

Banco Central de Chile
Documentos de Trabajo

Central Bank of Chile
Working Papers

N° 512

Diciembre 2008

**A SYSTEMIC APPROACH TO MONEY
DEMAND MODELING**

Mauricio Calani

J. Rodrigo Fuentes

Klaus Schmidt-Hebbel

La serie de Documentos de Trabajo en versión PDF puede obtenerse gratis en la dirección electrónica: <http://www.bcentral.cl/esp/estpub/estudios/dtbc>. Existe la posibilidad de solicitar una copia impresa con un costo de \$500 si es dentro de Chile y US\$12 si es para fuera de Chile. Las solicitudes se pueden hacer por fax: (56-2) 6702231 o a través de correo electrónico: bcch@bcentral.cl.

Working Papers in PDF format can be downloaded free of charge from: <http://www.bcentral.cl/eng/stdpub/studies/workingpaper>. Printed versions can be ordered individually for US\$12 per copy (for orders inside Chile the charge is Ch\$500.) Orders can be placed by fax: (56-2) 6702231 or e-mail: bcch@bcentral.cl.



BANCO CENTRAL DE CHILE

CENTRAL BANK OF CHILE

La serie Documentos de Trabajo es una publicación del Banco Central de Chile que divulga los trabajos de investigación económica realizados por profesionales de esta institución o encargados por ella a terceros. El objetivo de la serie es aportar al debate temas relevantes y presentar nuevos enfoques en el análisis de los mismos. La difusión de los Documentos de Trabajo sólo intenta facilitar el intercambio de ideas y dar a conocer investigaciones, con carácter preliminar, para su discusión y comentarios.

La publicación de los Documentos de Trabajo no está sujeta a la aprobación previa de los miembros del Consejo del Banco Central de Chile. Tanto el contenido de los Documentos de Trabajo como también los análisis y conclusiones que de ellos se deriven, son de exclusiva responsabilidad de su o sus autores y no reflejan necesariamente la opinión del Banco Central de Chile o de sus Consejeros.

The Working Papers series of the Central Bank of Chile disseminates economic research conducted by Central Bank staff or third parties under the sponsorship of the Bank. The purpose of the series is to contribute to the discussion of relevant issues and develop new analytical or empirical approaches in their analyses. The only aim of the Working Papers is to disseminate preliminary research for its discussion and comments.

Publication of Working Papers is not subject to previous approval by the members of the Board of the Central Bank. The views and conclusions presented in the papers are exclusively those of the author(s) and do not necessarily reflect the position of the Central Bank of Chile or of the Board members.

Documentos de Trabajo del Banco Central de Chile
Working Papers of the Central Bank of Chile
Agustinas 1180
Teléfono: (56-2) 6702475; Fax: (56-2) 6702231

**A SYSTEMIC APPROACH TO MONEY
DEMAND MODELING**

Mauricio Calani
Gerencia de Investigación Económica
Banco Central de Chile

J. Rodrigo Fuentes
Pontificia Universidad Católica
Instituto de Economía

Klaus Schmidt-Hebbel
Organisation for Economic Co-operation
and Development

Resumen

Este artículo utiliza un enfoque sistémico, basado en la teoría del consumidor, para modelar la demanda por activos líquidos (dinero). Usamos las sugerencias e implicancias de la teoría de agregación para estimar este sistema de demandas. Las estimaciones se hacen para especificaciones estáticas, dinámicas y con parámetros variantes. Nuestros resultados son robustos y coherentes teóricamente con las restricciones que impone la teoría del consumidor. El sistema se comporta como proveniente de un proceso de maximización de una función de utilidad bien comportada, heredando propiedades deseables en cualquier sistema de demandas. En nuestros resultados encontramos estabilidad en las elasticidades estimadas de tasas de interés y gasto total. También documentamos que tasas de mayor (menor) plazo están asociadas a activos menos (más) líquidos. Además documentamos que el vigoroso crecimiento del dinero M1 de los últimos años de la muestra no puede explicarse solamente por las bajas tasas de interés. Las implicancias de política son directas; hay una relación estable entre tasas de interés y dinero, pero este último no responde exclusivamente a las primeras.

Abstract

This paper uses a consumer theory-based systemic approach to model the demand for monetary liquid asset holdings. We implement the suggestions and caveats of aggregation theory for the estimation of a demand system for liquid assets (monies) in static, dynamic and time-varying parameters setups. Our results are robust and theoretically consistent with consumer theory restrictions, as system derived from a utility maximizing framework and a well-behaved utility function. In our estimations we find stability of interest-rate and total-expenditure elasticities, in contrast to previous literature. We also document evidence that long (short) maturity rates are associated to less (more) liquid assets and that the vigorous growth of M1 during the last five of years is not accounted for by low interest rates alone. Policy implications are straightforward; there is stable relationship between monies and interest rates, but the former do not respond exclusively to the latter.

The authors wish thank Álvaro García, Mariana García, Juan Díaz, Luis Salomó, and participants of the 2008 Meeting of the Chilean Economic Association and the 2008 Meeting of Latin American Economic Association for their useful comments. E-mail: mcalani@bcentral.cl; rfuentes@facepuc.cl; Klaus.SCHMIDT-HEBBEL@oecd.org.

1 Introduction

Money should be a kind of “natural” anchor for the longer term orientation of monetary policy. Assessment of the stability of the demand function for it is important, in theory and practice, not only for monetary policy implementation but also as a pre-condition for it to exert a *predictable* influence on other macroeconomic variables. Even more, money demand stability allows estimations to be valuable instruments for diagnosing possible financial anomalies, such as financial stress, bank run symptoms or unusual lack of trust in the financial system. Also, from a pure monetary policy perspective, these estimations should provide a quantitative benchmark for identifying longer-term risks to price stability, in contrast to short/medium-term risks identified with parallel projection models broadly used in central banking. After all, at least in the long run, inflation is simply a monetary phenomenon. (Issing, 2006).

Extensive work has been done in order to capture, stable and theoretically consistent specifications. Judd and Scading (1982) and Mies and Soto (2000) provide extensive surveys on such quest, for the U.S. and Chile respectively. Results, however, seem to favor instability for most specifications.

This article’s objectives are twofold. First, we present a fresh and more comprehensive perspective to modeling monetary assets holdings in the Chilean economy. We use the new monetary aggregates adopted in Chile as of 2006 (comparable with those of several other countries reported in Arraño, 2006). This approach is novel for four reasons; (i) We focus our attention on the rigorous micro-founded decisions of a representative optimizing agent that chooses a portfolio composed of several liquid assets, instead of an ad-hoc econometric specification, (ii) we use the results and caveats of aggregation theory and their direct link to consumer theory to aggregate different liquid assets into baskets that can be treated by the representative consumer as single meaningful goods, (iii) we examine the consumer-theory-based implications of such specifications through all estimations (add-up restrictions, homogeneity of degree zero in prices and symmetry), and (iv) we analyze this system of related demand equations in their static, dynamic (error correction) and time-varying-parameters estimation frameworks. The second objective of this article is to provide new elements to the discussion of the correct specification for money demand equations for Chile. De Gregorio (2003), Vergara (2003) and Restrepo (2003) discuss, on empirical grounds, if the correct specification for money demand equations should use interest-rate elasticities or semi-elasticities. Their main conclusion is that even though elasticities seem to work better for the period before 2002, both alternatives perform poorly for 2002 and 2003 (last years of their samples). We choose our specification, not from goodness of fit but from theory. We show, through time varying parameter estimation, that it is not interest rates elasticities instability or that explain recent unexpected growth of monetary assets. Instead, we offer evidence of interest-rate and real expenditure stable elasticities. Also, our system of related demand equations fulfills the requirements of any system that comes from a quasi-concave utility function with monetary services among its arguments. We also find

support to the observation made in the previous literature (De Gregorio, 2003; Adams, 2003) in which it is suggested that around 2001 there is a large recomposition of monetary assets from M2 to M1, however we argue against this being led exclusively by an era of low interest rates.

Following this introduction and motivation, section 2 presents our theoretical framework, section 3 presents some stylized facts and the data we use, the econometric approach is addressed in section 4, along with the results. Finally, section 5 presents conclusions and policy implications.

2 Demand for Money Theory

2.1 On the Precise Definition of Money

Demand for monetary asset holdings has an extensive background in the related literature. From the initial considerations of Goldfeld et al. (1976), money demand equations' stability and forecasting performance have been extensively studied. In the empirical literature, two groups of different approaches can be identified. The first one is based on the different strategies that aim to modeling uni-equationally stable demand functions for M1¹, assuming this monetary aggregate is best suited to fulfill the characteristics of the theoretical definition of money². A complete survey of this literature is beyond the scope of this paper (the interested reader is referred to see Judd and Scading (1982) and Mies and Soto (2000)). The second strand in empirical literature is less abundant. It assumes there is no single/narrow definition of money, but that different liquid assets fulfill imperfectly the requirements to be considered as "monies". Some work in this line can be found in Collins and Anderson (1998) and Ewis and Fisher (1984). This approach is more prone to favoring Friedman's affirmation that money has many, but imperfect, substitutes. Consequently the right definition of opportunity cost for each liquid asset is somewhat a matter of practice and good judgement. In this paper we favor the latter approach, modeling Chilean monetary asset holding in a system of interrelated demand equations for different liquid assets.

A caveat is worth mentioning at this point. Theoretical and empirical work have run, most of the time, in two distinct directions. All theoretical models use one (or several) of the *motives* for holding money to rationalize reasons that lead people to hold a "good" that cannot be consumed and has zero or negative profitability if seen as an investment asset. Empirical literature, however, has concentrated on improving statistical tools and specifications of money-demand estimation using national-account simple-sum aggregate measures of "money" as the right variable yielding monetary services with no care about whether such definition is consistent with the theory at hand. Exceptions being Serletis (1991),

¹M1A for Chile before 2006. See Arraño (2006) and Mies and Soto (2000)

²In the Keynesian tradition, three reasons for holding money are stated: Transactions Motive, Precautionary Motive and Speculative Motive.

Barnett (1980) and Serletis (2007)³.

As pointed out by Barnett (1980) it is unlikely that simple-sum monetary aggregates fulfill the properties we would ask for money demand specifications based on a utility maximizing framework (more on this next). Instead, he argues in favor of an economic-theory based monetary aggregate that is consistent with a framework in which individuals demand *flows of monetary services* from *monetary asset stocks*. We can understand these portfolios of stocks of liquid assets as durable goods, which are held not for themselves, but for the flow of services they provide. It is these services which enter the utility function, and what consumers seek by holding otherwise-useless stocks of low-profitable investment. See Donovan (1978); Serletis (1991); Barnett (1980).

In this paper we will be interested in demand for three baskets of liquid assets which we call monies, M_i ; where $i = 0, 1, 2$. Economic agents must then be able to treat M_i as a quantity of a meaningful single good in their decisions⁴ so that demand for stocks of these aggregates should be decided independently of their internal composition (see Barnett, 1980). To make this point more explicit, suppose we concentrate on the demand for M_j , which is composed by a bundle of k liquid assets $\{M_{j,1}, M_{j,2}, \dots, M_{j,k}\}$. We could find H different vectors of combinations of its components, that aggregated in some way, yield the same flow of monetary services. Individuals should be indifferent from choosing between any of these H combinations, since they all yield the same service, and hence, the same utility. A straightforward implication is that arbitrary changes in composition in monetary aggregates do not necessarily mean changes in the demand for monetary services they provide. Clearly, this is not true with simple-sum monetary aggregates (national account data) which add currency, demand deposits, long-term bonds among other assets with equal weight. Nobody would seriously consider that currency and bonds provide the same flow of monetary services, and yet using simple-sum aggregates assumes this is indeed the case by treating all components in each monetary aggregate as perfect substitutes.

