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OPTIMAL TAXATION WITH HETEROGENEOUS FIRMS

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Resumen

En este artículo estudiamos política tributaria óptima en un contexto donde las empresas difieren en productividad y deciden si producir después de comparar las ganancias que obtendrían después de impuestos versus una opción externa. Estudiamos un contexto donde el gobierno cobra impuestos al capital, a las ganancias de las empresas y al trabajo, pero no cobra impuestos a la opción externa. En este contexto, los impuestos pueden distorsionar la decisión de las firmas de entrar en el sector formal (margen extensivo) como así también sus decisiones de contratación de factores de producción una vez que decidieron producir (margen intensivo). Encontramos que el gobierno tiene incentivos a subsidiar costos para inducir a las empresas a producir. El impuesto óptimo a los ingresos derivados del capital es negativo, mientras que la tasa de impuesto corporativo es positiva y el signo del impuesto a los ingresos del trabajo es ambiguo.

Abstract

We study steady state optimal taxation in a context where firms differ in productivity and they decide whether to produce or not after comparing after-tax profits vis-à-vis an outside alternative option. The government taxes capital income, firms' profits and labor income, but does not tax the alternative outside option. In this context, taxation might distort the firms' decisions to participate in production (extensive margin) as well as their factor allocations once they decide to produce (intensive margin). We find that the government has incentives to subsidize costs to induce firms into production. The optimal capital income tax is negative while the corporate tax rate is positive and the sign of labor income tax is ambiguous.

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1 Introduction

In this paper we study steady state optimal (Ramsey) taxation in a context where firms are heterogeneous in the sense that they differ in their productivity and decide whether to enter into production or not. This decision is taken after comparing the after-tax profits obtained from production vis-*à*-vis an outside alternative option. The government finances an exogenous expenditure path using three tax instruments: capital income tax, labor income tax and a tax on firms' profits. Also, the government can issue debt to finance its expenditure but, crucially, we assume that the yields of the alternative option cannot be taxed.

In this context, taxation potentially distorts the firms' decision to participate in production, i.e. the extensive margin distortion, as well as the factor allocations of the firms already involved in production, i.e. the intensive margin distortion.

The distortions created in the extensive margin are key to our results. When there are firms that choose not to produce, it is optimal to set a negative tax on capital income and a positive tax on firms' profits. There is also a tendency to subsidize labor, although the sign of this tax depends on labor supply considerations. The intuition is that by subsidizing capital income, and possibly labor, the planner induces more firms into production and makes them taxable through the tax on profits.

However, when there is no distortion in the extensive margin, i.e. when all firms decide to produce or when the government can also tax the alternative option, the optimal capital income tax is equal to zero, as in the celebrated results of Chamley (1986) and Judd (1985). The tax on labor is also zero in this case and all tax revenues are raised using the tax on profits. The intuition is that, in this case, additional capital and labor is not socially desirable since they do not enlarge the taxable set of firms and thus it is optimal not to distort the intensive margins. The tax on profits does not distort any margin.

Our main analysis is conducted as if the government cannot tax the alternative option. However, we also analyze the case where it is possible to tax the alternative option to show the importance of the distortions on the extensive margin in the results.

The paper is related to several other studies in the literature. Chamley (1986) and Judd (1985) find that the optimal capital income taxation is equal to zero in steady state in a competitive environment. Following these studies, several papers showed that optimal capital income taxation is different from zero if the context is modified in some ways.

Correia (1996) extends Chamley (1986)'s result to an environment of incomplete taxation, where not all factors can be taxed. She finds that taxing capital would be an indirect way of taxing the untaxed factor and the sign of this tax would depend on the complementarity between capital and the untaxed factor.[1] Like Correia (1996), we consider a context of incomplete taxation where the government does not tax the alternative option, but differ in that we analyze an environment where firms are heterogeneous. Firm heterogeneity and the distortions created in the extensive margin are key to our results. In this setting, the planner has incentives to subsidize costs in order to tax firms that would not otherwise be taxed. Thus, the optimal capital income taxation is negative.

In the same vein as Correia (1996), other studies find that capital taxes should not be zero as a consequence of incomplete taxation. Jones et al (1993) study the issue with endogenous growth models, showing that including government expenditures as a productive input leads to an optimal tax rate different from zero. The reason is similar to Correia's explanation since government expenditure is not taxed. Jones et al (1997) also show that the zero income capital tax is no longer optimal when pure profits are generated. Their interpretation of this result is that taxing capital is a way of taxing pure profits in a setting where they cannot be taxed directly.

A second line of research related to our study includes Judd (1997), Judd (2002) and Coto-Martinez et al (2007). Judd (1997) and Judd (2002) argue that the optimal capital income tax rate is negative and the tax on profits is positive in a context of monopolistic competition. Coto-Martinez et al (2007) add entry and exit of firms to Judd's framework, where the entrance of new firms augment the general productivity of the economy but implies a waste of resources in the form of a fixed cost. In Coto-Martinez et al (2007) optimal taxes depend on the tax code available. When the available taxes are such that the government can control the number of firms through a tax on profits, it is optimal to subsidize capital to correct the markup distortion as in Judd and set a subsidy or a tax on profits depending on the aggregate returns to specialization. When the tax system does not allow to control the number of firms through profits taxation, they find that the optimal capital income taxation is zero if the returns to specialization are zero. The reason is that, in this case, it is not desirable to subsidize the entrance of new firms since there are only losses (fixed cost) associated with them.

We also find that it is optimal to subsidize capital but in a different context and for different reasons. Ours is a context of perfect competition (no markup distortions) and

[1] Correia (1996) suggests that similar results would be obtained if firms present decreasing returns to scale and profits cannot be taxed.

without aggregate returns to specialization. In the general case, where the yields of the alternative option cannot be taxed, it is optimal to subsidize capital, and possibly labor, to induce firms that are on the margin into production, making their taxation possible. As mentioned above, if the alternative option could be taxed at the same rate as profits the labor and capital taxes are zero. Thus, in our case, the planner provides a subsidy to capital in order to complete the tax system and not due to distortions created by imperfect competition or the presence of returns to specialization.

While, to the best of our knowledge, the papers most related to ours are those mentioned above, there are other papers that find an optimal capital income tax different from zero. Aiyagari (1995) makes the point in an economy with borrowing constraints, where precautionary savings leads to too much capital in steady state. The optimality of taxing capital income was also obtained in OLG models. Recent work using this approach includes Abel (2005) and Erosa and Gervais (2002). Abel (2005) focuses on a context of consumption externalities between generations and shows that taxing capital is a way to correct the no "internalization" of cohorts' consumption. Erosa and Gervais (2002) derives the optimality of capital taxation as a way of making taxes age-dependant. In a context of private information about agents' skills, Golosov et al (2003) show that it is optimal to have a wedge between the marginal benefit and marginal cost of investing, which is consistent with a positive tax on capital income.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 sets up the Ramsey problem; subsection 3.1 analyzes capital income taxation, subsection 3.2 sets the optimal labor income and profits tax and subsection 3.3 studies the case when the yields of the alternative option is taxable. Section 4 presents numerical examples that confirm our theoretical findings and provides quantitative impacts in the case of standard utility and production functions. Section 5 concludes the paper.

2 The Model

2.1 Firms

There is a set I of mass one of heterogeneous firms indexed by i that can operate and produce a single good. Firm heterogeneity comes from a productivity parameter, A_{it}, which is iid across time t and is distributed across firms with cumulative distribution $G(A_{it})$ with support $[A_l, A_u]$, where $0 < A_l < A_u < \infty$. Let k_{it} and l_{it} be capital and labor used by firm i in the production process in period t.

Capital is rented from the representative household each period at the rental rate r_t, which, in equilibrium, is the same for all firms. Firms pay an amount w_t as compensation for the use of labor, which is also common to all firms in equilibrium. Both, r_t and w_t are expressed in terms of the consumption good. Each firm production function presents decreasing returns to scale and is given by $A_{it} f(k_{it}, l_{it})$, where $f(k_{it}, l_{it})$ is strictly increasing, strictly concave and satisfies Inada conditions on k_{it} and l_{it}. We further assume that $f(k_{it}, l_{it})$ is homogeneous of degree $\theta < 1$ in (k_{it}, l_{it}). This last assumption is not important to the main results of the paper but simplifies the exposition.

Let $\phi > 0$ be an outside option, common to all firms, expressed in units of the single good that each firm would get in an alternative project not considered explicitly in the paper.

In each period, firm i must decide between entering the market to produce, or not entering. To make this decision, firms compare the after-tax profits derived from production and the yields of the alternative option. We make the following assumption about the taxes that a firm faces:

Assumption 1. *The government taxes firms' profits at a rate τ_t^u but cannot tax the parameter ϕ.*

This assumption implies that the tax system is incomplete and it is very important to our results as will become clear below. We analyze the consequences of dropping it in section 3.3.

The outside option could be interpreted in the same spirit as in Jovanovic (1982) who considers it as a "managerial ability" or "advantageous location" which is common to all firms. Note, however, that an implication of Assumption 1 is that we could also interpret ϕ as the return obtained by the firms in an informal sector where returns cannot be taxed by the government.

This last interpretation of ϕ deserves more discussion. It is common in the literature to model the informal sector as using labor more intensively than capital. See, for example, Turnovsky and Basher (2009), Ihrig and Moe (2004), Larrain and Poblete (2007) and Yuki (2007), where the informal sector is modeled using only labor while the formal sector uses both capital and labor. We model the outside option as not using any factor of production and yielding a fixed and common value ϕ. However, the trade offs analyzed in this paper and its results are robust to modeling the alternative activity as using a labor intensive production function as is done in the works mentioned above.[2]

Firms' profits derived from production are given by the product obtained minus payments to capital and labor. The rental rate of capital and the wage rate are determined in competitive markets and are the same for all firms. Thus, firms deciding to produce must obtain after-tax profits that are at least equal to ϕ. Hence, a firm solves the following static problem in period t:

$$\max\{\phi \quad ; \quad V_{it}\} \tag{1}$$
$$where \quad V_{it} = \max_{k_{it},l_{it}} \quad (1-\tau_t^u)[A_{it}f(k_{it},l_{it}) - r_t k_{it} - w_t l_{it}] = \max_{k_{it},l_{it}} \quad (1-\tau_t^u)(1-\theta)A_{it}f(k_{it},l_{it}),$$

the last term holds because of the homogeneity of the production function.

Let V_{lt} and V_{ut} be the function V_{it} evaluated at the lowest and highest productivity shocks, $A_{it} = A_l$ and $A_{it} = A_u$, respectively.

Assumption 2. $V_{ut} > \phi$.

This assumption assures entrance of a positive mass of firms into production. The solution to the firm's problem, given in equation (1), is stated in the next lemma.

Lemma 1. *There exists a threshold technology level A^* such that firms endowed with technology $A_{it} \geq A_t^*$ enter into production, while firms endowed with $A_{it} < A_t^*$ do not enter into production. When $V_{lt} \leq \phi$, the threshold A_t^* is interior and uniquely determined by:*

$$\max_{k_{it},l_{it}} \quad (1-\tau_t^u)(1-\theta)A_t^* f(k_{it},l_{it}) = \phi. \tag{2}$$

When $V_{lt} > \phi$, the threshold A_t^* is equal to A_l.

Proof. The function V_{it} in (1) is an increasing and continuous function of A_{it}.[3] Then, when V_{lt} is smaller than ϕ, there is a unique A_{it} that makes V_{it} equal to ϕ given our assumption that V_{ut} is always higher than ϕ. If V_{lt} were larger than ϕ, all firms would prefer to produce and the threshold A_t^* will be given by A_l. □

[2] For the sake of brevity we do not report this development in the paper. However, it is available from the authors upon request.

