

Banco Central de Chile
Documentos de Trabajo

Central Bank of Chile
Working Papers

N° 638

Agosto 2011

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WITH HETEROGENEOUS FIRMS AND
CORPORATE TAXATION**

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Documentos de Trabajo del Banco Central de Chile
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INVESTMENT DYNAMICS IN A DSGE MODEL WITH HETEROGENEOUS FIRMS AND CORPORATE TAXATION

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Abstract

In this paper I study a new business cycle fact recently documented by Bachmann and Bayer (2011): the dispersion of the distribution of investment rates across firms is procyclical. Using data from German firm, the authors find a correlation coefficient between the standard deviation of investment distribution and the cyclical component of output of 0.45. They also report a correlation coefficient for US economy of 0.33. Using a model similar to Khan and Thomas's (2003), that is standard to heterogeneous firms literature, I obtain a correlation coefficient of 0.57. In the model I also consider a government sector that collects taxes on corporate profits. In such model, with a corporate tax of 23.5%, which corresponds to German economy, I obtain a correlation coefficient of 0.46 and when I consider a corporate tax rate of 18.79% that corresponds to US economy I find a correlation coefficient of 0.51.

Resumen

En este trabajo estudio un nuevo hecho del ciclo económico recientemente documentado por Bachmann y Bayer (2011): la dispersión de la distribución de las tasas de inversión a través de las firmas es procíclica. Usando datos de firmas alemanas, los autores encuentran un coeficiente de correlación entre la desviación estándar de la distribución de inversión y el componente cíclico del producto de 0.45. Para los Estados Unidos, los autores reportan un coeficiente de correlación de 0.33. Usando un modelo similar al de Khan y Thomas (2003), que es estándar en la literatura de firmas heterogéneas, obtengo un coeficiente de correlación de 0.57. En el modelo también considero un gobierno que recauda impuestos sobre las ganancias corporativas. En tal modelo, con un impuesto corporativo de 23.5%, el cual corresponde a la economía alemana, obtengo un coeficiente de correlación de 0.46 y cuando considero un impuesto corporativo de 18.79% que corresponde a la economía de Estados Unidos, encuentro un coeficiente igual a 0.51.

This paper is my research project developed to obtain the Master Degree in Economics at the University of Chile. I would like to thank my advisor Alexandre Janiak and the other members of the committee, Nicolás Figueroa, Benjamin Villena and Sofia Bauducco for their suggestions and encouragement. Also, I would like to thank Luis Felipe Céspedes, Pierre-Olivier Gourinchas, Yan Carriere-Swallow, Carlos Medel and María Consuelo Edwards for their extremely helpful comments and suggestions. All remaining errors are mine. Usual disclaimers apply. E-mail: ssalgado@bcentral.cl.

1 Introduction

At firm level, investment is lumpy and infrequent. This has effects not only in the aggregate investment behavior (Bachmann, Caballero and Engel, 2010) but also on the distribution of investment across firms. In this paper I analyze the positive correlation between the dispersion of investment rates across firms and the business cycle recently documented by Bachmann and Bayer (2011) in a model where firms face fixed costs to capital adjustment and have to pay corporate taxes. Using a wide sample of German firms, Bachmann and Bayer found that the dispersion of the distribution of the investment rates across firms is procyclical. In other words, when the business cycle goes up, the dispersion of investment rates across firms, measured by its standard deviation, increases, and the opposite happens when the business cycle goes down. In particular, they documented that the correlation coefficient between the cross-sectional standard deviation of firms' investment rates and the cyclical component of aggregate output is 0.45. They also reported a correlation coefficient of 0.33 using a sample of United States' firms. Then, the authors analyzed whether a heterogeneous firms model can account for the empirical evidence. They found that their model economy implies a correlation of 0.58.

In this paper I discuss Bachmann and Bayer's (2011) results in the context of the model of Khan and Thomas (2003), which is standard to the literature of heterogeneous firms that face fixed adjustment costs. In this model, firms have to incur in idiosyncratic fixed costs to undertake capital investment. Because of these costs, in each period only a proportion of total firms will adjust their capital stock while others will let their capital depreciate. In this environment, heterogeneity arises endogenously and distributions of firms over capital and investment rates characterize the economy. Additionally, in this model there is a government sector that collects taxes over firms' corporate profits.

In this simple framework, procyclicality of investment rate dispersion emerges naturally. In order to gain some intuition, take the following example. Consider an economy that begins in a steady state where all firms are concentrated in the same level of capital (all the firms are identical). In this example (as in the model) capital is defined as the amount of services that the stock of capital provides. To make it simpler, assume that there is no depreciation of capital. In this stylized economy a positive productivity shock will induce firms to adjust their capital stock but, due to the adjustment costs, only those firms that observe that it is profitable

to change their capital will do that. As a consequence, the dispersion of firms over capital will increase and the same will happen with the distribution of firms over investment rates. Prior to the shock, firms' investment rate was equal to zero, but after that, some firms incur in positive investment while other do not. Therefore, the dispersion of investment rates across firms increases.

This intuition applies to both models, the one developed in Bachmann and Bayer's (2011) work and the model that I present in this paper. In order to make my results quantitatively comparable with Bachmann and Bayer's, I simulate my economy first considering a corporate tax rate equal to zero. I find a correlation coefficient between investment rate dispersion across firms and the cyclical component of output of 0.57. As it will become more clear in the following sections, I have obtained the same correlation found by Bachmann and Bayer (2011) in a simpler model.

The main finding of this paper is that the presence of a positive corporate tax rate dampens the correlation between the dispersion of investment rates distribution and the business cycle and helps to reduce the gap between empirical evidence and model results. A positive corporate tax rate reduces the procyclicality of investment rates dispersion across firms because it reduces the value of the firms compared with the adjustment cost, making them less willing to adjust, and also because it reduces the optimal capital to which firms adjust when they decide to do so. Consequently, when a productivity shock hits the economy, firms are less responsive than under a corporate tax rate equal to 0. In particular, I find that a model that considers a corporate tax rate of 18.79%, which is the actual corporate tax rate for the United States calculated by Djankov et al. (2008), implies a correlation coefficient of 0.51, and, if one considers a corporate tax rate of 23.5%, which corresponds to the economy of Germany, one obtains a correlation coefficient of 0.46 which is almost equal to data evidence found by Bachmann and Bayer (2011) for German firms.

The rest of the paper is organized as follows. In the following section review the literature related with the topic of this paper. The next four sections present the model, its solution and the numerical method used to simulate the model's dynamics. Section 7 shows the results and some of its implications. Section 8 concludes.

2 Literature Review

My work relates with existing literature in several ways. The effects of corporate tax rates on investment decisions have been thought coming via the user cost of capital and the price sensitivity of investment, which combines the impacts of interest rates, the tax burden, the depreciation rate and adjustment costs on capital accumulation. Chirinko et al. (1999) find a negative correlation between the user cost of capital and investment in a wide microeconomic sample.¹

Economists have debated these issues at length motivated by the idea that simple fully flexible models cannot account for investment fluctuations because they are too stylized. The first attempts to bridge that gap, and give more realistic assumptions to economic models, was the incorporation of a cost for adjusting the capital stock. According to these models, investment decisions are based on forward-looking considerations and rational expectations of future variables. Because investment implies a trade-off between current costs and future earnings, it seems clear that only those firms that anticipate good future economic conditions for their products will accumulate capital, while others will wait and let their capital depreciate, or may actively reduce their capital stock.² The literature on adjustment costs has focused mainly on convex adjustment costs in general and quadratic costs in particular. In these kinds of models, large increments in the capital stock generate bigger adjustment costs. Therefore, firms try to adjust their capital level to the optimum slowly. However, quadratic adjustment cost models were difficult to reconcile with microeconomic evidence, especially when economists did realize that most of the investment takes place in a single episode with long periods of inaction. Indeed, plant investment looked more lumpy, in the sense that it was accumulated in short bursts, than the investment patterns implied by quadratic-costs models. The empirical evidence on the lumpy behavior of investment comes mainly from Doms and Dunne (1998). Their work, based on a wide sample of American manufacturing plants, revealed that a big part of the investment that firms undertake in one year is made in a single event. However, these spikes of investment are difficult to reconcile with models where firms' investment patterns are smooth. Caballero et al. (1995) find that the lumpy behavior is better explained by a Ss -type model where investment is undertaken on noncontinuous patterns with large periods of inaction. However, the relevance of lumpy investment for DSGE models is still under debate.

¹See Hassett and Hubbard (1998) for an extensive literature review of these issues.

²Caballero (1999) reviews part of the discussion on this matter.

Thomas (2002) and Khan and Thomas (2003, 2008) find that adding fixed, non-convex, adjustment costs to an otherwise standard DSGE model does not improve its ability to replicate the main business cycle features of the data. Indeed, nonlinearities and lumpy investment disappear when firms are aggregated in a general equilibrium. The authors conclude that fixed adjustment costs are not relevant and quadratic-adjustment-costs models are able to replicate the main features of aggregate investment. Veracierto (2002), examining investment irreversibilities, finds similar results to Thomas's (2002). Nonetheless, Bachmann et al. (2010) find that the results of Khan and Thomas (2008) are not robust to changes of calibration. In a richer model, they find that lumpy investment does not disappear in general equilibrium and fixed adjustment costs are relevant to explain aggregate investment fluctuations. Other studies have documented the relevance of adjustment costs in related issues. For example, the work of Bachmann and Bayer (2011), that will be discussed in depth below in this section, argue that some empirical facts of investment distribution across firms can only be accounted for a model with fixed adjustment costs. The works of Bloom, Bond and Vaan Reenen (2007) and Bloom, Floetotto and Jaimovich (2010) discuss the role of uncertainty shocks and their effects of firms' investment behavior. In these models, firms face fixed adjustment costs that make them unwilling to invest when future conditions become uncertain.

