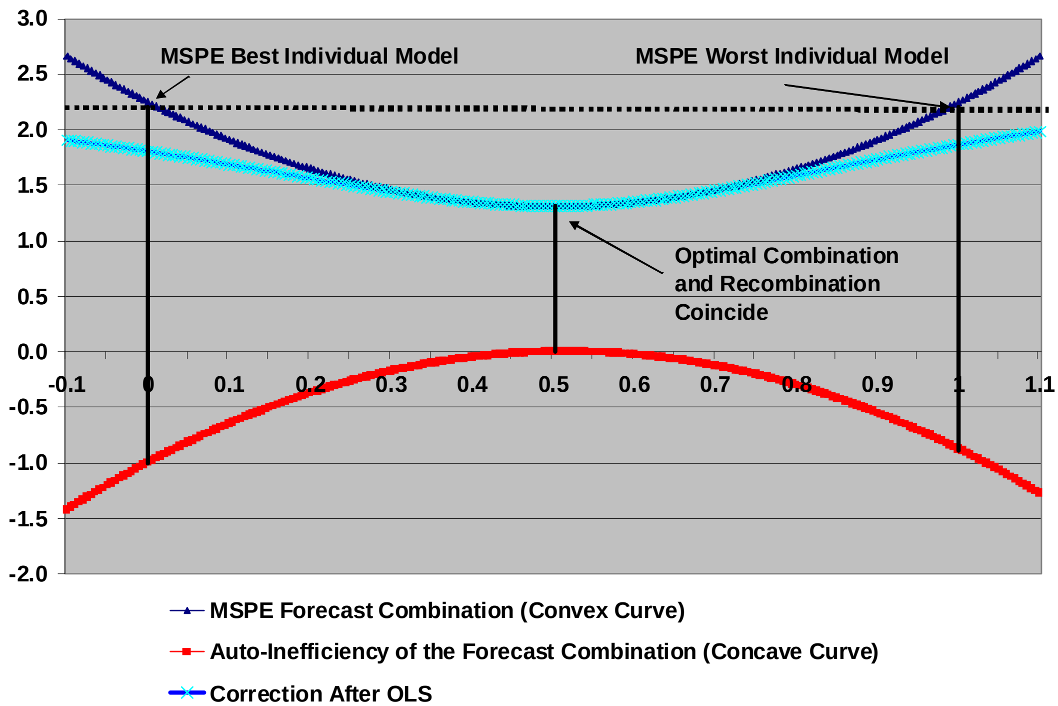
Pictures 2 and 4 correspond to virtuous examples. In Picture 2 we can see that for low values of λ we achieve both reductions in MSPE and auto-inefficiency (in absolute value) with respect to the more accurate individual forecast. Furthermore, within the region $(0,0.3)$ every single combination display lower MSPE and lower auto-inefficiency in absolute terms. A situation like this one is possible as long as

$$\begin{aligned}
\mathbb{E}\left[Y_2 u_2\right] &< 0 \\
\frac{d\left(\left[\mathbb{E}\left(u^C\right)^2-\mathbb{E}\left(u_2\right)^2\right]\right)|_{\lambda=0}}{d\lambda} &= 2\mathbb{E}\left[u_1-u_2\right]\left[u_2\right]<0 \\
\frac{d\left(\left[\mathbb{E}\left(Y^C u^C\right)^2-\mathbb{E}\left(Y_2 u_2\right)^2\right]\right)|_{\lambda=0}}{d\lambda} &= \mathbb{E}\left[u_2-u_1\right]\left[u_2-Y_2\right]>0
\end{aligned}$$

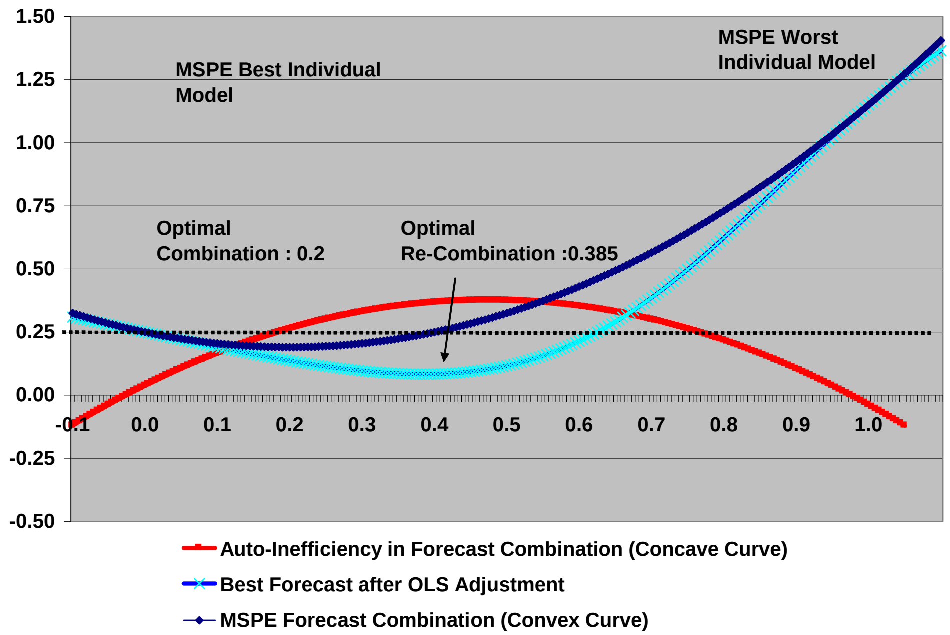
Picture 4 is also a particular case in which there is a relatively wide region for combinations to be approximately optimal and auto-efficient. This is a case in which every single combination brings reductions in MSPE and auto-inefficiency in absolute terms.

