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Sofia Bauducco

Francesco Caprioli

Documentos de Trabajo del Banco Central de Chile
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Agustinas 1180, Santiago, Chile
Teléfono: (56-2) 3882475; Fax: (56-2) 3882231

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OPTIMAL FISCAL POLICY IN A SMALL OPEN ECONOMY WITH LIMITED COMMITMENT

Sofia Bauducco
Banco Central de Chile

Francesco Caprioli
Banca d'Italiá

Abstract

We introduce limited commitment into a standard optimal fiscal policy model in small open economies. We consider the problem of a benevolent government that signs a risk-sharing contract with the rest of the world, and that has to choose optimally distortionary taxes on labor income, domestic debt and international debt. Both the home country and the rest of the world may have limited commitment, which means that they can leave the contract if they find it convenient. The contract is designed so that, at any point in time, neither party has incentives to exit. We define a small open emerging economy as one where the limited commitment problem is active in equilibrium. Conversely, a small open developed economy is an economy with full commitment. Our model is able to rationalize two stylized facts about fiscal policy in emerging economies: i) the volatility of tax revenues over GDP is higher in emerging economies than in developed ones; ii) the volatility of tax revenues over GDP is positively correlated with sovereign default risk.

Resumen

En este artículo introducimos compromiso parcial en un modelo estándar de política fiscal óptima para economías pequeñas y abiertas. Consideramos el problema de un planificador benevolente que suscribe un contrato de riesgo compartido con el resto del mundo, y que tiene que elegir de manera óptima impuestos distorsivos al ingreso por trabajo, deuda doméstica y deuda internacional. Tanto la economía doméstica como el resto del mundo pueden tener compromiso parcial, lo cual implica que pueden abandonar el contrato si lo estiman conveniente. El contrato es diseñado de manera que, en cualquier momento del tiempo, ninguna de las partes tiene incentivos a abandonarlo. Definimos una pequeña economía abierta emergente como una economía donde el problema de compromiso parcial está activo en equilibrio. Por el contrario, una pequeña economía abierta desarrollada es una economía con compromiso total. Nuestro modelo puede racionalizar dos hechos estilizados sobre política fiscal en economías emergentes: i) la volatilidad de la recaudación tributaria sobre PIB es mayor en economías emergentes que en economías desarrolladas, ii) la volatilidad de la recaudación tributaria sobre PIB está positivamente correlacionada con el riesgo de default soberano.

Corresponding author. Email: sbauducco@bcentral.cl . Address: Agustinas 1180, Santiago, Chile. Phone:(+562) 6702207.
Email: francesco.caprioli@bancaditalia.it

1 Introduction

The international evidence on tax revenues suggests that tax revenues over GDP are more volatile in small open emerging than in small open developed economies: Figure 1 shows the coefficient of variation of tax revenues over GDP for 28 developed economies (in red) and 25 emerging economies (in blue) for the period 1997-2009[1]. From this figure, we can conclude that the coefficient of variation of tax revenues over GDP in small open emerging economies almost doubles that of small open developed economies[2].

In addition, we find a positive correlation between the volatility of tax revenues over GDP and the EMBIG spread, which is the variable we have used to classify a country as a small open emerging economy. Figure 2 shows this correlation for the period 1997-2009, for 20 small open emerging economies. In Section 2 we explore this evidence more in depth and find that this correlation is positive and statistically significant even when controlling for the volatility of GDP or of government expenditure. We interpret the EMBIG spread, which is a measure of country risk premium, as a degree of the lack of commitment that the country has towards foreign sovereign obligations. Then, countries with more volatile series of tax revenues over GDP are countries with worse commitment problems towards its foreign sovereign debt.

In this paper we develop a model of optimal fiscal policy for a small open economy that is able to rationalize the previous set of facts. To this end, we introduce limited commitment in a model of international risk sharing *à la* Marcet and Marimon (1992) and Kehoe and Perri (2002), where the countries set their fiscal policy in a Ramsey fashion. We define a small open emerging economy as an economy that has entered the international financial market but that can default on its foreign bonds at some state of nature. Conversely, a developed economy is an economy that never defaults on its foreign obligations. We show that emerging economies have much more volatile tax revenues over GDP than developed economies[3]. Moreover, we show that, as the economy transits from being emerging to being developed, this volatility is reduced. Then, we are able to generate a positive correlation between the volatility of tax revenues over GDP and the degree of lack of commitment towards foreign obligations.

We specify the world economy to be populated by two countries: the home country and the rest of the world. The home country is populated by risk averse households. The fiscal authority has to finance an exogenous public expenditure shock either through distortionary labor income taxes or by issuance of internal and/or international debt. The rest of the world is inhabited by

[1] Small open emerging economies are defined as economies for which the EMBIG spread of JP Morgan is computed. Conversely, small open developed economies are economies that belong to the OECD and that do not pertain to the former group. We have also discarded the U.S. for being a large economy. Details on the data sources can be found in Appendix A.1.

[2] The mean for small open emerging economies is 11.14%, while in small open developed economies is 5.6%.

[3] In terms of the model, the volatility of tax revenues over GDP is equal to the volatility of labor tax rates. Therefore, when discussing the model, we focus on the volatility of tax rates and relate our results to the classic prescription of smooth taxation of Lucas and Stokey (1983).

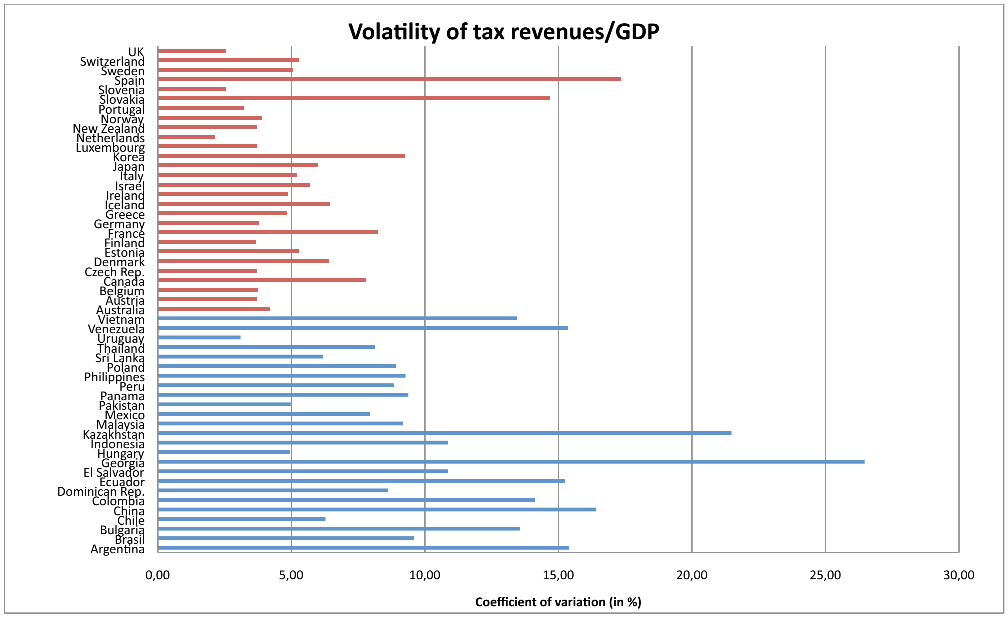

Figure 1: Volatility of tax revenues for emerging and developed countries

risk-neutral agents that receive a constant endowment and have to decide how much to consume and how much to borrow/lend in the international capital market. A contract, signed by the two countries, regulates international capital flows. We assume that, when a country enters this contract it has limited commitment, which implies that the country will terminate the contract if, for some state of nature, the outside option is more attractive than the continuation value of staying in it. Consequently, the contract needs to specify participation constraints that define adjustments in the allocations necessary to rule out default in equilibrium.

The presence of limited commitment lessens international risk sharing among countries. In consequence, when a large government expenditure shock hits the home country, a fraction of this expenditure has to be absorbed by tax revenues, and this fraction increases, the stronger the commitment problem is. Then, tax revenues over GDP are more volatile in emerging economies characterized by limited commitment than in developed economies that have full commitment.

A corollary of our analysis is that optimal fiscal policy in the presence of limited commitment should be procyclical. A (large) negative shock should be met by an increase in tax rates, and the converse holds for a (large) positive shock. We study the robustness of this assertion shutting down shocks to government expenditure and introducing productivity shocks. Our exercise suggests that a small deviation from full commitment is sufficient to turn fiscal policy from strongly countercyclical to procyclical. This result is in line with the recent discussion on the cyclical properties of fiscal policy for emerging economies when markets are incomplete. Examples of papers that go in this line are Riascos and Vegh (2003) and Cuadra et al. (2010).

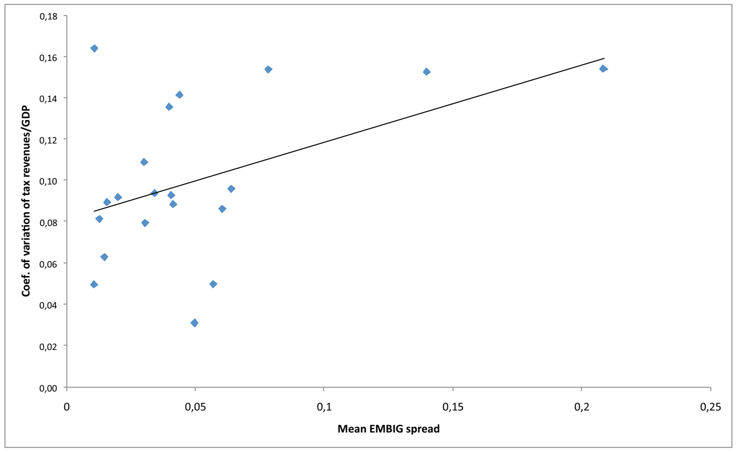

Figure 2: Volatility of tax revenues and country risk premium

Alternative explanations for the high volatility of tax rates observed in emerging economies rely on the quality of their institutions and the sources of tax collection. It is argued that emerging countries are more prone to switches in political and economic regimes that, almost by definition, translate into unstable tax systems. Moreover, in booms these countries often tax heavily those economic sectors that are responsible for the higher economic activity[4]. As a consequence, when economic conditions deteriorate, necessarily tax revenues go down dramatically. We are aware that these considerations are relevant sources of tax variability and that our study does not incorporate them in the analysis. However, we do not intend to provide an exhaustive description of such sources, but rather to focus on sovereign risk and incomplete international capital markets as possible causes for the high tax rate volatility of emerging economies.

Several papers have introduced the idea of limited commitment to study many important issues. Among others, Kehoe and Perri (2002) introduce credit arrangement between countries to reconcile international business cycle models with complete markets and the data, Krueger and Perri (2006) look at consumption inequality, Chien and Lee (2010) look at capital taxation in the long-run, Marcet and Marimon (1992) study the evolution of consumption, investment and output, and Kocherlakota (1996) analyzes the properties of efficient allocations in a model with symmetric information and two-sided lack of commitment. To our knowledge, none of them has focused on the impact of limited commitment on the volatility of optimal taxation.

In the recent years there have been some attempts to add default to dynamic macroeconomic models. A number of papers (Arellano (2008), Aguiar and Gopinath (2006)) have introduced sovereign default in otherwise standard business cycle models in order to quantitatively match some empirical regularities of small open emerging economies. More specifically, they adapt the

[4]As an example, in the recent years Argentina has been experiencing rapid export-led growth, mainly due to exports of commodities such as soya. In this period, the government's main source of tax revenues has come from taxation of these exports.

framework of Eaton and Gersovitz (1981) to a dynamic stochastic general equilibrium model. These models are usually able to explain with relative success the evolution of the interest rate, current account, output, consumption and the real exchange rate. Nevertheless, since they all consider endowment economies, they fail to capture the effects of default risk over the taxation scheme. Our contribution is to extend the analysis to be able to characterize the shape of fiscal policy and the links between the risk of default and taxes in a limited commitment framework.

The closest papers to ours are probably those by Pouzo (2008), Scholl (2009) and Mendoza and Oviedo (2006). The first paper studies the optimal taxation problem in a closed economy under incomplete markets allowing for default on internal debt. The second paper analyzes the problem of a donor that has to decide how much aid to give to a government that has an incentive to use these external resources to increase its own personal consumption without decreasing the distortive tax income it levies on private agents. Finally, the third paper documents the higher volatility of public revenues over GDP for developing countries[5]. However, the main focus of this paper is to explain other features of fiscal policy in developing economies, such as why government expenditure is procyclical in these economies. Therefore, the authors use this empirical observation as a justification to model fiscal revenues as a more volatile exogenous process in developing economies. None of these papers explains the higher volatility of tax revenues to GDP ratios in emerging economies than in developed ones.

The rest of the paper proceeds as follows. Complementing Figure 2, Section 2 provides some evidence on the positive correlation between sovereign risk and volatility of tax revenues as a fraction of GDP. Section 3 provides functional definitions of small open emerging and developed economies, in accordance to the model described in Section 4. Section 5 shows how the optimal fiscal plan is affected by the the presence of limited commitment in the case study of a perfectly anticipated one-time fiscal shock. In Section 6 we solve the model for the general case of an autocorrelated government expenditure shock. The cyclical properties of the optimal fiscal policy plan when there is limited commitment are analyzed in Section 7. Section 8 is devoted to show that our economy can be reinterpreted as one in which the government can issue debt subject to debt limits, both on internal and external debt[6]. Section 9 concludes.

[5] We have followed this work to produce Figure 1.

[6] In this section, we show that we can write the problem as one in which countries trade a complete set of instruments subject to debt limits. At first sight, it may seem unrealistic to assume that countries have access to a a complete set of foreign state-contingent bonds. However, countries can replicate the complete markets solution by issuing debt of different maturities, as Angeletos (2002) and Buera and Nicolini (2004) show.

2 Stylized facts

In this section we present some evidence showing that the volatility of tax revenues over GDP of a country is correlated with the country's risk premium[7].

We use annual data on tax revenues over GDP, total government expenditure over GDP and GDP from 1997 to 2009[8]. Our measure of volatility of tax revenues over GDP and total government expenditure over GDP is the coefficient of variation of the respective variables for the period considered. We use the coefficient of variation to take into account the difference in means of theses variables across countries.

In the analysis that follows, we need to control for the volatility of shocks that may be hitting these economies and causing the volatility of tax revenues to GDP to be high. In particular, we control for the volatility of the government expenditure shock and the volatility of GDP, which reflects shocks such as productivity shocks, as these may be important determinants of tax revenues. In the case of GDP, we compute the standard deviation of the cyclical component of (the log of) GDP. In order to extract the cyclical component, we filter the series using a Hodrick-Prescott filter with a smoothing parameter of 6.25, as recommended by Ravn and Uhlig (2002). As a measure of country risk premium, we use the mean of the EMBIG spread over the sample period for each country considered[9]. The EMBIG is an index computed by JPMorgan that tracks total returns for U.S dollar denominated debt instruments issued by emerging market sovereign and quasi-sovereign entities.

Figure 2 shows a scatter plot of the coefficients of variation of tax revenues over GDP and mean values of the EMBIG spread for the whole sample period, for those emerging countries for which data is available. As can be seen in the figure, there is a positive relation between country risk and volatility of tax revenues over GDP.

Next, we run a number of regressions to have a clearer idea of the correlation between these variables, and whether the relation between them is statistically significant[10]. We perform standard OLS regressions and, to account for possible outliers, we also run median regressions to obtain robust estimations. Table 1 shows the main results. We can observe in the table that the results show a positive relation between the volatility of tax revenues over GDP and the country risk premium. The relation between these variables is significant when not controlling for other variables, both in the case of OLS and median regression. This is also true when

[7] In the model depicted in the next sections, the marginal tax rate is equal to tax revenues over GDP. Although this identity is due to the specific tax structure and production function considered, due to data unavailability we cannot obtain marginal tax rates for the countries for which we perform the analysis. Consequently, we take tax revenues over GDP to be the best proxy available for marginal tax rates.

[8] See Appendix A.1 for a description of the data.

[9] The original spread is expressed in basis points. We have divided the variable by 10000 to express it in units.

[10] We do not intend to extract conclusions in terms of causality. Instead, we only derive conclusions in terms of correlations between the variables of interest. Although an analysis of causality would certainly be interesting, due to data availability this is not possible.

Table 1: Dependent variable: Volatility of tax revenues/GDP

	OLS				Median regression			
	1	2	3	4	1	2	3	4
Mean (EMBIG)	0.374**	0.339*	0.215	0.153	0.364**	0.241*	0.274**	0.143
	(0.169)	(0.18)	(0.172)	(0.182)	(0.132)	(0.118)	(0.12)	(0.199)
Std. Dev. GDP		0.355		0.52		0.901**		1.02
		(0.564)		(0.512)		(0.393)		(0.60)
Volatility Exp/GDP			0.491**	0.524**			0.256	0.423
			(0.232)	(0.234)			(0.226)	(0.368)
(Pseudo) R^2	0.215	0.232	0.378	0.416	0.176	0.217	0.227	0.252
Observations	20	20	20	20	20	20	20	20

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The sample period is 1997-2009, and it may vary for some countries due to data availability. The measures of volatility are computed over annual data. Volatility of tax revenues over GDP is computed as the coefficient of variation of tax revenues over GDP for the period under consideration. Mean (EMBIG) is the mean value over the sample period of the EMBIG spread for each country considered. Volatility of Exp/GDP is computed as the coefficient of variation of total government expenditures over GDP. Std. Dev. GDP is computed as the standard deviation of the cyclical component of (the log of) GDP. In order to extract the cyclical component, we filter the series using a Hodrick-Prescott filter with a smoothing parameter of 6.25.

the volatility of GDP is added as a control variable, although the coefficient associated to the country risk premium decreases when computing the median regression. Adding government expenditure over GDP as a control implies that Mean (EMBIG) is no longer significant under OLS. However, under median regression, the variable is still significant and the coefficient is of similar magnitude as in the previous case. Finally, when we control for both variables, the coefficients become non-significant in both cases.

The results previously depicted point to the fact that there is a positive relation between the volatility of tax revenues over GDP and the country risk premium. The fact that, when introducing volatility of total government expenditure over GDP as a control, the relation is no longer statistically significant (columns 3 and 4 of OLS analysis, and column 4 of median regression analysis) is probably due to the fact that the government expenditure includes interest rate payments for government liabilities, and these are more volatile, the higher the risk premium is. Moreover, the reduction in degrees of freedom when introducing regressors may be an issue with few observations, as is our case.