What is the alternative to using national account data?. We need to consider all liquid assets and aggregate them conveniently in new monetary aggregates (Economic Quantity Indexes), so that we assign the same value to combinations of assets yielding the same flow of monetary services. Thus, it would seem that unless we know the representative consumer's utility function we cannot possibly construct such Index. We follow Barnett (1980) and construct economic quantity indexes named Divisia Monetary Aggregates. These aggregates are numerically not too different to national-account monetary aggregates but they fulfill the economic-theory-based properties of consumer theory that can be found in Appendix B. In a nutshell, the theoretically consistent aggregator of monetary assets is the very utility function of monetary services $f(x)$, where x is a vector of its components. If we exploit the fact that this function can be assessed with consumer theory, we can achieve a non-parametric discrete

³In Chile there has been no exception. All previous work has more or less concentrated in M1 or M1A money demand (See Mies and Soto, 2003) and Soto and Tapia (2000)

⁴Our empirical analysis will focus on the demand for M_0 , $M_1 - M_0$ and $M_2 - M_1$ as the main monetary asset holdings.

approximation that depends only of prices and quantities.

$$d \log f(x) = \sum_{i=1}^k w_i d \log x_i \quad (1)$$

Appendix B derives this expression. For our analysis in section (4) we will also use Donovan’s (1978) (jorgensonian) user cost of money. This user cost is the appropriate measure of opportunity cost of a durable good; this is shown in Barnett (1980) and summarized in Appendix A.

2.2 Demand System for Monetary Assets

Once we have properly defined quantities and prices for different “monies” in the context of utility-maximizing behavior of a representative consumer, we are able of formulating a demand system that can provide quantitative estimation of price and income elasticities. In this section we formulate the specification we will estimate in section (4)

In the literature on systems of demand equations, one of the fundamental objectives has been to propose general enough specifications to be approximations to demand functions derived from any utility function. The most popular ones have been the Translog and the Rotterdam models (See Deaton and Muellbauer, 1980 and Serletis, 2007). In this paper we follow closely the model developed by Deaton and Muellbauer (1980), the Almost Ideal Demand System. The AIDS is based on a a cost function of the form

$$\log c(u, p) = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj}^* \log p_k \log p_j + u \beta_o \prod_k p_k^{\beta_k} \quad (2)$$

where α_i , β_i and γ_{ij}^* are parameters, $c(u, p)$ is total expenditure and p_j is the price of purchased item j . It can be shown that we can obtain the following demand system (using Shepard’s Lemma) in budget share form:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log \left\{ \frac{x}{P} \right\} \quad (3)$$

where x is total expenditure (duality implies it is equal to $c(u, p)$ of equation (2)); w_i is the share in total expenditure of item i ; $\gamma_{ij} = \frac{1}{2}(\gamma_{ij}^* + \gamma_{ji}^*)$ and $\log P$ is the resulting overall price index which is equal to $\log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_j \sum_k \gamma_{kj} \log p_k \log p_j$

From this setup we can see that we have defined a set of demand equations for shares of total budget expenditure on liquid assets that is a parametric nonlinear function on prices and real expenditure only. Demand functions in Equation (3) are first order approximations to any set of demand functions derived from utility-maximizing behavior.

A useful feature of such demand systems is that we can impose some constraints across equations; i.e. adding-up constraints: $\sum_{i=1}^k \alpha_i = 1$; $\sum_{i=1}^n \gamma_{ij} = 0$ and $\sum_{i=1}^n \beta_i = 0$. These conditions must be met at all times if $\sum w_i = 1$, which is true by construction. Consequently,

we have a singular demand system; a problem we deal with in section (4). Even more, if we wanted to impose more structure in equation (3) we would like to test and impose homogeneity of degree zero in nominal prices: $\sum_j \gamma_{ij} = 0$ ⁵.

Given such conditions, our system of demand equations is simply interpreted; in absence of changes in relative prices and expenditure, budget shares are constant. Recomposition of monetary aggregates holdings will take place if relative prices change, or “real” expenditure on such assets changes.

3 Data and Stylized Facts

This sections briefly characterizes two features of data. First document some stylized facts of monetary aggregates and make a comparison between simple-sum monetary aggregates and Divisia monetary aggregates. We give some details about the necessary steps to construct Divisia money, however, the interested reader is referred to [Anderson et al. \(1997\)](#). Second, it briefly discusses the data used in section (4).

3.1 Divisia Monetary Aggregates compared to Simple-Sum Monetary Aggregates

In this subsection we present some differences between simple sum monetary aggregates and Divisia monetary aggregates using Chilean money data. Similar comparisons for the U.S. can be found in [Anderson et al. \(1997a, 1997b, 1997c\)](#) and [Serletis \(2007\)](#).

The Central Bank of Chile publishes on a regular basis monetary aggregates that are constructed according to the procedure described in [Arraño \(2006\)](#). Such methodology has the advantage of being comparable across various countries. In fact, such comparability is the main motivation for the new definitions of money used in Chile from 2006. We will call such quantities, simple-sum monetary aggregates. In contrast, in this paper we consider Divisia monetary aggregates because of their strong microeconomic foundations. Figure (1) presents time series for both definitions for M1 and M2 monetary aggregates. The pattern of differences between the two definitions of money are very similar to those reported by [Serletis \(2007\)](#) and [Anderson et al. \(1997\)](#) for the U.S.

We also examine the implication of using new monetary aggregates on the velocity of circulation. We plot simple-sum and Divisia M1 and M2 velocities of circulation in Figure 2. It is evident that Divisia velocities rise by far less than simple-sum velocities in the whole sample period. Figure (2) also shows the Hodrick-Prescott filtered series of monthly

⁵Price index P , is also a function of prices as is total budget expenditure, both nominal variables. However their ratio is not. Common practice in micro-econometrics has been to approximate P through Stone’s price index (defined on section 4)

velocities⁶. Again, these stylized facts are similar to those found for the U.S. in previous studies.

In order to construct our series of monetary aggregates we use the real Törnqvist-Theil monetary services index (a chained quantity index formula) from aggregation theory which is a discrete approximation to the actual continuous Divisia formula (see [Appendix B](#)).

$$M_t = M_{t-1} \prod_{i=1}^n \left(\frac{m_{i,t}}{m_{i,t-1}} \right)^{\frac{1}{2}(w_{i,t} + w_{i,t-1})}$$

where m_i represents holding of the i^{th} monetary asset and w_i stands for the share in total expenditure y on the i^{th} monetary asset.⁷

3.2 Data Sources

Empirical work performed in section (4) uses data from the Central Bank of Chile and the Superintendence of Banks and Financial Institutions for the construction of monetary aggregates. Definitions of monetary aggregates follow [Arraño, 2006](#). Since we have three different liquid assets in equation (3), we need three prices for our specification in (3). We use nominal deposit interest rates, i.e. 30 to 89 days, 90 days to 1 year and 1 to 3 years maturities. Clearly there is no perfect interest rate to compute each price as defined in [Appendix B](#). However, it is a reasonable approximation to use interest rates associated with different maturities to model the demand for assets whose main difference is their liquidity. Even more, if in fact there is no a unique definition of money, but many imperfect substitutes there is no reason to rule out any interest rate in favor of another a-priori.⁸ Previous work has used mostly very short maturity interest rates (30 to 90 days and the policy rate seem to be the most popular. See [De Gregorio, 2003](#); [Adam, 2003](#); [Vergara, 2003](#) and [Mies and Soto, 2000](#).

4 Econometric Approach

In this we section present our approach to model in a systemic way the demand for a portfolio of monetary assets. We do so using the Linear Almost Ideal Demand System (LAIDS), the

⁶Nominal income for these computations are proxied through nominal GDP; built using the method suggested by [Chow and Lin \(1971\)](#). We construct nominal monthly GDP series from the monthly index of economic activity (IMACEC)

⁷For monetary assets m_j that do not exist until some period in the middle of the sample, the Törnqvist-Theil index is not defined ($m_{k,t-1}$ is zero and therefore the T-T index does not exist). Thus, for such periods

we use the real Fisher Ideal user cost index $P_t^F = P_{t-1}^F \sqrt{\frac{\sum_{j=1}^n \pi_{jt}^{real} m_{jt}^{nom}}{\sum_{j=1}^n \pi_{jt}^{real} m_{j,t-1}^{nom}} \frac{\sum_{j=1}^n \pi_{j,t-1}^{real} m_{jt}^{nom}}{\sum_{j=1}^n \pi_{j,t-1}^{real} m_{j,t-1}^{nom}}}$, definitions of

π_{jt}^{real} can be found in [Appendix A](#) which is well defined with new monetary assets.

⁸The benchmark asset (A) interest rate is taken to be the highest paying interest rate available in the economy. See [Anderson et al. \(1997b\)](#)

Error Correction (EC) LAIDS and the Time-Varying Parameter Long-Run LAIDS. Results are summarized at the end of the section.

4.1 System of demand equations and adding up restrictions

4.1.1 Static Linear AIDS

In general, we can proceed to estimate equation (3) substituting the endogenous price index P in it, and obtain

$$w_i = (\alpha_i \beta_i \alpha_0) + \sum_j \gamma_{ij} \log p_j + \beta_i \left\{ \log x - \sum_k \alpha_k \log p_k - \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k \log p_j \right\} \quad (4)$$

which can be estimated using NLLS or Maximum Likelihood.

It has been common practice, however, to estimate a linear approximation to this non-linear function using Stone's (1953) index: $\log P^* = \sum w_k \log p_k$ (Li et al, 2006).

If $P \cong \psi P^*$ (Deaton and Muellbauer, 1980), then (3) can be estimated as

$$w_i = (\alpha_i - \beta_i \log \psi) + \sum_j \gamma_{ij} \log p_j + \beta_i \log \left(\frac{x}{P^*} \right) \quad (5)$$

Let $\alpha_i^* = (\alpha_i - \beta_i \log \psi)$, then it is evident that for $\sum w_i = 1$ it is still required that $\sum \alpha_k^* = 1$ since $\sum \beta_i = 0$ and $\sum_i \gamma_{ij} = 0$. The linear approximation (5), to the non-linear expression in equation (4) has been proved to present very satisfactory performance (Buse, 1994).

We begin estimating each equation individually (5) by OLS and testing for homogeneity of degree zero in prices; $H_0 : \sum_j \gamma_{ij} = 0$. We perform such estimations for two sample periods; 1993 to 2007 and the period after which monetary policy interest rate target was changed to a nominal, rather than indexed, interest rate⁹; 2001 (August) to 2007. Results are shown in tables (1) to (4)

4.1.2 Error Correction Linear AIDS

Static specification of equation (3) assumes that individuals actually hold the amount of assets they desire to hold at every point in time. However adjustment costs or several other reasons could result in individuals adjusting holdings to desired ones with some lag¹⁰. Thus we consider an error correction specification of equation (3).

Thus we assume the error component u_{it} in equation (3) follows an autoregressive process.

$$\Lambda(L)u_t = \varepsilon_t \quad (6)$$

⁹Before August, 2001 the Central Bank of Chile operational interest rate target was a premium over the annual variation of Unidad de Fomento

¹⁰For instance, if agents face quadratic costs of adjustment of their portfolio; lagged dependent variables can be justified (Cuthbertson, 1985)

In practice, as suggested by Ng (1995), there is no need to model imperfect adjustment to equilibrium with extensive lag polinomials. Usually low order polinomials seem to work best ¹¹.