[3] By the envelope theorem $\frac{\partial V_{it}}{\partial A_{it}} = (1-\tau_t^u)f(k_{it},l_{it})$.

Firms demand capital and labor if they participate in production, however these factors are not needed if the firm is not engaged in production and participate in the alternative option. Thus, factor demands are functions of factor prices and the idiosyncratic shock and are generically given by:

$$\begin{aligned} k_{it} &= k_{it}(A_{it}, r_t, w_t) \quad if \quad A_t^* \leq A_{it}, \\ k_{it} &= 0 \quad if \qquad\quad A_t^* > A_{it} \end{aligned} \tag{3}$$

and

$$\begin{aligned} l_{it} &= l_{it}(A_{it}, r_t, w_t) \quad if \quad A_t^* \leq A_{it} \\ l_{it} &= 0 \quad if \qquad\quad A_t^* > A_{it}. \end{aligned} \tag{4}$$

Markets are competitive, capital and labor are paid their marginal productivity, and the rental rate and wage rate are the same for all firms. Hence, the rental and wage rates for the economy are given by: [4]

$$r_t = \frac{\int_{A_t^*}^{A_u} A_{it} f_k(k_{it}, l_{it}) dG(A_{it})}{1 - G(A_t^*)} \tag{5}$$

and

$$w_t = \frac{\int_{A_t^*}^{A_u} A_{it} f_l(k_{it}, l_{it}) dG(A_{it})}{1 - G(A_t^*)}. \tag{6}$$

where $1 - G(A_t^*)$ is the fraction of firms involved in production in period t.

The capital and labor demands for the economy follow from the aggregation of individual factor demands by all the firms that decide to produce; that is, all the firms that get a productivity shock higher than A_t^*. That is:

$$K_t^D = \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) \tag{7}$$

and

$$L_t^D = \int_{A_t^*}^{A_u} l_{it} dG(A_{it}). \tag{8}$$

[4]Each firm equates its marginal productivity of capital and labor to the interest rate and the wage rate respectively, that is, $r_t = A_{it} f_k(k_{it}, l_{it})$ and $w_t = A_{it} f_l(k_{it}, l_{it})$. Equations (5) and (6) follow by aggregation of these expressions among all firms that decide to produce.

2.2 The household

There is an infinitely lived representative household choosing a consumption path $\{c_t\}_{t=0}^{\infty}$ and a leisure path $\{h_t\}_{t=0}^{\infty}$ that maximizes:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{9}$$

where u(.) is strictly increasing, strictly concave and three times continuously differentiable in both arguments. We assume also that $u_{ch} \geq 0$. The household is endowed at time zero with an initial amount of capital K_0 and holds the initial stock of government bonds b_0. Each period, she decides how much to consume, how much to work, how much to invest in capital and government bonds to be held into next period. It is assumed that capital depreciates at rate δ while total time available to work and to rest is $\overline{H}$. Capital, K_t, is rented to firms in order to be used in the production process at the rental rate r_t. Labor, L_t, is also rented to firms at the rate w_t. The representative individual receives the after-tax profits that firms obtain in the production process, $\int_{A_t^*}^{A_u} V_{it} dG(A_{it})$, or alternatively, the returns of the outside option if firms do not engage in production, $\phi G(A_t^*)$. The government taxes the rental rate at rate τ_t^k, the wage rate at the rate τ_t^l, firms' profits at rate τ_t^u and issues one-period bonds, which pay a gross interest rate of R_t. Let b_t^d be the stock of bonds held by the representative household. Hence, each period the household faces the following budget constraint:

$$\begin{aligned} c_t + i_t + \frac{b_{t+1}^d}{R_t} &= \widetilde{r}_t K_t + \widetilde{w}_t L_t + \int_{A_t^*}^{A_u} V_{it} dG(A_{it}) + \phi G(A_t^*) + b_t^d && (10) \\ K_{t+1} &= (1-\delta)K_t + i_t && (11) \\ h_t + L_t &= \overline{H} && (12) \end{aligned}$$

where following Chamley (1986) we define the variables $\tilde{r}_t$ and $\tilde{w}_t$ as $\tilde{r}_t \equiv r_t(1-\tau_t^k)$ and $\tilde{w}_t \equiv w_t(1-\tau_t^l)$.

Note that the household's problem does not include explicit expressions concerning uncertainty. In fact, uncertainty in our model arises in the firm sector. While each firm faces an idiosyncratic shock, there is no aggregate uncertainty in this economy as the productivity parameter has the same distribution in each period.

The solution to the consumer's problem yields the standard optimality conditions which include the marginal rate of substitution between present and future consumption (13), the intratemporal rate of substitution between leisure and consumption (14), the non-arbitrage condition (15) and the transversality conditions for capital and government bonds.

$$U_c(t) = \beta U_c(t+1)(1+\widetilde{r}_{t+1}-\delta) \tag{13}$$

$$U_h(t) = U_c(t)\widetilde{w}_t \tag{14}$$

$$R_t = 1+\widetilde{r}_{t+1}-\delta \tag{15}$$

2.3 The government

As is usual in the optimal taxation literature the government collects taxes to finance an exogenous expenditure path $\{g_t\}_{t=0}^{\infty}$. We assume that government expenditure is wasteful; that is, it does not provide any utility to the consumer. As noted above, the government finances its expenditure by issuing bonds and levying flat-rate, time-varying taxes on capital income, on labor income and on firms' profits. To avoid the possibility that the government raises all revenues by taxing initial capital heavily not distorting the economy allocations, we make the standard assumption that the government takes the tax rate on capital income in the first period, τ_0^k, as given. We also assume that the government can commit itself to a given policy so we do not analyze commitment issues. Further,

Assumption 3. *We assume that* $\tau^u < 1$.

We consider the case where $\tau^u < 1$ because when $\tau^u = 1$ there would be no firms producing, making it impossible to collect revenues to finance fiscal expenditure.[5]

Hence the government's budget constraint period t is:

$$\tau_t^l w_t \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) + \tau_t^k r_t \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) + \tau_t^u \int_{A_t^*}^{A_u} [A_{it} f(k_{it}, l_{it}) - r_t k_{it} - w_t l_{it}] dG(A_{it}) + \frac{b_{t+1}^s}{R_t} - b_t^s \geq g_t \quad \forall t$$

The first term on the left hand side is the amount of taxes on labor income, the second is the amount raised from capital income while the third term corresponds to the

[5]In fact, when $\tau^u = 1$ we have $V_{it} = 0 < \phi, \quad \forall i$

taxes raised from firms' profits. Since the firms' production functions are homogeneous of degree θ and using the definitions of $(\widetilde{w}_t, \widetilde{r}_t)$, the government's budget constraint can be written as (see appendix):

$$\begin{aligned} & \tau_t^u(1-\theta)\int_{A_t^*}^{A_u} A_{it} f(k_{it}, l_{it}) dG(A_{it}) + \theta \int_{A_t^*}^{A_u} A_{it} f(k_{it}, l_{it}) dG(A_{it}) \\ + & \frac{b_{t+1}^s}{R_t} - b_t^s - \widetilde{w}_t \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) - \widetilde{r}_t \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) \geq g_t \quad \forall t \end{aligned} \tag{16}$$

2.4 Equilibrium

Given the description of our economy, we may state the following definition of equilibrium.

Definition 1. *A competitive equilibrium is a sequence of allocations $\{c_t, k_{it}, l_{it}, b_t\}_{t=0, i\in I}^{\infty}$, a sequence of prices $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, a government policy $\{\tau_t^k, \tau_t^l, \tau_t^u, g_t\}_{t=0}^{\infty}$ and a sequence of threshold technology levels $\{A_t^*\}_{t=0}^{\infty}$ such that:*

1. *the household maximizes* (9) *subject to (10), (11) and (12) taking K_0 and b_0 as given,*
2. *each firm solves* (1) *conditional on A_{it},*
3. *the sequence of threshold technology levels is determined by:*

   $$(1-\tau_t^u)(1-\theta)A_t^* f(k_t^*, l_t^*) \geq \phi \quad \forall t,$$

   where k_t^*, l_t^* are the optimal capital stock and labor demanded by the firm endowed with the threshold technology level,
4. *the government satisfies* (16)*,*
5. *the capital market clears, i.e.*

$$K_t = \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) \quad \forall t, \tag{17}$$

6. *the labor market clears, i.e.*

$$\overline{H} - h_t = L_t = \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) \quad \forall t, \tag{18}$$

7. *the bonds market clears*

$$b_t^s = b_t^d \quad \forall t, \tag{19}$$

8. *the goods market clears*

$$c_t + g_t + K_{t+1} = \int_{A_t^*}^{A_u} A_{it} f(k_{it}^d) dG(A_{it}) + \phi G(A_t^*) + (1-\delta)K_t \quad \forall t \tag{20}$$

3 The Ramsey Problem and the Optimal Taxes

Our goal is to characterize the tax rates that are consistent with the allocations in a second best steady state, assuming that the economy converges to this steady state in the long run. As is standard in the literature, the social planner will choose among the set of competitive equilibria available the one that maximizes the representative individual utility. The planner chooses the allocations, tax rates and threshold technology subject to goods market clearing, consumer budget constraints, government budget constraints and the individual's and firms' optimality conditions. Therefore, the planner solves the following problem:

$$
\begin{aligned}
L &= \max_{\{c_t,\tau_t^u,k_{it},l_{it},b_t,\tilde{w}_t,\tilde{r}_t,A_t^*\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big\{ u\left(c_t, \overline{H} - \int_{A_t^*}^{A_u} l_{it} dG(A_{it})\right) \\
&+ \lambda_t^1 \Big[\int_{A_t^*}^{A_u} A_{it} f(k_{it}, l_{it}) dG(A_{it}) + \phi G(A_t^*) + (1-\delta) \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) - c_t - g_t - \int_{A_{t+1}^*}^{A_u} k_{it+1} dG(A_{it+1}) \Big] + \\
&+ \lambda_t^2 \Big[\tau_t^u \int_{A_t^*}^{A_u} (1-\theta) A_{it} f(k_{it}, l_{it}) dG(A_{it}) + \int_{A_t^*}^{A_u} \theta A_{it} f(k_{it}, l_{it}) dG(A_{it}) \\
&- \tilde{r}_t \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) - \tilde{w}_t \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) + \frac{b_{t+1}}{1 + \tilde{r}_{t+1} - \delta} - b_t - g_t \Big] + \\
&+ \lambda_t^3 \left[u_c(t) - \beta u_c(t+1)(1 + \tilde{r}_{t+1} - \delta) \right] + \lambda_t^4 \left[u_h(t) - u_c(t) \tilde{w}_t \right] + \lambda_t^5 \left[(1-\tau_t^u)(1-\theta) A_t^* f(k_t^*, l_t^*) - \phi \right] \Big\}
\end{aligned} \tag{21}
$$

Note that the above Ramsey problem is written as in the "dual approach", similar to many papers in the literature. In stating the problem, we follow Chamley (1986) by including $\tilde{r}_t$ and $\tilde{w}_t$. Note that these expressions do not represent prices but replace the capital income tax and the labor income tax, respectively.

The first constraint in this problem is the goods market clearing, given by equation (20). The second is the government budget constraint (16) taking into account the non-arbitrage condition (15), while the third is the intertemporal consumption Euler equation (13). The fourth restriction is the intratemporal marginal rate of substitution between consumption and leisure (14). The last restriction indicates that the marginal firm (i.e. the least productive firm that decides to operate) must earn after-tax profits at least as large as the outside option in the alternative activity.[6] For expositional simplicity we include the optimal conditions of the Ramsey problem in the appendix.