Pictures 5 and 6 next correspond to $\Omega(3)$ and $\Omega(4)$ respectively. Both figures show cases in which λ^{**} represents a global solution to the optimal recombination problem in $(0, 1)$. The main difference between pictures 5 and 6 relies on the symmetry condition (15). In picture 5 the symmetry condition holds true and consequently λ^{**} coincides with λ^{*}. In picture 6, however, the symmetry condition does not hold true and therefore the optimal combination and recombination weights λ^{*} and λ^{**} are different.

Picture 5
The Optimal Combination and Optimal Recombination May Coincide



Picture 6
The Optimal Combination and Optimal Recombination May Differ

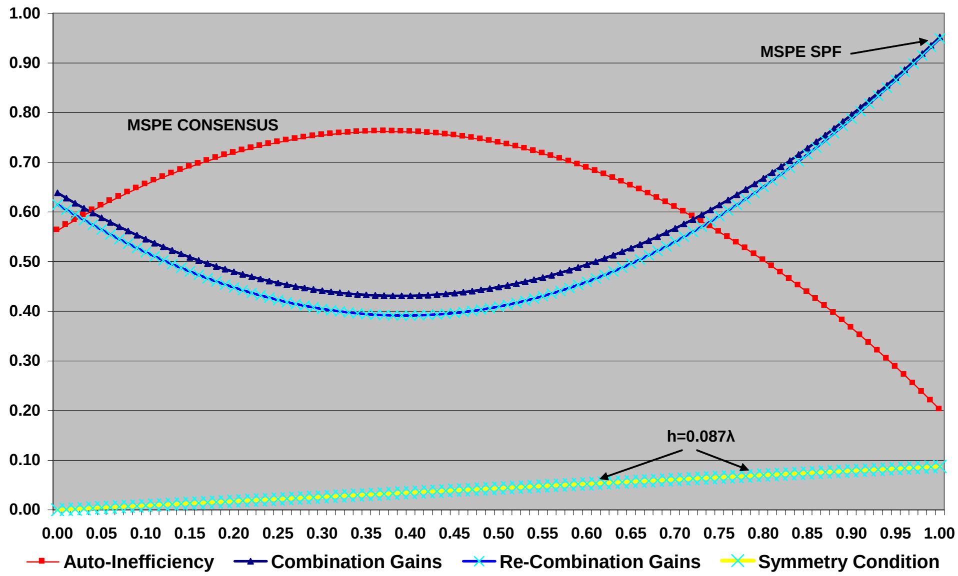


4 Empirical Illustration

In this section we present an empirical application in which we combine two sets of forecasts for Chilean year-on-year CPI inflation. The exercise is twofold. First we consider two sets of three month ahead forecasts for Chilean inflation: those reported by Consensus Economics and those coming from the Survey of Professional Forecasters (SPF) carried out at a monthly basis by the Central Bank of Chile. We consider the simple average of the individual forecasts reported by Consensus and the median of the forecasts coming from the SPF, which is information publicly available. Because the SFP is carried out at the beginning of each month while Consensus' survey is carried out at the middle of each month, we expect a little advantage of Consensus over the SPF. We have a sample of 98 monthly observations for the period November 2001-December 2009.

Picture 7 shows with a blue line that combination gains do exist for a wide region of the interval (0,1). With a simple combination, the MSPE can go down from 0.64 (Consensus MSPE) to 0.43. The red line, however, shows that every single combination is auto-inefficient. This allows for further improvement using an OLS recombination. This adjustment is shown in a light blue color. We see that this final recombination results in further reductions in MSPE.

Picture 7
Auto-Inefficiency when Combining Inflation Forecasts from the SPF and Consensus
Forecasting Horizon: 3 Months Ahead

