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OPTIMAL MONETARY POLICY RULES UNDER INFLATION RANGE TARGETING

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Resumen
En este trabajo calculamos y comparamos reglas óptimas de política monetaria (PM) bajo alternativas funciones objetivos (de pérdida) para un banco central. Obtenemos reglas de política monetaria óptimas tanto cuando existe un rango para el objetivo inflacionario como cuando hay asimetrías en las pérdidas asignadas a los desvíos de la inflación con respecto a la meta. Comparamos estas reglas con el caso de una función objetivo cuadratica que representa una meta de inflación puntual. Los resultados muestran que la agresividad de la PM bajo una meta rango depende crucialmente de la magnitud de la pérdida asignada a que la inflación este fuera del rango. Si el rango se interpreta como una meta de inflación ancha, la PM es siempre activa (no existen zonas de inacción), aunque es menos agresiva contra los shocks de inflación y de producto si el costo de salirse del rango no es muy elevado (vis-à-vis una meta puntual). Sin embrago, si el banco central es más estricto en relación al logro del objetivo inflacionario, entonces la PM es más agresiva aun cuando la economía se encuentre cerca de la parte central del rango. Finalmente, una función de pérdida asimétrica que castiga relativamente más las desviaciones positivas genera un sesgo contra el producto.

Abstract
We calculate and compare optimal monetary policy (MP) rules for a simple economy under alternative central bank objective (loss) functions. We compare both soft- and hard-edges range (zone) targeting as well as asymmetric loss-functions to a quadratic loss case. The latter represents the standard loss-function for point inflation targeting. The results show that MP aggressiveness under range targeting critically depends on how hard are the edges of this range. If a range is thought of as a thick point objective, MP is always active (there are no inaction zones), although it is less aggressive against inflation and output shocks if range edges are sufficiently soft (vis-à-vis a point target). Harder edges makes MP more aggressive even when the economy is close to the central part of the range. Finally, an asymmetric loss-function for inflation that penalizes positive deviations relatively more generates a bias against output.

We thank Pablo García, Luis Oscar Herrera, and Felipe Morandé and seminar participants at the Central Bank of Chile for useful comments. All remaining errors are our responsability. This paper presents the views of the authors and does not represent in any way positions or views of the Central Bank of Chile. Email: jpmedina@condor.central.cl and rvaldesx@condor.bcentral.cl.

1 Introduction

Central Banks (CBs) resort to alternative arrangements to formulate, conduct and communicate monetary policy (MP). One arrangement that has become increasingly popular is based on a target range for inflation.[1] In this setup the conduct of monetary policy is oriented to keep inflation inside pre-announced boundaries. For example, the CBs of Brazil, Canada, Israel, New Zealand, and Sweden all have target zones for inflation, with a width that ranges from 2 to 3 percent. Chile, in turn, has recently announced an inflation target range of 2 to 4% to be used starting in year 2001. One could further distinguish two alternative range arrangements, namely soft- and hard-edge ranges. Under the former, deviations from edges are considered as adverse as a deviation from a point target, whereas in the latter deviations from the target range are considered to be relatively more undesirable.[2] Other CBs organize monetary policy around a point target (e.g., the UK and Chile until the year 2000), and some, implicitly, evaluate upward deviations of inflation as been more costly than downward deviations. The latter is typically the case of a country trying to desinflate (e.g., Brazil, Colombia, and Israel nowadays and Chile during the 90s.).

In this paper we investigate how optimal monetary policy rules change under these alternatives formulations. In particular, we compare optimal policy rules derived under alternative policy objectives or loss-functions for a simple backward-looking dynamic economy. We are interested in answering four related questions: (i) Does range targeting (optimally) yield less aggressive MP reactions?[3] (ii) Are there relevant non-linearities for MP under range targeting (e.g., are there inaction zones)? (iii) What happens with optimal MP reactions if range edges are harder, that is, if deviations from the range are considered very undesirable events? and (iv) What are the implications of having an asymmetric objective, in which positive inflation deviations are relatively more undesirable? Our purpose is not only to answer these questions but provide quantitative measures so as to evaluate their economic relevance. For that purpose we consider a simple but realistic economy calibrated with Chilean data.

[1]This arrangement is part of a broader movement in monetary policy design in which the anchor is a target for inflation. See, e.g., Bernanke at al. (1999).

[2]One reason for considering a deviation from a target range as being relatively more costly is its impact on credibility.

[3]Other ways to make MP less aggressive include the incorporation of interest rate smoothness among CB objectives and the measurement of inflation over longer horizons (see, e.g., Nessén, 1999).

The economy for which we calculate optimal policy rules is extremely simple and is described by two linear equations: (i) an acceleratcionist Phillips curve; and (ii) an output gap equation. Monetary policy directly controls the real interest rate. The latter, in turn, affects output with a lag, while output affects inflation with another lag. For simplicity we do not consider any preferences over interest rate variability nor bounds for possible interest rate values. In general, the rules we derive will prescribe a more aggressive monetary policy than what a CB would normally follow. Accordingly, rather than taking the quantitative results we derive at face value, they should be analyzed relative to the baseline-case scenario of rules for a symmetric point inflation target (derived from a standard quadratic loss-function).

This paper is closely related to literature on optimal monetary policy rules developed by Svensson (1999), Ball (1999), McCallum (1998), Woodford (1999), among many others. It departs from an otherwise standard analysis by comparing the implications of alternative non-quadratic loss-functions. It is closely related to Orphanides and Wieland (1999), which also study the effects of having range targets for inflation. The key difference between that paper and ours is that they consider no lags in the effects of the output gap on prices, whereas we consider that this effect takes one period. In their model, MP has a one-period control lag (it affects inflation after one period), while in ours it has a two-period lag. This extra lag poses the difficulty of having a second state variable, although it makes the model more realistic.[4]

The main results we find are the following. In comparison to a quadratic loss-function, range targeting yields less aggressive MP. For example, after a 1% inflation shock interest rates increase by approximately one third less when there is a range targeting with soft edges than when there is point targeting. Optimal MP in this setup is always active thought. That is, even if inflation is well inside the range, if it is not right on target (i.e., in the middle of the target), interest rates should not be at their neutral level. MP moves in a preemptive way: because the likelihood that a shock will move the economy out of the range increases when it is not in the middle of it, it is optimal to move back the economy towards the center of the range. If range edges are relatively hard, range targeting implies a more aggressive MP.[5]

[4]Orphanides and Wieland (1999) also include in their analysis a non-linear Phillips curve and model uncertainty. We, in turn, focus our analysis on the implications of having alternative loss-functions.