In order to quantify the relation between the variables of interest, in Table 2 we report the correlation between the volatility of tax revenues over GDP and the country risk premium. From the table it is clear that the volatility of tax revenues over GDP and the country risk premium are positively correlated, even when controlling for the standard deviation of GDP

Table 2: Correlation and partial correlations

Z	$corr(X, Y\|Z)$
-	0.463
Std. Dev. GDP	0.415
Volatility Exp/GDP	0.291
Std. Dev. GDP, Volatility Exp/GDP	0.206

X = Volatility of tax revenues/GDP, Y = mean (EMBIG)

and the volatility of government expenditure over GDP: the partial correlation amounts to 0.2 when imposing both controls simultaneously.

To summarize, in this section we explore the relation between the volatility of tax revenues over GDP and the country risk premium. Our findings suggest that there is a positive correlation between these variables, even when controlling for possible shocks that may affect tax revenues, such as government expenditure shocks and productivity shocks.

3 Emerging and developed economies

Consider the case of a small economy that decides to engage itself in a risk-sharing contract with other countries by contracting transfers[11]. We assume that, when an economy enters the international financial market, it has no commitment mechanism embedded to it such that it can credibly commit to stay in a transfer contract at all times and states of the world. Therefore, an economy that decides to contract transfers with others may, at a certain point, want to leave the contract if the continuation value of such strategy exceeds the continuation value of staying in the contract. In order to prevent any party from leaving the contract unilaterally, the transfer contract must be designed so as to satisfy *participation constraints*:

Definition 1. *A participation constraint for country i is*

$$V_t^i(s^t) \equiv E_t \sum_{j=0}^{\infty} \beta^j u_t^{i,c}(s^t) \geq E_t \sum_{j=0}^{\infty} \beta^j u_t^{i,a}(s^t) \equiv V_t^{a,i}(s^t) \ \forall s^t, \tag{1}$$

where β is the discount factor of economy i, $u_t^{i,c}$ is the period utility received when staying in the contract, $u_t^{i,a}$ is the period utility received when exiting the contract[12] *and s^t is the history in period t.*

[11] In section 8 we show that these transfers can be reinterpreted as bonds traded in the international financial market, provided that these bonds meet certain restrictions.

[12] We are assuming that, once the country leaves the contract, it goes into autarky forever.

Next, we provide functional definitions of a small open emerging economy and a small open developed economy:

Definition 2. *A small open emerging economy is an economy that engages itself in transfer contracts with other countries and for which there exists some s^t such that restriction* (1) *binds.*

Definition 3. *A small open developed economy is an economy that engages itself in transfer contracts with other countries and for which restriction* (1) *never binds.*

The nature of the contract can be such that participation constraints such as (1) never bind, can occasionally be binding for $t \to \infty$, or occasionally bind for T periods until the stock of assets in each country is such that they never bind from T onwards; and one or other case arises in accordance to the parameters and functional forms specified for a given problem. Then, using Definitions 2 and 3, the first case corresponds to small open economies that are developed from the first period of the contract. Similarly, the second case corresponds to small open economies that remain emerging economies forever, and the third case to small open emerging economies at the beginning of the transfer contract, but that build up a stock of assets such that eventually they become developed.

4 The Model

In this section, we develop a model for a small open economy that decides to engage in an international risk-sharing contract. While the previous section described a general characterization of this type of contract, in the current section we make some simplifying assumptions to keep the analysis tractable and obtain an analytical characterization of the optimal contract and the fiscal plan associated to it.

We assume that the economy is constituted by two countries: the home country (HC) and the rest of the world (RW). The HC is populated by risk-averse agents, which enjoy consumption and leisure, and by a benevolent government that has to finance an exogenous and stochastic stream of public expenditure either by levying distortionary taxes, by issuing state-contingent domestic bonds, or by receiving transfers from the RW. The RW is populated by risk-neutral agents that receive a fixed endowment each period. These resources can be either consumed or lent to the HC. There is no uncertainty and no government in this country. We assume throughout the analysis that the discount factor of both countries is β.

4.1 The contract

The government of the HC can engage itself in a risk-sharing contract with the RW of the type described in Section 3. Denote T_t the transfers received by the HC at a given time t. There are three conditions that have to be met by $\{T_t\}_{t=0}^{\infty}$.

First, the expected present discounted value of transfers exchanged with the RW must equal zero:

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0. \tag{2}$$

If both the HC and the RW had an embedded commitment technology, in the sense that they could commit to honor the contract at any state of nature form period $t = 0$, this condition would rule out net redistribution of wealth between countries. We call this condition the *fairness condition*, since it implies that, given full commitment, *ex-ante* the contract is fair from an actuarial point of view[13].

If we assumed that the two parties in the contract have full commitment to pay back the debt contracted with each other, equation (2) would be the only condition regulating international flows. The allocations compatible with this situation will be our benchmark for comparison purposes. However, as explained in the previous section, we assume that upon entering the risk sharing contract, the countries have no intrinsic commitment technology. Then, we specify participation constraints for the HC and the RW. In the case of the HC, equation (1) becomes:

$$V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V_t^a \ \forall s^t. \tag{3}$$

We assume that if the government chooses to leave the contract at any given period, it is excluded from international markets from that moment on. Moreover, when the government defaults on its external obligations, it also defaults on its outstanding domestic debt. Consequently, the government is forced to run a balanced budget thereafter[14]. Alternative assumptions to identify the costs of default could be made, for example that, in case of default, the government cannot use external funds, but it still has access to the domestic bonds market to smooth the distortions caused by the expenditure shock. We have chosen the current specification for two reasons. First, this allows us to keep the problem tractable, both from an analytical and a numerical point of view. Second, this specification is consistent with the interpretation that the government is subject to debt limits, as shown in Section 8.

Notice that this participation constraint may bind only in "good times", i.e. for a low government expenditure shock. The reason for this is that, when a good shock hits the economy, the value of the outside option increases. In other words, in good times there is less need to resort to international risk sharing, so the continuation value of the contract decreases relative

[13]This condition implies that the contract is actuarially fair only if the RW has full commitment. This is due to the fact that, if the RW has limited commitment, the risk-free interest rate will not always be $1/\beta$ (see Section 8 for further details). This condition is useful because it allows us to pin down the allocations. However, one can impose other similar conditions that will yield different allocations.

[14]It follows that the only state variable influencing the outside option is the government expenditure shock. Therefore $V_t^a = V_t^a(g_t)$.

to the outside (autarky) option. This implies that the HC would want to default in good times. Although most theoretical models that incorporate default consider that this happens in bad times, Tomz and Wright (2007) provide evidence showing that a significant fraction of defaults have occurred in good times.

The participation constraint for the RW reads:

$$E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \;\; \forall s^t. \tag{4}$$

This condition is analogous to (3) and states that, at each point in time and for any contingency, the expected discounted value of future transfers the HC is going to receive cannot exceed an exogenous threshold value $\underline{B}$. Notice that this constraint may bind only in bad times, i.e., for high government expenditure shocks, when the HC should receive transfers from the RW to absorb the negative shock[15].

We use condition (4) because it is the natural specification of equation (1) for the risk-neutral RW, and the introduction of these two constraints links our work to the existing literature on limited commitment[16]. However, the qualitative results of the model are not altered by not imposing a participation constraint for the RW[17], or by considering alternative constraints that may fulfill a similar task as (4)[18].

As long as conditions (2), (3) and (4) are satisfied, the government of the HC can choose any given sequence $\{T_t\}_{t=0}^{\infty}$ to partially absorb its expenditure shocks. Given constraint (2) and the outside options V_t^a and $\underline{B}$, this contract is the one that maximizes risk-sharing among the HC and the RW.

4.2 Households in the HC

Households in the HC derive utility from consumption and leisure, and each period are endowed with one unit of time. The production function is linear in labor and one unit of labor produces one unit of the consumption good. Therefore, wages $w_t = 1 \; \forall t$. Households can save or borrow by trading one-period contingent liabilities with the government.

The representative agent in the HC maximizes her expected lifetime utility

[15] This restriction is meant to capture the fact that, for emerging economies, a sovereign debt contract can cease not only because the country defaults, but also because the international lender decides to stop lending money to the country. This may be due to contagion (in the case of international crises), uncertainty about the fundamentals of the emerging economy, fear of moral hazard issues, or simply because the lender cannot or does not want to transfer large sums of money to the HC.

[16] Abraham and Cárceles-Poveda (2006), Abraham and Cárceles-Poveda (2009), Alvarez and Jermann (2000), Marcet and Marimon (1992),Kehoe and Perri (2002) and Scholl (2009) are some of the papers that study different implications of introducing limited commitment in a similar fashion as we do.

[17] See Appendix A.6.

[18] One example is the constraint that we analyze in Appendix A.7 which imposes that the transfer T_t at a given time t cannot exceed a threshold $\underline{B'}$.

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

subject to the period-by-period budget constraint

$$b_{t-1}(g_t) + (1-\tau_t)(1-l_t) = c_t + \sum_{g^{t+1}|g^t} b_t(g_{t+1}) p_t^b(g_{t+1}), \tag{5}$$

where c_t is private consumption, l_t is leisure, $b_t(g_{t+1})$ denotes the amount of bonds issued at time t contingent on the government expenditure shock in period $t+1$, τ_t is the flat tax rate on labor earnings and $p_t^b(g_{t+1})$ is the price of a bond contingent on the expenditure shock realization in the next period.

The optimality condition with respect to the state-contingent bond is:

$$p_t^b(g_{t+1}) = \beta \frac{u_{c,t+1}(g^{t+1})}{u_{c,t}} \pi(g^{t+1}|g^t), \tag{6}$$

.

where $\pi(g^{t+1}|g^t)$ is the conditional probability of the government expenditure shock. Combining the optimality conditions with respect to consumption and leisure we obtain the intratemporal condition

$$1 - \tau_t = \frac{u_{l,t}}{u_{c,t}}. \tag{7}$$

4.3 Government of the HC

The government finances its exogenous stream of public expenditure $\{g_t\}_{t=0}^{\infty}$ by levying a distortionary tax on labor income, by trading one-period state-contingent bonds with domestic consumers and by contracting transfers with the RW. The government's budget constraint is

$$g_t = \tau_t(1-l_t) + \sum_{g^{t+1}|g^t} b_t(g_{t+1}) p_t^b(g_{t+1}) - b_{t-1}(g_t) + T_t. \tag{8}$$

4.4 Equilibrium

We proceed to define a *competitive equilibrium with transfers* in this economy.

Definition 4. *A competitive equilibrium with transfers is given by allocations $\{c, l\}$, a price system $\{p^b\}$, government policies $\{g, \tau^l, b\}$ and transfers T such that*[19]*:*

[19] We follow the notation of Ljungqvist and Sargent (2000) and use symbols without subscripts to denote the one-sided infinite sequence for the corresponding variable, e.g., $c \equiv \{c_t\}_{t=0}^{\infty}$.

1. *Given government policies, allocations and prices satisfy the household's optimality conditions* (5), (6) *and* (7).

2. *Given allocations and prices, government policies satisfy the sequence of government budget constraints* (8).

3. *Given allocations, prices and government policies, transfers satisfy conditions* (2), (3) *and* (4).

4. *Allocations and transfers satisfy the sequence of feasibility constraints:*

$$c_t + g_t = 1 - l_t + T_t. \tag{9}$$

4.5 Optimal policy

The government of the HC behaves as a benevolent Ramsey Planner and chooses tax rates, bonds and transfers $\{c_t, b_t, T_t\}_{t=0}^{\infty}$ in order to maximize the representative household's life-time expected utility, subject to the constraints imposed by the definition of competitive equilibrium.

Before studying the consequences of introducing limited commitment in terms of the optimal fiscal plan, it is instructive to analyze the benchmark scenario in which both the government in the HC and the RW have a full commitment technology from the beginning of the contract.

4.5.1 Full commitment

If both the HC and the RW can commit to honor their external obligations in all states of nature, conditions (3) and (4) need not be specified in the contract. Then, the problem of the Ramsey planner is

$$\max_{\{c_t, l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

s.t.

$$b_{-1} u_{c,0} = E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1 - l_t)), \tag{10}$$

$$c_t + g_t = 1 - l_t + T_t, \tag{11}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0. \tag{12}$$

Equation (10) is the intertemporal budget constraint of households, after imposing the transversality condition and plugging in the optimality conditions of the household's problem (6) and (7). This condition is known in the literature of optimal fiscal policy as the implementability condition.

The optimality conditions for $t \geq 1$[20] are:

$$u_{c,t} + \Delta(u_{cc,t}c_t + u_{c \cdot t} + u_{cl,t}(1 - l_t)) = \lambda, \tag{13}$$

$$u_{l,t} + \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda, \tag{14}$$

where λ is the multiplier associated with constraint (12), and Δ is the multiplier associated with the implementability condition (10). The next proposition characterizes the equilibrium.

Proposition 1. *Under full commitment, consumption, labor and taxes are constant $\forall t \geq 1$. Moreover, if $b_{-1} = 0$, $b_t(g_{t+1}) = 0$ $\forall t$, $\forall g_{t+1}$ and the government perfectly absorbs the public expenditure shocks through transfers T_t.*

Proof. Using optimality conditions (13) and (14), there are two equations to determine two unknowns, c_t and l_t, given the lagrange multipliers λ and Δ. Since these two equations are independent of the current shock g_t, the allocations are constant $\forall t \geq 1$. From the intratemporal optimality condition of households (7) it can be seen that the tax rate τ_t^l is also constant $\forall t \geq 1$. Finally, when $b_{-1} = 0$ the intertemporal budget constraint of households at time $t = 0$ (equation (10)) can be written as

$$\frac{1}{1-\beta}(u_c c - u_l(1 - l)) = 0.$$

Notice that, for any given time $t+1$, domestic bond holdings $b_t(g_{t+1})$ are obtained from the intertemporal budget constraint of households in that period, i.e.,

$$b_t(g_{t+1})u_{c,t+1} = E_{t+1} \sum_{j=0}^{\infty} \beta^j (u_{c,t+1+j}c_{t+1+j} - u_{l,t+1+j}(1 - l_{t+1+j})).$$

However, since the allocations are constant over time, it is the case that

$$b_t(g_{t+1})u_c = E_{t+1} \sum_{j=0}^{\infty} \beta^j (u_c c - u_l(1 - l)) = \frac{1}{1-\beta}(u_c c - u_l(1 - l)) = 0.$$

Therefore, $b_t(g_{t+1}) = 0$ $\forall g_{t+1}$ and, from the feasibility constraint (11), it follows that all fluctuations in g_t must be absorbed by T_t.

□

Proposition 1 illustrates the effect of full risk-sharing on the optimal fiscal policy plan: since

[20]Notice that if $b_{-1} \neq 0$ the Ramsey problem is not recursive for $t \geq 0$. This constitutes the standard source of time inconsistency in these type of optimal policy problem. However, the problem becomes recursive for $t \geq 1$. In the numerical exercises that follow we solve the problem taking into account that optimality conditions for $t = 0$ are different from the rest.

consumption and leisure are constant in time, the optimal tax rate is constant as well. The government in the HC uses transfers from the RW to completely absorb the shock. When g_t is higher than average, the government uses transfers to finance its expenditure; conversely, when g_t is below average, the government uses the proceeds from taxation to pay back transfers received in the past[21]. In this way, the RW provides full insurance to the domestic economy.

In Section 3 we have identified a small open developed economy to an economy with full commitment. From Proposition 1, we can conclude that, in the current setting, a small open developed economy is characterized by perfectly smooth tax rates.

4.5.2 Limited Commitment

We consider the case in which neither the government in the HC nor the RW can commit to stay in the contract in all states of the world. The problem of the Ramsey planner is identical to the one in the previous section, but now conditions (3) and (4) have to be explicitly taken into account:

$$\max_{\{c_t,l_t,T_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{15}$$

subject to

$$c_t + g_t = (1 - l_t) + T_t, \tag{16}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1 - l_t)) = u_{c^1,0}(b_{-1}), \tag{17}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0, \tag{18}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \ \forall t, \tag{19}$$

$$E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \ \forall t. \tag{20}$$

Since the participation constraint at time t (19) includes future endogenous variables that influence the current allocation, standard dynamic programming results do not apply directly. To overcome this problem we apply the approach described in Marcet and Marimon (2009) and write the Lagrangian as:

[21] In Appendix A.2 we study the case in which the utility function is logarithmic in its two arguments. In such a case, it is easy to see that transfers behave exactly as described here.

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t [(1+\gamma_t^1) u(c_t, l_t) - \psi_t (c_t + g_t - (1-l_t) - T_t)$$
$$- \mu_t^1 (V_t^a) + \mu_t^2 (\underline{B}) - \Delta(u_{c,t} c_t - u_{l,t}(1-l_t)) - T_t(\lambda + \gamma_t^2)] + \Delta(u_{c,0}(b_{-1})),$$

where

$$\gamma_t^1 = \gamma_{t-1}^1 + \mu_t^1; \quad \gamma_t^2 = \gamma_{t-1}^2 + \mu_t^2,$$

for $\gamma_{-1}^1 = 0$ and $\gamma_{-1}^2 = 0$. Δ is the Lagrange multiplier associated to equation (17), ψ_t is the Lagrange multiplier associated to equation (16), λ is the Lagrange multiplier associated to equation (18), μ_t^1 is the Lagrange multiplier associated to equation (19) and μ_t^2 is the Lagrange multiplier associated to equation (20). γ_t^1 and γ_t^2 are the sum of past Lagrange multipliers μ^1 and μ^2 respectively, and summarize all the past periods in which either constraint has been binding. Intuitively, γ^1 and γ^2 can be thought of as the collection of past compensations promised to each country so that it would not have incentives to leave the contract[22].