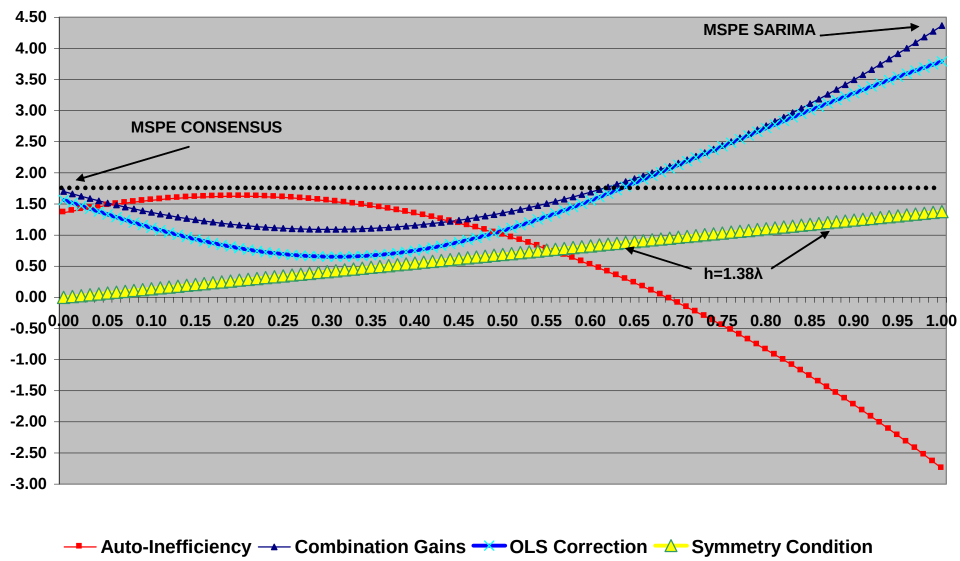


Second, we consider two sets of six month ahead forecasts for Chilean inflation: those reported by Consensus Economics and those coming from the simple average of eight SARIMA models fitted for Chilean inflation following the early work of Pincheira & García (2012). Each SARIMA model is estimated with rolling windows of forty observations. Out-of-sample forecasts are generated and thus we collect predictions six months ahead. The sample period is a little different than before. We work with 105 monthly observations from May 2001 until December 2009.

Picture 8 shows with a blue line that combination gains do exist for a wide region of the interval (0,1). With a simple combination, the MSPE can go down from 1.70 (Consensus MSPE) to 1.09. The red line, however, shows that almost every single combination is auto-inefficient. This allows for further improvement using an OLS recombination. This adjustment is shown in a light blue color. We see that this final recombination results in important reductions in MSPE. For instance, when

$\lambda = 0.30$, the MSPE of the traditional combination is 1.09, yet the MSPE of the adjusted recombination is only 0.65.

Picture 8
Auto-Inefficiency when Combining Inflation Forecasts from the SPF and Consensus
Forecasting Horizon: 6 Months Ahead



There is an important shortcoming in these two applications: we have used all the available data to compute sample estimates of the optimal weight and OLS adjustment. This, of course, is impossible in a real time application. We move towards a final exercise aimed at evaluating in a total out-of-sample fashion the benefits from combination and from the OLS adjustment. We consider again a rolling window of forty observations of forecasts and the target variable. Within these rolling windows we compute both the optimal combination using λ^* and the corresponding OLS adjustment. We then build a real time sequence of combined forecasts and OLS-adjusted forecasts. Table 1 next shows the result of this exercise.

Table 1
MSPE of Different Combination Strategies
Out-of-Sample Results for Chilean Inflation

	Consensus	Optimal Linear Combination	Optimal Recombination
3 months ahead	0.85	0.65	0.60
6 months ahead	2.69	1.84	1.19

Combination gains and re-combination gains are quite important, especially at longer horizons. This means that an OLS adjustment provides an useful way to remove the auto-inefficiency of forecast combinations.

5 Summary and Conclusions

It is well known that weighted averages of two competing forecasts may reduce Mean Squared Prediction Errors (MSPE) and generate certain problems. In this paper we take an in-depth view of one particular type of problem stemming from simple combination schemes. This problem is called forecast auto-inefficiency, and refers to the notion of inefficiency analyzed by Mincer and Zarnowitz (1969).

Under mild assumptions we show that linear convex forecast combinations are auto-inefficient with probability one, and therefore room for accuracy improvement is almost surely possible. This implies that greater reductions in MSPE are possible and has the additional implication that traditional optimal linear combination weights might be sub-optimal in a broader sense. We also show that certain symmetry condition is sufficient to ensure that the traditional combination scheme is optimal in this broader sense.

We also identify testable conditions under which every linear convex combination of two forecasts is auto-inefficient. In particular, we show that the process of taking averages of forecasts may induce inefficiencies in the combination, even when the individual forecasts are auto-efficient. The extent to which these inefficiencies are indeed relevant requires a case by case analysis. Nonetheless it is striking that in many applications in which a number of different forecasts are available, the combination of all of them seems to be the last step in the search of forecast accuracy, and no attempt to take advantage of potential inefficiencies stemming from the combination process is pursued.

We illustrate our findings with an empirical application in which two different forecast for Chilean CPI inflation are combined. In a totally out-of-sample exercise, we show that gains from combination may be huge, but that the auto-inefficiency induced by the combination may also be sizable. In our empirical application an OLS-adjustment seems to remove this inefficiency quite well.

The extension of our results to the combination of more than two forecasts seems conceptually straightforward, yet tedious and cumbersome. Further research may include the aforementioned extension to combinations of any number of forecasts, as well as a thorough empirical analysis of the benefits of our results in larger data sets.