Next, we explain the restrictions we need to impose in order to make the system, theoretically consistent and empirically possible to be estimated. We require a specification that (i) does not explain shares with their complements but with prices and total real expenditure and (ii) fulfills adding up restrictions at all times; i.e., $\sum_i w_{i,t} = 1, \forall t$. Let \mathbf{u}_t be the 3×1 vector containing the system errors $u_{i,t}$ for $i = 1, 2, 3$. Then, $\Lambda(L)\mathbf{u}_t = \epsilon_t$ where $\epsilon_t \sim \mathcal{N}(0, \sigma^2\Omega)$ and $\mathcal{E}(\epsilon_t \epsilon'_{t-j}) = 0$ for $j \neq 0$. If Λ were any different from a diagonal, then multiplying equation (3) by $\Lambda(L)$ to obtain the error correction specification would result in a specification in which shares are explained by their complements. This leads long run vectors different from the original AIDS specification we wish to obtain as long-run relationships. If in fact Λ is a diagonal, adding up restrictions require that $\sum_i \Lambda_{i,j} = 0$, otherwise $\sum_i w_i \neq 1$ ¹². However, given that extra holding of one monetary asset is the missing counterpart of the rest; i.e., $\sum_i u_{i,t} = 0$; if $\Lambda_{i,i} = \Lambda$, adding up results in $\Lambda \sum_i u_{i,t}$ which is equal to zero, fulfilling $\sum_i w_i = 1$. This structure is equivalent to admitting that there is an error correction representation for each equation in system 3, which is independent across equations but that the rate of adjustment is equal across equations. We estimate jointly the three equations and test whether $\Lambda_{i,i} = \Lambda$, a restriction which cannot be rejected and is later imposed ¹³.

Error components in equation (3) are found to follow AR(2) processes. We use standard information criteria to select the optimal lag structure for each $u_{i,t}$. All these criteria concur on the choice of the best autoregressive processes. M_1 money and M_2 money errors follow AR(2) processes and currency error follows an AR(3) process. For parsimony, we choose to estimate system (6) with $p = 2$. Results with $p = 3$ are however practically unchanged.

Thus, if we consider such autocorrelation structure we can use equation (6)

$$\Lambda(L)u_t = \epsilon_t$$

where ϵ_t is white noise and fulfills classic requirements for the error component. Pre-Multiplying Equation (3) with the operator $\Lambda(L) = 1 - \rho_1 L - \rho_2 L^2$ we can obtain the following specification

$$\begin{aligned} \Delta w_{i,t} = & -\rho_2 \Delta w_{i,t-1} + \sum_j [\gamma_j (\Delta \log p_{j,t} + \rho_2 \Delta \log p_{j,t-1})] + \beta \Delta \log \left\{ \frac{x}{P^*} \right\}_t + \beta \rho_2 \Delta \log \left\{ \frac{x}{P^*} \right\}_{t-1} + \\ & (1 - \rho_1 - \rho_2) \left\{ w_{i,t-1} - \alpha_i - \sum_j \gamma_j \log p_{j,t-1} - \beta \log x P^*_{t-1} \right\} + \epsilon_t \end{aligned} \quad (7)$$

¹¹This suggestion turns out to be true, although we do not initially constraint ourselves. We wish to find the AR(p) process which is consistent with ϵ_t being white noise

¹²The intuition is simple; the error in one equation must be found in the error of the rest of the equations of the system

¹³The chi-squared statistic associated with such restriction takes a value of 0.32 and a p-value of 0.98

Again, we need to know that this system of equations is singular for estimation purposes given that $\sum_i \Delta w_i = 0$, that is that a change in the share of asset i in the portfolio must encounter its counterpart in the rest of the $j, j \neq i$ assets. We allow ε_t to be correlated across equations but not across time (a condition we guarantee in our lag selections procedure). Thus $\mathcal{E}(\varepsilon_t \varepsilon_t') = \Omega \sigma^2$. Consequently we estimate equation (7) using Zellner's SURE procedure using Non linear Least Squares instead of OLS. Results are shown in Table (8) for the period ranging from 1993 to 2007. We check that the error component of such estimation is indeed white noise; as can be seen from figures 7) to (9).

At this point we check whether we accomplish homogeneity of degree zero in prices in the long-run vector of parameters; such tests are shown in Table (8). We can not reject homogeneity of degree zero for M_1 -share, we reject it marginally for M_2 -share and reject it for the currency share. This result is similar to our Static LAIDS specification from tables (5) and (6). Further, we check whether our system of demand equations comes from a quasi-concave utility function by checking the eigenvalues of the Slutsky Matrix. In practice it is easier to check the signs of $k_{ij} = p_i p_j s_{ij} / x$, the eigenvalues of which have the same signs as those of s_{ij} (Deaton and Muellbauer, 1980). We can express k_{ij} as $k_{ij} = \gamma_{ij} + \beta_i \beta_j \log \frac{x}{P} - w_i \delta_{ij} + w_i w_j$, being δ_{ij} the Kronecker delta. Eigenvalues calculated for sample averages and parameter estimates imply a semi-definite negative Slutsky Matrix ¹⁴.

4.1.3 Time Varying Parameters Estimation

The estimations of the two previous sub-sections have assumed that coefficients are constant in our sample. Vergara (2003), in a somewhat different specification for the demand for (simple sum) M1 money, finds that interest rate elasticities may be quite different in time. In particular, the author distinguishes three periods, in accordance to his sample; 1992-1998, 1999-2000 and 2001-2003. In each of these periods the interest rate elasticities, and semielasticities, are remarkably higher than those in the previous periods. In this section we examine more rigorously such exercise which is important in its own terms. Time instability would, at least in part, invalidate the results presented so far. We adopt a methodology that enables us to estimate, in a time-varying context, the specification given in equation(3).

There are at least four ways of analyzing elasticities that may vary in time. Each of them, facing a trade-off between simplicity and statistical rigourousness. The first is the approach taken by Vergara (2003) that consists of delimiting time-windows, performing estimations in such windows and comparing parameter estimates. The second way consists in estimating the equation in a recursive manner, adding one observation at the time, and examining the path of parameter estimates. This, however, is equivalent to the first approach with as many windows as the sample size minus the window length -each window nesting the previous one-. The third way to deal with time-varying parameter estimation is through the Kalman Filter. Using the Kalman Filter requires re-writing any set of equations in a state-space

¹⁴These eigenvalues are: -0.082, -0.315 and 0.000 (which is natural given the singularity of the system)

representation. In this case, the signal (or measurement) equation is the equation to be estimated in (3), and the state equations will consist of random-walk representations for the coefficients (elasticities). The Kalman Filter provides a step-by-step updating procedure that incorporates any new information to the estimation so as to minimize the Mean Squared Forecast Error in each step. However, the Kalman Filter, estimated through maximum likelihood, is very prone to exhibit the so called “pile-up” problem discussed in Stock and Watson (1998) when the variance of the state variables is dangerously close to zero. This problem results in singular matrices that cannot be inverted. Stock and Watson (1998) develop the median-unbiased estimators of ratios of state variables variances to signal variable variances, in a two step procedure. In simple terms, this procedure ensures that state-equation variances can approach zero but never reach it, since it is linked to other non-zero-approaching (signal equation) variances. In practice, for identification purposes, it is useful to have as many signal equations as ratios of variances. This procedure, is not feasible in our specification in (3) since we have three signal equations and 15 random walk parameters whose variances are dangerously close to zero; making estimates extremely sensitive to the procedure.

An alternative, fourth way, in which we can assess time-varying coefficients estimation is using semi-nonparametric estimation procedure for a SURE specification, as proposed by Orbe et al. (2003). They develop an algorithm to estimate time-varying coefficients for a system of equations, in which we can impose restrictions across equations. Notice that our sample (consisting of 180 observations) requires estimating $180 \times 5 \times 3$ coefficients (180 periods in time, 5 coefficients in each equation and 3 equations). We present a simple summary of what this method does.

Let us consider the m^{th} equation in a SURE system like (3). Static and Error Correction estimations assume

$$w_{m,t} = \alpha_m + \sum_j \gamma_{m,j} \log p_{j,t} + \beta_m \log \left\{ \frac{x_t}{P_t} \right\} \quad (8)$$

In this context, however, we will assume

$$w_{m,t} = \alpha_{m,t} + \sum_j \gamma_{m,j,t} \log p_{j,t} + \beta_{m,t} \log \left\{ \frac{x_t}{P_t} \right\} \quad (9)$$

and we will make standard assumptions for SURE models for the error component. We allow for heteroscedasticity and for time-varying contemporary correlations structure. We will also assume that any coefficient $\beta_{mit} = f_{mit}(t/T)$ is a smooth function such that $f_{mi}(\cdot) \in \mathcal{C}^2[0, 1]$ for all $i = 1, \dots, 5$ and all $m = 1, 2, 3$.

The estimators for each period in time are obtained by minimizing the sum of squares of smoothed residuals for each equation m .

$$S_{m,r}(\beta_{m,r}) = \sum_{t=1}^T K_{mrt} (w_{mt} - \alpha_{mr} - \sum_j \gamma_{mjt} \log p_{jt} - \beta_{mr} \log \left\{ \frac{x_t}{P_t} \right\})^2 \quad (10)$$

where $K_{mrt} = (Th_m)^{-1}K((r - t)/Th_m)$ is a kernel function. The parameter h_m is the bandwidth that will regulate the smoothness. Note that the kernel function is univariate and thus is not subject to the curse of dimensionality. As proposed by Orbe et al. (2003) we use an Epanechnikov kernel. We further impose homogeneity of degree zero in our estimation. Even though Orbe et al. (2003) present a very useful algorithm to perform this type of estimation, they do not provide asymptotic analysis to derive confidence intervals. It is critical to notice that due to the usage of a univariate kernel in time, sample size is unlikely to tend to infinity, making asymptotic analysis of reduced importance. This is why we use a bootstrap procedure to derive a small-sample first-order approximation of confidence intervals. We use paired bootstrap (making random draws on time rather than on the variables themselves). In order to simulate the fact that it is more likely that observations around time r are more informative about it rather than further ones, we use a rejection method in the kernel estimate probability function (Hall, 1994) to assign higher probability of being drawn in each iteration to observations that are closer to r . Confidence intervals are those proposed by Hall (1994).¹⁵ Next, we present our main results, a discussion on our findings and proceed to conclude in the next section.

4.2 Results

We perform several estimation procedures to assess the different values price elasticities, which are our greatest concern, could take. First we present static estimations and then we compare such results with dynamic error-correction estimations. Finally we compare those results with the insights of time-varying parameter estimation.

Tables (1) and (2) present simple OLS estimates of demand system (3) without any constraint or systemic estimation procedures - each equation is estimated individually with and without a trend component. Columns 1, 3 and 5 show the trend-augmented equation and columns 2, 4 and 6 show the trend-less equation estimation. It can be seen that including a trend component, although always small and significant, makes no difference for price elasticities; being the largest change in estimation the value of $\beta_{3,2}$; from 0.2190 to 0.1993¹⁶. For completeness we present trend and without-trend estimations for Tables (1) to (6). We will focus our attention to trend-less results though. From Linear AIDS estimation we can conclude that own-price elasticities take negative values¹⁷ as we would expect for any own-price elasticity. The coefficient associated to total real expenditure is negative and significant for currency share (w_s) and positive and different from zero for less liquid assets (w_1 and w_2); i.e. currency is an inferior good and less liquid assets are superior goods. At this point we test for price-homogeneity of degree zero. For each equation we present the sum of price elasticities and the associated standard error. Homogeneity of degree zero cannot be rejected

¹⁵MatLab codes for estimation and Bootstrap procedures are available upon request.