3.1 Optimal capital income taxation

We will next state the planner's optimal condition concerning the ith firm's capital stock, k_{it}. The optimality condition evaluated in steady state is:[7]

[6]We do not post the consumer budget constraint because it is redundant by Walras law.

[7]Follows from equation (53) in the appendix, dividing both sides by β^t and taking out time indexes (since we analyze steady state).

$$\lambda_1 \left[A_i f_k(k_i, l_i) + (1-\delta) - \frac{1}{\beta} \right] g(A_i) + \lambda_2 \left[\tau^u (1-\theta) A_i f_k(k_i, l_i) + \theta A_i f_k(k_i, l_i) - \widetilde{r} \right] g(A_i)$$
$$+ \quad \lambda_5 (1-\tau^u)(1-\theta) A^* f_k(k^*, l^*) \mathbf{1} \left[A_i = A^* \right] = 0, \tag{22}$$

where $\mathbf{1}\left[A_i = A^*\right]$ equals to one if firm i is the marginal firm and is zero otherwise.

The first term of this optimality condition indicates the marginal social value of the increase in output derived from the marginal increase in capital by firm i net of investment cost, while the second term is the social valuation of the increase in tax revenues derived from the increase in capital. The last term indicates the social value of an additional unit of capital invested by the marginal firm, which relaxes the participation constraint, enlarging the set of firms that are involved in production and, thus, are taxable.

The last term involving λ_5 refers to the distortion in the extensive margin and is key to our results. The first two terms related to λ_1 and λ_2 appear also in previous analysis where the optimal capital tax is equal to zero (e.g. Chamley 1985). The valuation given to the investment made by the marginal firm provides an additional benefit derived from an additional unit of capital.

Similarly, the first order condition with respect to τ^u, evaluated in steady state, yields the following expression:

$$\lambda_2 \int_{A^*}^{A_u} A_i (1-\theta) f(k_i, l_i) dG(A^i) = \lambda_5 A^* (1-\theta) f(k^*, l^*). \tag{23}$$

This expression also highlights the extensive margin distortion produced by a change in τ^u. It balances the marginal social cost of raising τ^u, given by the social value of displacing the marginal firm out of production, the term on the right hand side containing λ_5, with the social value of raising government revenues trough the increase in this tax rate.

Integrating (22) over all the firms involved in production and using (23) , we obtain the following expression for τ^k (see appendix):

$$\tau^k \quad = \quad \left[\frac{(1-\theta)(1-\tau^u)}{1-G(A^*)} \right] \left[\frac{\lambda_5}{\lambda_1 + \lambda_2} \right] \left[S_Y - 1 \right], \tag{24}$$

where $S_Y \equiv \frac{(1-\theta)A^* f(k^*,l^*)}{(1-\theta)\int_{A^*}^{A_u} A_i f(k_i,l_i) \frac{dG(A^i)}{1-G(A^*)}}$ is the share of production (profits) of the marginal firm in total production (profits). Note that S_Y is positive but less than one since the marginal firm has lower production (profits) than the rest of the firms involved in production.

Equation (24) is not a reduced form expression for τ^k. In fact, it depends on other endogenous variables such as A^*, $\frac{\lambda_5}{\lambda_1+\lambda_2}$ and S_Y. However, it allows us to obtain some intuition about the sign of τ^k. Firstly, note that $\tau^k \leq 0$. This result holds because (1) $\tau^u < 1$ if not, no firm would be involved in production, (2) λ_1, λ_2 and λ_5 are non negative and (3) $S_Y < 1$.

Note the relevance of the extensive margin distortion in our results. If λ_5 were equal to zero the optimal capital income tax would be zero as in the classical results of Chamley (1986) and Judd (1985). In this case, all firms are involved in production and there is no distortion in the extensive margin.[8]

However, if there are firms not involved in production, such that λ_5 is positive, it is optimal to subsidize capital income. This incentive to subsidize capital income comes from the extra social benefit derived from the additional unit of capital employed by the marginal firm (the term related to λ_5 in (22) discussed above). By setting τ^k less than zero the steady state rental rate faced by firms, r, is depressed.[9] This provides incentives to firms that are not producing, but that are in the neighborhood of doing so, to enter into production and allows the government to obtain revenue from them through the tax on profits. This would be the case if $\tau^u > 0$, a result that will be shown below.

As will be shown below, it is key to our results that the yields from the alternative option cannot be taxed at the same rate as profits. In our case, the government cannot obtain fiscal revenues from firms not involved in production. By inducing some firms into production through a decrease in the interest rate, the planner enlarges the set of firms that can be taxed using a tax on profits. The planner completes the tax system, at least partially.

Note that when there is no heterogeneity between firms, i.e. $S_Y = 1$, the optimal capital tax rate is zero. However, this situation is considered in the above discussion since

[8]If all firms are involved in production λ_5 is equal to zero by complementary slackness.

[9]By the consumer's Euler condition, equation (13), in a steady state, we have:

$$1 = \beta(1 + r(1 - \tau^k) - \delta).$$

in this case all firms would be producing; if all firms were in the alternative option there would not be taxation and the analysis loses relevance.

We can summarize the findings of this section in the following proposition:

Proposition 1. *It is optimal to set τ^k less than zero if and only if not all firms are producing. In the case that all firms are involved in production, the optimal τ^k is zero.*

Proof. See the discussion above.

3.2 Optimal labor tax and optimal tax on profits

In this section, we focus on the planner's choice of the tax rate on labor and profits. As in the case of the expression concerning τ^k, the expressions we obtain next are not reduced form solutions for those taxes as they will depend on other endogenous variables. However, as above, we will be able to obtain the signs and intuition about the economic determinants involved.

The optimal labor tax is obtained as follows. Similarly to the optimality condition of capital stock given in equation (22), we may obtain the optimality condition with respect to the allocation of labor in the i^{th} firm; which yields the following when evaluated in steady state:[10]

$$\begin{aligned} & [-u_h + \lambda_3 u_{ch}(\widetilde{r} - \delta) - \lambda_4 u_{hh} + \lambda_4 u_{ch}\widetilde{w}]\, g(A_i) \\ + \quad & \lambda_1 A_i f_l(k_i, l_i) g(A_i) + \lambda_2 \left[\tau^u (1-\theta) A_i f_l(k_i, l_i) + \theta A_i f_l(k_i, l_i) - \widetilde{w}\right] g(A_i) \\ + \quad & \lambda_5 (1-\tau^u)(1-\theta) A_i f_l(k_i^*, l_i^*) \mathbf{1}\left[A_i = A^*\right] = 0, \end{aligned} \tag{25}$$

where, as before, $\mathbf{1}\left[A_i = A^*\right]$ takes the value of one if the firm i is the marginal firm and zero otherwise.

The social planner balances the social benefit of increasing output through a marginal increase in labor (the term involving λ_1), the social value of increasing tax revenues (the term involving λ_2) and the social value of the change in the marginal firm's profits (the term involving λ_5) with the marginal social costs of increasing labor that is given by the direct effect in the utility of the consumer and the effects in the marginal rates of substi-

[10]Follows from dividing both sides of equation (54) in the appendix by β^t and dropping the time indexes since we are in steady state.

tution (first term of (25)). Similar effects were present in the derivation of (22), with the exception of the last one.

Integrating this expression over all the firms involved in production and using the first order conditions with respect to c_t , $\tilde{w}_t$ and $\tilde{r}_t$ (equations (55) to (57) in the appendix) we obtain:

$$\tau^l\left[\lambda_2\left(1+\sigma_{hh}+\sigma_{ch}\right)+u_c\right] = \lambda_2\left[\frac{(1-\theta)(1-\tau^u)}{S_Y}\left(S_Y-1\right)+\left(\sigma_{hh}+\sigma_{cc}+\sigma_{ch}\right)\right] \quad (26)$$

where $\sigma_{cc}=\frac{-u_{cc}\widetilde{c}}{u_c}$, $\sigma_{ch}=\frac{u_{ch}\widetilde{c}}{u_h}$, $\sigma_{hh}=\frac{-u_{hh}(1-h)}{u_h}$ and $\widetilde{c}=\left[\widetilde{r}(\int_{A^*}^{A_u}k_i dG(A_i)+b_{ss}\beta)+\widetilde{w}\int_{A^*}^{A_u}l_i dG(A_i)\right]$ is total individual's income -excluding firm's transfers- in steady state.[11] Note that $\sigma_{cc},\sigma_{ch},\sigma_{hh}>0$. The term in parenthesis that multiplies τ^l is positive, so the sign of this tax depends on the sign of the right hand side of (26). The first term on the right hand side has similar components to the expression obtained for τ^k in (24, and is negative. The second term is the sum of σ_{cc},σ_{ch} and σ_{hh}, which is related to the concavity of the utility function and is positive.

Equation (26) shows that there are two forces involved in the determination of the sign of the optimal labor tax rate. On the one hand, and similar to the case of τ^k, there is an incentive to subsidize firms' production costs (the first part of the right hand side on (26)) to induce firms into production, which allows the government to collect fiscal revenue from these additional firms through the corporate tax. On the other hand, there is a second term that considers the impact on the individual's utility of distorting leisure that shows that the more concave is the utility function, the more likely the optimal labor tax to be positive.

Further, note that if the whole set of firms is involved in production $\lambda_5=0$ and, from equation (23), $\lambda_2=0$. It follows from equation (26) that the optimal τ^l is zero, as in the case of capital taxation. The intuition is that as the whole set of firms is already involved

[11] Also, note that

$$\beta b_{ss}=(1+\widetilde{r}-\delta)b_{ss}$$

is the gross return on bonds in steady state and

$$(1-\beta)b_{ss}=\frac{\widetilde{r}-\delta}{1+\widetilde{r}-\delta}b_{ss}=\frac{R-1}{R}b_{ss}$$

is the bonds' interest payment expressed in units of this period.

in production, the planner's incentives to subsidize firms' costs disappear. Fiscal revenue will be obtained from firms' profits, as will be shown below. As a result, the optimal policy is to set $\tau^l = 0$ to avoid distortions in the marginal rates of substitution.

We next focus on obtaining the optimal tax on profits, τ^u. We will initially analyze the case in which there are firms involved in production while others obtain ϕ in the alternative outside option. In this case A^* is interior, i.e. $A_l < A^* < A_u$ and $\lambda_5 > 0$.

To obtain τ^u note that using equations (25) and the first order condition with respect to A^* (equation (58) in the appendix) yields:

$$\frac{\tau^u}{1-\tau^u} = \frac{\lambda_5}{\lambda_1 + \lambda_2}\left[\frac{\theta(1-S_Y)}{1-G(A^*)} + \frac{1}{A^* g(A^*)}\right]. \tag{27}$$

Again, this expression is not a closed form solution since it depends on other endogenous variables. However, it is enough to determine the sign of τ^u, which is positive since the right hand side of (27) is positive for the reasons mentioned above.

We analyze next the case in which A^* is not interior, i.e. $A_l = A^*$. In this case equation (27) cannot be applied in the analysis since it was obtained using the first order condition with respect to A^*, equation (58) in the appendix, that is no longer valid in the case that $A^* = A_l$. To obtain τ^u in this corner case note that we must satisfy the government budget constraint, which in steady state is:[12]

$$\tau^u \int_{A_l}^{A_u} (1-\theta) A_i f(k_i, l_i) dG(A_i) + \frac{b_{ss}}{1+\widetilde{r}-\delta} - b_{ss} = g$$

where b_{ss} is the level of government bonds in steady state. It follows that:

$$\tau^u = \frac{g + (1-\beta) b_{ss}}{(1-\theta)\int_{A_l}^{A_u} A_i f(k_i, l_i) dG(A_i)}. \tag{28}$$

Note that in the case where all firms are involved in production, the sign of τ^u depends on fiscal expenditure, g, plus bond interest payments in steady state, $(1-\beta)b_{ss}$. We can summarize the preceding discussion in the following proposition.