I focus on the dynamic effects of corporate taxation on aggregate macroeconomics variables in a general equilibrium framework. A big part of the research in the field has focused on the impact of distortionary taxation in a single firm's decision problem in partial equilibrium. These models ignore firms' heterogeneity, which may be important for understanding the impact of different tax rates. However, besides this paper, other important advances have been made for studying these issues in general equilibrium. For instance, Gourio and Miao (2010a, 2010b) study the effects of capital and dividend taxation on investment and find that changes taxation could have had a negative effect on investment in the United States. They estimate that an unexpected and temporary reduction of the capital tax rate reduces the investment level by 11% in the short run while the steady-state level of macroeconomic aggregates is unchanged. The main differences of my paper with their study are that the heterogeneity in their model comes from different histories of idiosyncratic shocks that firms face and that they are not considering aggregate uncertainty. That simplifies a lot the computation of the model because the distribution of firms over state variables does not enter in the firm's maximization problem. In the model that I

present here, heterogeneity comes from differences in the adjustment costs that firms face to undertake capital changes. Since in each period only those firms that expect that adjusting to the optimal capital level will be profitable will do so, while others will let their capital depreciate. For this reason, the economy is characterized by a distribution of firms over capital and because aggregate capital and price level depend of this distribution, it has to be considered in the firms' decision problem. In another paper, Miao and Wang (2009) study the effects of corporate taxation on investment distribution across firms in a model where firms face convex and non-convex adjustment costs. They show that an anticipated decrease in the future corporate income tax rate raises investment and the adjustment rate immediately, while an anticipated increase in the future investment tax credit reduces investment and the adjustment rate initially. The main difference between their research and mine is that they analyze the effects of permanent and transitory changes on corporate taxation only in the long run steady-state of the economy, while I analyze both, the steady state effects of corporate tax changes and the impact of corporate taxation on aggregate investment dynamic over the business cycle.

To end this section I will explain the main findings of Bachmann and Bayer's (2011). These authors document a novel business cycle fact: the dispersion of the investment rate across firms is procyclical. Using a panel data set of 30,000 German firms, they study the business cycle proprieties of the cross-section distribution of firms over investment rates. Their main empirical results are the following. First, across 2-digit industries, there is a positive association between the cyclical component of the extensive margin of investment (the proportion of firms incurring in investment activities each period) and the cyclical component of the investment rate dispersion. This means that, when the economic cycle goes up, the number of firms that increase their capital stock moves in the same direction and, at the same time, the distribution of firms over investment becomes more disperse. Second, in the goods-producing sectors, where one could expect adjustment costs to be more important, the dispersion of the investment rate across firms is procyclical. Again this means that, when a shock hits the economy, some firms adjust their capital while others do not, and this effect makes the dispersion of the investment rates across firms higher. Third, the procyclicality of the dispersion of the investment rate declines with the size of the firms. Their most important result is the relation between the dispersion of the distribution of investment rates across firms and the business cycle. They find a correlation coefficient between the standard deviation of the investment rate

across firms and the cyclical component of aggregate output of 0.45.

Bachmann and Bayer also report this correlation for the US economy. Using a data set of American firms from Compustat, they find a correlation coefficient of 0.33. This is lower than the correlation found in the sample of German firms, however, it should be considered as a lower bound. Compustat covers only publicly traded and mostly large firms.³ Since large firms are less responsive to changes in economic environment, the correlation coefficient between the dispersion of the distribution of investment rates across firms and the cyclical component of output should be lower in this group of firms compared with a larger sample that considers smaller firms.

It is possible that the results observed by Bachmann and Bayer (2011) are guided by a selection bias. The sample that they use to obtain their main statistics considers entry and exit of firms and, therefore, the procyclicality of the dispersion of investment rates may be explained by the fact that, during a boom more firms of different size enter the productive sector, incurring in positive investment, and during a bust, less profitable firms disappear and the continuing firms do not invest. However, as Clementi and Palazzo (2010) have shown in a model with entry and exit of firms, the procyclicality of the dispersion of firms' investment rates is reinforced.

Based on their empirical results, Bachmann and Bayer (2011) turn to analyze if a DSGE model with firms that face fixed adjustment costs, like the model that I present here, is able to account for the empirical evidence. Their model economy is in the same vein of Khan and Thomas' (2003) model; however, they introduce additional features that make it much more complicated. In particular, additional to the fixed adjustment costs and the aggregate uncertainty, in their model firms face idiosyncratic shocks and a second-moment shock that is modelled as a countercyclical time-varying conditional standard deviation of aggregate shocks. Their baseline results are presented in table 1. The first and second columns show the correlation coefficient between the dispersion of the distribution of investment rates across firms found by Bachmann and Bayer for US and German firms. The third and fourth columns show the results of the model with and without second moment shocks, respectively. The key result is that, although the dispersion of the distribution of investment rates is procyclical, a model without second-moment shocks overshoots the correlation considerably. Only with these countercyclical second-moment shocks, the model estimates are closer to data evidence. Consequently, for

³Bachmann and Bayer indicate that firms present in Compustat are larger than the 5% of largest firms that are present in the sample that they use to obtain their results for German economy.

their results, the introduction of a countercyclical time-varying standard deviation of aggregate productivity shocks is crucial.

The intuition of the dispersion of the investment rates procyclicality deserves some explanation. In this class of models, where firms face fixed adjustment costs, aggregate investment combines an intensive margin (the difference between the current capital stock of a particular firm and its desired optimal capital) with an extensive margin (the number of firms that undertake capital adjustment). This distinction cannot be made in a one-shock standard real business cycle model without costs to adjustment because, in that case, all firms adjust their capital to the same target and the distribution of firms over investment rates collapses: all firms invest the same amount of resources each period. To fix ideas, consider a case where the intensive margin is irrelevant (for instance, assume that firms can only decide between adjusting a fix amount of capital or letting their stock depreciate). In this case, the distribution of firms over investment rates (and capital) is only determined by the changes in the proportion of firms undertaking investment operations. In this particular case, investment and its dispersion is increasing in the fraction of firms adjusting. Therefore, if investment is procyclical, the dispersion across firms of investment is also procyclical. Why do the results of Bachmann and Bayer (2011) imply a higher response of the dispersion of firms over investment rate to aggregate shocks than data evidence? Part of the explanation lies in the presence of aggregate and idiosyncratic productivity shocks. Consider a case where firms face a high realization of the aggregate productivity shock and only two firms decide to adjust. If those firms have a different capital stock but only face aggregate productivity shocks, both firms adjust to the same level of capital stock. The other firms will not adjust, and therefore, the dispersion of the distribution of firms' investment rates will increase. This effect is reinforced in the presence of idiosyncratic productivity shock because the same firms will not adjust their capital stock to the same level. Therefore, in this case the dispersion of the distribution of investment rates across firms will increase even more. Finally, the reason why second-moment shocks help Bachmann and Bayer (2011) to match empirical evidence is simple. In their case, second-moment shocks are modelled as a countercyclical process that reduces the dispersion of aggregate shocks at higher realizations and this dampens the correlation between the cyclical component of output and the dispersion of the distribution of investment rates across firms.

Table 1: Correlation of cyclical component of output and investment dispersion across firms

United States	Germany	With 2d Order Shocks	Without 2d Order Shocks
0.33	0.45	0.58	0.87

Source: Bachmann and Bayer (2011).

3 The Model

Here I describe the model that complements the work of Khan and Thomas (2003) adding a government that collects a tax on the firms' profits. For simplicity, I assume that the government transfers to households all the resources that it collects in a lump-sum manner. In this economy, there is a continuum of firms that face a time-varying fixed cost to undertake capital adjustments. In any period, if a firm wants to change its capital level, it has to pay a fixed cost that is independent of the size of the adjustment. Because each firm faces idiosyncratic adjustment costs, only those firms that anticipate that adjusting to the optimal capital stock will be profitable will do so, incurring in positive investment, while other firms will let their capital depreciate, and will have an investment equal to zero. As a result, there are distributions of firms over capital and over investment rates that characterize the economy.

3.1 Firms

In this economy, firms compete in a perfect market and share the same diminishing returns to scale production technology that uses capital and labor to produce a single good denoted by y . The production function is:

$$y = zk^\theta l^\nu, \quad (1)$$

where $(\theta + \nu) < 1$ and z is the total factor productivity shock. As usual, I assume that z follows a Markov Chain with J states, where $z \in \{z_1, z_2, \dots, z_J\}$, and

$$Pr(z' = z_j | z = z_i) \equiv \pi_{ij} \geq 0.$$

Here the $'$ represents the value of a variable one period ahead. In any period a firm is characterized by its level of capital, k , and the independent distributed idiosyncratic adjustment cost, denoted by $\xi \in [0, B]$, drawn from a distribution function $G(\xi)$ that does not change over time or across firms. Here B is the upper bound of the distribution of adjustment costs, $G(\xi)$. After production, the firm must

decide whether to adjust its capital to a certain level, k' , or let its capital depreciate at a rate of δ .

