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# FORWARD PREMIUM PUZZLE AND TERM STRUCTURE OF INTEREST RATES: THE CASE OF NEW ZEALAND

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## Abstract

Using monthly data for the United States dollar – New Zealand dollar exchange rate, this paper revisits the forward premium puzzle and applies a discrete no-arbitrage affine model of the term structure of interest rates to obtain historical estimates of the time-varying foreign exchange risk premium. The two-factor model is estimated via maximum likelihood for the period 1995-2006. The results of this study demonstrate that the modeled risk premium satisfies the required Fama's conditions, and its inclusion in an extended GARCH(1,1) model is significant in explaining both the mean and the volatility of the exchange rate. However, consistently with the extant literature, the estimated risk premium does not preclude the presence of the forward premium anomaly. Lastly, out-of-sample forecasts of the exchange rate for different specifications and time periods reveal that predictions of the proposed model for the exchange rate are far from the accuracy of a simple random walk specification.

## Resumen

Utilizando datos mensuales de la paridad entre el dólar de Estados Unidos y el dólar de Nueva Zelanda, este estudio revisa el puzzle del premio *forward* y aplica un modelo discreto de no arbitraje para la estructura de tasas de interés para obtener estimaciones históricas del premio por riesgo variable del tipo de cambio. Para ello, se estima un modelo de dos factores por máxima verosimilitud para el período 1995-2006. Los resultados de este estudio demuestran que el premio por riesgo estimado satisface las condiciones de Fama, y que su inclusión en una versión extendida del modelo GARCH(1,1) es significativa para explicar la media y la volatilidad del tipo de cambio. Sin embargo, y en línea con la literatura existente, el premio por riesgo estimado no evita la presencia de la anomalía del premio *forward*. Finalmente, las predicciones fuera de muestra del tipo de cambio para diferentes especificaciones y periodos de tiempo revelan que las predicciones del modelo propuesto en este estudio están lejos de la exactitud de un simple modelo de camino aleatorio.

## 1 INTRODUCTION

In recent years, numerous attempts have been made to resolve the long-standing empirical anomaly found in the biasedness of the forward exchange rate as a predictor of the future spot rate. In theory, Uncovered Interest Rate Parity (UIP) states that high yield economies are compensating investors for the risk of depreciation in their currency, therefore an increase (decrease) in domestic interest rates versus the foreign interest rates at time  $t$  signals an expected depreciation (appreciation) of the home currency versus the foreign currency in the future. Otherwise, investors would be able to make positive expected profits, called in the literature as carry trade returns, by borrowing in the lower interest rate currency, lending in the higher currency, and at maturity translating the payoff to the original currency to pay off the obligation.

Large carry trade returns in empirical portfolios document the failure of the UIP (Burnside, Eichenbaum, Kleshchelski and Rebelo, 2008). This departure in the UIP has been attributed basically to two sorts of explanations. The first kind refers to a possible breakdown of rational expectations, including models in the line of peso problems, irrational expectations and speculative bubbles<sup>1</sup>. The second argument, discussed by Fama (1984), introduces the existence of a time-varying risk premium as a consequence of the risk-averse behavior into the standard rational expectation model. Risk aversion among investors may lead them to demand higher foreign exchange risk premium in a foreign currency when the interest rate differential increases. In other words, the risk premium can vary over time and its effects on the exchange rate movements may overcome the effects of the interest rate differential. In this way, the forward premium puzzle emerges from the fact that the UIP equation omits the risk premium as an explanatory variable, and therefore the parameter associated to the forward premium (or interest rate differential) becomes negatively biased because it is capturing the negative impact of the risk premium into the depreciation rate.

The challenge of the modern literature has been to model, under alternative specifications, a risk premium sufficiently volatile in order to account for the high variation in the exchange rate. Recent studies, based on a multiple-currency setting of the term structure of the interest rates, have found new representations of the risk premium (Backus, Foresi and Telmer, 2001; Brennan and Xia, 2006; Dong, 2006; Benati, 2006; Graveline, 2006). The advantage of this framework is that relies on a reduced number of theoretical assumptions being an intermediate approach between traditional asset pricing models with a large number of restrictions and simple time series specifications. Research to date has tended to apply this method mainly to the most traded currencies pairs in the foreign exchange market, and there is not general agreement about the ability of this new framework to generate a risk premium consistent with the forward premium anomaly.

The main objective of this paper is to examine a more general model for the foreign exchange risk premium by fully exploiting the information of the term structure of interest rate for two countries. In particular, the key research question of this study is whether the estimated risk premium has power to explain the exchange rate movements and thus account for the forward premium anomaly in the context of a lesser traded currency parity. It is important to note that this paper tests the forward premium anomaly - and thus the failure in the UIP - only under

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<sup>1</sup> For a survey of the role of expectation see Jongen, Verschoor and Wolff (2008).

the risk premium explanation. Therefore, with the aim of seeking a time-varying risk premium in the foreign exchange market, rational expectations are strongly assumed<sup>2</sup>.

The theoretical framework of this investigation relies on a discrete no-arbitrage model that associates pricing kernels for different currencies with the future movements in the exchange rate. Following the work of Ang and Piazzesi (2003) and Dong (2006), the representation of the pricing kernel for each currency is established as a function of the short term interest rate and the market prices of risk. Thus, exchange rate depreciation not only depends on the interest rate differential but also is determined by the risk premium which at the same time is related to the market prices of risk. By assuming that the same factors that control the risk premium in the bond market of each country influence the risk premium in the foreign exchange market, the term structure of the interest rate has valuable information to characterize the market prices of risk. Therefore, an arbitrage-free affine model of the term structure of the interest rate in each country is employed to obtain the latent factors that identify the market prices of risk. The main advantage of this specification is that uses a small set of factors for characterizing the entire yield curve. However, a potential drawback is that its estimation many times requires the imposition of additional restrictions over the simple no-arbitrage assumption, reducing the forecasting performance for a number of currency rates (Diez de los Rios, 2009).

The empirical analysis is carried out in the bond and foreign exchange markets of New Zealand Dollar (NZD) and United States Dollar (USD) using monthly data for the period January 1995 to December 2007. Market prices of risk are inferred from the zero-coupon yield curve, using two unobservable factors which later are associated with the level and slope of the yield curve. Generalizing the study by Backus et al. (2001) and Brennan and Xia (2006), this paper includes a model with a set of eight different maturities of zero-coupon yields for the domestic and the foreign bond markets. Maximum likelihood estimation and numerical procedures are applied to generate the model parameters, whose results provide a good fitting to the actual yields data for both countries. Based on these outcomes, a time-varying risk premium is approximated as a quadratic function of the market prices of risk for each currency. The first important conclusion is that this definition of the foreign exchange risk premium satisfies the required Fama's (1984) conditions to replicate the forward premium anomaly.

Findings of this paper confirm the presence of the anomaly in the USD-NZD exchange rate, previously reported in the literature (Rae, 2000). A simple regression of the exchange rate return on the forward premium provides a negative and significant slope. Moreover, including the risk premium as an independent variable in an extended GARCH(1,1) model deepens the anomaly: the slope parameter is still negative and of higher magnitude in absolute terms. Ultimately, out-of-sample forecasting exercises report the not surprising result of a low prediction power of this model in comparison with a simple random walk specification for the foreign exchange rate.

The rest of the paper unfolds in the parts outlined here. The section which follows reviews the extant literature about the application of affine models for term structure of interest rates to the foreign exchange market and it describes the main contributions of this piece of research. Section 3 presents an affine model for foreign exchange returns and term structure of interest rates and its application to the valuation of exchange rate depreciation and risk

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<sup>2</sup> Engel (1996) stresses that the expression  $f_t - E_t s_{t+1}$  can be interpreted as risk premium only if agents have rational expectations.

premium. Section 4 describes the data and its main stylized facts. The methodology of the parameters estimation is also discussed with a complete description of the restrictions imposed in the maximum likelihood procedure. Section 5 analyses the empirical results of adding a time-varying risk premium in the forward premium regression and discusses its predictive power for the level of exchange rate. Finally conclusions and potential limitations of this study are presented in section 6.

## 2 LITERATURE REVIEW

The use of no-arbitrage affine models to a two-currency problem is relatively new in the exchange rate literature. Previously, foreign exchange risk has been studied with econometric models based on traditional general-equilibrium in line with Lucas (1982) or statistical settings that exploit the time series properties of the currency prices. However, as in the equity premium puzzle, the majority of these studies have failed to explain the risk premium with plausible levels of risk aversion parameters (Engel, 1996).

In discrete-time affine models of the term structure, introduced by Duffie and Kan (1996), yields are affine or linear functions of a set of unobservable variables or latent factors. One advantage of this specification is that it limits the assumptions only to a no-arbitrage condition in financial markets. Moreover, its linearity facilitates the extension of this framework to the currency price market. Preceding studies have included a linear relationship between the pricing kernel and some *observable* factors; however, the selection of these observable factors has been rather arbitrary and in general related to macroeconomic variables, such as: consumption growth, inflation rate, change in the short interest rate, and the long-short interest differential. This class of models is known as multivariate GARCH-in-mean and the main inconvenience when using them is the estimation of conditional covariance, which becomes extremely arduous if there are more than three factors (Cuthbertson and Nitzsche, 2004). What is more, Backus et al. (2001, p.288) demonstrate that GARCH-in-mean based on the conditional variance of the depreciation rate violates the symmetry condition and for this reason these models tend to fail in explaining the foreign exchange risk premium.

Alternatively, risk premium has been estimated with *unobservable* factors in affine models of the term structure. Vasicek (1977) and Cox, Ingersoll, and Ross (1985) are examples of single-factor models where the conditional covariance is a linear function of the unobservable factor, usually assumed to be the short interest rate. This structure though simple imposes some restrictions on the shape of the yield curve and the strong condition that all bond returns are perfectly correlated. An example of this model in the foreign exchange market is provided by Bansal (1997) who applies a single-factor term structure framework to analyze the forward anomaly, concluding that it is not possible to explain the negative slope coefficient in the exchange rate regression.

On account of the limitations generated by one-single factor models, researchers have focused their analysis to a multifactor setting. These models offer greater flexibility, although the challenge is the number of state variables to enter in the estimation and their economic interpretation. For instance, Ang and Piazzesi (2003) estimate a three-factor model of the term structure including not only unobservable factors but also macroeconomic variables. More specifically, they consider the inflation rate and real activity as additional variables in the term structure equation and use a factor representation of the pricing kernel. Nevertheless, the authors show that in the long end of the yield curve, and for long horizon predictions, unobservable factors continue to be the most important elements for the exchange rate variability. Consequently, this paper only considers unobservable factors in the affine model and not extends toward macroeconomic variables.

Recent literature employing two-country version of affine models to estimate consistent foreign exchange risk premiums embraces, inter alia, Backus et al. (2001), Benati (2006),

Brennan and Xia (2006), Wu (2007), and Graveline (2006). Although this approach has been one of the most promising streams to continue the analysis of the foreign exchange risk, empirical studies have failed to explain the forward premium anomaly.

Backus et al. (2001) formulate a discrete two-country version of the term structure model for currency-specific pricing kernels and translate Fama's (1984) conditions for risk premium into restrictions on pricing kernels. Based on the theory and data from the US Dollar-Sterling Pound exchange rates for the period 1974-1994, they find that the three-factor model could explain a negative slope in the forward-bias regression, but it is not successful in accounting for the anomaly, since it must either allow for some positive probability of negative interest rates or for asymmetric effects of state prices on interest rates in different currencies. In a more recent sample, Benati (2006) employs a similar representation for the pricing kernels in the USD-GBP exchange rate for the period 1980-2004 and he also concludes that it fails to explain the forward premium anomaly. Additionally, the author deduces that this model brings virtually no forecasting power for the depreciation rate. Wu (2007) uses data of countries that form the major currency blocs and shows that the slope coefficient on the interest rate differential is significantly negative. He suggests that the failure of dynamic term-structure models to produce a theoretically consistent UIP is due to the fact that foreign exchange markets are not fully integrated with the bond markets.

On the other hand, Brennan and Xia (2006) center their analysis on an *essentially affine model* of the term structure for the US Dollar, Canadian Dollar, Deutsche Mark, British Pound and Japanese Yen for the period 1985-2002<sup>3</sup>. They obtain a foreign exchange risk premium that satisfies the Fama's (1984) necessary conditions but the puzzle remains evident in all the parities that involve CAD and Yen currencies. An appealing finding of Brennan and Xia is that the foreign exchange risk premium can be approximated as a function of the domestic and foreign market prices of risk, and for to test this hypothesis the authors formulate an extended GARCH(1,1) model. This paper follows closely this formulation with the aim of estimating a risk-premium-adjusted version of the UIP.

Further extensions have been incorporated to the simple affine model in the line of including others variables than the unobservable factors or exploiting the information of a different market than the bond returns. For instance, Dong (2006) incorporates macroeconomic variables as factors in a term structure model for explaining the foreign exchange risk premium and the dynamic of exchange rates following the representation of Ang and Piazzesi (2003). Using monthly data for the German Mark and US Dollar parity for the period 1983-1998, he reveals an important role of the macroeconomic representation in the risk premium for matching deviations from UIP and an improved forecasting performance of the model for the exchange rate. In the same way, Chabi-Yo and Yang (2006) consider macroeconomic variables in the determination of the term structure of the interest rate and the exchange rate between Canadian Dollar and US Dollar from 1980 to 2006. Both studies, Dong and Chabi-Yo and Yang, are successful in justifying the dynamics of the exchange rate with macro aggregates. They conclude that the correlation between the model-implied depreciation rate and that subtracted from the data is between 20 and 60%, and the correlation between the model-implied exchange risk premium and its counterpart from the data is 25%.