[12]When $\tau^k = \tau^l = 0$.

Proposition 2. *In the case that some firms are not involved in production, the optimal tax on profits is positive while the sign of the optimal labor tax remains ambiguous. However, in the case that all firms are involved in production, the optimal tax on profits is different from zero while the labor tax is zero.*

3.3 Allowing Taxation on the Yields of the Alternative Option.

We have analyzed a setup where the planner faces a problem in which (1) there are heterogeneous firms and (2) there is an alternative outside option available to firms whose return is not taxable. In this setup, we have shown that $\tau^k \leq 0$ and $\tau^u > 0$ while the sign of τ^l is ambiguous. We have also shown that in the case that all firms choose to be involved in production, the optimal taxes are $\tau^k = \tau^l = 0$ and $\tau^u \neq 0$.

We will argue next that these results depend crucially on the absence of taxation of the outside option at the same rate as profits derived from production. To understand the importance of this assumption, we will allow next for taxation of ϕ at the same rate that is applied to profits obtained in production, τ^u. In that case, our Ramsey problem would be modified as follows:

$$\begin{aligned} L \; = & \max_{\{c_t,\tau_t^u,k_{it},l_{it},b_t,\tilde{w}_t,\tilde{r}_t,A_t^*\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big\{ u\left(c_t, \overline{H} - \int_{A_t^*}^{A_u} l_{it} dG(A_{it})\right) \\ + & \; \mu_t^1 \Big[\int_{A_t^*}^{A_u} A_{it} f(k_{it}, l_{it}) dG(A_{it}) + \phi G(A_t^*) + (1-\delta) \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) - c_t - g_t - \int_{A_{t+1}^*}^{A_u} k_{it+1} dG(A_{it+1}) \Big] + \\ + & \; \mu_t^2 \Big[\tau_t^u \phi G(A_t^*) + \tau_t^u \int_{A_t^*}^{A_u} (1-\theta) A_{it} f(k_{it}, l_{it}) dG(A_{it}) + \int_{A_t^*}^{A_u} \theta A_{it} f(k_{it}, l_{it}) dG(A_{it}) \\ - & \; \tilde{r}_t \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) - \tilde{w}_t \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) + \frac{b_{t+1}}{1+\tilde{r}_{t+1}-\delta} - b_t - g_t \Big] + \\ + & \; \mu_t^3 \left[u_c(t) - \beta u_c(t+1)(1+\tilde{r}_{t+1}-\delta) \right] + \mu_t^4 \left[u_h(t) - u_c(t)\tilde{w}_t \right] + \mu_t^5 (1-\tau_t^u) \left[(1-\theta) A_t^* f(k_t^*, l_t^*) - \phi \right] \Big\} \end{aligned} \tag{29}$$

Problem (29) differs from problem (21) in two ways. First, the government budget constraint includes the taxation of the yields from the outside option, $\tau_t^u \phi G(A_t^*)$, and second, the marginal firm's entry decision differs, as a firm obtains a return $(1-\tau_t^u)\phi$ if it chooses not to participate in production. We will next obtain the optimal taxes in this case. The next proposition states the results.

Proposition 3. *If we allow taxation of the outside option at the rate τ^u, the optimal tax rates are $\tau^k = \tau^l = 0$ while $\tau^u \neq 0$.*

Proof. We focus initially on the case of an interior solution for A_t^*. The first order condition with respect to τ^u, evaluated in steady state is:[13]

$$\mu^2\left[\phi G(A_t^*) + \int_{A_t^*}^{A_u} (1-\theta)A_{it}f(k_{it},l_{it})dG(A_{it})\right] = \mu^5\left[(1-\theta)A_t^* f(k_t^*,l_t^*) - \phi\right]. \tag{30}$$

Since in an interior solution $(1-\theta)A_t^* f(k_t^*,l_t^*) = \phi$ it follows that $\mu_2 = 0$. Using this condition, and the optimality condition with respect to capital stock, we obtain:

$$\tau^k = -\frac{(1-\theta)(1-\tau^u)}{1-G(A^*)}\frac{\mu_5}{\mu_1} \leq 0. \tag{31}$$

On the other hand, using the optimality condition with respect to A^*, the optimality condition on labor and the result concerning μ_2, we get:

$$\tau^k = \frac{(1-\theta)(1-\tau^u)}{rk^*}\frac{\mu_5}{\mu_1}\left[\frac{wl^*}{1-G(A^*)} + \frac{A^* f(k^*,l^*)}{A^* g(A^*)}\right] \geq 0. \tag{32}$$

Note that (31) implies $\tau^k \leq 0$ while (32) implies $\tau^k \geq 0$. It follows that $\tau^k = 0$. Further, the optimality condition with respect to labor implies:

$$\tau^l\left(u_c + \mu_2(1+\sigma_{hh}+\sigma_{ch})\right) = \mu_2\frac{(1-\theta)(1-\tau^u)}{S_Y}\left[S_Y - 1\right] + \mu_2\left[\sigma_{hh}+\sigma_{cc}+\sigma_{ch}\right]. \tag{33}$$

But since $\mu_2 = 0$, it follows that (33) implies $\tau^l = 0$. Finally, to satisfy the government budget constraint we require:

$$\tau^u = \frac{g + (1-\beta)b_{ss}}{(1-\theta)\int_{A^*}^{A_u} A_i f(k_i,l_i)dG(A_i)}, \tag{34}$$

[13] We do not post in the appendix the derivations since they are similar to the ones obtained in the case where ϕ is not taxable.

where b_{ss} is the level of government bonds in steady state.

We will now establish the results in the case of no interior solution, i.e. $A^* = A_l$. In this case $\mu_5 = 0$ and by the optimality condition on τ^u, we have $\mu_2 = 0$. Trivially by (32) and (33), we obtain $\tau^k = \tau^l = 0$ and by (34), we get $\tau^u \neq 0$. □

Proposition 3 illustrates the importance of the impossibility of taxing firms' outside option at the same rate as profits in our results: if we allow taxation of the proceeds of the alternative activity at the same rate as profits, capital income and labor taxes would be zero. In this case the tax on profits, τ^u, is non distortive since it does not affect either the intensive or the extensive margins because in both sectors there is a common tax rate.[14]

We can relate this result to the ones obtained previously that highlight the relevance of the extensive margins and the completeness of the tax system. The capital and labor income tax rates are zero when all firms are involved in production and/or the yields of the alternative option are taxable. That is, when there is no distortion in the extensive margin derived from the tax on profits. When there is such a distortion because the alternative option cannot be taxed and some firms choose not to produce, the planner subsidizes capital and possibly labor to induce firms into production, making them taxable. In this way, the planner completes, at least partially, the tax system.

4 Numerical Examples

We now use numerical methods to simulate calibrated versions of the model we have analyzed. We will use these results to confirm the validity of our analytical expressions and shed additional light on our results. We use the following standard functional form for the utility function:

$$U = \frac{\left(c_t^{\Phi 2}(\overline{H} - l_t)^{1-\Phi 2}\right)^{\Phi 3}}{\Phi_3} \tag{35}$$

The production function of a firm in operation:

[14] Note that if the tax on the alternative option, call it τ^ϕ, were different from the tax on profits a change in this tax would affect the extensive margin. However, our analysis of non taxation of the alternative option holds if we redefine the tax on profits as $1 - \tau^\pi = \frac{1-\tau^u}{1-\tau^\phi}$.

$$Y_{it} = A_{it}(k_{it}^{\alpha} l_{it}^{1-\alpha})^{\theta} \tag{36}$$

General Method

In our simulations, we consider the long-run steady state of the Ramsey equilibrium. To obtain our results, we follow the procedure developed in Schmidt-Gohré and Uribe (2006). In the notation that follows, we eliminate the time index since we analyze the steady state. Let $\mathcal{F}(x, \Gamma)$ be the first order condition of the Ramsey problem defined in (21), where x are the variables and Γ are the parameters of the problem. Optimality requires $\mathcal{F}(x, \Gamma) = 0$. Our goal is to obtain x_{ss} -where ss indicates steady state- such that $\mathcal{F}(x_{ss}, \Gamma) = 0$. To obtain the solution, we use the symbolic Matlab toolbox and implement the following algorithm:

1. Guess an initial candidate vector x_{ss}^{j} and choose criteria $\delta > 0, \upsilon > 0$, where j indicates iteration.

2. Compute the direction s^{j} to modify the initial candidate vector x_{ss}^{j}. The direction is chosen as in the steepest decent method, [15]

   $$s^{j} = -\nabla\mathcal{F}(x_{ss}^{j}, \Gamma)'$$

   where $\nabla\mathcal{F}(x_{ss}^{j}, \Gamma)$ is the Jacobian of $\mathcal{F}(x, \Gamma)$ evaluated at x_{ss}^{j} and ′ indicates transpose.

3. Solve for the line-step criterion, λ_j, as in $\lambda_j = argim_{\lambda} \quad \mathcal{F}(x_{ss}^{j} + \lambda s^{j}, \Gamma)$.

4. Compute the update x_{ss}^{j+1} as in $x_{ss}^{j+1} = x_{ss}^{j} + \lambda_j s^{j}$.

5. If $\|x_{ss}^{j} - x_{ss}^{j+1}\| < \upsilon(1 + \|x_{ss}^{j}\|)$ continue to next step, otherwise go back to step 2.

6. If $\|\nabla\mathcal{F}(x_{ss}^{j+1}, \Gamma)\| < \delta(1 + \|\mathcal{F}(x_{ss}^{j+1}, \Gamma)\|)$ stop and report success, otherwise report failure.

[15] We alternatively used the Broyden-Fltecher-Goldfarb-Shannon method but we found no significative differences in our results.

Calculation of the guess x_{ss}^j

To implement the numerical procedure, we require a candidate steady state vector in the j^{th} iteration, x_{ss}^j, which is calculated as follows. We discretize the number of firms and we include tax rates plus labor supply as initial guesses, i.e. $(\tau_{ss}^{u,j}, \tau_{ss}^{l,j}, \tau_{ss}^{k,j}, l_{ss}^j)$. The rest of the variables evaluated in the steady state are obtained by using the following algorithm in iteration j. Using the consumer's intertemporal optimality condition evaluated in steady state we get:

$$\widetilde{r}_{ss}^j = \frac{1}{\beta} - (1-\delta) \tag{37}$$

We next obtain labor demand by each firm. Labor market clearing condition requires:

$$l_{ss}^j = \sum_{i=1}^{N} l_i^j \tag{38}$$

where N is the discrete number of total firms in the economy. Further, note that each firm's labor demand (if it decides to operate) is:[16]:

$$l_i^j = \left[\frac{A_i \alpha\theta}{r}\right]^{\frac{1}{1-\theta}} \left(\frac{k_i}{l_i}\right)^{\frac{\alpha\theta-1}{1-\theta}} = \left[\frac{A_i \alpha\theta}{r}\right]^{\frac{1}{1-\theta}} \left(\frac{\alpha}{1-\alpha}\frac{w}{r}\right)^{\frac{\alpha\theta-1}{1-\theta}} \tag{42}$$

Note that equations (38) and (42) provide labor demand as a function of the labor supply guess and productivity parameters:

[16] In fact, each firm's optimality conditions are

$$A_i \alpha\theta k_i^{\alpha\theta-1} l_i^{(1-\alpha)\theta} = r \tag{39}$$

$$A_i (1-\alpha)\theta k_i^{\alpha\theta} l_i^{(1-\alpha)\theta-1} = w \tag{40}$$

which imply:

$$\frac{k_i}{l_i} = \frac{\alpha}{1-\alpha}\frac{w}{r} \tag{41}$$

Replacing equation (41) in (39) is (42).