The aggregate state of this economy is defined as (z, μ) where μ is the distribution of firms over capital, k , in the support K . This distribution evolves over time according to the mapping Γ , which varies with the evolution of aggregate productivity, $\mu' = \Gamma(\mu)$. In the following section I will explain how this distribution evolves over time in the model. Following similar nomenclature as Khan and Thomas (2003), let $v^1(k, \xi; z, \mu)$ be the expected discounted value of a firm having the capital level of k , facing a fixed adjustment cost of ξ and an aggregate state of (z, μ) . With this, I can define the expected value of a firm prior the realization of the adjustment cost but after the determination of z as:

$$v^o(k; z, \mu) = \int_0^B v^1(k, \xi; z, \mu) G(d\xi). \quad (2)$$

In this economy, all firms pay a corporate tax for their profits at rate τ_c that does not change over time. Then, taken as given the evolution of the distribution of firms over capital, μ , the problem that the firm solves can be defined by the following functional form:

$$v^1(k, \xi; z, \mu) = \max_l \left[(1 - \tau_c)(zk^\theta l^\nu - w(z, \mu)l) + (1 - \delta)k + \delta\tau_c k \right] + \max \left[v_a^0(z, \mu); v_{na}^0(k; z, \mu) \right], \quad (3)$$

where $v_a^0(z, \mu)$ and $v_{na}^0(k, z, \mu)$ are the value of the firm in the next period if it adjusts its capital level and the value of the firms if does not adjust, respectively, and are defined as:

$$v_a^0(z, \mu) = -\xi w(z, \mu) + \max_{k'} \left(-\gamma k' + E_{z'|z} d_{z'}(z, \mu) v^0(k'; z', \mu') \right), \quad (4)$$

and

$$v_{na}^0(k; z, \mu) = -(1 - \delta)k + \left(E_{z'|z} d_{z'}(z, \mu) v^0\left(\frac{(1 - \delta)k}{\gamma}; z', \mu'\right) \right), \quad (5)$$

where $E_{z'|z}$ is the expected next period's value of the productivity shock given the information of the current period, $d_{z'}(z, \mu)$ is the stochastic discount factor applied by a firm expecting a shock z' when the current productivity is z and γ is the rate of exogenous technological progress.⁴ Just for notational convenience I have

⁴ Following King and Rebelo (1999) I assume that the efficiency units of labor grow at the exogenous rate of $\gamma^{1-\theta} - 1$ where θ is capital's share of aggregate output. This implies that growth trend of output is $\gamma - 1$. For this

considered that each firm sells its capital at the end of the period and re purchases it at the beginning of the next. Also, I have supposed that the adjustment cost, ξ , is denominated in hours of labor. As in Gourio and Miao (2010a, 2010b), I have assumed that firms do not pay corporate taxes for investment expenditure and there are depreciation allowances represented by the term $\tau\delta k$ in equation (3). The firm chooses next period's capital level and the amount of labor given the wage $w(z, \mu)$. Note that in equation (5), the next period's capital of a firm that does not adjust is equal to $k' = \frac{1-\delta}{\gamma}k$. That happens because, when a firm lets its capital depreciate, the capital in the next period is also less productive than the new capital of firms that have adjusted because aggregate labor productivity grows at the rate of γ (see King and Rebelo, 1999).

3.2 Households

In this economy there is a continuum of identical families that live infinitely, that are the owners of the firms and receive profits for the shares that they hold. In any period households have to choose their consumption level, how much labor to supply and how many shares they want to hold for the next period. Following Hansen (1985) and Rogerson (1988), I assume that individuals have to choose employment lotteries and have access to complete financial markets that allow them to fully diversify idiosyncratic risk. This implies that the economy behaves as if there were a representative household with an instantaneous utility function given by,

$$u(c, N) = \log(C) + A(1 - N). \quad (6)$$

Here C is the consumption level and A is the value of the marginal disutility of labor, N . Therefore, the households' utility maximization problem can be described as:

$$W(\lambda; z, \mu) = \max_{C, N, \lambda'} [\log(C) + A(1 - N) + \beta E_{z'|z} W(\lambda'; z', \mu')] , \quad (7)$$

subject to

$$C + \int_K \rho(k; z, \mu) \lambda'(dk) \leq w(z, \mu)N + \int_K v^0(k; z, \mu) \lambda(dk) + Tr, \quad (8)$$

reason, I require that next period's units of capital be measured relative to the efficiency units of labor available at that time. See King and Rebelo (1999) for more details.

where λ is the number of shares that the household holds and ρ is their price, Tr is the amount of transferences that households receive from the government and β is a discount factor. Households receive earnings from their labor, the profits of the firms and government transferences, and spend those resources in consumption and new shares represented by the second term in the left-hand side of the budget constraint.

Let $c(\lambda; z, \mu)$ be the allocation of current consumption that maximizes households' utility, $n^*(\lambda; z, \mu)$ their optimal labor supply and $\Lambda(k, \lambda; z, \mu)$ the number of shares that the households purchase for the next period.

3.3 Recursive Equilibrium

In this model economy a Recursive Equilibrium is a set of functions

$$(w, d_{z'|z}, \rho, v^1, l^*, k^*, W, c, n^*, \Lambda),$$

such that

1. v^1 satisfies equation (3) to equation (5) and (l^*, k^*) are the policy functions of the firms;
2. W satisfies equation (7) and equation (8) and (c, n^*, Λ) are the associated policy functions of the households;
3. Shares market clears,

$$\Lambda(k', \lambda; z, \mu) = \mu'(k') = \int_{(k, \xi) | k' = k^*(k, \xi; z, \mu)} G(\xi) \mu(dk);$$

4. Labor market clears,

$$n^*(\lambda; z, \mu) = \int_K (l^*(k; z, \mu) + \int_0^B \xi \Phi \left(\frac{1 - \delta}{\gamma} k - k^*(\xi; z, \mu) G(d\xi) \right) \mu(dk),$$

where $\Phi(x) = 0$ if $x = 0$ and $\Phi = 1$ if $x \neq 0$;

5. Goods market clears,

$$c(\lambda; z, \mu) = \int_K \int_0^B (zk^\theta (l^*(k; z, \mu))^\nu + (1 - \delta)k - \gamma k^*(k, \xi; z, \mu)) G(d\xi) \mu(dk);$$

6. Government budget is in equilibrium,

$$Tr = \int_K \tau_c (zk^\theta (l^*(k; z, \mu))^\nu - w(z, \mu) l^*(k; z, \mu)) \mu(dk).$$

4 Model Solution

Here I explain how the model is solved. From the households' utility maximization problem one can obtain the following optimality conditions (see the optimization appendix for details):

$$w(z, \mu) = \frac{A}{1/C}, \quad (9)$$

$$d_{z'}(z, \mu) = \beta. \quad (10)$$

Using these conditions one can compute a single Bellman equation for solving the problem of households and firms at the same time. For simplicity I will assume, like Khan and Thomas (2003), that the price that firms use to value their output is equal to households' marginal utility of consumption, $p(z, \mu) = 1/C$. Therefore, equation (9) can be expressed as $w(z, \mu) = A/p(z, \mu)$, and I can rewrite equation (3) as:

$$V^1(k, \xi; z, \mu) = \max_l \left[((1 - \tau_c)(zk^\theta l^\nu - w(z, \mu)l) + (1 - \delta)k + \delta\tau_c k)p(z, \mu) \right] \quad (11)$$

$$+ \max \{V_a^0(z, \mu); V_{na}^0(k; z, \mu)\},$$

where

$$V_a^0(z, \mu) = -\xi A + \max_{k'} (-\gamma k' p(z, \mu) + E_{z'|z} \beta V^0(k'; z', \mu')), \quad (12)$$

$$V_{na}^0(k; z, \mu) = -(1 - \delta)k p(z, \mu) + \left(E_{z'|z} \beta V^0\left(\frac{(1 - \delta)}{\gamma} k; z', \mu'\right) \right), \quad (13)$$

and

$$V^0(k; z, \mu) = \int_0^B V^1(k, \xi; z, \mu) G(d\xi).$$

Expressions (12) and (13) will be the base of the solution of the model.

Because the problem incorporates a discrete decision (each firm must choose whether to adjust or not to adjust its capital) there is a non-linearity that impedes to obtain a closed-form solution of the model. Therefore I proceed numerically to obtain my results. As is usual in this literature, I will employ non-linear techniques that build on Krusell and Smith (1997, 1998). In order to solve its optimization problem each firm has to know, first, the price level of the economy and, second, the

distribution of firms over capital. But, since the distribution of firms over capital is continuous, it cannot be used as a state variable in the firms' optimization problem. Therefore, I will assume that, when a firm makes its decision it does not observe the complete distribution of firms over capital, but only the aggregate capital stock of the economy. Specifically, a firm only knows the aggregate capital level, denominated by K , and infers both next period's aggregate capital level, K' , and the current price level, P , using the following simple OLS rules for each aggregate productivity state $j \in J$:

$$\log(K') = \beta_{0K,j} + \beta_{1K,j} \log(K), \quad (14)$$

$$\log(P) = \beta_{0P,j} + \beta_{1P,j} \log(K). \quad (15)$$

Therefore, after a firm has observed the productivity shock and current aggregate capital level, it can infer both the current aggregate price and the next period's aggregate capital. With these variables, the firm can make its decision regarding labor and next period's capital. The optimal labor decision, $l^*(k; z, \mu)$, comes from the first-order condition with respect to labor in (11),

$$\frac{A}{P} = z\nu k^\theta l^{\nu-1}. \quad (16)$$

The optimal decision for next period's capital level comes from solving equation (12). Suppose that k^* is the capital level that maximizes equation (12) given current aggregate capital and productivity shock. Then, a firm will adjust its capital level to k^* only if the expected value of the firm minus the fixed adjustment cost is higher than the value of the firm if it allows its capital depreciate. Let $\widehat{\xi}(z, \mu)$ be the value of the adjustment cost that makes the firm indifferent between adjusting and not adjusting its capital level. Then, making equal (12) to (13) one gets,

$$\begin{aligned} & \left[-\widehat{\xi}(z, K)A + \max_{k'} (-\gamma k' p(z, K) + E_{z'|z} \beta V^0(k'; z', K')) \right] - \\ & \left[-(1-\delta)kp(z, K) + \left(E_{z'|z} \beta V^0\left(\frac{(1-\delta)}{\gamma}k; z', K'\right) \right) \right] = 0. \end{aligned} \quad (17)$$

Note that I have replace μ , for the aggregate capital K . Next, I define

$$\bar{\xi}(z, K) = \min\{B, \max\{\widehat{\xi}(z, K), 0\}\}, \quad (18)$$

so that $0 \leq \bar{\xi}(z, K) \leq B$. Consequently, only those firms with idiosyncratic adjust-

ment cost below $\bar{\xi}(z, K)$ will adjust their capital level to k^* .