Alternatively, Graveline (2006) uses exchange rate option prices and the term structure of the interest rates to estimate a dynamic arbitrage-free pricing model for the exchange rate in US

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<sup>3</sup> *Essentially affine models* allow for independent variations in risk premium and interest rates.

Dollar, British Pound, and Euro for the period 2001-2005<sup>4</sup>. He notices that using at-the-money option prices to estimate the model brings valuable information about exchange rate volatility and the risk premium in exchange rate returns, a finding that previous studies usually fail to explain. Panigirtzoglou (2004) uses option prices to estimate market prices of risk in a forward looking framework but he does not estimate explicitly the pricing kernels, since his study relies on historical and constant values of the correlation between pricing kernels.

Existent literature of the premium puzzle on lesser traded currencies, including New Zealand Dollar, is scarce and none of these studies bring into play the term structure of interest rate as the estimation model. For instance, Poghosyan, Kočenda and Zemčik (2008) develop stochastic discount factors in Armenia using a GARCH-in-mean method, but as discussed before, this model presents serious limitations to estimate a consistent risk premium. In the case of New Zealand, Rae (2000) confirms the presence of the anomaly in the parity USD-NZD for the period 1985-2000 and associates a time varying risk premium with the volatility in the NZD spot market. However, his estimates of the depreciation equation with this new factor do not resolve the puzzle. In another effort, Hawkesby, Smith and Tether (2003) approximate the New Zealand currency risk premium as a residual by subtracting from the interest rate differential the default and liquidity risk and the expected changes in exchange rates. Their results imply that over the nineties New Zealand faced a significant currency risk premium versus the United States.

This paper adds several contributions to the extant literature. Firstly, it provides empirical evidence of the forward premium puzzle in a new market. While a large and growing body of literature has investigated the implications for the five most traded currencies, to the author best knowledge, this is the first paper applied to New Zealand market in the context of affine models. Compared to other countries, New Zealand is a relatively small economy ranked in the place 53th in terms of Gross Domestic Product (World Bank, 2009) but its currency market is between the sixteen more actively traded. The high level of interest rates has contributed to be one of the favorite currencies, together with the Australian Dollar, for carry trade activity against the Japanese currency (Galati, Heath and McGuire, 2007). Secondly, recognizing that the failure of Rae (2000) in accounting for the anomaly in the New Zealand market could be based on a poor definition of the foreign exchange risk premium, this paper proposes an enhanced representation for the risk premium by considering the term structure of the interest rates and the stochastic discount factor model. Thirdly, different from previous studies that have used affine models to understand the risk premium (Backus et al., 2001; Brennan and Xia, 2006), the methodology applied in this research exploits the whole information embedded in the yield curve, and a set of eight different maturities of the zero-coupon yields are the source for the risk premium estimation in the bond and foreign exchange market.

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<sup>4</sup> An alternative article employing option prices to test the forward premium puzzle is Nikolaou and Sarno (2006); however, they do not analyze the anomaly in the context of affine term structure models.

### 3 THEORETICAL FRAMEWORK

This section defines the forward premium puzzle and pricing kernels and presents the affine models for exchange rates and term structure of interest rates. Particular attention is paid to the application of affine models to the valuation of exchange rate depreciation and risk premium.

#### 3.1 Forward premium puzzle

Theoretically, under the UIP market participants are risk-neutral and form their expectations rationally, therefore an increase (decrease) in domestic interest rates versus the foreign country interest rates at time  $t$  signals an expected depreciation (appreciation) of the home currency versus the foreign currency in the future. The UIP is generally represented by the following equation:

$$s_{t+n} - s_t = \beta_0 + \beta_1(r_{t,n} - r_{t,n}^*) + v_{t+n} \quad (1)$$

Where,  $s_{t+n} - s_t$  is the logarithmic change in the spot exchange rate (domestic price of foreign currency) over  $n$  periods,  $r_{t,n} - r_{t,n}^*$  is the difference between the  $n$  period domestic and foreign country interest rates, respectively, and  $v_{t+n}$  is a rational expectations error term. Under no-arbitrage conditions, the UIP can be extended to the Covered Interest Rate Parity (CIP), where the forward market is used as a hedge against the fluctuations in spot rates, and it is identified as the Unbiased Expectation Hypothesis (UEH) equation and is given by:

$$s_{t+1} - s_t = \beta_0 + \beta_1(f_{t,1} - s_t) + v_{t+1} \quad (2)$$

Where  $f_{t,1}$  is the logarithm of the forward rate at time  $t$  for delivery in  $n = 1$  period. The null hypothesis in testing this equation is that  $\beta_0 = 0, \beta_1 = 1$ , and the error term has a conditional mean of zero. The forward premium puzzle is based on the fact that empirical testing has found  $\beta_1$  to be significantly different from unity, and in many cases with negative values (Engel, 1996).

Understanding that  $s_{t+1} - s_t$  is the dependent variable and  $f_{t,1} - s_t$  the regressor, the slope parameter  $\beta_1$  in equation (2) might be expressed as:

$$\beta_1 = \frac{Cov(f_{t,1} - s_t, s_{t+1} - s_t)}{Var(f_{t,1} - s_t)} \quad (3)$$

At time  $t$ , the future value of the spot rate ( $s_{t+1}$ ) it is unknown. However, the assumption of rational expectation implies that:  $s_{t+1} - s_t = E_t s_{t+1} - s_t + \xi_{t+1}$ , where  $E_t$  represents the conditional expectations and  $\xi_{t+1} \sim \mathcal{N}(0, I)$ . Therefore, equation (3) can be defined as<sup>5</sup>:

$$\beta_1 = \frac{Cov(f_{t,1} - s_t, E_t s_{t+1} - s_t)}{Var(f_{t,1} - s_t)} \quad (4)$$

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<sup>5</sup> Considering that  $Cov(f_{t,1} - s_t, \xi_{t+1}) = 0$ .

In line with the work of Fama (1984), it is possible to divide the forward premium ( $f_{t,1} - s_t$ ) into two different components: (i)  $q_t$ , the conditional mean of the rate of depreciation ( $E_t s_{t+1} - s_t$ ) and (ii)  $p_t$ , the expected excess of return of the exchange rate, also called risk premium ( $f_{t,1} - E_t s_{t+1}$ ), such that:

$$\begin{aligned} f_{t,1} - s_t &= (f_{t,1} - E_t s_{t+1}) + (E_t s_{t+1} - s_t) \\ &\equiv p_t + q_t \end{aligned} \quad (5)$$

Replacing these parameters in equation (4) brings the following relationship:

$$\begin{aligned} \beta_1 &= \frac{Cov(p_t + q_t, q_t)}{Var(p_t + q_t)} \\ &= \frac{Cov(p_t, q_t) + Var(q_t)}{Var(p_t + q_t)} \end{aligned} \quad (6)$$

From equation (6) is obvious that  $\beta_1 < 0$  if  $Cov(p_t, q_t) + Var(q_t) < 0$ , which Fama (1984) translate into two necessary conditions on the moments of the foreign exchange risk premium. First, the covariance of the foreign exchange risk premium and the expected depreciation rate (or interest rate differential) should be negative and second, the variance of the risk premium must be higher than the variance of the expected depreciation rate:

$$Cov(p_t, q_t) < 0 \quad (7)$$

$$Var(p_t) > Var(q_t) \quad (8)$$

### 3.2 Pricing kernel

In a world without uncertainty, future payoffs can be translated into present value by the standard rational valuation formula, where the discount factor is the inverse of the risk-free rate. However, with risky assets, the discount factor must be adjusted by specific risks and its value is random or stochastic since it is not known with certainty in the current period.

The specific definition of the stochastic discount factor, also called the pricing kernel, depends on the asset pricing model. Backus et al. (2001, p. 283) refers to pricing kernel as "...essentially an intertemporal price. It represents the probability-weighted cost of receiving a state-contingent payoff sometime in the future". Under factor models, the pricing kernel could be defined as a linear function of the market portfolio (CAPM) or a set of macroeconomic factors (APT). In the same way, under consumption-based models (consumption-CAPM), the pricing kernel is defined as the marginal rate of substitution of current for future (discounted) consumption,  $M_{t+1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)}$ , and it depends on the preferences of agents between consumption today and consumption tomorrow (Cochrane, 2005; Cuthberston and Nitzsche, 2004). In the absence of a generally accepted asset pricing model, many authors have used direct specification for the pricing kernel coming from affine models, as it is discussed in the following subsection.

If a no-arbitrage environment is considered, there is a unique minimum variance pricing kernel,  $M_{t+1}$ , such that the price at time  $t$  of any payoff in the future is given by (Harrison and Kreps, 1979):

$$P_t = E_t[M_{t+1}X_{t+1}] \quad (9)$$

Where the nominal future payoff of the financial asset  $X_{t+1}$  is the summation of the price at the end of the period and any cash flow generated by the asset between period  $t$  and  $t + 1$  (e.g. dividends, coupons). Additionally, the gross return is defined by  $R_{t+1} \equiv \frac{X_{t+1}}{P_t}$ , then equation (9) can be written as:

$$1 = E_t[M_{t+1}R_{t+1}] \quad (10)$$

This equation is the basis for asset pricing models: only assuming no-arbitrage opportunities, the existence of a pricing kernel is assured and it can be used to price any asset in the economy. In an international asset pricing model with integrated markets, formula (10) must also be satisfied by a foreign currency return, with a foreign pricing kernel given by  $M_{t+1}^*$  such that:

$$1 = E_t[M_{t+1}^*R_{t+1}^*] \quad (11)$$

The foreign currency return (New Zealand Dollars) may be priced in terms of the domestic currency (US dollars) as:  $R_{t+1}^* = \frac{S_t}{S_{t+1}}R_{t+1}$ , where  $S_t$  denotes the spot exchange rate between USD and NZD. Therefore, under no arbitrage (law of one price) equation (10) must be equal to (11) once the foreign return definition has been incorporated, resulting in the following expression:

$$E_t[M_{t+1}R_{t+1}] = E_t\left[M_{t+1}^* \frac{S_t}{S_{t+1}} R_{t+1}\right] \quad (12)$$

The previous equation implies that there is a strong relationship between the rate of depreciation and the ratio of pricing kernels for both currencies. Backus et al. (2001) show that under complete markets for currencies and state-contingent claims, the following relationship must be hold<sup>6</sup>:

$$\frac{M_{t+1}^*}{M_{t+1}} = \frac{S_{t+1}}{S_t} \quad (13)$$

Or equivalently in logarithm terms:

$$s_{t+1} - s_t = m_{t+1}^* - m_{t+1} \quad (14)$$

In other words, given processes for the rate of depreciation and the domestic pricing kernel, it is possible to use equation (14) to derive a foreign pricing kernel consistent with the Stochastic Discount Factor (SDF) model. Alternatively, with the definition of the domestic and foreign pricing kernels, the exchange rate change can be obtained. In summary, these three variables are dependent and knowing the process of two of them permit to infer the third one.

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<sup>6</sup> The equivalent approach is developed in Graveline (2006) and Dong (2006).

### 3.3 Affine models for foreign exchange returns

Affine or linear models in the bond pricing theory have been broadly employed in the recent financial literature, in particular for its simplicity in finding close form solutions. This section presents an adaptation to currencies of a discrete-time essential affine model, following the work of Ang and Piazzesi (2003) and Dong (2006). In these papers, the authors use the following discrete factor representation of the nominal pricing kernel for both the domestic and the foreign economy:

$$M_{t+1} = e^{m_{t+1}} = e^{-r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1}} \quad (15)$$

$$M_{t+1}^* = e^{m_{t+1}^*} = e^{-r_t^* - \frac{1}{2} \lambda_t^{*'} \lambda_t^* - \lambda_t^{*'} \varepsilon_{t+1}} \quad (16)$$

Where,  $m_{t+1}$  is the logarithm of the nominal pricing kernel,  $r_t$  is the nominal short rate,  $\lambda_t$  is a  $k \times 1$  vector with the time-varying market prices of risk associated with the source of uncertainty  $\varepsilon_t$ , and  $\varepsilon_{t+1}$  is a  $k \times 1$  vector of shocks to the unobservable state variables,  $Z_t$ <sup>7</sup>.  $\lambda_t$  is called the market price of risk given that it describes how  $m_{t+1}$  responds to the shock  $\varepsilon_t$  and is also referred as the volatility of the pricing kernel since it corresponds to the excess return per unit of volatility. The number of latent factors ( $k$ ) will be determined using statistic procedures (principal components). The specific process of each variable is explained as follows.