[5]Thus, there is an apparent trade-off between the width of the range and the form of its edges. The width generates less aggressive responses whereas harder edges generate more activism.

If the loss-function is the same as with point targeting, but with a discrete jump to zero in the inflation target range, then optimal MP does not change in any important way. If the loss-function is asymmetric, penalizing to a larger extent positive inflation deviations, optimal MP involves higher average interest rates. This implies that this loss-function generates (on average) a negative output gap.

The paper is organized as follows. Section 2 presents the basic model of the economy, discusses the procedure we use to transform a continuous economy into a discrete one, and reviews the dynamic programming framework we use to solve for the optimal policy rules. Section 3 compares the optimal policy rules derived under alternative loss-functions. Finally, section 4 concludes.

2 General Framework

This section presents the framework we use to calculate and compare optimal monetary policy rules under alternative loss-functions. It describes the economy, CB preferences, and the method we use to find optimal MP rules.

2.1 The Economy

We consider a simple dynamic backward-looking economy described by the following two equations:

$$\pi_t = \pi_{t-1} + \alpha_y y_{t-1} + \varepsilon_t^{\pi} \tag{1}$$

and

$$y_t = \beta_y y_{t-1} + \beta_r r_{t-1} + \varepsilon_t^{y} \tag{2}$$

where π_t is the gap between inflation in period t and its long-run target, y_t is the output gap, r_t is the real (or indexed) interest rate (measured as deviation with respect to its long run level), and ε_t^{π} and ε_t^{y} are (possibly correlated) serially uncorrelated mean-zero stochastic shocks. α_y, α_r, and β_y are constant parameters.

These two equations are similar to those presented in Ball (1999) and Svensson (1997). Equation (1) is a standard acceleracionist Phillips curve. Equation (2) is a standard output gap equation (a dynamic IS curve). Below we present an estimation of these equations using data of the Chilean economy. Notice that in this set-up monetary policy has a two-period control-lag over inflation.

At time t the CB problem is to choose a sequence of interest rates $\{r_{t+\tau}\}_{\tau=0}^{\infty}$ so as to minimize the following expected intertemporal loss-function:

$$\mathbf{L}_t = E\left[\sum_{\tau=0}^{\infty} \delta^{\tau}\mathbf{l}(x_{t+\tau})\right], \tag{3}$$

where δ is a discount factor, $\mathbf{l}(.)$ is an instantaneous loss-function, and x_t is a vector with the state variables, $x_t = (\pi_t, y_t)'$.

We seek to characterize the interest rate sequence through a time-invariant (probably non-linear) policy function for alternative instantaneous loss-functions. In particular we seek to characterize and compare optimal policy (reaction) functions when the loss-function is of the following type:

$$\mathbf{l}(x_t) = \begin{cases} a_L\left(\pi_t - c\bar{\pi}_L\right)^2 + by_t^2 & \text{for } \pi_t \in (-\infty, \bar{\pi}_L] \\ by_t^2 & \text{for } \pi_t \in (\bar{\pi}_L, \bar{\pi}_H) \\ a_H\left(\pi_t - c\bar{\pi}_H\right)^2 + by_t^2 & \text{for } \pi_t \in [\bar{\pi}_H, \infty), \end{cases} \tag{4}$$

where b, c, a_j and $\bar{\pi}_j$ $(j = L, H)$ are constants.

This loss-function includes the standard quadratic function with weights a and b in inflation and output gap variability. In this case $\bar{\pi}_L = \bar{\pi}_K = 0$ and $a_L = a_H$. It also includes less standard set-ups such as target zones (e.g., $\bar{\pi}_L < 0$, $\bar{\pi}_K > 0$, and $c = 1$) and asymmetric weights (e.g., $a_L < a_H$). Table 1 presents the summary of the five baseline cases we consider in this paper.[6]

Figures 1 and 2 show the alternative loss-functions described in table 1. The quadratic case is meant to represent the standard point-target framework, whereas those with a zero-value in the $(-1, 1)$ range represent alternative range targeting setups. Furthermore, we include an asymmetric objective function to represent larger costs of positive inflation deviations.[7]

In sum, the problem is to choose a sequence $\{r_{t+\tau}\}_{\tau=0}^{\infty}$ in order to minimize (3) subject to (1) and (2). Under some regularity conditions, this problem has a solution which can be represented by a time-invariant policy function mapping inflation and output gap

[6] Of course, one could also consider other parameters. In Medina and Valdés (1999) we consider the case of a one-sided objective.

[7] Other asymmetries that may in general arise include asymmetric shocks and asymmetric responses of the economy.