Given the firms' next period capital level decision, I can define the evolution of the distribution of firms over capital. For those firms that decide to adjust their capital:

$$\mu'(k) = \int_K G(\bar{\xi}(k; z, K)) \mu(dk) + \left[1 - G(\bar{\xi}(\frac{1-\gamma}{\delta}k; z, K)) \right] \mu(\frac{\delta}{1-\gamma}k), \quad (19)$$

while for those that do not adjust their capital,

$$\mu'(k) = \left[1 - G(\bar{\xi}(\frac{1-\gamma}{\delta}k; z, K)) \right] \mu(\frac{\delta}{1-\gamma}k). \quad (20)$$

The first term on the right-hand side of equation (19) corresponds to those firms that have drawn an idiosyncratic adjustment cost below $\bar{\xi}(z, K)$ and adjust to $k' = k^*$, while the second term corresponds to those firms that have drawn a higher adjustment cost, have let their capital depreciate and, as a consequence, have reached a level of $k' = k^*$ that is just equal to the optimal level. The term in the right-hand side of equation (20) corresponds to non-adjusters that have a k' equal to $((1-\delta)/\gamma)k$.

Finally, I can define the amount of resources that the government collects in each period as:

$$Tr = \int_K \tau_c(zk^\theta l^\nu - w(z, K)l) \mu(dk). \quad (21)$$

Then I can obtain the levels of output, consumption, investment and total labor using the following expressions:

$$Y = \int_K zk^\theta l^\nu \mu(dk), \quad (22)$$

$$I = \int_K [\gamma k^*(z, K) - (1-\delta)k] G(\bar{\xi}(k; z, K)) \mu(dk), \quad (23)$$

$$C = Y - I, \quad (24)$$

and

$$N = \int_K \left[l^*(k; z, K) + \int_0^{\bar{\xi}} \xi G(d\xi) \right] \mu(dk), \quad (25)$$

where the second term comes from the fact that in this model, adjustment costs are denominated in units of labor and only those firms that obtain an adjustment cost below $\bar{\xi}(z, K)$ change their capital stock.

5 Numerical Method

In this section I explain in detail the numerical procedure used to solve the model. Since the firm's problem does not have an analytical solution I have to find it using an optimization algorithm and make a discretization over certain variables. In particular, I use value function iteration to solve the problem of equation (11) over a multidimensional grid of points where the state variables are the idiosyncratic capital level, the aggregate level of capital and the productivity shock. The specific steps are the following:

1. First, guess a set of parameters for β_P and β_K , define a grid of points for z , k and K and use equations (14) and (15) to solve equation (11) for each point on the grid.
2. Second, simulate the economy for T periods saving the distribution of firms over capital in each of them. In order to solve the model in each period, first I generate an initial distribution of firms and calculate the aggregate capital, which in turn is used for calculating next period's aggregate capital level using equation (14). That defines both the value of the firm if it adjusts its capital as well as the value of the firm if it does not adjust, for any current aggregate price level. Nevertheless, equation (15) is never used for solving the price level of the economy, instead the equilibrium price will be calculated iteratively. For doing this, first I guess a price and then verify if that price is equal to the price implied by the optimality conditions of the household's maximization problem, such that the guess is equal to the marginal utility of consumption. Let p^e be the guess of the price level of the economy. With that price one can solve for the labor demand of each firm, l^* , next period's optimal capital level, k^* , and the decision rule, $\bar{\xi}(z, K)$, given the aggregate productivity shock z . Then, use equations (22), (23) and (24) to obtain a new price level, $p^v = 1/C$. If p^v is near p^e , save that price level and continue with the simulation, but, if they are sufficiently different, given some convergence criteria, update p^e using

$$p^e = \chi p^e + (1 - \chi)p^v,$$

where $0 < \chi < 1$.

3. Third, use equations (19) and (20) to update the distribution of firms over capital. Following these steps one can obtain an endogenous path for the aggregate capital and aggregate price level that can be used in the next step.
4. Finally, update firms' forecasting rules using the path of the aggregate variables. In particular, recalculate the values of β_P , β_K using OLS and evaluate if the new parameters are equal to the initial guess. If that is the case, the convergence is complete, so use the simulated aggregate variables to calculate the results. If it is not, update the values of the parameters and return to step 1. In order to ensure that the distribution of firms comes from the ergodic set, in each iteration update the initial distribution of firms over capital using the last distribution of firms from the previous iteration.

This procedure, although imposing a high computational burden, generates accurate predictions for the aggregate variables. Indeed, the R^2 of the OLS regressions are very high (up to 99%) when convergence has been reached (see the numerical appendix for details).

6 Parametrization

The model solution requires the selection of several parameters. Here I use similar values to Khan and Thomas (2003) and take the corporate tax rate from Djankov et al. (2010). The authors report a corporate tax rate for the United States economy's, τ_c , of 18.19%, the value that I use in my simulation. I fix the length of the period to one year, which allows me to use establishment-level investment data provided by Doms and Dunne (1998) in the parametrization of the adjustment cost distribution. The value of A , the marginal disutility of labor in the utility function, is chosen to match a 20% of time dedicated to labor each year. I fix the mean growth rate of technological progress, γ , at 1.6%, pick a value of β to imply an average interest rate of 6.5% (King and Rebelo, 1999) and a rate of capital depreciation that implies an average investment-to-capital ratio of 7.6%. For θ and ν I use values of 0.325 and 0.58, respectively. I set the parameters for the shock process according to Khan and Thomas (2003). In particular, I assume that the shock is a first-order autoregressive process with a persistence parameter of 0.9225 and a variance of innovations of 0.0134.⁵ Finally, I have to choose parameters for the adjustment

⁵See the Numerical Appendix for details.

cost distribution. I assume that the adjustment costs are uniform with cumulative distribution $G(\xi) = \xi/B$, where B is the upper bound of the distribution. That bound is selected to match three results of Doms and Dunne’s (1998) research: 1) in one year, lumpy investors (those that increase their capital stock by more than 30% of their previous capital level) account for 25% percent of total investment in the economy, 2) these lumpy investors are a very small proportion of total firms, roughly 8%, and 3) the other 75% of the investment is carried out by firms with an investment level below 10% of their previous capital level. A value of $B = 0.002$ almost matches these observations. Table 2 shows the parameters used to simulate the model.

Table 2: Parameters									
γ	β	δ	θ	ν	A	ρ	σ_ε	B	τ_c
1.016	0.954	0.06	0.325	0.58	3.614	0.9225	0.0134	0.002	18.19%

7 Results

7.1 Cyclical Properties of the Model Economy

In this section I begin the exposition of the results of the model. Table 3 shows the volatility, the contemporaneous correlations with output and the first-order autocorrelation of the key aggregate variables of my model economy. These results are obtained using the algorithm of the previous section and the parameters of table 2. All aggregate series are in logs and have been HP-filtered using a smoothing parameter of 100. In table 3, columns labelled Data correspond to empirical evidence of the cyclical component of the United States macroeconomic aggregates calculated by King and Rebelo (1999), while those labelled Model show my model’s results. In the face of aggregate productivity shocks, my model economy exhibits an output volatility of 1.84, which is almost equal to data evidence. The others main macroeconomic aggregates show similar business cycle dispersion to standard representative agents models (see Khan and Thomas (2003) for a discussion).

In the light of this evidence, one can ask what is the contribution of corporate tax and the firms’ heterogeneity generated by fixed adjustment costs. To gauge that contribution I compare the results of three additional exercises. In the first case I set the value of $\tau_c = 0$ and eliminate the heterogeneity setting the value of B , the upper bound of the distribution of idiosyncratic adjustment costs, near

Table 3: Business cycle estimates of the model with $\tau_c = 0$

	Standard Deviation		Stand. Deviation Rel. to Output		Contemporaneous Correlation		First Order AutoCorrelations	
	Data	Model	Data	Model	Data	Model	Data	Model
Y	1.81	1.84	1.00	1.00	1.00	1.00	0.84	0.94
C	1.35	0.92	0.75	0.50	0.88	0.94	0.80	0.98
I	5.30	6.07	2.93	3.30	0.80	0.97	0.87	0.82
N	1.79	1.03	0.99	0.56	0.88	0.95	0.88	0.76
w	1.68	0.92	0.93	0.50	0.12	0.94	0.66	0.98
r	0.30	0.89	0.17	0.48	- 0.35	0.66	0.60	0.20

zero.⁶ In that case, firms adjust their capital each period and the distribution of firms over capital collapses to a unique level given the state of the productivity shock. This happens because firms' investment decisions are based on the value of $\bar{\xi}(z, \mu)$, the adjustment cost level that makes firms indifferent between adjusting their capital stock or letting it depreciate. Since $\bar{\xi}(z, \mu)$ is always between 0 and B (see equation (18)), the probability of an adjustment cost being higher than $\bar{\xi}(z, \mu)$ is 0 if B is near 0 and, therefore, all firms will find that it is profitable to adjust each period. Additionally, since the only source of heterogeneity between firms are the idiosyncratic adjustment costs, all firms adjust to the same level of capital and the distribution collapses. The second and third exercises considers two cases; first, I keep B near 0 and set the value of $\tau_c = 18.79\%$ and, second, I set the corporate tax rate equal to 0 again and fix the value of B as in table 2. The comparison between all these numerical exercises will give a measure of the marginal contribution of each element to my business cycle estimates. The results are shown in table 4.⁷