¹⁶Intercept estimates, as expected, are changed due to the close link between an intercept and a trend

¹⁷Recall our warning about imputation of prices to money components.

for the demand of non-currency monetary assets in M1 and can be rejected (in some cases only marginally) for currency and non-M1 monetary assets in M2.

Tables (3) and (4) perform the same set of estimations imposing homogeneity of degree zero. Thus estimation is performed on relative prices¹⁸. We can always recover the constrained estimator or $\gamma_{i,3}$ as the negative of the sum of the coefficients associated to $\frac{\log p_1}{\log p_3}$ and $\frac{\log p_2}{\log p_3}$. Imposing homogeneity makes practically no changes in parameter estimates. The largest and only noticeable large change is the estimation of $\gamma_{1,1}$, from -0.0068 to 0.0533. Below, we show that this latter value is robust to specification and to other econometric approaches, particularly time-varying parameter estimation.

Tables (5) and (6) perform the same estimation of system (3) using Zellner's SURE procedure for to sample periods; i.e. the complete sample period (Table 5) and the one after the Central Bank of Chile began using a nominal, rather than indexed, interest rate as main policy instrument (Table 6). The results presented in these tables only confirm our previous assessments, with efficiency gains (as the method promises). Table (5) shows that own price elasticities are negative and statistically different from zero. Again, currency can be assessed as an inferior good in contrast to less liquid monetary assets that exhibit real-expenditure positive elasticities and can be assessed as superior goods.

Next, we consider the possibility imperfect portfolio adjustment, which leads us to consider an error-correction representation of equation (3) as in equation (7) that we reproduce for convenience.

$$\begin{aligned} \Delta w_{i,t} = & -\rho_2 \Delta w_{i,t-1} + \sum_j [\gamma_j (\Delta \log p_{j,t} + \rho_2 \Delta \log p_{j,t-1})] + \beta \Delta \log \left\{ \frac{x}{P^*} \right\}_t + \beta \rho_2 \Delta \log \left\{ \frac{x}{P^*} \right\}_{t-1} + \\ & (1 - \rho_1 - \rho_2) \left\{ w_{i,t-1} - \alpha_i - \sum_j \gamma_j \log p_{j,t-1} - \beta \log x P^*_{t-1} \right\} + \varepsilon_t \end{aligned}$$

As stated in section (4) (and for the reasons exposed therein) we begin by checking that $\Lambda(L) = \Lambda_1 L + \Lambda_2 L^2$ has a particular structure consistent with a demand system of portfolio shares. We test (and cannot reject) that Λ_j is a diagonal and can be written as $\rho_j I$. We begin by analyzing the residuals in equation (3), which exhibit clear patterns of autocorrelation (that justify the error correction representation). Figures (4) to (6) show correlograms (correlations and partial autocorrelations) for the three vectors of residuals. Table (7) shows three different criteria we use to select the best autoregressive process to characterize u_j for $j = C, M1, M2$. We use a second order lag polinomial and then check for $\epsilon_{jt} = \Lambda(L)u_{jt}$ to be characterized as white noise proceses. Figures (7 to 9) show correlograms for ϵ_{jt} , which are well behaved. Now that we have checked the plausibility of the estructure we require for $\Lambda(L)$, we estimate equation (7) imposing such structure with Zellner's SURE approach. Results of these estimations are shown in Table (8). We are specially interested

¹⁸If we wish to impose $\gamma_{i1} + \gamma_{i2} + \gamma_{i3} = 0$ in $w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log \left\{ \frac{x}{P} \right\}$ then we can simply estimate $w_i = \alpha_i + \gamma_{i1} \frac{\log p_1}{\log p_3} + \gamma_{i2} \frac{\log p_2}{\log p_3} + \beta_i \log \left\{ \frac{x}{P} \right\}$

in price elasticities (i.e. opportunity cost relative the investing in the benchmark asset) and total real expenditure elasticities. First, currency share holdings exhibit a negative relation (γ_1 and γ_2) with P_1 (30 to 90 days rate) and P_2 (90 days to one year rate) and a positive (γ_3) to higher maturity rates (1 to 3 years). The total-real-expenditure elasticity is negative and statistically different from zero, a result that classifies currency as an inferior good. Column 2 of Table (8) shows results for non-currency holdings share of assets which belong to M1. In this case γ_2 and γ_3 are negative and significant and γ_1 is positive and also different from zero. Non-currency M1 money is classified as a superior good. Finally, in the case of portfolio share of non-M1 holdings of assets belonging to M2, γ_1 and γ_3 are negative (although γ_1 is almost five times larger than γ_3) and γ_2 is large and positive, meaning higher maturity deposits do respond positively to higher paying interest rates). Total real expenditure elasticity is positive, meaning these assets can be considered a superior (normal) good. Results of tests of homogeneity of degree zero in prices are also presented at the end of Table (8); they confirm that we cannot reject homogeneity for the demand for portfolio shares of non-currency monetary assets in M1. The same is not true for the other two equations. In all cases, regression residuals follow white noise processes.

Finally, with our set of money demand estimations we analyze two hypotheses that have been previously proposed: (i) Price elasticities and semi-elasticities (of M1 monetary assets) seem to have changed over time (Vergara, 2003 and De Gregorio, 2003) (ii) M1 vigorous growth that has been remarkably higher than other aggregates is perfectly consistent and explained by low interest rates; that is, low interest rates have actually led to a change in portfolio from less liquid assets to more liquid monetary assets (De Gregorio, 2003). Our system of portfolio demand equations plus time-varying parameters estimation techniques are a perfect instruments to address such questions.

Time varying parameter estimation procedures are powerful tools for analyzing the properties of parameters in specifications in which one suspects variable omission (or structural non-abrupt regime changes). We allow for such imperfection. Results are presented with figures, due to the infeasibility of presenting tables for $180 \times 5 \times 3$ parameters.

Figures (10) to (15) present the estimates of the procedures explained above. Figures (10) and (11) show the estimates of the path of parameter estimates for the currency-share equation. From these figures we can clearly see any source of instability is more likely be found in the intercept rather than in price-elasticities, which seem very stable. Figures (12) and (13) show similar characteristics for the second equation (comprising M1 beyond currency). We can see from these estimates, that in the latter period, confidence intervals for price (interest rate) elasticities are wider. This is not surprising, since in year 2001 the Central Bank of Chile decided to target a nominal interest rate rather than the real ex-ante rate it had targeted until then. This change in policy reduced the variance of interest rates, which explains broader confidence intervals. Finally, Figures (14) and (15) show the estimates for the equation containing M2 components beyond those already included in M1.

All previous work estimating demands for money in Chile has assumed (implicitly) the intercept to be constant. Thus, forecasts using demand equation estimates, systematically

(over)underestimated monetary aggregates. Two reasons are most likely to be behind these change, not explained by variables in the model. First, bank competition and penetration in Chile after the Asian Crisis had become rather fierce; banking penetration allowed people to more easily hold demand deposits and time-deposits than before. Second, it is more likely that time deposits (at least nominal and short-termed) demand is influenced by the variability of the observed interest rate. Nominalization, by sharply reducing interest rates' volatility, also reduced to a great deal the risk of holding in short-term assets ¹⁹.

This analysis supports the conclusions and analysis performed for the static and error-correction representation presented above. Indeed, parameter estimates in such frameworks are very much alike the figures for price-elasticities. Intercept estimates, of course, will be an average of the time-varying intercepts presented in this section. This results, also validate our conclusion of the estimation of Equation (3) as a demand system fulfilling homogeneity, adding up restrictions and representing a first order approximation of a demand system derived from a quasi-concave utility function.

5 Conclusions

Much effort has been devoted to the estimation of empirically plausible and theoretically consistent functional forms of demand for money. Empirical fit and stability, have been the most valuable attributes in this literature, at least for Chile. During the 1990's, money demand estimations appeared to behave properly. It is in the 2000's however, that forecast performance was rather poor, more liquid assets' growth rates were steadily higher than prices' growth rates and a new discussion on the instability of money demand was brought back alive.

Not only did well known arguments (such as those proposed for the U.S.) were proposed, but also a debate around the appropriate specification of these demand equations had arisen. Some authors argued in favor of including more variables to these estimations in order to capture possible variable omissions, others focused on the implications of assuming constant elasticities rather than semi-elasticities. However, any choice, still led to very different estimates for different time periods.

This paper seeks to contribute new elements to such discussion. We propose the application of rigorous consumer theory to the very definition of monies and prices, and the specification of a system of demand equations for them, in a way that we are able to impose several theoretical restrictions.

We provide quantitative results for a portfolio system of demand equations expressed in shares, in a static and dynamic - error correction representation. Results of these set of estimations are theoretically plausible as stemming from a well behaved utility function. We further contribute to the literature by estimating the system of demand equations in a semi-nonparametric time-varying parameter context. To our knowledge this has not been

¹⁹For a full description and analysis of the effects of the "Nominalization", see [Fuentes et al. \(2003\)](#).

applied to money demand estimation for Chile or any other country. We use an estimation algorithm for estimating SURE models in which we can impose consumer-theory restrictions.

Our results show that total-expenditure and interest rate elasticities are very stable in the sample 1993 to 2007, using monthly data. The source of the documented instability arises from the estimation of the intercept; a feature that cannot be seen from any other econometric methodology that imposes constant parameters. We can draw several implications of our results. There is a portfolio recomposition following the Asian Crisis that only gets more vigorous after 2001 - which is in line to the evidence that defines such period as “unstable”-. This recomposition, that favors less-liquid types of money and is captured in the intercept, is evidence on the lack of missing elements for the estimation of money demand equations. Although arbitrary, the natural explanations are financial innovation and vigorous bank penetration in the Chilean economy, nominalization of the monetary policy rate target can also be a possible (and not excluding) explanation. From 2001, interest rate variability reduced dramatically. Less uncertainty is very likely to reduce holdings of cash and liquid assets in favor of less liquid assets for any optimizing agent, an effect which would only reinforce the financial innovation hypothesis.