$$l_{i,ss}^{j} = \left[\frac{A_i^{\frac{1}{1-\theta}}}{\sum_{z=1}^{N} A_z^{\frac{1}{1-\theta}}}\right] l_{ss}^{j} \quad i = 1, ..., N \tag{43}$$

Similarly capital demand, if the firm operates, is:

$$k_{i,ss}^{j} = \frac{\frac{\widetilde{r}_{ss}^{j}}{1-\tau_{ss}^{k}}}{(A_i \theta \alpha (l_{i,ss}^{j})^{\theta-1})^{\frac{1}{\alpha\theta-1}}} \tag{44}$$

We define next an indicator function equal to one when the firm decides to operate by using:

$$\mathbb{1}_i(operation)^j = 1 - \frac{(\arctan(\frac{-(1-\tau^u)A_i(k_{i,ss}^{j,\alpha} l_{i,ss}^{j,1-\alpha})^{\theta}+\phi}{\epsilon}) + \pi/2)}{\pi} \tag{45}$$

where $\mathbb{1}_i(operation)^j$ is the indicator function which is equal to one if the i^{th} firm operates and zero otherwise, while $\epsilon > 0$ is a parameter. Note that in the above indicator function k_i^j and l_i^j represent capital and labor demand if the i^{th} firm operates. These demands are defined in (43) and (44). The parameter ϵ determines the shape of the function, the smaller is ϵ the less smooth is the function. An example of this function is shown in figure (1). It shows the case of a firm that faces $(\tau^u = 0.1, \phi = 1)$. In the figure we treat capital and labor demand as exogenous. Obviously in our model, these two last variables are endogenous. However, in the figure we treat them as exogenous to describe the way the function works out. In the figure, $A_i \approx 1.1$ is a threshold level: if the firm draws a productivity parameter larger than the threshold level, the firm's after-tax profits are larger than ϕ and the firm operates. Clearly, the smaller is ϵ the more the function resembles an indicator function.

[Insert figure 1 about here]

Note that if a firm does not operate, it does not demand either labor or capital. Therefore, we update labor and capital demand by firms as in:

$$k_{i,ss}^{j} = \left\{ \begin{array}{cc} k_{i,ss}^{j} & \mathbb{1}_i(operation)^j = 1 \\ 0 & \mathbb{1}_i(operation)^j = 0 \end{array} \right\} \tag{46}$$

$$l_{i,ss}^{j} = \left\{ \begin{array}{cc} l_{i,ss}^{j} & \mathbb{1}_i(operation)^j = 1 \\ 0 & \mathbb{1}_i(operation)^j = 0 \end{array} \right\} \tag{47}$$

Using the clearing of the goods market, we obtain consumption in steady state:

$$c_{ss}^{j} = \sum_i \mathbb{1}_i(operation) A_i (k_i^{\alpha} l_i^{1-\alpha})^{\theta} + \sum_i (1 - \mathbb{1}_i(operation))\phi - \delta \sum_i \mathbb{1}_i(operation) k_i - G \tag{48}$$

where G is fiscal expenditure. Next note that marginal utilities of consumption and leisure in steady state are:

$$U_c^j = \Phi 2 \left((c_{ss}^j)^{\Phi 2} (\overline{H} - l_{ss}^j)^{1-\Phi 2} \right)^{\Phi 3 - 1} \left((c_{ss}^j)^{\Phi 2 - 1} (\overline{H} - l_{ss}^j)^{1-\Phi 2} \right) \tag{49}$$

$$U_h^j = (1 - \Phi 2) \left((c_{ss}^j)^{\Phi 2} (\overline{H} - l_{ss}^j)^{1-\Phi 2} \right)^{\Phi 3 - 1} \left((c_{ss}^j)^{\Phi 2} (\overline{H} - l_{ss}^j)^{-\Phi 2} \right) \tag{50}$$

It follows that to satisfy the intratemporal marginal rate of substitution between consumption and leisure, total labor supply is[17]:

$$l_{ss}^j = \overline{H} - \frac{(c_{ss}^j)}{\widetilde{w}^j} \left[\frac{1 - \Phi 2}{\Phi 2} \right] \tag{51}$$

Equation (51) allows us to update our guess on labor supply. Let $l_{ss}^{j,updated}$ be the update obtained from (51). If $\|l_{ss}^j - l_{ss}^{j,updated}\| < \vartheta$, where $\vartheta > 0$ is a convergence criterion, we have computed the candidate vector x_{ss}^j. Otherwise, we go back to (37) and recompute the steady state variables in the j^{th} iteration, using the updated labor supply.

Equations (37), (43) to (51) and the tax guesses allow us to obtain our candidate vector x_{ss}^j in the j^{th} iteration.

Parameters and results

We set the following parameters: $\beta = 0.9906$, $\delta = 0.05$, $\alpha = 0.36$, $\Phi_2 = 0.75$, $\Phi 3 = 1$,

[17] In this step, we require $\widetilde{w}^j$ which is computed as:

$$\widetilde{w}^j = A_N (1 - \alpha) \theta k_N^{\alpha\theta} l_N^{(1-\alpha)\theta - 1} (1 - \tau_{ss}^{l,j})$$

where N indicates the firm with the larger productivity parameter which by assumption (2) is always involved in production.

$\epsilon = 10^{-100}$. These parameters are consistent with the values reported Schmidt-Gohré and Uribe (2006). We discretize productivity in 50 equidistant points in the range $[1, 5]$, each of the points with 2% probability. Hence, our numerical exercises will have 50 different types of firms.

Figures (2) to (4) show the results concerning after-tax profits, labor demand and capital demand per type of firm for $\theta = \{0.7; 0.85; 0.9\}$ and $\phi = 1.5$. In the figures, when a firm's after-tax profit is equal to ϕ, the firm is not involved in production. As shown in the figures, optimal tax rates differ. In line with our theoretical discussion, we obtain in the three cases $\tau^u > 0, \tau^k$ and $\tau^l < 0$. In general, the larger is θ the larger is τ^u and the larger are the subsidies to the capital and labor income, τ^k and τ^l. Further, figure (2) shows also that the larger is θ, the larger is the fraction of firms with after-tax profits equal to ϕ, i.e. the larger is the fraction of firms not involved in production. Figures (3) and (4) show, respectively, labor and capital demand per firm. The larger is θ, the larger is the increase in labor and capital demand as productivity rises, conditional on the firm being in operation. These results hold because the larger is θ, the more elastic is the marginal cost of the firm and therefore the larger is the output and factor demand responses to the change in productivity.

[Insert figures (2) to (4) about here]

Table (1) shows the results for different values of ϕ. The first three columns of the table present the firm's exogenous parameters, the next three columns show the preference parameters and the last four columns show the results, including the fraction of firms involved in production, $1 - G(A^*)$. In the table, fiscal expenditure is set at a level of 500, which corresponds to 7% of output when $(\phi = 0, g = 0)$, i.e. the case in which all firms are involved in production and there are no fiscal distortions.

The table shows that, conditional on ϕ, a larger θ in most of the cases is associated with a larger corporate tax rate and larger capital and labor subsidies. Similarly, the fraction of firms involved in production decreases. These results are in line with the results obtained in figures (2) to (4) but as shown in the table, they also apply to the cases of $\phi = \{1; 1.5; 2\}$. Intuitively, the larger is θ, holding constant other parameters, the lower are firms' profits and the larger must be the corporate tax rate to raise revenues. On the other hand,the larger is θ, larger subsidies to production costs are required to complete the tax system, i.e. provide incentive to firms to produce. Finally, the result concerning a lower fraction of firms involved in production is easily explained because a larger θ is associated with larger factor payments and lower after-tax firms' profits.

[Insert table (1) about here]

Table (2) provides sensitivity analysis. In the table we use the cases: $(\phi, \theta) = (1, 0.85)$ and $(\phi, \theta) = (1, 0.9)$ as benchmark. We initially include in the table a larger fiscal expenditure. We set fiscal expenditure at 600 which corresponds to approximately 8.5% of output when all firms are involved in production and there are no fiscal distortions. While the signs of the tax rates continue to be $\tau^u > 0, \tau^k, \tau^l < 0$, there is not a unique response of the tax rates to the increase in fiscal expenditure. On one hand, when $\theta = 0.85$, the magnitudes of both the subsidies and the corporate tax rate decrease and as a result, the fraction of firms becomes larger. In that case, the base of collection, in terms of the number of firms, increases. On the other hand, when $\theta = 0.9$, the corporate tax rate marginally increases while the labor tax rate becomes larger and the capital income tax rate approaches zero. In this case, the fraction of firms involved in production remains stable while the increase in labor subsidy is small compared to the decrease in capital income tax rate, i.e. the increase in tax revenues is obtained holding constant the base of the tax collection and decreasing net subsidies.

We next increase the capital share in the production function by setting $\alpha = 0.4$. In this case, the optimal subsidy in capital income becomes larger, both in the case of $\theta = \{0.85; 0.9\}$ while the labor subsidy and the corporate tax rate approach zero. As a result, the number of firms involved in production is larger. In this case, the firm's capital demand becomes more elastic providing more incentives to the planner to depress the rate of return faced by firms to induce more firms into production. Since the number of firms involved in production -which is a component of the tax base- rises, the planner might depress the labor subsidy and the corporate tax rate.

Finally, we provide a sensitivity analysis of the response of optimal tax rates vis-$\grave{a}$-vis the parameters of the utility function, (Φ_2, Φ_3). On one hand, when we increase Φ_2, the triplet of taxes approaches zero while the fraction of firms involved in production increases. In this case, distortions in leisure are more relevant, and the planner reacts by setting a smaller subsidy to labor income. To satisfy the government budget constraint the planner reacts by providing incentives to new firms to enter into production by setting lower distortions -through a lower corporate tax rate- in the extensive margin decision. On the other hand, when we set $\Phi_3 = 0.75$, i.e. the utility function becomes more concave, we obtain mixed results. In the case of $\theta = 0.85$, the capital income subsidy approaches zero while the magnitude of the labor income subsidy and the magnitude of the corporate tax rate increase and subsequently the fraction of firms involved in production decreases. When $\theta = 0.9$, the converse holds. The consequence of this last set of results is that the planner uses the optimal tax rates such that fluctuations in the fraction of firms involved in production is diminished, as a way of decreasing fluctuations in consumption and labor supply (leisure).

[Insert table (2) about here]

5 Conclusions

In this paper we have introduced heterogeneous firms to study optimal taxation. Firms differ in their productivity and have to decide if they want to produce in each period after comparing their expected profits vis-$\grave{a}$-vis an outside option that is not taxable, i.e. our environment is one of incomplete taxation (Correia (1996)). To finance its expenditure the government relies on capital income, labor and profits taxation. The presence of heterogeneous firms implies two kinds of possible distortions from taxes. They may affect the extensive margin decisions (i.e. whether to produce or not) and the intensive margin decisions (i.e. optimal allocation given that they decide to produce).

We have shown that the results depend on whether all firms decide to produce or the less productive firms decide not to produce. In the first case, there is no distortion in the extensive margin and the social planner will not tax capital, which replicates Chamley (1986) and Judd (1985) results of not taxing capital income in the long run. Also, in this case, it is optimal not to tax labor and leave the tax on profits as the only source of fiscal revenues.

However, when there are firms that are not so productive or when the outside option is lucrative enough such that some firms prefer not to produce, it is optimal to subsidize capital. The sign of the tax on labor income is ambiguous depending on the distortions it creates in the labor supply.