Table 4: Business cycle estimates of the model with different levels τ_c y B

	$\tau_c = 0$ B = 0				$\tau_c = 0$ B = 0.002				$\tau_c = 18.79$ B = 0			
	Std	Rel	Contem	1st	Std	Rel	Contem	1st	Std	Rel	Contem	1st
Y	1.88	1.00	1.00	0.93	1.89	1.00	1.00	0.93	1.84	1.00	1.00	0.94
C	1.18	0.63	0.73	0.74	0.89	0.47	0.91	0.98	1.08	0.59	0.72	0.96
I	6.79	3.61	0.87	0.96	6.42	3.40	0.97	0.81	6.95	3.78	0.89	0.77
N	1.09	0.58	0.95	0.75	1.13	0.60	0.95	0.76	1.03	0.56	0.94	0.77
w	1.18	0.63	0.73	0.96	0.89	0.47	0.91	0.98	1.08	0.59	0.72	0.96
r	1.31	0.70	0.49	0.07	0.83	0.44	0.68	0.25	1.28	0.70	0.47	- 0.12

Note that in all cases presented in table 4, the standard deviation of output and investment are higher than my baseline estimates. When the model does not consider any friction (as in the case where τ_c and B are equal to zero), the volatility of output is 1.88 while the volatility of investment is 6.79. That happens because

⁶Note that B cannot be 0 because I have supposed that the distribution function of idiosyncratic adjustment costs is uniform.

⁷In table 4 column *Std* corresponds to the standard deviation of each variable, *Rel* is the standard deviation of each variable relative to output standard deviation, *Contem* is the contemporaneous correlation with output and *1st* is the first output autocorrelation of each variable.

without the friction generated by the adjustment costs, firms change their capital stock each period and, therefore, the volatility of aggregate investment increases. Consequently, output volatility and consumption volatility also increase. The introduction of adjustment costs has the predicted effect: since firms are less willing to adjust as a response to a shock due to the fixed cost that they have to pay, the volatility of investment declines. Besides, output and consumption volatility also have to decline because the aggregate capital is less volatile. In the case where only the corporate tax rate is considered (last section of table 4), the model estimates are quite in line with empirical evidence.

However, because the focus of this work is to analyze the role of adjustment costs and heterogeneity, I consider the case with adjustment costs and a positive corporate tax rate as my basic benchmark. Consequently, in the following sections I will take the results presented in table 3 as my baseline estimation. Although the consumption volatility is lower compared with the other estimates of my model under different parametrizations, it represents fairly well the cyclical behavior of investment and output, which is the main focus of this paper. Moreover, as I will explain in the following section in more detail, if there are no idiosyncratic adjustment costs, firms will concentrate in the same capital stock and will adjust their capital each period to the same optimal level. As a consequence, the distribution of firms over capital will collapse to that optimal capital level and the statistics related with the dispersion of the distribution of firms over investment rates cannot be calculated.

7.2 Investment Dispersion Across the Cycle

7.2.1 Simple Stylized Example

Now I evaluate the correlation of investment dispersion across firms and output cyclicity using the model of section 2. First, note that, compared with the model of Bachmann and Bayer (2011), the model presented here lacks idiosyncratic uncertainty and second-moment shocks. Second, the intuition explained in the previous section applies also to the simpler model like the one that I have presented in this paper. In order to make this idea more clear, consider a very stylized case where firms are distributed only over two different capital levels, k_0 and k_1 , like the distribution depicted in figure 1. In this example I use a depreciation rate equal to zero. Now, consider that a shock hits the economy and the optimal capital is given by k_2 . Finally, consider that 50% of the firms draw an idiosyncratic adjustment cost,

ξ , below $\bar{\xi}(k, \mu)$, so that, 50% of firms adjust their capital. The next period's distribution of firms over capital will look like the one depicted in figure 2. Clearly, the dispersion of firms over capital increases and the same happens with the distribution of firms over investment. Figure 3 depicts the distribution of investment after the shock in this simplified example. The cross sectional dispersion of the investment has increased after the shock. Note also that this result holds only if there are capital adjustment costs. Without them, all firms would adjust to the optimal capital k_2 instantly and the capital dispersion across firms would be equal to zero after the initial shock. The dispersion increases only for one period and then it is zero independent of the new shocks that may hit the economy. Something similar happens with the distribution of investment rates. Its dispersion increases only in the period when the first shock hits the economy. After that, because all the firms have the same capital stock, all invest the same amount of resources that only depends on the magnitude of the new shocks that impact the economy. In that case, all firms move together and the dispersion is 0. Moreover, if the capital adjustment cost is the same for all firms, the dispersion of investment across firms is again equal to 0. In that case, after the initial shock, all firms would be in one of two states: either all firms adjust their capital to the optimal because the profits obtained using the optimal capital level offset the losses incurred in the adjustment process, or non firm adjusts its capital stock. In both states the dispersion of investment across firms is 0 after the initial shock because all firms invest the same positive amount in the first case, or 0 in the second case. Note that in a model with convex adjustment costs one can obtain similar results to those of a non-convex adjustment costs. In this paper I am not considering such kind of adjustment costs for the following reasons. First, the distributional effects would be weaker. In these models, firms are unwilling to incur in large adjustments because that implies large adjustment costs. Therefore, after a shock firms only adjust partially to the new target. After the shock, all firms would change their capital stock to a new level that is between their initial level and their target and, the distribution of firms over capital level would have the same dispersion of the initial capital distribution. In terms of the distribution of firms over investment rates, since all firms adjust their capital, it is clear that the distribution would look less disperse than the one presented in figure 3. Second, convex adjustment costs are not consistent with the microeconomic evidence of lumpy investment discussed before, which is the main focus of this paper.

7.2.2 Steady-State Results

One can use the model of the previous section for studying the last intuition more formally. Consider as an example a economy that has faced an average aggregate productivity shock for several periods, so that the economy has converged to a steady state. Here I have fixed the value of τ_c equal to 0. In this economy, a steady state implies that the aggregate variables are constant over time although at firm level there are still capital adjustments. In other words, the aggregate levels of capital, labor and output are constant, but, because the capital depreciates and average productivity increases, firms observe that their capital is farther from the optimal capital the longer the time that has passed from the last adjustment. The economy does not collapse to a distribution of firms of the same size because adjustment costs prevent that. Therefore, in a steady state, aggregate capital is constant, but at the firm level there are still capital changes. That can be appreciated in figure 5. The horizontal axis represents different capital levels. The green line is the adjustment hazard rate that shows, in the right-hand vertical axis, the probability that a firm has to adjust its capital level to the optimal, while the blue line shows in the left-hand axis the distribution of firms over capital (the proportion of total mass of firms that have a particular capital level). The part of the distribution of firms over capital that is under the green line is the proportion of firms that adjust their capital level each period. Note that the adjustment hazard rate is centered on the optimal capital level given the aggregate productivity. That happens because those firms that have a capital level that is equal to the optimal capital given the aggregate productivity shock, have an adjustment probability equal to zero. In this example, the optimal capital level and the proportion of firms that undertake investment each period are shown in the first row of fourth and fifth columns of table 5.

Now, consider that a positive shock of productivity hits the economy. The second panel of figure 5 shows the distribution of firms over capital in the same period that the shocks arrives. First note that the target capital increases from 1.12 to 1.18. That represents an increase of 6% as the last row of table 5 shows. Now, note from the second panel of figure 5 that after the shock, a higher proportion of firms are under the adjustment hazard rate line and more firms are willing to adjust. In particular, the proportion of adjusters increases 9%. More relevant, the proportion of lumpy adjusters increases dramatically by almost 180% (last column of table 5). Those firms change their capital for the next period after the shock, when

aggregate productivity is on its average again and the target capital has returned to its original value of 1.12. That is shown in the last panel of figure 5, which depicts a more disperse distribution of firms over capital due to the increase in the extensive margin (the proportion of firms adjusting their capital) and the intensive margin (the temporal increase in the target capital). The same can be appreciated in figure 6 that shows the distribution of firms over investment before and after the shock. Clearly, the distribution is more disperse after the shock, as shown in the second panel.

Table 5: Distributional effects of a positive shock

Time	Standard Deviation Capital Distribution	Standard Deviation Investment Distribution	Target Capital	% Adjusters	% Lumpy Adjusters
t	0.1736	0.0097	1.1209	0.3807	0.1098
t+1	0.1872	0.0119	1.1835	0.4135	0.3065
% change	8%	23%	6%	9%	179%

Now consider that this economy is in steady state again and receives a negative shock of similar magnitude to the positive shock discussed before. The results are shown in table 6 and in figures 7 and 8. Note that in this case there are still some firms that are willing to adjust (there is a proportion of firms that are under the adjustment hazard rate after the negative shock) but both the optimal capital and the proportion of adjusting firms decrease in comparison with the situation prior to the shock. Note also that the proportion of lumpy adjusters is almost 0 in this case. Moreover, the reduction is, in absolute terms, stronger than the increase generated by the positive shock. The consequence is that the distribution of firms over capital now is thicker and the dispersion decreases, as shown in the first column of table 6 in the second panel of figure 8.