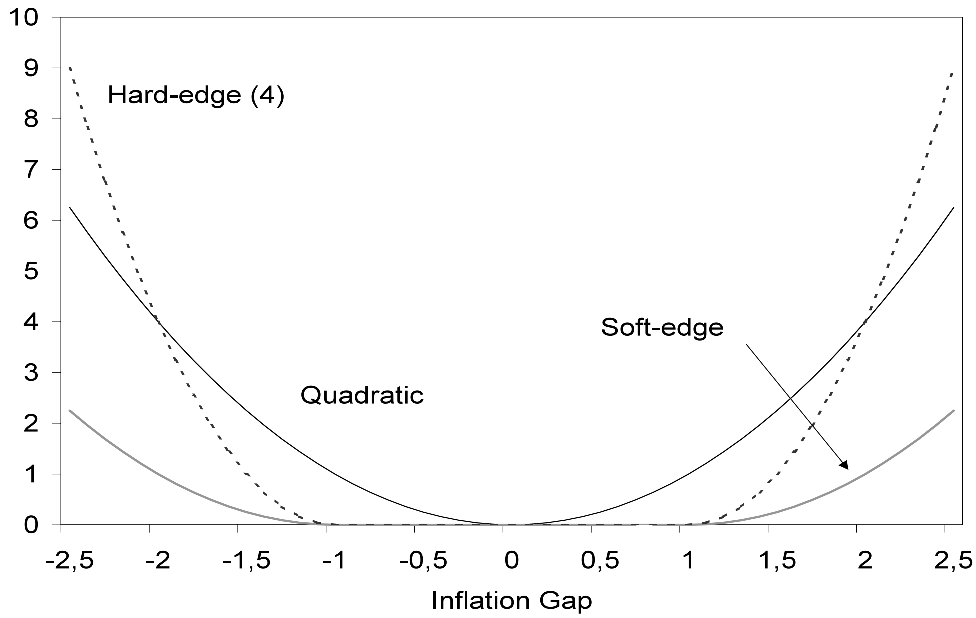

Figure 1: Loss-Functions under Range Targeting

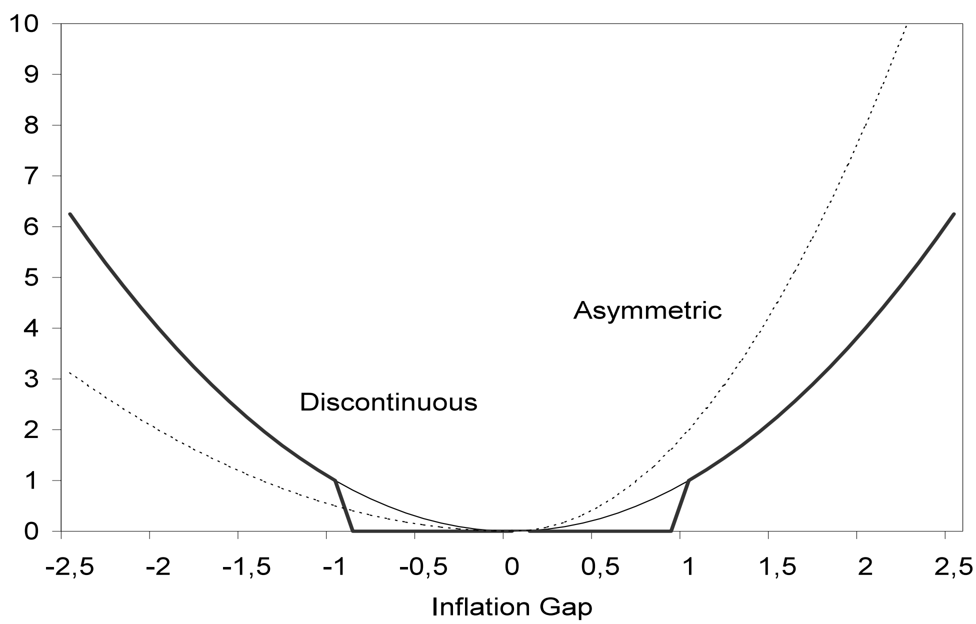

Figure 2: Asymmetric and Discontinuous Loss-Functions

Table 1: Alternative Loss-Functions

	Quadratic	Range with soft edges	Range with hard edges	Range with discrete edges	Asymmetric hawk
a_L	1.0	1.0	1.25/4.0	1.0	0.5
a_H	1.0	1.0	1.25/4.0	1.0	2.0
b	0.5	0.5	0.5	0.5	0.5
c	1.0	1.0	1.0	0.0	1.0
$\bar{\pi}_L$	0	-1	-1	-1	0
$\bar{\pi}_H$	0	$+1$	$+1$	$+1$	0
δ	0.95	0.95	0.95	0.95	0.95

onto the real interest rate, $r_t = h(x_t)$, where x_t is the vector of state variables (see, e.g., Sargent and Ljungqvist, 1999). In this case $x_t = (\pi_t, y_t)'$.

Unfortunately, for the class of loss-functions we consider it is not possible to find a closed-form solution for the function $h(.)$. Only in the well known case of a quadratic problem it is possible to find a vector F such that $r_t = F'x_t$ yields the optimal solution. We therefore have to resort to numerical methods. In particular, we change the original problem to one in which we constrain the states and control variables to be a discrete and finite set of points. We then apply standard dynamic programming techniques.[8]

2.2 Discretization of the Continuous-State Space

In order to transform the continuous economy described by equations (1) and (2) and the control variable r_t into a discrete-state space economy we proceed as follows. We first assume that the economy can be in m possible states collected in a set $\Gamma = \{(\pi_1, y_1), (\pi_2, y_2), \ldots, (\pi_m, y_m)\}$. We further assume that the interest rate r_t can take n different values arranged in a set $\Psi = \{r_1, r_2, \ldots, r_n\}$.

To preserve the dynamic and stochastic properties embedded in equations (1) and (2) we calculate transition probabilities between states that depend on the interest rate. In this way, monetary policy can be thought of as choosing alternative transition matrices of a Markov chain of the economy.

[8] Judd (1998) analyzes discrete-state space dynamic programming.

Assume that the shocks ε_t^π and ε_t^y follow a bivariate normal distribution with variance-covariance matrix Σ. It is then straightforward to calculate the conditional distribution of the vector $x_t = (\pi_t, y_t)'$ conditional on $(\pi_{t-1}, y_{t-1})'$ and r_{t-1} as the bivariate normal distribution with mean:

$$E\left(x_t \setminus x_{t-1}, r_{t-1}\right) = E\left(\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} \setminus \pi_{t-1}, y_{t-1}, r_{t-1}\right) = \begin{bmatrix} \pi_{t-1} + \alpha_y y_{t-1} \\ \beta_y y_{t-1} + \beta_r r_{t-1} \end{bmatrix},$$

and variance-covariance matrix

$$Var\left(x_t \setminus \pi_{t-1}, y_{t-1}, r_{t-1}\right) = \Sigma.$$

To construct the transition matrix associated to each interest rate in Ψ we divide a rectangle $[a_\pi, b_\pi] \times [a_y, b_y] \subseteq \Re^2$ into m equal-size rectangles $\theta_1, \theta_2, \ldots, \theta_m$ and consider that the center of each of these rectangles form the set Γ. For each feasible interest rate $r_k \in \Psi$ we can define the transition probability p_{ij}^k as the likelihood of a movement from state (π_i, y_i) at $t-1$ to state (π_j, y_j) one period ahead given that $r_{t-1} = r_k$. This probability is given by:

$$\begin{aligned} p_{ij}^k &= \Pr\left[(\pi_t, y_t)' \in \theta_j \setminus (\pi_{t-1}, y_{t-1}) = (\pi_i, y_i), r_{t-1} = r_k\right] \\ &= \left(\int_{\theta_j} \frac{1}{2\pi\Sigma^{1/2}} \exp\left[-\frac{\left(x - \bar{x}_i^k\right)' \Sigma \left(x - \bar{x}_i^k\right)}{2}\right] dx\right) \div \\ &\int_{[a_\pi, b_\pi]\times[a_y, b_y]} \frac{1}{2\pi\Sigma^{1/2}} \exp\left[-\frac{\left(x - \bar{x}_i^k\right)' \Sigma \left(x - \bar{x}_i^k\right)}{2}\right] dx, \end{aligned}$$

where

$$\bar{x}_i^k = E\left(\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} \setminus \pi_{t-1} = \pi_i, y_{t-1} = y_i, r_{t-1} = r_k\right).$$

Armed with this set of transition matrices we next find a solution of the original problem using standard dynamic programming. Clearly, because we consider a subset of the possible states of the economy, this discretization is only an approximation. Moreover, it should be clear that in the neighborhood of the borders of the rectangle $[a_\pi, b_\pi] \times [a_y, b_y]$ the approximation is far from accurate, both because the probability of moving further away from the center of the rectangle is assumed to be zero and because the center of the small rectangles θ_j at the borders do not properly represent the values that the system can take outside the (large) rectangle. When we calculate optimal policy rules we use the complete rectangle, but only consider the neighborhood of its center when we analyze and compare the implications of alternative loss-functions.