State variables are unobservable or latent factors, represented by a first order Vector Autoregressive process (VAR):

$$Z_t = \mu + \Phi Z_{t-1} + \Sigma \varepsilon_t \quad (17)$$

Where  $\Phi$  is a stable matrix with positive diagonal elements and  $\Sigma$  is a diagonal matrix with the time variation of the volatilities of the state variables.  $\varepsilon_t \sim \mathcal{N}(0, I)$  is a structural shock to the latent factors and are assumed to be uncorrelated  $E(\varepsilon_{t,i}, \varepsilon_{t,j}) = 0$ . The short term interest rate and the time-varying market prices of risk are assumed to be an affine function of the state variables, with the following parameters<sup>8</sup>:

$$r_t = \delta_0 + \delta_1' Z_t \quad (18)$$

$$\lambda_t = \lambda_0 + \lambda_1 Z_t \quad (19)$$

Where  $\delta_0$  is a constant term,  $\delta_1$  is a  $1 \times k$  vector,  $\lambda_0$  is a  $k \times 1$  vector and  $\lambda_1$  a  $k \times k$  matrix. Models where the market prices of risk depend directly on the latent factors, and not only through factor volatilities, are richer dynamic term structure models (Dai and Singleton, 2002).

With the preceding formulation and the definition in equation (5), it is possible to express the conditional mean of the depreciation rate ( $q_t$ ) and the risk premium ( $p_t$ ) in terms of domestic and foreign pricing kernels, which at the same time are function of the market prices of risk. Thus, the expected rate of depreciation is characterized by:

<sup>7</sup> The foreign variables, represented with an asterisk, have the same meaning as the domestic ones.

<sup>8</sup> Equations (17) to (19) have a similar representation for the foreign variables, but they are not included here for simplicity.

$$\begin{aligned}
q_t &= E_t s_{t+1} - s_t = E_t m_{t+1}^* - E_t m_{t+1} \\
&= r_t - r_t^* + \frac{1}{2}(\lambda_t' \lambda_t - \lambda_t^{*'} \lambda_t^*)
\end{aligned} \tag{20}$$

It is easy to note that if investors are non risk-neutral ( $\lambda_t \neq 0$  and  $\lambda_t^* \neq 0$ ), the depreciation rate is different to the spread in interest rates and the equation (20) is essentially the UIP with an additional term corresponding to the foreign exchange risk premium. In this model, exchange rate is heteroskedastic since the market prices of risk affect not only the drift but also the volatility<sup>9</sup>.

On the other hand, the risk premium becomes:

$$\begin{aligned}
p_t &= f_{t,1} - E_t s_{t+1} = r_t - r_t^* - E_t s_{t+1} + s_t \\
&= -\frac{1}{2}(\lambda_t' \lambda_t - \lambda_t^{*'} \lambda_t^*)
\end{aligned} \tag{21}$$

Strictly speaking,  $p_t$  denotes the negative of the risk premium term. Dong (2006) emphasizes that UIP remains if market prices of risk are zero, and then the risk premium is equal to zero, or if market prices of risk are constant, and so the risk premium is constant.

Reinterpreting Fama (1984) conditions in terms of market prices of risk, equations (7) and (8) imply that market prices of risk must be more volatile, and negatively correlated, than the interest rate differential.

### 3.4 Bond pricing

So far, the definitions of depreciation rate and risk premium are functions of unknown variables: the market prices of risk. By assuming that the same factors that determine the risk premium in the bond market of each country help to determine the risk premium in the foreign exchange market, the term structure of the interest rate has valuable information to characterize the market prices of risk. Hence, a discrete-time affine model of the term structure of interest rates is discussed in this subsection. Under this model, introduced by Duffie and Kan (1996), yields are affine or linear functions of a set of state variables. Fixed-income securities are easy to price with this framework, and term-structure models are equivalent to time-series models for the stochastic discount factor (Campbell, Lo and Mackinlay, 2007).

Assuming that  $P_t^n$  is the price of a pure discount bond at time  $t$ , with maturity  $n$  and total payment of \$1 at maturity. At the end of the next period ( $t + 1$ ), the price of this bond will be:  $P_{t+1}^{n-1}$ . Note that after one period, the remaining maturity of the bond is  $n - 1$ . As any other assets return, the holding period gross return of this bond satisfies the pricing formula (10) such that:

$$1 = E_t[M_{t+1}R_{t+1,n}] \tag{22}$$

And replacing for the definition of return  $R_{t+1,n} = \frac{P_{t+1}^{n-1}}{P_t^n}$ , this expression becomes:

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<sup>9</sup> Before taking expectation, the depreciation rate is equal to:  $r_t - r_t^* + \frac{1}{2}(\lambda_t' \lambda_t - \lambda_t^{*'} \lambda_t^*) + (\lambda_t' - \lambda_t^{*}')\varepsilon_{t+1}$ , where  $\lambda_t$  and  $\lambda_t^{*'}$  are also part of the volatility term  $(\lambda_t' - \lambda_t^{*}')\varepsilon_{t+1}$ .

$$P_t^n = E_t[M_{t+1}P_{t+1}^{n-1}] \quad (23)$$

Ang and Piazzesi (2003) identify that this discrete-time Gaussian model is part of the class of affine term structure models in which bond prices are exponential linear functions of the latent factors,  $Z_t$ , and it is represented by (Appendix A):

$$P_t^n = e^{A_n + B_n' Z_t} \quad (24)$$

It is easy to demonstrate that the yield to maturity, continuously compounded, is also a function of the latent factors:

$$Y_{t,n} = -\frac{\log P_t^n}{n} = -\tilde{A}_n - \tilde{B}_n' Z_t \quad (25)$$

Where  $\tilde{A}_n = \frac{A_n}{n}$  and  $\tilde{B}_n = \frac{B_n}{n}$ . Considering the definition of  $M_{t+1}$  as in equation (15), the coefficients or factor loadings can be estimated as:

$$A_{n+1} = -\delta_0 + A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}B_n'\Sigma\Sigma'B_n \quad (26)$$

$$B_{n+1} = -\delta_1' + B_n'(\Phi - \Sigma\lambda_1) \quad (27)$$

Where  $A_n$  and  $B_n$  are  $k \times 1$  vectors. Details of how to derive these difference equations are provided in Appendix A. Under the Expectation Hypothesis of the yield curve, long rates depend on expectation of short rates and on the term premium that is constant for all maturities. On the other hand, under the SDF model long rates depend on a risk premium that varies with the conditional covariance between the pricing kernel and a sequence of long rates of different maturities, and it is consistent with no-arbitrage opportunities. As it can be seen, market prices of risk parameters ( $\lambda_0, \lambda_1$ ) affect in different ways to the yield curve.  $\lambda_0$  only impacts the long-run average of yields, because is part of the constant coefficient of the yield equation ( $A_{n+1}$ ) but not from the slope equation. Conversely,  $\lambda_1$  determines the slope of the curve ( $B_{n+1}$ ), and thus affect the time-variation of the risk premium.

The term structure model developed in this paper follows closely the exposition of Ang and Piazzesi (2003) and Dong (2006), except for one important difference: the affine model in this study supposes that all movements in bond prices are covered only by latent factors and it is not extended toward macro variables. According to the authors, macro variables may give an economic interpretation to the driving factors of bond yields; however, Ang and Piazzesi (2003) demonstrate that for longer maturities unobservable factors are still the most important in explaining the bond yield volatility.

## 4 ESTIMATION

This section describes the data and its main stylized facts. The methodology of the parameters estimation is also discussed with a complete description of the restrictions imposed in the maximum likelihood procedure.

### 4.1 Data and descriptive statistics

Monthly data of USD–NZD exchange rate and nominal zero-coupon yields to eight different maturities for each economy are obtained from Datastream database. Values correspond to the end of the month close quotes. The data spans the period January 1995 to December 2007, totaling 156 monthly observations, and it is limited to the availability of NZD yield curve information. The last fourteen months - November 2006 to December 2007 - are excluded from the estimation in order to implement an out-of-sample forecasting exercise. Monthly frequency is employed because this study assumes that the one-month yield is the observable short term rate. However, working with monthly frequency could prevent capturing short periods of turbulence and speculative actions in the foreign exchange market; besides, the moderate number of observations might generate some finite sample bias. Consequently, the inference process has been carried out considering the Newey and West (1987) correction in order to ensure consistency in the parameter estimates, although the finite sample bias could remain<sup>10</sup>.

Both New Zealand and North American yields are presented in Figures 1 and 2, for the following maturities: 1, 3, 6, 12, 24, 36, 60 and 120 months. Rates are annualized and continuously compounded. When zero-coupon yields are not directly observed in the market, they have to be extracted from coupon paying bonds and their calculation is not exempt of errors. In general, the process involves the estimation of a constant rate that makes equal the market price of a bond with the present value of the streams of coupon payments and redemption value, for a given maturity. Nevertheless, this rate, called yield to maturity (YTM), is not a correct measure of the return of a bond. It assumes that investors are able to reinvest the coupon payments at the same rate in every single period over the life of the bond, and it corresponds to an average rate, usually higher or lower than the effective rate in each period. These problems are accentuated for yields of longer maturities.

Some stylized facts are identified from the basic statistics of the yield curves (Table 1). Firstly, both countries present upward sloping yield curve. For instance, one-month USD (NZD) yield is 4.13% (6.9%) and increases to 5.98% (7.38%) for a 10-year horizon. Secondly, standard deviation decreases with longer maturity. This effect is particularly notorious for the NZD yield curve which presents almost a 50% reduction in the estimated volatility between the shortest and the longest maturity. Finally, autocorrelation of bond yields is above 0.89 for each horizon, and the correlation between yields of different maturities is also highly persistent, in especial for yields with closer maturities (Table 2). The shortest and longest yield for USD (NZD) have just a 0.68 (0.46) correlation coefficient, whereas the correlation with the closest maturity yield is always above 0.8. Jarque-Bera test rejects the null hypothesis of normality only for longer maturities; in consequence, the

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<sup>10</sup> Small sample distributions based on bootstrapping methodology have been used in previous studies to overcome this bias. In general, they agree in a high concordance between the small sample estimates and the real data (Dong, 2006).

assumption of normal distribution is reasonable for modeling the dynamic of the yield curve (Ang and Piazzesi, 2003).

In terms of exchange rate data, the New Zealand Dollar is a floated currency from March 1985 when its initial rate was set at 0.4444 United States Dollars (USD)<sup>11</sup>. Since that time, the parity has fluctuated freely and nowadays it is the sixteenth currency more traded in the world (BIS, 2007). During the sample period, the parity reached a maximum value of 0.7819 USD per NZD at the end of 2007, and it fell down to 0.3973 USD per NZD in 2001 (Figure 3). Its sample statistics (skewness: -0.36 and kurtosis: 1.89) provides evidence of an empirical distribution far from a normal distribution (Table 3). As a consequence, Jarque-Bera test rejects the null hypothesis of normality (Jarque-Bera statistic=11.4, Prob. =0.003).

Table 3 also depicts selected sample moments of the depreciation rate of the USD-NZD parity and the forward premium, where the rate of depreciation is computed as the difference in the logarithm of exchange rate for the month  $t + 1$  and the month  $t$ , and the forward premium is approximated as the difference in the logarithm of the one-month USD and NZD interest rate, assuming that CIP holds<sup>12</sup>. The first inference from their sample moments is that the average value of the depreciation rate is almost zero (0.1%), but it has higher volatility (3.3%) than the forward premium (0.1%), consistent with the findings of previous literature (Backus et al., 2001; Brennan and Xia, 2006). Alternatively, forward premium exhibits high persistence, with an autocorrelation coefficient of 0.96, whereas the depreciation rate autocorrelation is almost inexistent. Under the CIP, the forward rate and as a consequence the forward premium should be an optimal predictor of the exchange rate changes; however, the variance of the forward premium is not higher enough to account for the extreme variation of the depreciation rate (Figure 4). This result underpins the existence of a time-varying risk premium that captures the unexplained volatility of the exchange rate. Aside from the elevated variation in the exchange rate, there is also evidence of volatility clustering. In the course of the first two years (1996-1997) the volatility in the USD-NZD rate reached an average of 1.6% and it was mostly decreasing after the Reserve Bank of New Zealand set up the Trade-weighted Index (TWI) as its main policy lever<sup>13</sup>. The next period (1998-2002) is depicted by a continuous increment on the volatility following the Asian crisis, with an average of 2.6%. Lastly, the period from 2003 to 2007 is highly volatile with levels close of 3.2%. Observed data on exchange rate volatility confirms the assumption of heteroskedasticity of the model.

Empirical literature has shown that both spot and future exchange rates are well characterized by a unit root process (Ballie and Bollerslev, 1989), and the USD-NZD parity is consistent with this feature. Results for unit root tests according to Augmented DF (Dickey and Fuller, 1979), PP (Phillips and Perron, 1988) and KPSS (Kwaitkowski, Phillips, Schmidt and Shin, 1992) are presented in Table 4. For ADF and PP, the null hypothesis of unit root cannot be rejected for the level of exchange rate and yields to different maturities, but it is strongly rejected for the first difference of the variables. Likewise, KPSS test rejects the null of stationary process for the level of the series, but this hypothesis cannot be rejected when the test is performed on first differences. Although stationary process is rejected for the level of yields, theory suggests that interest rates cannot follow a unit root process since are functions

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<sup>11</sup> Due to the indirect quotation of the New Zealand Dollar, the exchange rate definition sets the US Dollar as the domestic currency.