Table 6: Distributional effects of a negative shock

Time	Standard Deviation Capital Distribution	Standard Deviation Investment Distribution	Target Capital	% Adjusters	% Lumpy Adjusters
t	0.1736	0.0097	1.1209	0.3807	0.1098
t+1	0.1540	0.0078	1.0646	0.2991	0.0225
% change	-11%	-20%	-5%	-21%	-80%

7.2.3 Dynamic Results

Keeping the last idea in mind, now I turn to the quantitative evaluation of the correlation between investment dispersion and the cyclical component of output. In order to make my results directly comparable to those of Bachmann and Bayer (2011), I set the corporate tax rate equal to zero. The central section of table

4 shows the volatility, correlation with output and first-order autocorrelation of the key aggregate variables. First note that, compared with the model with a positive corporate tax rate, the economy is more volatile in terms of the cyclical component standard deviation of output and investment. However, my model yields similar results to other models in the literature (see Khan and Thomas, 2003). Table 7 shows the correlation between the dispersion of investment distribution and the cyclical component of output for US and Germany. I also show three additional statistics that are relevant for Bachmann and Bayer's (2011) results. In the table, $\rho_{\sigma,y}$ is the correlation coefficient between the investment dispersion and the cyclical component of aggregate output for US and German economies, and $\rho_{g,y}$ is the correlation of value added growth dispersion across firms and the cyclical component of output for the German economy.⁸ Comparing the second column with the third one gets the main point of this section: in order to obtain a positive correlation between investment distribution dispersion and the cyclical component of output, the key element is the presence idiosyncratic fixed adjustment costs. Using a standard fixed adjustment cost model, I can obtain the same results of a richer model like the one of Bachmann and Bayer (2011): my model implies a higher correlation coefficient than the results obtained for US and Germany economies. Note that, in terms of the correlation between the proportion of adjusters and the cyclical component of output (the second row in table 7), I obtain similar correlations as Bachmann and Bayer (2011); however, my estimate related with the proportion of lumpy adjusters is smaller. As I will show in the following section, this result changes if I consider a positive corporate tax rate. An additional divergence between my results and Bachmann and Bayer's (2011) estimates is related with value added growth dispersion of firms. In their results as well in the data, the correlation coefficient between this variable and the cyclical component of output is negative. However, my estimate is near 0.⁹

Table 7: Bachmann and Bayer's (2011) and model's results with $\tau_c = 0$

	United States	Germany	Bachmann and Bayer (2011)	This paper with $\tau_c = 0$
$\rho_{y,\sigma}$	0.33	0.45	0.58	0.57
$\rho_{y,g}$	-	-0.45	-0.42	-0.06
% Adjusters	-	0.73	0.42	0.46
% Lumpy Adjusters	-	0.61	0.38	0.02

Source: Bachmann and Bayer (2011) and author's estimates.

⁸I do not have statistics of this correlation or the other estimates for US economy.

⁹Here I have used the same definition of adjusters and lumpy adjusters as Bachmann and Bayer (2011). A lumpy investor is such firm with an investment rate of $|\frac{\dot{i}_{i,y}}{0.5(k_{i,t} + k_{i,t+1})}| > 0.2$. This convention was first used by Cooper and Haltiwanger (2006).

7.3 Corporate Tax Rate and Investment Dispersion

Can a model that considers a government that collects taxes on corporate profits make better predictions about the procyclicality of the standard deviation of the distribution of investment? My results suggest that it really can. In order to gain some intuition, first, consider the effect that an increase in the corporate tax rate has on the firms' investment decisions. On one hand, there are effects on the intensive margin of capital adjustment. A higher corporate tax rate decreases the optimal level of capital that maximizes equation (12). This happens because a higher corporate tax rate reduces the value of the firm in all periods and, as a consequence, there is a reduction of the earnings that a firm obtains per unit of investment. To clarify this, consider the first-order condition with respect to capital of equation (12), which is given by

$$\frac{\partial V_a^0}{\partial k'} = -\gamma p(z, \mu) + \frac{\partial E_{z'|z} \beta V^0(k'; z', \mu')}{\partial k'} = 0. \quad (26)$$

Abstracting from general equilibrium effects in marginal changes of the capital stock of a particular firm, clearly if $V^0(k'; z', \mu')$ decreases, k' has to decrease also in order to keep the first-order condition, since $V^0(k'; z', \mu')$ is increasing in k .

On the other hand, a higher corporate tax rate makes firms less willing to adjust in the presence of a shock, positive or negative. This is the extensive margin. This happens because the reduction in the value of the firm implies that the profits are lower compared with the adjustment cost, and consequently, the adjustment cost that makes firms indifferent between adjusting or not adjusting decreases. That can be appreciated in equation (17). There, the value of V^0 decreases independently of the adjustment decision of the firms in a proportion that should be near to τ_k . Therefore, the proportion of firms that draw an adjustment cost higher than $\bar{\xi}(z, \mu)$ increases and with this, the fraction of adjusters decreases. Now, the joint effect of the intensive and the extensive margins over the procyclicality of the dispersion of investment distribution across firms comes from the following. Consider that, after a positive shock, firms adjust to an optimal capital that is lower than the optimal capital to which firms would adjust in the case of a corporate tax rate equal to zero, simply because the next period's value of the firms has decreased. Therefore, firms do not adjust to k_2 in figure 2 but to k_3 in figure 4. Moreover, for the reasons explained above, the fraction of firms that adjust decreases. Both

effects are depicted in figure 4 where I have considered that only 40% of firms have adjusted their capital stock. Clearly, compared with figure 2, the distribution of firms over capital depicted in figure 4 shows lower dispersion and the same happens with the distribution of investment across firms. Consequently, the procyclicality of investment dispersion is lower the higher is the corporate tax rate.

7.3.1 Steady-State Results

I can perform a similar analysis to the one presented in previous section. Beginning from a steady-state situation, but now assuming a positive corporate tax rate equal to 18.79%, I can evaluate the distributional effects of a positive and a negative productivity shock. The results are shown in tables 8 and 9 and figures 9 to 12. As expected, the distributional effects of a positive or negative shock are weaker than the case with a corporate tax rate equal to 0 (see table 5 and 6). Note that in this case, the dispersion of the distribution of firms over investment rates respond less to the positive shock. This happens even if the percentage change in the target capital and the proportion of investors is equal in both cases (compare the third and fourth columns of tables 5 and 8). The main difference is in the proportion of lumpy adjusters: while, with a corporate tax rate of zero, the number of firms that invest a huge amount of resources grows dramatically (179%), in this case, with a corporate tax rate of 18.79%, the increase, though large, is smaller (112%). Consequently, all the reduction in the response of investment dispersion across firms to a positive productivity shock can be associated with the smaller proportion of firms that undertake large adjustments after the shock. Something similar happens when the economy faces a negative productivity shock: the proportion of lumpy adjusters decreases less in this case than in the case with a corporate tax rate equal to 0.

Table 8: Distributional effects of a positive shock and $\tau_c = 18.79\%$					
Time	Standard Deviation Capital Distribution	Standard Deviation Investment Distribution	Target Capital	% Adjusters	% Lumpy Adjusters
t	0.1411	0.0074	1.0771	0.3343	0.1792
t+1	0.1523	0.0080	1.1397	0.3660	0.3798
% change	8%	8%	6%	9%	112%

7.3.2 Dynamic Results in the Presence of Corporate Taxes

Now I turn to the effects that the presence of corporate taxation has on business cycle proprieties of the model. The results are shown in table 10. The first three

Table 9: Distributional effects of a negative shock and $\tau_c = 18.79\%$					
Time	Standard Deviation Capital Distribution	Standard Deviation Investment Distribution	Target Capital	% Adjusters	% Lumpy Adjusters
t	0.1411	0.0074	1.0771	0.3343	0.1792
t+1	0.1321	0.0061	1.0208	0.2628	0.0663
% change	-6%	-18%	-5%	-21%	-63%

columns show the results of Bachmann and Bayer (2011) already presented, and the last four show the results for different values of τ_c . Note how the correlation coefficient between the dispersion of the distribution of investment rates across firms and the cyclical component of output declines the higher is the corporate tax rate, in line with the intuition presented before (first row of table 10). When τ_c is equal to 18.79%, which is the effective corporate tax rate calculated by Djankov et al. (2008) for the United States' economy, the correlation is 0.51, which is 12% lower than the original estimates with a corporate tax equal to 0. Now, when I consider a corporate tax of 23.5%, which corresponds to the effective corporate tax rate of the German economy, the gap between model's estimates and data evidence is almost zero. Note also that, in terms of the correlation between the fraction of adjusters and lumpy adjusters, my estimates are very close to Bachmann and Bayer's (2011) results. This can also be appreciated in the table 11 where I show the correlation implied by the model relative to the data. Note that, for the correlation coefficient, the difference of Bachmann and Bayer's (2011) results with data is 30% but is less than 2% in the last column.

Table 10: Correlation of dispersion of investment and the cyclical component of output

	Bachmann and Bayer (2011)			This Paper		
	United States	Germany	Model	$\tau_c = 0$	18.79%	23.5%
$\rho_{y,\sigma}$	0.33	0.45	0.58	0.57	0.51	0.46
% Adjusters	-	0.73	0.45	0.46	0.39	0.42
% Lumpy Adjusters	-	0.61	0.38	0.02	0.34	0.43

Source: Bachmann and Bayer (2011) and author's estimates

Table 11: Model correlations relative to data evidence for Germany				
	Bachmann and Bayer (2011)		This Paper	
	Model	$\tau_c = 0$	18.79%	23.5%
$\rho_{y,\sigma}$	1.29	1.28	1.13	1.02
% Adjusters	0.62	0.63	0.54	0.58
% Lumpy Adjusters	0.62	0.03	0.55	0.69

Source: Bachmann and Bayer (2011) and author's estimates

My results suggest that a model that considers corporate taxation is able to close the gap considerably. The discrepancies between model results and empirical

evidence are reduced especially when I consider the evidence for Germany. However, there is an important caveat: the calibration that I have used refers to US economy and not for Germany economy like Bachmann and Bayer's model. Though some parameters are very similar, the main differences lie in the calibration of the upper bound of the distribution of adjustment costs. Table 10 shows the correlation for United States' firms which is quite lower than the model's results. However, as I said, the results have to be considered as a lower bound. Additional research has to be made in order to obtain a better estimation of the correlation between investment dispersion across firms and the business cycle for the United States' economy.