6 References

1. *Aiolfi M, C. Capistrán and A. Timmermann (2011): "Forecast Combinations". In The Oxford Handbook of Economic Forecasting, Edited by Michael Clements and David Hendry.*

2. *Aiolfi M. and A. Timmermann (2006): "Persistence in Forecasting Performance and Conditional Combination Strategies". Journal of Econometrics. 135: 31-53.*

3. *Batchelor R. and P. Dua (1995): "Forecaster diversity and the benefits of combining forecasts". Management Science. January 41(1): 68-75.*

4. *Bates J and C. Granger. (1969): "The combination of forecasts". Operational Research Quarterly, 20, 451-68.*

5. *Capistrán C. and A. Timmermann (2009): "Forecast Combination with Entry and Exit of Experts". Journal of Business, Economics and Statistics, 27: 429-440*

6. *Chong, Y. and D. Hendry (1986): "Econometric Evaluation of Linear Macroeconomic Models". Review of Economic Studies, 53, 671-690.*

7. *Clemen. R. (1986): "Linear constraints and the efficiency of combined forecasts". Journal of Forecasting, 5:31-38.*

8. *Clements M. and D. Harvey (2011): "Combining Probability Forecasts". International Journal of Forecasting, 27(2): 208-223.*

9. *Clements, M. and D. Hendry (1993): "On the Limitations of Comparing Mean Square Forecast Errors". Journal of Forecasting, 12: 617-637.*

10. *Costantini M and R. Kunst (2011): "Combining forecasts based on multiple encompassing tests in a macroeconomic core system". Journal of Forecasting, 30(6): 579-596, September.*

11. *Diebold F. (1988): "Serial correlation and the combination of forecasts". Journal of Business, Economics and Statistics 6(1): 105-111.*

12. *Diebold, F and R. Mariano (1995): "Comparing Predictive Accuracy". Journal of Business and Economic Statistics 13: 253-263.*

13. *Elliot, G. and A. Timmermann (2008): "Economic Forecasting". Journal of Economic Literature 46(1): 3-56.*

14. *Granger, C. and P. Newbold. (1973): "Some Comments on the Evaluation of Economic Forecasts". Applied Economics, 5: 35-47.*

15. *Granger, C. and P. Newbold. (1986): "Forecasting Economic Time Series". (2nd ed.), Orlando, FL: Academic Press.*

16. *Granger C. and R. Ramanathan (1984): "Improved methods of combining forecasts". Journal of Forecasting 3: 197-204.*

17. *Hansen B. (2008): "Least Squares Forecast Averaging". Journal of Econometrics, 146: 342 350.*

18. *Harvey, D., Leybourne S. and P. Newbold (1998): "Tests for Forecast Encompassing". Journal of Business & Economic Statistics, 16(2): 54-259.*

19. *Kolassa. (2011): "Combining exponential smoothing forecasts using Akaike weights". International Journal of Forecasting 27(2): 238-251.*

20. *Mincer, J. and V. Zarnowitz (1969): "The Evaluation of Economic Forecasts". In J.Mincer (ed.) Economic Forecasts and Expectations, National Bureau of Economic Research,New York.*

21. *Newbold, P. and C. Granger (1974): "Experience With Forecasting Univariate Time Series and the Combination of Forecasts". Journal of the Royal Statistical Society, Ser. A, 137:131-165.*

22. *Patton, A. and A. Timmermann (2012): "Forecast Rationality Tests Based on Multi-Horizon Bounds". Journal of Business & Economic Statistics, forthcoming.*

23. *Pincheira P. and A. García (2012): "En busca de un buen marco de referencia predictivo para la inflación en Chile". El Trimestre Económico 313 (enero-marzo de 2012).*

24. *Pincheira P. (2012) "Shrinkage Based Tests of Predictability". Journal of Forecasting, forthcoming.*

25. *Poncela P. J. Rodriguez, R. Sánchez-Mangas and E. Senra (2011): " Forecast combinations through dimension reduction techniques". International Journal of Forecasting 27(2): 224-237.*

26. *Stock J. and M. Watson (2004): "Combination forecasts of output growth in a seven-country data set". Journal of Forecasting, 23(6): pages 405–430, September 2004.*

27. *Timmermannn A. (2006): "Forecast Combinations". In Handbook of Economic Forecasting, Elliott G, Granger C, Timmermannn A (eds): 135-194.*

28. *West K.(1996): "Asymptotic Inference About Predictive Ability". Econometrica 64: 1067-1084.*

29. *White H. (2000). "A Reality Check for Data Snooping". Econometrica 68: 1097-1126.*

30. *Wright, J. (2008). "Bayesian Model Averaging and exchange rate forecasts". Journal of Econometrics. 146(2): 329-341, October.*

7 Appendix

7.1 Lemmas

Lemma 11

$$\begin{aligned}\mathbb{E}[Y^C u^C] &= -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\ \mathbb{E}[Y^C u^C] &= \lambda^2 \mathbb{E}\left[Y_1 u_1\right] + (1-\lambda)^2 \mathbb{E}\left[Y_2 u_2\right] + \lambda(1-\lambda)\mathbb{E}\left[Y_1 u_2 + Y_{12} u_1\right]\end{aligned}$$