<sup>12</sup> Interest rates are divided by 12 in order to convert them in monthly rates.

<sup>13</sup> In January 1999, the Reserve Bank revised the method used to calculate the Trade-Weighted Index (TWI) measure of the New Zealand dollar and announced plans to reweight the TWI annually.

of the natural interest rate and the expected inflation rate, both series with a mean-reverting behavior. Besides, empirical findings of unit root in interest rates might be the consequence of a small sample problem. Cochrane (2005, p.199) points that "...in an estimate and test that uses the level of interest rates the asymptotic distribution theory might be a bad approximation to the correct finite sample distribution theory". Consequently, this study considers interest rates as stationary series, and the analysis will be based on the level of the interest rates instead of their first differences.

#### 4.2 Estimation methodology

The estimation procedure encompasses three steps: (i) testing of the UEH; (ii) modeling of the foreign exchange risk premium and other parameters of interest using bond yields information; and (iii) estimation of an extended version of the UIP that include the risk premium estimates into the depreciation rate equation.

##### *Testing the Unbiased Expectation Hypothesis*

The first model consists in a regression of the one-month depreciation rate onto the forward premium. This is estimated by Ordinary Least Squares (OLS) according to equation (2). The forward premium has been replaced by the one-month interest rate differential, as suggested by the UIP. Results of this estimation are discussed in section 5.1.

##### *Modeling the foreign exchange risk premium*

In the second step, the number of parameters to be estimated comes from the system (17)-(19), both for United States and New Zealand, and are represented by the vector  $\Psi = \{\lambda_0, \lambda_1, \delta_0, \delta_1, \mu, \Phi, \Sigma, \Omega\}$ , where  $\Omega$  is the matrix of errors for the non observable yields. Vector  $\Psi$  is calculated by Maximum Likelihood Estimation (MLE) using equation (25) and taking for granted the likelihood function derived in Ang and Piazzesi (2003). The numerical optimization of the likelihood function is computed via the MATLAB subroutine `fminsearch.m`. MLE is asymptotically efficient, although its finite-sample properties in the context of affine models are not clear (Duffee and Stanton, 2004).

The estimation of  $\Psi$  requires to know the number of state variables ( $Z_t$ ) which is approximated by principal components analysis. Sahut and Mili (2008) also apply principal components to determine the number of common factors that influence risk premium on international bond markets. According to eigenvalues of zero-coupon yields, the first two components explain 99.1% of the USD yields variation and 98.3% of the NZD yields (Table 5). Diebold, Rudebusch and Aruoba (2006) argue that two factors are enough to capture the shape of the yield curve, since the third factor is usually associated to heteroskedasticity. Previous studies have related these latent factors with the "level" and "slope" of the yield curve (Dai and Singleton, 2000)<sup>14</sup>. However, these two state variables are still unobservable. For identification purpose, this paper follows Chen and Scott (1993) approach. This method is faster and more efficient than alternative ones (Kalman's (1960) filter) and it has been broadly used by earlier studies<sup>15</sup>. As the number of zero-coupon yields of different maturities ( $N = 8$ ) exceeds the number of unobservable factors ( $k = 2$ ), Chen and Scott approach implies that certain yields are considered to be fully identified without measurement errors

<sup>14</sup> In the case of three-factor models, the last factor is associated with the "curvature" of the yield curve.

<sup>15</sup> For example, Ang and Piazzesi (2003) follow Chen and Scott method in their three-factor model and select the 1, 12 and 60-month yields to be measured without error.

$(Y^{ne}_{t,n})$ , and the remaining yields are expected to present miscalculation ( $Y^e_{t,n}$ ), and they can be represented by the following equations:

$$Y^{ne}_{t,n} = -\tilde{A}_n - \tilde{B}'_n Z_t \quad (28)$$

$$Y^e_{t,n} = -\tilde{A}_n - \tilde{B}'_n Z_t + \Omega_t \quad (29)$$

In particular, the existence of two latent factors entails the selection of two yields to be measured accurately: the 3- and 60-month yields<sup>16</sup>. Therefore,  $ne = \{3, 60\}$ ,  $e = \{1, 6, 12, 24, 36, 120\}$  and  $\Omega_t$  is an *IID* error uncorrelated across  $Y^e_{t,n}$ . For a given vector  $\Psi$ , the latent factors  $Z_t$  can be inferred from the precisely observed yields  $Y^{ne}_{t,n}$  inverting equation (28), such that:

$$Z_t = \tilde{B}_n^{-1}(-\tilde{A}_n - Y^{ne}_{t,n}) \quad (30)$$

This procedure looks straightforward, though it is highly complex because of the elevated number of parameters, many of which are non-linear and subject to several restrictions. Thus, the optimization of the likelihood function requires appropriate starting values in order to achieve convergence in the estimation, especially when the function seems to have more than one local maximum. To reduce these complexities, the following restrictions are imposed to the starting values.

For the market prices of risk, starting values for  $\lambda_0$  and  $\lambda_1$  are restricted to be zero which is equivalent to assume that UIP holds. Additionally, in the short rate formulation (Eq.18), the constant coefficient  $\delta_0$  represents the unconditional mean of the one-month yield, with values of 0.34% for USD and 0.57% for NZD. The slope  $\delta_1$  is constrained to be a matrix of ones, following the canonical representation of an affine term structure model of Dai and Singleton (2000, 2002). Finally, a general parameterization of the state variables process is adopted: the constant  $\mu$  is imposed to be zero; the variance matrix  $\Sigma$  is a diagonal matrix; and the  $2 \times 2$  matrix  $\Phi$  is lower-triangular, whose elements are estimated using a first order VAR of  $Y^{ne}_{t,n}$ . However, both yields included in  $Y^{ne}_{t,n}$  have a correlation coefficient far from zero (0.86 for USD and 0.72 for NZD) and violate the orthogonality condition of the latent factors. Hence, the second latent factor is approximated by the spread of the five-year and the three-month yield, and the VAR model is estimated with the three-month yield and this spread.

MLE is performed as a past dependence optimization. That is, with the starting values presented in Appendix B, a first set of estimated parameters ( $\Psi^I$ ) is obtained in conjunction with their minimum and maximum boundaries. Afterward, a second optimization process employs  $\Psi^I$  as starting values and produces a second set of optimal parameters  $\Psi^{II}$ . This routine is repeated four times before the final coefficients are reached. At last, these parameters are employed in the factor loadings equations (26)-(27) for the yield curve of each country.

#### *Estimating an extended version of the UIP*

The third and final step in the estimation methodology is the inclusion of the risk premium in the exchange rate regression. In line with the work of Brennan and Xia (2006), an extended GARCH(1,1) is estimated. This specification includes as independent variables the forward

<sup>16</sup> One-month yield is not selected because it has liquidity problems, while the ten-year yield exhibits high term premium.

premium and the time-varying risk premium that has been computed as a quadratic function of the domestic and foreign market prices of risk. Due to the high persistence in the regressors, levels, leads and lags of their first difference are added<sup>17</sup>.

The mean equation of the spot rate can be expressed as follows:

$$\begin{aligned}
s_{t+1} - s_t = & c_0 + c_1(f_t - s_t) + c_2\lambda'_t\lambda_t + c_3\lambda_t^*\lambda_t^* + \sum_{p=j}^J c_{4,p}\Delta(f_{t-p} - s_{t-p}) \\
& + \sum_{p=j}^J c_{5,p}\Delta\lambda'_{t-p}\lambda_{t-p} + \sum_{p=j}^J c_{6,p}\Delta\lambda_{t-p}^*\lambda_{t-p}^* + \omega_t
\end{aligned} \tag{31}$$

Where  $\Delta$  is the first difference operator and  $\omega_t$  is the error term. If the true values of  $\lambda_t$  and  $\lambda_t^*$  were known, the parameters  $c_2$  and  $c_3$  should be restricted to  $c_2 = -c_3 = \frac{1}{2}$ .

On the other hand, under the no-arbitrage hypothesis in integrated markets, the variance of the error term is expected to be also a function of the market prices of risk, such that:

$$\sigma_{s,t}^2 = d_0 + d_1\omega_{t-1}^2 + d_2\sigma_{s,t-1}^2 + d_3\lambda'_t\lambda_t + d_4\lambda_t^*\lambda_t^* \tag{32}$$

The null hypothesis is  $d_1 = d_2 = 0$ , which implies that GARCH effects are not relevant and the spot rate volatility is only driven by the foreign exchange risk premium, as the theoretical relation entails (Brennan and Xia, 2006).

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<sup>17</sup> Brennan and Xia (2006) include two leads and two lags arguing that the Johansen (1991) test rejects the null hypothesis of no cointegration between the depreciation rate and the market prices of risk.

## 5 EMPIRICAL RESULTS

This section analyses the empirical results of a simple forward premium regression and an extended version incorporating a time-varying risk premium as independent variable. The predictive power of the last model for the level of exchange rate is also discussed.

### 5.1 Basic forward premium regression

A simple OLS model explaining the depreciation rate in terms of the forward premium for the period January 1995 to October 2006 is presented in Table 6<sup>18</sup>. The first conclusion that emerges from the estimated parameters is that the forward premium anomaly is also present in the USD-NZD foreign market. The coefficient of the one-month forward premium is different from one, and even negative (-3.222) at 10% level. This estimation seems more negative than previous studies on USD-NZD parity. Rae (2000) obtains a coefficient of -1.465 for the period August 1986 to April 2000, while Gibbs, Grimes and Harrison (1990) estimate -2.32 over the period July 1986 to June 1990. The general rejection of the unbiased hypothesis could be related with the statistical features of the exchange rate discussed in section 4.1; however, recent evidence demonstrates that even using an improved statistical methodology that accounts for both non-stationarity and non-normality in exchange rates does not resolve the anomaly for a set of forward rates and horizons (Aggarwal, Lin and Mohanty, 2008).

Predictability of the exchange rate is supported in view of the fact that F-test rejects the hypothesis of both coefficients (constant and slope) being equal to zero<sup>19</sup>. What is more, the presence of volatility clustering in the exchange rate variations might facilitate its prediction once some patterns are identified. These features shed lights into potential limitations of the simple OLS method. Non-normality together with ARCH effects on the exchange rate produces OLS estimates that are consistent but not efficient<sup>20</sup>.

In the pursuit of superior models that capture accurately the exchange rate properties, an affine model of the term structure of interest rates is analyzed. The next subsection presents the estimated parameters and subsection 5.3 discusses the results of including a time-varying risk premium in the exchange rate equation.

### 5.2 Foreign exchange risk premium

The estimated parameters of the vector  $\Psi$  are presented in Table 7. The table also reports their respective standard errors which are calculated by Hessian matrices over the period January 1995 to December 2007<sup>21</sup>. Panel A in Table 7 contains the market prices of risk for USD and NZD<sup>22</sup>. The vector  $\lambda_0$  has one significant parameter corresponding to the first factor for United States, whereas both estimates for New Zealand are not different from zero. Slope

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<sup>18</sup> Similar regressions to different maturities of interest rates are calculated (Table 6). The forward premium bias is observed even in longer horizons where is expected to see higher influence of fundamental variables. Literature exploiting the term structure of forward premiums to explicate the future changes in spot rates includes Clarida and Taylor (1997), Clarida, Taylor, Sarno and Valente (2003), and Nucci (2003).

<sup>19</sup> The F-statistic is 2.8 and it is rejected at 10% of significance.

<sup>20</sup> Rae (2000) estimates an ARCH(4) process obtaining once more a negative slope coefficient.

<sup>21</sup> Note that the estimation is executed with all the sample period because the vector  $\Psi$  is an independent variable in the GARCH(1,1) and it does not affect the forecasting exercise.

<sup>22</sup>  $\lambda_t^1$  and  $\lambda_t^2$  are the basic inputs for the estimated time-varying risk premium.

coefficients  $\lambda_1$  are not statistically significant. In terms of magnitude, the  $\lambda_0$  parameters for United States are almost three times bigger than for New Zealand, but the opposite pattern is observed in the case of  $\lambda_1$ . Historical estimates of market prices of risk for each country are noticeably different both in levels and trends (Figure 5). USD has an annual average of 148%, much higher than the 16% for NZD. In terms of trend, both countries exhibit a reduction during 1999-2000; however, for USD this started at the end of 2000 and it remained until 2004. On the other hand, NZD market price of risk has displayed more variability: it was appreciably low until 1998 then it increased all of a sudden reaching its peak value at the beginning of 1999, and after a second increment in 2002-2003, it returned to its previous levels. Rae (2000) associates this rise with large forecasting errors around that time owing to the collapse of currencies in Asia, an event which also hit the NZD, and the easing of monetary policy by the New Zealand Reserve Bank.

Panel B in Table 7 reports the values for the factor dynamics included in matrix  $\Phi$ . It can be seen that latent factors are very persistent. For both countries the parameter associated with the lag in the latent factor is close to unity: the lag parameter of  $Z_1(Z_2)$  in USD is 0.98 (0.97), and for NZD the same coefficient is 0.99 (0.96). Furthermore, lagged values of  $Z_1$  are statistically significant for the process of  $Z_2$ . Finally, Panel C shows the volatility matrix  $\Sigma$  for the latent factors.