References

- ANDERSON, R., B. JONES AND T. NESMITH(1997a). "Introduction to the St. Louis Monetary Services Index Project." *Federal Reserve Bank of St. Louis Review* 79(1) pp.25-9.
- ANDERSON, R., B. JONES AND T. NESMITH(1997b). "Monetary Aggregation Theory and Statistical Index Numbers." *Federal Reserve Bank of St. Louis Review* 79(1) pp.31-51.
- ANDERSON, R., B. JONES AND T. NESMITH(1997c). "Building New Monetary Services Indices: concepts, Data and Methods." *Federal Reserve Bank of St. Louis Review* 79(1) pp.53-82.
- ARRAÑO, ERIKA(2006). "Agregados Monetarios: Nuevas Definiciones." Serie de Estudios Económicos Estadísticos del Banco Central de Chile No. 53.
- BARNETT, WILLIAM A.(1978). "The User Cost of Money" *Economics Letters* 1 pp.145-49.
- BARNETT, WILLIAM A.(1980). 'Economic monetary aggregates an application of index number and aggregation theory." *Journal of Econometrics* 14 pp.11-48.
- BARNETT, W.A., D. FISHER AND A. SERLETIS(1992). 'Consumer Theory and the Demand for Money" *Journal of Economic Literature* 30 pp.2086-119.
- BUSE, A. (1994). "Evaluating the linearized almost ideal demand system," *American Journal of Agricultural Economics* 76 pp.781-93.
- CHOW, GC. AND A. LIN (1971). "Best Linear Unbiased Interpolation, Distribution, and Extrapolation of Time Series by Related Series" *The Review of Economics and Statistics* 53(4) pp.372-75.
- CLEMENTS, K AND P. NGUYEN (1980). "Economic Monetary Aggregates Comment" *Journal of Econometrics* 14 pp.49-53.
- CUTHBERTSON, KEITH (1985). *The Supply and Demand for Money* Oxford: Basil Blackwell
- DE GREGORIO, JOSÉ (2003). "Dinero e Inflación: En qu estamos?" *Economía Chilena* 6(1):5-19.
- DONOVAN, DONALD J. (1978). "Modelling the Demand for Liquid Assets: An Application to Canada" *IMF Staff papers* 25 pp.676-704.
- FUENTES, R., A. JARA, K. SCHMIDT-HEBBEL Y M. TAPIA(2003). "La Nominalización de la Política Monetaria en Chile: Una Evaluación." *Economía Chilena* 60(2): 5-27.
- GOLDFELD, S., D. FAND AND W.C. BRAINARD (1976). "The Case of the Missing Money" *Papers on Economic Activity* 3 pp.683-739.
- HALL, PETER (1994). "Methodology and Theory for the Bootstrap" *Handbook of Econometrics* 4: 2341-81. Edited by R. Engle and D. McFadden.
- ISSING, OTMAR (2006). "The ECB's Monetary Policy Strategy; Why did we choose a two Pillar Approach?" Manuscript, 4th ECB Central Banking Conference.

- JUDD, J.P. AND P. SCADDING (1982). "The Search for a Stable Money Demand Function." *Journal of Economic Literature* 20(3): 999-1023.
- LI, G. AND H. SONG (2006). "Time varying parameter and fixed parameter linear AIDS: An application to tourism demand forecasting" *International Journal of Forecasting* 22 pp.57-71.
- MIES, V. Y R. SOTO (2003). "Demanda por Dinero: Teoría, Evidencia y Resultados," *Economía Chilena* 3 pp.5-32.
- OFFENBACHER, EDWARD (1980). "Economic Monetary Aggregates - Comment" *Journal of Econometrics* 14 pp.55-6.
- ORBE, S., E. FERREIRA AND J. RODRIGUEZ-POO (2003). "An algorithm to estimate time varying parameter SURE models under different types of restrictions" *Computational Statistics and Data Analysis* 42(3): 363-83
- RESTREPO, JORGE (2003). "Demanda de Dinero para Transacciones en Chile" *Economía Chilena* 5 pp.95-104.
- SERLETIS, APOSTOLOS (1991). "Modeling the Demand for Consumption Goods and Liquid Assets" *Journal of Macroeconomics* 13 pp.435-57.
- SERLETIS, APOSTOLOS (2007). *The Demand for Money*. Springer
- SOTO, R. Y M. TAPIA (2000). "Cointegración Estacional en la Demanda por Dinero." *Economía Chilena* 3(3): 57-71.
- STOCK, J. AND M. WATSON (1998). "Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model" *Journal of the American Statistical Association*:93-441.
- VERGARA, RODRIGO (2003). "El Dinero como Indicador de Política Monetaria en Chile" *Cuadernos de Economía* 40 pp.707-15.

Appendix A: The Price of Money

In this section we make a brief summary of what is extensively discussed in [Barnett \(1978\)](#) and [Barnett \(1980\)](#). For this, assume a representative consumer that can decide allocate resources to consumption, i distinct monetary assets and another kind of profitable investment for each period:

$$\max u(m_t, m_{t+1}, m_{t+2}, \dots, m_{t+T}; c_t, c_{t+1}, c_{t+2}, \dots, c_{t+T}, \frac{A_{t+T}}{p_{t+T}^*}) \quad (11)$$

subject to:

$$p_s^* c_s \leq w_s L_s + \sum_{i=n}^N \left[(1 + r_{i,s-1}) p_{s-1}^* m_{i,s-1} - p_s^* m_{i,s} \right] + \left[(1 + R_{s-1}) p_{s-1}^* A_{s-1} - p_s^* A_s \right] \quad (12)$$

where

- $m_{i,t}$: Real monetary asset holding of asset i in t
- p_s : Prices of goods and services in period s
- c_s : Consumption of goods and services in period s
- $r_{i,s}$: Nominal yield for holding monetary asset i in s (paid at the beginning of $s + 1$)
- A_s : Bond holdings during period s
- R_s : Expected one-period yield on assets accumulated to transfer wealth between periods
- L_s : Exogenous labor supply in s
- w_s : Wage during period s

and let

$$\begin{aligned} \rho_s &= 1, & s = t \\ &= \prod_{u=t}^{s-1} (1 + R_u), & t + 1 \leq s \leq t + T \end{aligned} \quad (13)$$

be the discount factor for discounting period s transactions. If we solve for A_s in Equation 12, back-substitute for A_s starting from A_{t+T} and work down to A_t , we can represent the T budget constraints in a single wealth constraint of the form:

$$\begin{aligned} \sum_{s=t}^{t+T} \frac{p_s^*}{\rho_s} x_s + \sum_{s=t}^{t+T} \sum_{i=1}^n \left[\frac{p_s^*}{\rho_s} - \frac{p_s^*(1 + r_{i,s})}{\rho_{s+1}} \right] m_{i,s} + \sum_{i=1}^n \frac{p_{t+T}(1 + r_{i,t+T})}{\rho_{t+T+1}} m_{i,t+T} + \frac{p_{t+T}^*}{\rho_{t+T}} A_{t+T} = \\ \sum_{s=t}^{t+T} \frac{w_s}{\rho_s} L_s + \sum_{i=1}^n (1 + r_{i,t-1}) p_{t-1}^* m_{i,t-1} + (1 + R_{t-1}) A_{t-1} p_{t-1}^* \end{aligned}$$

from which we can see that the user cost for holding m_i monetary assets ($i = 1, 2, \dots, n$) is

$$\left[\frac{p_s^*}{\rho_s} - \frac{p_s^*(1 + r_{i,s})}{\rho_{s+1}} \right]$$

thus, the price for nominal monetary asset i : $M_i = m_i p^*$ is [Donovan's \(1978\)](#) proposition of the Jorgensonian user cost.

$$p_{i,t} = \frac{R_t - r_{i,t}}{1 + R_t}$$

which are the prices for distinct monetary assets we adopted in this paper. Note that interest rates are nominal, thus inflationary expectations appear

Appendix B: Divisia Money and Consumer Theory

In this section we summarize a simple argument: Divisia Monetary Aggregates are consistent with (i) Consumer Theory and therefore with any adding-up restrictions of demand systems and (ii) [Donovan's \(1978\)](#) and [Barnett's \(1978\)](#) discussion of the Jorgensonian user-cost of money. For a detailed discussion of aggregation theory see [Barnett \(1980\)](#), [Clements and Nguyen \(1980\)](#), [Offenbacher \(1980\)](#) and [Serletis \(2007\)](#).

Assume an individual representative consumer whose utility function depends upon a vector of consumption goods, leisure, and a vector of monetary assets.

$$u = u(\mathbf{c}, l, \mathbf{x}) \tag{14}$$

Further, suppose this utility function is weakly separable in arguments and there exists a monetary services aggregator $f(\mathbf{x})$, that is a function of the vector of stocks of monetary assets; \mathbf{x} . Weak separability implies that we can re-write equation 14 as:

$$u = v(\mathbf{c}, l, f(\mathbf{x})) \tag{15}$$

and that optimization is carried out in a two-step process (recursive separability) as outlined in [Collins and Anderson \(1998\)](#) and [Serletis \(2007\)](#). Thus the representative consumer begins by deciding the expenditure on the services of monetary assets, y ; and then solves the following problem

$$\max_{\{\mathbf{x}\}} f(\mathbf{x}) \quad \text{subject to} \quad \mathbf{p}'\mathbf{x} = y \tag{16}$$

where \mathbf{p} is a vector of monetary asset user costs defined in [Appendix A](#). First order necessary conditions of the associated Lagrangian, imply

$$\frac{\partial f(x)}{\partial x_i} - \lambda p_i = 0 \tag{17}$$

$$y - \sum_{i=1}^n p_i x_i = 0$$

where λ is the Lagrange multiplier of the constrained optimization problem.

Notice that total differential of $f(x)$ is

$$df(x) = \sum_{i=1}^n \left(\frac{\partial f(x)}{\partial x_i} \right) dx_i$$

and from equation 17 (first order conditions) we can express the total differential as

$$df(x) = \sum_{i=1}^n \lambda p_i dx_i \quad (18)$$

We will assume that this aggregator function that maps monetary asset holdings (EQI) to a monetary service utility function; $f(x)$ is linearly homogeneous.²⁰ We define $\mathcal{P}(p)$ as the price index such that,

$$\mathcal{P}(p)f(x) = \sum_{i=1}^n p_i x_i = y$$

We need a way to relate $\mathcal{P}(p)$ to equation 18. Barnett, Fisher and Serletis (1992) show in a somewhat complicated way that $\lambda = 1/\mathcal{P}(p)$. Here, we propose a more intuitive and straightforward way to achieve the same result. We exploit the linear-homogeneity property of $f(x)$ ²¹. From equation 17 we see that

$$\begin{aligned} \frac{\partial f(x)}{\partial x_i} - \lambda p_i &= 0 \\ x_i \frac{\partial f(x)}{\partial x_i} - \lambda x_i p_i &= 0 \\ \sum_{i=1}^n x_i \frac{\partial f(x)}{\partial x_i} - \lambda \sum_{i=1}^n x_i p_i &= 0 \end{aligned}$$

But if $f(x)$ is homogeneous of degree one, then Euler's equation must be met; $\sum_{i=1}^n \frac{\partial f(x)}{\partial x_i} x_i = f(x)$, thus

$$f(x) - \lambda y = 0 \quad \Rightarrow \quad 1/\lambda = \mathcal{P}(p) \quad (19)$$

Using this result in equation 18 we can see that

$$df(x) = \sum_{i=1}^n \left(\frac{1}{\mathcal{P}(p)} \right) p_i dx_i$$

and finally,

$$d \log f(x) = \sum_{i=1}^n w_i^* d \log x_i \quad (20)$$

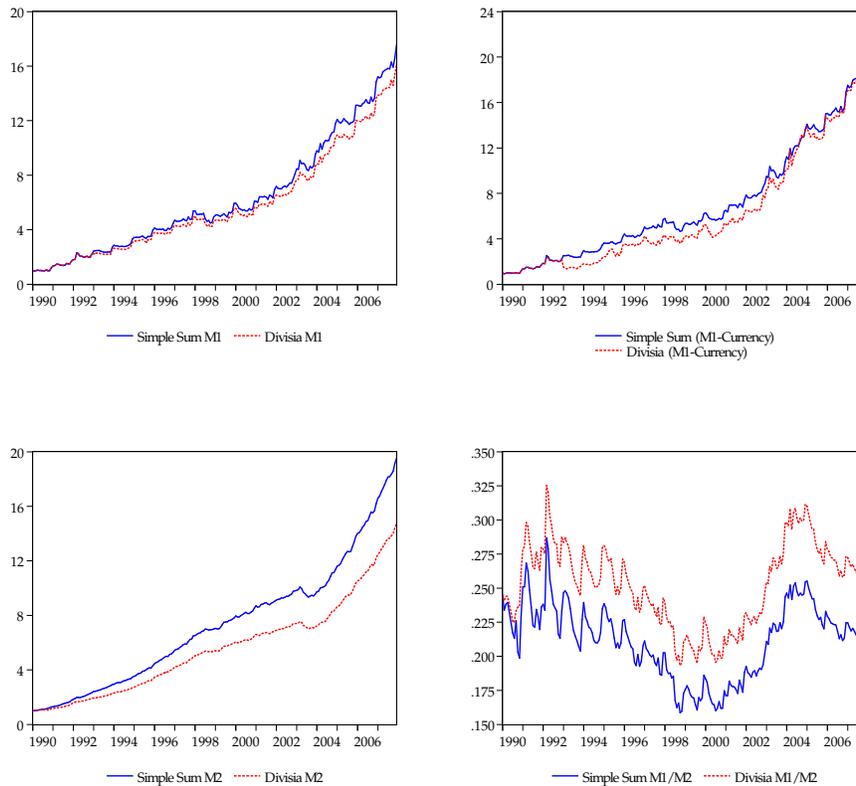
which is the Divisia index expressed in log-growth rates.