2.3 Discrete-State Space Dynamic Programming

With the economy represented in a discrete-state space, the original CB problem becomes:

$$
\begin{aligned}
&\min_{\{r_{t+\tau}\}_{\tau=0}^{\infty}} \; E\sum_{t=0}^{\infty}\delta^t l(\pi_t, y_t) \\
&s.t. \quad (\pi_{t+1}, y_{t+1}) \in \Gamma = \{(\pi_1, y_1), \ldots, (\pi_m, y_m)\} \\
&\qquad\;\; r_t \in \Psi = \{r_1, \ldots, r_k\} \\
&\qquad\;\; \Pr\left[(\pi_t, y_t) = (\pi_j, y_j) \setminus (\pi_{t-1}, y_{t-1}) = (\pi_i, y_i), r_{t-1} = r_k\right] = p_{ij}^k
\end{aligned}
$$

where $\sum_{j=1}^{m} p_{ij}^k = 1$, for all $i = 1, \ldots, m$ and for all $k = 1, \ldots, n$.

The Bellman equation associated to this problem is:

$$
V(\pi_t, y_t) = \min_{r' \in \Psi}\left\{l(\pi_t, y_t) + \delta E\left[V(\pi_{t+1}, y_{t+1}) \setminus (\pi_i, y_i), r'\right]\right\} \tag{5}
$$

which, given the transition probabilities, can be written as:

$$
V(\pi_i, y_i) = \min_{r_k \in \Psi}\left\{l(\pi_i, y_i) + \delta\sum_{j=1}^{m} p_{ij}^k V(\pi_j, y_j)\right\}, \tag{6}
$$

for all $i = 1, \ldots, m$ and for all $k = 1, \ldots, n$. Here $V(\pi_t, y_t)$ is the optimal value of the objective function starting from an inflation π_t and output gap y_t. The solution of the CB problem is characterized by a value function $V(.,.)$ that satisfies (6) and an associated policy function $r' = h(\pi, y)$ mapping the state of the economy (π, y) onto an optimal choice of interest rate. Since (6) satisfies Blackwell's sufficient conditions for a contraction (see Stockey and Lucas, 1989, p. 54), it has a unique solution $V^*(.,.)$.

In cases in which the discrete-state space is of a small size it is not difficult to solve the Bellman equation applying an iterative algorithm. Define v to be vector in $\Re^m$ and $T[.]$ to be an operator that maps a vector v into a new vector $T[v] = (tv_1, tv_2, \ldots, tv_m)'$ in which each element tv_i is given by:

$$
tv_i = l(\pi_i, y_i) + \min\left\{\sum_{j=1}^{m} p_{ij}^1 v_j, \sum_{j=1}^{m} p_{ij}^2 v_j, \ldots, \sum_{j=1}^{m} p_{ij}^n v_j\right\}. \tag{7}
$$

Thus, the Bellman equation can be represented by

$$
v = T[v],
$$

which can be solved by iterating until convergence the following recursion:

$$(v)_{s+1} = T\left[(v)_s\right].$$

If v^* is the value function of the problem, then the optimal policy function h satisfies:

$$r' = h(\pi, y) \in \arg\min_{r_k \in \Psi} \left\{ \sum_{j=1}^{m} p_{ij}^k v_j^* \right\}.$$

That is, for each pair (π_t, y_t), the function h yields the optimal interest rate r_t such that the CB discounted intertemporal loss-function is minimized.

3 Comparison of Alternative Loss-Functions

This section calculates and compares optimal monetary policy rules for different loss-functions using the methodology described above. In order to provide realistic results it uses an estimation of the economy described by equations (1) and (2) using Chilean data. We then use the model presented above to calculate policy functions for alternative objective functions and compare them to the standard quadratic loss-function.

3.1 Estimation/Calibration

We estimate equations (1) and (2) using Chilean quarterly and semi-annual data on core inflation and output gap (calculated with an HP filter) from 1986 to 1998. Table 3 presents the basic results.[9] It also presents the parameters we finally use in the simulations. The interest rate corresponds to the short-term indexed rate that the Central Bank of Chile has used for monetary policy (PRBC90 in 1986-1995 and the overnight interbank interest rate thereafter).

While we broadly use the OLS estimates for elasticities, we consider a lower innovation volatility in our simulations. In the case of inflation, the main reason for this assumption is that this variable displays significant ARCH effects. In fact, a standard ARCH-LM test on the equations of table 2 yield a p-value of 0.06. Furthermore, as Magendzo (1998) has documented, this volatility has a positive and stable relation with the

[9] It is worth mentioning that we also considered in the estimation a one-period lagged direct effect of interest rates on inflation. This was meant to capture the standard open-economy transmission mechanism from monetary policy to inflation through the exchange rate. The results do not show results that can be considered different from zero.

Table 3: Parameters Estimation: Chile 1986–1998

	Quarterly Data	Semi-Annual Data	Calibration
	Phillips Curve		
α_y	0.49	0.54	0.50
	(0.20)	(0.16)	
$\sigma(\varepsilon_t^\pi)$	0.030	0.011	0.008
$\bar{R}^2$	0.08	0.43	
	Output Gap		
β_y	0.61	0.32	0.60
	(0.07)	(0.11)	
β_r	–0.43	–0.55	–0.50
	(0.16)	(0.32)	
$\sigma(\varepsilon_t^y)$	0.015	0.015	0.010
$\bar{R}^2$	0.46	0.23	
$corr(\varepsilon_t^\pi, \varepsilon_t^y)$	–0.05	0.21	0.00

Notes: OLS estimates. Newey-West consistent standard errors in parenthesis

inflation level. In order to verify this effect we estimate the same equation (1) but using the ratio between actual and trend-inflation instead of actual inflation. The estimate still shows a significant effect of the output gap on changes in (relative) inflation. More importantly, the standard error of the innovations of this estimation is approximately 0.25. Considering this standard deviation of 0.25 and an annual trend inflation of around 3% we calibrate the model using a quarterly standard deviation of 0.8% (for annualized inflation). As for output, we consider a standard deviation of 1.0% to broadly take into account the effect of other known output determinants such as mining and fiscal policy.