From equation (17), the unobservable factors for each country are estimated. For both economies,  $Z_1$  exhibits higher volatility than  $Z_2$ , and it is considerable higher than the figures of the second factor (Figures 6 and 7). As mentioned before, latent factors can be associated with some features of the yield curve. In particular,  $Z_1$  is highly correlated with the “level” of the yield curve (correlation coefficient of 0.97 for USD and 0.95 for NZD), and  $Z_2$  is associated to the “spread” (correlation of 0.39 for USD and 0.37 for NZD), where “level” is defined as the average of the 3-month and the 5-year yields, and “spread” is the difference between the 5-year and 3-month yields.

After that, factor loadings of the term structure are computed following equations (26) and (27). The slope coefficient  $\tilde{B}_n$  represents the response of yields to changes in the latent factors and it hinges on the market prices of risk, the dynamic of the latent factors and the sensibility of the short rate to the unobservable factors. Estimated values of  $\tilde{B}_n$  are depicted in Figures 8 and 9. With the aim of facilitate the interpretation, the values symbolize one standard deviation of the respective factor (Ang and Piazzesi, 2003). The first inference of the Figures is that the slope coefficients offer similar patterns in both economies, though USD coefficients seem mildly higher. Secondly, slope coefficient related to  $Z_1$  is upward sloping, with decreasing rates. Thus, the sensibility to  $Z_1$  shocks is stronger in shorter yields and becomes smooth at longer maturities. Conversely, the second factor parameter is downward sloping and flatter for yields with prolonged maturities.

Afterward, the implied yields from the model are estimated and a simple eyeballing test suggests an extraordinary accuracy in the model-generated values (Figures 10 and 11). In both countries the estimated yields follow closely the observed ones and as it is reported in Table 8, the measurement errors ( $\Omega_t$ ) of the six yields assumed to be measured with inaccuracy are almost zero. The greater error is generated in the one-month yield estimation for USD with a mean value of 2 basis points (bp), and the rest quantities are lesser than 0.5 bp. Likewise, the standard deviation is in the range 1 to 3 bp. Overall, these results indicate the high goodness-of-fit of the model.

The ultimate and most important implication of the two-country affine term structure model is the foreign exchange risk premium derivation. From equation (21), the risk premium is equal to  $\frac{1}{2}(\lambda_t' \lambda_t - \lambda_t^{*'} \lambda_t^*)$ , which is an affine or linear function of the latent factors that have been extracted from the domestic and foreign bond returns. The USD-NZD conditional mean of the depreciation rate and its components are plotted in Figure 12. For construction, the time-varying risk premium ( $-p_t$ ) and the conditional mean of the depreciation rate ( $q_t$ ) are similar. The only difference between both series is the interest rate differential which is mainly negative, therefore  $q_t$  is to some extent smaller than the absolute value of  $p_t$ . In terms of size, the time-varying risk premium for USD-NZD is significantly higher than the interest rate differential accounting for the fact that during this period US investors have required a positive premium to invest in the New Zealand currency (Hawkesby, Smith and Tether, 2003; Cappiello and Panigirtzoglou, 2008).

A well-behaved estimate of the foreign exchange risk premium must satisfy the Fama's (1984) necessary conditions. To recall, these conditions encompass a negative correlation between the risk premium and the expected depreciation rate, and at the same time, a higher volatility of the former respect to the latter. Table 9 contains the estimated moments of both variables. The mean value of the risk premium is more volatile than depreciation rate, and both are negatively correlated, with a coefficient close to unity (-0.985). As a consequence, the extant estimate of risk premium based on bond prices would be a good candidate to account for the forward premium puzzle.

### 5.3 Extended forward premium regression

This subsection revises if the forward premium anomaly remains once a time-varying risk premium has been included. In first place, a simple OLS regression that incorporates both the forward premium and the risk premium in the UEH equation is estimated (Table 10). The components of the risk premium, this is the market prices of risk of each country, are included in a restricted way, assuming that  $\beta_2 = -\beta_3$  (Panel A) and avoiding this restriction (Panel B). Results for the restricted equation over the period January 1995 to October 2006 show that the forward premium anomaly deepens since the coefficient associated with the slope is further from unity (-3.705). Moreover, the risk premium coefficient is negative, contradicting the theoretical value. If the parameters are not restricted,  $\beta_2 \neq -\beta_3$ , the relevant coefficients are statistically insignificant.

Following Brennan and Xia (2006) a second approach is adopted, which corresponds to a GARCH(1,1) model represented by the equations (31) and (32). Coefficients are estimated by maximum likelihood using the Bollerslev-Wooldridge-heteroskedasticity consistent covariance matrix over the period January 1995 to October 2006. The first two leads and lags of the independent variables are entered into the equation in order to capture their eminent persistence<sup>23</sup>.

Results are presented in Table 11. Firstly, the mean equation depends significantly on the level of the forward premium ( $f_{t,1} - s_t$ ), but the estimated parameter is still negative (-8.283) and higher, in absolute terms, than the simple unbiasedness equation (Table 11, Panel A). Secondly, the risk premium term, which is estimated by both the domestic and foreign squared market prices of risks, has the expected sign on the domestic parameter but it is not

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<sup>23</sup> Depreciation rate is stationary, but forward premium and domestic and foreign  $\lambda_t^2$  are integrated in order 1. Johansen (1991) test cannot reject the existence of one cointegrating vector between them.

significant; while the foreign parameter is significant but it has the wrong sign. In spite of this, the joint Wald test rejects the null of both parameters being equal to zero. Some of the coefficients associated with the leads and lags of the first difference of each independent variable are also statistically significant. These findings are in line with Brennan and Xia (2006) results, who report that market prices of risk seem to have an explanatory power on the exchange rate equation, but the forward premium anomaly still remains because the risk premium is estimated with errors.

Turning to the volatility equation, Table 11, Panel B shows that both the domestic and foreign market prices of risk have a positive and significant impact on the exchange rate volatility. Domestic  $\lambda_t$  is significant at 5% and the foreign at 10% level. However, this result does not preclude the existence of GARCH effects: GARCH term is still a relevant factor, though the ARCH term is not more significant at traditional levels. Overall, the null hypothesis of  $d_1 = d_2 = 0$  is strongly rejected, implying that the risk premium term is not sufficient to explain the exchange rate volatility as the theoretical relationship entails. This conclusion adheres to the recent literature that emphasizes that accounting for a risk premium is not sufficient to explain the bias unless some specific time-series properties are imposed on the risk premium process (Gospodinov, 2009).

#### 5.4 Exchange rate forecasting

A classical finding in the literature has been that forward rate is an imperfect predictor of future movements in the spot rates and current foreign exchange models have failed to produce superior predictions than a simple random walk process<sup>24</sup>. Therefore, an interesting question at this point is if the affine term model with the time-varying risk premium could improve the exchange rate forecasting.

There is some evidence that no-arbitrage models improve out-of-sample forecasting performance for yield curve (Favero, Niu and Sala, 2006; Ang and Piazzesi, 2003); however, there is less support to the fact that the same model could enhance the exchange rate prediction. Diez de los Rios (2009) assesses the out-of-sample predictability of the exchange rate using two-country affine term structure model. The author finds that this model has a lower root mean squared error (RMSE), and thus a better prediction performance, than random walk only in the case of Sterling pound and Canadian Dollar rates, but it does a poor job in forecasting the German mark/Euro and the Swiss Franc, all of them against the US Dollar. Benati (2006) estimates a two-affine term structure model for the GBP-USD rate and concludes that this model has virtually no forecasting power for the depreciation rate.

Using the estimated coefficients from the extended GARCH(1,1), an out-of-sample forecasting exercise is carried out for the period November 2006 to October 2007<sup>25</sup>. The model is used to forecast the spot rate at 1-, 3-, 6- and 12-month horizons. Dynamic forecasting is performed which allows a recursive evaluation, re-estimating the parameters with previously forecasted values of the dependent variables at each new data point. Forecasting results of the time-varying risk premium model are compared with the forecasts from a simple unbiased equation (UEH) and from a naïve random walk process (RW), where:

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<sup>24</sup> Meese and Rogoff (1983). On the other hand, Clarida and Taylor (1997) and Clarida et al. (2003) find that VEC models out-perform the random walk forecast. Nucci (2003), however, reports mixed results as the VECM estimates beat the random walk only in one case of out three for the different currencies in the dollar market.

<sup>25</sup> The last two months of 2007 are missed because of the two order lead operator in eq. (31).

$$E_t[S_{t+1}/S_t, S_{t-1} \dots] = S_t \quad (33)$$

That is, the naïve random walk requires no estimation as the best forecast of a variable at period  $t + 1$  is its previous value at period  $t$ . Predictions of each model are assessed with the traditional criteria in the forecasting literature: root mean squared error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). For more details see Appendix C.

Table 12 presents the results for the dynamic out-of-sample forecasting. For each prediction criterion, the model with the lower value is the best predictor for the exchange rate. A general result is that RW model is the most accurate forecast of the exchange rate for all horizons and for all criteria, while GARCH(1,1) exhibits almost zero prediction power, with even weaker predictions than the UEH model (Figure 13). Another interesting issue is that the prediction error increases proportionally with the horizon. Thus, in case of one-month prediction, the GARCH(1,1) model has an MSE just 1.6 times the RW value, but for 12-month the error prediction in the GARCH(1,1) is 4.1 times higher than the RW. The previous finding implies that over 50% of the predictions of the GARCH(1,1) model at 12-month horizon are considerably different from the actual exchange rate. This is not a striking result since most structural models have been tested and they cannot beat the RW.

## 6 CONCLUDING REMARKS

This paper has studied the application of a two-country affine model for the foreign exchange risk premium. The aim of this investigation was to assess the ability of the estimated risk premium to explain the eminent volatility of the exchange rate and thus account for the forward premium anomaly.

The following conclusions can be drawn from the empirical analysis on the US Dollar - New Zealand Dollar parity and interest rates for the period 1995-2007. Firstly, simple equations of the depreciation rates into the forward premium demonstrated that the forward rate in the USD-NZD market is a biased predictor of future changes in the spot exchange rate; result broadly reported in the case of other currencies. A second major finding is that the estimated affine model of term structure of interest rate produces high level of fitting to the actual values of yields and therefore, it generates a well-behaved risk premium term that is consistent with the Fama's (1984) necessary conditions. Thirdly, the estimated risk premium is a significant variable for the mean and volatility equations of the exchange rate. However, similar to traditional asset pricing models, the affine framework falls short to produce an appropriate measure of the risk premium that takes into account the forward premium anomaly. Finally, the predictive power of affine models is highly overwhelmed by the random walk forecast of the exchange rate.

The current findings add to a growing body of literature on affine models applied to exchange rate markets (Backus et al., 2001; Benati, 2006; Brennan and Xia, 2006; Graveline, 2006; Wu, 2007). Taken together, these results suggest that the promising semi-structured model, represented by the two-country affine specification, has some limitations to correctly describe the forward premium anomaly.

It is important to note that the rejection of the hypothesis could be explained as a methodological issue. Indeed, more than a failure of affine models, the inability of the risk premium to resolve the anomaly could be driven by the fact that deviations from the simple UIP are caused by expectations errors. Chakraborty and Evans (2008) demonstrate that perpetual learning can explain the forward-premium puzzle and replicate other features of the data. Therefore, further research should be undertaken in this area.

In the same way, special caution must be applied to the fact that this study lies in a small sample size (156 observations), making difficult the extension of results to other periods. Bootstrapping procedure is suggested in order to account for the potential small sample bias. Furthermore, this affine specification could ignore nonlinear connections between the risk premium and the foreign exchange return. Inci (2007) constructs a nonlinear model for the US-Swiss term structure with better performance than traditional affine models in accounting for the currency market anomaly. Other authors that investigate nonlinearities in the forward premium regression include Baillie and Kiliç (2006), Sarantis (2006), Sarno, Valente and Leon (2006) and Liu and Sercu (2009). Finally, interesting extensions of this study would be the analysis of a different market than bond returns for the definition of the stochastic discount factor, like the use of currency option prices (Bakshi, Carr and Wu, 2008; Graveline, 2006) or the inclusion of macroeconomic variables as additional variables in the dynamics of the yield curve (Chabi-Yo and Yang, 2006; Dong, 2006).