It can be shown that, for $t \geq 1$[23], the solution to the problem stated above is given by time-invariant policy functions that depend on the *augmented* state space $\mathcal{G} \times \Gamma^1 \times \Gamma^2$, where $G = \{g_1, g_2, \ldots, g_n\}$ is the set of all possible realizations of the public expenditure shock g_t and Γ^1 and Γ^2 are the sets of all possible realizations of the costate variables γ^1 and γ^2, respectively. Therefore,

$$\begin{bmatrix} c_t \\ l_t \\ T_t \\ \mu_t^1 \\ \mu_t^2 \end{bmatrix} = H(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) \qquad \forall t \geq 1.$$

More specifically, the government's optimality conditions for $t \geq 1$ are:

$$u_{c,t}(1+\gamma_t^1) - \psi_t - \Delta(u_{cc,t} c_t + u_{c,t} - u_{cl,t}(1-l_t)) = 0, \tag{21}$$

$$u_{l,t}(1+\gamma_t^1) - \psi_t - \Delta(u_{cl,t} c_t + u_{l,t} - u_{ll,t}(1-l_t)) = 0, \tag{22}$$

[22] Strictly speaking, it can be the case that a participation constraint is binding, and yet its associated Lagrange multiplier is zero. However, in this case, the fact that the participation constraint binds would not alter the allocations and, consequently, there would not be a change in γ.

[23] Once again, for $t = 0$ the optimality conditions of the problem are different. Applying Marcet and Marimon (2009), the problem only becomes recursive from $t \geq 1$ onwards.

$$\psi_t = \lambda + \gamma_t^2. \tag{23}$$

Other optimality conditions are equations (16) to (20) and:

$$\mu_t^1 (E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) - V^a(g_t)) = 0, \tag{24}$$

$$\mu_t^2 (E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} - \underline{B}) = 0, \tag{25}$$

$$\gamma_t^1 = \mu_t^1 + \gamma_{t-1}^1; \quad \mu_t^1 \geq 0, \tag{26}$$

$$\gamma_t^2 = \mu_t^2 + \gamma_{t-1}^2; \quad \mu_t^2 \geq 0. \tag{27}$$

From (21), (22) and (23) it is immediate to see that now the presence of γ_{t-1}^1 and γ_{t-1}^2 makes the allocations state-dependent. Moreover, since γ_{t-1}^1 and γ_{t-1}^2 are functions of all the past shocks hitting the economy, the allocations are actually history-dependent.

Notice also that the presence of these Lagrange multipliers makes the cost of distortionary taxation state-dependent. While in the full-commitment case this cost is constant over time and across states, in the limited commitment case it changes depending on the incentives to default that the HC and the RW have[24]. We will discuss this is further detail in Section 8.

The next proposition characterizes the equilibrium for a logarithmic utility function.

Proposition 2. *Consider a utility function logarithmic in consumption and leisure and separable in the two arguments:*

$$u(c_t, l_t) = \alpha log(c_t) + \delta log(l_t) \tag{28}$$

with $\alpha > 0$ *and* $\delta > 0$. *Define* $t < t'$*:*

1. *If the participation constraint* (19) *binds such that* $\gamma_t^1 < \gamma_{t'}^1$, *then* $c_t < c_{t'}$, $l_t < l_{t'}$ *and* $\tau_t > \tau_{t'}$.

2. *If the participation constraint* (20) *binds such that* $\gamma_t^2 < \gamma_{t'}^2$, *then* $c_t > c_{t'}$, $l_t > l_{t'}$ *and* $\tau_t < \tau_{t'}$.

Proof. See Appendix A.3. □

Proposition 2 states the way the allocations and tax rates adjust in order to make the contract incentive-compatible for the HC and the RW. As long as neither participation constraint binds, consumption and leisure remain constant.

[24] It can be shown that, in the full commitment case, this cost is given by Δ, while in the limited commitment one is determined by $\frac{\Delta}{1+\gamma_t^1}$.

In order to gain intuition, consider the case in which, at period t', the participation constraint of the HC (equation (19)) is not satisfied when $\mu^1_{t'} = 0$. Then, as the HC has incentives to go into autarky, the contract has to be such that the expected lifetime utility of households of the HC increases so as to make (19) hold with equality. For this to be the case, $\mu^1_{t'} > 0$, and consequently $c_{t'}$ and $l_{t'}$ jump upwards. Moreover, because the utility function is strictly concave, it is efficient to increase consumption and leisure permanently through a higher $\gamma^1_{t'}$, rather than to increase them substantially for only one period. The increase in utility also dictates a decrease in tax rates so as to increase the net income of households. The intuition for the case in which the participation constraint (20) of the RW binds is exactly analogous to the previous one.

The fact that tax rates now depend on γ^1_t and γ^2_t implies that, if equations (19) and (20) are effectively binding at certain periods, tax rates are more volatile than under the full commitment scenario. Then, a small open emerging economy is characterized by more volatile tax rates than a small open developed one. Moreover, the deeper the commitment problem is, the more often the participation constraints bind, and the higher the volatility of tax rates is. This feature of the model allows to rationalize the positive correlation studied in Section 2 between a measure of lack of commitment, such as the EMBIG spread, and the volatility of tax rates.

Finally, notice that if a small open economy that enters the international financial market starts the transfer contract as an emerging economy but eventually becomes developed, it will have a very volatile tax schedule in the initial periods. As time passes by and the economy accumulates assets, tax rates will become less volatile, until the point in which the country reaches full commitment and can be regarded as developed. From that point onwards, tax rates will remain perfectly flat.

5 An example of labor tax-smoothing

To better understand the impact of limited commitment on the ability of the government to smooth taxes, in this section we analyze the case study of a perfectly anticipated government expenditure shock. The example follows closely one of the examples of tax smoothing in Lucas and Stokey (1983).

Suppose that government expenditure is known to be constant and equal to 0 in all periods except in T, when $g_T > 0$. In order to simplify the analysis, throughout this section we assume that $\underline{B}$ is large so that the RW never has incentives to leave the contract. Moreover, we assume that $b_{-1} = 0$ and that households have a logarithmic utility function as (28).

Since equilibrium allocations depend on γ^1_t, understanding the dynamics of the incentives to default is crucial. The next proposition states that, given the assumptions previously made, the participation constraint (3) binds only at $t = T + 1$[25].:

[25] The reader may wonder why the participation constraint binds just after the shock. The reason is that agents

Table 3: Parameter values

Preferences	$\alpha = \delta = 1$	
Intertemporal discount factor	$\beta = 0.98$	
Government expenditure	T = 10	$g_T = 0.2$

Proposition 3. *Suppose that the government expenditure is known to be constant and equal to* 0 *in all periods except in* T*, when* $g_T > 0$*. Assume further that* $b_{-1} = 0$*. Then, the participation constraint* (19) *binds exactly in period* $T + 1$*.*

Proof. See Appendix A.4. □

In terms of Definitions 2 and 3, this country can be classified as an emerging economy until period $T + 1$, and as a developed economy from period $T + 2$.

From the results of Proposition 2 we can characterize the allocations for $t < T + 1 \leq t'$. Given that $\gamma_t^1 < \gamma_{t'}^1$, it follows that $c_t < c_{t'}$, $l_t < l_{t'}$ and $\tau_t > \tau_{t'}$[26]. The limited commitment by the government exerts a permanent effect on the tax rate and alters its entire dynamics, since the tax rate level after the shock is permanently lower than before the shock.

The intuition for this result is as follows. Since at $T + 1$ the continuation value of staying in the contract has to increase in order to prevent default, utility of households in the HC has to increase. By the intratemporal optimality condition, a positive tax rate implies that the marginal utility of consumption is higher than the marginal utility of leisure. Therefore, increasing consumption is relatively more efficient than increasing labor and, as a consequence, the tax rate decreases.

5.1 The example in numbers

In this section we solve numerically the example depicted above. Table 3 contains the parameter values used in the simulation.

Figures 3 and 4 show the evolution of the allocations c_t, l_t, the tax rate τ_t, international capital flows T_t, domestic bonds b_t and the costate variable γ_t^1. We compare the allocations with limited commitment to the ones under full commitment . There are two forces determining the dynamics of the economy. On one side, the government has to finance the higher and expected expenditure outflow at T in the most efficient way; on the other, the participation constraint has to be satisfied. For $t \leq T$ the higher and expected shock at T keeps the continuation value of autarky low, and for this reason leaving the contract is not optimal. Therefore, before T

know the bad shock will happen in T, so this decreases the outside option value in every period before the shock effectively takes place. Once the shock is over, the autarky value goes up.

[26] In Appendix A.5 we show that $\Delta < 0$ in this case.

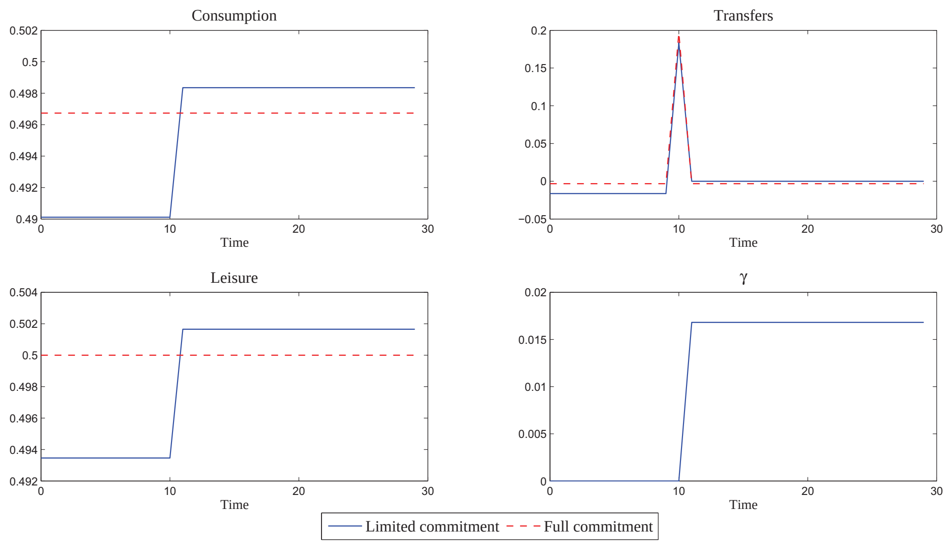

Figure 3: Example: $g_t = 0$ for $t \neq T$, and $g_T > 0$

the government accumulates assets towards the RW, and uses them to finance part of the high expenditure outflow in T. The remaining part is covered both through tax revenues and through transfers from the RW. After the high shock has taken place, the outside option value increases. In order to prevent default, the government lowers the tax rate to allow domestic households to enjoy a higher level of consumption and leisure. Moreover, from $T+1$ onwards, the transfers with the RW are zero, and the utility of households is equal to the utility level in autarky.

Notice the difference with the full commitment scenario, where the allocations are constant and transfers absorb completely the shock. The high inflow in period T is repaid forever by the government through small outflows after the shock. Taxes remain constant *even in period T* and do not react to the shock at all. The limited commitment feature constraints the amount of insurance offered by international capital markets, and perfect risk-sharing among countries is no longer possible. In this way, since risk-sharing is limited, markets become *endogenously incomplete*. Consequently, the negative expenditure shock has to be absorbed through external debt and higher tax revenues in the initial periods.

6 Numerical results

In this section we proceed to solve the model numerically assuming that the government spending follows an AR(1) process. We calibrate the parameters of this process to the argentinean economy. The purpose of the exercise is threefold. First, the issue of whether the participation constraints actually bind in equilibrium depends on the parameters of the model and, in particular, on the stochastic process for g_t. Therefore, it is important to check that, for

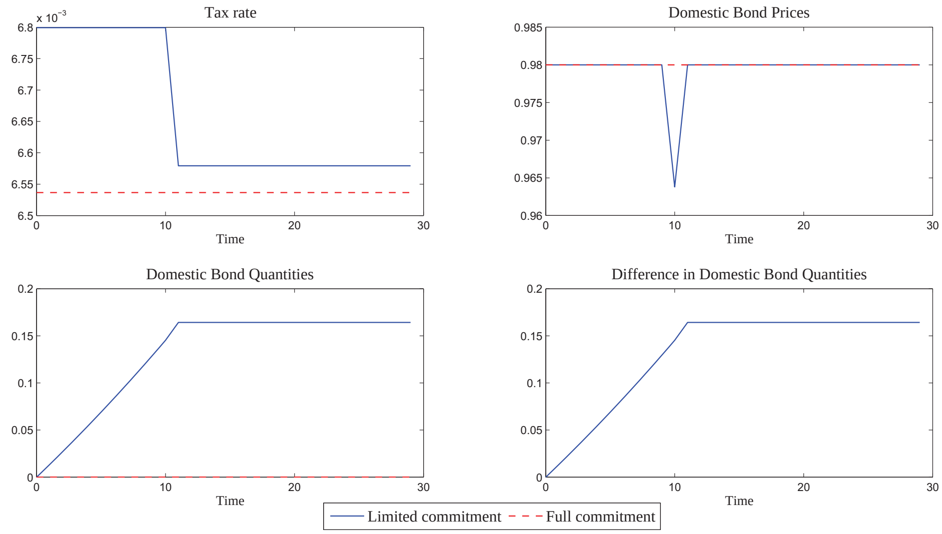

Figure 4: Example: $g_t = 0$ for $t \neq T$, and $g_T > 0$

a reasonable parameterization, the mechanisms of the model depicted in the previous section are at work. Second, we quantitatively assess the implications of the model for the case study of Argentina and provide a characterization of the long-run allocations by studying the behavior of the costate variables γ^1_{t-1} and γ^2_{t-1}. Finally, we measure the welfare losses associated to the presence of endogenously incomplete markets due to the lack of full commitment.

Throughout this section, we assume that the transfer contract is bounded by participation constraints (3) and (4). In Appendix A.6 we show that, for the current parameterization, the qualitative implications of the model are not altered when we do not impose the participation constraint (4) as a restriction to the contract. This scenario corresponds to the case in which the RW never leaves the contract or limits its transfers to the HC.

6.1 Parameterization

We use quarterly series of current government expenditure net of interest payments plus gross government investment as our measure of government expenditure for the period 1993-I to 2005-IV[27]. The data is available from Argentina's Ministry of Finance. We estimate an AR(1) process in levels and find that $\hat{\rho} = 0.9107$ for the following specification

$$g_t = \alpha + \rho g_{t-1} + \epsilon_t$$

In the data for Argentina, the coefficient of variation is 0.1320. We estimate the mean of g_t as the value of g_t in steady state, given the mean of $\frac{g_t}{GDP_t}$ in the data. This value is

[27] All series have been deflated using the GDP deflator.

Table 4: Parameter values

Preferences	$\alpha = \delta = 1$
Intertemporal discount factor	$\beta = 0.98$
Government expenditure process	$g_t = \alpha + \rho g_{t-1} + \epsilon_t$
$g*$	$0.1820 * 0.5$
ρ^g	0.9107
σ_g^2	$0.1320 * 0.091$
$\underline{B}$	0.0402
b_{-1}	0.034

$\overline{\frac{g}{GDP}} = 0.182$. We consider that $1 - l_t$ is roughly $\frac{1}{2}$, which is the value of output in autarky. Then $\overline{g} = 0.5 * 0.182 = 0.0901$ and $\sigma_g^2 = (0.1320 * 0.0901)^2$.

Finally, we need to calibrate the initial level of domestic public debt b_{-1} and the limit to transfers from the rest of the world, $\underline{B}$. To compute a value for the first concept, we need to consider public debt held by nationals. Since we do not have data on debt ownership, we consider debt issued in national currency as a proxy for debt held by nationals. We do not have data on debt issued in national currency before the fourth quarter of 1993, so we compute the mean percentage of public debt issued in national currency for the period IV-1993 to IV-2004, which is about 10%[28]. Then, we multiply the total public debt of the first quarter of 1993 divided by annual GDP by this percentage and multiply it by approximate annual GDP of the model: $0.17 * 0.1 * 2 = 0.034$.

The calibration of $\underline{B}$ is more cumbersome. First, notice that T_t can be interpreted as the change in public debt contracted with foreign creditors in a given period. We will use debt contracted with the IMF, so that $T_t = b_t^{IMF} - b_{t-1}^{IMF}$, where b_t^{IMF} is debt over annualized GDP from the data, multiplied by the approximate annual GDP of the model. Then $\underline{B}$ is computed from the data in the following manner:

$$\underline{B} = E_{I-93} \sum_{j=0}^{\infty} \beta^j \tilde{T}_{t+j} \tag{29}$$

where $\tilde{T}_t$ is the simulated change in debt with the IMF in a given period. To simulate the series $\{\tilde{T}_{t+j}\}_{j=0}^{\infty}$ we estimate an AR(1) process for b_{t+j}^{IMF} from the data and construct $\tilde{T}_t$, according to the definition of transfers previously specified. Finally, we compute the expectation in (29) as the mean of 10000 replications of discounted sums of transfers, of 1000 periods each, where the initial level of debt $b_t^{IMF} = b_{I-93}^{IMF}$ is taken from the data[29].

[28] We do not consider data for the year 2005 because during that period, as part of the debt renegotiation after the sovereign default of 2001, a large fraction of public debt originally issued in US dollars was re-denominated in argentinean pesos.

[29] We only consider data until the year 2000 because, during 2001 and specially in the months prior to the

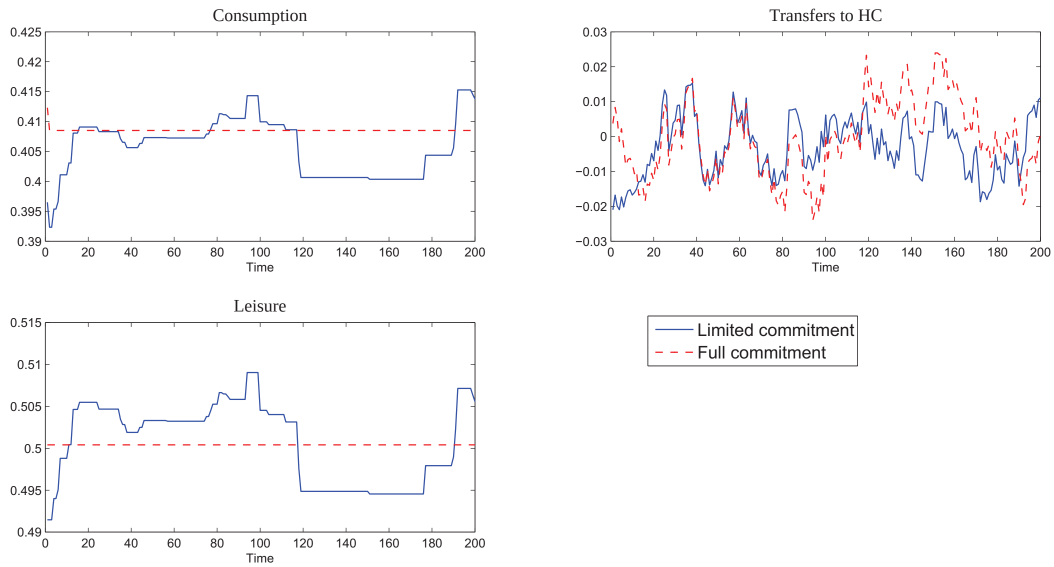

Figure 5: Allocations

6.2 Results

Figures 5, 6 and 7 show the allocations, co-state variables and fiscal variables, respectively, for a particular realization of the government expenditure shock, for the case in which the government of the HC and the RW have limited commitment (solid line). For comparison purposes, we show the same variables under full commitment (dashed line). Appendix A.10 explains the computational algorithm used to solve the model[30].