3.2 Quadratic Preferences

In order to evaluate the accuracy of our discretization of the economy we compare the optimal policy rules for a quadratic loss-function that results from using the algorithm presented in last section and the results from a standard dynamic programming procedure using the continous-state space economy. For the latter we use the standard linear regulator problem solution (see, e.g., Sargent and Ljungqvist, 1999).

Notice that because output is not that persistent, monetary policy has to be quite active in order to affect inflation. We construct grids for inflation and output in the $[-5, +5]$ range with an increment of 0.25 and 0.5 respectively. We search for optimal interest rates in the $[-17, +17]$ range with increments of 33 basis points at each point of the grid.[10]

Figure 3 presents the results of this exercise. The top panels (A and B) show the optimal interest rate for an inflation and output gap deviation, respectively. Both assume that the other variable has a deviation equal to zero. The bottom panels show the same functions but assuming a positive (one percent) deviation of the other variable. As mentioned above, the accuracy of the results is limited at the borders of the grid we consider. However, they are surprisingly accurate once we move away from these borders. Indeed, only in the policy response to extreme inflation deviations the two reaction functions diverge. In what follows we present results using deviations for the $[-3, +3]$ range.

3.3 Range Targeting

Is monetary policy more or less aggressive with point or range targets? The answer obviously depends on how hard are the edges of the range under consideration. Figure 4 shows optimal policy reaction functions for a soft-edge target range with a 2% width. To facilitate comparisons it also presents the optimal reaction functions for a quadratic loss-function. The soft-edge range is represented by a loss-function with quadratic losses in the edges with the same parameters as the quadratic case (see table 1 and figures 1 and 2 for details). It can be though of as being a point target with a "thick" target.

The results show that, indeed, monetary policy is less aggressive with a range target with soft-edges. For all practical purposes, the reaction functions are piece-wise linear: when inflation lies in the $(-1.5, 1.5)$ range the policy rule is a line with a slope 30% smaller than the quadratic case. This means that MP is one third less aggressive when there is a target range. Outside the $(-1.5, 1.5)$ range the MP reaction function is still linear, with a similar slope, but starts at a higher level (in absolute value). The reaction function against output shocks is also piece-wise linear. It is almost equal to the policy function in the quadratic case for gaps in the $(-1.5, 1.5)$ range, an less aggressive outside that range (by approximately 80 basis points). The magnitude of these differences is

[10] These grids imply that there are more than 58 million p_{ij}^k's.