8 Conclusions

In this paper I have studied a new business cycle fact documented by Bachmann and Bayer (2011): the dispersion of the distribution of the investment rate across firms is procyclical. Using a wide sample of German firms they find a correlation coefficient between the cyclical component of output and the standard deviation of the distribution of investment rates across firms of 0.45. The correlation coefficient for US economy is equal to 0.33. Here I have shown that a model similar to Kahn and Thomas (2003), which is standard to heterogeneous firms literature, can account well for that correlation. The model predicts a correlation coefficient between investment distribution dispersion and the cyclical component of output of 0.57. Moreover, I find that adding a government sector that levies taxes on corporate profits, the model can account for the correlation found by Bachmann and Bayer (2011) much better. In particular, I find that considering a corporate tax rate of 18.79%, that is the effective corporate tax rate of the United States, the correlation is reduced to 0.51, and when I consider a corporate tax rate of 23.5%, which corresponds to the German economy, the correlation coefficient predicted by my model is 0.46, which is almost equal to empirical evidence. The key element to obtain a positive correlation between firms' investment rate dispersion and the business cycle is the presence of idiosyncratic adjustment costs. Without such friction, the model cannot match the empirical evidence.

9 Appendix

Optimization Appendix

Here I show the optimality conditions of households' optimization problem. These optimality conditions are used to obtain equations (9) y (10) in the text. The problem of the representative household is

$$W(\lambda; z, \mu) = \max_{C, N, \lambda'} [\log(C) + A(1 - N) + \beta E_{z'|z} W(\lambda'; z', \mu')] , \quad (27)$$

subject to

$$C + \int_K \rho(k; z, \mu) \lambda'(dk) \leq w(z, \mu)N + \int_K v^0(k; z, \mu) \lambda(dk) + Tr, \quad (28)$$

that are equations (7) y (8) in the text. With this I can define the following Lagrangian.

$$\begin{aligned} N &= (\log(C) + A(1 - N) + \beta E_{z'|z} W(\lambda'; z', \mu')) - \\ \Omega &\left(C + \int_K \rho(k; z, \mu) \lambda'(dk) - w(z, \mu)N - \int_K v^0(k; z, \mu)(dk) - Tr \right). \end{aligned} \quad (29)$$

The first-order conditions are the following:

$$\frac{\partial N}{\partial C} = \frac{1}{C} - \Omega = 0, \quad (30)$$

$$\frac{\partial N}{\partial N} = -A + \Omega w = 0, \quad (31)$$

$$\frac{\partial N}{\partial \lambda'} = \beta \frac{\partial E_{z'|z} W(\lambda'; z', \mu')}{\partial \lambda'} - \Omega \rho(k; z, \mu) = 0. \quad (32)$$

From equations (30) and (31), one gets

$$\frac{1}{C} = \frac{A}{w},$$

but I have supposed that $P = \frac{1}{C}$. Therefore,

$$w = \frac{A}{P}, \quad (33)$$

which is equation (9) in the text. Now, to obtain equation (10) in the text, first

note that the partial derivative of expression (29) with respect to λ is

$$\frac{\partial N}{\partial \lambda} = -\Omega(v^0(k; z, \mu)). \quad (34)$$

Using the Envelope Theorem one gets that the partial derivative respect with λ in the next period is given by

$$\frac{\partial N}{\partial \lambda'} = \Omega'(v^0(k'; z', \mu')). \quad (35)$$

Replacing equation (35) in equation (32), yields

$$\beta \Omega'(v^0(k'; z', \mu')) - \Omega \rho(k; z, \mu) = 0, \quad (36)$$

Then, note that $d_{z'}(z, \mu)$ is the discount factor of the firms, that is equal to $\frac{\rho(k; z, \mu)}{(v^0(k'; z', \mu'))}$ and in steady-state $\Omega' = \Omega$. Using this in equation (36) one gets

$$d_{z'}(z, \mu) = \beta, \quad (37)$$

which is equation (10) in the text.

Numerical Appendix

In this section I describe in more detail the numerical procedure used to solve the model. The model solution relies on two key assumptions: 1) each firm does not observe the entire distribution of firms over capital but only the aggregate level, and 2) firms forecast the aggregate price and next period's aggregate capital using the simple rules defined by equations (14) and (15). The first assumption allows me to replace μ by K in the firm's problem and reduces the dynamic programming problem to a three-dimensional state space $(k; z, K)$. Since the employment problem has an analytical solution, there is essentially just one continuous control variable, k' . I have discretized the state space as follows:

1. For k , the individual capital level, I use 400 equi-spaced points on the interval $[0.1, 5]$. I tried with a grid in logs and using a grid with more points at lower levels of capital, where the curvature of the value function is higher, but the results did not change.

2. For K , the aggregate capital level, I used 50 equi-spaced points on the interval $[0.7, 1.4]$.
3. For z , the aggregate productivity shock, I used the same grid as Khan and Thomas (2003). I assume an exogenous logarithmic autoregressive process of the form,

$$\log(z') = \rho \log(z) + \varepsilon, \quad \varepsilon \sim n(0, \sigma_\varepsilon). \quad (38)$$

In Khan and Thomas (2003), the parameter of persistence, ρ and the log-normal innovations, σ_ε are chosen to be consistent with measured Solow residuals from the US economy between the years 1953 and 1997. Then, this process is discretized using a grid of 5 points which are:

$$z = [0.9328; 0.9658; 1.00; 1.0354; 1.0720].$$

These parameters are similar to those chosen by Bachmann and Bayer (2011) to simulate their model.

Since I allow for continuous control, k and K can take any value. However, I can only compute the value function for each combination of points of the grids described above. That arises two problems: 1) the aggregate capital calculated from the simulated distribution of firms may not be included in the grid, and 2) the value of k' that maximizes (12) may not be on the grid either. In order to solve these problems I use two additional procedures. I solve the first by interpolating the value function for in-between points. This is done using a multidimensional cubic splines based on the `spapi` Matlab function with four knots for each dimension (see Judd, 1998). I solve the second problem using Golden Section Search (see Miranda and Fackler (2002) for details).

For the simulation, I draw one random series for the aggregate productivity level and fix it across models. I simulate the economy for $T = 1500$ periods and discard the first 500 periods for calculating the main statistics. I do so in order to ensure that results are not driven by the initial conditions of the economy, especially by the initial distribution of firms over capital.

For solving the equilibrium price level I used guess-and-verify with a convergence criteria of 10^{-5} . In some cases, especially when the forecasting rules are far from

their convergence level, the optimal price level cannot be found. In those cases, I broke the iteration procedure after 2000 iterations and used the last price level to continue the simulation. However, the differences in absolute value between the guess and the price level implied by the model were never higher than 10^{-3} .

Finally, the convergence criteria for the forecasting rules parameters was 10^{-5} . When the convergence was reached, the forecasting rules were very accurate in predicting the aggregate price level and next period's capital level: the R^2 of the OLS estimation is higher than 99%. In table 12, I show the R^2 of forecasting rules in the baseline case.

Table 12: Forecasting rules

R_P^2	R_K^2	β_{0P}	β_{1P}	β_{0K}	β_{1K}
0.9972	0.9993	1.2034	-0.4848	-0.0195	0.8261
0.9986	0.9942	1.1810	-0.4810	-0.0071	0.8251
0.9993	0.9953	1.1584	-0.4789	0.0054	0.8226
0.9991	0.9953	1.1355	-0.4767	0.0181	0.8207
0.9994	0.9945	1.1123	-0.4738	0.0308	0.8204

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Figure 1: Initial distribution