We can observe from Figure 6 that both participation constraints bind often for this particular realization of the shock. This causes the allocations to jump upwards and downwards, and consequently to be more volatile than in the benchmark case of full commitment. It is clear from Figure 7 that the tax rate is also more volatile than in the full commitment case. Moreover, in the limited commitment case the government of the HC makes active use of the domestic bond market to improve its ability to smooth taxes.

Tables 5 and 6 show some statistics obtained by simulating the model for 1000 realizations of

sovereign default of 2001, the debt contracted with the IMF grew from 3278 million US dollars (III-2000) to 14592 million US dollars (III-2001). This large increase in the debt contracted with the IMF is due to the profound economic and political crisis that the country went through during that period, and does not reflect the normal evolution of debt in the previous more stable years.

[30]We only show the first 200 periods of the simulation because, for this simulation and this time span, the costate variables remain within the grid for which the model has been solved. Notice that, as explained in Section 3, depending on the parameterization, a country can be an emerging economy at the beginning of the contract and become developed in time. Consequently, in this section, the first periods are the ones of interest. In Section 6.3 we study the long-run properties of the allocations.

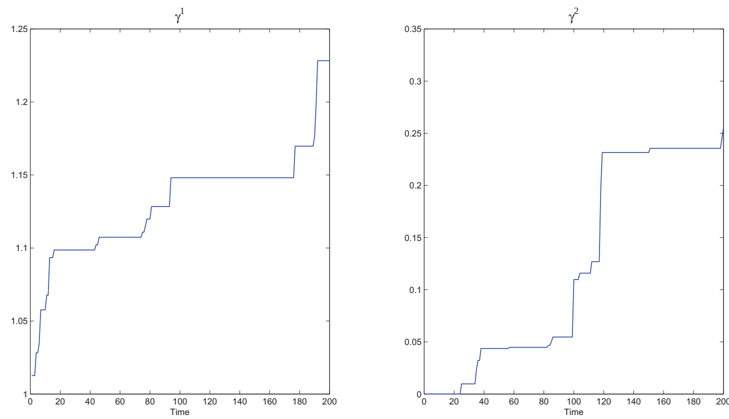

Figure 6: Costate variables

the shock of 100 periods each[31]. Notice first that, while the average values of the allocations are roughly the same with limited commitment as with full commitment, their coefficient of variation is much higher in the first case: the coefficient of variation of tax rates is more than three times higher under limited commitment. In the full commitment case, if $b_{-1} = 0$, consumption, leisure and the tax rate would be constant from period 0. The fact that the initial stock of domestic government debt is not zero makes consumption and the tax rate different in $t = 0$.

Table 6 shows the correlation between the allocations and tax rate with the government expenditure shock. The table shows that consumption and leisure are negatively correlated with the shock, whereas the tax rate is positively correlated. The reason for these results hinges on the relation between the shock and the participation constraints. In our setup, a low realization of g_t usually implies that the HC has to transfer resources to the RW. Therefore, for low values of g_t, the value of the outside option of the HC $V^a(g_t)$ is increased relative to the continuation value of staying in the contract. This translates into the participation constraint of the HC being binding for low realizations of the public expenditure shock, for some configurations of γ^1_{t-1} and γ^2_{t-1}. From Proposition 2 we can conclude that consumption and leisure are likely to jump up and taxes are likely to go down when the public expenditure shock is relatively low.

Conversely, the participation constraint of the RW binds when high values of g_t are realized, for some configurations of γ^1_{t-1} and γ^2_{t-1}, since it is in this case that transfers to the HC are positive. Again, using Proposition 2 we know that consumption and leisure are likely to go down and taxes are likely to go up when the public expenditure shock is relatively high.

The results illustrated in Table 6 point to the fact that, in this framework, optimal fiscal policy is procyclical, in the sense that tax rates are increased in bad times (high g_t) and are

[31] Once more, we consider only the first 100 periods in order to make sure that the costate variables do not exit the grid used to solve the model.

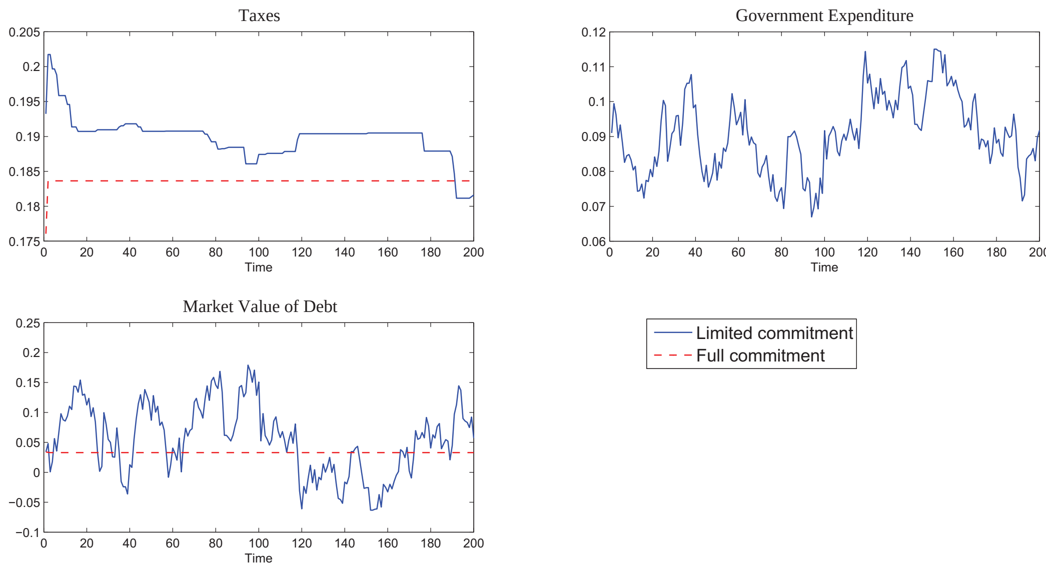

Figure 7: Fiscal variables

Table 5: Statistics of allocations for the first 100 periods - 1000 simulations

	Limited Comm.		Full Comm.	
	Mean	Coef. Var.	Mean	Coef. Var.
consumption	0.4050	1.05%	0.4085	0.09%
leisure	0.5014	0.74%	0.5004	0%
labor tax rate	0.1924	1.47%	0.1836	0.41%

decreased in good times (low g_t). However, given that periods with high g_t in which the participation constraint of the RW is binding are also periods in which domestic output increases, this result is at least debatable. In Section 7 we explore the implications of the model when we introduce a productivity shock and keep the government expenditure constant. We confirm that fiscal policy is procyclical, in the sense that the correlation between output and tax rates is negative.

6.3 Allocations in the long run

A question that arises naturally from the analysis of the previous section is whether, in the long run, the economy remains as an emerging economy or becomes developed. The latter possibility corresponds to the case that, in a given time period $t = T$, the costate variables γ^1_{t-1} and γ^2_{t-1} reach certain values such that neither participation constraint is ever binding for $t > T$. On the other hand, the former possibility requires the costate variables to keep increasing over

Table 6: Correlation with shock

	Limited Comm.	Full Comm.
	$Corr(x,g)$	$Corr(x,g)$
consumption	-0.4391	0
leisure	-0.4656	0
labor tax rate	0.3182	0

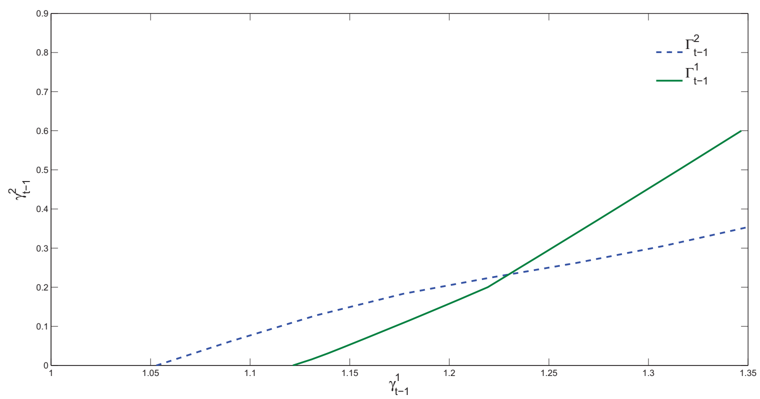

Figure 8: Long-run analysis

time. In this case, the equilibrium would not be stationary because the costate variables would remain unbounded; however, consumption, leisure and the tax rate would fluctuate around their means. Which of the two scenarios prevails is a quantitative issue.

To answer this question, call Γ^1_{t-1} the minimum value of the costate γ^1_{t-1} for a given γ^2_{t-1} such that, for any possible realization of the shock g_t, the participation constraint of the HC is satisfied. Define Γ^2_{t-1} in a similar fashion. Then,

$$\text{argmin}_{\Gamma^1_{t-1}} \; W(g_t, \Gamma^1_{t-1}, \gamma^2_{t-1}) = E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a_t(g_t) \quad \forall g_t \in \{g_{min}, g^{max}\}$$

$$\text{argmin}_{\Gamma^2_{t-1}} \; W(g_t, \gamma^1_{t-1}, \Gamma^2_{t-1}) = E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \quad \forall g_t \in \{g_{min}, g^{max}\}$$

Figure 8 shows the computed Γ^1_{t-1} and Γ^2_{t-1} for our previous parameterization[32]. Notice that, for any configuration of γ^1_{t-1} and γ^2_{t-1} that lies in the region to the right of the solid line, the participation constraint of the HC is never binding. Similarly, for any configuration of γ^1_{t-1}

[32] For the computation, we extend the original grid for γ^1_{t-1} and γ^2_{t-1} to characterize the behavior of the costate variables on a larger set. Although this implies a reduction in precision, in this subsection we are only interested in the characterization of the long run, and not in the precision of the simulations.

and γ^2_{t-1} that lies in the region above the dashed line, the participation constraint of the RW is never binding.

The fact that both lines cross implies that the multipliers will increase until a given time period $t = T$, in which the Γ^1_{t-1} and the Γ^2_{t-1} lines meet. At that point, the costate variables have reached the area in which the participation constraints are never binding again, the equilibrium is stationary and the economy becomes developed. In particular, for $t \geq T$ consumption, leisure and the tax rate are constant, and lifetime expected discounted utility of households when the HC stays in the contract is equal to the highest possible value of the outside option, which will correspond to the value of autarky when the government expenditure shock is the lowest possible.

6.4 Welfare costs

In this subsection we quantify the welfare cost associated to the presence of limited commitment. We compute this cost as the permanent percentage increase in consumption that we would have to give to a household in order for it to be indifferent between living in a world in which the HC and the RW have full commitment, and one with limited commitment. That is, the welfare cost is given by a fraction χ of consumption such that:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{FC}, l_t^{FC}) = E_0 \sum_{t=0}^{\infty} \beta^t u((1+\chi)c_t^{LC}, l_t^{LC}),$$

where c_t^{FC} and l_t^{FC} correspond to allocations under full commitment, and c_t^{LC} and l_t^{LC} to allocations under limited commitment. Given the logarithmic utility function (28), χ can be easily computed from:

$$log(1+\chi) = \frac{1-\beta}{\alpha} \left(E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{FC}, l_t^{FC}) - E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{LC}, l_t^{LC}) \right).$$

In order to compute the expectations that appear on the right hand side of the expression above, we use the simulations previously performed to produce Tables 5 and 6. We only use the first 100 periods so as to make sure that the costate variables do not exit the grid used to solve the model. Following the discussion in Section 6.3, we assume that from $t = 100$ the participation constraints of the HC and the RW are no longer binding and, consequently, the expected discounted lifetime utility of households for $t \geq 100$ is equal to $V^a(g_{min})$.

We obtain a welfare cost $\chi = 0.0093$, which means that a household would need to receive a permanent increase in consumption of 0.93% in order for it to be indifferent between living in a small open developed economy and living in a small open emerging one. This measure corresponds to the welfare loss associated with the endogenously incomplete markets that arise due to the lack of full commitment, and quantifies the cost in terms of consumption of imperfect risk sharing.

This measure is higher than the classical computation of Lucas (1987), who calculates the welfare costs of business cycles as the gain from eliminating all consumption fluctuations using a logarithmic utility function, and estimates this gain to be about 0.0088% of consumption[33]. However, our measure is in line with Aiyagari et al. (2002), who compute the welfare costs of incomplete markets in a model of optimal taxation without state-contingent debt and find this cost to be as high as 0.96% in one of their examples.

7 Procyclicality of fiscal policy

In the recent past there has been a significant interest in studying the cyclical properties of fiscal policy in emerging countries, both from an empirical as well as a theoretical perspective. For this reason, in this section we assess whether our model generates procyclical or countercyclical fiscal policy. By procyclical fiscal policy, we understand higher public expenditure and lower tax rates in good times, when GDP is relatively high (countercyclical tax rates), and lower public expenditure and higher tax rates in bad times, when GDP is relatively low (procyclical tax rates).

7.1 Evidence and some theory

A number of authors have documented the fact that the government expenditure appears to be procyclical in emerging economies, whereas it is countercyclical or acyclical in developed economies[34]. Despite this broad literature, there is little evidence on the cyclical properties of tax rates. The main reason for this is data availability: while data on government consumption is fairly easy to obtain, data on tax rates is very scarce. Mendoza et al. (1994) compute time series of effective tax rates on consumption, capital income and labor income for G-7 countries using data on tax revenues and national accounts. Unfortunately, these series are hard to construct in the case of emerging economies, as usually the information on revenues is not disaggregated enough. However, as Cuadra et al. (2010) point out, several episodes suggest that in emerging countries tax rates behave according to a procyclical fiscal policy plan[35].

As Ilzetzki and Vegh (2008) point out, the literature has followed two main paths to explain the procyclicality of fiscal policy in emerging economies. One set of papers exploits political economy arguments based on the idea that good times encourage fiscal leniency and rent-seeking

[33] Other studies such as Otrok (2001) have estimated similar figures (around 0.0044% of consumption) by allowing for potential time-non-separabilities in preferences and by requiring that preferences be consistent with observed fluctuations in a general equilibrium model of business cycles.

[34] Examples of papers that deal with this issue are Gavin and Perotti (1997), Talvi and Vegh (2005), Kaminsky et al. (2004) and Ilzetzki and Vegh (2008). For a careful review of this literature, see Cuadra et al. (2010).

[35] See Cuadra et al. (2010) for a description of such episodes.

Table 7: Statistics of allocations - Productivity shock

	Limited Commitment	Full Commitment
corr(y_t,τ_t)	-0.20	0.981
Coef. Var.(τ_t)	8.20%	1.53%

activities. Examples of such papers are Lane and Tornell (1999), Talvi and Vegh (2005) and Alesina et al. (2008). Alternatively, Riascos and Vegh (2003) and Cuadra et al. (2010) develop models that rely on the notion that emerging economies have imperfect access to financial markets that prevents them from borrowing in bad times. The intuition behind the mechanism that links this imperfect access to credit markets to procyclical fiscal policy is simple: when the government has limited ability to issue debt during crises, it will have to reduce public spending, increase tax rates, or a combination of both. Our paper contributes to this latter strand of literature.

7.2 Cyclical properties of fiscal policy in the presence of limited commitment

In order to be able to clearly characterize the cyclical properties of tax rates in the presence of limited commitment, in this section we keep government spending fixed and instead consider that the source of fluctuations in the economy is a productivity shock. The production function now is described by

$$y_t = exp(z_t)(1 - l_t)$$

where $z_t = \rho^z z_{t-1} + \epsilon_t$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. The rest of the model remains as in Section 4.

We solve the model numerically and perform 1000 simulations of 200 periods each to obtain the statistics reported in Table 7. In this case we take ad hoc parameter values to characterize the productivity process and government spending, as we are only interested in analyzing the qualitative behavior of the main variables. To this end, we specify $\bar{g} = 0.06$, $\rho^z = 0.9$ and $\sigma_z = 0.025$.

Table 7 shows some statistics for the relevant variables, for both the cases of limited as well as full commitment. With limited commitment, the correlation of tax rates with output is negative, which implies that tax rates decrease when output is high and increase when output is low. This corresponds to the notion that tax rates are countercyclical (so fiscal policy is said to be procyclical) for emerging economies. For small open developed economies, we find that tax rates should be procyclical (and fiscal policy countercyclical).

In the current setup, bad times are times in which the RW may have incentives to default, as these are times in which transfers to the HC are positive[36]. In order for the RW to have incentives to stay in the contract, transfers to the HC have to be decreased (with respect to the full commitment case), which implies that part of the negative shock has to be absorbed by increasing tax rates[37]. Finally, by observing the coefficient of variation of the tax rate, we confirm the result of previous sections that the volatility of tax rates is significantly increased in the presence of limited commitment.

8 Borrowing constraints

In this section we show that it is possible to reinterpret the problem depicted in previous sections as one in which the the HC and the RW trade one-period state-contingent bonds in the international financial market, but their trading is limited by borrowing constraints. To do so, we follow the strategy proposed by Alvarez and Jermann (2000) and Abraham and Cárceles-Poveda (2009)[38]. We show that, if we impose limits on international borrowing only, the allocations obtained in Section 4 and the ones obtained in the setup of this section do not coincide. An additional constraint on the value of domestic debt that the government of the HC can issue is required.

In what follows, we present the problem of the HC and the RW when, instead of transferring resources among them, they trade state-contingent bonds in the international financial market. We will denote with a superscript 1 variables corresponding to the HC, and with superscript 2 variables corresponding to the RW. Let $Z_t^1(g_{t+1})$ be an international one-period bond bought at t by the government of the HC contingent on next period's realization of the government expenditure shock. Symmetrically, call $Z_t^2(g_{t+1})$ an international one-period bond bought at t by households of the RW contingent on next period's realization of the government expenditure shock. Denote the price of these bonds by $q_t(g_{t+1})$, and assume that there are lower bounds, denoted by $A_t^1(g_{t+1})$ and $A_t^2(g_{t+1})$, on the amount of bonds that the government of the HC and the households in the RW can hold, respectively.