²⁰It is not an unreasonable assumption to make, because if such condition failed, then the growth rate of the aggregate would differ from the growth rates of its components even if all of them were growing at the same rate. Such assumption is proposed by Serletis (2007).

²¹We assume degree-one homogeneity of such $f(x)$

Appendix C: Figures and Tables

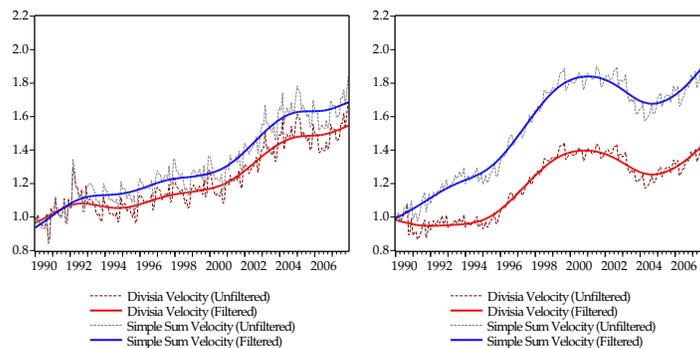
Figure 1: Divisia and Simple Sum Monetary Aggregates



Source: Authors' calculation and Central Bank of Chile

Note: Sub figures show correspondingly; (Upper Left) M1 Simple-Sum and Divisia Monetary Aggregates, (Upper Right) Simple-Sum and Divisia Demand Deposits plus "a la vista" deposits, (Lower Left) M2 Simple-Sum and Divisia Monetary Aggregates, (Lower Right) Ratio of $\frac{M1^{SSum}}{M2^{SSum}}$ compared to $\frac{M1^{Div}}{M2^{Div}}$

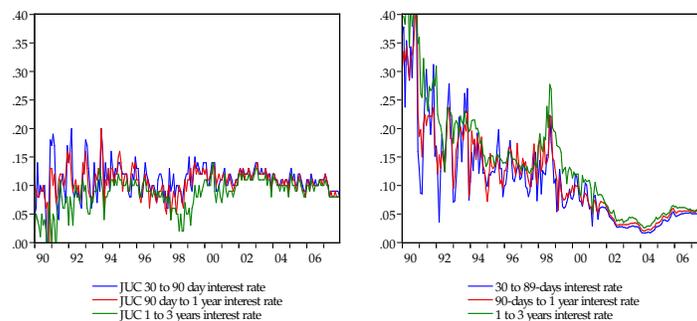
Figure 2: Divisia and Simple Sum Monetary Aggregate Velocities



Source: Authors' calculation and Central Bank of Chile

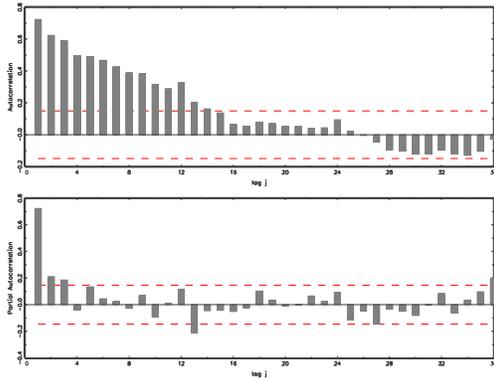
Note: Velocities are calculated using nominal money and nominal monthly GDP. Velocities are also scaled to unity for the initial period.

Figure 3: Jorgensonian User Costs compared to Interest Rates



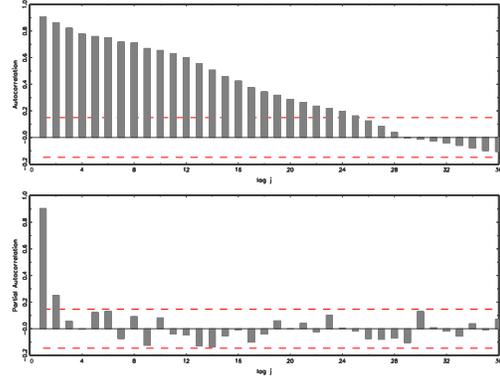
Source: Authors' calculation

Figure 4: Correlogram $u_{currency}$



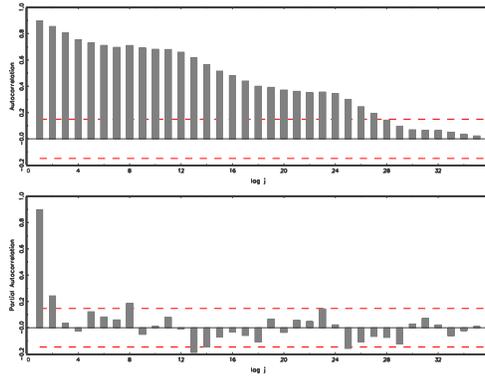
Source: Authors' calculation

Figure 5: Correlogram u_{M1}



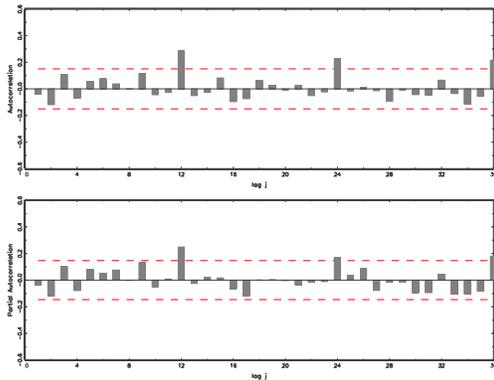
Source: Authors' calculation

Figure 6: Correlogram u_{M2}



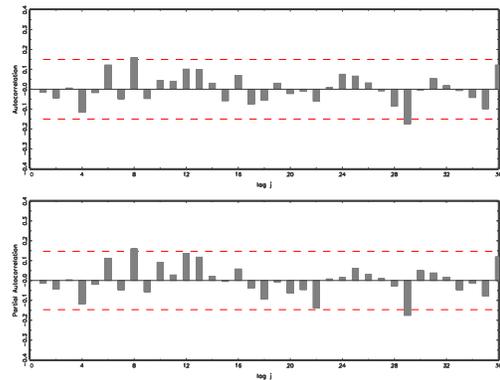
Source: Authors' calculation

Figure 7: Correlogram $\varepsilon_{t,c}$



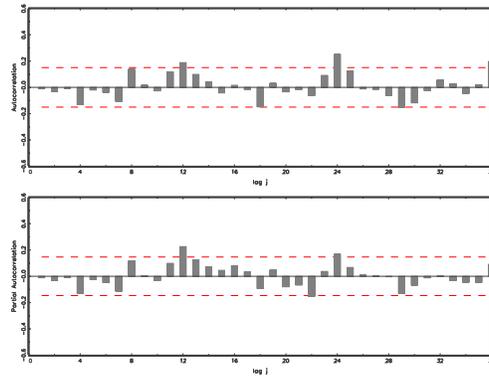
Source: Authors' calculation

Figure 8: Correlogram $\varepsilon_{t,M1}$



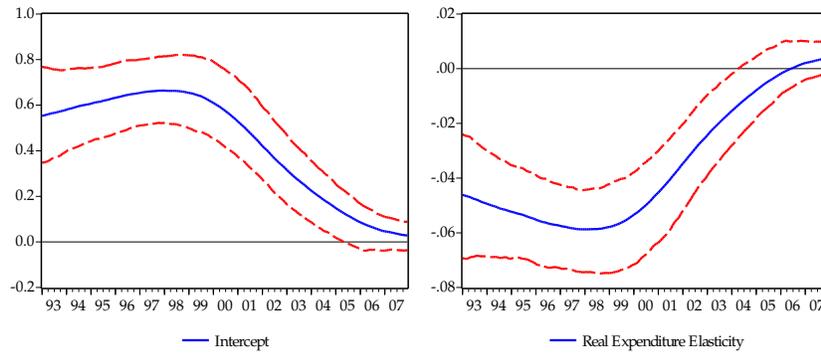
Source: Authors' calculation

Figure 9: Correlogram $\varepsilon_{t,M2}$



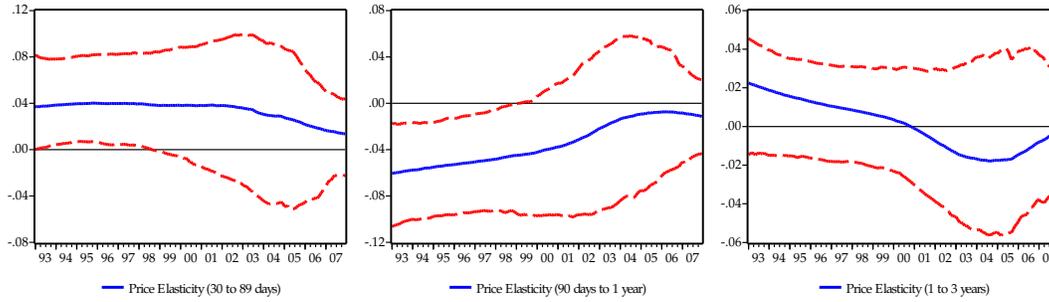
Source: Authors' calculation

Figure 10: Currency Share (w_c): Constant (α_r) and real expenditure elasticity (β_r)