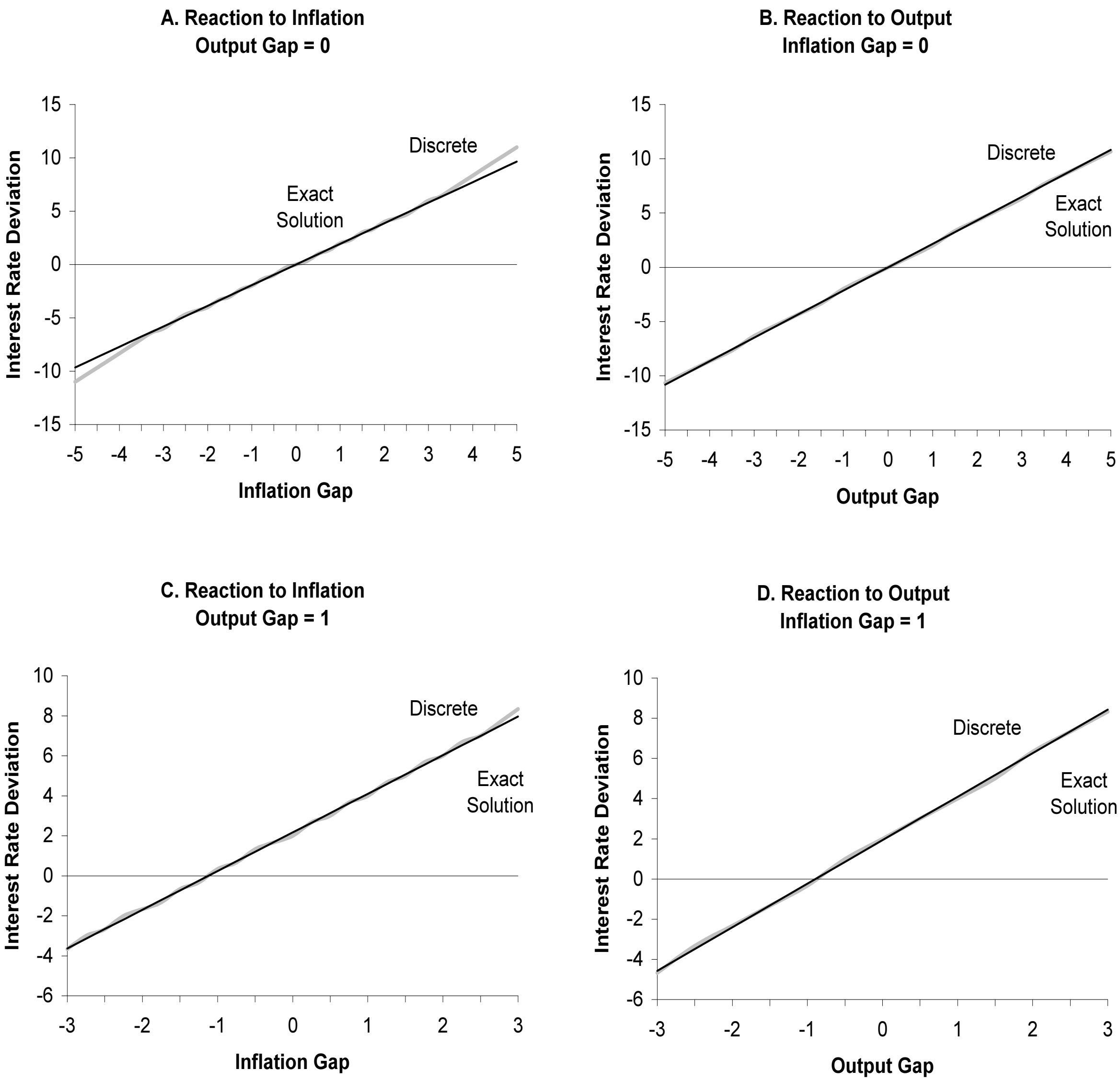

Figure 3: Optimal Policy Functions with a Quadratic Loss-function

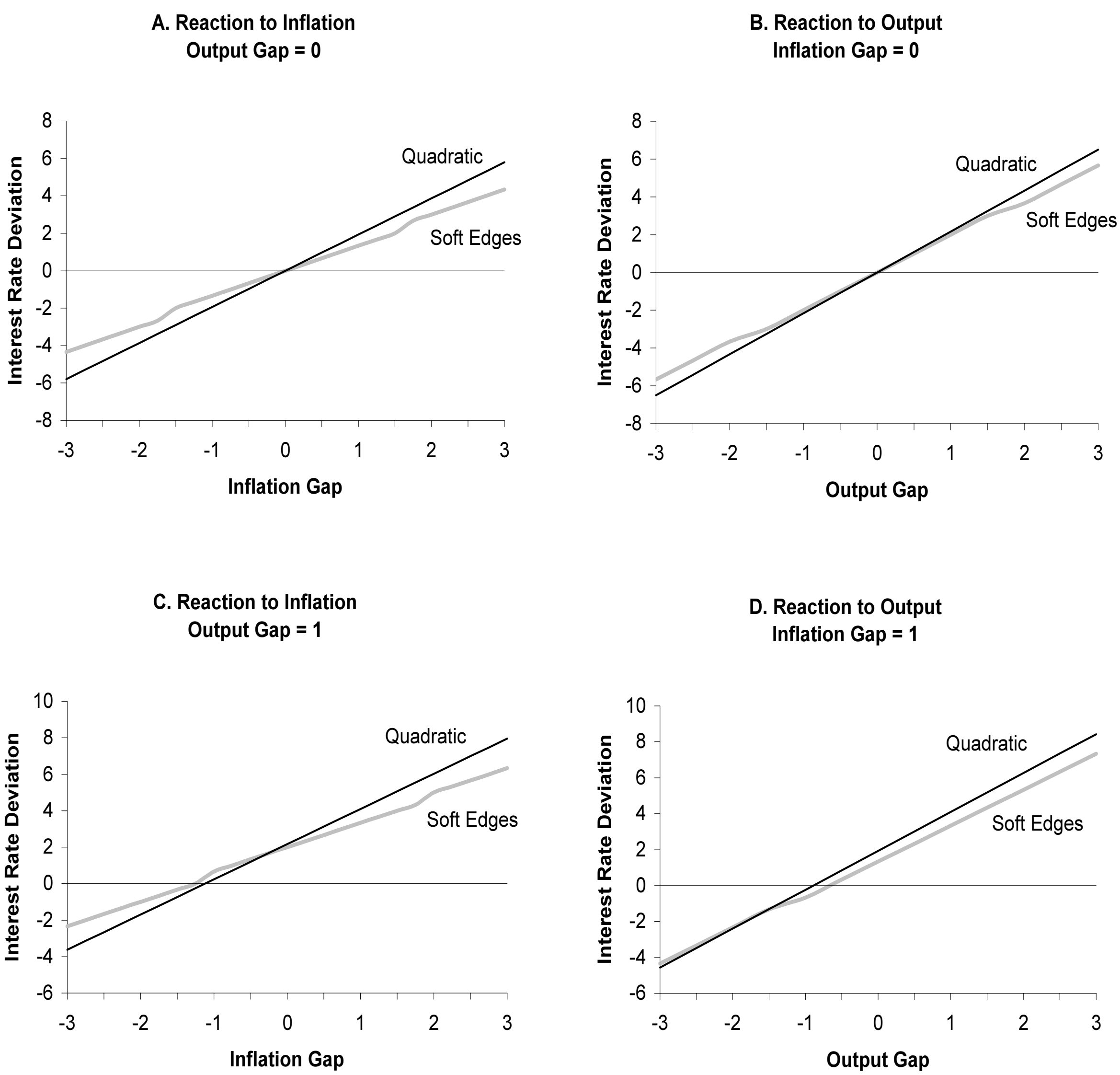

Figure 4: Optimal Policy Functions under Soft-edge Range Targeting

not large, although economically meaningful.

The results also show that MP is always active. Even when an inflation shock leaves the economy well inside the range target, if its not right on the middle of it, interest rates should not be equal to their neutral level. The intuition of this result is that MP has to move in a preemptive way: because new shocks are coming in the future, being close to the borders of the range is risky. Thus, the CB has to move interest rates in such a way that the economy gets close to the middle of the target range.

Figure 5 shows the results for two alternative cases in which range edges are harder ($a = 1.25$ and $a = 4$). In these cases policy functions have some non linearities, although they are not substantial. When $a = 1.25$, that is a range deviation are considered 25% worse than point deviations, optimal policy is still less aggressive with a target range than with a point target. When range deviations are four-fold worse than point deviations policy is more aggressive. Numerically, following a 2% inflation shock, interest rates respond by approximately 10% more than in the quadratic case. When shocks are in the neighborhood of the target, differences are less important and alternative loss-functions yield similar outcomes (so far as edges are relatively harder).

Figure 6 presents the case of a range with discontinuous-edges, in which the loss-function is the standard quadratic function that collapses (discontinuously) to zero inside the range $[-1, 1]$. This could be thought of as a range that permits some limited flexibility, although outside the range losses are the same as in the point range case. In other words, small deviations are not considered undesirable, whereas larger deviations are as costly as in the case of a quadratic loss-function. The result shows that from an economic point of view this range does not yield any room for a less aggressive monetary policy. Indeed, the reaction functions in both cases are practically equal.

The intuition behind this result is simple. Because the economy is subject to continuous shocks, is very unlikely that the economy will be very close to the middle of the range in a period by period basis. Therefore, it is immaterial whether inflation deviations are considered neutral inside the target range. As a way to prevent movements out of this range, MP has to be equally active.

3.4 Asymmetric Losses

If positive inflation deviations are considered as more costly than negative ones (e.g., because of credibility problems or because the economy is following a desinflation s-