The problem of the households in the HC is exactly identical to the one described in Section 4.2, so we do not reproduce it here. The problem of the government in the HC is slightly different from the one in previous sections. In order to finance its public expenditure, in addition

[36] In order to understand the mechanisms at work in our model, it is useful to think about the contract between the HC and the RW as an insurance contract. During bad times, the RW has to provide insurance to the HC through positive transfers. Conversely, in good times the HC has to pay a fee to entitle it to the insurance agreement.

[37] Notice that this intuition holds no matter what the nature of the shock we wish to consider is, since, by interpreting the contract as an insurance contract, a "bad time" will be associated with $T_t > 0$.

[38] In Appendix A.8 we show that the government's problem coincides with the one of an international institution in charge of distributing resources among the HC and the RW, taking into account the aggregate resource constraint, the implementability condition of the HC, and the fact that countries have limited commitment. Therefore, the problem laid out in section 4 can be thought of as one in which a central planner determines the constrained efficient allocations.

to distortionary taxes on labor income and domestic bonds, now the government has available one-period state-contingent bonds traded with the RW. Therefore, the budget constraint of the government is:

$$g_t + \sum_{g^{t+1}|g^t} Z_t^1(g_{t+1})q_t(g_{t+1}) - Z_{t-1}^1(g_t) = \tau_t(1-l_t) + \sum_{g^{t+1}|g^t} b_t(g_{t+1})p_t^b(g_{t+1}) - b_{t-1}(g_t) \quad (30)$$

The government faces a constraint on the amount of debt that can issue in the international financial market:

$$Z_t^1(g_{t+1}) \geq A_t^1(g_{t+1}) \quad (31)$$

Assume households in the RW receive a fixed endowment y every period. The problem of households in the RW that trade bonds with the government in the HC now is

$$\max_{\{c_t^2, Z_t^2\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t c_t^2 \quad (32)$$

s.t.

$$y + Z_{t-1}^2(g_t) = c_t^2 + \sum_{g_{t+1}} q_t(g_{t+1})Z_t^2(g_{t+1}) \quad (33)$$

$$Z_t^2(g_{t+1}) \geq A_t^2(g_{t+1}) \quad (34)$$

Notice that the RW is also constrained in the amount of debt it can trade with the HC. The optimality conditions of this problem are equation (33) and

$$q_t(g_{t+1}) = \beta\pi(g^{t+1}|g^t) + \omega_t^2 \quad (35)$$

$$\omega_t^2(Z_t^2(g_{t+1}) - A_t^2(g_{t+1})) = 0 \quad (36)$$

$$\omega_t^2 \geq 0 \quad (37)$$

where ω_t^2 is the Lagrange multiplier associated to the borrowing constraint (34).

Definition 5. *A competitive equilibrium with borrowing constraints is given by allocations* $\{c^1, c^2, l\}$*, a price system* $\{p^b, q\}$*, government policies* $\{g, \tau, b\}$ *and international bonds* $\{Z^1, Z^2\}$ *such that:*

1. *Given prices and government policies, allocations c and l satisfy the HC household's optimality condition* (5), (6) *and* (7).

2. *Given allocations and prices, government policies and bonds Z^1 satisfy the sequence of government budget constraints* (30) *and borrowing constraints* (31)*.*

3. *Prices q and bonds Z^2 satisfy the RW optimality conditions* (34) *and* (35)*.*

4. *Allocations satisfy the sequence of feasibility constraints:*

$$c_t^1 + g_t + \sum_{g^{t+1}|g^t} Z_t^1(g_{t+1})q_t(g_{t+1}) = 1 - l_t + Z_{t-1}^1(g_t) \tag{38}$$

$$c_t^2 + \sum_{g^{t+1}|g^t} Z_t^2(g_{t+1})q_t(g_{t+1}) = y + Z_{t-1}^2(g_t) \tag{39}$$

5. *International financial markets clear:*

$$Z_t^1(g_{t+1}) + Z_t^2(g_{t+1}) = 0$$

We need to specify borrowing constraints that prevent default by prohibiting agents from accumulating more contingent debt than they are willing to pay back, but at the same time allow as much risk-sharing as possible. Define first

$$V_t^1(Z_{t-1}^1(g_t), g_t) = u(c_t^1, l_t) + \beta E_t V_{t+1}^1(Z_t^1(g_{t+1}), g_{t+1})$$

$$V_t^2(Z_{t-1}^2(g_t), g_t) = c_t^2 + \beta E_t V_{t+1}^2(Z_t^2(g_{t+1}), g_{t+1})$$

We define the notion of borrowing constraints that are not too tight:

Definition 6. *An equilibrium has borrowing constraints that are not too tight if*

$$V_{t+1}^1(A_t^1(g_{t+1}), g_{t+1}) = V_{t+1}^a \quad \forall t \geq 0,\ \forall g_{t+1} \in \{g_{min}, g^{max}\}$$

and

$$V_{t+1}^2(A_t^2(g_{t+1}), g_{t+1}) = \underline{B} \quad \forall t \geq 0,\ \forall g_{t+1} \in \{g_{min}, g^{max}\}$$

where V_{t+1}^a and $\underline{B}$ are defined as in Section 4.1.

As Alvarez and Jermann (2000) explain, borrowing constraints that satisfy these conditions prevent both parties in the contract to accumulate more debt than they are willing to repay. At the same time, they are the loosest possible constraints that can be imposed such that default does not occur in equilibrium. In other words, imposing borrowing constraints $A_t^1(g_{t+1})$ and $A_t^2(g_{t+1})$ allows for as much risk sharing as possible, given the option to default that the HC and the RW have. Then,

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V_t^a(g_t) \quad \text{and} \quad E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) = V_t^a(g_t) \iff Z_{t-1}^1(g_t) = A_{t-1}^1(g_t)$$

$$E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \quad \text{and} \quad E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} = \underline{B} \iff Z_{t-1}^2(g_t) = A_{t-1}^2(g_t)$$

We introduce the concept of high implied interest rates, which guarantees finiteness of the value of the endowment implied by a given allocation.

Definition 7. *An allocation $\{c_t^i, l_t, T_t\}$ for $i = 1, 2$ has high implied interest rates if:*

$$\sum_{t=0}^{\infty} \sum_{g^t} Q_0(g^t|g_0)(c_t^1 + c_t^2) < \infty$$

where

$$Q_0(g^t|g_0) = q_0(g_0, g_1) \cdot q_1(g^1, g_2) \dots q_t(g^{t-1}, g_t)$$

$$q_{t+1}(g^t, g_{t+1}) = \beta\pi(g_{t+1}|g^t) max \left\{ \frac{u_{c^1,t+1} - \frac{\Delta}{1+\gamma_{t+1}^1}(u_{cc^1,t+1}c_{t+1}^1 + u_{c^1,t+1} - u_{c^1 l,t+1}(1 - l_{t+1}))}{u_{c^1,t} - \frac{\Delta}{1+\gamma_t^1}(u_{cc^1,t}c_t^1 + u_{c^1,t} - u_{c^1 l,t}(1 - l_t))}, \beta \right\} \tag{40}$$

Proposition 4. *Let $\{c_t^1, c_t^2, l^t\}_{t=0}^{\infty}$ be a constrained efficient allocation with high implied interest rates. Then the constrained efficient allocations cannot be decentralized as a competitive equilibrium with borrowing constraints that are not too tight.*

Proof. The proof is immediate. Write the problem of the government in the HC as

$$\max_{\{c_t^1, l_t, Z_t^1\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^1, l_t) \tag{41}$$

s.t.

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c^1,t}c_t^1 - u_{l,t}(1 - l_t)) = b_{-1}u_{c^1,0} \tag{42}$$

$$c_t^1 + g_t + \sum_{g_{t+1}} q_t(g_{t+1})Z_t^1(g_{t+1}) = 1 - l_t + Z_{t-1}^1(g_t) \tag{43}$$

$$Z_t^1(g_{t+1}) \geq A_t^1(g_{t+1}) \tag{44}$$

Taking the first order conditions of the problem (41)-(44) with respect to c_t^1 and l_t

$$u_{c_t^1} - \tilde{\Delta}(u_{c_t^1}c_t^1 + u_{cc_t^1}c_t^1 - u_{c^1 l,t}(1 - l_t)) = \lambda_{1,t} \tag{45}$$

$$u_{l_t} - \tilde{\Delta}(u_{c^1 l,t} c_t^1 + u_{l,t} l_t - u_{ll,t}(1 - l_t)) = \lambda_{1,t} \tag{46}$$

Clearly, the allocations satisfying equations (45)-(46) cannot coincide with the solution of the system of equations (21)-(22) because, in the latter case, the weight attached to the term $u_{c_t^1} c_t^1 + u_{cc_t^1} c_t^1 + u_{c^1 l,t}(1 - l_t)$ is constant and equal to $\tilde{\Delta}$, while in the former it is given by $\frac{\Delta}{1+\gamma_t^1}$ and varies over time.

□

Proposition 4 states that the economy with transfers among countries cannot be reinterpreted as an economy in which there are international bond markets and limits to international debt issuance only.

From the proof of the proposition, it is again evident what has already been pointed out in Section 4.5.2. When there is full commitment, the cost of distortionary taxation is given by the Lagrange multiplier associated to the implementability constraint, Δ. This cost is constant due to the presence of complete bond markets. However, when we relax the assumption of full commitment and consider instead the case in which the government of the HC has limited commitment, the cost of distortionary taxation becomes state-dependent and is given by $\frac{\Delta}{1+\gamma_t^1}$. The reason for this is that now the government faces endogenously incomplete international bond markets[39]. Since allocations and tax rates vary permanently every time the participation constraint of the HC binds, so does the burden of taxation.

The previous discussion leads us to impose borrowing constraints on the value of domestic debt in addition to the constraints on international debt[40]. In this case, the problem of the government is given by equations (41)-(44) and

$$b_{t-1}^1(g_t) u_{c^1,t} = E_t \sum_{j=0}^{\infty} \beta^j (u_{c^1,t+j} c_{t+j}^1 - u_{l,t+j}(1 - l_{t+j})) \leq B_{t-1}(g_t) \tag{47}$$

The next proposition states that, in this case, it is possible to establish a mapping between the economy with transfers and the one with borrowing constraints on domestic as well as international debt.

Proposition 5. *Let* $\{c_t^1, c_t^2, l^t\}_{t=0}^{\infty}$ *be a constrained efficient allocation with high implied interest rates. Further, assume that the Ramsey planner of the HC is subject to domestic debt limits* $\{B_t\}_{t=0}^{\infty}$*. Then the constrained efficient allocation can be decentralized as a competitive equilibrium with borrowing constraints that are not too tight.*

Proof. See Appendix A.9 □

[39] A similar result is obtained in the incomplete markets literature (see Aiyagari et al. (2002)).

[40] Sleet (2004) also defines a borrowing constraint in terms of the value of debt.

This result provides a rationale for our specification of the outside option of the government in the HC. In Section 4 we assumed that if the government defaulted, it would lose access to the international and domestic bond markets and would remain in financial autarky thereafter. It therefore seems natural to impose constraints on the amount of debt that it can issue in both markets.

9 Conclusions

A key issue in macroeconomics is the study of the optimal determination of the tax rate schedule when the government has to finance (stochastic) public expenditure and only has available distortionary tools. Under this restriction, a benevolent planner seeks to minimize the intertemporal and intratemporal distortions caused by taxes. Since consumption should be smooth, a general result is that taxes should also be smooth across time and states.

When considering a small open developed economy that can borrow from international risk-neutral lenders, this result is amplified because there is perfect risk-sharing. Consumption and leisure are perfectly flat, thus the tax rate is also flat. The domestic public expenditure shock is absorbed completely by external debt and there is no role for domestic debt.

When we relax the assumption of full commitment to include small open emerging economies in the analysis, perfect risk-sharing is no longer possible. The presence of limited commitment lessens the ability of the government to fully insure against the public expenditure shock through use of international capital markets. Consequently, the government has to resort to taxes and internal debt in order to absorb part of the shock.

Our simulation results show that the volatility of the tax rate increases substantially when there is limited commitment. Considering a government expenditure shock as the only source of fluctuations generates a coefficient of variation around 3 times higher under limited commitment than under full commitment. Moreover, tax rates are countercyclical, so fiscal policy is procyclical.

The results presented in the paper suggest that the volatility of tax rates observed in emerging countries is not necessarily an outcome related (only) to reckless policy-making, as one could think a priori. We have shown that, in order to establish the optimal fiscal policy plan in emerging countries, it is important to take into account the degree of commitment that the economy has towards its external obligations, as this element is crucial in determining the extent of risk-sharing that can be achieved.

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A Appendix

A.1 Data

We use annual data on tax revenues over GDP, total government expenditure over GDP and GDP from 1997 to 2009, and data on the EMBIG spread for the same period. For all countries, the fiscal variables correspond to the central government. For those countries for which we do not have data for the whole sample period, we take the largest sub-sample period for which data is available.

The criterion used to include a country as a small open emerging economy was to include all countries for which the EMBIG or EMBI+ spread is computed. For small open developed economies, we included all OECD countries except those for which the EMBIG/EMBI+ spread is computed, and except the USA, which can be regarded as a large economy.

Tables 8 and 9 show the small open emerging and developed countries, respectively, in our sample, the data sources for fiscal variables and GDP, and the time period for which data is available. Those emerging economies for which there is not specified a data source for real GDP correspond to countries that have been used to produce Figure 1 but that have not been included in the empirical analysis of Section 2 because of lack of sufficient data on the EMBIG spread.

Fiscal data from national sources, such as central banks or ministries of finance, usually report data on fiscal variables according to specific definitions for each country. Therefore, in order to have comparable data across countries, we restricted our data sources to international institutions. ECLAC corresponds to the United Nations Economic Commission for Latin America, IFS stands for International Financial Statistics database and ADB stands for Asian Development Bank[41].

A.2 Optimal policy under full commitment: logarithmic utility

In this section we show a particular case of Proposition 1 when the utility function of households is logarithmic both in consumption and leisure. This corresponds to the utility function used for the numerical exercises in the paper.

Consider a utility function of the form:

$$u(c_t, l_t) = \alpha \; log(c_t) + \delta \; log(l_t)$$

with $\alpha > 0$ and $\delta > 0$. Assume that initial wealth $b_{-1} = 0$. Then the allocations and government policies can be easily computed from the optimality conditions (10) to (14). From the intertemporal budget constraint of households (10) it can be derived that:

[41] African countries have not been considered because the African Development Bank only reports data on total government revenues, but not on tax revenues.

Table 8: Data sources and sample periods - Emerging economies

Country	Source		Sample period
	Fiscal Variables	Real GDP	
Argentina	ECLAC	IFS	1997-2009
Brazil	ECLAC	IFS	1997-2009
Bulgaria	Eurostat	IFS	1997-2009
Chile	ECLAC	IFS	1999-2009
China	ADB	IFS	1997-2009
Colombia	ECLAC	IFS	1997-2009
Dominican Rep.	ECLAC	IFS	2001-2009
Ecuador	ECLAC	ECLAC	1997-2009
El Salvador	ECLAC	IFS	1997-2009
Georgia	ADB		1997-2009
Hungary	Eurostat	IFS	1999-2009
Indonesia	ADB		1997-2009
Kazakhstan	ADB		1997-2009
Malaysia	ADB	IFS	1997-2009
Mexico	ECLAC	IFS	1997-2009
Pakistan	ADB	IFS	2001-2009
Panama	ECLAC	ECLAC	1997-2009
Peru	ECLAC	IFS	1997-2009
Philippines	ADB	IFS	1997-2009
Poland	Eurostat	IFS	1997-2009
Sri Lanka	ADB		1997-2009
Thailand	ADB	IFS	1997-2009
Uruguay	ECLAC	IFS	2001-2009
Venezuela	ECLAC	ECLAC	1997-2009
Vietnam	ADB		1997-2009

Table 9: Data sources and sample periods - Developed economies

Country	Source of fiscal variables	Sample period
Australia	ADB	1997-2009
Austria	Eurostat	1997-2009
Belgium	Eurostat	1997-2009
Canada	OECD	1997-2008
Czech Rep.	Eurostat	1997-2009
Denmarc	Eurostat	1997-2009
Estonia	Eurostat	1997-2009
Finland	Eurostat	1997-2008
France	Eurostat	1997-2009
Germany	Eurostat	1997-2009
Greece	Eurostat	1997-2009
Iceland	Eurostat	1997-2008
Ireland	Eurostat	1997-2008
Israel	OECD	1997-2009
Italy	Eurostat	1997-2009
Japan	ADB	1997-2008
Korea	ADB	1997-2009
Luxembourg	Eurostat	1997-2008
Netherlands	Eurostat	1997-2009
New Zealand	ADB	1997-2008
Norway	Eurostat	1997-2009
Portugal	Eurostat	1997-2009
Slovakia	Eurostat	1997-2008
Slovenia	Eurostat	1997-2008
Spain	Eurostat	1997-2009
Sweden	Eurostat	1997-2009
Switzerland	Eurostat	1997-2009
UK	Eurostat	1997-2008

$$l = \frac{\delta}{\alpha + \delta} \tag{48}$$

Plugging this expression in (13), $c = \frac{\alpha}{\lambda}$. Combining this expression for consumption, together with (48), (11) and (12) we arrive to the following expression:

$$\frac{1}{1-\beta}\left(\frac{\alpha}{\lambda} - 1 + \frac{\delta}{\alpha + \delta}\right) + E_0 \sum_{t=0}^{\infty} \beta^t g_t = 0$$

Define the last term of the previous expression as

$$E_0 \sum_{t=0}^{\infty} \beta^t g_t \equiv \frac{1}{1-\beta}\tilde{g}$$

where $\tilde{g}$ is known at $t = 0$. Then

$$\lambda = \frac{\alpha + \delta}{1 - \frac{\alpha+\delta}{\alpha}\tilde{g}}$$

Substituting in the expression for c, we obtain

$$c = \frac{\alpha - (\alpha + \delta)\tilde{g}}{\alpha + \delta} \tag{49}$$

From the feasibility constraint (11), transfers are given by the difference between the actual realization of public expenditure g_t and its expected discounted value $\tilde{g}$:

$$T_t = g_t - \tilde{g} \tag{50}$$

Finally, from the intratemporal optimality condition of households (7) we can obtain an expression for the tax rate:

$$\tau = \frac{(\alpha + \delta)}{\alpha}\tilde{g} \tag{51}$$

A.3 Proof of Proposition 2

In order to prove Proposition 2, we first need to establish some intermediate results. We begin with a discussion about the sign of Δ, the Lagrange multiplier associated to the intertemporal budget constraint in the Ramsey planner's problem.