The intuition of the subsidy is related to the government's inability to tax the firms' alternative option. By subsidizing production costs, the government induces firms to produce, making it possible to tax their profits. That is, in the second best, the planner is partially completing the tax instruments by taxing firms that would not be taxed if they remain in the alternative option. In this respect, firm heterogeneity is key to the results.

We have also analyzed the case where the government can tax the yields from the alternative option at the same rate it taxes profits from firms that are involved in production. In this case the tax system is complete and there is no need to induce firms into production by subsidizing. Tax on profits are now non distortive and it is optimal to set tax on capital and labor income equal to zero.

Appendix

•Derivation of equation 16

Since the production functions are homogeneous of degree θ, the government budget constraint is:

$$\tau_t^l w_t \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) + \tau_t^k r_t \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) + \tau_t^u \int_{A_t^*}^{A_u} A_{it}(1-\theta) f(k_{it}, l_{it}) dG(A_{it}) + \frac{b_{t+1}^s}{R_t} - b_t^s \geq g_t \quad \forall t$$

Further note that

$$\begin{aligned}
\tau_t^l w_t \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) &= w_t \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) - \widetilde{w_t} \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) \\
\tau_t^k r_t \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) &= r_t \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) - \widetilde{r}_t \int_{A_t^*}^{A_u} k_{it} dG(A_{it}).
\end{aligned}$$

Replacing in the government budget constraint,

$$\begin{aligned}
& \int_{A_t^*}^{A_u} (w_t l_{it} + r_t k_{it}) dG(A_{it}) - \widetilde{w_t} \int_{A_t^*}^{A_u} l_{it} dG(A_{it}) - \widetilde{r}_t \int_{A_t^*}^{A_u} k_{it} dG(A_{it}) + \tau_t^u (1-\theta) \int_{A_t^*}^{A_u} A_{it} f(k_{it}, l_{it}) dG(A_{it} \\
+ \quad & \frac{b_{t+1}^s}{R_t} - b_t^s \geq g_t \quad \forall t
\end{aligned}$$

Finally, since $w_t l_{it} + r_t k_{it} = \theta A_{it} f(k_{it}, l_{it})$, we get equation (16) in the text.

•**Optimality conditions of Ramsey problem in equation (21)**.

The optimality condition with respect to k_{it} is:

$$
\begin{aligned}
& \lambda_{1t}\beta^t \left[A_{it} f_k(k_{it}, l_{it}) + (1-\delta)\right] g(A_{it}) + \lambda_{2t}\beta^t \left[\tau_t^u (1-\theta) A_{it} f_k(k_{it}, l_{it}) + \theta A_{it} f_k(k_{it}, l_{it}) - \widetilde{r}_t\right] g(A_{it}) \\
+ \quad & \lambda_{5t}\beta^t (1-\tau_t^u)(1-\theta) A_t^* f_k(k_t^*, l_t^*) \mathbf{1}\left[A_{it} = A_t^*\right] = \beta^{t-1} \qquad (53)
\end{aligned}
$$

The optimality condition with respect to l_{it} is:

$$
\begin{aligned}
& \beta^t \left[-u_h(t) + \lambda_{3t} u_{ch}(t)(\widetilde{r}_t - \delta) - \lambda_{4t} u_{hh}(t) + \lambda_{4t} u_{ch}(t)\widetilde{w}_t\right] g(A_{it}) \\
+ \quad & \beta^t \lambda_{1t} A_{it} f_l(k_{it}, l_{it}) g(A_{it}) + \beta^t \lambda_{2t} \left[\tau_t^u (1-\theta) A_{it} f_l(k_{it}, l_{it}) + \theta A_{it} f_l(k_{it}, l_{it}) - \widetilde{w}_t\right] g(A_{it}) \\
+ \quad & \beta^t \lambda_{5t} (1-\tau_t^u)(1-\theta) A_t^* f_l(k_t^*, l_t^*) \mathbf{1}\left[A_{it} = A_t^*\right] = 0 \qquad (54)
\end{aligned}
$$

The optimality condition with respect to $(c_t, A_t^*, \widetilde{w_t}, \widetilde{r}_t)$ evaluated in steady state are:

$$
\begin{aligned}
[c] \quad &: \quad u_c(c,h) - \lambda_1 + u_{cc}(c,h)\left[\lambda_3(-\widetilde{r} + \delta) - \lambda_4 \widetilde{w}\right] = 0 \qquad (55) \\
[\widetilde{w}] \quad &: \quad -\lambda_2 \int_{A^*}^{A_u} l_i dG(A_t) - \lambda_4 u_c(c,h) = 0 \qquad (56) \\
[\widetilde{r}] \quad &: \quad -\lambda_2 \int_{A^*}^{A_u} k_i dG(A_t) - \lambda_2 \frac{b}{1 + \widetilde{r} - \delta} - \lambda_3 u_c(c,h) = 0 \qquad (57) \\
[A^*] \quad &: \quad \left[u_h - \lambda_3 u_{ch}(\widetilde{r} - \delta) - \lambda_4 u_{ch}\widetilde{w} + \lambda_4 u_{hh}\right] l^* g(A^*) + \lambda_1 \left[-1 + (1-\tau^u)(1-\theta)\right] A^* f(k^*, l^*) g(A^*) + \\
& \quad + \lambda_1 \widetilde{r} k^* g(A^*) + \lambda_2 \left[-\tau^u(1-\theta) - \theta\right] A^* f(k^*, l^*) g(A^*) + \lambda_2 \left[\widetilde{r} k^* + \widetilde{w} l^*\right] g(A^*) \\
& \quad + \lambda_5 (1-\tau^u)(1-\theta) f(k^*, l^*) = 0 \qquad (58)
\end{aligned}
$$

•Derivation of equation (24):

Integrating (22) with respect to the firms involved in production, we have:

$$\begin{aligned}
& \lambda_1 \left[\int_{A_t^*}^{A_u} A_i f_k(k_i, l_i) dG(A_i) + \left((1-\delta) - \frac{1}{\beta} \right) (1 - G(A_t^*)) \right] \\
+ \quad & \lambda_2 \left[\tau^u (1-\theta) \int_{A_t^*}^{A_u} A_i f_k(k_i, l_i) dG(A_i) + \theta \int_{A_t^*}^{A_u} A_i f_k(k_i, l_i) dG(A_i) - \widetilde{r}(1 - G(A_t^*)) \right] \\
+ \quad & \lambda_5 (1-\tau^u)(1-\theta) A^* f_k(k^*, l^*) = 0 \qquad (59)
\end{aligned}$$

Adding and substracting $\lambda_2 \int_{A_t^*}^{A_u} A_i f_k(k_i, l_i) dG(A_i)$ and dividing by $1 - G(A_t^*)$, we obtain:

$$\begin{aligned}
& \lambda_1 \left[\frac{\int_{A_t^*}^{A_u} A_i f_k(k_i, l_i) dG(A_i)}{1 - G(A_t^*)} + \left((1-\delta) - \frac{1}{\beta} \right) \right] \\
+ \quad & \lambda_2 \left[(\tau^u - 1)(1-\theta) \frac{\int_{A_t^*}^{A_u} A_i f_k(k_i, l_i) dG(A_i)}{1 - G(A_t^*)} + \frac{\int_{A_t^*}^{A_u} A_i f_k(k_i, l_i) dG(A_i)}{1 - G(A_t^*)} - \widetilde{r} \right] \\
+ \quad & \frac{\lambda_5}{1 - G(A_t^*)} (1-\tau^u)(1-\theta) A^* f_k(k^*, l^*) = 0 \qquad (60)
\end{aligned}$$

Since $\beta(1 + \widetilde{r}_t - \delta) = 1$ and using (5):

$$\lambda_1 [r - \widetilde{r}] + \lambda_2 [(\tau^u - 1)(1-\theta) r] + \lambda_2 [r - \widetilde{r}] + \frac{\lambda_5}{1 - G(A_t^*)} (1-\tau^u)(1-\theta) r = 0 \qquad (61)$$

Dividing by r:

$$\lambda_1 \tau^k + \lambda_2 [(\tau^u - 1)(1-\theta)] + \lambda_2 \tau^k + \frac{\lambda_5}{1 - G(A_t^*)} (1-\tau^u)(1-\theta) = 0 \qquad (62)$$

Using (23) in (62):

$$\tau^k = \left[\frac{\lambda_5}{\lambda_1 + \lambda_2} \right] \left[\frac{(1-\tau^u)(1-\theta)}{1 - G(A_t^*)} \right] [S_Y - 1] \qquad (63)$$

Where (63) is equation (24) in the text.

•Derivation of equation (26):

Integrating (25) with respect to the firms involved in production, we have:

$$\begin{aligned} & [-u_h + \lambda_3 u_{ch}(\widetilde{r} - \delta) - \lambda_4 u_{hh} + \lambda_4 u_{ch}\widetilde{w}]\,(1 - G(A^*)) + \lambda_1 \int_{A^*}^{A_u} A_i f_l(k_i, l_i) dG(A_i) \\ + & \lambda_2 \left[\tau^u (1-\theta) \int_{A^*}^{A_u} A_i f_l(k_i, l_i) dG(A_i) + \theta \int_{A^*}^{A_u} A_i f_l(k_i, l_i) dG(A_i) - \widetilde{w}(1 - G(A^*))\right] \\ + & \lambda_5 (1-\tau^u)(1-\theta) A^* f_l(k^*, l^*) = 0 \end{aligned} \quad (64)$$

Adding and substracting $\lambda_2 \int_{A_t^*}^{A_u} A_i f_l(k_i, l_i) g(A_i)$ and dividing by $1 - G(A^*)$, we obtain:

$$\begin{aligned} & [-u_h + \lambda_3 u_{ch}(\widetilde{r} - \delta) - \lambda_4 u_{hh} + \lambda_4 u_{ch}\widetilde{w}] \\ + & \lambda_1 \frac{\int_{A^*}^{A_u} A_i f_l(k_i, l_i) g(A_i)}{1 - G(A^*)} + \lambda_2 \left[(\tau^u - 1)(1-\theta) \frac{\int_{A^*}^{A_u} A_i f_l(k_i, l_i) dG(A_i)}{1 - G(A^*)}\right] + \lambda_2 \left[\frac{\int_{A^*}^{A_u} A_i f_l(k_i, l_i) g(A_i)}{1 - G(A^*)} - \widetilde{w}\right] \\ + & \frac{\lambda_5}{1 - G(A^*)} (1-\tau^u)(1-\theta) A^* f_l(k^*, l^*) = 0 \end{aligned} \quad (65)$$

Using (6):

$$\begin{aligned} & [u_h - \lambda_3 u_{ch}(\widetilde{r} - \delta) + \lambda_4 u_{hh} - \lambda_4 u_{ch}\widetilde{w}] \\ = & \lambda_1 w + \lambda_2 [(\tau^u - 1)(1-\theta) w] + \lambda_2 w \tau^l + \frac{\lambda_5}{1 - G(A^*)} (1-\tau^u)(1-\theta) w \end{aligned} \quad (66)$$

It follows that:

$$\tau^l = \frac{[u_h - \lambda_3 u_{ch}(\widetilde{r}-\delta) + \lambda_4 u_{hh} - \lambda_4 u_{ch}\widetilde{w}]}{\lambda_2 w} - \frac{\lambda_1}{\lambda_2} + \left[1 - \frac{\lambda_5}{\lambda_2(1-G(A^*))}\right](1-\tau^u)(1-\theta) \quad (67)$$