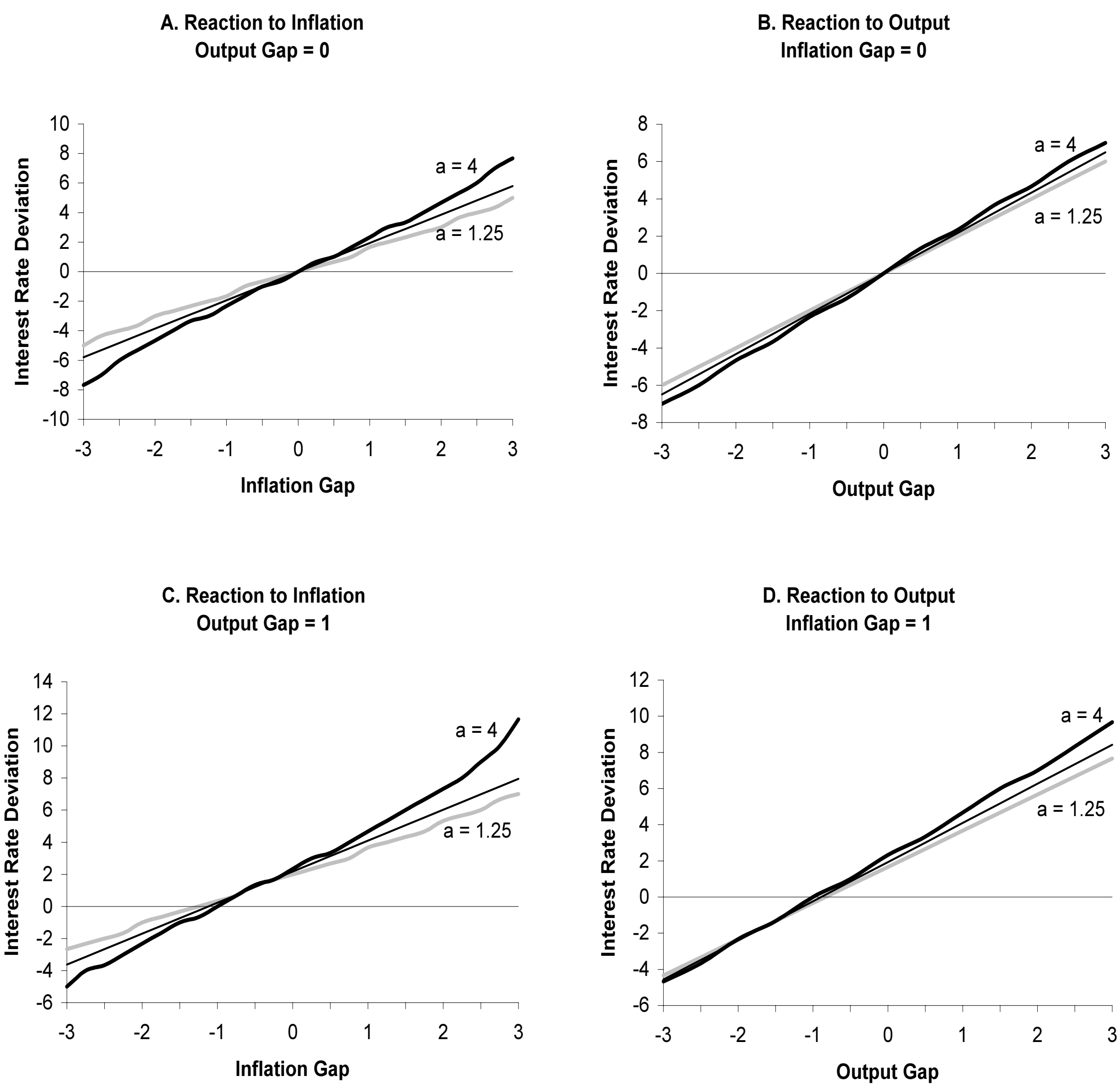

Figure 5: Optimal Policy Functions with Hard-edge Range Targeting

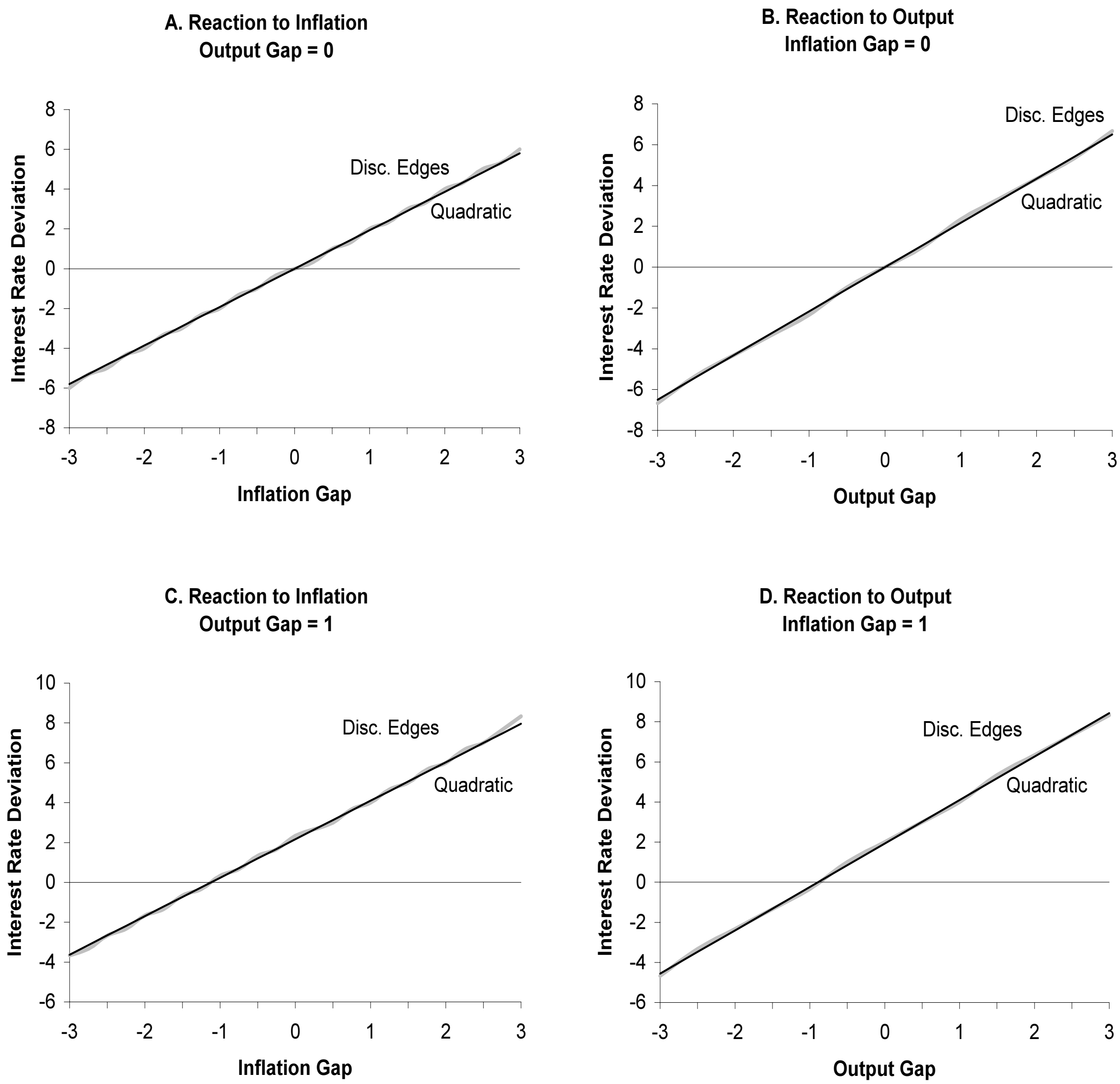

Figure 6: Optimal Policy Functions with Discrete Loss-function

trategy), the loss-function will be asymmetric, penalizing to a larger extent positive deviations. Figure 7 plots the policy functions for the case in which there is a point target but positive deviations are considered twice as much costly as negative deviations (that is $a_L = 1, a_H = 2$).