## 7 APPENDICES

### 7.1 Appendix A: Bond pricing coefficients

Starting from equation (23), Ang and Piazzesi (2003) demonstrate that when  $n = 1$  the price of one-period bond is:

$$\begin{aligned}
 P_t^{n=1} &= E_t[M_{t+1}P_{t+1}^{1-1}] & (B1) \\
 &= E_t[e^{m_{t+1}} \cdot 1] \\
 &= e^{[E_t(m_{t+1}) + \frac{1}{2}Var_t(m_{t+1})]} \\
 &= e^{-r_t} \\
 &= e^{[-\delta_0 - \delta_1'Z_t]} \\
 &= e^{[A_1 + B_1'Z_t]}
 \end{aligned}$$

Where  $A_1 = -\delta_0$  and  $B_1 = -\delta_1$ . Given that the price of a  $n$ -period bond is  $P_t^n = e^{[A_n + B_n'Z_t]}$ , in the same way, the price of an  $n + 1$ -period bond is:

$$\begin{aligned}
 P_t^{n+1} &= E_t[M_{t+1}P_{t+1}^n] & (B2) \\
 &= E_t[e^{m_{t+1} + A_n + B_n'Z_{t+1}}] \\
 &= E_t\left[e^{-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1} + A_n + B_n'Z_{t+1}}\right] \\
 &= e^{-r_t - \frac{1}{2}\lambda_t'\lambda_t + A_n} E_t\left[e^{-\lambda_t'\varepsilon_{t+1} + B_n'(\mu + \phi Z_t + \Sigma\varepsilon_{t+1})}\right] \\
 &= e^{-\delta_0 - \delta_1'Z_t - \frac{1}{2}\lambda_t'\lambda_t + A_n + B_n'\mu + B_n'\phi Z_t} E_t\left[e^{(\lambda' - B_n'\Sigma)\varepsilon_{t+1}}\right] \\
 &= e^{-\delta_0 - \delta_1'Z_t - \frac{1}{2}\lambda_t'\lambda_t + A_n + B_n'\mu + B_n'\phi Z_t} e^{\frac{1}{2}(\lambda' - B_n'\Sigma)(\lambda' - B_n'\Sigma)'} \\
 &= e^{-\delta_0 - \delta_1'Z_t - \frac{1}{2}\lambda_t'\lambda_t + A_n + B_n'\mu + B_n'\phi Z_t + \frac{1}{2}\lambda_t'\lambda_t - B_n'\Sigma(\lambda_0 - \lambda_1 Z_t) + \frac{1}{2}B_n'\Sigma\Sigma'B_n} \\
 &= e^{-\delta_0 + A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}B_n'\Sigma\Sigma'B_n + (-\delta_1' + B_n'(\Phi - \Sigma\lambda_1))Z_t}
 \end{aligned}$$

Previous result relies on the assumption that  $\varepsilon_t \sim \mathcal{N}IID(0, I)$ .

### 7.2 Appendix B: Starting values

Parameter	USD	NZD
$\lambda_0$	0.0000	0.0000
$\lambda_0$	0.0000	0.0000
$\lambda_1$	0.0000	0.0000
$\Phi$	0.9970	0.9980
$\Phi$	-0.0004	0.0015
$\Phi$	0.9733	0.9379
$\Sigma$	0.0136	0.0136
$\Sigma$	0.0164	0.0164
$\Omega$	0.0005	0.0000
$\Omega$	0.0001	0.0001
$\Omega$	-0.0002	0.0001
$\Omega$	0.0002	0.0001
$\Omega$	0.0001	0.0001
$\Omega$	0.0002	0.0002
$\delta_0$	0.0034	0.0057

### 7.3 Appendix C: Forecasting criteria

Basic measures to determine the forecast accuracy are described in this appendix (Brooks, 2002). Assume that  $T_1$  is the first observation of the out-of-sample forecast;  $T$  is the total sample size, that include the in-sample size (1 to  $T_1 - 1$ ) and the out-of-sample size;  $h$  is the time ahead forecasting horizon and  $\hat{s}_{t,h}$  is the  $h$ -ahead forecast of the actual variable  $s_t$ .

#### *Mean Squared Error (MSE)*

Provide a quadratic loss function and it is useful when the larger forecast errors are more severe than smaller errors.

$$MSE = \frac{1}{T - (T_1 - 1)} \sum_{t=T_1}^T (s_{t+h} - \hat{s}_{t,h})^2 \quad (D1)$$

#### *Mean Absolute Error (MAE)*

Measures the average absolute forecast errors and it is useful in the presence of outliers.

$$MAE = \frac{1}{T - (T_1 - 1)} \sum_{t=T_1}^T |s_{t+h} - \hat{s}_{t,h}| \quad (D2)$$

#### *Mean Absolute Percentage Error (MAPE)*

It can be interpreted as a percentage error, with values range from 0 to 100.

$$MAPE = \frac{100}{T - (T_1 - 1)} \sum_{t=T_1}^T \left| \frac{s_{t+h} - \hat{s}_{t,h}}{s_{t+h}} \right| \quad (D3)$$

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## 9 TABLES AND FIGURES

**Table 1: Descriptive statistics of yield curves**

Zero-coupon yields for USD and NZD are displayed in a monthly basis to eight different maturities (1, 3, 6, 12, 24, 36, 60 and 120 months). Information comes from Datastream. The sample period is January 1995 - December 2007.

Panel A: USD yield curve								
	1M	3M	6M	1Y	2Y	3Y	5Y	10Y
Mean	4.13	4.41	4.48	4.65	4.96	5.19	5.51	5.98
Median	4.90	5.31	5.29	5.26	5.24	5.32	5.49	5.91
Maximum	6.78	6.85	7.07	7.76	8.33	8.40	8.37	8.39
Minimum	1.09	1.11	1.12	1.19	1.50	1.89	2.71	3.96
Standard Deviation	1.68	1.82	1.81	1.78	1.61	1.45	1.22	1.00
Skewness	-0.63	-0.71	-0.70	-0.63	-0.47	-0.33	-0.10	0.22
Kurtosis	2.05	1.97	2.03	2.14	2.29	2.33	2.24	2.02
Jarque-Bera	16.02	20.10	18.96	15.11	8.88	5.69	3.96	7.49
Probability	0.00	0.00	0.00	0.00	0.01	0.06	0.14	0.02
Autocorrelation	0.98	0.99	0.99	0.98	0.97	0.96	0.94	0.93

Panel B: NZD yield curve								
	1M	3M	6M	1Y	2Y	3Y	5Y	10Y
Mean	6.90	6.99	7.03	7.14	7.21	7.26	7.31	7.38
Median	6.69	6.95	7.05	7.14	7.23	7.20	7.17	7.33
Maximum	10.06	10.18	10.19	9.83	9.68	9.49	9.27	9.19
Minimum	3.62	4.09	4.41	4.84	5.25	5.39	5.38	5.61
Standard Deviation	1.54	1.46	1.40	1.29	1.04	0.91	0.79	0.71
Skewness	0.19	0.17	0.14	0.06	0.17	0.25	0.24	0.42
Kurtosis	2.12	2.09	2.06	1.98	2.27	2.43	2.56	2.83
Jarque-Bera	5.96	6.12	6.20	6.78	4.24	3.79	2.77	4.70
Probability	0.05	0.05	0.04	0.03	0.12	0.15	0.25	0.10
Autocorrelation	0.95	0.95	0.95	0.94	0.92	0.91	0.90	0.89

**Table 2: Correlation matrix of yield curves**

Cross-correlation between zero-coupon yields to different maturities (1, 3, 6, 12, 24, 36, 60 and 120 months). Information comes from Datastream. The sample period is January 1995-December 2007.

Panel A: USD yield curve								
	1M	3M	6M	1Y	2Y	3Y	5Y	10Y
1M	1.00							
3M	0.98	1.00						
6M	0.97	1.00	1.00					
1Y	0.96	0.98	0.99	1.00				
2Y	0.92	0.95	0.97	0.99	1.00			
3Y	0.88	0.92	0.94	0.96	0.99	1.00		
5Y	0.82	0.86	0.88	0.91	0.96	0.99	1.00	
10Y	0.68	0.72	0.74	0.79	0.86	0.91	0.96	1.00

Panel B: NZD yield curve								
	1M	3M	6M	1Y	2Y	3Y	5Y	10Y
1M	1.00							
3M	0.99	1.00						
6M	0.98	1.00	1.00					
1Y	0.95	0.97	0.99	1.00				
2Y	0.87	0.91	0.94	0.97	1.00			
3Y	0.80	0.84	0.87	0.92	0.98	1.00		
5Y	0.67	0.72	0.75	0.80	0.91	0.96	1.00	
10Y	0.46	0.51	0.53	0.58	0.72	0.81	0.93	1.00

**Table 3: Exchange rate statistics**

This table presents descriptive statistics for the level of the exchange rate, the first difference of the logarithm of the USD-NZD rate, and the forward premium which is approximated as the one-month interest rate differential between USD and NZD, expressed in logarithms. The sample period is January 1995-December 2007.

	Spot rate $S_t$ : USD-NZD	Depreciation rate $s_{t+1} - s_t$	Forward premium $f_t - s_t$
Mean	0.596	0.114	-0.231
Median	0.631	0.263	-0.268
Maximum	0.782	8.697	0.110
Minimum	0.397	-9.246	-0.430
Standard Deviation	10.383	3.307	0.138
Skewness	-0.364	-0.316	0.830
Kurtosis	1.894	3.258	2.592
Jarque-Bera	11.405	3.017	19.008
Probability	0.003	0.221	0.000
Autocorrelation	0.957	-0.009	0.963

**Table 4: Unit root tests**

Three unit root tests are presented: Augmented Dickey Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS). Lag selection for ADF is based on Akaike's (1974) information criterion with a maximum of 13 lags, including a constant but not a tendency in the regression. Bandwidth for PP and KPSS tests is based on Newey-West (1987) using Bartlett kernel. Asymptotic critical values for KPSS test are: 0.739, 0.463, and 0.347 at 1%, 5% and 10% of level of significance, respectively. The sample period is January 1995-December 2007.

	ADF						PP						KPSS					
	Levels			First differences			Levels			First differences			Levels		First differences			
	Lags	t-Stat	Prob.	Lags	t-Stat	Prob.	Band.	t-Stat	Prob.	Band.	t-Stat	Prob.	Band.	t-Stat	Band.	t-Stat		
USD-NZD	3	-0.77	0.823	2	-5.89	0.000	4	-0.66	0.852	4	-12.50	0	10	0.37	4	0.38		
NZD yield																		
1M	7	-1.91	0.326	6	-5.14	0.000	5	-2.01	0.283	5	-12.77	0	10	0.41	4	0.18		
3M	9	-2.05	0.265	8	-4.67	0.000	6	-2.15	0.225	4	-10.06	0	10	0.42	6	0.24		
6M	6	-2.11	0.241	5	-4.95	0.000	6	-2.20	0.207	3	-9.98	0	10	0.41	5	0.24		
1Y	2	-2.14	0.229	1	-9.64	0.000	4	-2.18	0.215	0	-9.64	0	10	0.41	4	0.27		
2Y	2	-2.49	0.119	1	-10.10	0.000	4	-2.60	0.096	1	-10.09	0	9	0.53	3	0.26		
3Y	1	-2.88	0.050	0	-10.42	0.000	3	-2.79	0.062	2	-10.43	0	9	0.63	1	0.25		
5Y	6	-1.99	0.290	5	-6.78	0.000	2	-2.92	0.045	2	-12.07	0	9	0.79	2	0.14		
10Y	6	-2.22	0.199	5	-6.86	0.000	2	-3.21	0.022	3	-13.11	0	9	1.01	4	0.10		
USD yield																		
1M	6	-2.16	0.223	5	-2.88	0.050	7	-1.32	0.618	7	-11.49	0	10	0.41	7	0.18		
3M	3	-1.74	0.409	2	-3.84	0.003	8	-1.49	0.535	6	-7.29	0	10	0.55	8	0.23		
6M	8	-2.42	0.137	7	-4.14	0.001	8	-1.65	0.454	6	-7.53	0	10	0.55	8	0.22		
1Y	3	-1.75	0.403	0	-8.50	0.000	7	-1.90	0.332	5	-8.65	0	10	0.59	7	0.21		
2Y	1	-1.89	0.338	0	-10.22	0.000	6	-2.14	0.228	4	-10.28	0	10	0.68	5	0.16		
3Y	1	-2.06	0.261	0	-10.74	0.000	5	-2.24	0.194	3	-10.72	0	10	0.77	5	0.13		
5Y	0	-2.28	0.181	1	-9.40	0.000	4	-2.33	0.164	3	-11.21	0	10	0.91	3	0.10		
10Y	0	-2.38	0.149	1	-10.07	0.000	4	-2.39	0.146	3	-11.93	0	10	1.10	3	0.07		

**Table 5: Principal components for yield curves**

Contribution of each of the eight components for USD and NZD yield curve is presented in decreasing order, starting from the factor with higher eigenvalue (Comp1). The cumulative proportion corresponds to the summation of the variance proportion up to the  $n$ th eigenvalue. The sample period is January 1995-December 2007.

Panel A: USD yield curve								
	Comp1	Comp2	Comp3	Comp4	Comp5	Comp6	Comp7	Comp8
Eigenvalue	7.385	0.543	0.048	0.020	0.003	0.001	0.0002	0.0001
Variance proportion	0.923	0.068	0.006	0.003	0.0004	0.0001	0.0000	0.0000
Cumulative proportion	0.923	0.991	0.997	1.000	1.000	1.000	1.000	1.000

Panel B: NZD yield curve								
	Comp1	Comp2	Comp3	Comp4	Comp5	Comp6	Comp7	Comp8
Eigenvalue	6.881	0.984	0.112	0.013	0.006	0.003	0.001	0.0005
Variance proportion	0.860	0.123	0.014	0.002	0.001	0.0004	0.0001	0.0001
Cumulative proportion	0.860	0.983	0.997	0.999	0.999	1.000	1.000	1.000

**Table 6: Forward premium regression**

Depreciation rate and forward premium are expressed in logarithm terms. The forward premium regression, estimated by OLS, is of the form:  $s_{t+n} - s_t = \beta_0 + \beta_1(f_{t,n} - s_t) + v_{t+n}$ , where  $n$  corresponds to the horizon of estimation, from one month to ten years. Numbers in parenthesis are Newey-West HAC standard errors (lags=4). An asterisk represents a coefficient significant at 10% and two asterisks at 5% level. The adjusted sample is January 1995 - October 2006.