A.3.1 The Ramsey problem with limited commitment

For ease of exposition, we will assume that only the HC has limited commitment. Since the problem of the Ramsey planner is identical to the one in Section 4.5.2, but without imposing constraint (20), we do not reproduce it here.

The optimality conditions for $t \geq 1$ are:

$$u_{c,t}(1+\gamma_t^1) - \psi_t - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1-l_t)) = 0 \tag{52}$$

$$u_{l,t}(1+\gamma_t^1) - \psi_t - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1-l_t)) = 0 \tag{53}$$

$$\psi_t - \lambda = 0 \tag{54}$$

and equations (16), (17), (18), (19), (24) and (26) in the main text.

Multiplying equations (52) and (53) by c_t and $-(1-l_t)$ respectively, and summing:

$$\begin{aligned}&(1+\gamma_t^1 - \Delta)(u_{c,t}c_t - u_{l,t}(1-l_t)) - \psi_t(c_t - (1-l_t)) \\ &- \Delta\underbrace{(u_{cc,t}c_t^2 - 2u_{cl,t}(1-l_t)c_t + u_{ll,t}(1-l_t)^2)}_{A_t} = 0\end{aligned} \tag{55}$$

Notice that given that the utility function is strictly concave, expression A_t is strictly negative. By a similar procedure we can write down an equivalent expression at $t = 0$:

$$\begin{aligned}&(1+\gamma_0^1 - \Delta)(u_{c,0}(c_0 - b_{-1}) - u_{l,0}(1-l_0)) - \psi_0(c_0 - (1-l_0) - b_{-1}) \\ &- \Delta\underbrace{(u_{cc,0}(c_0 - b_{-1})^2 - 2u_{cl,0}(1-l_0)(c_0 - b_{-1}) + u_{ll,0}(1-l_0)^2)}_{A_0} = 0\end{aligned} \tag{56}$$

Multiplying (55) by $\beta^t\pi(s^t)$[42], summing over t and s^t and adding expression (56):

$$\begin{aligned}&E_0\sum_{t=0}^{\infty}\beta^t(1+\gamma_t^1 - \Delta)(u_{c,t}c_t - u_{l,t}(1-l_t)) - (1+\gamma_0 - \Delta)u_{c,0}b_{-1} \\ &- \Delta Q - E_0\sum_{t=0}^{\infty}\beta^t\psi_t(c_t - (1-l_t)) + \psi_0 b_{-1} = 0\end{aligned}$$

where Q is the expected value of the sum of negative quadratic terms A_t. Using the implementability constraint (17) and the resource constraint (16) we obtain equation (57)

[42] $\pi(s^t)$ is the probability of history s^t taking place given that the event s_0 has been observed.

$$
\begin{aligned}
&E_0 \sum_{t=0}^{\infty} \beta^t (\gamma_t^1 - \gamma_0^1)(u_{c,t}((1-l_t) + T_t - g_t) - u_{l,t}(1-l_t)) \\
&- \Delta Q + E_0 \sum_{t=0}^{\infty} \beta^t \psi_t (g_t - T_t) + \psi_0 b_{-1} = 0
\end{aligned}
\tag{57}
$$

For later purposes, using the intratemporal optimality condition of households (7) we can re-express this equation as[43].

$$
E_0 \sum_{t=0}^{\infty} \beta^t (\gamma_t^1 - \gamma_0^1) u_{c,t} (\tau_t (1-l_t) - g_t + T_t) - \Delta Q + E_0 \sum_{t=0}^{\infty} \beta^t \lambda g_t + \lambda b_{-1} = 0 \tag{58}
$$

Notice that, in the case of full commitment, expression (57) simplifies to

$$
-\Delta Q + \lambda \left(E_0 \sum_{t=0}^{\infty} \beta^t g_t + b_{-1} \right) = 0 \tag{59}
$$

Since $\lambda = \psi_t > 0 \ \forall t$, it is straightforward to see that when the present value of all government expenditures exceeds the value of any initial government wealth, the Lagrange multiplier $\Delta < 0$. As usual, this Lagrange multiplier can be interpreted as the marginal cost, in terms of utility, of raising government revenues through distortionary taxation.

In the presence of limited commitment, however, there is an extra term involving the costate variable γ_t^1 which prevents us from applying the same reasoning. Nevertheless, we will show that $\Delta < 0$ for the specific example of Section 5, and we will assume this result extends to the general setup. In the numerical exercise we perform in Section 6 we confirm that this assumption holds.

We show now under which conditions $\Delta = 0$. Setting $\Delta = 0$, from equations (52) and (53) we know that

$$
u_{c,t}(1+\gamma_t) = u_{l,t}(1+\gamma_t) \tag{60}
$$

$$
u_{c,t} = u_{l,t} \tag{61}
$$

This last expression and equation (7) in the text imply that $\tau_t = 0 \ \forall t$. Inserting these results into equation (58):

$$
E_0 \sum_{t=0}^{\infty} \beta^t (\gamma_t - \gamma_0)(u_{c,t}(T_t - g_t) + E_0 \sum_{t=0}^{\infty} \beta^t \lambda (g_t - T_t) + \lambda b_{-1} = 0
$$

[43]Notice that if the participation constraint was never binding, then $\gamma_t^1 = \gamma_0^1 = 0$ and we would recover an identical condition to the one obtained in the Lucas and Stokey (1983) model.

Using (60)

$$\Rightarrow E_0 \sum_{t=0}^{\infty} \beta^t(-\gamma_t + \gamma_0 + 1 + \gamma_t)u_{c,t}(g_t - T_t) + u_{c,0}(1 + \gamma_0)b_{-1} = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} \beta^t \frac{u_{c,t}}{u_{c,0}}(g_t - T_t) = -b_{-1} \tag{62}$$

We can rewrite (62) as

$$\sum_{t=0}^{\infty} \sum_{s^t} p_t^0(g_t - T_t) = -b_{-1} = b_{-1}^g \tag{63}$$

where p_t^0 is the price of a hypothetical bond issued in period 0 with maturity in period t contingent on the realization of s_t. Equation (63) states that when the government's initial claims b_{-1}^g against the private sector equal the present-value of all future government expenditures net of transfers, the Lagrange multiplier Δ is zero. Since the government does not need to resort to any distortionary taxation, the household's present-value budget does not exert any additional constraining effect on welfare maximization beyond what is already present through the economy's technology.

Finally, we will follow Ljungqvist and Sargent (2000) and assume that if the government's initial claims against the private sector were to exceed the present value of future government expenditures, the government would return its excess financial wealth as lump-sum transfers and Δ would remain to be zero.

A.3.2 Proof of Proposition 2

We begin by proving the first part of the Proposition. Given a logarithmic utility function as (28), optimality conditions (21) to (23) become

$$\frac{\alpha}{c_t}(1 + \gamma_t^1) - (\lambda + \gamma_t^2) - \Delta\left(-\frac{\alpha}{c_t^2}c_t + \frac{\alpha}{c_t}\right) = 0$$

$$\Longrightarrow c_t = \frac{\alpha(1 + \gamma_t^1)}{\lambda + \gamma_t^2} \tag{64}$$

$$\frac{\delta}{l_t}(1 + \gamma_t^1) - (\lambda + \gamma_t^2) - \Delta\left(\frac{\delta}{l_t} + \frac{\delta}{l_t^2}(1 - l_t)\right) = 0$$

$$\Longrightarrow l_t = \frac{\delta(1 + \gamma_t^1) \pm \sqrt{\delta^2(1 + \gamma_t^1)^2 - 4\Delta\delta(\lambda + \gamma_t^2)}}{2(\lambda + \gamma_t^2)} \tag{65}$$

Notice from equation (65) that if $\Delta < 0$ then we need to take the square root with positive sign in order to have $l_t > 0$. To show that consumption and leisure increase with γ_t^1, we take the derivatives of c_t and l_t with respect to γ_t^1

$$\frac{\partial c_t}{\partial \gamma_t^1} = \frac{\alpha}{\lambda + \gamma_t^2} > 0$$

$$\frac{\partial l_t}{\partial \gamma_t^1} = \frac{\delta + \left(\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)\right)^{-\frac{1}{2}} \delta^2(1+\gamma_t^1)}{2(\lambda+\gamma_t^2)} > 0$$

We can write the intratemporal optimality condition of households (7) as

$$\tau_t = \frac{u_{c,t} - u_{l,t}}{u_{c,t}} = 1 - \frac{\delta c_t}{\alpha l_t} \tag{66}$$

Given $t < t'$, assume $\gamma_t^1 < \gamma_{t'}^1$ while $\gamma_t^2 = \gamma_{t'}^2$. Now we compare the tax rates at t and t', and show that τ_t decreases with γ_t^1 by contradiction. Then, using (66)

$$\tau_{t'} - \tau_t = \frac{\delta}{\alpha}\left(\frac{c_t}{l_t} - \frac{c_{t'}}{l_{t'}}\right) > 0$$

It follows that it must be the case that $c_t l_{t'} - c_{t'} l_t > 0$. After some algebra this condition translates into

$$\left(\frac{1+\gamma_t^1}{1+\gamma_{t'}^1}\right)^2 > \frac{\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}{\delta^2(1+\gamma_{t'}^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}$$

$$(1+\gamma_t^1)^2 > (1+\gamma_{t'}^1)^2$$

which is clearly a contradiction. Thus, τ_t increases with γ_t^1.

Now we proceed to prove the second part of the proposition. We can immediately check that c_t decreases with γ_t^2 by taking partial derivates:

$$\frac{\partial c_t}{\partial \gamma_t^2} = -\frac{\alpha(1+\gamma_t^1)}{(\lambda+\gamma_t^2)^2} < 0$$

Suppose l_t is an increasing function of γ_t^2. Then the partial derivative of l_t w.r.t γ_t^2 must be positive

$$\frac{\partial l_t}{\partial \gamma_t^2} = \frac{-2\Delta\delta(\lambda+\gamma_t^2)A^{-\frac{1}{2}} - \delta(1+\gamma_t^1) - A^{\frac{1}{2}}}{2(\lambda+\gamma_t^2)^2} > 0$$

where $A = \delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)$. This last expression implies that

$$-2\Delta\delta(\lambda+\gamma_t^2) > \delta(1+\gamma_t^1)A^{\frac{1}{2}} + A$$

Then,

$$2\Delta\delta(\lambda+\gamma_t^2)-\delta^2(1+\gamma_t^1)^2 > A^{\frac{1}{2}}\delta(1+\gamma_t^1)$$

Since the left hand side of the previous expression is negative, while the right hand side is positive, this statement is clearly a contradiction. Then it must be the case that l_t is a decreasing function of γ_t^2.

Finally, suppose that $t' > t$, $\gamma_{t'}^2 > \gamma_t^2$ but $\gamma_{t'}^1 = \gamma_t^1$. Assume that τ_t is a decreasing function of γ_t^2. Then, using (66), it must be the case that

$$u_{c,t'}u_{l,t} < u_{c,t}u_{l,t'}$$

This implies that

$$\begin{aligned} &\frac{\delta(1+\gamma_t^1)+\sqrt{\delta^2(1+\gamma_t^1)^2-4\delta\Delta(\lambda+\gamma_{t'}^2)}}{2(\lambda+\gamma_{t'}^2)}\frac{\alpha(1+\gamma_t^1)}{\lambda+\gamma_t^2} < \\ &\frac{\delta(1+\gamma_t^1)+\sqrt{\delta^2(1+\gamma_t^1)^2-4\delta\Delta(\lambda+\gamma_t^2)}}{2(\lambda+\gamma_t^2)}\frac{\alpha(1+\gamma_t^1)}{\lambda+\gamma_{t'}^2} \end{aligned} \tag{67}$$

Simplifying and remembering that $\Delta < 0$, the previous inequality is a contradiction. Therefore, τ_t increases with γ_t^2. This completes the proof.

A.4 Proof of Proposition 3

Notice first that at $t = 0$ and for $\gamma_0^1 = 0$, the continuation value of staying in the contract has to be (weakly) greater than the value of the outside option (financial autarky):

$$\sum_{t=0}^{\infty}\beta^t u(c_t, l_t) \geq \sum_{t=0}^{\infty}\beta^t u(c_{t,A}, l_{t,A}) \tag{68}$$

The reason for this statement is that, for the government, subscribing the contract with the rest of the world represents the possibility to do risk-sharing and, consequently, to smooth consumption of domestic households. Since utility is concave, a smoother consumption path translates into a higher life-time utility value. Obviously, this result hinges on the fact that the initial debt of the government is zero and that equation (18) must hold[44].

Now we show that equation (19) holds with strict inequality for $1 \leq t \leq T$. It is important to bear in mind that the allocations can change in time only due to a different γ_t^1. Since $\gamma_{t-1}^1 \leq \gamma_t^1$

[44] If, for example, the initial level of government debt b_{-1} was very high, then the government could find it optimal to default on this debt and run a balanced budget thereafter. On the other hand, if condition (18) was not imposed, then the contract could mean a redistribution of resources from the HC to the RW that could potentially lead the HC to have incentives not to accept the contract.

$\forall t$, then $u(c_{t-1}, l_{t-1}) \leq u(c_t, l_t)$. Assume that $\mu_1^1 > 0$. This implies that, if μ_1^1 was equal to zero, the participation constraint would be violated, that is,

$$u(c_0, l_0) + \sum_{t=2}^{T-1} \beta^{t-1} u(c_t, l_t) + \beta^{T-1} u(c_T, l_T) + \sum_{t'=T+1}^{\infty} \beta^{t'-1} u(c_{t'}, l_{t'}) < \sum_{t=0}^{T-2} \beta^t u(c_A, l_A) + \beta^{T-1} u(c_{A'}, l_{A'}) + \sum_{t=T}^{\infty} \beta^t u(c_A, l_A) \tag{69}$$

Equation (68) can be rewritten as

$$\sum_{t=0}^{T} \beta^t u(c_t, l_t) + \sum_{t'=T+1}^{\infty} \beta^{t'} u(c_{t'}, l_{t'}) > \sum_{t=0}^{T-1} \beta^t u(c_A, l_A) + \beta^T u(c_{A'}, l_{A'}) + \sum_{t=T+1}^{\infty} \beta^t u(c_A, l_A) \tag{70}$$

Subtracting (70) from (69):

$$\begin{aligned} &\beta[u(c_2, l_2) - u(c_1, l_1)] + \beta^2[u(c_3, l_3) - u(c_2, l_2)] + \ldots + \beta^{T-1}[u(c_T, l_T) - u(c_{T-1}, l_{T-1})] + \\ &\beta^T[u(c_{T+1}, l_{T+1}) - u(c_T, l_T)] + \beta^{T+1}[u(c_{T+2}, l_{T+2}) - u(c_{T+1}, l_{T+1})] + \ldots \\ &< \beta^{T-1}[u(c_{A'}, l_{A'}) - u(c_A, l_A)] + \beta^T[u(c_A, l_A) - u(c_{A'}, l_{A'})] \end{aligned} \tag{71}$$

Reordering terms we arrive at:

$$\begin{aligned} &\beta \underbrace{[u(c_2, l_2) - u(c_1, l_1)]}_{\geq 0} + \beta^2 \underbrace{[u(c_3, l_3) - u(c_2, l_2)]}_{\geq 0} + \ldots + \beta^{T-1} \underbrace{[u(c_T, l_T) - u(c_{T-1}, l_{T-1})]}_{\geq 0} + \\ &+ \beta^T \underbrace{[u(c_{T+1}, l_{T+1}) - u(c_{T-1}, l_{T-1})]}_{\geq 0} + \beta^{T+1} \underbrace{[u(c_{T+2}, l_{T+2}) - u(c_{T+1}, l_{T+1})]}_{\geq 0} + \ldots \\ &< \underbrace{[u(c_{A'}, l_{A'}) - u(c_A, l_A)]}_{<0} \underbrace{(\beta^{T-1} - \beta^T)}_{>0} \end{aligned} \tag{72}$$

Expression (72) is clearly a contradiction, since the left hand side of the inequality is greater or equal than 0, but the right hand side is strictly smaller than 0. We conclude then that it cannot be that $\mu_1^1 > 0$. Therefore, equation (19) is not binding in period $t = 1$. The same reasoning can be extended to periods $t = 2, 3, \ldots, T$, so $\gamma_t^1 = \gamma_0^1 = 0$ for $t = 1, 2, ..., T$ and the allocations $\{c_t\}_{t=0}^T, \{l_t\}_{t=0}^T$ are constant.

Notice that, from $T+1$ onwards, $g_t = 0$ so the allocations do not change. Therefore, $\gamma_t^1 = \gamma_{T+1}^1$ for $t = T+2, T+3, \ldots, \infty$.

Finally, we show that $\mu_{T+1}^1 > 0$[45]. We prove this by contradiction. Assume that $\mu_{T+1} = 0$. From the previous discussion, this implies that $\gamma_t^1 = 0\ \forall t$. Then the allocations are identical to the case of full commitment, and from the results of Section A.2, we know that $T_t < 0$ for $t \neq T$ and $T_T > 0$. Thus, from the feasibility constraint (16) we can see that $c_{T+1} < c_A$ and $l_{T+1} < l_A$. But this implies that utility $u(c_{T+1}, l_{T+1}) < u(c_A, l_A)$, so

$$\frac{1}{1-\beta}u(c_{T+1}, l_{T+1}) < \frac{1}{1-\beta}u(c_A, l_A)$$

$$\sum_{j=0}^{\infty}\beta^j u(c_{T+1+j}, l_{T+1+j}) < \sum_{j=0}^{\infty}\beta^j u(c_A, l_A)$$

which clearly contradicts with the fact that $\mu_{T+1}^1 = 0$. Therefore, it must be the case that $\mu_{T+1}^1 > 0$. This completes the proof.