Proof. Let us derive the first expression

$$\begin{aligned}\mathbb{E}[Y^C u^C] &= \mathbb{E}[\lambda Y_1 + (1-\lambda) Y_2]\left[\lambda u_1 + (1-\lambda) u_2\right] \\ \mathbb{E}[Y^C u^C] &= \mathbb{E}\left[\lambda\left[Y_1 - Y_2\right] + Y_2\right]\left[\lambda\left[u_1 - u_2\right] + u_2\right] \\ &= \lambda^2 \mathbb{E}\left[\left(Y_1 - Y_2\right)\left(u_1 - u_2\right)\right] + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2\right] + \\ &\quad + \lambda \mathbb{E}\left[u_1 - u_2\right]\left[Y_2\right] + \mathbb{E}\left[Y_2 u_2\right]\end{aligned}$$

Notice that

$$u_1 - u_2 = (Y - Y_1) - (Y - Y_2) = (Y_2 - Y_1) = -\left(Y_1 - Y_2\right)$$

therefore

$$\begin{aligned}\mathbb{E}[Y^C u^C] &= -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2\right] + \\ &\quad - \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[Y_2\right] + \mathbb{E}\left[Y_2 u_2\right]\end{aligned}$$

so finally we have

$$\mathbb{E}[Y^C u^C] = -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2 u_2\right]$$

Let us derive the second expression. From

$$\mathbb{E}[Y^C u^C] = -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2 u_2\right]$$

we have

$$\begin{aligned}\mathbb{E}[Y^C u^C] &= -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - u_1\right] + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\ &= -\lambda^2 \mathbb{E}\left[Y_1 u_2 - Y_1 u_1 - Y_2 u_2 + Y_2 u_1\right] + \\ &\quad + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2\right] - \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\ &= -\lambda^2 \mathbb{E}\left[Y_1 u_2 - Y_1 u_1 - Y_2 u_2 + Y_2 u_1\right] + \\ &\quad + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2\right] - \lambda \mathbb{E}\left[u_2 - u_1\right]\left[Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\ &= -\lambda^2 \mathbb{E}\left[Y_1 u_2 - Y_1 u_1 - Y_2 u_2 + Y_2 u_1\right] + \\ &\quad + \lambda \mathbb{E}\left[Y_1 u_2\right] - \lambda \mathbb{E}\left[Y_2 u_2\right] - \lambda \mathbb{E}\left[u_2 Y_2\right] + \lambda \mathbb{E}\left[u_1 Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\ &= -\lambda^2 \mathbb{E}\left[Y_1 u_2 - Y_1 u_1 - Y_2 u_2 + Y_2 u_1\right] + \lambda \mathbb{E}\left[Y_1 u_2\right] + \\ &\quad + \lambda \mathbb{E}\left[Y_2 u_1\right] + \mathbb{E}\left[u_2 Y_2\right]\{-2\lambda + 1\}\end{aligned}$$

therefore

$$\begin{aligned}\mathbb{E}[Y^C u^C] &= \mathbb{E}[Y_1 u_1]\lambda^2 + \mathbb{E}[Y_2 u_2]\left\{\lambda^2 - 2\lambda + 1\right\} + \left\{\lambda - \lambda^2\right\}\mathbb{E}[Y_1 u_2 + Y_2 u_1] \\ &= \lambda^2\mathbb{E}[Y_1 u_1] + (1-\lambda)^2\mathbb{E}[Y_2 u_2] + \lambda(1-\lambda)\mathbb{E}[Y_1 u_2 + Y_2 u_1]\end{aligned}$$

■

Lemma 12

$$\begin{aligned}Cov(Y^C, u^C) &= -\lambda^2 V[Y_1 - Y_2] + \lambda Cov(Y_1 - Y_2, u_2 - Y_2) + Cov(Y_2, u_2) \\ Cov[Y^C, u^C] &= \lambda^2 Cov(Y_1, u_1) + (1-\lambda)^2 Cov(Y_2, u_2) + \lambda(1-\lambda) Cov(Y_1, u_2) + \\ &\quad + Cov(Y_2, u_1)\end{aligned}$$

Proof. Let us derive the first expression

$$\begin{aligned}\mathbb{C}ov(Y^C, u^C) &= \mathbb{C}ov(\lambda Y_1 + (1-\lambda)Y_2, \lambda u_1 + (1-\lambda)u_2) \\ \mathbb{C}ov(Y^C, u^C) &= \mathbb{C}ov(\lambda[Y_1 - Y_2] + Y_2, \lambda[u_1 - u_2] + u_2) \\ &= \lambda^2\mathbb{C}ov(Y_1 - Y_2, u_1 - u_2) + \lambda\mathbb{C}ov(Y_1 - Y_2, u_2) + \\ &\quad + \lambda\mathbb{C}ov(u_1 - u_2, Y_2) + \mathbb{C}ov(Y_2, u_2)\end{aligned}$$

Notice that

$$u_1 - u_2 = (Y - Y_1) - (Y - Y_2) = (Y_2 - Y_1) = -(Y_1 - Y_2)$$

therefore

$$\begin{aligned}\mathbb{C}ov(Y^C, u^C) &= -\lambda^2\mathbb{V}[Y_1 - Y_2] + \lambda\mathbb{C}ov(Y_1 - Y_2, u_2) + \\ &\quad - \lambda\mathbb{C}ov(Y_1 - Y_2, Y_2) + \mathbb{C}ov(Y_2, u_2)\end{aligned}$$

so finally we have

$$\mathbb{C}ov(Y^C, u^C) = -\lambda^2\mathbb{V}[Y_1 - Y_2] + \lambda\mathbb{C}ov(Y_1 - Y_2, u_2 - Y_2) + \mathbb{C}ov(Y_2, u_2)$$