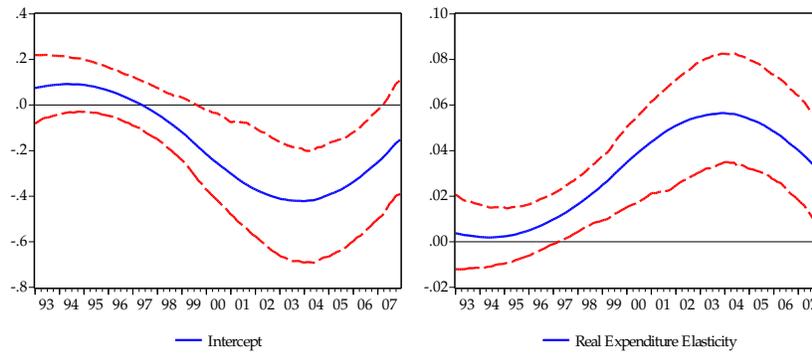
Source: Authors' calculation

Figure 11: Currency Share (w_c): Price Elasticities



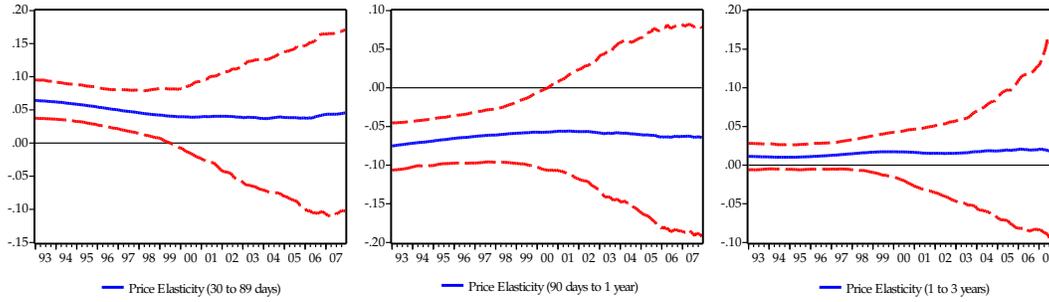
Source: Authors' calculation

Figure 12: M1-Currency Share(w_{M1}): Constant (α_r) and real expenditure elasticity (β_r)



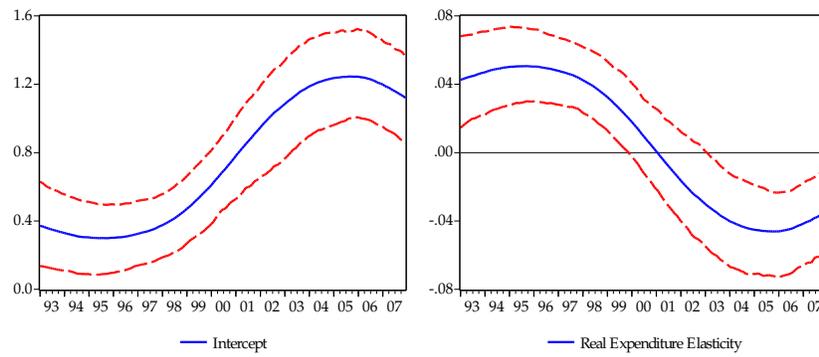
Source: Authors' calculation

Figure 13: M1-Currency Share (w_{M1}): Price Elasticities



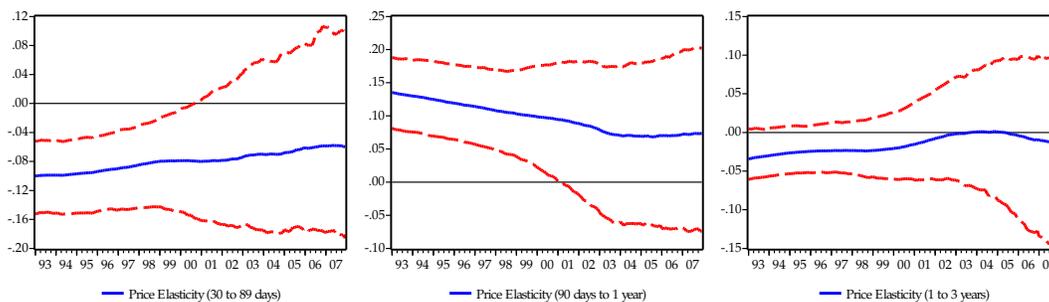
Source: Authors' calculation

Figure 14: M2-M1 Share (w_{M2}): Constant (α_r) and real expenditure elasticity (β_r)



Source: Authors' calculation

Figure 15: Currency Share (w_{M2}): Price Elasticities



Source: Authors' calculation

Table 1: Linear AIDS

Dependent Variable: Expenditure Share w_i												
Sample Period: 1993m1 2007m12												
Estimation Method: OLS												
	Currency Share				Demand Deposits (M1-Currency)				Fix Term Savings Deposits (M2-M1)			
	(1)		(2)		(3)		(4)		(5)		(6)	
α_i^*	0.5335	***	0.3941	***	0.9297	***	-0.1966	***	-0.4632	***	0.8026	***
	(0.048)		(0.014)		(0.093)		(0.036)		(0.101)		(0.040)	
$\log p_{3089}$	-0.0061		-0.0068		0.0985	***	0.0931	***	-0.0924	***	-0.0863	***
	(0.004)		(0.004)		(0.009)		(0.012)		(0.010)		(0.014)	
$\log p_{90t01}$	-0.0944	***	-0.0922	***	-0.1246	***	-0.1071	***	0.2190	***	0.1993	***
	(0.004)		(0.004)		(0.009)		(0.012)		(0.009)		(0.013)	
$\log p_{1t03}$	0.0128	***	0.0115	***	0.0365	***	0.0262	***	-0.0493	***	-0.0377	***
	(0.002)		(0.002)		(0.005)		(0.007)		(0.005)		(0.007)	
$\log \frac{p}{P^*}$	-0.0679	***	-0.0510	***	-0.0998	***	0.0366	***	0.1677	***	0.0144	***
	(0.005)		(0.001)		(0.011)		(0.003)		(0.012)		(0.003)	
Trend	0.0002	***			0.001546	***			-0.0017	***		
	(0.000)				(0.000)				(0.000)			
$\sum_j \gamma_{ij}$	-0.0877		-0.0877		0.0104		0.0122		0.0773		0.0752	
	(0.003)		(0.003)		(0.006)		(0.0009)		(0.007)		(0.010)	
$\sum_j \gamma_{ij} \neq 0$	Yes		Yes		No		No		Yes		Yes	
R^2	0.940		0.937		0.787		0.592		0.789		0.584	
S.E.E.	0.009		0.009		0.017		0.023		0.018		0.026	
Log likelihood	600.5743		596.0244		482.7575		424.3518		467.5727		406.3384	

Table 2: Linear AIDS. Post Nominalization Sample

Dependent Variable: Expenditure Share w_i												
Sample Period: 2001m9 2007m12												
Estimation Method: OLS												
	Currency Share				Demand Deposits (M1-Currency)				Fix Term Savings Deposits (M2-M1)			
	(1)		(2)		(3)		(4)		(5)		(6)	
α_i^*	-0.1333	***	0.0117		2.1533	***	-0.300080	***	-1.0199	***	1.2884	***
	(0.048)		(0.0170)		(0.1672)		(0.1133)		(0.15929)		(0.1069)	
$\log p_{3089}$	0.0194	*	0.0199	*	0.1706	***	0.1626	**	-0.1901	***	-0.1825	***
	(0.010)		(0.0108)		(0.0345)		(0.0723)		(0.0329)		(0.0681)	
$\log p_{90to1}$	-0.0518	***	-0.0541	***	-0.1292	***	-0.091107		0.1810	***	0.1452	**
	(0.009)		(0.0097)		(0.0310)		(0.0646)		(0.0295)		(0.0610)	
$\log p_{1to3}$	0.0039		0.0064		-0.0402		-0.0815		0.0363		0.0752	
	(0.009)		(0.0098)		(0.0315)		(0.0657)		(0.0300)		(0.0620)	
$\log \frac{p}{P^*}$	0.0170	***	-0.0006		-0.2568	***	0.041988	***	0.2397	***	-0.0414	***
	(0.006)		(0.0020)		(0.0203)		(0.0135)		(0.0193)		(0.0127)	
Trend	-0.0002	***			0.0036	***			-0.0033	***		
	(0.000)				(0.0002)				(0.0002)			
$\sum_j \gamma_{ij}$	-0.0285		-0.0278		0.0012		-0.0100		0.0272		0.0378	
	(0.003)		(0.004)		(0.013)		(0.026)		(0.012)		(0.025)	
$\sum_j \gamma_{ij} \neq 0$	Marginally		Yes		No		No		Marginally		No	
R^2	0.676		0.632		0.838		0.280		0.864		0.409	
S.E.E.	0.003		0.003		0.011		0.023		0.010		0.021	
Log likelihood	331.2257		326.3687		238.868		182.259		242.5825		186.6801	

Table 3: Homogeneous Linear AIDS

Dependent Variable: Expenditure Share w_i
 Sample Period: 1993m1 2007m12

Homogeneity imposed

Estimation Method: OLS

	(1)	Currency Share	(2)	Demand Deposits (M1-Currency)	(3)	(4)	Fix Term Savings Deposits (M2-M1)	(5)	(6)			
α_i^*	0.6961 (0.102)	***	0.5808 (0.024)	***	0.9105 (0.092)	***	-0.2226 (0.030)	***	-0.6065 (0.128)	***	0.6418 (0.038)	***
$\log \frac{p_{3089}}{p_{1to3}}$	0.0539 (0.009)	***	0.0533 (0.008)	***	0.0914 (0.007)	***	0.0847 (0.010)	***	-0.1453 (0.011)	***	-0.1379 (0.013)	***
$\log \frac{p_{90to1}}{p_{1to3}}$	-0.0760 (0.009)	***	-0.0743 (0.009)	***	-0.1268 (0.009)	***	-0.1095 (0.012)	***	0.2028 (0.012)	***	0.1838 (0.015)	***
$\log \frac{x}{p^*}$	-0.0639 (0.012)	***	-0.0500 (0.002)	***	-0.1003 (0.011)	***	0.036 (0.003)	***	0.1642 (0.015)	***	0.0136 (0.003)	***
Trend	0.0002 (0.000)				0.0016 (0.000)	***			-0.0017 (0.000)	***		
R^2	0.723		0.721		0.784		0.588		0.655		0.456	
S.E.E.	0.019		0.062		0.017		0.023		0.023		0.029	
Log likelihood	463.47		462.79		481.54		423.48		423.26		382.34	

Table 4: Homogeneous Linear AIDS: Post Nominalization Sample

Dependent Variable: Expenditure Share w_i
 Sample Period: 2001m9 2007m12

Homogeneity imposed

Estimation Method: OLS

	(1)	Currency Share	(2)	Demand Deposits (M1-Currency)	(3)	(4)	Fix Term Savings Deposits (M2-M1)	(5)	(6)			
α_i^*	-0.1405 (0.066)	**	-0.0149 (0.021)		2.1536 (0.166)	***	-0.3097 (0.109)	***	-1.0130 (0.163)	***	1.3245 (0.105)	***
$\log \frac{p_{3089}}{p_{1to3}}$	0.0500 (0.012)	***	0.0500 (0.012)	***	0.1693 (0.031)	***	0.1735 (0.065)	**	-0.2195 (0.030)	***	-0.2235 (0.062)	***
$\log \frac{p_{90to1}}{p_{1to3}}$	-0.0491 (0.012)	***	-0.0511 (0.012)	***	-0.1293 (0.030)	***	-0.0900 (0.064)		0.1784 (0.030)	***	0.1411 (0.061)	**
$\log \frac{x}{p^*}$	0.0234 (0.007)	***	0.0080 (0.002)	***	-0.2571 (0.019)	***	0.0451 (0.010)	***	0.2337 (0.019)	***	-0.0530 (0.010)	***
Trend	-0.0001 (0.000)	**			0.0036 (0.000)	***			-0.0034 (0.000)	***		
R^2	0.418		0.385		0.838		0.279		0.855		0.390	
S.E.E.	0.004		0.004		0.011		0.023		0.011		0.034	
Log likelihood	308.96		306.86		238.86		182.18		239.98		185.50	

Table 5: Linear AIDS - System Estimation

Dependent Variable: Expenditure Share w_i
 Sample Period: 1993m1 2007m12
 Estimation Method: SUR - Full Information Maximum Likelihood