Dependent variable: $s_{t+n} - s_t$				
Model	$\beta_0$	$\beta_1$	$R^2$	Observations
1-month	-0.007 (0.004)	-3.222* (1.730)	0.021	141
3-month	-0.010** (0.004)	-4.669** (1.747)	0.122	139
6-month	-0.011** (0.004)	-5.309** (1.548)	0.262	136
1-year	-0.012** (0.003)	-5.882** (1.275)	0.430	130
2-year	-0.012** (0.002)	-6.830** (0.874)	0.597	118
3-year	-0.011** (0.001)	-6.620** (1.081)	0.517	106
5-year	-0.006** (0.002)	-3.785** (1.176)	0.179	82
10-year	-0.002** (0.0002)	-1.818** (0.240)	0.803	22

**Table 7: Estimates from the affine term structure model**

Panel A reports the estimates for USD and NZD market prices of risk,  $\lambda_t = \lambda_0 + \lambda_1 Z_t$ . Panel B presents the estimates for the factor loadings of the latent factors, and panel C the volatility matrix of the latent factors, following the process:  $Z_t = \mu + \phi Z_{t-1} + \Sigma \varepsilon_t$ . Numbers in parenthesis are standard errors estimated by Hessian matrices. An asterisk represents a coefficient significant at 10% and two asterisks at 5% level. The sample period is January 1995-December 2007.

Panel A: Market prices of risk: $\lambda_t = \lambda_0 + \lambda_1 Z_t$							
USD				NZD			
	$\lambda_0$	$\lambda_1$			$\lambda_0^*$	$\lambda_1^*$	
		$Z_1$	$Z_2$			$Z_1^*$	$Z_2^*$
$Z_1$	0.110** (0.014)	4.672 (14.264)	0.125 (3.356)	$Z_1^*$	0.028 (0.022)	8.953 (5.656)	0.729 (0.938)
$Z_2$	-0.314 (0.542)	0.041 (0.783)	-6.018 (44.907)	$Z_2^*$	-0.170 (0.166)	1.046 (1.610)	53.714 (60.673)

Panel B: Factor dynamics of latent factors: $\Phi$					
USD			NZD		
	$Z_1$	$Z_2$		$Z_1^*$	$Z_2^*$
$Z_{1,t-1}$	0.980** (0.004)	9.992** (0.662)	$Z_{1,t-1}^*$	0.992** (0.001)	42.811** (2.474)
$Z_{2,t-1}$		0.969** (0.012)	$Z_{2,t-1}^*$		0.961** (0.019)

Panel C: Volatility matrix of the latent factors: $\Sigma \times 10^4$			
	USD		NZD
$Z_1$	-0.021** (0.000)	$Z_1^*$	-0.005** (0.000)
$Z_2$	1.660** (0.501)	$Z_2^*$	2.819** (0.173)

**Table 8: Measurement errors statistics**

Errors for yields assumed to be measured inaccurately are reported, where the error is defined according to the following equation:  $Y_{t,n}^e = -\tilde{A}_n - \tilde{B}_n' Z_t + \Omega_t$ . 3-month and 5-year yields are excluded because they have errors equal to zero. Mean and standard errors values are multiplied for  $10^4$  in order to represent basis points. The sample period is January 1995-December 2007.

Panel A: USD yield curve						
	1M	6M	1Y	2Y	3Y	10Y
Mean	-1.838	-0.119	-0.101	0.192	0.186	0.173
Standard errors	(3.204)	(0.948)	(1.808)	(1.713)	(1.022)	(1.695)

Panel B: NZD yield curve						
	1M	6M	1Y	2Y	3Y	10Y
Mean	-0.490	0.109	0.474	0.306	0.217	0.428
Standard errors	(1.531)	(0.902)	(1.840)	(1.629)	(1.185)	(1.739)

**Table 9: Conditions for the forward premium anomaly**

Statistical moments for the risk premium  $-p_t = \frac{1}{2}(\lambda_t' \lambda_t - \lambda_t^{*'} \lambda_t^*)$  and the expected depreciation rate  $q_t = r_t - r_t^* + \frac{1}{2}(\lambda_t' \lambda_t - \lambda_t^{*'} \lambda_t^*)$  are exhibited. These moments are the basis for the Fama's (1984) necessary conditions. Standard deviations are presented in percentage. The sample period is January 1995-December 2007.

	Values
Standard deviation $p_t$	0.8121
Standard deviation $q_t$	0.8107
Correlation $p_t, q_t$	-0.985

**Table 10: Extended forward premium regression**

One-month depreciation rate and one-month forward premium are expressed in logarithm terms. Panel A reports an extended version of the UEH equation that includes the risk premium term, assuming that  $\beta_2 = -\beta_3$ . Panel B ignores this assumption and includes separately both components of the risk premium. Numbers in parenthesis are Newey-West HAC standard errors (lags=4). An asterisk represents a coefficient significant at 10% and two asterisks at 5% level. The adjusted sample is January 1995 - October 2006.

Panel A: Restricted parameters					
$s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t) + \beta_2(\lambda_t' \lambda_t - \lambda_t^{*'} \lambda_t^*) + v_{t+1}$					
Dependent variable: $s_{t+1} - s_t$					
$\beta_0$	$\beta_1$	$\beta_2$	$R^2$	Observations	
0.039** (0.015)	-3.705** (1.554)	-0.869** (0.292)	0.068	141	

Panel B: Unrestricted parameters					
$s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t) + \beta_2 \lambda_t' \lambda_t + \beta_3 \lambda_t^{*'} \lambda_t^* + v_{t+1}$					
Dependent variable: $s_{t+1} - s_t$					
$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$R^2$	Observations
0.052 (0.112)	-3.337 (3.731)	-0.528 (0.813)	0.393 (0.370)	0.068	141

**Table 11: GARCH(1,1) results**

Table reports the mean and volatility equations for the one-month change in exchange rate. Robust standard errors are obtained from Bollerslev-Wooldridge heteroskedasticity consistent covariance matrix. An asterisk represents a coefficient significant at 10% and two asterisks at 5% level. The adjusted sample is January 1995 - October 2006.

Panel A: Mean equation

$$s_{t+1} - s_t = c_0 + c_1(f_t - s_t) + c_2\lambda'_t\lambda_t + c_3\lambda_t^*\lambda_t^* + \sum_{p=-2}^2 c_{4,p}\Delta(f_{t-p} - s_{t-p}) + \sum_{p=-2}^2 c_{5,p}\Delta\lambda'_{t-p}\lambda_{t-p} + \sum_{p=-2}^2 c_{6,p}\Delta\lambda^*_{t-p}\lambda^*_{t-p} + \omega_t$$

	Coefficients	Standard errors
$c_0$	-0.071	0.128
$(f_t - s_t)$	-8.283**	3.965
$\lambda'_t\lambda_t$	0.325	0.912
$\lambda_t^*\lambda_t^*$	0.883*	0.521
$\Delta(f_{t-2} - s_{t-2})$	3.671**	6.989
$\Delta(f_{t-1} - s_{t-1})$	-6.222	6.106
$\Delta(f_t - s_t)$	17.413	7.090
$\Delta(f_{t+1} - s_{t+1})$	-2.841	7.669
$\Delta(f_{t-1} - s_{t-1})$	-5.401	6.157
$\Delta\lambda'_{t-2}\lambda_{t-2}$	10.707**	5.349
$\Delta\lambda'_{t-1}\lambda_{t-1}$	0.408	4.402
$\Delta\lambda'_t\lambda_t$	-4.770	5.574
$\Delta\lambda'_{t+1}\lambda_{t+1}$	-7.665	4.955
$\Delta\lambda'_{t+2}\lambda_{t+2}$	-3.425	4.975
$\Delta\lambda^*_{t-2}\lambda^*_{t-2}$	-0.339	0.818
$\Delta\lambda^*_{t-1}\lambda^*_{t-1}$	0.557	0.734
$\Delta\lambda^*_t\lambda^*_t$	-1.191	0.766
$\Delta\lambda^*_{t+1}\lambda^*_{t+1}$	-0.394	0.655
$\Delta\lambda^*_{t+2}\lambda^*_{t+2}$	-0.555	0.862
$R^2$	0.174	
Adj. $R^2$	0.007	
Observations	138	
Wald test Prob.: $c_2 = c_3 = 0$	0.0011	

Panel B: Variance equation

$$\sigma_{s,t}^2 = d_0 + d_1\omega_{t-1}^2 + d_2\sigma_{s,t-1}^2 + d_3\lambda'_t\lambda_t + d_4\lambda_t^*\lambda_t^*$$

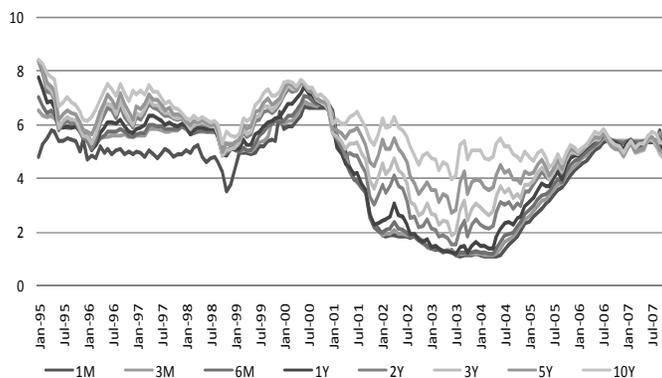
	Coefficients	Standard errors
$d_0$	0.0000**	0.0000
$\omega_{t-1}^2$	-0.0406	0.0353
$\sigma_{s,t-1}^2$	0.9981**	0.0359
$\lambda'_t\lambda_t$	0.0003**	0.0001
$\lambda_t^*\lambda_t^*$	0.0017*	0.0009
Wald test Prob.: $d_1 = d_2 = 0$	0.0000	

**Table 12: Results from out-of-sample forecasts**

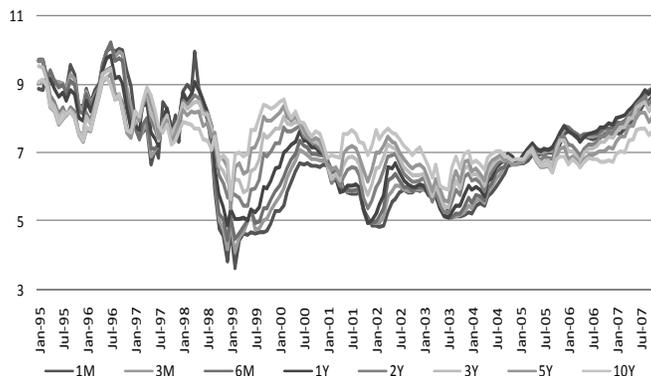
This table presents three forecasting criteria for a simple Random Walk specification  $S_{t+1} = S_t$ , the unbiased expectation hypothesis equation  $s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t) + v_{t+1}$  and the extended GARCH(1,1) model represented by the system (31)-(32). The forecasting horizon is October-November 2006 for 1-month forecast; October 2006-January 2007 for the 3-month forecast; October 2006-April 2007 for 6-month forecast and October 2006 - October 2007 for the 12-month forecast.