Let us derive the second expression. From

$$\mathbb{C}ov(Y^C, u^C) = -\lambda^2\mathbb{V}[Y_1 - Y_2] + \lambda\mathbb{C}ov(Y_1 - Y_2, u_2 - Y_2) + \mathbb{C}ov(Y_2, u_2)$$

we have

$$
\begin{aligned}
\mathbb{C}ov(Y^C, u^C) &= -\lambda^2 \mathbb{V}\left[Y_1 - Y_2\right] + \lambda \mathbb{C}ov(Y_1 - Y_2, u_2 - Y_2) + \mathbb{C}ov(Y_2, u_2) \\
&= -\lambda^2 \mathbb{C}ov(Y_1 - Y_2, u_2 - u_1) + \\
&\quad + \lambda \mathbb{C}ov(Y_1 - Y_2, u_2) - \lambda \mathbb{C}ov(Y_1 - Y_2, Y_2) + \mathbb{C}ov(Y_2, u_2) \\
&= -\lambda^2 \mathbb{C}ov(Y_1 - Y_2, u_2 - u_1) + \\
&\quad + \lambda \mathbb{C}ov(Y_1 - Y_2, u_2) + \lambda \mathbb{C}ov(u_1 - u_2, Y_2) + \mathbb{C}ov(Y_2, u_2) \\
&= -\lambda^2 \left[\mathbb{C}ov(Y_1, u_2) - \mathbb{C}ov(Y_1, u_1) - \mathbb{C}ov(Y_2, u_2) + \mathbb{C}ov(u_1, Y_2)\right] + \\
&\quad + \lambda \mathbb{C}ov(Y_1, u_2) - \lambda \mathbb{C}ov(Y_2, u_2) + \lambda \mathbb{C}ov(u_1, Y_2) - \lambda \mathbb{C}ov(u_2, Y_2) + \\
&\quad + \mathbb{C}ov(Y_2, u_2) \\
&= -\lambda^2 \left[\mathbb{C}ov(Y_1, u_2) - \mathbb{C}ov(Y_1, u_1) - \mathbb{C}ov(Y_2, u_2) + \mathbb{C}ov(u_1, Y_2)\right] + \\
&\quad + \lambda \mathbb{C}ov(Y_1, u_2) - 2\lambda \mathbb{C}ov(Y_2, u_2) + \lambda \mathbb{C}ov(u_1, Y_2) + \mathbb{C}ov(Y_2, u_2) \\
&= -\lambda^2 \left[\mathbb{C}ov(Y_1, u_2) - \mathbb{C}ov(Y_1, u_1) - \mathbb{C}ov(Y_2, u_2) + \mathbb{C}ov(u_1, Y_2)\right] + \\
&\quad + \lambda \mathbb{C}ov(Y_1, u_2) + \lambda \mathbb{C}ov(u_1, Y_2) + \mathbb{C}ov(Y_2, u_2) \left\{-2\lambda + 1\right\}
\end{aligned}
$$

therefore

$$
\begin{aligned}
Cov(Y^C, u^C) &= Cov(Y_1, u_1)\lambda^2 + Cov(Y_2, u_2)\left\{\lambda^2 - 2\lambda + 1\right\} + \\
&\quad + \left\{\lambda - \lambda^2\right\}\left[Cov(Y_1, u_2) + Cov(u_1, Y_2)\right] \\
&= \lambda^2 Cov(Y_1, u_1) + (1-\lambda)^2 Cov(Y_2, u_2) + \\
&\quad + \lambda(1-\lambda)\left[Cov(Y_1, u_2) + Cov(u_1, Y_2)\right]
\end{aligned}
$$

■

7.2 Proof of Proposition 9

We will use the following notation:

$$
\begin{aligned}
f(\lambda) &\equiv \mathbb{E}\left(u^C\right)^2 = \lambda^2 \mathbb{E}\left[u_1 - u_2\right]^2 + 2\lambda \mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] + \mathbb{E}\left(u_2\right)^2 \\
g(\lambda) &\equiv \mathbb{E}\left[Y^C u^C\right] = -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\
&= -\lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + \lambda \mathbb{S} - 2\lambda \mathbb{E}\left[Y_1 - Y_2\right]\left[Y_2\right] + \mathbb{E}\left[Y_2 u_2\right] \\
h(\lambda) &\equiv \mathbb{V}\left(Y^C\right) = \lambda^2 \mathbb{E}\left[Y_1 - Y_2\right]^2 + 2\lambda \mathbb{E}\left[\left(Y_1 - Y_2\right) Y_2\right] + \mathbb{V}\left(Y_2\right) \\
\mathbb{S} &= \mathbb{E}\left[Y_1 - Y_2\right] Y
\end{aligned}
$$