The result, as expected, is that monetary policy is, on average, tighter than in the quadratic (symmetric) case. Even when inflation is on target and the output gap is zero interest rates have a positive bias. In general, reaction functions are still linear with respect to output deviations (although at a higher level), whereas they are clearly non-linear with respect to inflation, specially for large positive inflation deviations. When inflation is very low, the reaction function is parallel to function of the quadratic case (in this case, because $a_L = 1$, they are practically equal). But when inflation is positive interest rates are increasingly higher in the former case.

Because interest rates are on average higher in the economy with asymmetric preferences it is also the case the output gap is, on average, smaller. That is, there is a bias against output.

4 Concluding Remarks

This paper has investigated how do optimal MP rules change when the CB considers alternative loss functions. In particular, it has evaluated the implications of considering a range target for inflation in a simple model calibrated to the Chilean economy.

The answers that this paper provides to the four questions we stated in the introduction of the paper are the following. First, inflation range targeting indeed yields aggressive MP responses so far as the range edges are not hard. If these edges are sufficiently hard a range target involves more aggressive optimal MP reactions. Numerically, a soft-edge range implies that interest rates move a third less than in the case of a point target. If the loss-function is the same as in the case of a point target but with losses being equal to zero when inflation lies inside the target range, then optimal MP does not change.

Second, there are some non linearities, but their economic relevance is not high. Interestingly, there no inaction zones for MP when there is range targeting. That is, MP should always react to shocks, even if inflation is well inside the target range.

Third, if inflation deviations from the target range are considered as highly undesirable events —i.e., the target range has hard-edges— then MP is more aggressive than

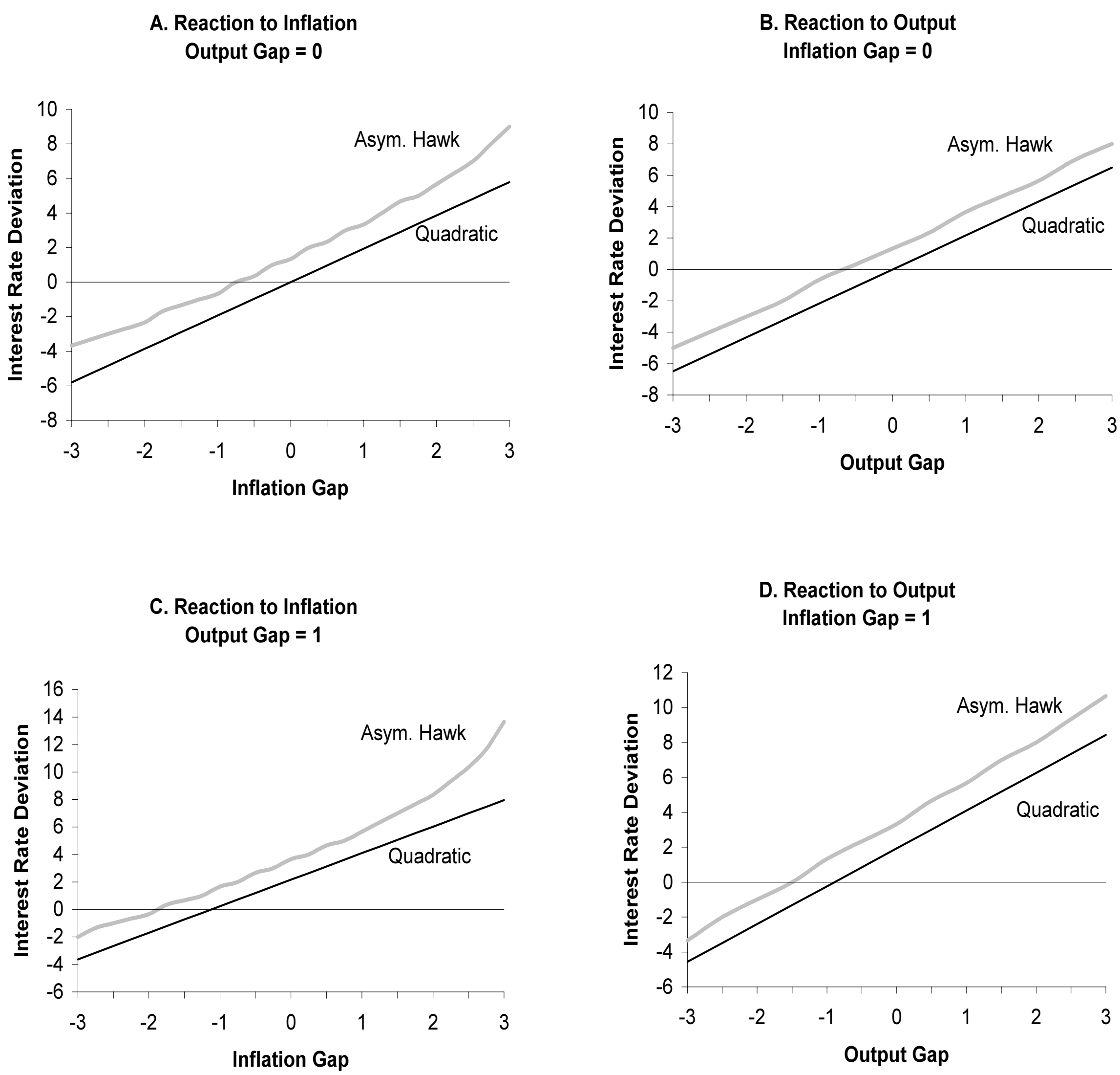

Figure 7: Optimal Policy Functions with an Asymmetric Loss-function

in the case of a point target, in which deviations are always undesirable, but to a less extent. Therefore, range targeting does not imply a less active MP. The optimal policy will be more or less aggressive depending on how hard are the range edges.

Finally, if positive inflation deviations are considered relatively more undesirable, optimal monetary policy is tighter than when the objective is symmetric. This means that in this environment the average output gap is negative.

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