Horizon	Random walk	UEH equation	GARCH(1,1)
<b>Root Mean Squared Error</b>			
1 month	0.0216	0.0237	0.0346
3 month	0.0225	0.0524	0.0742
6 month	0.0294	0.0596	0.0965
12 month	0.0420	0.1055	0.1736
<b>Mean Absolute Error</b>			
1 month	0.0216	0.0237	0.0346
3 month	0.0225	0.0484	0.0688
6 month	0.0268	0.0546	0.0892
12 month	0.0348	0.0936	0.1558
<b>Mean Absolute Percentage Error</b>			
1 month	5.481	6.015	8.771
3 month	6.092	13.378	19.016
6 month	7.460	15.414	25.076
12 month	11.010	31.611	52.258

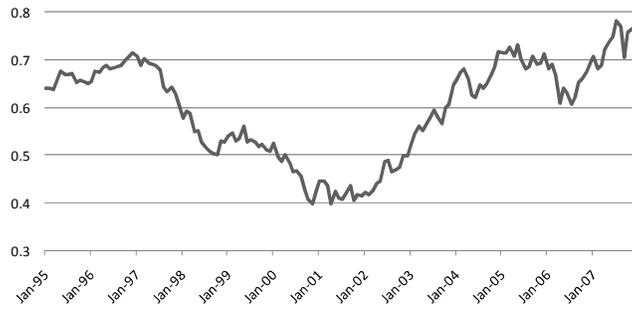
**Figure 1: USD yield curve**  
(Annualized percentage)



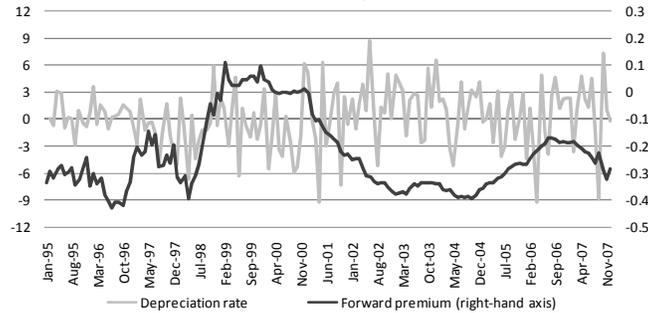
**Figure 2: NZD yield curve**  
(Annualized percentage)



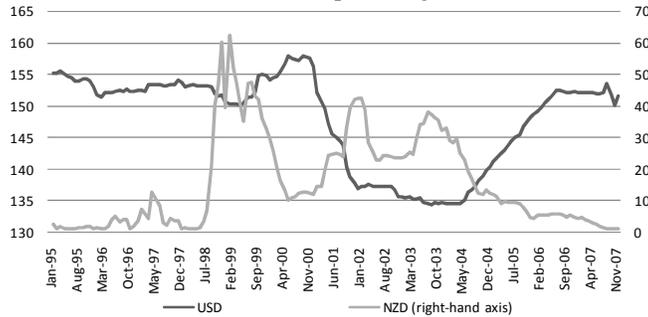
**Figure 3: USD-NZD exchange rate**  
(US Dollar per NZ Dollar)



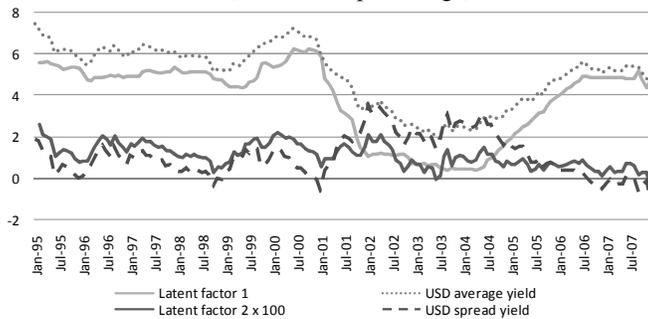
**Figure 4: One-month depreciation rate and forward premium**  
(Percentage)



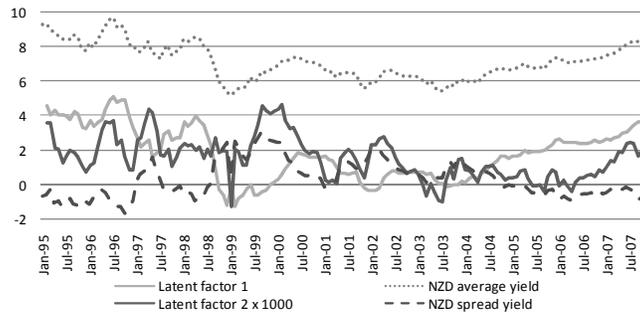
**Figure 5: Market prices of risk**  
(Annualized percentage)



**Figure 6: State variables for USD**  
(Annualized percentage)

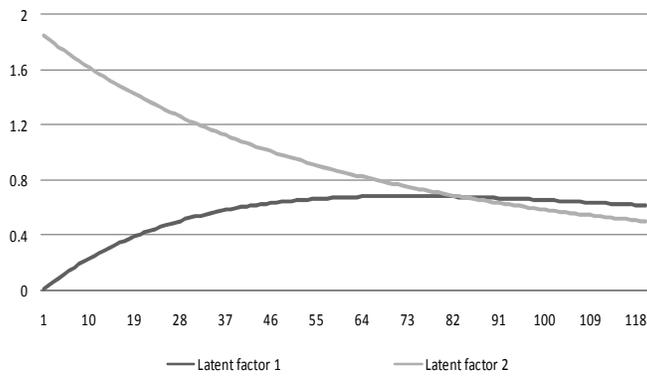


**Figure 7: State variables for NZD**  
(Annualized percentage)



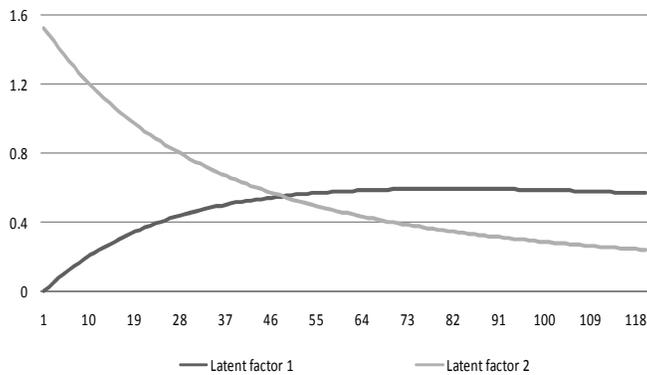
**Figure 8: Factor loadings for USD yield curve**

(Y-axis represents one standard deviation of the respective latent factor, and X-axis is the time to maturity in monthly basis)

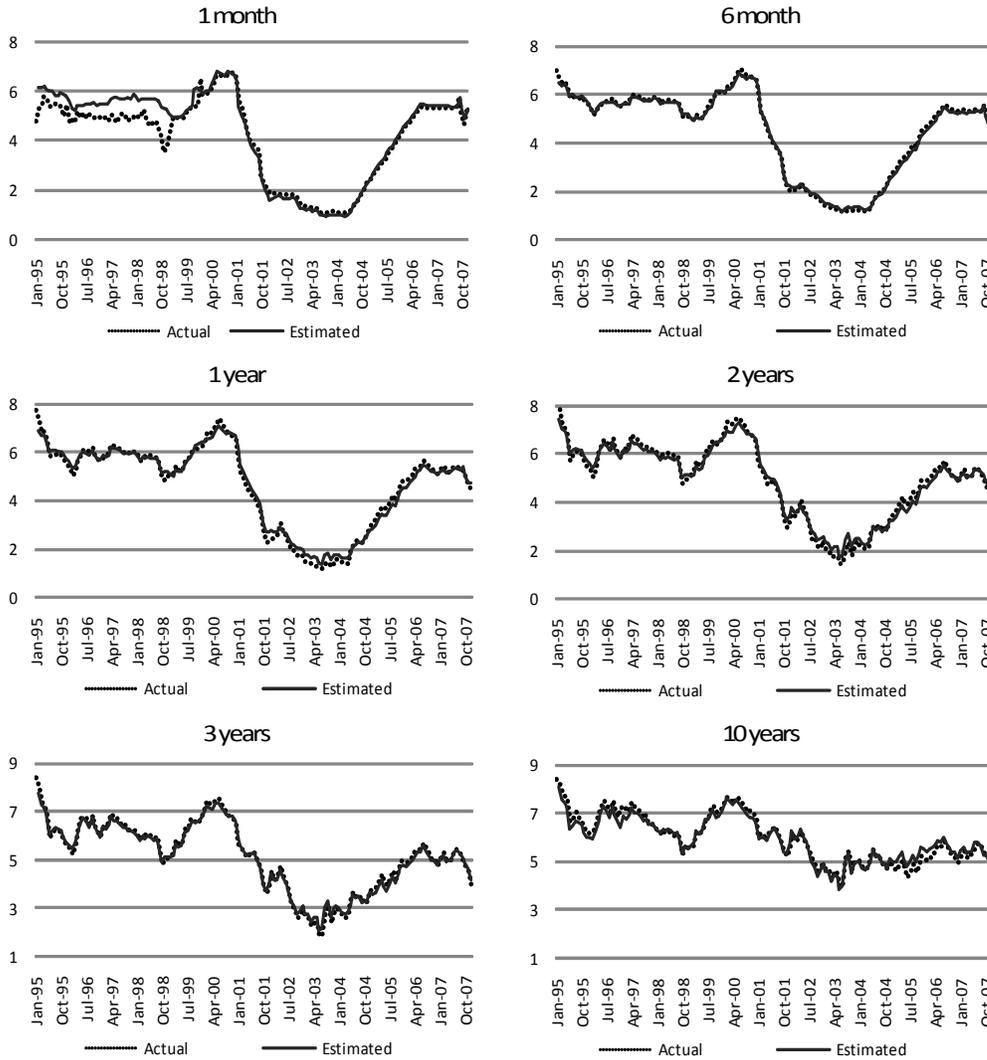


**Figure 9: Factor loadings for NZD yield curve**

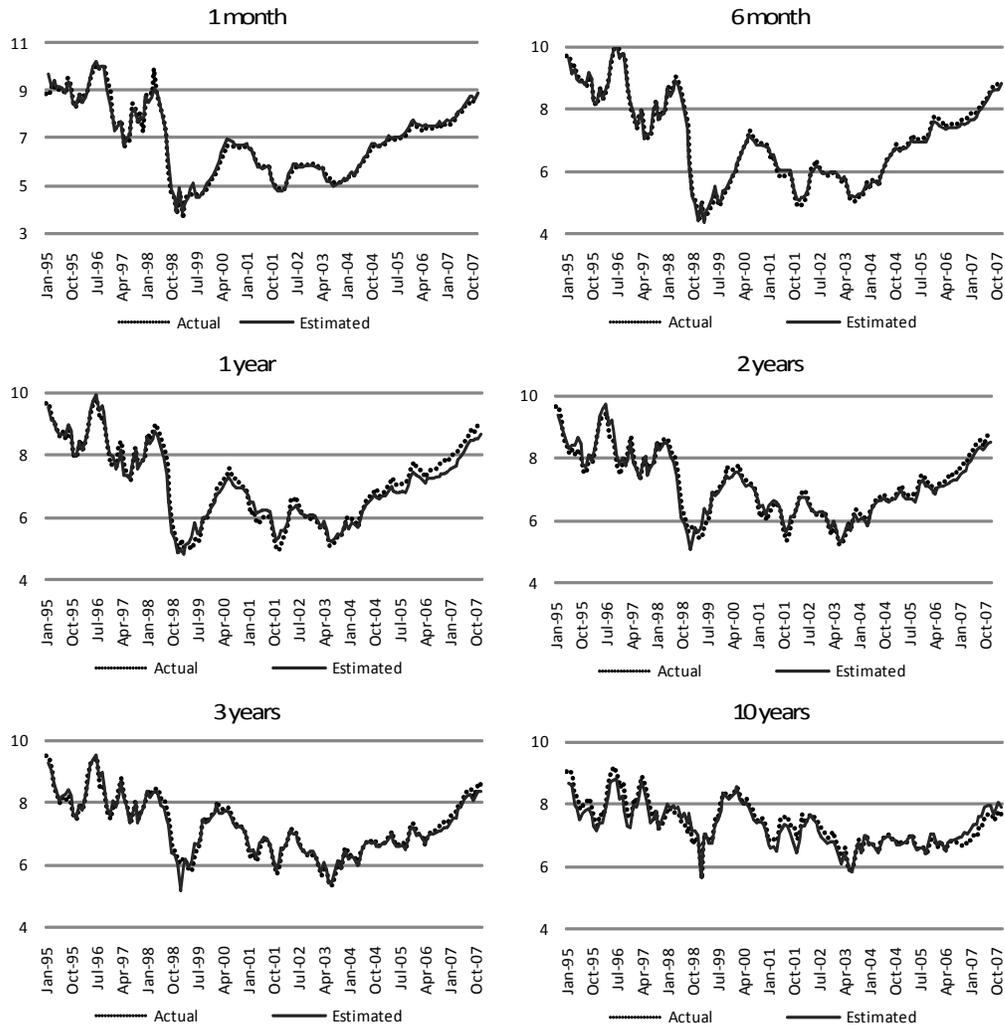
(Y-axis represents one standard deviation of the respective latent factor, and X-axis is the time to maturity in monthly basis)



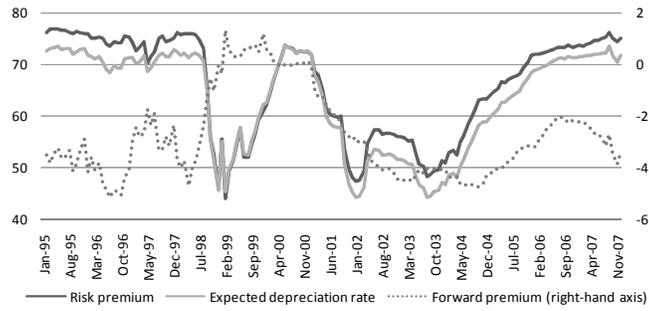
**Figure 10: Modeled and observed yields for USD**  
(Annualized percentage)



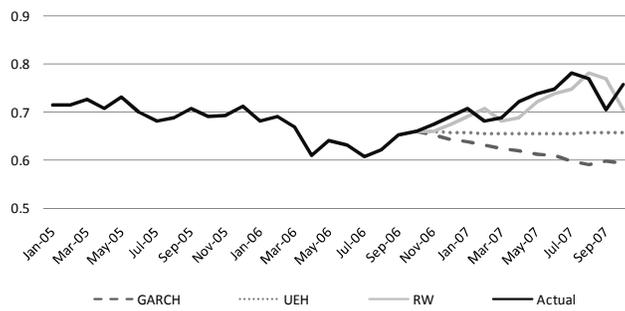
**Figure 11: Modeled and observed yields for NZD**  
(Annualized percentage)



**Figure 12: Risk premium and depreciation rate**  
(Annualized percentage)



**Figure 13: Forecast for the exchange rate**  
(US Dollar per NZ Dollar)



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