It follows that

$$\begin{aligned}
f'(\lambda) &= 2\lambda\mathbb{E}\left[u_1 - u_2\right]^2 - 2\mathbb{E}\left[Y_1 - Y_2\right]u_2 \\
&= 2\lambda\mathbb{E}\left[u_1 - u_2\right]^2 + 2\mathbb{E}\left[Y_1 - Y_2\right]Y_2 - 2\mathbb{S} \\
g'(\lambda) &= -2\lambda\mathbb{E}\left[u_1 - u_2\right]^2 + \mathbb{E}\left[Y_1 - Y_2\right]\left[u_2 - Y_2\right] \\
&= -2\lambda\mathbb{E}\left[u_1 - u_2\right]^2 - 2\mathbb{E}\left[Y_1 - Y_2\right]\left[Y_2\right] + \mathbb{S} \\
h'(\lambda) &= 2\lambda\mathbb{E}\left[u_1 - u_2\right]^2 + 2\mathbb{E}\left[Y_1 - Y_2\right]\left[Y_2\right]
\end{aligned}$$

therefore

$$f'(\lambda) + h'(\lambda) + 2g'(\lambda) = 0 \tag{21}$$

or

$$\begin{aligned}
f' + g' &= -\mathbb{S} \\
f' - h' &= -2\mathbb{S}
\end{aligned}$$

The critical points of $\mathbb{E}\left[u^{**}\right]^2$ for $\lambda \in (0, 1)$ must satisfy:

$$\frac{\partial\mathbb{E}\left[u^{**}\right]^2}{\partial\lambda} = 0$$

which can be written as

$$\frac{\partial\left(f(\lambda) - g^2(\lambda)/h(\lambda)\right)}{\partial\lambda} = 0$$

so we have

$$f' - \left[\frac{2gg'h - g^2h'}{h^2}\right] = 0$$

or

$$h^2f' - 2gg'h + g^2h' = 0$$

using

$$f' + g' = -\mathbb{S}$$

we have

$$\begin{aligned}
h^2f' + 2g(\mathbb{S}+f')h + g^2(f' + 2\mathbb{S}) &= 0 \\
h^2f' + 2g\mathbb{S}h + 2gf'h + g^2f' + 2\mathbb{S}g^2 &= 0 \\
h^2f' + 2gf'h + g^2f' + 2g\mathbb{S}h + 2\mathbb{S}g^2 &= 0 \\
f'\left[h + g\right]^2 + 2g\mathbb{S}\left[h + g\right] &= 0 \\
\left[h + g\right]\left[f'\left[h + g\right] + 2g\mathbb{S}\right] &= 0
\end{aligned}$$

now

$$\begin{aligned}
h+g &= \lambda\mathbb{E}\left[Y_1-Y_2\right]\left[u_2-Y_2\right]+\mathbb{E}\left[Y_2u_2\right]+2\lambda\mathbb{E}\left[\left(Y_1-Y_2\right)Y_2\right]+\mathbb{V}\left(Y_2\right)\\
&= \lambda\mathbb{E}\left[Y_1-Y_2\right]\left[Y-2Y_2\right]+\mathbb{E}\left[Y_2u_2\right]+2\lambda\mathbb{E}\left[\left(Y_1-Y_2\right)Y_2\right]+\mathbb{V}\left(Y_2\right)\\
&= \lambda\mathbb{S}+\mathbb{E}\left[Y_2u_2\right]+\mathbb{V}\left(Y_2\right)\\
&= \lambda\mathbb{S}+\mathbb{C}ov(Y_2,Y)
\end{aligned}$$

on the other hand,

$$\begin{aligned}
f'\left[h+g\right] &= \left[2\lambda\mathbb{E}\left[u_1-u_2\right]^2+2\mathbb{E}\left[Y_1-Y_2\right]Y_2-2\mathbb{S}\right]\left[\lambda\mathbb{S}+\mathbb{C}ov(Y_2,Y)\right]\\
2g\mathbb{S} &= \left[-2\lambda^2\mathbb{E}\left[Y_1-Y_2\right]^2+2\lambda\mathbb{S}-4\lambda\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right]+2\mathbb{E}\left[Y_2u_2\right]\right]\mathbb{S}\\
&= -2\mathbb{S}\lambda^2\mathbb{E}\left[Y_1-Y_2\right]^2+2\lambda\mathbb{S}^2-4\lambda\mathbb{S}\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right]+2\mathbb{S}\mathbb{E}\left[Y_2u_2\right]
\end{aligned}$$