A.5 Proof that $\Delta < 0$ in Section 5

Since in the example of Section 5 we have a full analytical characterization of the equilibrium, it is possible to determine the sign of Δ.

Given our assumption about the government expenditure shock and the result of Proposition 3, equation (58) can be written as

$$\sum_{t=T+1}^{\infty}\beta^t \gamma_{T+1} u_{\bar{c}}(\bar{\tau}(1-\bar{l}) + \bar{T}) - \Delta Q + \beta^T \lambda g_T = 0 \tag{73}$$

where $\bar{c}, \bar{l}, \bar{\tau}$ and $\bar{T}$ are the constant allocations and fiscal variables from $t = T+1$ onwards. In order to determine the sign of the first term of the previous expression, we recall the period by period budget constraint of the government for $t \geq T+1$:

$$(\beta - 1)\bar{b}^G = \bar{\tau}(1-\bar{l}) + \bar{T}$$

The sign of the first term of equation (73) depends on whether government bonds are positive or negative after the big shock has taken place. From equation (17)

[45] Notice that, given that our shock in this example is not a Markov process, neither γ_t nor the allocations c_t and l_t are time-invariant functions of the state variables g_t, γ_{t-1} but, on the contrary, they depend on t.

$$
\begin{aligned}
&\sum_{j=0}^{T}\beta^t(u_{\tilde{c}}\tilde{c}-u_{\tilde{l}}(1-\tilde{l}))+\sum_{j=T+1}^{\infty}\beta^t(u_{\bar{c}}\bar{c}-u_{\bar{l}}(1-\bar{l}))=0\\
\Rightarrow\ &\frac{1-\beta^{T+1}}{1-\beta}\left(\alpha-\frac{\delta}{\tilde{l}}(1-\tilde{l})\right)+\frac{\beta^{T+1}}{1-\beta}\left(\alpha-\frac{\delta}{\bar{l}}(1-\bar{l})\right)=0 \qquad (74)
\end{aligned}
$$

where $\tilde{c}$ and $\tilde{l}$ are the constant allocations from $t = 0$ to $t = T$. We know that the participation constraint binds in period $T + 1$ and consequently $\bar{l} > \tilde{l}$. But this implies that

$$
\begin{aligned}
\alpha-\frac{\delta}{\tilde{l}}(1-\tilde{l})<0\\
\alpha-\frac{\delta}{\bar{l}}(1-\bar{l})>0 \qquad (75)
\end{aligned}
$$

because the two terms of (74) have to add up to zero. Now we recover b_t for $t \geq T+1$ from the intertemporal budget constraint (17) of households at time $T + 1$:

$$
\begin{aligned}
u_{\bar{c}}\bar{b}=&\sum_{j=0}^{\infty}\beta^j(u_{\bar{c}}\bar{c}-u_{\bar{l}}(1-\bar{l}))=\frac{1}{1-\beta}(u_{\bar{c}}\bar{c}-u_{\bar{l}}(1-\bar{l}))\\
=&\frac{1}{1-\beta}\left(\alpha-\frac{\delta}{\bar{l}}(1-\bar{l})\right)>0
\end{aligned}
$$

If $\bar{b} > 0$, $\bar{b}^G < 0$ so the first term in equation (73) is positive. From this equation it is immediate to see that $\Delta < 0$.

A.6 One-sided limited commitment

In Section 4.1 in the main text we considered the possibility that both the HC and the RW have no commitment towards each other when signing a transfer contract. In this section, we assume instead that only the HC has limited commitment but that the RW has full commitment.

The problem of the government in this case is to maximize (15) subject to (16)-(19). Then, Proposition 2 still holds for the case in which the participation constraint of the HC binds, and tax rates are more volatile for small open emerging economies that have limited commitment than for small open developed economies that have full commitment.

A.6.1 Numerical exercise

In this section we perform a numerical exercise analogous to that of Section 6. We specify the same functional forms and parameter values used in that section, but now we do not consider

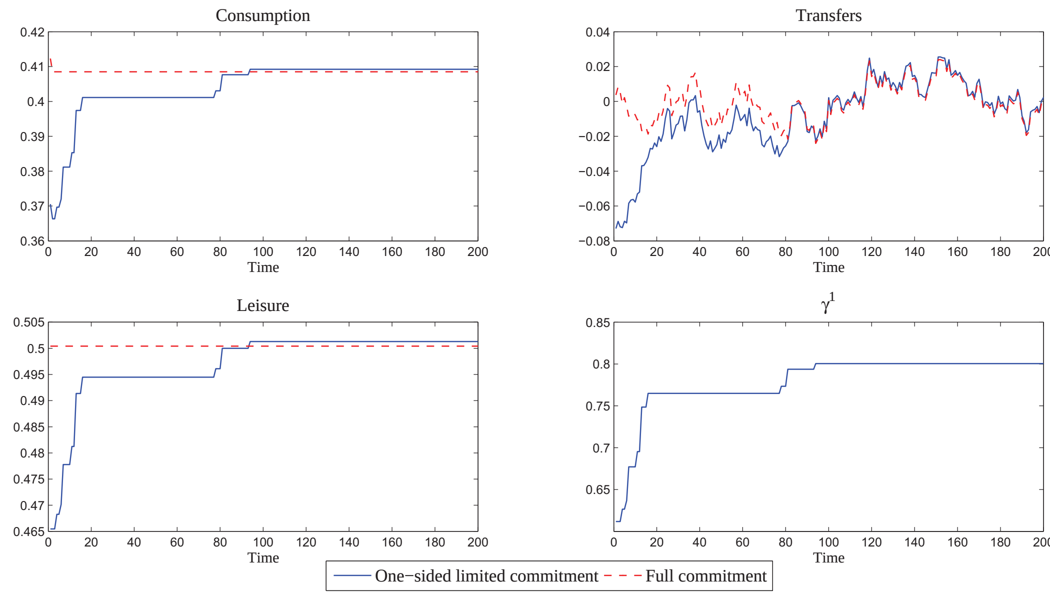

Figure 9: Allocations - One sided limited commitment

Table 10: Statistics of allocations for the first 100 periods - 1000 simulations

	Limited Comm.		Full Comm.	
	Mean	Coef. Var.	Mean	Coef. Var.
consumption	0.3952	2.53%	0.4085	0.09%
leisure	0.4895	1.73%	0.5004	0%
labor tax rate	0.1929	3.44%	0. 1836	0.41%

constraint (20) as a restriction of the maximization problem.

Figures 9 and 10 show the allocations, co-state variable and fiscal variables for the same realization of the government expenditure shock that we used to illustrate the model in Section 6.2. Table 10 shows the statistics of the allocations for the first 100 periods, for 1000 simulations.

It is clear from the table that the labor tax rate is much more volatile under one-sided limited commitment that under full commitment: the coefficient of variation is about 7 times higher in the former case. Comparing Table 5 to Table 10, we can observe that, while the mean of the allocations are very similar under two-sided or one-sided limited commitment, the allocations are more volatile in the latter case. Finally, from inspection of Figures 9 and 10, we can see that, in the time span considered, the allocations reach a stationary level for which the participation constraint of the HC never binds again. After this point, the economy becomes developed and there is perfect risk-sharing.

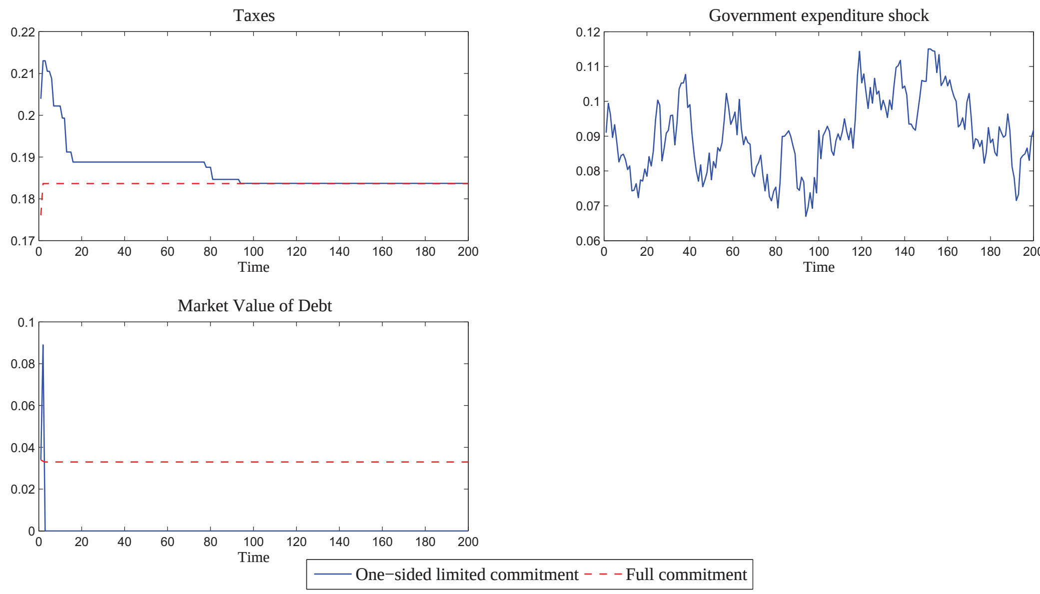

Figure 10: Fiscal variables - One sided limited commitment

A.7 An alternative participation constraint for the RW

In Section 4.1 in the main text we argued that constraint (4) was meant to capture the idea that emerging economies may see their international debt contracts interrupted by reasons other than the decision to default from the own country. In this section we propose a participation constraint for the RW which, although being alternative to constraint (4), conveys a similar idea to the one just described. In particular, the constraint we consider here is:

$$T_t \leq \underline{B} \tag{76}$$

This constraint implies that the resources that the RW has to give to the HC in a given period cannot exceed a given quantity $\underline{B}$. The rationale for this is that we may think that foreign lenders may not be willing to lend large amounts to emerging economies for reasons such as contagion, uncertainty about fundamentals, considerations of moral hazard and/or efficiency in the use of the loans.

The problem of the government becomes

$$\max_{\{c_t, l_t, T_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to

$$c_t + g_t = (1 - l_t) + T_t \tag{77}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1 - l_t)) = u_{c^1,0}(b_{-1}) \tag{78}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0 \tag{79}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \tag{80}$$

$$T_t \leq \underline{B} \tag{81}$$

Notice that now, contrary to what happens in the baseline case, constraint (81) does not impose the need to add a costate variable γ_t^2. Constraint (80), however, still requires us to expand the state space in order to transform the problem into a recursive one.

The Lagrangian of the problem is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t [(1 + \gamma_t^1) u(c_t, l_t) - \psi_t (c_t + g_t - (1 - l_t) - T_t)$$
$$- \mu_t^1 (V_t^a) - \mu_t^2 (T_t - \underline{B}) - \Delta(u_{c,t} c_t - u_{l,t}(1 - l_t)) - \lambda T_t] + \Delta(u_{c,0}(b_{-1}))$$

where

$$\gamma_t^1 = \gamma_{t-1}^1 + \mu_t^1$$

for $\gamma_{-1}^1 = 0$. The government's optimality conditions for $t \geq 1$ are:

$$u_{c,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cc,t} c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = 0 \tag{82}$$

$$u_{l,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cl,t} c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = 0 \tag{83}$$

$$\psi_t = \lambda + \mu_t^2 \tag{84}$$

Other optimality conditions are equations (77), (80),

$$\mu_t^1 (E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) - V^a(g_t)) = 0, \qquad \mu_t^1 \geq 0$$

$$\mu_t^2 (T_t - \underline{B}) = 0, \qquad \mu_t^2 \geq 0$$

$$\gamma_t^1 = \mu_t^1 + \gamma_{t-1}^1$$

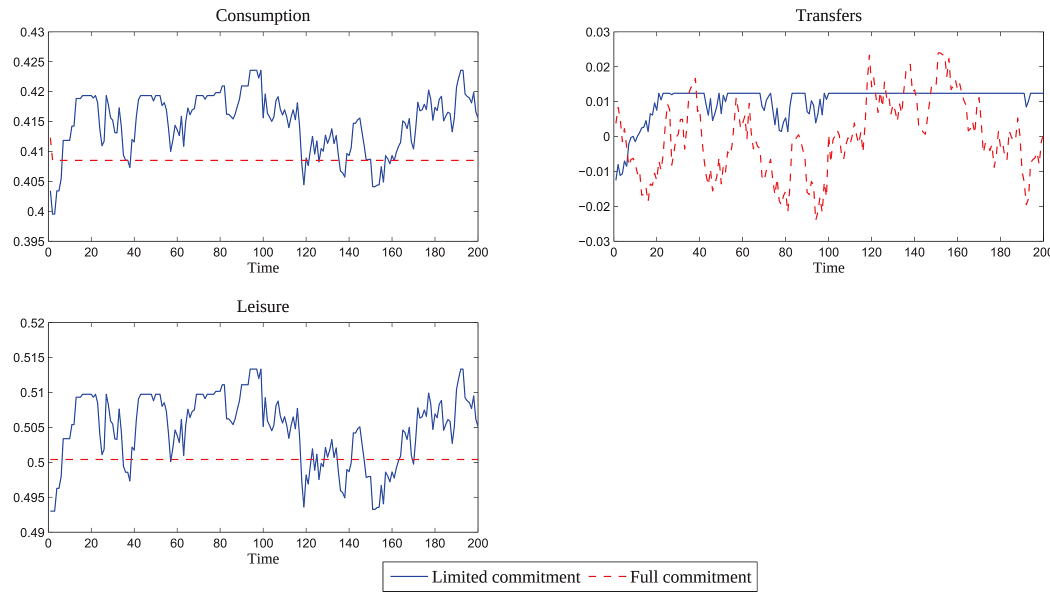

Figure 11: Allocations - Alternative participation constraint

The second part of Proposition 2 is slightly modified by considering this alternative participation constraint, while the first part remains unchanged. In particular, now it is the case that, whenever constraint (76) binds, consumption and leisure go down, and taxes go up, but only during the periods for which the constraint is effectively binding. In other words, the fact that constraint (76) binds in period T does not have permanent effects over the allocations.

A.7.1 Numerical exercise

We solve the model previously described using the same parameterization of Section 6 except for the value of $\underline{B}$. As explained in the main text, T_t can be interpreted as the change in public debt contracted with foreign creditors in a given period. Since we use data on debt contracted with the IMF, $T_t = b_t^{IMF} - b_{t-1}^{IMF}$, where b_t^{IMF} is debt over annualized GDP from the data, multiplied by the approximate annual GDP of the model. Then, we parameterize $\underline{B}$ as

$$\underline{B} = \max \{T_t\}_{\text{t=I-93}}^{\text{III-2000}} = 0.01239$$

Analogous to the results shown in the main text, Figures 11, 12 and 13 show the allocations, co-state variable and Lagrange multiplier associated to the participation constraints, and fiscal variables for the same realization of the government expenditure shock that we used to illustrate the model in Section 6.2.

At first sight, it is clear that the qualitative results of the model are not changed if we consider

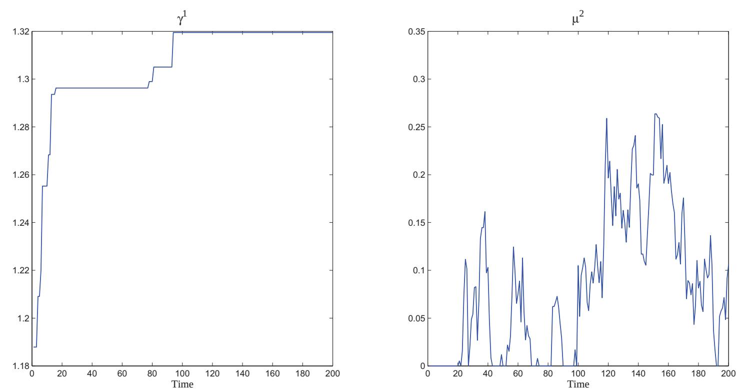

Figure 12: Costate and Lagrange multiplier - Alternative participation constraint

Table 11: Statistics of allocations for the first 100 periods - 1000 simulations

	Limited Comm.		Full Comm.	
	Mean	St.Dev.	Mean	St.Dev.
consumption	0.4135	0.0051	0.4085	3.7734e-04
leisure	0.5041	0.0048	0.5004	0
labor tax rate	0.1798	0.0027	0.1836	7.5405e-04

equation (76) instead of (4) as the relevant participation constraint for the RW. This provides evidence that our model is robust to different specifications of the participation constraint, as it implies that it is the possibility that the parties have to leave the contract which induces the variability in the tax rate, and not the particular participation constraint considered.

There is, however, a subtle difference between the two approaches. In the case depicted in this section, the volatility of tax rates prevails even in the long run. Notice from Figure 12 that γ_t^1 reaches a certain value in period $t = T$ and, after that, it stays constant. This is because, for $t \geq T$, the value of the costate variable is such that the participation constraint of the HC never binds again. However, as μ_t^2 continues to fluctuate, so do the allocations and the fiscal variables. The reason for this is that constraint (76) does not impose history dependence on the problem since it only involves a variable at time t, but not future variables[46].

Table 11 shows some statistics obtained by simulating the model for 1000 realizations of the shock of 100 periods each. The government expenditure shocks are the same generated to compute the statistics of Table 5, and we only use the first 100 periods of each simulation to be able to compare the results reported in the two tables. Tables 5 and 11 show very similar statistics for the limited commitment case and the full commitment one[47]. We confirm,

[46]On the contrary, constraint (4) does impose history dependence, which is captured by the costate variable γ_t^2.