Using (23) in (67):

$$\tau^l = \frac{[u_h - \lambda_3 u_{ch}(\widetilde{r}-\delta) + \lambda_4 u_{hh} - \lambda_4 u_{ch}\widetilde{w}]}{\lambda_2 w} - \frac{\lambda_1}{\lambda_2} + \left[\frac{S_Y - 1}{S_Y}\right](1-\tau^u)(1-\theta) \quad (68)$$

Note that using (55) to (58):

$$\begin{aligned}
\frac{[u_h - \lambda_3 u_{ch}(\widetilde{r}-\delta) + \lambda_4 u_{hh} - \lambda_4 u_{ch}\widetilde{w}]}{\lambda_2 w} - \frac{\lambda_1}{\lambda_2} &= \frac{u_h}{\lambda_2 w} + \frac{u_{ch}}{u_c w}\left(\int_{A^*}^{A_u} k_i dG(A_i) + \frac{b}{1+\widetilde{r}-\delta}\right)(\widetilde{r}-\delta) \\
&- \frac{u_{hh}\int_{A^*}^{A_u} k_i dG(A_i)}{u_c w} + \frac{u_{ch}\int_{A^*}^{A_u} k_i dG(A_i)\widetilde{w}}{u_c w} \\
&- \frac{u_c}{\lambda_2} - \frac{u_{cc}}{u_c}\left(\int_{A^*}^{A_u} k_i dG(A_i) + \frac{b}{1+\widetilde{r}-\delta}\right)(\widetilde{r}-\delta) \\
&- \frac{u_{cc}}{u_c}\left(\int_{A^*}^{A_u} l_i dG(A_i)\right)\widetilde{w} \qquad (69)
\end{aligned}$$

Since $u_c\widetilde{w} = u_h$, we have:

$$\begin{aligned}
\frac{[u_h - \lambda_3 u_{ch}(\widetilde{r}-\delta) + \lambda_4 u_{hh} - \lambda_4 u_{ch}\widetilde{w}]}{\lambda_2 w} - \frac{\lambda_1}{\lambda_2} &= -\tau^l \frac{u_c}{\lambda_2} \\
-\frac{u_{cc}}{u_c}\left[\left(\int_{A^*}^{A_u} k_i dG(A_i) + \frac{b}{1+\widetilde{r}-\delta}\right)(\widetilde{r}-\delta) + \left(\int_{A^*}^{A_u} l_i dG(A_i)\right)\widetilde{w}\right] & \\
+\frac{u_{ch}}{u_c}\left[\left(\int_{A^*}^{A_u} k_i dG(A_i) + \frac{b}{1+\widetilde{r}-\delta}\right)(\widetilde{r}-\delta) + \left(\int_{A^*}^{A_u} l_i dG(A_i)\right)\widetilde{w}\right](1-\tau^l) & \\
-\frac{u_{hh}\int_{A^*}^{A_u} l_i dG(A_i)}{u_h}(1-\tau^l) &
\end{aligned}$$

Further $\beta(1+\widetilde{r}_t-\delta) = 1$ implies $\frac{b}{1+\widetilde{r}-\delta} = b\beta$. Let $\sigma_{cc} = \frac{-u_{cc}\widetilde{c}}{u_c}$, $\sigma_{ch} = \frac{u_{ch}\widetilde{c}}{u_h}$, $\sigma(h) = \frac{-u_{hh}(1-h)}{u_h}$ where $\widetilde{c} = \left[\widetilde{r}(\int_{A^*}^{A_u} k_i dG(A_i) + b_{ss}\beta) + \widetilde{w}\int_{A^*}^{A_u} l_i dG(A_i)\right]$ is total individual's income -excluding firm's transfers- in steady state. It follows:

$$\frac{[u_h - \lambda_3 u_{ch}(\widetilde{r} - \delta) + \lambda_4 u_{hh} - \lambda_4 u_{ch}\widetilde{w}]}{\lambda_2 w} - \frac{\lambda_1}{\lambda_2} = -\tau^l \frac{u_c}{\lambda_2} + \sigma_{cc} + \sigma_{ch}(1-\tau^l) + \sigma_{hh}(1-\tau^l) \quad (70)$$

Replacing (70) in (68):

$$\begin{aligned} \tau^l &= -\tau^l \frac{u_c}{\lambda_2} + \sigma_{cc} + \sigma_{ch}(1-\tau^l) - \sigma_{hh}(1-\tau^l) + \left[\frac{S_Y - 1}{S_Y}\right](1-\tau^u)(1-\theta) \\ \Rightarrow \tau^l \left(1 + \frac{u_c}{\lambda_2} + \sigma_{ch} + \sigma_{hh}\right) &= (\sigma_{cc} + \sigma_{ch} + \sigma_{hh}) + \left[\frac{S_Y - 1}{S_Y}\right](1-\tau^u)(1-\theta) \quad (71) \end{aligned}$$

This is equation (26) in the text.

•Derivation of equation (27):

Replacing (66) in (58):

$$\begin{aligned} &\lambda_1 w l^* g(A^*) + \lambda_2 \left[(\tau^u - 1)(1-\theta) w l^* g(A^*)\right] + \lambda_2 w \tau^l l^* g(A^*) + \frac{\lambda_5}{1 - G(A^*)}(1-\tau^u)(1-\theta) w l^* g(A^*) \\ + &\lambda_1 \left[-1 + (1-\tau^u)(1-\theta)\right] A^* f(k^*, l^*) g(A^*) + \lambda_1 \widetilde{r} k^* g(A^*) \\ + &\lambda_2 \left[-\tau^u(1-\theta) - \theta\right] A^* f(k^*, l^*) g(A^*) + \lambda_2 \left[\widetilde{r} k^* + \widetilde{w} l^*\right] g(A^*) + \lambda_5 (1-\tau^u)(1-\theta) f(k^*, l^*) = 0 \quad (72) \end{aligned}$$

Using $\widetilde{r} = r(1-\tau^k)$ and additional steps of algebra:

$$\begin{aligned} &(\lambda_1 + \lambda_2)\left[w l^* + r k^*\right] g(A^*) + \lambda_2 \left[(\tau^u - 1)(1-\theta) w l^* g(A^*)\right] \\ + &(\lambda_1 + \lambda_2)\left[-1 + (1-\tau^u)(1-\theta)\right] A^* f(k^*, l^*) g(A^*) - (\lambda_1 + \lambda_2) r \tau^k k^* g(A^*) \\ + &\lambda_5 (1-\tau^u)(1-\theta) f(k^*, l^*) + \frac{\lambda_5}{1 - G(A^*)}(1-\tau^u)(1-\theta) w l^* g(A^*) = 0 \quad (73) \end{aligned}$$

It follows:

$$
\begin{aligned}
& (\lambda_1+\lambda_2)\theta A^* f(k^*,l^*)g(A^*) + \left[-\lambda_2 + \frac{\lambda_5}{1-G(A^*)}\right]\left[(1-\tau^u)(1-\theta)wl^* g(A^*)\right] \\
+ \quad & (\lambda_1+\lambda_2)\left[-1+(1-\tau^u)(1-\theta)\right]A^* f(k^*,l^*)g(A^*) - \frac{\lambda_5}{1-G(A^*)}(1-\tau^u)(1-\theta)(S_Y-1)rk^* g(A^*) \\
+ \quad & \lambda_5(1-\tau^u)(1-\theta)f(k^*,l^*) = 0
\end{aligned}
$$

Since (23) implies $\lambda_2 = \frac{\lambda_5}{1-G(A^*)}S_Y$, we have:

$$
\begin{aligned}
& -(\lambda_1+\lambda_2)\tau^u(1-\theta)A^* f(k^*,l^*)g(A^*) + \frac{\lambda_5}{1-G(A^*)}(1-\tau^u)(1-\theta)\left[1-S_Y\right]\theta A^* f(k^*,l^*)g(A^*) \\
+ \quad & \frac{\lambda_5}{A^* g(A^*)}(1-\tau^u)(1-\theta)A^* f(k^*,l^*) = 0
\end{aligned}
$$

Dividing by $(\lambda_1+\lambda_2)(1-\tau^u)(1-\theta)A^* f(k^*,l^*)$, we finally obtain:

$$
\frac{\tau^u}{1-\tau^u} = \frac{\lambda_5}{\lambda_1+\lambda_2}\left[\frac{\theta(1-S_Y)}{1-G(A^*)} + \frac{1}{A^* g(A^*)}\right] \tag{74}
$$

which is (27) in the text.

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Table 1: Optimal policies in steady state

ϕ	θ	α	Φ_2	Φ_3	g	$1-G(A^*)$	τ^k	τ^l	τ^u
1	0.7	0.36	0.75	1	500	0.66	-0.0358	0.0021	0.0166
1	0.75	0.36	0.75	1	500	0.60	-0.233	-0.1563	0.183
1	0.8	0.36	0.75	1	500	0.54	-0.3294	-0.0655	0.1561
1	0.85	0.36	0.75	1	500	0.44	-0.3935	-0.2635	0.2812
1	0.9	0.36	0.75	1	500	0.30	-0.3816	-1.0553	0.5319
1.5	0.7	0.36	0.75	1	500	0.60	-0.0731	-0.0041	0.0492
1.5	0.8	0.36	0.75	1	500	0.48	-0.2945	-0.0616	0.1484
1.5	0.85	0.36	0.75	1	500	0.40	-0.399	-0.0855	0.2109
1.5	0.9	0.36	0.75	1	500	0.30	-0.4242	-0.3199	0.2822
2	0.7	0.36	0.75	1	500	0.54	-0.0878	0.0188	0.0679
2	0.85	0.36	0.75	1	500	0.36	-0.2483	-0.5221	0.2147
2	0.9	0.36	0.75	1	500	0.28	-0.3197	-0.0866	0.1728

Table 2: Optimal policies in steady state, sensibility analysis

ϕ	θ	α	Φ_2	Φ_3	g	$1-G(A^*)$	τ^k	τ^l	τ^u
1	0.85	0.36	0.75	1	500	0.44	-0.3935	-0.2635	0.2812
1	0.9	0.36	0.75	1	500	0.30	-0.3816	-1.0553	0.5319
1	0.85	0.36	0.75	1	600	0.46	-0.0109	-0.0514	0.0219
1	0.9	0.36	0.75	1	600	0.30	-0.2141	-1.1232	0.565
1	0.85	0.4	0.75	1	500	0.48	-0.4368	-0.1017	0.2317
1	0.9	0.4	0.75	1	500	0.36	-0.3923	-0.2923	0.3191
1	0.85	0.36	0.85	1	500	0.46	-0.2905	-0.0546	0.1434
1	0.9	0.36	0.85	1	500	0.34	-0.3349	-0.2621	0.2708
1	0.85	0.36	0.75	0.75	500	0.40	-0.1416	-2.5552	0.4611
1	0.9	0.36	0.75	0.75	500	0.34	-0.4456	-0.2417	0.2637