therefore

$$\begin{aligned}
f'\left[h+g\right]+2g\mathbb{S} &= -2\lambda\mathbb{S}\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right]+\\
&\quad+\left[2\lambda\mathbb{E}\left[u_1-u_2\right]^2+2\mathbb{E}\left[Y_1-Y_2\right]Y_2-2\mathbb{S}\right]\mathbb{C}ov(Y_2,Y)+2\mathbb{S}\mathbb{E}\left[Y_2u_2\right]\\
&= 2\lambda\left(\mathbb{C}ov(Y_2,Y)\mathbb{E}\left[u_1-u_2\right]^2-\mathbb{S}\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right]\right)+\\
&\quad+2\left[\mathbb{E}\left[Y_1-Y_2\right]Y_2-\mathbb{S}\right]\mathbb{C}ov(Y_2,Y)+2\mathbb{S}\mathbb{E}\left[Y_2u_2\right]\\
&= 2\lambda\left(\mathbb{C}ov(Y_2,Y)\mathbb{E}\left[u_1-u_2\right]^2-\mathbb{S}\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right]\right)+\\
&\quad+2\left[\mathbb{E}\left[Y_1-Y_2\right]Y_2-\mathbb{E}\left[Y_1-Y_2\right]Y\right]\mathbb{C}ov(Y_2,Y)+2\mathbb{S}\mathbb{E}\left[Y_2u_2\right]\\
&= 2\lambda\left(\mathbb{C}ov(Y_2,Y)\mathbb{E}\left[u_1-u_2\right]^2-\mathbb{S}\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right]\right)+\\
&\quad+2\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2-Y\right]\mathbb{C}ov(Y_2,Y)+2\mathbb{S}\mathbb{E}\left[Y_2u_2\right]\\
&= 2\lambda\left(\mathbb{C}ov(Y_2,Y)\mathbb{E}\left[u_1-u_2\right]^2-\mathbb{S}\mathbb{E}\left[Y_1-Y_2\right]\left[Y_2\right]\right)+\\
&\quad+2\mathbb{E}\left[u_1-u_2\right]\left[u_2\right]\mathbb{C}ov(Y_2,Y)+2\mathbb{S}\mathbb{E}\left[Y_2u_2\right]
\end{aligned}$$

Let us recall that our first order condition is given by

$$\left[h+g\right]\left[f'\left[h+g\right]+2g\mathbb{S}\right]=0$$

It follows that our critical values are the roots of

$$\left[h+g\right]$$

and

$$\left[f'\left[h+g\right]+2g\mathbb{S}\right]$$

But the only root of $h+g$ satisfies

$$\lambda\mathbb{S}+\mathbb{C}ov(Y_2,Y)=0$$

which means that our first critical value λ_1 is given by

$$\lambda_1 \; = -\frac{\mathbb{C}ov(Y_2, Y)}{\mathbb{S}}$$

To analyze if this critical value is a local minimum we explore the following second derivative:

$$\begin{aligned}
\frac{\partial^2\left(f(\lambda) - g^2(\lambda)/h(\lambda)\right)}{\partial\lambda^2} &= \frac{\partial}{\partial\lambda}\left[\frac{\partial\left(f(\lambda) - g^2(\lambda)/h(\lambda)\right)}{\partial\lambda}\right] \\
&= \frac{\partial}{\partial\lambda}\left[\frac{[h+g]\left[f'[h+g] + 2g\mathbb{S}\right]}{h^2}\right] \\
&= \frac{\partial}{\partial\lambda}\left[\frac{\left[f'[h+g]^2 + 2g\mathbb{S}[h+g]\right]}{h^2}\right] \\
&= \left[\frac{f''h^2 - 2f'hh'}{h^4}\right][h+g]^2 + 2\frac{f'}{h^2}[h+g][h'+g'] + \\
&\quad +2\mathbb{S}\left[\frac{g'h^2 - 2ghh'}{h^4}\right][h+g] + 2\mathbb{S}\frac{g}{h^2}[h'+g']
\end{aligned}$$

It follows that for λ_1 we have $h + g = 0$, therefore

$$\frac{\partial\mathbb{E}\left[u^{**}\right]^2}{\partial\lambda}_{|\lambda=\lambda_1} = 2\mathbb{S}\frac{g}{h^2}\left[h'+g'\right]_{|\lambda=\lambda^*} = 2\mathbb{S}^2\frac{g}{h^2}_{|\lambda=\lambda^*}$$

but

$$h(\lambda_1) + g(\lambda_1) = 0$$

so

$$g(\lambda_1) = -h(\lambda_1) = -\mathbb{V}\left(\lambda_1(Y_1 - Y_2) + Y_2\right) < 0$$

therefore λ_1 is a local maximum of our target function and it cannot be a solution of our problem. We must then focus on the roots of $[f'[h+g] + 2g\mathbb{S}]$. They satisfy:

$$2\lambda\left(\mathbb{C}ov(Y_2,Y)\mathbb{E}\left[u_1 - u_2\right]^2 - \mathbb{S}\mathbb{E}\left[Y_1 - Y_2\right]\left[Y_2\right]\right) + 2\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right]\mathbb{C}ov(Y_2,Y) + 2\mathbb{S}\mathbb{E}\left[Y_2 u_2\right] = 0$$

which means that, given that assumption (19) hold true, our second critical value λ_2 is given by

$$\lambda_2 \; = -\frac{\mathbb{C}ov(Y_2,Y)\mathbb{E}\left[u_1 - u_2\right]\left[u_2\right] + \mathbb{S}\mathbb{E}\left[Y_2 u_2\right]}{\mathbb{C}ov(Y_2,Y)\mathbb{E}\left[u_1 - u_2\right]^2 - \mathbb{S}\mathbb{E}\left[Y_1 - Y_2\right]\left[Y_2\right]}$$

Assumption (20) ensures that both roots λ_1 and λ_2 will not coincide. Because the first order condition is a quadratic form with two different real roots, one of them is bounded to be a local maximum and the other one is bounded to be a local minimum. Given that we already proved that λ_1 is a maximum then λ_2 must be a local minimum.

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