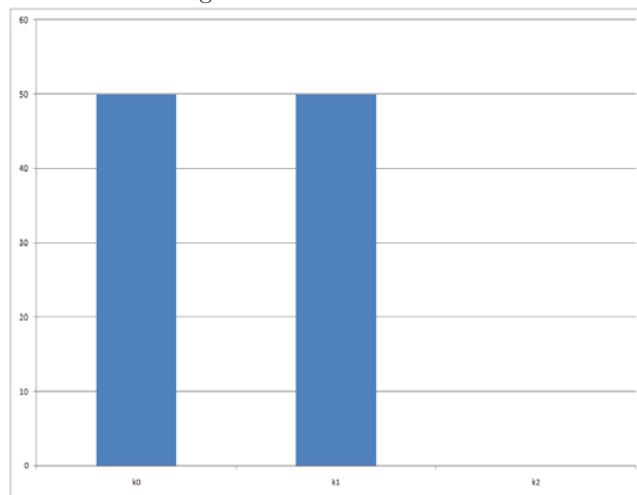


Figure 2: Distribution after a shock

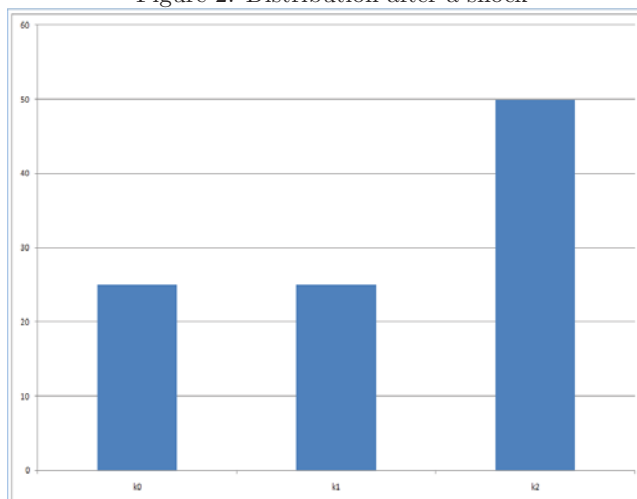


Figure 3: Investment distribution after a shock

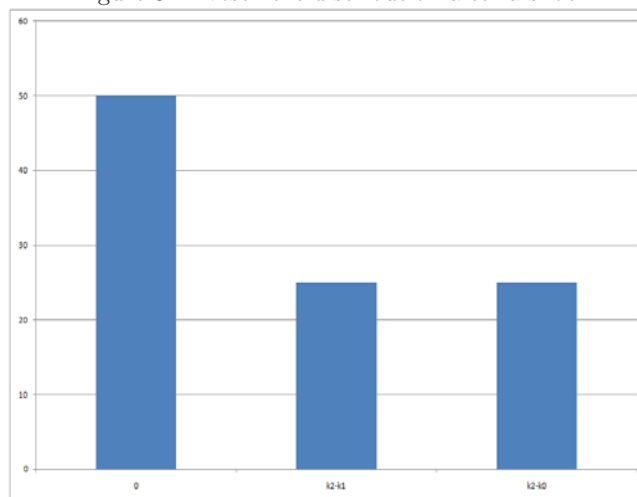


Figure 4: Distribution after the shock with $\tau_c > 0$

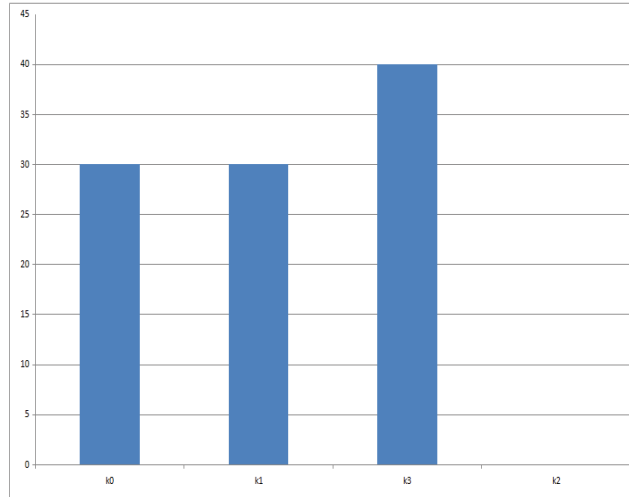


Figure 5: Distributional effects of a positive shock - capital distribution

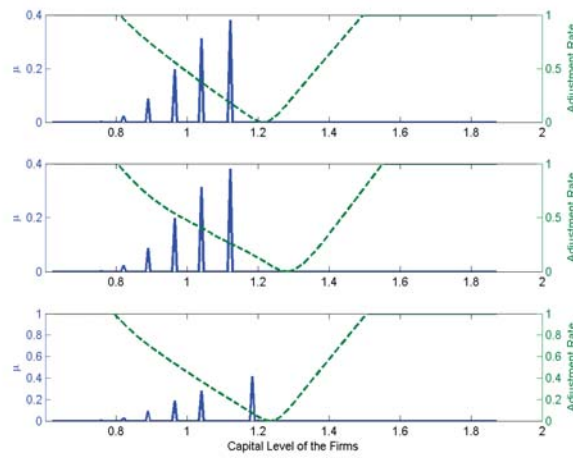


Figure 6: Distributional effects of a positive shock - investment distribution

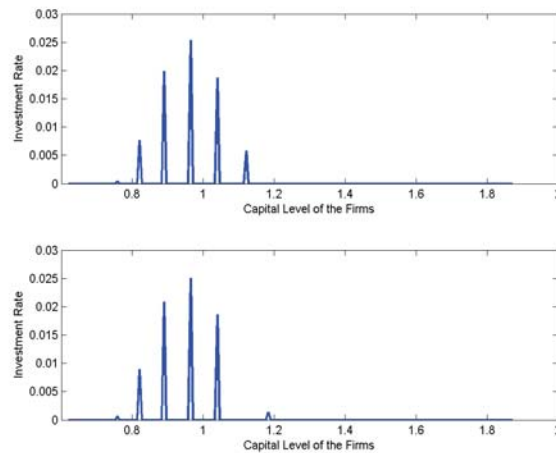


Figure 7: Distributional effects of a negative shock - capital distribution

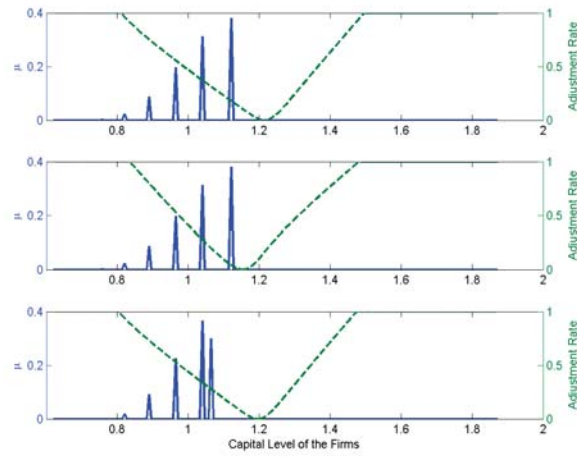


Figure 8: Distributional effects of a negative shock - investment distribution

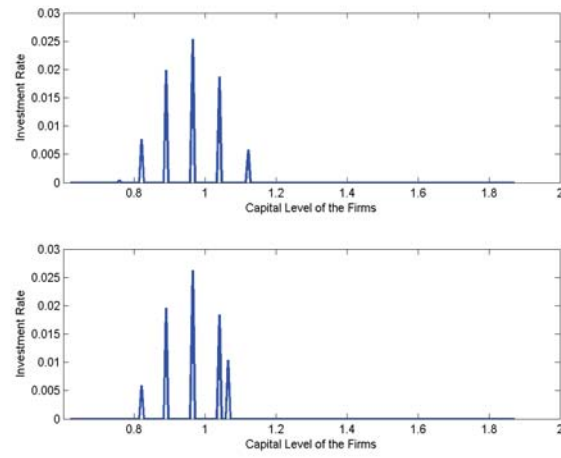


Figure 9: Distributional effects of a positive shock - capital distribution - $\tau_c = 18.79\%$

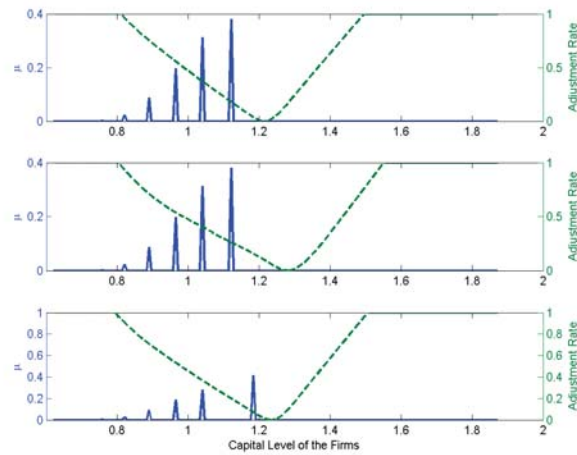


Figure 10: Distributional effects of a positive shock - investment distribution - $\tau_c = 18.79\%$

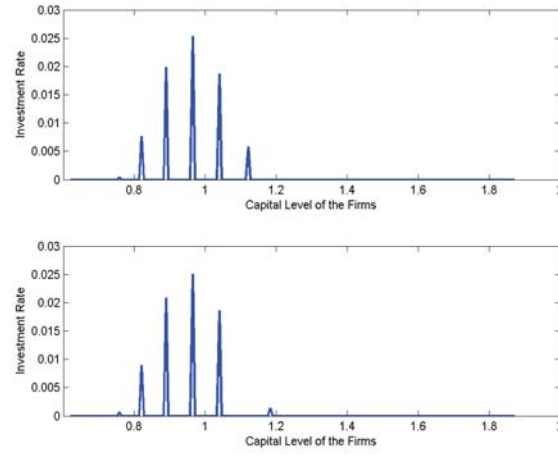


Figure 11: Distributional effects of a negative shock - capital distribution - $\tau_c = 18.79\%$

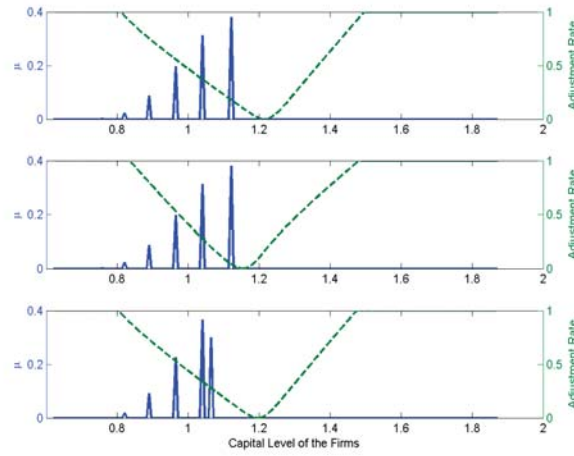
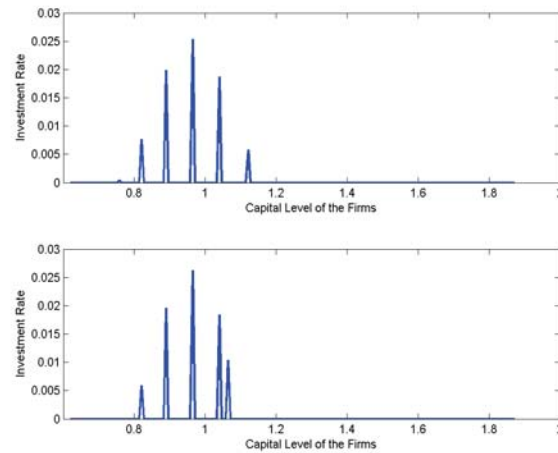


Figure 12: Distributional effects of a negative shock - investment distribution - $\tau_c = 18.79\%$



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