Figure 1: Indicator function, $\mathbb{1}_i(operation) = 1 - \frac{(\arctan(\frac{-(1-\tau^u)A_i(k_i^{\alpha}h_i^{1-\alpha})^{\theta}+\phi}{\epsilon})+\pi/2)}{\pi}$

$$k_i = 1.5, h_i = 0.7, \phi = 1, \tau^u = 0.1, \alpha = 0.36, \theta = 0.4$$

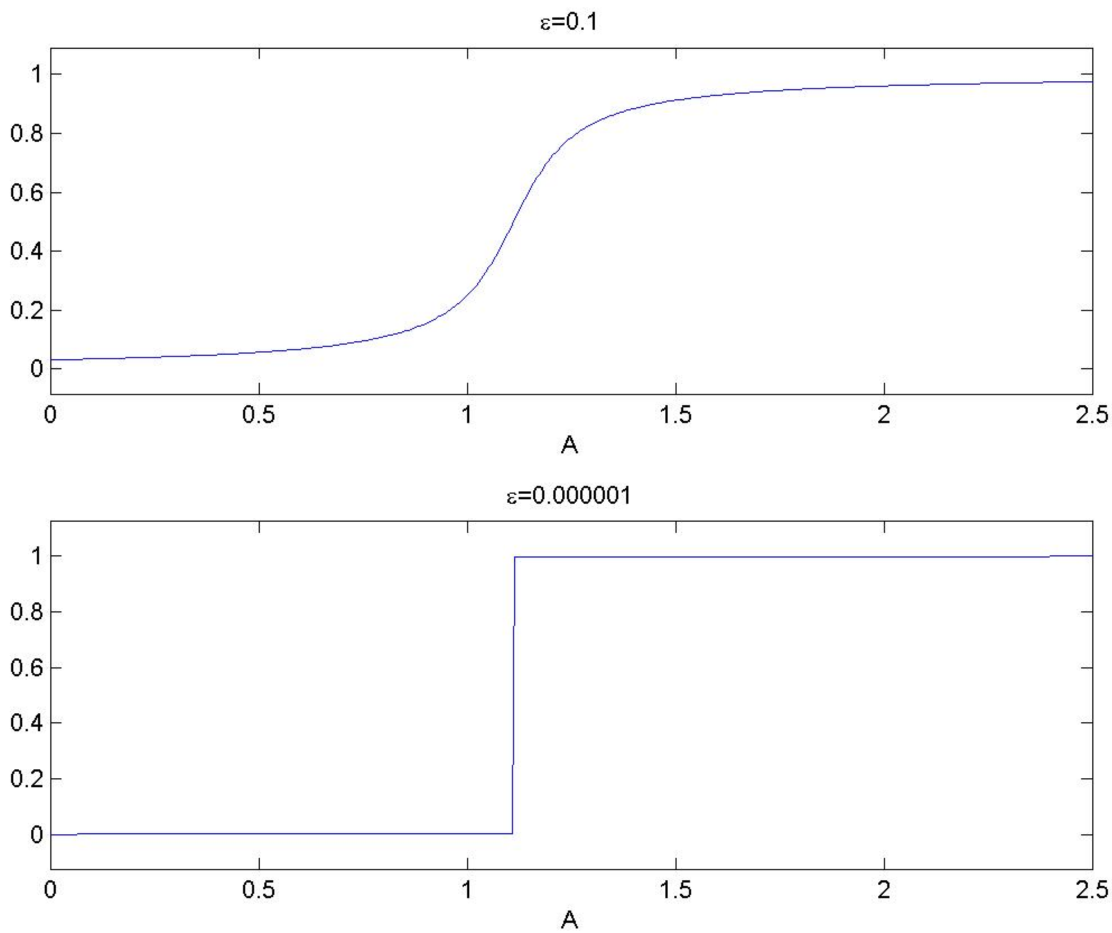


Figure 2: Steady state after-tax profits per firm

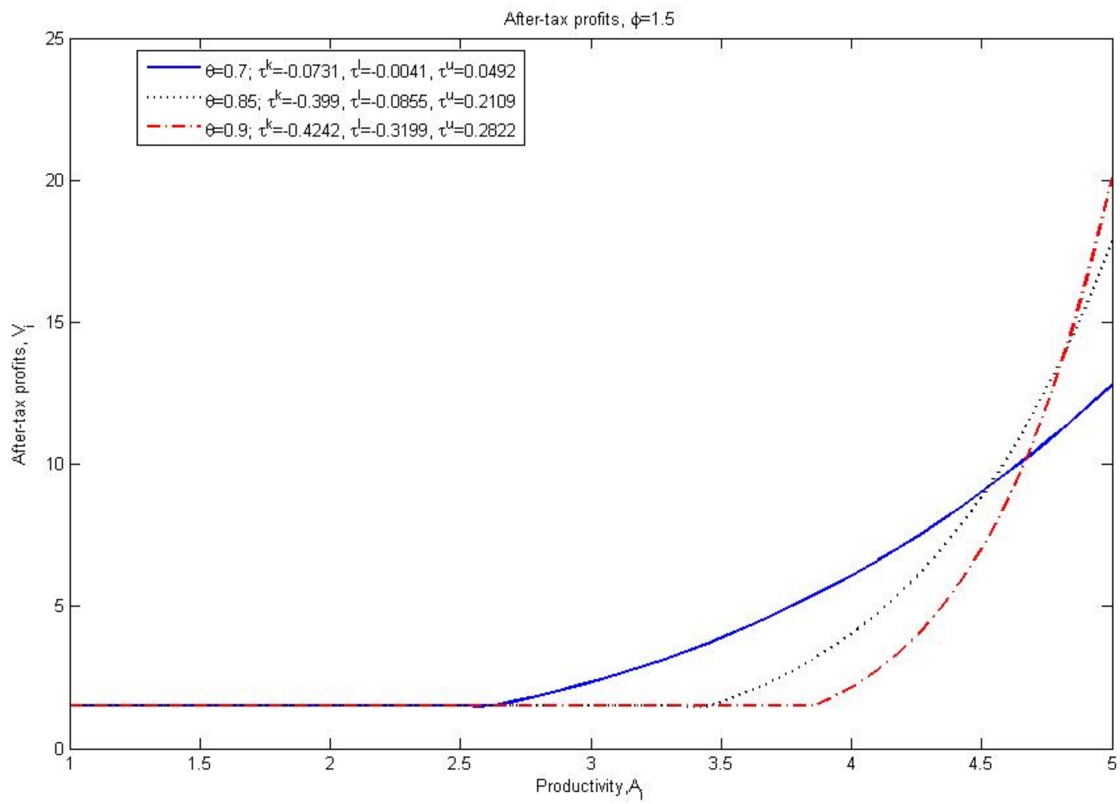


Figure 3: Steady state labor demand per firm

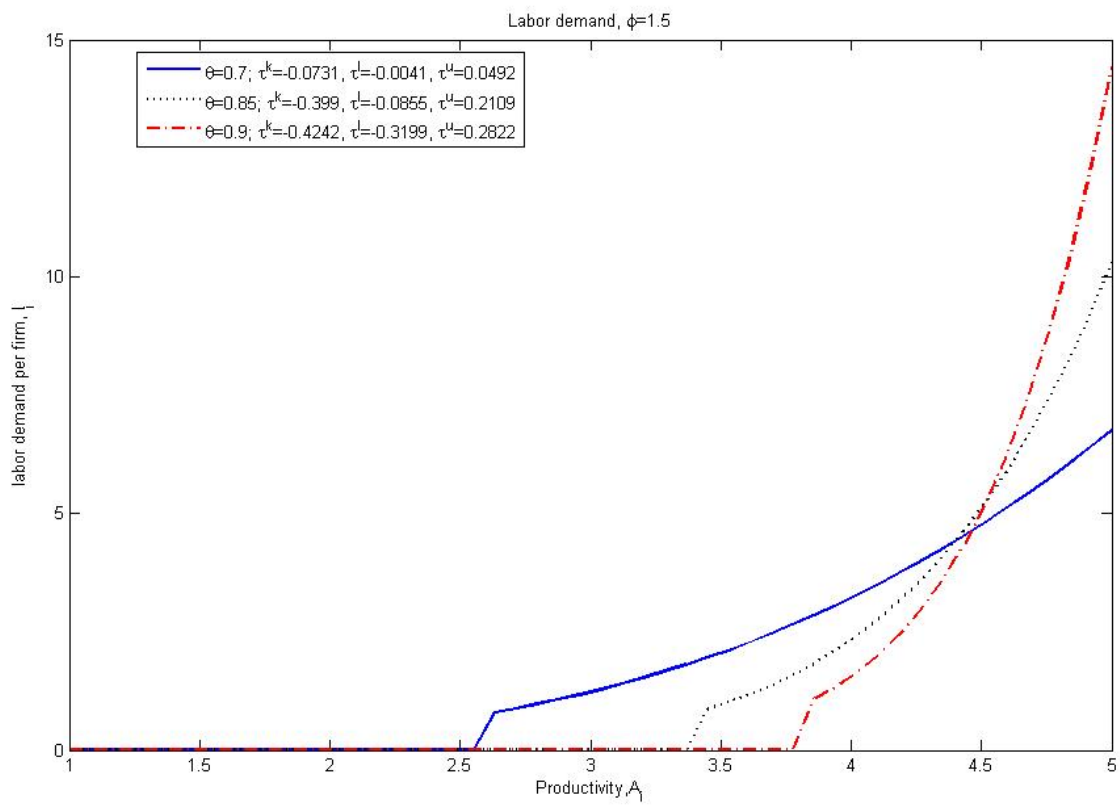
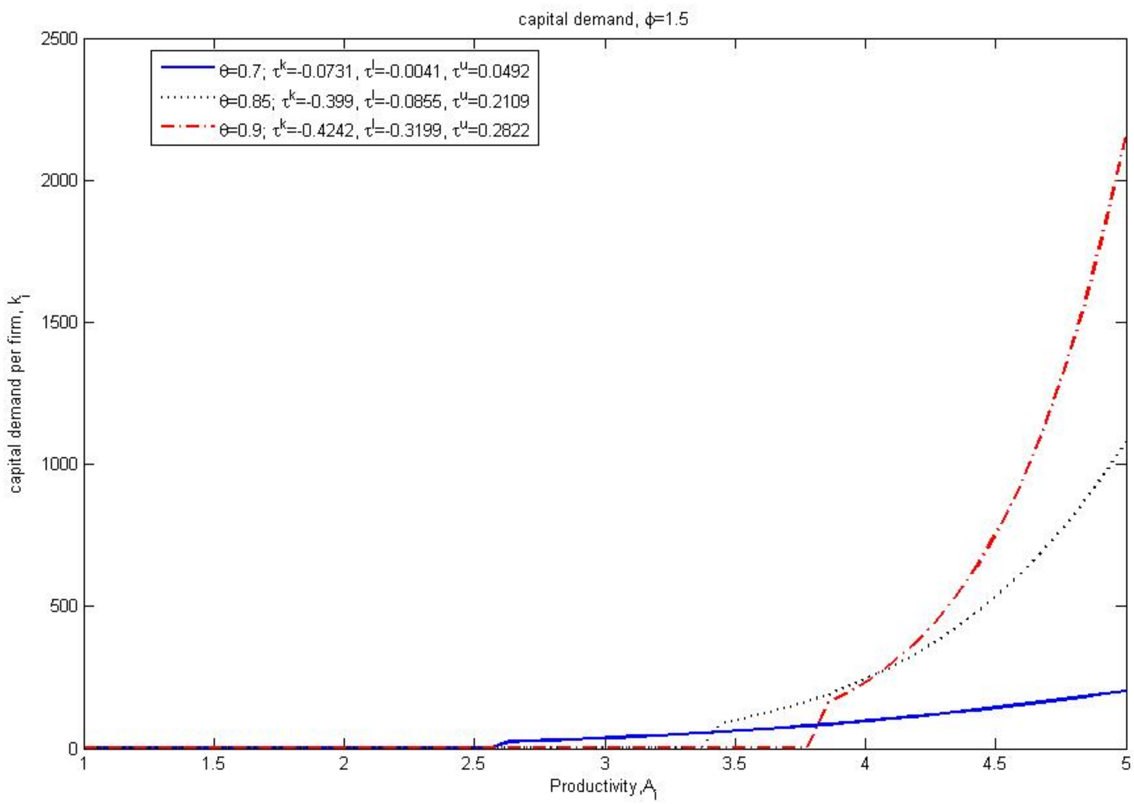


Figure 4: Steady state capital demand per firm



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