	Currency Share		Demand Deposits (M1-Currency)		Fix Term Savings Deposits (M2-M1)	
	(1)	(2)	(3)	(4)	(5)	(6)
α_i^*	0.5334 *** (0.061)	0.394049 *** (0.016)	0.9297 *** (0.128)	-0.196608 *** (0.060)	-0.4631 *** (0.149)	0.8026 *** (0.062)
$\log p_{3089}$	-0.0061 (0.005)	-0.006799 (0.005)	0.0985 (0.011)	0.093113 *** (0.020)	-0.0924 *** (0.013)	-0.0863 *** (0.019)
$\log p_{90t01}$	-0.0943 *** (0.006)	-0.092192 *** (0.006)	-0.1246 *** (0.011)	-0.107091 *** (0.017)	0.2189 *** (0.013)	0.1993 *** (0.018)
$\log p_{1t03}$	0.0128 *** (0.004)	0.011528 *** (0.003)	0.0364 *** (0.005)	0.026162 *** (0.009)	-0.0492 *** (0.008)	-0.0377 *** (0.011)
$\log \frac{p}{P^*}$	-0.0679 *** (0.007)	-0.051013 *** (0.001)	-0.0998 *** (0.014)	0.036585 *** (0.005)	0.1677 *** (0.017)	0.0144 *** (0.006)
Trend	0.0001 ** (0.000)		0.0015 *** (0.000)		-0.0016 *** (0.000)	
$\sum_j \gamma_{ij}$	-0.0877 (0.004)	-0.0875 (0.004)	0.0104 (0.006)	0.0122 (0.011)	0.0773 (0.008)	0.0753 (0.011)
$\sum_j \gamma_{ij} \neq 0$	Yes	Yes	No	No	Yes	Yes
R^2	0.940	0.937	0.787	0.592	-	-
S.E.E.	0.009	0.009	0.017	0.023	-	-
Log likelihood	1083.954	1021.19	1083.954	1021.19	1083.954	1021.19

Table 6: Linear AIDS - System Estimation. Post Nominalization Sample

Dependent Variable: Expenditure Share w_i
 Sample Period: 2001m9 2007m12
 Estimation Method: SUR - Full Information Maximum Likelihood

	Currency Share		Demand Deposits (M1-Currency)		Fix Term Savings Deposits (M2-M1)	
	(1)	(2)	(3)	(4)	(5)	(6)
α_i^*	-0.1334 *** (0.047)	0.0117 (0.016)	2.1535 *** (0.000)	-0.3001 *** (0.109)	-1.0201 *** (0.152)	1.2884 *** (0.010)
$\log p_{3089}$	0.0194 ** (0.009)	0.0199 ** (0.010)	0.1706 *** (0.033)	0.1626 ** (0.069)	-0.1901 *** (0.031)	-0.1825 *** (0.065)
$\log p_{90t01}$	-0.0518 *** (0.008)	-0.0541 *** (0.009)	-0.1292 *** (0.029)	-0.0911 *** (0.062)	0.1810 *** (0.028)	0.1452 ** (0.058)
$\log p_{1t03}$	0.0039 (0.008)	0.0064 (0.009)	-0.0402 (0.030)	-0.0815 (0.063)	0.0363 (0.028)	0.0752 (0.059)
$\log \frac{p}{P^*}$	0.0171 *** (0.005)	-0.0006 (0.001)	-0.2568 *** (0.019)	0.0420 *** (0.013)	0.2398 *** (0.018)	-0.0414 *** (0.012)
Trend	-0.0002 (0.000)	***	0.0036 (0.000)	***	-0.0033 (0.000)	***
$\sum_j \gamma_{ij}$	-0.0285 (0.003)	-0.0278 (0.003)	0.0012 (0.012)	-0.0100 (0.025)	0.0272 (0.011)	0.0378 (0.024)
$\sum_j \gamma_{ij} \neq 0$	Yes	Yes	No	No	No	No
R^2	0.676	0.632	0.838	0.280	-	-
S.E.E.	0.003	0.003	0.011	0.023	-	-
Log likelihood	573.8113	516.8388	573.8113	516.8388	573.8113	516.8388

Table 7: Lag Order Selection of S-LAIDS u_i i : Currency, M1 and M2

L^i	Error u_{curr}		
	Schwarz criterion	Akaike criterion	Hannan-Quinn criter.
1	-7.341	-7.305	-7.326
2	-7.377	-7.323	-7.355
3	-7.402	-7.330	-7.373
4	-7.392	-7.302	-7.355

L^i	Error u_{M1}		
	Schwarz criterion	Akaike criterion	Hannan-Quinn criter.
1	-6.424	-6.388	-6.410
2	-6.493	-6.439	-6.471
3	-6.485	-6.413	-6.456
4	-6.474	-6.384	-6.437

L^i	Error u_{M2}		
	Schwarz criterion	Akaike criterion	Hannan-Quinn criter.
1	-6.159	-6.123	-6.145
2	-6.219	-6.165	-6.197
3	-6.209	-6.137	-6.180
4	-6.198	-6.108	-6.162

Source: Authors' calculation

Note: Errors u_i are those from the static LAIDS. We choose lag-order 2 for the Error Correction specification

Table 8: Error Correction L-AIDS. Full Sample

Dependent Variable: Expenditure Share w_i
Sample Period: 1993m01 2007m12
Estimation Method: Non Linear Least Squares - Error Correction SUR

	w_C		w_{M1}		w_{M2}	
ρ_1	0.8548	***	0.8548	***	0.8548	***
	(0.045)		(0.045)		(0.045)	
ρ_2	0.1607	***	0.1607	***	0.1607	***
	(0.043)		(0.043)		(0.043)	
γ_1	-0.0081	***	0.0988	***	-0.0907	***
	(0.003)		(0.003)		(0.004)	
γ_2	-0.0944	***	-0.0877	***	0.1821	***
	(0.003)		(0.003)		(0.0042)	
γ_3	0.0324	***	-0.0092	**	-0.0233	***
	(0.004)		(0.005)		(0.0062)	
β	-0.1207	***	0.0471	***	0.0736	***
	(0.014)		(0.017)		(0.0219)	
α	1.1816	***	-0.3486	*	0.1670	
	(0.176)		(0.186)		(0.253)	
R^2	0.9201		0.8786		0.9253	
SEE	0.0059		0.0069		0.0091	
$\sum_j \gamma_{ij} = 0$	-0.0701		0.0019		0.0681	
	(0.004)		(0.004)		(0.005)	
Homogeneity	No		Yes		No	

Source: Authors' calculation

Table 9: Error Correction L-AIDS. Post Nominalization Sample

Dependent Variable: Expenditure Share w_i
Sample Period: 2001m9 2007m12 (Nominal MPR)
Estimation Method: Non Linear Least Squares - Error Correction SUR

	w_c		w_{M1}		w_{M2}	
ρ_1	0.8134	***	0.8134	***	0.8134	***
	(0.089)		(0.089)		(0.089)	
ρ_2	0.2611	***	0.2611	***	0.2611	***
	(0.082)		(0.082)		(0.082)	
γ_1	0.0086		0.1357	***	-0.1443	***
	(0.007)		(0.016)		(0.018)	
γ_2	-0.0446	***	-0.1357	***	0.1803	***
	(0.005)		(0.013)		(0.014)	
γ_3	0.0071		-0.0034		-0.0037	
	(0.006)		(0.015)		(0.017)	
β	-0.0050		0.0294		-0.0244	
	(0.01)		(0.025)		(0.028)	
α	0.0554		-0.1420		1.0862	***
	(0.105)		(0.264)		(0.295)	
R^2	0.7241		0.7241		0.7242	
SEE	0.0025		0.0060		0.0025	
$\sum_j \gamma_{ij} = 0$	-0.0290	0	0.0324	0	-0.0034	0
	(0.003)		(0.008)		(0.007)	
Homogeneity	No		Yes		No	

Source: Authors' calculation

**Documentos de Trabajo
Banco Central de Chile**

**Working Papers
Central Bank of Chile**

NÚMEROS ANTERIORES

PAST ISSUES

La serie de Documentos de Trabajo en versión PDF puede obtenerse gratis en la dirección electrónica: www.bcentral.cl/esp/estpub/estudios/dtbc. Existe la posibilidad de solicitar una copia impresa con un costo de \$500 si es dentro de Chile y US\$12 si es para fuera de Chile. Las solicitudes se pueden hacer por fax: (56-2) 6702231 o a través de correo electrónico: bcch@bcentral.cl.

Working Papers in PDF format can be downloaded free of charge from: www.bcentral.cl/eng/stdpub/studies/workingpaper. Printed versions can be ordered individually for US\$12 per copy (for orders inside Chile the charge is Ch\$500.) Orders can be placed by fax: (56-2) 6702231 or e-mail: bcch@bcentral.cl.

- | | |
|---|----------------|
| DTBC-511
Forecasting Inflation in Difficult Times
Juan Díaz y Gustavo Leyva | Diciembre 2008 |
| DTBC-510
Overoptimism, Boom-Bust Cycles, and Monetary Policy in Small Open Economies
Manuel Marfán, Juan Pablo Medina y Claudio Soto | Diciembre 2008 |
| DTBC-509
Monetary Policy Under Uncertainty and Learning: An Overview
Klaus Schmidt-Hebbel y Carl E. Walsh | Diciembre 2008 |
| DTBC-508
Estimación de Var Bayesianos para la Economía Chilena
Patricio Jaramillo | Diciembre 2008 |
| DTBC-507
Chile's Growth and Development: Leadership, Policy-Making Process, Policies, and Results
Klaus Schmidt-Hebbel | Diciembre 2008 |
| DTBC-506
Exit in Developing Countries: Economic Reforms and Plant Heterogeneity
Roberto Álvarez y Sebastián Vergara | Diciembre 2008 |
| DTBC-505
Evolución De La Persistencia Inflacionaria En Chile
Pablo Pincheira | Diciembre 2008 |
| DTBC-504
Robust Learning Stability with Operational Monetary Policy Rules
George W. Evans y Seppo Honkapohja | Noviembre 2008 |

DTBC-503 Riesgo de Crédito de la Banca Rodrigo Alfaro, Daniel Calvo y Daniel Oda	Noviembre 2008
DTBC-502 Determinacy, Learnability, And Plausibility In Monetary Policy Analysis: Additional Results Bennett T. McCallum	Octubre 2008
DTBC-501 Expectations, Learning, And Monetary Policy: An Overview Of Recent Research George W. Evans y Seppo Honkapohja	Octubre 2008
DTBC-500 Higher Order Properties of the Symmetrically Normalized Instrumental Variable Estimator Rodrigo Alfaro	Octubre 2008
DTBC-499 Imperfect Knowledge And The Pitfalls Of Optimal Control Monetary Policy Athanasios Orphanides y John C. Williams	Octubre 2008
DTBC-498 Macroeconomic And Monetary Policies From The Eductive Viewpoint Roger Guesnerie	Octubre 2008
DTBC-497 Macroeconomía, Política Monetaria y Patrimonio del Banco Central Jorge Restrepo, Luis Salomó y Rodrigo Valdés	Octubre 2008
DTBC-496 Microeconomic Evidence of Nominal Wage Rigidity in Chile Marcus Cobb y Luis Opazo	Octubre 2008
DTBC 495 A Sticky-Information General Equilibrium Model for Policy Analysis Ricardo Reis	Octubre 2008