[47]The statistics for the full commitment case are identical in the two exercises. This is because the parameterization used here is the same as the one used in Section 6, except for the value of $\underline{B}$, which is only relevant in the

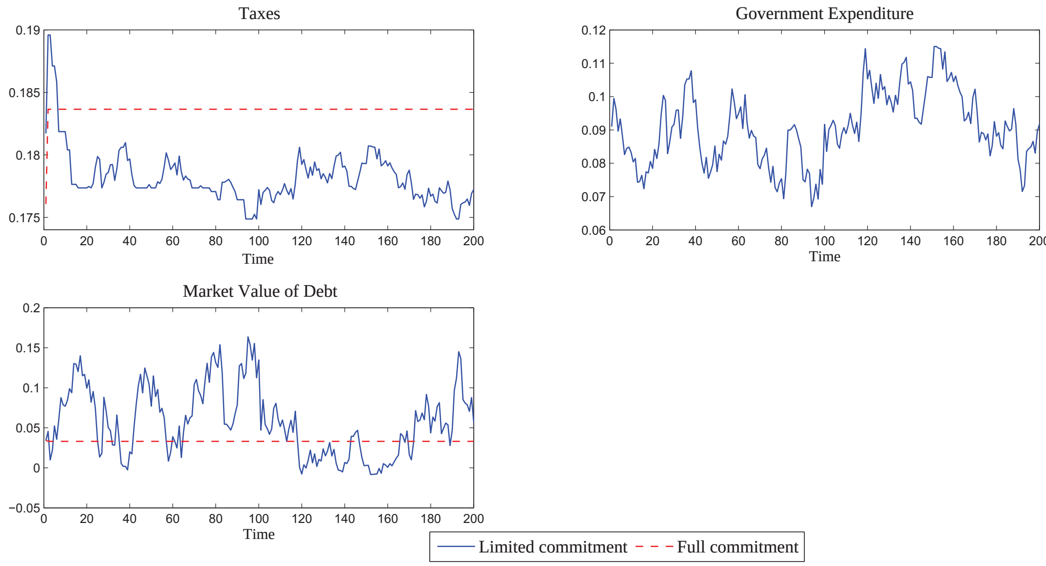

Figure 13: Fiscal variables - Alternative participation constraint

therefore, that the use of the alternative participation constraint (76) modifies only slightly the results reported in the main text.

A.8 The International Institution Problem and the Government Problem: Equivalence of Results

Suppose that there exists an international financial institution that distributes resources among the HC and the RW, taking into account the aggregate feasibility constraint (16), the implementability condition (17), and participation constraints (19) and (20). Assume for simplicity that $b_{-1} = 0$. The Lagrangian associated to the international institution is

$$\begin{aligned} \max_{\{c_t^1, c_t^2, T_t\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t (\eta u(c_t^1, l_t^1) + (1-\eta) u(c_t^2)) \\ +\tilde{\mu}_{1,t} & (E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}^1, l_{t+j}^1) - V_t^{1,a}) + \tilde{\mu}_{2,t}(\underline{B} - E_t \sum_{j=0}^{\infty} \beta^j T_{t+j}) \\ -\tilde{\Delta} & (u_{c^1,t} c_t^1 - u_{l_{1_t}} (1 - l_t^1)) - \tilde{\psi}_t (c_t^1 + c_t^2 + g - (1 - l_t^1 + y))) \end{aligned} \tag{85}$$

where η is the Pareto weight that the international institution assigns to the HC and y is a fixed endowment that households of the RW receive every period. Since by assumption households in the RW are risk-neutral, $u(c_t^2) = c_t^2$. The feasibility constraint of the RW implies

limited commitment case.

that $c_t^2 = y - T_t$. Substituting this into (85) and applying Marcet and Marimon (2009) we can recast problem (85) as

$$\max_{\{c_t^1, c_t^2, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t ((\eta + \tilde{\gamma}_{1,t}) u(c_t^1, l_t^1) + (1-\eta)(y - T_t)$$
$$-\tilde{\mu}_{1,t} V_t^{1,a} + \tilde{\mu}_{2,t} \underline{B} - \tilde{\gamma}_{2,t} T_t - \tilde{\Delta}(u_{c^1,t} c_t^1 - u_{l_{1_t}}(1 - l_t^1)) - \tilde{\psi}_t (c_t^1 + g - (1 - l_t^1 + T_t)) \tag{86}$$

Dividing each term by η does not change the solution, since η is a constant. Let $\frac{\tilde{\gamma}_t^i}{\eta} \equiv \gamma_t^i$, for $i = 1, 2$, $\frac{\tilde{\Delta}}{\eta} \equiv \Delta$ and $\frac{\tilde{\psi}_t}{\eta} \equiv \psi_t$. The first-order conditions are

$$u_{c,t}(1 + \tilde{\gamma}_t^1) - \psi_t - \Delta(u_{cc,t} c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = 0 \tag{87}$$

$$u_{l,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cl,t} c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = 0 \tag{88}$$

$$\psi_t = \frac{1-\eta}{\eta} + \gamma_t^2 \tag{89}$$

where $c_t \equiv c_t^1$. Calling $\lambda \equiv \frac{1-\eta}{\eta}$ makes the system of equations (87)-(89) coincide with (21)-(23).

A.9 Proof of Proposition 5

First notice that when we introduce (47) as one of the constraints of the Ramsey planner's problem, the problem becomes non-recursive because future endogenous variables appear in a constraint valid in period t. Once more, we need to apply the recursive contract's approach of Marcet and Marimon (2009) to write a recursive problem which solution coincides with the one of the original problem. The Lagrangian of this new problem can be written as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{ u(c_t^1, l_t^1) - (\tilde{\Delta} + \Lambda_{1,t})(u_{c^1,t} c_t^1 - u_{l^1,t}(1 - l_t^1)) + \lambda_{1,t} B_{t-1}(g_t) - \lambda_{2,t}(A_t^1(g_{t+1})$$
$$- Z_t^1(g_{t+1})) + \lambda_{3,t}(1 - l_t^1 - \sum_{g_{t+1}} q_t Z_t^1(g_{t+1}) + Z_{t-1}^1(g_t) - c_t^1 - g_t) + \tilde{\Delta} b_{-1} u_{c^1,0} \}$$

where $\Lambda_{1,t} = \Lambda_{1,t-1} + \lambda_{1,t}$ is a costate variable representing the sum of all past Lagrange multipliers λ_1 attached to constraint (47), and its initial condition is $\Lambda_{1,-1} = 0$. This costate variable keeps track of all past periods in which the constraint on the value of the domestic debt has been binding.

The optimality conditions are

$$u_{c^1,t} - (\tilde{\Delta} + \Lambda_{1,t})(u_{cc^1,t}c_t^1 + u_{c^1,t} - u_{c^1l,t}(1 - l_t)) = \lambda_{3,t} \tag{90}$$

$$u_{l^1,t} - (\tilde{\Delta} + \Lambda_{1,t})(u_{c^1l,t}c_t^1 + u_{l^1,t} - u_{ll^1,t}(1 - l_t^1)) = \lambda_{3,t} \tag{91}$$

$$q_t(g_{t+1}) = \frac{\beta\lambda_{3,t+1}\pi(g^{t+1}|g^t) + \lambda_{2,t}}{\lambda_{3,t}} \tag{92}$$

The problem of households in the RW and the optimality conditions associated to it are given by (32)-(34) and (35)-(37), respectively.

Following Alvarez and Jermann (2000), we prove the proposition by construction. We consider three possible scenarios:

1. Neither the HC's nor the RW's participation constraints are binding in $t+1$, $\forall g' = g_{t+1}$
2. The RW's participation constraint is binding in $t+1$ for $g' = g_{t+1}$
3. The HC's participation constraint is binding in $t+1$ for $g' = g_{t+1}$

1. **Neither the HC's nor the RW's participation constraints are binding in $t+1$, $\forall g' = g_{t+1}$**

   Assume for simplicity that neither the HC not the RW have ever been borrowing constrained and, consequently, $\Lambda_{1,t} = 0$. Set debt limits $A_t^1(g')$, $A_t^2(g')$ and $B_t(g')$ to be very large in absolute value such that constraints (31), (34) and (47) do not bind for $g' = g_{t+1}$. Then multipliers $\lambda_{1,t+1} = \lambda_{2,t} = \omega_t^2 = 0$. Given the allocations, equations (35) and (92) define prices. Notice that, from these two equations and the optimality conditions (90) and (91), it has to be the case that $c_t^1 = c_{t+1}^1$ and $l_t = l_{t+1}$. The allocations of the problem of Section 4 satisfy these conditions. Notice also that, given the assumptions made, $\gamma_t^1 = \gamma_t^2 = \mu_{t+1}^1 = \mu_{t+1}^2 = 0$. Finally, from the optimality conditions of the two problems, (21)-(23) and (90)-(92), it is easy to see that it has to be the case that $\lambda_{3,t} = \lambda$. The initial level of international bonds $Z_{-1}^1 = -Z_{-1}^2$ is chosen so that this equality holds.

2. **The RW's participation constraint is binding in $t+1$ for $g' = g_{t+1}$**

   From the definition of high implied interest rates, and given the allocations, the price of the bond is determined by equation (40). From equations (35) and (92), it is clear that the borrowing constraint of the RW is binding, while the borrowing constraint of the HC is not. Therefore, we set the debt limit $A_t^2(g')$ to be equal to the holding of the corresponding bond. We will explain later how such holdings are determined. The rest of the debt limits,

$A_t^1(g')$ and $B_t(g')$, are again set to be very large in absolute value so that constraints (31) and (47) do not bind for $g' = g_{t+1}$. From (21)-(23) and (90)-(92), we know that $\lambda_{3,t+1} > \lambda_{3,t}$ because $\gamma_{t+1}^2 > 0$, which is exactly what the pricing equation is reflecting.

3. **The HC's participation constraint is binding in $t+1$ for $g' = g_{t+1}$**

   From the definition of high implied interest rates, and given the allocations, the price of the bond is determined by equation (40). From equations (35) and (92), it is clear that the borrowing constraint of the HC is binding, while the borrowing constraint of the RW is not. Also, given the allocations, it is clear from (21)-(23) and (90)-(92) that $\lambda_{3,t+1} < \lambda_{3,t}$ because $\gamma_{t+1}^1 > 0$. From equation (92) we can conclude that $\lambda_{2,t} > 0$. Consequently, we set the debt limit $A_t^1(g')$ to be equal to the holding of the corresponding international bond. Moreover, we set the limit $B_t(g')$ to be equal to the holding of the domestic bond, which is already known from the allocations of Section 4. The debt limit $A_t^2(g')$ is set to be very large in absolute value so that constraint (34) does not bind.

   Notice that equations (21) and (22) imply that:

$$u_{c,t}(1+\gamma_t^1) - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1-l_t)) = u_{l,t}(1+\gamma_t^1) - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1-l_t)) \quad (93)$$

   On the other hand, equations (90) and (91) imply:

$$u_{c^1,t} - (\Delta+\Lambda_{1,t})(u_{cc^1,t}c_t^1 + u_{c^1,t} - u_{c^1l,t}(1-l_t)) = u_{l^1,t} - (\Delta+\Lambda_{1,t})(u_{c^1l,t}c_t^1 + u_{l^1,t} - u_{ll^1,t}(1-l_t^1)) \quad (94)$$

   If we did not introduce a limit of the value of domestic debt, then the allocations that solve (93) would never solve (94), because the possibility of default in the first case changes the marginal rate of substitution between consumption and labor, but the binding limits on international debt alone would not do so in the second case. Therefore, we need to introduce a limit on the value of domestic debt as well.

Finally, we need to determine the holdings of the corresponding international bonds. From the budget constraints of the HC and the RW, (43) and (33) respectively, and given prices and allocations, iterate forward to obtain the holding of the international bond for each possible realization of the public expenditure shock g_t. This ensures that c_t^1, l_t and c_t^2 are budget feasible. It is easy to see that, if constructed in this way, $Z_t^1(g') + Z_t^2(g') = 0 \; \forall g'$.

A.10 Computational algorithm

Following Christiano and Fisher (2000), we solve the model by a projection method in which we parameterize the expectations associated to the two participation constraints (19) and (20). We use linear splines to approximate the expectations. Our problem entails three state variables corresponding to the government expenditure shock and the two costate variables associated to the participation constraints of the HC and the RW respectively. We construct the grid for the approximation by considering 4 points for each state variable, which implies that the grid has 64 points in total. The intervals in which we approximate the solution are the following: $g_t \in [\bar{g} - 2\sigma_g, \bar{g} + 2\sigma_g]$, $\gamma_t^1 \in [0, 1.2]$ and $\gamma_t^2 \in [0, 0.3]$.

The algorithm is as follows:

1. For a given guess for Δ and λ, obtain the initial guess of the parameters that approximate the expectations in the left hand side of equations (19) and (20) by considering that the participation constraints are never binding. The allocations are obtained by solving the optimality conditions for $\gamma_t^1 = \gamma_t^2 = 0$. Once we have the allocations, we can write the expectations recursively. In particular, consider the expectation in the left-hand side of the participation constraint for the HC. Then, $E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \equiv V(g_t, \gamma_{t-1}^1 = 0, \gamma_{t-1}^2 = 0)$ and

$$V(g_t, \gamma_{t-1}^1 = 0, \gamma_{t-1}^2 = 0) = u(c_t, l_t) + \beta E_t V(g_{t+1}, \gamma_t^1 = 0, \gamma_t^2 = 0) \qquad (95)$$

For a given guess for $V(g_t, \gamma_{t-1}^1 = 0, \gamma_{t-1}^2 = 0)$ and for the allocations previously obtained, we iterate on (95) to obtain $E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j})$. We do the same to obtain $E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \equiv U(g_t, \gamma_{t-1}^1 = 0, \gamma_{t-1}^2 = 0)$.

   (a) Given the guess for the parameters that approximate expectations and the guesses for Δ and λ, we obtain the equilibrium allocations in the following manner: assume that neither the participation constraint of the HC nor the one of the RW bind in period t. Then $\mu_t^1 = \mu_t^2 = 0$. Then compute c_t, l_t and T_t from the optimality conditions of the problem:

$$u_{c,t}(1 + \gamma_{t-1}^1) - \Delta(u_{cc,t} c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = \lambda + \gamma_{t-1}^2$$

$$u_{l,t}(1 + \gamma_{t-1}^1) - \Delta(u_{cl,t} c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda + \gamma_{t-1}^2$$

$$c_t + g_t = (1 - l_t) + T_t$$

(b) Given $\mu_t^1 = \mu_t^2 = 0$, compute a large grid at $t+1$ for every possible realization of g_{t+1}, given g_t. Then compute $V(g_{t+1}, \gamma_t^1 = \gamma_{t-1}^1, \gamma_t^2 = \gamma_{t-1}^2)$ and $U(g_{t+1}, \gamma_t^1 = \gamma_{t-1}^1, \gamma_t^2 = \gamma_{t-1}^2)$. Compute

$$A_t^1 = E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) = u(c_t, l_t) + \beta E_t V(g_t, \gamma_t^1 = \gamma_{t-1}^1, \gamma_t^2 = \gamma_{t-1}^2)$$

$$A_t^2 = E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} = T_t + \beta E_t U(g_{t+1}, \gamma_t^1 = \gamma_{t-1}^1, \gamma_t^2 = \gamma_{t-1}^2)$$

(c) Using A_t^1, check whether the participation constraint of the HC is satisfied or not. If it is, proceed to the next step, otherwise recompute the allocations c_t, l_t, T_t and μ_t^1 from optimality conditions

$$u_{c,t}(1 + \gamma_{t-1}^1 + \mu_t^1) - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = \lambda + \gamma_{t-1}^2$$

$$u_{l,t}(1 + \gamma_{t-1}^1 + \mu_t^1) - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda + \gamma_{t-1}^2$$

$$c_t + g_t = (1 - l_t) + T_t$$

$$u(c_t, l_t) + \beta E_t V(g_t, \gamma_t^1 = \gamma_{t-1}^1 + \mu_t^1, \gamma_t^2 = \gamma_{t-1}^2) = V^a(g_t)$$

(d) Using A_t^2, check whether the participation constraint of the RW is satisfied or not. If it is, proceed to the next step, otherwise recompute the allocations c_t, l_t, T_t and μ_t^2 from optimality conditions

$$u_{c,t}(1 + \gamma_{t-1}^1) - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = \lambda + \gamma_{t-1}^2 + \mu_t^2$$

$$u_{l,t}(1 + \gamma_{t-1}^1) - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda + \gamma_{t-1}^2 + \mu_t^2$$

$$c_t + g_t = (1 - l_t) + T_t$$

$$T_t + \beta E_t U(g_{t+1}, \gamma_t^1 = \gamma_{t-1}^1, \gamma_t^2 = \gamma_{t-1}^2 + \mu_t^2) = \underline{B}$$

(e) Given c_t, l_t, T_t, μ_t^1 and μ_t^2, compute a large grid at $t+1$ for every possible realization of g_{t+1}, given g_t. Then compute $V(g_{t+1}, \gamma_t^1 = \gamma_{t-1}^1, \gamma_t^2 = \gamma_{t-1}^2)$ and $U(g_{t+1}, \gamma_t^1 = \gamma_{t-1}^1, \gamma_t^2 = \gamma_{t-1}^2)$. Compute

$$V^*(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) = u(c_t, l_t) + \beta E_t V(g_{t+1}, \gamma_t^1 = \gamma_{t-1}^1, \gamma_t^2 = \gamma_{t-1}^2)$$

$$U^*(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) = T_t + \beta E_t U(g_{t+1}, \gamma_t^1 = \gamma_{t-1}^1, \gamma_t^2 = \gamma_{t-1}^2)$$

(f) Compute residuals

$$r^V = V^*(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) - V(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) \tag{96}$$

$$r^U = U^*(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) - U(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) \tag{97}$$

(g) Solve the system of nonlinear equations given by (96) and (97) using a nonlinear equation solver such as Broyden's algorithm.

2. Once the equilibrium parameters for a guess of Δ and λ have been obtained, compute the allocations at $t = 0$ from the optimality conditions of the problem. Notice that the FOCs with respect to c_0 and l_0 are:

$$u_{c,0}(1 + \gamma_0^1) - \Delta(u_{cc,0}c_0 + u_{c,0} - u_{cl,0}(1 - l_0) - u_{cc,0}b_{-1}) = \lambda + \gamma_0^2$$

$$u_{l,0}(1 + \gamma_0^1) - \Delta(u_{cl,0}c_0 + u_{l,0} - u_{ll,0}(1 - l_0) - u_{cl,0}b_{-1}) = \lambda + \gamma_0^2$$

Notice that, as before, it is necessary to check whether either the participation constraint of the HC or of the RW are violated. In such a case, the allocations have to be recalculated to make the corresponding participation constraint bind.

3. Compute residuals

$$r^\Delta = E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t}c_t - u_{l,t}(1 - l_t)) - u_{c,0}b_{-1} \tag{98}$$

$$r^\lambda = E_0 \sum_{t=0}^{\infty} \beta^t T_t \tag{99}$$

4. Solve the system of nonlinear equations given by (98) and (99) using a fine grid search for Δ and λ.

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